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Comments on string theory backgrounds with non-relativistic conformal symmetry

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Abstract: We consider non-relativistic conformal quantum mechanical theories that arise by doing discrete light cone quantization of field theories. If the field theory has a gravity dual, then the conformal quantum mechanical theory can have a gravity dual description in a suitable finite temperature and finite density regime. Using this we compute the thermodynamic properties of the system. We give an explicit example where we display both the conformal quantum mechanical theory as well as the gravity dual. We also discuss the string theory embedding of certain backgrounds with non-relativistic conformal symmetry that were recently discussed. Using this, we construct finite temperature and finite density solutions, with asymptotic non-relativistic conformal symmetry. In addition, we derive consistent Kaluza-Klein truncations of type IIB supergravity to a five dimensional theory with massive vector fields.

Keywords: AdS-CFT Correspondence, Gauge-gravity correspondence, Black Holes in String Theory.

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Contents

1. Introduction

Recently some gravity backgrounds with non-relativistic conformal symmetry were discussed [1 − 4].[1] The idea is that such backgrounds would be gravity duals of conformal quantum mechanical systems, which are useful for describing certain condensed matter systems (see [1, 2, 7] for references).

Here we make some comments on these constructions. These constructions involve performing the discrete light cone quantization (DLCQ) of certain field theories and their gravity duals. As it is well known, the DLCQ of a field theory gives a non-relativistic system. Thus the DLCQ of a conformal field theory is expected to give a conformal quantum mechanical system. On the other hand, DLCQ quantization is subtle. In particular, when we perform DLCQ of gravity backgrounds we cannot naively apply the gravity approximation, since there is a circle becoming very small. Fortunately, if one considers a sector with large non-zero light-cone momentum one can find regions in the geometry where the circle has a non-zero size so that computations can be trusted. In this paper we will discuss some backgrounds that arise in string theory that display a non-relativistic conformal symmetry. One particular example where we know the conformal quantum mechanical theory and the corresponding gravity background is the theory that arises when we do the DLCQ of the M5 brane theory. In this case the conformal quantum mechanics was discussed in [8] and we will review it below. We will discuss some gravity backgrounds that can be used to perform computations which can be trusted and are predictions for results in the conformal quantum mechanical theory. Conformal quantum mechanical systems were studied in the context of black hole physics and AdS_2, see [9] for a review and further references.

The backgrounds considered in [1, 2] enjoy non-relativistic conformal symmetry even before taking the DLCQ limit. In this paper we embed these backgrounds in string theory and discuss their deformation at finite temperature and finite density. In one particular case we can relate these backgrounds to certain non-commutative dipole theories studied in [10]. There is a simple procedure that allows us to introduce such non-commutativity both in the field theory and in gravity [11]. A non-relativistic quantum mechanical system is expected to arise when we perform a DLCQ quantization of such a theory. Due to the non-commutative interpretation of the background we find that certain quantities are independent of the non-commutativity in the planar approximation, since non-commutativity does not change certain planar diagrams [12]. Thus for many observables computations in the backgrounds [1, 2] are the same as in asymptotically AdS backgrounds with x^- compactified.

We also discuss consistent type IIB Kaluza-Klein reductions of $AdS_5 \times Y$ backgrounds, where Y is a Sasaki-Einstein manifold, to five-dimensional systems involving massive vector fields. Our motivation is that these truncations admit solutions with (asymptotic) non-relativistic conformal symmetries, of the type discussed in [1, 2]. Such Kaluza-Klein reductions might be useful also for other purposes.

The contents of the paper are organised as follows: we start in section 2 by discussing how we obtain a quantum mechanical system with non-relativistic conformal symmetry via

[1] For earlier work on non-relativistic conformal structures see e.g. [5, 6] and references therein.

the discrete light cone quantisation of relativistic field theories. We also discuss subtleties inherent in the construction. In section 3 we construct the gravity dual of these systems, which can be trusted when we put sufficient amount of momentum along the compactified dimension so that it becomes a spacelike circle with large radius. In section 4 we study the consistent Kaluza-Klein reductions of type IIB supergravity on S^5 or any other Sasaki-Einstein space which retain massive gauge fields, and we show that these reductions can be used to construct some of the supergravity solutions discussed in section 3. We conclude the paper with a short discussion in section 5. We have a few appendices where the reader can find the details of the calculation.

Note added: the authors learned just before completion that there will appear two papers, one by A. Adams, K. Balasubramanian, and J. McGreevy [13] and another by C. P. Herzog, M. Rangamani and S. F. Ross [14] which have some overlap with our present paper.

2. Non-relativistic theories from DLCQ

2.1 DLCQ of relativistic theories in Minkowski space

It is well known that the light cone quantization of a relativistic theory looks like a non-relativistic theory. Choosing light cone coordinates $x^{\pm} = t \pm x^3$ we see that the mass shell condition for a massive particle looks like

$$-p_+ = \frac{\vec{p}^2}{(-4p_-)} + \frac{m^2}{(-4p_-)} \tag{2.1}$$

which looks like the energy of a non-relativistic particle of mass $M \sim -p_-$ in a constant potential. We find it useful to write things in terms of $p_{\pm}$ with the lower index since that is the momentum that is canonically conjugate to $x^{\pm}$ translations. One minor disadvantage is that they are negative definite. Thus our $p_- = -p^+/2$ if we start with the ordinary Minkowski metric, $\mathrm{d}s^2 = -\mathrm{d}x^+\mathrm{d}x^- + \mathrm{d}\vec{x}^2$.

In a relativistic theory p_- is a continuous variable. We can make it discrete by compactifying the light cone direction $x^- \sim x^- + 2\pi r^-$ [15]. We then find that p_- is quantized as

$$-p_- = \frac{N}{r^-} \tag{2.2}$$

where $N \geq 0$. This is called "discrete light cone quantization". Note that the parameter r^- can be changed by doing a boost in the $+-$ directions, which is a symmetry of the relativistic theory. This boost is broken by the compactification. However, the fact that it is a symmetry in the original theory implies that theories with different values of r^- are related by a simple rescaling of the generators. Though in the formulas below we will keep r^- explicitly, one could set $r^- = 1$ without loss of generality.

If, in addition, the relativistic theory is conformal invariant, then this procedure would formally lead to a conformal invariant quantum mechanical theory, with a symmetry group which is called the "Schrödinger group".[2] This is simply the subset of the conformal generators which commute with p_-. See appendix A for some details. This fact was already noted in [8] where the DLCQ of the theory on M5 brane was studied.

As explained in [16], DLCQ is very subtle. One has to be careful about the zero modes. In general these zero modes (modes with $p_- = 0$) are described by an interacting theory which is obtained by taking the original theory and placing it on a very small spatial circle of vanishing size. Thus, one has to solve the problem of a field theory in one less dimension. For example, if we start with 3+1 dimensional $\mathcal{N} = 4$ super Yang Mills, the zero mode dynamics is described by a 2+1 dimensional conformal field theory which is the IR limit of 2+1 dimensional super Yang Mills. This is also the theory that lives on M2 branes. Thus, the proper analysis of the dynamics of a DLCQ theory is fairly non-trivial, but it can be done in principle. This point should be kept in mind when we discuss various theories in this paper. Proposals for the DLCQ of $\mathcal{N} = 4$ super Yang Mills were made in [17, 18]. We will not give a totally explicit description of the field theory side in this case, leaving a complete analysis of this issue for the future. Note that in this case we get a family of conformal quantum mechanical systems that arise by taking different expectation values of A_- (and the dual photon) [17, 18].

The DLCQ procedure we outlined above is a way of generating examples of conformal quantum mechanical systems. Writing out explicitly the quantum mechanical system requires a proper analysis of the zero modes. This is an important issue for understanding precisely the nature of the corresponding non-relativistic quantum mechanical system. In particular, one would like to write down the Schrödinger equation for the quantum mechanical system. We discuss one specific example below.

Finally, note that the discussion in [19] that links AdS_3 to AdS_2 and a possible conformal quantum mechanical dual can be interpreted as a DLCQ quantization of AdS_3 in a sector with nonzero P_-.

If the parent relativistic theory has a gravity dual which is weakly coupled one can hope to have a gravity description of the corresponding conformal quantum mechanical system. For example, if the parent theory is $\mathcal{N} = 4$ super Yang Mills, then it also has a dual description as a gravity (or string) theory on $\mathrm{AdS}_5 \times S^5$ when the 't Hooft coupling $g^2_{\mathrm{YM}} k$ is large. We denote by k the rank of the gauge group in order not to confuse it with N, which is the number of quanta of the light cone momentum. Thus, one would hope that by performing the DLCQ procedure on both sides one would get a strongly coupled quantum mechanical system that is dual to a weakly coupled gravity solution. The DLCQ limit of the gravity dual is simply given by identifying the x^- direction in the bulk [4, 3]. One should remember, though, that this identification is not as innocent as it looks. When we periodically identify the x^- direction in the bulk we are performing a drastic change in the theory. For example, the correct graviton scattering amplitude in the DLCQ theory and the naive one (obtained by truncating tree level graviton scattering amplitudes in the theory before the DLCQ) are not the same [20].

[2] The name comes from the fact that it is the dynamical symmetry group of the ordinary Schrödinger equation for a free particle.

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The situation changes for the better when we introduce a large amount of momentum $N \sim -p_- r^-$ in the DLCQ direction. In fact, one is interested in the sector of the theory with non-zero values of N. In the bulk, this has the effect of making the size of the x^- circle spacelike in some interesting regions of the geometry. Thus, for large enough P_-, or large enough P_- density, one can indeed use the gravity description for computing certain properties of the system. We will describe this in more detail when we talk about the gravity solutions. Let us first describe a specific example and some simple variations of this construction.

2.2 DLCQ description of the M5 brane theory

There is one case where a fairly explicit description of the DLCQ is available in the literature. It is the case of the M5 brane theory, which is a 5+1 dimensional conformal field theory. In DLCQ with N units of momentum, this becomes a certain conformal quantum mechanics theory constructed as follows [8]. We start with k fivebranes and N units of momentum along a compactified spatial direction. The small radius limit, which leads to the DLCQ description, forces us to perform a duality to end up with N D0 branes and k D4 branes, and to take the low energy limit of this system. The result is a quantum mechanics theory which is a sigma model on the Higgs branch of a certain theory with 8 supercharges. It is the quantum mechanics on the moduli space of N-instantons.

This system has four-dimensional Galilean invariance and conformal invariance, so it has the Schrödinger symmetry, but it is quite unlike the non-relativistic conformal system which is discussed recently in the literature, e.g. N fermions interacting via contact interaction. Namely, each of the N instantons is not pointlike but has a size parameter which is affected by the dilatation; there is no obvious second-quantized framework, and so on.

The resulting quantum mechanics is the following. We start with a $\mathrm{U}(N)$ gauge theory with an adjoint hypermultiplet and k fundamental hypermultiplets. The bosonic variables involve two complex adjoint matrices X, $\tilde{X}$ and k complex scalars q_i in the $\mathbf{N}$ of $\mathrm{U}(N)$ and k scalars $\tilde{q}^i$ in the $\bar{\mathbf{N}}$ of $\mathrm{U}(N)$. These fields are constrained by

$$[X, X^\dagger] - [\tilde{X}, \tilde{X}^\dagger] + q_i q_i^\dagger - (\tilde{q}^i)^\dagger (\tilde{q}^i) = 0\,, \qquad\qquad [X, \tilde{X}] + q_i \tilde{q}^i = 0. \tag{2.3}$$

and we quotient by $\mathrm{U}(N)$ gauge transformations. This gives a space of $4Nk$ real dimensions which is a hyperkähler manifold. The metric is the induced metric in the ambient space. It can be found as follows: one constructs the Kähler potential of the ambient space $C = |q_i|^2 + |\tilde{q}^i|^2 + |X|^2 + |\tilde{X}|^2$, which also gives the Kähler potential of the metric in the moduli space, after choosing complex coordinates for the moduli. C is also the special conformal generator in the quantum mechanical theory. This defines a conformal quantum mechanics, which becomes superconformal once we include also the fermionic degrees of freedom [8]. For further details on the definition of the quantum mechanical theory and its symmetries see [8]. See also [9] for a nice review on conformal and superconformal quantum mechanics.

2.3 DLCQ of a conformal field theory on a plane wave background

A small variation of the preceding theme is to start with a relativistic theory on a plane wave background

$$\mathrm{d}s^2 = -\mathrm{d}x^+\mathrm{d}x^- - \vec{x}^{\,2}(\mathrm{d}x^+) + \mathrm{d}\vec{x}^{\,2} \tag{2.4}$$

where $\vec{x}$ stands for the transverse spatial directions. For a general field theory this also gives us a quantum mechanical system now with particles in a harmonic oscillator potential. The Hamiltonian is then

$$-p_+ = \frac{\vec{p}^{\,2}}{(-4p_-)} + (-p_-)\vec{x}^{\,2} \ . \tag{2.5}$$

The isometries of the plane wave are symmetries acting on the quantum mechanical theory. If the field theory is conformal we have further symmetries and the resulting theory is precisely the same as the one we obtained when we started from flat space but with a different choice of Hamiltonian [21]. This can be understood considering the $\mathrm{SL}(2,\mathbb{R})$ subgroup of the Schrödringer group which includes the Hamiltonian H the dilatation D and the special conformal transformation C. Then the Hamiltonian on the plane wave background is

$$H_{\rm osc} = L_0 = \frac{1}{2}(H+C) \ . \tag{2.6}$$

Another variation of the idea is to take the following form of the plane wave metric when the transverse direction is two-dimensional (or more generally even dimensional):

$$\mathrm{d}s^2 = -\mathrm{d}x^+(\mathrm{d}x^- - 2\rho^2\mathrm{d}\hat{\psi}) + \mathrm{d}\rho^2 + \rho^2\mathrm{d}\hat{\psi}^2 \,, \tag{2.7}$$

where we took $\vec{x} = (\rho\cos\psi, \rho\sin\psi)$ and chose $\hat{\psi} = \psi - x^+$. A particle with fixed p_- moving on this metric reduces to a non-relativistic particle moving in the transverse space in the presence of a constant magnetic field $\mathrm{d}(\rho^2\mathrm{d}\psi)$ with no potential. The new Hamiltonian is related to the one above by

$$H_{\rm mag} = H_{\rm osc} - J \,, \tag{2.8}$$

where J is the angular momentum associated to the rotation in the $\vec{x}$ plane.

2.4 DLCQ of dipole theories

Another variation is to consider a certain non-commutative theory, called a "dipole theory" [10] (see also [11, 22, 23]). This is a theory were the field multiplication is defined via a star product. In order to define the star product we use the conserved charge p_- and also another global symmetry charge, Q of the system. The star product is then defined as follows

$$f * g = \mathrm{e}^{i2\pi\sigma(P_-^f Q^g - P_-^g Q^f)} fg \,, \tag{2.9}$$

where fg is the ordinary product and σ is an arbitrary parameter. (P_-^f, Q^f) and (P_-^g, Q^g) are the values of P_- and Q for f and g respectively. We are imagining that f and g have well defined values for both charges P_- and Q, and we can get the product for more general functions of f and g by the ordinary distributive property of the product.

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If the original theory has a symmetry that commutes with the two charges that appear in the definition of the star product (2.9), then it will also be a symmetry of the theory after the star product deformation. The generators of the conformal group that commute with P_- are the Schrödinger subgroup. Thus we have the Schrödinger symmetry even before compactifying the coordinate x^-. If we also compactify x^- and perform a DLCQ quantization we expect to get a non-relativistic conformal system. Our reason for introducing these exotic theories is that, in some cases, their gravity duals, derived in [10, 11], are given by the metrics introduced in [1, 2]. For more details on these theories see [10, 11, 22, 23] .

3. DLCQ of string or M theory backgrounds

3.1 DLCQ of AdS

As we saw in the previous section, one can construct a system with Schrödinger symmetry by taking the DLCQ of a relativistic conformal theory. If the relativistic CFT is dual to AdS we should then consider the DLCQ of AdS space. We can write the AdS metric in the Poincaré patch as

$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{r^2} + r^2\left(-\mathrm{d}x^+\mathrm{d}x^- + \mathrm{d}\vec{x}^{\,2}\right) \tag{3.1}$$

and take $x^- \sim x^- + 2\pi r^-$. This fact has been noticed also in the recent papers [4, 3].

Since we are taking a circle to have zero size, we cannot trust this geometry to do computations. A similar situation arises when we consider the DLCQ of eleven dimensional supergravity. In that case the conjectured correct description is in terms of a quantum mechanical theory given by N D0 branes [24] (see also [25]). This description is not the same as the one we get by taking the naive DLCQ of the gravity theory. For example, the scattering amplitude of three gravitons to three gravitons in the naive supergravity approximation gives a different answer than in the matrix model [20]. In fact, when one defines the DLCQ limit carefully, as a limit of the theory on a very small spatial circle, one finds that the correct answer is given by the D0 matrix quantum mechanics and not by supergravity [26].[3]

Fortunately, not all is lost. Of course, what we really want to do is to put N units of momentum on this space. When N is small, the dynamics cannot be computed in terms of particles moving in the metric (3.1), for reasons we have explained. Notice that this is true even when the radius of AdS is large. In fact, the correct description of N units of momentum in the DLCQ of type IIB string theory in flat space is in terms of the field theory that lives on N M2 branes on T^2 [27].

However, if we put a large amount of momentum, then the backreaction implies that the size of x^- will be non-zero in some regions and we will be able to trust the metric. More concretely, we can consider the black three brane metric describing a finite temperature

[3] Of course, the BFSS conjecture [25] is the idea that a suitable large N limit allows one to recover the results with no compactification of the light like circle.

system

$$\mathrm{d}s^2 = \frac{1}{1-\frac{r_0^4}{r^4}}\frac{\mathrm{d}r^2}{r^2} + r^2\left[-\mathrm{d}x^+\mathrm{d}x^- + \frac{r_0^4}{4r^4}\left(\lambda^{-1}\mathrm{d}x^+ + \lambda\mathrm{d}x^-\right)^2 + \mathrm{d}\vec{x}^{\,2}\right] . \tag{3.2}$$

This is simply a boosted version of the ordinary black brane metric, in the near-horizon region. We can compute the density of $-P_-$, which we interpret as the "particle density", and the energy density. We obtain (see appendix C.2)

$$\frac{N}{V_2} = \frac{r^-(-P_-)}{V_2} = \frac{R_{\rm AdS}^3}{G_N^5}\frac{(r^-)^2\lambda^2 r_0^4}{8}, \qquad \frac{H}{V_2} = \frac{(-P_+)}{V_2} = \frac{R_{\rm AdS}^3}{G_N^5}\frac{r_0^4}{16}r^- , \tag{3.3}$$

where V_2 is the volume of the two spatial dimensions.

We can see that by tuning λ and r_0 we get different values of particle density as well as energy density. We can also write this in terms of the temperature and chemical potential of the non-relativistic system

$$\frac{1}{T} = \frac{\pi\lambda}{r_0}, \qquad \frac{\mu_N}{T} = \frac{\pi}{r_0 r^-\lambda} . \tag{3.4}$$

The entropy is given by

$$S = \frac{R_{\rm AdS}^3}{4G_N^5}\lambda(2\pi r^-)\frac{r_0^3}{2}V_2 . \tag{3.5}$$

and these thermodynamic quantities satisfy the first law

$$\delta H = T\delta S - \mu_N\delta N . \tag{3.6}$$

The physical value of the radius of the x^- circle is

$$\frac{R^-_{\rm phys}}{l_s} = \frac{R_{\rm AdS}}{l_s}\frac{r_0}{r}\frac{\lambda r^- r_0}{2} , \tag{3.7}$$

therefore we can trust the gravity description as long as $\frac{R^-_{\rm phys}}{l_s} \gg 1$. When it becomes smaller than one we might be able to do a T-duality and use an alternative description also. Notice that this size becomes small as we approach the boundary of AdS, $r \to \infty$. Thus, we cannot trust the metric near the boundary. It might be possible that after performing suitable U-dualities we might find a metric we can trust.

Of course this solution implies that the thermodynamic properties can be simply extracted from the thermodynamic properties of the ordinary black D3 brane. We are only looking at the same system in light cone gauge, so we just need to translate all quantities to light cone gauge.

3.2 DLCQ of AdS$_5$ with a plane wave boundary: harmonic potential

In this subsection we consider the gravity dual of the field theory with plane wave boundary conditions, so that it is the gravity dual to the field theory on the plane wave. Since the plane wave metric is conformal to flat space the metric is simply AdS, which can be sliced in a way that make the plane wave at the boundary more manifest. One way to find the slicing is to start with $\mathbb{R} \times S^3$ and take the Penrose limit for a particle moving with large angular momentum in one of the angles of S^3. Namely, we write the AdS metric in global coordinates,

$$\mathrm{d}s^2 \;=\; -(1+r^2)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{1+r^2} + r^2 \mathrm{d}s^2(S^3)\,, \tag{3.8}$$

which has $\mathbb{R} \times S^3$ as a boundary. Here $\mathrm{d}s^2(S^3) = \mathrm{d}\theta^2 + \cos^2\theta \mathrm{d}\varphi^2 + \sin^2\theta \mathrm{d}\psi^2$. We then define

$$x^+ \;=\; t\,, \qquad \frac{x^-}{2R^2} \;=\; t - \varphi\,, \qquad \theta \;=\; \frac{\rho}{R}\,, \qquad r = Ry \tag{3.9}$$

and we take the $R \to \infty$ limit, keeping $x^\pm$ and ρ, y fixed. We find the metric

$$\mathrm{d}s^2 \;=\; \frac{\mathrm{d}y^2}{y^2} + y^2\left(-\mathrm{d}x^+\mathrm{d}x^- - \rho^2(\mathrm{d}x^+)^2 + \mathrm{d}\rho^2 + \rho^2\mathrm{d}\psi^2\right) - (\mathrm{d}x^+)^2\,. \tag{3.10}$$

Notice that it is not necessary to take this limit to obtain (3.10), and we can also obtain it directly by writing AdS space in the appropriate coordinates[4] (see also [3]). Explicitly, after the further change of coordinates

$$\sinh r \;=\; \rho y\,, \qquad z \;=\; \rho^2 + \frac{1}{y^2}\,, \tag{3.11}$$

the metric (3.10) becomes

$$\mathrm{d}s^2 \;=\; \sinh^2 r\,\mathrm{d}\psi^2 + \mathrm{d}r^2 + \cosh^2 r\left[-\left(\mathrm{d}x^+ + \frac{\mathrm{d}x^-}{2z}\right)^2 + \frac{(\mathrm{d}x^-)^2 + \mathrm{d}z^2}{4z^2}\right]\,. \tag{3.12}$$

This is AdS_5 sliced by AdS_3, which is the metric in square brackets. However, obtaining the metric (3.10) as a limit will be useful later. The metric in (3.10) cannot be trusted if x^- is compact, since x^- is a null direction.

We would now like to add some x^- momentum and also raise the temperature of the system so that we have a black hole in a space which is asymptotic to (3.10). We obtain this black hole by starting with the five-dimensional Kerr-AdS black hole [28] (see also [29]) and performing a limit similar to the limit we performed above (3.9). We describe this in detail in appendix B. The final metric can be written as

$$\begin{aligned}\mathrm{d}s^2 \;=\;& \frac{(r^2+\sin^2\theta)\mathrm{d}r^2}{(1+r^2)^2-2m} - (1+r^2\sin^2\theta)(\mathrm{d}x^+)^2 - \lambda(1+r^2)\cos^2\theta\mathrm{d}x^+\mathrm{d}x^- \\ &+ \frac{(r^2+\sin^2\theta)\mathrm{d}\theta^2}{\cos^2\theta} + r^2\sin^2\theta\mathrm{d}\psi^2 + m\frac{(-2\mathrm{d}x^+ + (\mathrm{d}x^+ - \lambda\mathrm{d}x^-)\cos^2\theta)^2}{2(r^2+\sin^2\theta)}\,.\end{aligned} \tag{3.13}$$

This metric depends only on one non-trivial parameter m. The parameter λ can be absorbed into the redefinition of x^- but it is convenient to keep it because it represents the amount of boost one performs when we take the limit. We can bring this metric to a form which asymptotes to (3.10) via the coordinate change:

$$\begin{aligned}\lambda^{-1}y^2 \;&=\; (r^2+1)\cos^2\theta\,, \\ \rho^2 y^2 \;&=\; r^2\sin^2\theta\,.\end{aligned} \tag{3.14}$$

[4] In fact, (3.10) can be transformed into the metric recently discussed in [4] (*cf.* equation (35) in that reference) by an obvious change of coordinates.

This is analogous to the coordinate change employed in [28] to display the AdS asymptotics of the Kerr-AdS black hole. The result does not have a simple analytic form. As an expansion in the new radial variable y, it is given by

$$\begin{aligned}
\mathrm{d}s^2 = &\left(1 - \frac{2m}{(1+\lambda\rho^2)^2}\frac{\lambda^2}{y^4}\right)^{-1}\frac{\mathrm{d}y^2}{y^2} + y^2\left(-\mathrm{d}x^+\mathrm{d}x^- - \rho^2(\mathrm{d}x^+)^2 + \mathrm{d}\rho^2 + \rho^2\mathrm{d}\psi^2\right) \\
&- (\mathrm{d}x^+)^2 + \frac{\lambda m}{y^2}\frac{\left((1+2\lambda\rho^2)\mathrm{d}x^+ + \lambda\mathrm{d}x^-\right)^2}{2(1+\lambda\rho^2)^3} + \mathcal{O}(y^{-4}) \qquad (3.15)
\end{aligned}$$

where we kept only terms of order lower than $\mathcal{O}(y^{-4})$ with respect to the $m=0$ solution.

The P_- and P_+ can be calculated by starting from the expressions for the energy and the angular momentum in [29] (also see [30]) and taking the appropriate limit, see appendix B. We find

$$H = -P_+ = \frac{R^3_{\mathrm{AdS}}}{G^5_N}(2\pi r^-)\lambda\frac{(1+r_H^2)^2}{16}, \qquad (3.16)$$

$$N = -P_- r^- = \frac{R^3_{\mathrm{AdS}}}{G^5_N}(2\pi r^-)r^-\lambda^2\frac{(1+r_H^2)^2}{32}. \qquad (3.17)$$

Recall that the Hamiltonian here is the oscillator Hamiltonian H_{osc}.

The horizon radius r_H is given by the solution of $(r_H^2+1)^2 - 2m = 0$. As $r_H \to 0$ the metric becomes singular. For $r_H = 0$ (or $2m = 1$) the energy and particle densities from (3.16), (3.17) are non-zero. For smaller densities or energies this black hole would not be a good description. However, for $m = 0$ the metric (3.13) does go to the plane wave metric (3.10).[5]

The Killing vector degenerating at the horizon is

$$\partial_+ + \frac{1}{\lambda}\frac{r_H^2-1}{r_H^2+1}\partial_- . \qquad (3.18)$$

Then the temperature and the chemical potential are

$$T = \frac{r_H}{\pi}, \qquad \mu_N = \frac{1}{r^-\lambda}\frac{r_H^2-1}{r_H^2+1}. \qquad (3.19)$$

The entropy of the system is

$$S = \frac{R^3_{\mathrm{AdS}}}{4G^5_N}(2\pi r^-)\frac{\pi}{2}\lambda r_H(r_H^2+1), \qquad (3.20)$$

and one can check again the first law

$$\delta H = T\delta S - \mu_N\delta N \qquad (3.21)$$

is satisfied, as it should be. We can also compute the size of the circle x^- at the horizon which is

$$\frac{R^-_{\mathrm{phys}}}{l_p} = \frac{R_{\mathrm{AdS}}}{l_p}r^-\lambda\frac{(r_H^2+1)\cos^2\theta}{2\sqrt{r_H^2+\sin^2\theta}}. \qquad (3.22)$$

[5]This is reminiscent to what happens for the BTZ black hole.

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This becomes zero when $\cos\theta \to 0$. In fact, the circle x^- shrinks at $\theta = \pi/2$ at any value of r as is evident from (3.15). In the coordinates ρ, y introduced in (3.14) these points correspond to the limit $y \to 0$, $\rho \to \infty$. Since we do not have a large contribution to the thermodynamic quantities from this region we assume that we can ignore this part of the metric.

3.3 DLCQ of AdS$_5$ with a plane wave boundary: magnetic field

Here we briefly discuss the gravity dual of the field theory with plane wave boundary conditions in the coordinates (2.7) which lead to a non relativistic particle in a magnetic field when we fix p_-. The Hamiltonian of this system is obtained shifting the oscillator Hamiltonian with the angular momentum on the plane, as in 2.8. The metric is obtained by performing a limit similar to the one discussed in the previous subsection, starting with the two-parameter Kerr-AdS black hole [28]. This is discussed in appendix B.2 and here we present the final result

$$\begin{aligned} \mathrm{d}s^2 =& \left(1 - \frac{2m\lambda^4}{y^4} + \frac{2m\lambda^6}{y^6}\right)^{-1} \frac{\mathrm{d}y^2}{y^2} + y^2 \left[-\mathrm{d}x^+(\mathrm{d}x^- - 2\rho^2 \mathrm{d}\hat{\psi}) + (\mathrm{d}\rho^2 + \rho^2 \mathrm{d}\hat{\psi}^2) \right] \\ & - (\mathrm{d}x^+)^2 + \frac{m\lambda^6}{2y^2} \left[\mathrm{d}x^- + \frac{\mathrm{d}x^+}{\lambda^2} - 2\rho^2 \mathrm{d}\hat{\psi} \right]^2 . \end{aligned} \tag{3.23}$$

The temperature and the chemical potential are given by

$$T = \frac{1}{2\pi} \frac{2y_H^2 - 3\lambda^2}{\lambda\sqrt{y_H^2 - \lambda^2}}, \qquad \mu_N = \frac{y_H^2 - 2\lambda^2}{r^- y_H^2 \lambda^2}, \tag{3.24}$$

and the entropy is

$$S = \frac{R_{\mathrm{AdS}}^3}{4G_N^5} (2\pi r^-) V_2 \frac{1}{2} \frac{y_H^4 \lambda}{\sqrt{y_H^2 - \lambda^2}}, \tag{3.25}$$

where the horizon is at $y = y_H$ with

$$1 - \frac{2m\lambda^4}{y_H^4} + \frac{2m\lambda^4}{y_H^6} = 0 . \tag{3.26}$$

The $P_\pm$ densities are constant (see appendix B.2), thus the charges are infinite on the $\vec{x}$ plane, and proportional to the volume V_2, as is in the case of the translation invariant black three brane metric. In fact, after Kaluza-Klein reducing along x^- we see that we get a translation invariant metric with a constant Kaluza-Klein magnetic field. The particle number and energy per unit volume are given by

$$\frac{N}{V_2} = \frac{R_{\mathrm{AdS}}^3}{4G_N^5} r^- m\lambda^6, \qquad \frac{H}{V_2} = \frac{R_{\mathrm{AdS}}^3}{4G_N^5} r^- \frac{m\lambda^4}{2}, \tag{3.27}$$

respectively. Recall that the Hamiltonian here is given by $H_{\mathrm{mag}} = H_{\mathrm{osc}} - J$. This implies that we can view this system as the system on with the Harmonic oscillator hamiltonian but at a critical value of the chemical potential for the spin in the transverse plane. One could find a metric interpolating between (3.13) and (3.23) by adding a general chemical potential for the spin. Such metrics should correspond to more general limits of the rotating Kerr black hole [28].

3.4 DLCQ of AdS$_7$ with a plane wave boundary

In this subsection we repeat the previous discussion for the AdS$_7$ case. We write the S^5 as $\mathrm{d}s^2(S^5) = \mathrm{d}\theta^2 + \cos^2\theta \mathrm{d}\varphi^2 + \sin^2\theta \mathrm{d}\Omega_3^2$. We now start with AdS$_7$ in coordinates where the boundary is $\mathbb{R} \times S^5$. After performing the rescaling (3.9) we get a metric similar to (3.10) except that $\mathrm{d}\psi^2 \to \mathrm{d}\Omega_3^2$. Starting from the seven dimensional Kerr-AdS black hole and performing the same limit (3.9) we get a metric very similar to (3.13) except for $m \to m/r^2$ and $\mathrm{d}\psi^2 \to \mathrm{d}\Omega_3^2$,

$$\begin{aligned} \mathrm{d}s^2 = &\frac{(r^2+\sin^2\theta)\mathrm{d}r^2}{(1+r^2)^2 - \frac{2m}{r^2}} - (1+r^2\sin^2\theta)(\mathrm{d}x^+)^2 - \lambda(1+r^2)\cos^2\theta \mathrm{d}x^+\mathrm{d}x^- \\ &+\frac{(r^2+\sin^2\theta)\mathrm{d}\theta^2}{\cos^2\theta} + r^2\sin^2\theta \mathrm{d}\Omega_3^2 + \frac{m}{r^2}\frac{(-2\mathrm{d}x^+ + (\mathrm{d}x^+ - \lambda\mathrm{d}x^-)\cos^2\theta)^2}{2(r^2+\sin^2\theta)}\,. \end{aligned} \tag{3.28}$$

We can also compute the entropy as a function of the temperature and the chemical potential. The expressions for the temperature and chemical potential are

$$T = \frac{1}{2\pi}\left(3r_H + \frac{1}{r_H}\right), \qquad \mu_N = \frac{1}{r^-\lambda}\frac{r_H^2 - 1}{r_H^2+1}\,, \tag{3.29}$$

where now the horizon radius obeys the equation $(r_H^2+1)^2 - 2m/r_H^2 = 0$. The values of the energy and the momentum can be uniquely fixed by first noting that $E \propto \lambda m$ and $N \propto \lambda^2 m$ from the asymptotic form of the metric, and second by demanding that the first law is satisfied. We obtain

$$H = -P_+ = \frac{R_{\mathrm{AdS}}^5}{G_N^7}(2\pi r^-)\lambda\frac{\pi}{16}r_H^2(1+r_H^2)^2\,, \tag{3.30}$$

$$N = -P_- r^- = \frac{R_{\mathrm{AdS}}^5}{G_N^7}(2\pi r^-)r^-\lambda^2\frac{\pi}{64}r_H^2(1+r_H^2)^2\,. \tag{3.31}$$

The entropy is

$$S = \frac{R_{\mathrm{AdS}}^5}{4G_N^7}(2\pi r^-)\lambda\frac{\pi^2}{4}r_H^3(1+r_H^2)\,. \tag{3.32}$$

In this case we see that as $r_H \to 0$ the energy and the particle number both go to zero. On the other hand, we see that the temperature in (3.29) has a minimum. This suggests that the thermal ensemble will display a phase transition similar to the Hawking-Page we see when we treat AdS in global coordinates [31, 32]. In fact, for a given temperature and chemical potential there is another solution which is simply thermal AdS space with a gas of particles (we can only trust this last solution if we take x^- to be non-compact). Comparing their free energies we find a phase transition at $r_H = 1$. Of course, the black hole is the stable solution for $r_H > 1$. We can evaluate this quantities in the particular case of AdS$_7 \times S^4$ background of M theory. In that case we find that

$$\frac{R_{\mathrm{AdS}}^5}{4G_N^7} = \frac{R_{\mathrm{AdS}}^5 R_{S^4}^4 V_{S^4}}{4G_N^{11}} = \frac{4k^3}{3\pi^2}\,, \qquad \frac{R_{\mathrm{AdS}}}{l_p} = 2\frac{R_{S^4}}{l_p} = 2(\pi k)^{1/3}\,, \tag{3.33}$$

where $G_N^{11} = 16\pi^7 l_p^9$ and k is the number of M5 branes. As in the five-dimensional case, the x^- circle shrinks when $\theta = \pi/2$. For moderate temperatures, with r_H of order one, we can trust the M-theory background near the horizon in the regime $k \gg 1$ and $N^3/k^7 \gg 1$. By reducing to type IIA we can extend the region of validity of the above formulas to $k \gg 1$, $N/k \gg 1$. The existence of the IIA string theory dual suggests that this conformal quantum mechanics theory has an interesting 't Hooft limit.

These quantities should describe the thermodynamic properties of the conformal matrix quantum mechanics described in [8] and reviewed in section 2.2. That quantum mechanics theory is characterized by only two discrete parameters k and N. Here k is the number of fivebranes and N is the amount of momentum. When we consider the Hamiltonian $H_{\rm osc} = \frac{1}{2}(H+C)$ we expect that the temperature would also be a non-trivial parameter since there is an energy gap of order one that is given by the confining potential. In the above metric the temperature translates directly into r_H. Finally, we see that the combination $r^-\lambda$, at fixed temperature, fixes the chemical potential for the light cone momentum N. Thus the metrics we have depend on the right number of parameters. Here r^- is a trivial parameter and can be set to one without loss of generality.

3.5 DLCQ of the dipole theory and its gravity dual

In this section we describe in more detail a particular example of the dipole theories introduced in [10, 22, 23]. As we explained above these theories are based on the non-commutative $*$ product in (2.9). As a particular example we can consider starting with $\mathcal{N} = 4$ super Yang Mills in 3+1 dimensions. This theory has an SO(6) R symmetry that rotates six scalars, ϕ_i, and their fermionic partners. We can consider the particular U(1) symmetry that rotates all pairs of scalars. Defining $W_j = \phi_j + i\phi_{j+3}$, $j = 1, 2, 3$, the symmetry acts by $W_j \to e^{i\alpha} W_j$. Planar diagrams in non-commutative theories are particularly simple. The only difference between the planar diagrams in the ordinary theory and the theory with the star product is an overall phase depending only on the external particles [12] (see also [33]). Thus, if we consider a quantity such as the free energy, which does not involve any external particles, then the final result is the same as in the ordinary theory.

The gravity duals of such theories are easy to construct. We simply need to do a particular transformation on the gravity solution. This transformation has the following origin. In the original solution we have two commuting isometry directions, x^- and an angle φ that is conjugate to the charge Q we defined above. If we naively imagine doing a dimensional reduction of the ten dimensional gravity theory on these two dimensions we get a theory in eight dimensions that has an $SL(2, \mathbb{R})$ symmetry. This $SL(2, \mathbb{R})$ symmetry is a symmetry in the eight dimensional gravity theory but it is not a symmetry of the ten dimensional configuration. In fact, it maps ordinary solutions into the solutions we want. This transformation is the following simple set of steps. We first do a T-duality in the direction φ to a T-dual direction $\tilde{\varphi}$. We then do a shift of coordinates $x^- \to x^- + \sigma\tilde{\varphi}$. Finally, we do a T-duality again on $\tilde{\varphi}$ into φ. The step where we did a shift of x^- is not a symmetry of the theory since $\tilde{\varphi}$ is periodic but x^- is not. Nevertheless this operation generates a new solution which is another solution of the gravity equations which is not equivalent via legal U-dualities to the original one. This procedure was applied in [11] to obtain the corresponding gravity solutions. We perform these steps more explicitly in appendix C. We start with $\mathrm{AdS}_5 \times S^5$, where the AdS_5 metric is written as in (3.1). The final type IIB background that we obtain through this procedure is

$$\begin{aligned} \mathrm{d}s^2 &= -\sigma^2 r^4 (\mathrm{d}x^+)^2 + r^2 \left[-\mathrm{d}x^+\mathrm{d}x^- + \mathrm{d}\vec{x}^{\,2}\right] + \frac{\mathrm{d}r^2}{r^2} + \mathrm{d}s^2(\mathbb{CP}^2) + \eta^2, \\ B^{\mathrm{NS}} &= \sigma r^2 \mathrm{d}x^+ \wedge \eta\,, \end{aligned} \tag{3.34}$$

and the rest of the fields have the same form as they had before we applied the transformation. Here $\eta = \mathrm{d}\phi + P$ is the one-form present in any Sasaki-Einstein manifold, which is dual to the Reeb vector field $\partial/\partial\phi$. In the case of S^5, this generates simultaneous rotation of the scalars as discussed above. Indeed, in (3.34) we can replace the $\mathbb{CP}^2$ space with any local Kähler-Einstein space B_{KE}, so that

$$\mathrm{d}s^2(Y) = \mathrm{d}s^2(B_{\mathrm{KE}}) + \eta^2 \tag{3.35}$$

is a Sasaki-Einstein metric on Y, where $\mathrm{d}\eta/2 = \omega$ is the Kähler two-form on B_{KE}. These backgrounds do not preserve any supersymmetry. See appendix E.

The five-dimensional part of this metric has the same form as the ones considered in [1, 2]. These metrics have the feature that they break the symmetries to the Schrödinger symmetry even before compactifying the direction x^-. However, if we do not compactify the direction x^- we have a theory with continuous eigenvalues for P_- so that there is no sense in which we have a non-relativistic system. On the other hand, if we do not compactify x^- we can definitely trust the metric (3.34), at least for values of r that are not too large.

However, we wanted to perform the DLCQ of this theory. When we compactify x^- we find that we can no longer trust computations in the metric (3.34). As before, we could consider a configurations with finite P_-. A simple case arises when we consider a finite temperature configuration. We can obtain the corresponding metric if we start with the black brane solution (3.2) and perform the TsT transformation that lead to (3.34). The final metric in the Einstein frame is (see appendix C for details)

$$\begin{aligned} \mathrm{d}s^2 = \mathrm{e}^{\frac{3}{2}\Phi} r^2 \Bigg[& \left(-1 + \frac{r_0^4}{2r^4}\right) \mathrm{d}x^+\mathrm{d}x^- + \frac{r_0^4}{4r^4}\left(\lambda^2 (\mathrm{d}x^-)^2 + \lambda^{-2}(\mathrm{d}x^+)^2\right) \\ & - \sigma^2 r^2 \left(1 - \frac{r_0^4}{r^4}\right)(\mathrm{d}x^+)^2\Bigg] + \mathrm{e}^{-\frac{\Phi}{2}} r^2 \left[\frac{1}{r^4 - r_0^4}\,\mathrm{d}r^2 + \mathrm{d}\vec{x}^2\right] \\ & + \mathrm{e}^{-\frac{\Phi}{2}} \mathrm{d}s^2(B_{\mathrm{KE}}) + \mathrm{e}^{\frac{3}{2}\Phi}\eta^2 \end{aligned} \tag{3.36}$$

with dilaton and B-field given by

$$\begin{aligned} \mathrm{e}^{-2\Phi} &= 1 + \sigma^2\lambda^2 \frac{r_0^4}{r^2}\,, \\ B^{\mathrm{NS}} &= \sigma \frac{r^2}{2} \mathrm{e}^{2\Phi} \left[\left(2 - \frac{r_0^4}{r^4}\right)\mathrm{d}x^+ - r_0^4 \lambda^2\, \mathrm{d}x^-\right] \wedge \eta\,, \end{aligned} \tag{3.37}$$

where σ is the parameter used in the shift, $x^- \to x^- + \sigma\tilde{\varphi}$. By construction, the solution has the desired asymptotic form at infinity as $r \to \infty$. However, there is now a horizon at some finite value of the coordinate r, inherited from the asymptotically-AdS black three-brane metric (3.2).

We can now compute the energy and momentum of this solution, as well as its entropy. All these results are independent of σ, which is related to the non-commutativity parameter. In fact we simply get the results in (3.3), (3.4), (3.5). The reason is that the TsT transformation is a symmetry of the eight dimensional gravity theory and this eight dimensional theory is all that we are using for computing these quantities.[6] See appendix C.2 for a more detailed explanation. As explained in the beginning of this subsection, this is due in the field theory language to the fact that the planar diagrams are the same in the two theories. Of course, it is not a symmetry of the full theory, so we expect that at order $1/k^2$ the two theories would yield different answers, where k is the rank of the gauge group.

4. Consistent truncations

In this section we construct two different consistent truncations of type IIB supergravity with massive vector fields. As we will see, one of them admits the black hole background (3.36), (3.37), among its solutions. Usually consistent truncations involve massless vector fields. These truncations can be used to generate other solutions which have the same symmetries as certain conformal quantum mechanical theories. We think that these truncations are interesting in their own right and could perhaps be useful for the construction of other solutions. We have two truncations which we will discuss in turn. Both are consistent truncations of the type IIB equations for the bosonic fields. It might be possible that they can be supersymmetrized. We will state here the final results and leave details to appendix D.

Before proceeding, let us recall how the backgrounds with non-relativistic conformal symmetry

$$\mathrm{d}s^2(M_z) \;=\; -\sigma^2 r^{2z}(\mathrm{d}x^+)^2 + \frac{\mathrm{d}r^2}{r^2} + r^2\left(-\mathrm{d}x^+\mathrm{d}x^- + \mathrm{d}\vec{x}^{\,2}\right) \tag{4.1}$$

arise from AdS gravity with a massive gauge field. This metric is a deformation of the AdS_{d+3} metric and clearly is only invariant under a subgroup of $\mathrm{SO}(d+2,2)$, which for generic values of z is the non-relativistic dilatation group, enhanced to the Schrödinger group for $z=2$. See appendix A. These metrics are solutions to the equations of motion derived from the action [1, 2]

$$S \;=\; \int \mathrm{d}^{d+2}x\,\mathrm{d}r\sqrt{-g}\left(R - 2\Lambda - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{m^2}{2}A_\mu A^\mu\right)\;. \tag{4.2}$$

In particular, the ansatz $A_+ \propto r^z$ solves the equations of motion, provided

$$\Lambda \;=\; -\frac{1}{2}(d+1)(d+2)\,, \qquad\qquad m^2 \;=\; z(z+d)\;. \tag{4.3}$$

In the following subsections we will describe consistent truncations of type IIB supergravity, containing the action (4.2), where $d=2$, and $z=2,4$ respectively.

[6] We are also using the fact that the TsT transformation leaves invariant the eight dimensional Kaluza-Klein gauge fields associated to the two charges.

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4.1 Non-linear Kaluza-Klein reduction with $m^2 = 8$

The ansatz we consider here is suggested by the observation that in any background of the type $\mathrm{AdS}_5 \times Y$, where Y is a Sasaki-Einstein manifold, there exist certain Kaluza-Klein vector fields with mass squared precisely $m^2 = 8$. These arise from standard Kaluza-Klein reduction of the NS B field and the RR C_2 field along the one-form $\eta = \mathrm{d}\phi + P$ that exists on Y [34].

We then consider type IIB supergravity where only the metric, the dilaton Φ, the five-form field F_5 and the NSNS three-form $H = \mathrm{d}B$ are non-trivial. The equations of motion in the Einstein frame are[7]

$$\begin{aligned}
R_{MN} &= \frac{1}{2}\partial_M \Phi \partial_N \Phi + \frac{1}{96} F_{MABCD} F_N{}^{ABCD} \\
&\quad + \frac{1}{4}\mathrm{e}^{-\Phi}\left(H_{MAB} H_N{}^{AB} - \frac{1}{12} g_{MN} H_{ABC} H^{ABC} \right), \qquad (4.4) \\
\Box_{10}\Phi &= -\frac{1}{12}\mathrm{e}^{-\Phi} H_{ABC} H^{ABC}\,, \\
\mathrm{d}(\mathrm{e}^{-\Phi} \star H) &= 0\,, \\
F_5 &= \star F_5, \qquad \mathrm{d}F_5 = 0\,, \qquad H \wedge F_5 = 0\,, \qquad (4.5)
\end{aligned}$$

where $\Box_{10} = \nabla^M \partial_M$. The final equation arises from the equation of motion of the RR 3-form, which has been set to zero. Our ansatz is

$$\begin{aligned}
\mathrm{d}s^2_{10} &= \mathrm{e}^{-\frac{2}{3}(4U+V)} \mathrm{d}s^2(M) + \mathrm{e}^{2U}\mathrm{d}s^2(B_{\mathrm{KE}}) + \mathrm{e}^{2V}\eta^2\,, \\
B &= A \wedge \eta + \theta\, \omega\,, \\
F_5 &= (1+\star) G_5 \qquad \text{where} \quad G_5 = 4\mathrm{e}^{-4U-V}\,\mathrm{vol}(M)\,, \qquad (4.6)
\end{aligned}$$

where $\mathrm{d}s^2(B_{\mathrm{KE}}) + \eta^2$ is a Sasaki-Einstein metric. Here, $\mathrm{d}s^2(M)$ is the Lorentzian metric of the five-dimensional part M and $\mathrm{vol}(M)$ denotes its volume form. We use indices x^a ($a = 0, \ldots, 4$) to denote the directions along M and x^i ($i = 1, \ldots, 4$) for those along B_{KE}. U, V, θ and Φ are scalar functions, and A is a one-form on M, respectively. The warp factor in front of $\mathrm{d}s^2(M)$ is inserted to obtain the reduced equations in the five-dimensional Einstein frame. This ansatz is closely related to the one used in [35], where they had scalars U, V, θ and Φ, but not the gauge field A. They also had non-trivial RR 2-form, which we do not have.

Our choices for A and F_5 satisfy the equations (4.5) automatically. Then defining $F = \mathrm{d}A$, the field strength $H = \mathrm{d}B$ of the two-form B reads

$$H \;=\; F \wedge \eta - (2A - \mathrm{d}\theta) \wedge \omega\ . \qquad (4.7)$$

The gauge transformation of the B-field $B \to B + \mathrm{d}(\chi \wedge \omega)$ with a function χ on M induces the five dimensional gauge transformation

$$A \;\to\; A + \mathrm{d}\chi\,, \qquad \theta \to \theta - 2\chi\,, \qquad (4.8)$$

[7]We use capital Roman letters for the ten-dimensional indices.

which shows that θ is the Stückelberg field giving mass to the gauge field A. We choose the gauge $\theta = 0$ for simplicity.

The equations of motion (4.5) follow from the Type IIB action

$$S = \frac{1}{2}\int \mathrm{d}^{10}x\sqrt{-g}\Big[R - \frac{1}{2}\partial_A\Phi\partial^A\Phi \\ - \frac{1}{2\cdot 3!}\mathrm{e}^{-\Phi}H_{ABC}H^{ABC} - \frac{1}{2\cdot 5!}G_{(5)\,ABCDE}G_{(5)}{}^{ABCDE}\Big] \quad (4.9)$$

by varying g_{MN}, B and Φ. Inserting the ansatz (4.6) in (4.9) and integrating over the internal directions, we obtain the five-dimensional action

$$S = \frac{1}{2}\int \mathrm{d}^5x\sqrt{-g}\Big[R + 24\mathrm{e}^{-u-4v} - 4\mathrm{e}^{-6u-4v} - 8\mathrm{e}^{-10v} - 5\partial_a u\partial^a u - \frac{15}{2}\partial_a v\partial^a v \quad (4.10) \\ -\frac{1}{2}\partial_a\Phi\partial^a\Phi - \frac{1}{4}\mathrm{e}^{-\Phi+4u+v}F_{ab}F^{ab} - 4\mathrm{e}^{-\Phi-2u-3v}A_aA^a\Big],$$

where we defined $u = \frac{2}{5}(U-V)$ and $v = \frac{4}{15}(4U+V)$ to diagonalize the kinetic terms for the scalars. The five-dimensional equations of motion which follow from the action above are

$$\begin{aligned}
R_{ab} =& -\frac{1}{3}(24\mathrm{e}^{-u-4v} - 4\mathrm{e}^{-6u-4v} - 8\mathrm{e}^{-10v})g_{ab} \\
&+ \frac{1}{2}\partial_a\Phi\partial_b\Phi + 5\partial_a u\partial_b u + \frac{15}{2}\partial_a v\partial_b v \\
&+ \frac{1}{2}\mathrm{e}^{-\Phi+4u+v}\left(F_{ac}F_b^c - \frac{1}{6}g_{ab}F_{cd}F^{cd}\right) + 4g_{ab}\mathrm{e}^{-\Phi-2u-3v}A_cA^c, \\
\mathrm{d}(\mathrm{e}^{-\Phi+4u+v}\star_5 F) =& -8\mathrm{e}^{-\Phi-2u-3v}\star_5 A, \\
\Box_5\Phi =& -\frac{1}{4}\mathrm{e}^{-\Phi+4u+v}F_{ab}F^{ab} - 4\mathrm{e}^{-\Phi-2u-3v}A_aA^a, \\
10\Box_5 u =& 24(\mathrm{e}^{-u-4v} - \mathrm{e}^{-6u-4v}) \\
&+ \mathrm{e}^{-\Phi+4u+v}F_{ab}F^{ab} - 8\mathrm{e}^{-\Phi-2u-3v}A_aA^a, \\
15\Box_5 v =& 16(6\mathrm{e}^{-u-4v} - \mathrm{e}^{-6u-4v} - 5\mathrm{e}^{-10v}) \\
&+ \frac{1}{4}\mathrm{e}^{-\Phi+4u+v}F_{ab}F^{ab} - 12\mathrm{e}^{-\Phi-2u-3v}A_aA^a
\end{aligned} \quad (4.11)$$

where $\Box_5 = \nabla^a\partial_a$ is the five-dimensional d'Alembertian.

Interestingly, this non-linear Kaluza-Klein reduction is consistent: any solution to the five-dimensional equations of motion (4.11) can be lifted to a solution of the ten-dimensional equations of motion of type IIB supergravity, using the ansatz (4.6). The details can be found in appendix D.1. The masses of the excitations around the AdS_5 background are

$$m_A^2 = 8\,, \qquad m_\Phi^2 = 0\,, \qquad m_u^2 = 12\,, \quad \text{and} \quad m_v^2 = 32\,. \quad (4.12)$$

Notice that one can write a "superpotential" (in the Hamilton-Jacobi sense) for the scalar fields u, v [35]. This may be useful for studying holographic renormalisation of the system, see e.g. [36 – 39].

Background with $z = 2$. As promised, we can now see that a particular solution to the equations of motion (4.11) is obtained by setting $\Phi = u = v = 0$, taking (4.1) with $z = 2$ as metric on M and the gauge field to be

$$A = \sigma r^2 \mathrm{d}x^+ \, . \tag{4.13}$$

Indeed, setting to zero the scalar fields, the action (4.10) reduces to the action (4.2) discussed in [1, 2]. We then recover in a different way the solution (3.34) previously obtained via the TsT transformation. As already mentioned, this solution does not preserve any supersymmetry. The derivation of this is relegated to appendix E.

In fact, it can be checked that also the black hole background (3.36), (3.37) arises as a solution to the equations of motion (4.11), with non trivial dilaton and scalar fields u, v. This provides an explicit check that this is indeed a solution of type IIB supergravity. There may be other interesting solutions to the equations (4.11), with non-relativistic symmetry or otherwise. We leave a more complete analysis for future work.

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4.2 Non-linear Kaluza-Klein reduction with $m^2 = 24$

Following the logic of section 4.1, it is natural to consider other special Kaluza-Klein modes on Y, that could be promoted to full non-linear consistent reductions. After having considered two-form modes, we should look at Kaluza-Klein modes coming from the RR four-form potential, as well as the metric. In fact, it is well known that these modes mix and come in pairs [40] with mass eigenvalues

$$m^2_{\pm} = \mu + 4 \pm 4\sqrt{\mu + 1}\,, \tag{4.14}$$

where μ is the eigenvalue of the Laplacian on the massive three-form ω_3 in Y on which the RR potential is reduced, that is $C_4 = A \wedge \omega_3$. For any Killing vector field in Y, there exists a mode with eigenvalue $\mu = 8$ [34, 41]. Thus, for each Killing vector, one obtains a massless mode as expected, but also a massive mode, with mass $m^2_+ = 24$. It has been shown [42, 43] that the massless KK mode associated to the Reeb Killing vector may be promoted to a non-linear truncation, yielding minimal gauged supergravity. In the following, we will demonstrate that in fact it is possible to include in the truncation also the massive gauge field, at least at the level of bosonic fields.

We then consider a background with only the metric and the five-form field non-trivial, with the following ansatz:

$$\mathrm{d}s^2 = \mathrm{e}^{-\frac{2}{3}(4U+V)}\mathrm{d}s^2(M) + \mathrm{e}^{2U}\mathrm{d}s^2(B_{\mathrm{KE}}) + \mathrm{e}^{2V}(\eta + \mathcal{A})^2\,, \tag{4.15}$$

$$F_5 = (1 + \star_{10})\left[2\omega^2 \wedge (\eta + \mathcal{A}) + 2\omega^2 \wedge \mathbf{A} - \omega \wedge (\eta + \mathcal{A}) \wedge \mathbb{F}\right] \, . \tag{4.16}$$

Here we gauged the Reeb isometry direction η by a connection one-form $\mathcal{A}$ on M, and we will denote its curvature by $\mathcal{F} = \mathrm{d}\mathcal{A}$. The ansatz for F_5, which has one-form $\mathbf{A}$ and two-form $\mathbb{F}$ on M, requires some explanation. The $2\omega^2 \wedge \eta$ term is chosen to have $\int_Y F_5 = 4$ which is the value for the $\mathrm{AdS}_5 \times Y$ background. The structure $2\omega^2 \wedge \mathbf{A} - \omega \wedge \eta \wedge \mathbb{F}$ is suggested by the discussion above that we should take $C_4 \sim \omega \wedge \eta \wedge \mathbf{A}$, therefore the combination

$$\mathrm{d}(\omega \wedge \eta \wedge \mathbf{A}) = 2\omega^2 \wedge \mathbf{A} - \omega \wedge \eta \wedge \mathbf{F} \tag{4.17}$$

with $\mathbf{F} = \mathrm{d}\mathbf{A}$ should be in the expansion of F_5. The ansatz is then made gauge-invariant by the replacement $\eta \to \eta + \mathcal{A}$, which imposes

$$\mathbb{F} \;=\; \mathcal{F} + \mathbf{F} \tag{4.18}$$

from the ω^2 component of the closure of F_5. This ansatz gives again a consistent reduction, which is an extension of the consistent reduction to the (bosonic sector of the) minimal gauged supergravity [42, 43]. The details of the calculation are explained in appendix D.2.

Here we only discuss the equations of motion of the vector fields $\mathcal{A}$ and $\mathbf{A}$, one of which comes from the ω component of $\mathrm{d}F_5 = 0$,

$$\mathrm{d}(\mathrm{e}^{-\frac{4}{3}(U+V)} \star_5 \mathbb{F}) = -8\mathrm{e}^{-8U} \star_5 \mathbf{A} + \mathcal{F} \wedge \mathbb{F} \;. \tag{4.19}$$

In addition, the 10d Einstein equation in (4.5), with $(A,B) = (a,\phi)$ gives the relation

$$\mathrm{d}(\mathrm{e}^{\frac{8}{3}(U+V)} \star_5 \mathcal{F}) \;=\; 16\mathrm{e}^{-8U} \star_5 \mathbf{A} + \mathbb{F} \wedge \mathbb{F} \;. \tag{4.20}$$

These two equations, as well as the equations for the 5d metric and the scalars, follow from the action

$$\begin{aligned} S = \frac{1}{2}\int \mathrm{d}^5x\sqrt{-g}\Big[&R + 24\mathrm{e}^{-u-4v} - 4\mathrm{e}^{-6u-4v} - 8\mathrm{e}^{-10v} - 5\partial_a u\partial^a u - \frac{15}{2}\partial_a v\partial^a v \\ &- \frac{1}{4}\mathrm{e}^{-4u+4v}\mathcal{F}_{ab}\mathcal{F}^{ab} - \frac{1}{2}\mathrm{e}^{2u-2v}\mathbb{F}_{ab}\mathbb{F}^{ab} - 8\mathrm{e}^{-4u-6v}\mathbf{A}_a\mathbf{A}^a\Big] + \frac{1}{2}\int \mathcal{A}\wedge\mathbb{F}\wedge\mathbb{F}\,, \end{aligned} \tag{4.21}$$

where $u = \frac{2}{5}(U-V)$ and $v = \frac{4}{15}(4U+V)$. As we explain in appendix D.2, this is a consistent reduction.

When $u = v = 0$, the kinetic term of the gauge fields can be diagonalized, with the full action becoming

$$S_{\text{vector}} = \frac{1}{2}\int \mathrm{d}^5x\sqrt{-g}\left[-\frac{3}{4}(\mathcal{F} + \frac{2}{3}\mathbf{F})_{ab}(\mathcal{F} + \frac{2}{3}\mathbf{F})^{ab} - \frac{1}{6}\mathbf{F}_{ab}\mathbf{F}^{ab} - 8\mathbf{A}_a\mathbf{A}^a\right] + S_{\text{CS}} \tag{4.22}$$

where

$$S_{\text{CS}} \;=\; \frac{1}{2}\int \mathcal{A}\wedge(\mathcal{F}+\mathbf{F})\wedge(\mathcal{F}+\mathbf{F}) \;. \tag{4.23}$$

Therefore we have one massless mode $\mathcal{A} + \frac{2}{3}\mathbf{A}$, which appears in the ordinary gauged supergravity, and one massive mode $\mathbf{A}$ with $m^2_{\mathbf{A}} = 24$.

Background with $z = 4$. As an example, let us construct a simple solution to the equations of motion which follow from (4.21). We can set $u = v = 0$ consistently if $|\mathbf{A}|^2 = |\mathcal{F}|^2 = |\mathbb{F}|^2 = 0$, then we can choose $\mathbf{A} = -\frac{3}{2}\mathcal{A}$ to set the massless gauge field to be zero. The action then becomes (4.2) with the null condition $|A|^2 = |F|^2 = 0$ as was the case for the background with $z = 2$. Therefore one solution is obtained by taking (4.1) with $z = 4$ as metric on M and

$$\mathcal{A} \;=\; \sigma\, r^4 \mathrm{d}x^+ \;. \tag{4.24}$$

We then find that the following is a solutions of type IIB supergravity

$$\begin{aligned} \mathrm{d}s^2 &= -\sigma^2 r^8 (\mathrm{d}x^+)^2 + \frac{\mathrm{d}r^2}{r^2} + r^2\left(-\mathrm{d}x^+\mathrm{d}x^- + \mathrm{d}\vec{x}^{\,2}\right) \\ &\qquad\qquad +\mathrm{d}s^2(B_{\mathrm{KE}}) + (\eta + \sigma r^4 \mathrm{d}x^+)^2 \,, \end{aligned} \tag{4.25}$$

$$\begin{aligned} F_5 &= 4\,\mathrm{vol}(M) + 2\omega\wedge\omega\wedge\eta \\ &\qquad\qquad -\sigma \mathrm{d}x^+ \wedge \left[\omega_C\wedge\omega_C + \mathrm{d}x^1\wedge\mathrm{d}x^2\wedge\mathrm{d}(r^6\eta)\right] \,, \end{aligned} \tag{4.26}$$

where recall $\mathrm{d}\eta = 2\omega$ and we defined

$$\omega_C = \frac{1}{2}\mathrm{d}(r^2\eta)\,, \tag{4.27}$$

which is the Kähler two-form on the Calabi-Yau cone $C(Y)$, $\mathrm{d}s^2(C(Y)) = \mathrm{d}r^2 + r^2\mathrm{d}s^2(Y)$. Notice the analogy with the background (3.34). In particular, we can think of the background here as a deformation of $\mathrm{AdS}_5\times Y$, with deformation parameter σ. However, in this case, the solution cannot be related to an AdS background by a simple TsT transformation.

This is a background enjoying non-relativistic symmetry with exponent $z = 4$. Based on the analogy with the $z = 2$ case, it is plausible that black hole geometries with (4.25), (4.26) as asymptotic boundary conditions arise as solutions to the equations with non-zero scalar fields. Since these are not accessible by simple duality transformations, it would be interesting to try to find such black hole solutions, which should be dual to non-relativistic systems with $z = 4$. It would also be nice to understand the field theory duals of these solutions. Moreover, it is possible that many more solutions could be found, with completely different applications.

5. Conclusions

In this paper we have considered some conformal quantum mechanical systems that arise by performing the DLCQ of conformal field theories. We have considered several aspects of the gravitational geometries that arise via this procedure. We have emphasized that we cannot trust the gravity solutions that we get by performing the DLCQ of AdS space. The correct DLCQ description is usually more involved. Fortunately, in cases with sufficiently high light cone momentum N, or high momentum density, we can trust the geometry at least in some region. Situations were the metric can only be trusted in some region of the geometry are common. For example, in the gravity dual of D0 brane quantum mechanics the geometry can be trusted only in certain regions, see [44].[8] In particular, this allows to compute certain correlation functions, as long as the length scales at which we compute them lie in the region where the computation is dominated by the gravity result. On the other hand, if N is small we cannot trust results computed on the AdS metric with x^- identified. Of course, if the modes we consider are BPS, it is possible that we get the right answer even if we do the naive computation.

We considered thermal configurations and we concluded that we can trust the geometries near the horizon if the momentum density is large enough. In this way we computed

[8] In this case the gravity results were compared with numerical simulations in [45, 46].

the thermodynamic properties of the dual quantum mechanical theories. Thus, we conclude that for the solutions with translationally invariant horizons, in which the hydrodynamic regime makes sense, the transport properties will have the usual gravity values. In particular, it is simple to see that the bulk viscosity coefficient vanishes, as dictated by conformal invariance [7, 47], and the shear viscosity has its universal value [48, 49].

We have given one concrete example of a quantum mechanical theory that we can analyze using these gravity duals. This is the conformal quantum mechanical theory describing the DLCQ of M5 branes [8]. This is a sigma model whose target space is the moduli space of N instantons in $\mathrm{U}(k)$ and it is given in terms of the ADHM construction reviewed in section 2.2. Naively one might have expected that the quantum mechanical description would involve a system of N particles moving in four spatial dimensions. However, the quantum mechanical theory has $4Nk$ variables, but realizing the Schrödinger group in 4+1 dimensions [8]. This highlights the subtle nature of DLCQ. The thermal properties of this theory were computed in section 3.4. For large enough k and N we can trust the gravity computations. The black hole geometries were obtained by taking a simple limit of the Kerr-AdS black holes [28, 29]. The quantum mechanics theory does not look particularly similar to the fermions at unitarity, which was the initial motivation for [1, 2]. However, it is interesting that we get some concrete conformal quantum mechanical theories that can be studied in this fashion. Maybe this point of view might shed some light on the mysterious connection between AdS_2 and conformal quantum mechanics.

We have also observed that some of the backgrounds introduced in [1, 2] can be realized in string theory by considering the gravity duals of the dipole non-commutative deformation of conformal field theories [10]. In this case the gravity description with and without the deformation are very closely related. In fact, if we consider observables with zero external charge and momentum the results are the same as in the ordinary theory.

We have also found certain consistent Kaluza-Klein reductions of type IIB supergravity involving massive vector fields. These reductions have also allowed us to find an embedding for a solution with non relativistic conformal symmetry with a dynamical exponent $z = 4$, which is different from the Schrödringer value, $z = 2$. It would be nice to find the field theory interpretation of these solutions. These Kaluza-Klein reductions are probably useful in their own right, independently of any applications to the backgrounds in [1, 2].

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A. Non-relativistic conformal symmetries

Let us summarise here the non-relativistic conformal groups and their embeddings to the relativistic counterpart [1, 2, 8]. Recall the Galilean algebra in d spatial dimensions has the Hamiltonian H, momenta P_i, rotations M_{ij}, Galilean boosts K_i and the mass operator M which commutes with everything. Less obvious commutators are

$$[P_i, K_j] = -i\delta_{ij}M\,, \qquad [H, K_i] = -iP_i\,. \tag{A.1}$$

This algebra can be embedded in the Poincaré algebra with $d+1$ spatial dimensions and one timelike dimension. We denote its generator by putting a tilde on the symbols, thus $\tilde{M}^{\mu\nu}$ are the rotations and $\tilde{P}^\mu$ are the translations. The Greek indices take values $0, 1, \ldots d+1$. The Galilean algebra is obtained by introducing the light-cone coordinates

$$x^\pm = x^0 \pm x^{d+1} \tag{A.2}$$

and retaining the subalgebra commuting with $\tilde{P}_-$, which is interpreted as the mass operator M. The identification is given by

$$M = -\tilde{P}_-\,, \quad H = -\tilde{P}_+\,, \quad P_i = \tilde{P}_i\,, \quad M_{ij} = \tilde{M}_{ij}\,, \quad K_i = \tilde{M}_{-i}\,. \tag{A.3}$$

This embedding is well-known in the context of discrete light-cone quantisation, as recalled in the main text.

The Galilean algebra may be extended to the non-relativistic conformal group, by including a dilatation generator D, whose non-zero commutators are

$$\begin{aligned} [D, P_i] &= -iP_i\,, & [D, H] &= -izH\,, \\ [D, K_i] &= i(z-1)K_i\,, & [D, M] &= i(z-2)M\,. \end{aligned} \tag{A.4}$$

The constant z is referred to as the "dynamical exponent" in the condensed matter literature, and reflects the freedom to scale differently time and space coordinates [2]. In the special case that $z = 2$, there exists a further extension of the group, obtained by adding a special conformal transformation with generator C. The new non-zero commutators are then given by

$$[C, P_i] = iK_i\,, \qquad [D, C] = 2iC\,, \qquad [H, C] = iD\,, \tag{A.5}$$

and the resulting group is called the Schrödinger group. Notice that this contains an $\mathrm{SL}(2, \mathbb{R})$ subgroup generated by H, D, C. This can be conveniently presented in terms of an "oscillator Hamiltonian" $H_{\rm osc} = L_0 = \frac{1}{2}(H + C)$ and the raising/lowering operators $L_\pm = \frac{1}{2}(H - C \mp iD)$ [50].

The Schrödinger algebra is obtained, just as in the case of the Galilean algebra, as the sub-algebra of generators commuting with $\tilde{P}_-$. One identifies the generators as follows:

$$D = \tilde{D} + B\,, \qquad C = -\tilde{K}_-\,. \tag{A.6}$$

where $\tilde{D}$ and $\tilde{K}^\mu$ are the dilatation and special conformal transformations of the relativistic conformal group. $B = 2\tilde{M}_{-+}$ is the generator of the boost normalised so that $x^\pm$ has eigenvalue ± 1. For $z \neq 2$, the non-relativistic dilatation is realized as the combination

$$D = \tilde{D} + (z-1)B \tag{A.7}$$

in the relativistic conformal algebra.

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B. Black holes with plane wave asymptotics

B.1 Harmonic potential metric

We start with the one-parameter five dimensional Kerr-AdS metric originally found in [28], and write this in the form presented in [29]. Namely, the metric is

$$\begin{aligned} ds^2 = & -\frac{\Delta_r}{r^2 + a^2\sin^2\theta}\left(dt - \frac{a}{\Xi}\cos^2\theta d\phi\right)^2 + \frac{r^2 + a^2\sin^2\theta}{\Delta_r}dr^2 + \\ & +\frac{r^2 + a^2\sin^2\theta}{\Delta_\theta}d\theta^2 + \frac{\Delta_\theta\cos^2\theta}{r^2 + a^2\sin^2\theta}\left(a dt - \frac{r^2 + a^2}{\Xi}d\phi\right)^2 + r^2\sin^2\theta d\psi^2 \;. \end{aligned} \tag{B.1}$$

where

$$\Delta_r = (r^2 + a^2)(1 + r^2) - 2m, \qquad \Delta_\theta = 1 - a^2\sin^2\theta, \qquad \Xi = 1 - a^2. \tag{B.2}$$

Note that our θ is their $\pi/2 - \theta$, i.e. $\sin\theta$ and $\cos\theta$ are flipped with respect to theirs. This metric has $J_\phi \neq 0$, $J_\psi = 0$. We need to have $|a| < 1$ to have space-like θ direction. The energy and the angular momenta are [29] (with $R^3_{\mathrm{AdS}}/G^5_N = 1$)

$$E = \frac{\pi m(3 - a^2)}{4(1 - a^2)^2}\,, \qquad J_\phi = \frac{\pi m a}{4(1 - a^2)}\,, \qquad J_\psi = 0\;. \tag{B.3}$$

This metric is asymptotically AdS_5, but in the coordinate system above the boundary is a rotating Einstein universe. To display the $\mathbb{R} \times S^3$ boundary, we need the coordinate change [28]

$$(1 - a^2)\hat r^2\cos^2\hat\theta = (r^2 + a^2)\cos^2\theta, \qquad \hat r^2\sin^2\hat\theta = r^2\sin^2\theta\,, \qquad \hat\phi = \phi + at\;. \tag{B.4}$$

Then the asymptotic form becomes

$$\begin{aligned} ds^2 = & -(1 + \hat r^2)dt^2 + \frac{d\hat r^2}{1 + \hat r^2 - \frac{2m}{\Delta_{\hat\theta}}} + \hat r^2\left(d\hat\theta^2 + \cos^2\hat\theta d\hat\phi^2 + \sin^2\hat\theta d\hat\psi^2\right) \\ & +\frac{2m}{\hat r^2(1 - a^2\sin^2\hat\theta)^3}(dt - a\cos^2\hat\theta d\hat\phi)^2 + \cdots\;. \end{aligned} \tag{B.5}$$

The boundary stress-energy tensor in this coordinate system is calculated in [30].

Since we want to obtain the asymptotic plane wave metric from the $\mathbb{R} \times S^3$ metric, we perform the limit (3.9). Namely, we define

$$t = x^+\,, \qquad t - \hat\phi = \frac{x^-}{2R^2}, \quad \hat\theta = \frac{\rho}{R}, \quad \hat r = Ry\;. \tag{B.6}$$

and let $R \to \infty$, keeping $x^\pm$, ρ and y fixed. We also need to scale E and J accordingly. The energy is conjugate to t translations and the angular momentum J to $\hat\phi$ translations. The coordinate change (B.6) implies that $-P_+ = E - J$ and $-P_- = \frac{J}{2R^2}$. However, in addition we need to take into account that when we compactify x^- we are effectively shrinking the $\hat\phi$ circle from radius one to radius $r^-/(2R^2)$. This rescales the energy and the angular momentum since the configuration is translation invariant along $\hat{\phi}$. The final expressions are

$$-P_+ = \frac{r^-}{2R^2}(E-J)\,, \qquad -P_- = \frac{r^-}{2R^2}\frac{J}{2R^2}. \tag{B.7}$$

We see from (B.3) that in order to get a finite limit we need to take $a \to 1$. Thus we end up doing the scalings

$$t = x^+\,, \qquad \phi = -\frac{x^-}{2R^2} + (1-a)x^+\,, \qquad 1-a = \frac{1}{2\lambda R^2}\,, \qquad R\to\infty \tag{B.8}$$

and r and θ are not scaled. Asymptotically, they are related to the scaled variables ρ and y by

$$\lambda^{-1}y^2 = (r^2+1)\cos^2\theta\,, \qquad y^2\rho^2 = r^2\sin^2\theta. \tag{B.9}$$

The limiting metric was given in (3.13). We can see that the λ dependence can be removed by rescaling x^-, but it is convenient to keep λ for some purposes. We can compute $P_\pm$ by taking the limits of (B.7). We find

$$-P_- = \pi\lambda^2 r^-\frac{m}{8}\,, \qquad -P_+ = \pi\lambda r^-\frac{m}{4}\,, \tag{B.10}$$

which lead to (3.17).

We have not found a simple analytic form of the metric above in the asymptotically plane wave coordinates, but for large y we can expand it as in (3.15). The boundary stress-energy tensor is then given by $T_{ab} = \frac{\lambda m}{(1+\lambda\rho^2)^3}S_{ab}$ where

$$\begin{aligned} &S_{++} = 1+3\lambda\rho^2+3\lambda^2\rho^4\,, && S_{+-} = \frac{\lambda(1+3\lambda\rho^2)}{2}\,, && S_{--} = \lambda^2\,,\\ &S_{\rho\rho} = \lambda(1+\lambda\rho^2\;)\,, && S_{\psi\psi} = \rho^2 S_{\rho\rho}\,, && \end{aligned} \tag{B.11}$$

which is traceless, as it should be.

In order to obtain the metric for the case of the black hole in AdS_7 we simply need to start from the general rotating black holes in seven dimension as written in [29] (see also [51]). The metrics are very similar up to the replacement $m \to m/r^2$ and $d\psi^2 \to d\Omega_3^2$. The scalings we need to do are precisely as in (B.8) and the resulting metric is (3.28). The temperature has a different expression because we replaced the constant parameter m by m/r^2 which depends on r. The expressions for the temperature and chemical potential can be found in [29] and we get (3.29) after taking the limit. Similarly, the expressions for the energy and angular momentum that are given in [29] (with $R^5_{\mathrm{AdS}}/G^7_N = 1$)

$$E = \frac{m\pi^2}{4(1-a^2)}\left(\frac{1}{(1-a^2)}+\frac{3}{2}\right)\,, \qquad J = \frac{ma\pi^2}{4(1-a^2)^2}\,, \tag{B.12}$$

and lead to the following values for P_+ and P_-

$$-P_- = \frac{\pi^2}{16}\lambda^2 m r^-\,, \qquad -P_+ = \frac{\pi^2}{4}\lambda m r^-\,. \tag{B.13}$$

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B.2 Magnetic field metric

We consider here a different limit, obtained starting with the general two-parameter Kerr-AdS metric [28]. In the limit, the two parameters become trivial, so for simplicity we discuss the case that the two parameters are equal. We start with the metric in the form presented in [52], and set $a = b$ and $q = 0$. We also set $g = 1$ without loss of generality. The metric is

$$\begin{aligned} ds^2 &= -\frac{1+r^2}{1-a^2}dt^2 + \frac{2m}{(r^2+a^2)(1-a^2)^2}\left[dt - a(\cos^2\theta d\phi + \sin^2\theta d\psi)\right]^2 \\ &\quad + (r^2+a^2)\left[\frac{dr^2}{\Delta_r} + \frac{1}{1-a^2}\left(d\theta^2 + \cos^2\theta d\phi^2 + \sin^2\theta d\psi^2\right)\right] , \end{aligned} \tag{B.14}$$

where

$$\Delta_r = \frac{(r^2+a^2)^2(1+r^2)}{r^2} - 2m . \tag{B.15}$$

Note we switched the variables ψ and ϕ in [52], which is equivalent to the replacement $\theta \to \pi/2 - \theta$. Now let us introduce the following definitions

$$\theta = \frac{\rho}{R}, \quad t = x^+, \quad \phi = x^+ - \frac{x^-}{2R^2}, \quad a = 1 - \frac{1}{2\lambda^2 R^2}, \tag{B.16}$$

and then take the limit $R \to \infty$. We then get the following result for the metric in this limit

$$\begin{aligned} ds^2 &= (1+r^2)\frac{dr^2}{\Delta_r} + \lambda^2(1+r^2)\left[-dx^+dx^- - \rho^2(dx^+)^2 + d\rho^2 + \rho^2 d\psi^2\right] - (dx^+)^2 \\ &\quad + \frac{2m\lambda^4}{(1+r^2)}\left[\frac{dx^-}{2} + \frac{1}{2\lambda^2}dx^+ + \rho^2(dx^+ - d\psi)\right]^2 , \end{aligned} \tag{B.17}$$

where

$$\Delta_r = \frac{(1+r^2)^3}{r^2} - 2m . \tag{B.18}$$

Changing the radial coordinate and the time by

$$y^2 = \lambda^2(r^2+1), \qquad \hat\psi = \psi - x^+ , \tag{B.19}$$

we arrive at the metric

$$\begin{aligned} ds^2 &= \left(1 - \frac{2m\lambda^4}{y^4} + \frac{2m\lambda^6}{y^6}\right)^{-1}\frac{dy^2}{y^2} + y^2\left[-dx^+(dx^- - 2\rho^2 d\hat\psi) + (d\rho^2 + \rho^2 d\hat\psi^2)\right] \\ &\quad - (dx^+)^2 + \frac{m\lambda^6}{2y^2}\left[dx^- + \frac{dx^+}{\lambda^2} - 2\rho^2 d\hat\psi\right]^2 , \end{aligned} \tag{B.20}$$

which is the metric (3.23) in the main text. In the large $y \to \infty$ limit, the metric approaches the pp-wave form (3.10). We find that the horizon is at $y = y_H$ where

$$1 - \frac{2m\lambda^4}{y_H^4} + \frac{2m\lambda^4}{y_H^6} = 0 . \tag{B.21}$$

which is the same as for the original Kerr-AdS black hole metric [52], before the limit.

The boundary stress-energy tensor computed in the usual way is then given by $T_{ab} = mS_{ab}$, where

$$\begin{aligned} &S_{++} = \lambda^2\,, && S_{+-} = \lambda^4/2\,, && S_{--} = 1\,, && S_{+\hat{\psi}} = -\rho^2\lambda^4\,, \\ &S_{-\hat{\psi}} = -2\rho^2\lambda^6\,, && S_{\rho\rho} = \lambda^4, && S_{\hat{\psi}\hat{\psi}} = \rho^2\lambda^4(1+4\rho^2\lambda^2)\,, && \end{aligned} \tag{B.22}$$

which is traceless. It is straightforward to repeat this procedure for the seven dimensional case.

C. Black holes with Schrödinger asymptotics

The background (3.34) should be regarded as the gravity dual of the vacuum of the non-commutative dipole conformal field theory. According to the AdS/CFT correspondence then, several physical properties may be computed holographically from gravity duals at finite temperature or with finite density of states. In this appendix we address a couple of issues. First we describe how to construct a large class of explicit solutions with asymptotic Schrödinger symmetry. Then we explain how to extract physical quantities from such kind of solutions, with non-standard (e.g. non-AdS) asymptotics. In particular, we discuss the issues arising in trying to define conserved charges (e.g. the analogue of ADM mass) and we propose how to circumvent them. The key point is a certain duality transformation, to which we now turn.

C.1 TsT transformation

It turns out that the background (3.34) may be obtained from a TsT [53] transformation of $\mathrm{AdS}_5\times Y$. This was discussed in [10] for the case of $Y = S^5$ and it extends straightforwardly to any Y.

Quite generally, the TsT transformation is a solution generating technique in the context of type II supergravity. Consider a background with two isometries along coordinates φ^1 and φ^2. The TsT transformation consists of three steps: firstly, we perform a T-duality along φ^1 and introduce the dualised direction $\tilde{\varphi}^1$. Secondly, we make the coordinate shift $\varphi^2 \to \varphi^2 + c\tilde{\varphi}^1$, and thirdly we perform another T-duality along $\tilde{\varphi}^1$. The first and the third steps are dualities and as such do not change the physics; the second step might change the periodicities of $\tilde{\varphi}^1$ and φ^2, but it is guaranteed to give a new solution at the level of supergravity equations of motion.

Let us apply this transformation to a general type IIB solution of the form

$$\begin{aligned} \mathrm{d}s^2 &= g_{ab}\mathrm{d}x^a\mathrm{d}x^b + \mathrm{d}s^2(\tilde{B}) + h^2(\mathrm{d}\varphi + \tilde{P})^2\,, \\ F_5 &= 4\left(\mathrm{vol}(M_5) + \mathrm{vol}(\tilde{B})\wedge h(\mathrm{d}\varphi+\tilde{P})\right)\,, \end{aligned} \tag{C.1}$$

where φ is a Killing direction in the five-dimensional space Y, $\mathrm{d}s^2(\tilde{B})$ is the metric of the four-dimensional space transverse to ∂_φ, h^2 is the norm of ∂_φ, and $\tilde{P}$ is a connection one-form on $\tilde{B}$. We assume all other fields are zero.

For our purposes, we T-dualise φ into $\tilde{\varphi}$, then shift a light-cone coordinate $x^- \to x^- + \sigma\tilde{\varphi}$, and T-dualise $\tilde{\varphi}$ back to φ. The resulting solution in the string frame is

$$\begin{aligned}\mathrm{d}s^2 = \Big[&g_{ab}\mathrm{d}x^a\mathrm{d}x^b - \sigma^2 \mathrm{e}^{2(\Phi-\Phi_0)}h^2 g_{a-}g_{b-}\mathrm{d}x^a\mathrm{d}x^b + \\ &+\mathrm{d}s^2(\tilde{B}) + \mathrm{e}^{2(\Phi-\Phi_0)}h^2(\mathrm{d}\varphi + \tilde{P})^2\Big]\,,\end{aligned} \tag{C.2}$$

$$\begin{aligned}F_5 &= 4\left(\mathrm{vol}(M_5) + \mathrm{vol}(\tilde{B})\wedge h(\mathrm{d}\varphi+\tilde{P})\right)\,,\\ B &= \sigma \mathrm{e}^{2(\Phi-\Phi_0)}h^2 g_{a-}\mathrm{d}x^a \wedge (\mathrm{d}\varphi + \tilde{P})\,,\\ \mathrm{e}^{-2(\Phi-\Phi_0)} &= 1+\sigma^2 h^2 g_{--}\,.\end{aligned} \tag{C.3}$$

where Φ_0 is the value of the dilaton before the duality.

If we start from the $\mathrm{AdS}_5 \times Y$ background, where AdS_5 is written in the Poincaré patch, and choose the isometry φ used to be the Reeb direction ϕ (or direction associated to the $\mathrm{U}(1)_R$ symmetry of the $\mathcal{N}=1$ dual CFT), where $h^2 = 1$, the metric after the TsT transformation is simply the solution (3.34). The term $-r^4(\mathrm{d}x^+)^2$, which is crucial for breaking the relativistic conformal group to the Schrödinger group, is generated through the term $g_{a-}g_{b-}\mathrm{d}x^a\mathrm{d}x^b$. In general, the solution obtained above when φ is the Reeb direction falls inside our ansatz (4.6) and as such gives a solution of the equations of motion derived from the 5d action. Thus, we have that any solution of the 5d equations, with vanishing scalars and gauge field, may be transformed to another solution

$$\begin{aligned}\mathrm{d}s^2(M) &= \mathrm{e}^{-\frac{2\Phi}{3}}g_{ab}\mathrm{d}x^a\mathrm{d}x^b - \mathrm{e}^{\frac{4}{3}\Phi}\sigma^2 g_{a-}g_{b-}\mathrm{d}x^a\mathrm{d}x^b\,,\\ A &= \sigma \mathrm{e}^{2\Phi}g_{a-}\mathrm{d}x^a\,,\\ \mathrm{e}^{-2\Phi} &= 1+\sigma^2 g_{--}\,,\\ U &= -\frac{1}{4}\Phi\,, \qquad V = \frac{3}{4}\Phi\,,\end{aligned} \tag{C.4}$$

as long as g_{--} and g_{a-} do not depend on the coordinate x^- (and we have set $\Phi_0 = 0$ for simplicity). In particular, this is true for AdS_5, as well as the asymptotically AdS_5 black hole metric. In general, applicability of the TsT transformation explained here and the Kaluza-Klein reduction given in section 4.1 is complementary: the TsT transformation can be used to obtain solutions with non-constant h^2 but our Kaluza-Klein reduction allows us to lift arbitrary solutions of five-dimensional equations of motion to ten-dimensional solutions.

Notice that it is straightforward to extend the procedure described above to more complicated black hole geometries, with asymptotic Schrödinger symmetry. For example, one can apply the TsT transformation to the R-charged asymptotically AdS_5 black holes constructed in [42].

C.2 Conserved charges and thermodynamic properties

We will now discuss some properties of the black hole metric (3.36) derived in subsection 3.5. This can be most simply done by first Kaluza-Klein reducing the metric along the φ and x^- directions to an eight dimensional solution. After this Kaluza-Klein reduction both the

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original black three brane metric (3.2) and the solution (3.36) are related by a symmetry of the eight dimensional gravity theory. Then, it can be checked explicitly that horizon is not affected by the TsT transformation, and in particular the Hawking temperature T, entropy S, and chemical potential μ_N, are unchanged [54]. We emphasize that it is not a symmetry of the full theory. But since we are computing quantities just in the gravity theory the results will be the same in both cases.

Let us next address the important issue of computing conserved charges in backgrounds with asymptotic Schrödinger symmetry such as (3.34). For backgrounds that are asymptotically AdS there are several methods based on the Fefferman-Graham expansion of the metric near the AdS boundary (see e.g. [55, 56, 36, 57]). In particular, there exist coordinates such that any metric which is asymptotically AdS takes the form

$$ds_5^2 = \frac{dr^2}{r^2} + r^2\gamma_{ab}dx^a dx^b \tag{C.5}$$

and γ_{ab} has an expansion of the form

$$\gamma_{ab} = \gamma_{ab}^{(0)} + \frac{1}{r^2}\gamma_{ab}^{(2)} + \frac{1}{r^4}\gamma_{ab}^{(4)} + h_{ab}^{(4)}\frac{1}{r^4}\log\frac{1}{r^2} + \cdots \tag{C.6}$$

Here $\gamma_{ab}^{(2)}$ is determined in terms of $\gamma_{ab}^{(0)}$ while $\gamma_{ab}^{(4)}$ captures the leading deformation with respect to the vacuum. The coefficient $h_{ab}^{(4)}$ is related to the Weyl anomaly [58] and all other terms are determined recursively in terms of these [36].

This expansion provides good control over the asymptotic behavior of the metric, allowing to subtract the infinities consistently. Then one can add suitable local counterterms and define the renormalised boundary energy-momentum tensor as

$$T_{ab} = \lim_{\epsilon\to 0}\frac{1}{8\pi G_N\epsilon^2}\frac{2}{\sqrt{-\gamma}}\frac{\delta}{\delta\gamma^{ab}}(S + S_{ct}) \tag{C.7}$$

where the integrals are evaluated at a finite distance from the boundary $r = 1/\epsilon$. Notice this is simply

$$T_{ab} = \gamma_{ab}^{(4)} \tag{C.8}$$

in the case $\gamma_{ab}^{(0)} = \eta_{ab}$ (then we also have $\gamma_{ab}^{(2)} = h_{ab}^{(4)} = 0$). For example, using this method, the energy-momentum for the non-extremal D3-brane metric (3.2) can be easily computed and is given by

$$T_{++} = \frac{1}{2}\lambda^{-2}r_0^4\,, \qquad T_{--} = \frac{1}{2}\lambda^2 r_0^4\,, \qquad T_{+-} = \frac{1}{4}r_0^4\,, \qquad T_{ij} = \frac{1}{4}r_0^4\delta_{ij}\,. \tag{C.9}$$

In general, the conserved charges associated to a Killing vector ξ^a are constructed as

$$Q_\xi = \int_\Sigma d^3x\sqrt{\sigma}\,\xi^b T_{ab}u^a \tag{C.10}$$

where Σ is a space-like surface with unit normal vector u^a, and is σ_{ab} the induced metric on it. These are easily extracted from the usual ADM decomposition of the boundary metric, with respect to a chosen time coordinate

$$ds^2 = -N_\Sigma^2 dt^2 + \sigma_{ab}(dx^a + N_\Sigma^a dt)(dx^b + N_\Sigma^b dt)\,. \tag{C.11}$$

Everywhere in the paper we slice at constant x^+. This is the natural choice dictated by the embedding of the Schrödinger group into the Poincaré group. We can then compute the (non-relativistic) energy, associated to the Killing vector $\partial/\partial x^+$

$$H = -P_+ = \int_\Sigma \mathrm{d}^3x\sqrt{\sigma}(u^+T_{++} + u^-T_{-+})\,, \tag{C.12}$$

and the mass (particle number), associated to the Killing vector $\partial/\partial x^-$

$$\frac{N}{r^-} = -P_- = \int_\Sigma \mathrm{d}^3x\sqrt{\sigma}(u^+T_{+-} + u^-T_{--})\,. \tag{C.13}$$

There are some obstacles to applying this program to metrics which asymptote to the Schrödinger background[9]

$$\mathrm{d}s^2 = \frac{\mathrm{d}r^2}{r^2} + r^2\left(-\mathrm{d}x^+\mathrm{d}x^- + \mathrm{d}x^i\mathrm{d}x^i\right) - \sigma^2r^4(\mathrm{d}x^+)^2\,. \tag{C.14}$$

Firstly, an analogue of the Fefferman-Graham expansion for this kind of asymptotic is not known. In particular, recall that (C.14) is not an Einstein metric. Moreover, it is not a priori clear how to define the boundary, since the leading term is one dimensional. Secondly, the full solution is intrinsically ten dimensional, involving both squashing of the internal geometry, as well as non-trivial RR and NS fields. Application of holographic renormalisation techniques to ten dimensional geometries is not very well developed (however, see [59, 60]). We expect that the five-dimensional truncations that we derived in this paper will be important in formulating a holographic renormalisation procedure, using for instance the Hamilton-Jacobi approach [38, 39]. Of course the whole discussion really makes sense only when x^- is non-compact, otherwise we cannot trust the metric near the boundary.

We leave a systematic treatment for future work and instead circumvent the problem taking advantage of the TsT transformation.[10] This transforms the original metric g_{ab} into a new metric given by (C.2). The proper Fefferman-Graham expansion of this metric should be such that it coincides with the corresponding expansion for the original metric. In other words, given the new metric $\tilde{g}_{ab}$ in (C.2) which was in the string frame, we can find the original metric g_{ab} as

$$g_{ab} = \tilde{g}_{ab} + \sigma^2h^2\frac{\tilde{g}_{-a}\tilde{g}_{-b}}{(1 - h^2\sigma^2\tilde{g}_{--})} \tag{C.15}$$

Thus, if someone gives us the metric $\tilde{g}_{ab}$, we find the metric g_{ab} and we perform the usual Fefferman-Graham expansion (C.6), reading off the stress tensor as in (C.8). The parameters σ and h can be read off from the other components of the metric and from the B field.

Then it is natural to *define* the boundary energy-momentum tensor using $\gamma^{(4)}$ via (C.8). Notice that indeed this prescription satisfies the first law of black hole thermodynamics,

[9]More generally, for any value of the exponent z in (4.1).

[10]The original version of this paragraph had mistakes, we thank Y. Oz and S. Yankielowicz for pointing them out.

which is a check that it is correct. A field theoretical argument was also given in section 3.5, which supports our interpretation here. It would be worthwhile to further justify this prescription from the gravity point of view, and to develop the holographic renormalisation with this asymptotics.

D. Details of the consistent truncations

D.1 Reduction with $m^2 = 8$

We provide the details of the Kaluza-Klein reduction discussed in section 4.1, leading to (4.10). Let us first use the metric ansatz of the form

$$\mathrm{d}s_{10}^2 \;=\; \mathrm{d}s^2(M)_{\mathrm{string}} + \mathrm{e}^{2U}\mathrm{d}s^2(B_{\mathrm{KE}}) + \mathrm{e}^{2V}(\eta + \mathcal{A})^2 \;. \tag{D.1}$$

Note that the five-dimensional part of the metric differs from what is used in (4.6) by a Weyl transformation. $\mathcal{A}$ is a connection on M which will be set to zero in this subsection. It is included here because we turn it non-zero in the reduction with $m^2 = 24$.

The Ricci tensor has the following components in the flat indices:

$$R_{ab} = R^{(5)}_{ab} - 4(\partial_a U\partial_b U + \nabla_a\partial_b U) - (\partial_a V\partial_b V + \nabla_a\partial_b V) - \frac{1}{2}\mathrm{e}^{2V}\mathcal{F}_{ac}\mathcal{F}_b{}^c \,, \tag{D.2}$$

$$R_{ij} = \delta_{ij}(6\mathrm{e}^{-2U} - 2\mathrm{e}^{-4U+2V} - 4\partial_a U\partial^a U - \partial_a U\partial^a V - \Box_5 U)\,, \tag{D.3}$$

$$R_{\phi\phi} = 4\mathrm{e}^{-4U+2V} - 4\partial_a U\partial^a V - \partial_a V\partial^a V - \Box_5 V + \frac{1}{4}\mathrm{e}^{2V}\mathcal{F}_{ab}\mathcal{F}^{ab}\,, \tag{D.4}$$

$$R_{ai} = R_{i\phi} = 0\,, \tag{D.5}$$

$$R_{a\phi} = -\frac{1}{2}\mathrm{e}^{-4U-2V}\nabla^b\mathrm{e}^{4U+3V}\mathcal{F}_{ba}\,, \tag{D.6}$$

where $R^{(5)}_{ab}$ is the Ricci tensor of $\mathrm{d}s^2_M$ and the covariant derivatives are with respect to $\mathrm{d}s^2_M$; $\mathcal{F} = \mathrm{d}\mathcal{A}$ is the curvature of $\mathcal{A}$. For $Y = S^5$ or $Y = T^{1,1}$, vanishing of R_{ai} and $R_{i\phi}$ immediately follows from the symmetry of the Kähler-Einstein base $\mathbb{CP}^2$ or $\mathbb{CP}^1 \times \mathbb{CP}^1$ respectively. For generic Sasaki-Einsteins one needs to calculate explicitly to see that they vanish.

The field strength H of the two-form B is

$$H \;=\; F\wedge\eta - 2A\wedge J\,, \tag{D.7}$$

so we have

$$H_{ABC}H^{ABC} \;=\; 3\mathrm{e}^{-2V}F_{ab}F^{ab} + 48\mathrm{e}^{-4U}A_aA^a \;. \tag{D.8}$$

Plugging these into the 10-dimensional action (4.9), we obtain the following 5d action:

$$\begin{aligned} S \;=\; \frac{1}{2}\int \mathrm{d}^5x\sqrt{-g}\mathrm{e}^{4U+V}\Big[&R^{(5)} + 24\mathrm{e}^{-2U} - 4\mathrm{e}^{-4U+2V} - 8\mathrm{e}^{-8U-2V} - \frac{1}{2}\partial_a\Phi\partial^a\Phi \\ &+ 12\partial_a U\partial^a U + 8\partial_a U\partial^a V - \frac{1}{4}\mathrm{e}^{-\Phi-2V}F_{ab}F^{ab} - 4\mathrm{e}^{-\Phi-4U}A_aA^a\Big]\;. \end{aligned} \tag{D.9}$$

By construction, any ansatz of our form (4.6) which is a solution of 10d equations of motion, gives a 5d metric $\mathrm{d}s^2(M)$, the scalars U, V and Φ and the gauge field A which solve the equations of motion of the 5d action (4.10). An interesting fact is that the converse is true, i.e. given a solution to the equations of motion for (4.10), the 10d fields constructed along our ansatz (4.6) automatically solve the 10d equations of motion. In other words, ours is a consistent reduction including massive gauge fields and scalars.

Indeed, the F_5 is self-dual and closed by construction, $F_5 \wedge H = 0$ is also automatic; the Φ equation of motion is exactly the same in 5d and in 10d; moreover it is easily verified that H satisfies $\mathrm{d} \star H = 0$ if A solves the Proca equation which follows from (4.10).

Therefore the only thing to check is the equations of motion of the metric, (4.4). Let us denote the equations as

$$0 = E_{MN} \equiv R_{MN} - (\text{right hand side of (4.4)}) \,. \tag{D.10}$$

The "off-diagonal" parts, i.e. E_{MN} with $(M, N) = (a, i)$, (a, ϕ), and (i, ϕ), are automatically zero because R_{MN}, $\partial_M \Phi \partial_N \Phi$, $H_{MAB} H_N{}^{AB}$ and $F_{MABCD} F_N{}^{ABCD}$ are all automatically zero. Moreover, E_{ij} is proportional to δ_{ij}, $E_{ij} = E_B \delta_{ij}$. So the non-trivial equations are

$$E_{ab} = 0 \,, \quad E_B = 0 \,, \quad E_{\phi\phi} = 0 \,. \tag{D.11}$$

However, a direct calculation shows that $E_{ab} = 0$ is the equations of motion of the 5d metric $\mathrm{d}s^2_M$, $E_B = 0$ is the one for the scalar U, and $E_{\phi\phi} = 0$ is the one for the scalar V. It is as it should be, because E_B is by definition the variation of the action with respect to the size of the Kähler-Einstein base, which is controlled by U, etc. This concludes the proof that the reduction is consistent.

Let us change the 5d action to the 5d Einstein frame, which is the form presented in the main part of the paper. This can be achieved by

$$\mathrm{d}s^2(M)_{\text{string}} = \mathrm{e}^{-\frac{8}{3}U - \frac{2}{3}V} \mathrm{d}s^2(M) \,. \tag{D.12}$$

Then the action becomes

$$\begin{aligned} S = \frac{1}{2} \int \mathrm{d}^5 x \sqrt{-g} \Big[& R + 24 \mathrm{e}^{-\frac{14}{3}U - \frac{2}{3}V} - 4 \mathrm{e}^{-\frac{20}{3}U + \frac{4}{3}V} - 8 \mathrm{e}^{-\frac{32}{3}U - \frac{8}{3}V} \\ & - \frac{1}{2} \partial_a \Phi \partial^a \Phi - \frac{28}{3} \partial_a U \partial^a U - \frac{8}{3} \partial_a U \partial^a V - \frac{4}{3} \partial_a V \partial^a V \\ & - \frac{1}{4} \mathrm{e}^{-\Phi + \frac{8}{3}U - \frac{4}{3}V} F_{ab} F^{ab} - 4 \mathrm{e}^{-\Phi - 4U} A_a A^a \Big] \,. \end{aligned} \tag{D.13}$$

Finally one diagonalises the scalar kinetic term by setting $u = \frac{2}{5}(U - V)$ and $v = \frac{4}{15}(4U + V)$ to arrive at (4.10). The final result agrees with the result in [35] and in appendix C of [61] after setting Φ and A to zero, where the Kaluza-Klein reduction with the metric, U and V was performed. Their scalars q, f are related to ours by a factor of two, $f = u/2$ and $q = v/2$.

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D.2 Reduction with $m^2 = 24$

The ansatz we consider is

$$\mathrm{d}s^2 = \mathrm{d}s^2(M)_{\text{string}} + \mathrm{e}^{2U}\mathrm{d}s^2(B_{\text{KE}}) + \mathrm{e}^{2V}(\eta + \mathcal{A})^2\,, \tag{D.14}$$

$$\begin{aligned} F_5 = {} & 4\mathrm{e}^{-4U-V}\,\mathrm{vol}(M) + 4\mathrm{e}^{-4U+V}(\eta+\mathcal{A})\wedge\star_5\mathbf{A} + \mathrm{e}^{-V}\omega\wedge\star_5\mathbb{F} \\ & + 2\omega^2\wedge(\eta+\mathcal{A}) + 2\omega^2\wedge\mathbf{A} - \omega\wedge(\eta+\mathcal{A})\wedge\mathbb{F}\,, \end{aligned} \tag{D.15}$$

and all other fields vanishing. We denote $\mathcal{F} = \mathrm{d}\mathcal{A}$ and $\mathbf{F} = \mathrm{d}\mathbf{A}$. We will go to the five-dimensional Einstein frame in the last step. $F_5 = \star F_5$ holds by construction. $\mathrm{d}F_5 = 0$ imposes

$$\begin{aligned} \mathrm{d}(\mathrm{e}^{-4U+V}\star_5\mathbf{A}) &= 0\,, && (\eta \text{ component}) \\ \mathrm{d}(\mathrm{e}^{-V}\star_5\mathbb{F}) &= -8\mathrm{e}^{-4U+V}\star_5\mathbf{A} + \mathcal{F}\wedge\mathbb{F}\,, && (\omega \text{ component}) \\ \mathrm{d}\mathbb{F} &= 0\,, && (\omega\wedge\eta \text{ component}) \\ \mathbb{F} &= \mathcal{F} + \mathbf{F}\,, && (\omega\wedge\omega \text{ component})\,. \end{aligned} \tag{D.16}$$

The components of the Ricci tensor was tabulated in (D.2),...,(D.6). The Einstein equation in ten dimensions is given by $R_{MN} = Q_{MN}$ where $Q_{MN} \equiv \frac{1}{96}F_{MABCD}F_N{}^{ABCD}$, which has the following values in the flat indices:

$$\begin{aligned} Q_{ab} &= -4\mathrm{e}^{-8U-2V}\eta_{ab} \\ &\quad +4\mathrm{e}^{-8U}(2\mathbf{A}_a\mathbf{A}_b - \eta_{ab}\mathbf{A}_c\mathbf{A}^c) + \frac{1}{4}\mathrm{e}^{-4U-2V}(4\mathbb{F}_{ac}\mathbb{F}_b{}^c - \eta_{ab}\mathbb{F}_{cd}\mathbb{F}^{cd})\,, \end{aligned} \tag{D.17}$$

$$Q_{ij} = \delta_{ij}\left(4\mathrm{e}^{-8U-2V} + 4\mathrm{e}^{-8U}\mathbf{A}_a\mathbf{A}^a\right)\,, \tag{D.18}$$

$$Q_{\phi\phi} = 4\mathrm{e}^{-8U-2V} - 4\mathrm{e}^{-8U}\mathbf{A}_a\mathbf{A}^a + \frac{1}{4}\mathrm{e}^{-4U-2V}\mathbb{F}_{ab}\mathbb{F}^{ab}\,, \tag{D.19}$$

$$Q_{ai} = Q_{i\phi} = 0\,, \tag{D.20}$$

$$Q_{a\phi} = 8\mathrm{e}^{-8U-V}\mathbf{A}_a - \frac{1}{8}\mathrm{e}^{-4U-2V}\epsilon_{abcde}\mathbb{F}^{bc}\mathbb{F}^{de}\,. \tag{D.21}$$

Then $R_{a\phi} = Q_{a\phi}$ gives

$$\mathrm{d}(\mathrm{e}^{4U+3V}\star_5\mathcal{F}) = 16\mathrm{e}^{-4U+V}\star_5\mathbf{A} + \mathbb{F}\wedge\mathbb{F}\,, \tag{D.22}$$

$R_{ab} = Q_{ab}$ is

$$\begin{aligned} R^{(5)}_{ab} = {} & 4(\partial_a U\partial_b U + \nabla_a\partial_b U) + (\partial_a V\partial_b V + \nabla_a\partial_b V) - 4\mathrm{e}^{-8U-2V}\eta_{ab} \\ & + \frac{1}{2}\mathrm{e}^{2V}\mathcal{F}_{ac}\mathcal{F}_b{}^c + 4\mathrm{e}^{-8U}(2\mathbf{A}_a\mathbf{A}_b - \delta_{ab}\mathbf{A}_c\mathbf{A}^c) + \frac{1}{4}\mathrm{e}^{-4U-2V}(4\mathbb{F}_{ac}\mathbb{F}_b{}^c - \delta_{ab}\mathbb{F}_{cd}\mathbb{F}^{cd})\,, \end{aligned} \tag{D.23}$$

and finally $R_{ij} = Q_{ij}$ and $R_{\phi\phi} = Q_{\phi\phi}$ give respectively

$$\Box_5 U = 6\mathrm{e}^{-2U} - 2\mathrm{e}^{-4U+2V} - 4\partial_a U\partial^a U - \partial_a U\partial^a V - 4\mathrm{e}^{-8U-2V} - 4\mathrm{e}^{-8U}\mathbf{A}_a\mathbf{A}^a\,, \tag{D.24}$$

$$\begin{aligned} \Box_5 V = {} & 4\mathrm{e}^{-4U+2V} - 4\partial_a U\partial^a V - \partial_a V\partial^a V + \frac{1}{4}\mathrm{e}^{2V}\mathcal{F}_{ab}\mathcal{F}^{ab} - 4\mathrm{e}^{-8U-2V} \\ & + 4\mathrm{e}^{-8U}\mathbf{A}_a\mathbf{A}^a - \frac{1}{4}\mathrm{e}^{-4U-2V}\mathbb{F}_{ab}\mathbb{F}^{ab}\,. \end{aligned} \tag{D.25}$$

This set of equations comes from the 5d action

$$S = \frac{1}{2}\int \mathrm{d}^5x\sqrt{-g}\mathrm{e}^{4U+V}\Big[R^{(5)} + 24\mathrm{e}^{-2U} - 4\mathrm{e}^{-4U+2V} - 8\mathrm{e}^{-8U-2V} + 12\partial_a U\partial^a U + 8\partial_a U\partial^a V$$
$$- \frac{1}{4}\mathrm{e}^{2V}\mathcal{F}_{ab}\mathcal{F}^{ab} - \frac{1}{2}\mathrm{e}^{-4U-2V}\mathbb{F}_{ab}\mathbb{F}^{ab} - 8\mathrm{e}^{-8U}\mathbf{A}_a\mathbf{A}^a\Big] + \frac{1}{2}\int \mathcal{A}\wedge\mathbb{F}\wedge\mathbb{F}\,. \quad \text{(D.26)}$$

To check the equations of motion, the following formula is useful:

$$\frac{1}{\sqrt{-g}}\frac{\delta}{\delta g^{ab}}\int \mathrm{d}^n x\sqrt{-g}XR \;=\; X(R_{ab} - \frac{1}{2}g_{ab}R) - \nabla_a\partial_b X + g_{ab}\Box X\,, \quad \text{(D.27)}$$

where X is a scalar field. Changing to the Einstein frame in five dimensions, we obtain the action (4.21).

E. Supersymmetry analysis of the Schrödinger vacuum

Here we study the supersymmetry preserved by the background (3.34). Supersymmetric extensions of the non-relativistic conformal groups were studied in [8, 62 – 65]. Hereafter we denote the generators of the supertranslation and the special superconformal transformations of the relativistic superconformal algebra by $Q_\pm$, $S_\pm$, respectively. The subscripts $\pm$ shows the charge under the boost, so we have for example $\{Q_+, Q_+\} \propto \tilde{P}_+$. We find that the solution does not preserve any supersymmetry.[11]

Let us first recall the supersymmetry variations of the fermions in the theory. We follow the notations in [66]. In particular, we combine two Majorana-Weyl spinors $\epsilon^{1,2}$ of type IIB supergravity into a complex Weyl spinor $\epsilon = \epsilon^1 + i\epsilon^2$ and ϵ^c is its complex conjugate, $\epsilon^c = \epsilon^1 - i\epsilon^2$. Then the gravitino transformation law in the Einstein frame is

$$\delta\psi_M = D_M\epsilon - \frac{i}{96}\mathrm{e}^{-\Phi/2}\left(\Gamma_M{}^{ABC}H_{ABC} - 9\Gamma^{AB}H_{MAB}\right)\epsilon^c + \frac{i}{192}\Gamma^{ABCD}F_{MABCD}\epsilon\,, \quad \text{(E.1)}$$

and that for the dilatino is

$$\delta\lambda \;=\; -\frac{1}{24}\mathrm{e}^{-\Phi/2}\Gamma^{ABC}H_{ABC}\epsilon\,, \quad \text{(E.2)}$$

where we have set the RR-axion and the RR 2-form to zero since they do not appear in our ansätze.

Let us determine first the supersymmetry preserved by the $\mathrm{AdS}_5\times S^5$ compactification of type IIB string theory, where the x^- direction is compactified to break the relativistic conformal symmetry to the Schrödinger symmetry. The gravitino variation is

$$\delta\psi_\mu \;=\; \partial_\mu\epsilon - \frac{1}{2}\Gamma_\mu M\epsilon\,, \qquad M \;\equiv\; \left(\Gamma_4 - \frac{i}{4}F_{01234}\Gamma^{01234}\right) \quad \text{(E.3)}$$

for $\mu = 0,1,2,3$. Recalling $F_{01234} = 4$ in our convention, the operator M above vanishes in the sector where $i\Gamma^{0123} = 1$, which fixes the chirality of ϵ, where ϵ is independent of $x^{+,-,2,3}$. Let us denote this "Q sector" temporarily.

[11] We thank the authors of [14] for pointing out the error in our statement in the first version of this paper.

The commutator of two supersymmetry transformation gives

$$[\delta_\epsilon, \delta_{\epsilon'}] \propto (\epsilon\gamma^M\epsilon')\partial_M; \tag{E.4}$$

so the commutator of two ϵ's in the sector Q gives a diffeomorphism which is independent of $x^{+,-,2,3}$. They correspond to the translations $\tilde{P}^\mu$ which in turn means the sector Q generates supertranslations $Q_\pm$, and other spinors with opposite chirality $i\Gamma^{0123} = -1$ correspond to the special superconformal generators $S_\pm$.

Let us compactify the x^- direction, $x^- \sim x^- + 2\pi r^-$. We can choose boundary conditions for the fermion, but under the most naive one the preserved supersymmetries are independent of x^-. Then (E.3) requires

$$\Gamma^+(1 - i\Gamma^{0123})\epsilon = 0 \ . \tag{E.5}$$

Therefore the preserved supersymmetry generators are $Q_\pm$ and S_-, and the superalgebra of the background preserves 3/4 of the original supersymmetry.

Now we will show that the Schrödinger vacuum (3.34) does not preserve supersymmetry. We will show that a Killing spinor does not exist. Using the value of the H field, we see that setting to zero the dilatino variation we get the condition

$$0 = \delta\lambda = -\frac{\sigma}{2}\Gamma^+\not{\psi}_C\epsilon \ . \tag{E.6}$$

This imposes $\Gamma^+\epsilon = 0$ because $\not{\psi}_C$ is clearly invertible. Then, consider the equation that arises from the $\delta\psi_+$ part of the gravitino variation

$$0 = \delta\psi_+ = \frac{1}{r}\partial_+\epsilon - \frac{1}{2}\Gamma_+ M\epsilon + \frac{3i\sigma}{8}\not{\psi}_C\epsilon^c. \tag{E.7}$$

The compatibility of $\Gamma^+\epsilon = 0$ and the evolution along x^+ imposes $i\Gamma^{0123}\epsilon = \epsilon$. It implies $i\Gamma^{0123}\epsilon^c = -\epsilon^c$, which means that we cannot impose $i\Gamma^{0123}\epsilon(x^+) = \epsilon(x^+)$ for all x^+. Therefore there is no supersymmetry preserved by this background.

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