

TWG15: Teaching mathematics with resources and technology

Introduction to the papers of TWG15: Teaching mathematics with resources and technology

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Introduction

For the second successive CERME two groups addressed mathematics education research concerning technology. TWG15 focused on issues concerning teaching, teacher education and professional development, whereas TWG16 focused on students' learning with technologies and software and task design issues (see Drijvers, Faggiano, Gerianou & Weigand, Introduction to TWG16 in this volume).

TWG15 engaged in work that was stimulated by contributions in the form of 19 research papers and 6 posters that had responded to the call, which had highlighted the following themes:

- The specific knowledge, skills and attributes required for efficient/effective mathematics teaching with technologies and resources.
- The design and evaluation of initial teacher education and teacher professional development programmes that embed these knowledge, skills and attributes – to include programmes that involve teachers' working and learning in online communities.
- Theoretical and methodological approaches to describe the identification/evolution of teachers' practices (and of effective practices) in the design and use of technology and resources in mathematics education.
- Theory and practice related to the formative/summative assessment of mathematical knowledge in a technological environment.

The work of TWG15 drew upon research from 17 countries: Australia, Austria, Denmark, Faroe Islands, France, Israel, Italy, Lebanon, Germany, Greece, Norway, Palestine, Spain, Sweden, Turkey, United Kingdom and USA.

TWG15 themes

The contributions to TWG15 were grouped according to the following themes: large-scale professional development through online courses; technology-mediated assessment of students' mathematical learning; establishing quality criteria for digital mathematics tasks; understanding teacher perspectives on technology use; in-service teachers' knowledge and practice; pre-service teachers' knowledge and practice; and the advancement of theories on technology use in mathematics education.

The TWG15 was organised as follows:

1. A research paper by Kimeswenger was selected to be the focus of a single TWG session as it highlighted a new issue for the TWG, which was the development of quality criteria to support the selection of (dynamic) digital resources for teaching mathematics.
2. Two symposia, to address the themes: on-line large-scale professional development courses; and digital assessment of students' mathematical learning. These included selected papers that were presented by the main author, followed by an invited reaction by one participant.
3. The remaining papers were grouped by theme and presented as individual short presentations by the main author, followed by individual reaction by another invited participant.

In all cases, the discussion was opened to the whole group (in small groups of 6-8 participants), which provided the opportunity for explicit links to be made with the topics of the poster submissions by both the TWG leads and participants, and to encourage all participants to share their own knowledge and experience during discursive work. Brief feedback from these small groups was collected at the end of each session.

Large-scale professional development: Online courses

The papers by Hohenwarter et al., Taranto et al. and Panero et al. focused on the design and early evaluation of three large-scale professional development online courses that had been designed for participants from Austria-Romania-Turkey, Italy and French-speaking countries respectively. Central to all three courses was the objective to offer practicing teachers an opportunity to develop their uses of technology in mathematics classrooms. These courses were described as either 'open online' (OOC) or 'massive open online' (MOOC), where the word 'massive' implied that there were no geographical boundaries nor limits to teachers' registration and participation, although the language of the course was a limiting factor.

The three courses used theoretical frames in different ways. Hohenwarter et al. adopted Koehler and Mishra's Technological, Pedagogical and Content Knowledge model (TPACK, Koehler & Mishra, 2005) to inform their course design. Panero et al. and Taranto et al. sought to network the theories of meta-didactical transposition (Arzarello et al., 2014), documentational genesis (Gueudet & Trouche, 2009) and communities of practice (Wenger, 1998) to understand the collaborative work of teachers in the online context as seen through their productions.

The invited reaction given by Bretscher stimulated a discussion that raised the following issues: defining and understanding 'participation' within open online courses; specific design features of (M)OOCs for teachers of mathematics and the balance between technological and mathematical content; the appropriateness of (M)OOCs for the (large-scale) professional development of teachers; and how research methodologies might need be developed to assess the impact of (M)OOCs on teachers' classroom practices.

Assessing students' mathematical learning

The papers by Sikko et al., Chenevotot-Quentin et al. and Olsher & Yerushalmy were centered on the use of technology in classes by teachers and students, for activities and assessment. Even if referring to different school levels, they focused on the technology as a means to support teachers' assessment activities. In the first case, Sikko et al. presented the use of motion sensors in the Norwegian primary

school classroom to support pupils' construction of meanings for functions and their graphs. This work was set in the context of a large European project (Formative Assessment in Science and Mathematics Education, FaSMEd) aimed at researching the use of technology for formative assessment. In the second case, Chenevotot-Quentin and colleagues showed the use of a technological tool for the assessment of lower secondary school students' learning of numbers and equations. Their technological tool is applied in a way that is consistent with an epistemological analysis of the topics and with the theoretical perspective of the Chevallard's Anthropological Theory of Didactics (ATD, Chevallard 1985). Olsher & Yerushalmy presented a platform where students respond to geometrical tasks using a dynamic geometry environment, which are then classified within the platform according to their geometrical dynamicity. From the teacher's perspective, the three papers presented in this session engaged the participating teachers with professional considerations in diverse and deep ways: as designers of tasks for assessment, as teachers while teaching; and as observers of students. The papers highlighted the importance of teachers' and researchers' collaborative work in the design and evaluation of such resources for the classroom.

The discussion at the end of the presentation, stimulated by the invited reaction given by Yerushalmy focused on: the potential impacts of online formative assessment on teaching; the nature of online mathematics tasks and their formative/summative assessment; possible theoretical frameworks to support design and evaluation; automatization of students' responses and subsequent feedback to students/teachers; high stakes testing; and issues of design.

Quality criteria for digital mathematics tasks

The paper by Kimeswenger problematized the existence of online platforms that host many thousands of user-generated digital resources for teaching mathematics, which presents a particular challenge for (other) teachers as they seek to locate suitable resources that meet their individual requirements. The author described a project in its early stages that seeks to develop a research-informed set of criteria to support different methodologies for user-review. The research focused on the views of 'experts' with respect to the existence of quality criteria alongside their personal descriptions of the 'educational value' of digital mathematical resources, concluding eight quality dimensions. This has led to an exploration of the possible correlation between resources that are highly rated as other users have decided that they have a 'high-quality author' and those that are identified by users as containing 'high quality material'.

As anticipated, the TWG15 participants were most interested by, and animated to discuss, the issues raised by this paper, given that many had themselves been involved in the design of open educational resources or worked alongside teachers to try to support them to make thoughtful resource selections. This discussion concerned: the authors of quality criteria and the mathematical cultures/content/values on which such criteria might be based; the role of a consumer-led approach (i.e., 'likes' by teachers?) or a community-led approach; and, given the vast number of available resources, the usefulness of new algorithms that might automatically score 'quality', based on developed criteria

Technology integration: Understanding teacher perspectives

The two papers by Abboud & Rogalski and Bretscher both addressed aspects of technology integration into 'ordinary' secondary mathematics classrooms in France and England respectively.

Whilst Abboud & Rogalski analyzed videos of lessons at distance using an ‘ergonomic’ theoretical approach (Robert & Rogalski, 2005) that highlighted tensions and disturbances in the observed practice, Bretscher used classroom observation and interviews to research aspects of a teacher’s mathematical knowledge for teaching with technology. These two papers instigated a critical discussion within the working group that was revisited several times during the conference as TWG15 sought to understand, and question the use of the word ‘ordinary’ to describe teachers (and their classrooms) within research studies. For some this referred to experienced teachers who are dependent on their own ability to (re)design lesson with technology (as in Abboud & Rogalski). For others, it referred to teachers who are required to adapt their teaching to their situation and institutional constraints in a world of changing digital tools. There was a general agreement that teachers who are involved in research studies/projects/communities concerning the use of technology in mathematics were rarely ‘ordinary’. One helpful description that was offered described the set of teachers who were not yet aware of their own instrumental genesis with new technologies (or that of their students), which seemed to resonate with many of the researchers in the TWG. The TWG15 participants concluded that ‘ordinary’ was an unhelpful descriptor and this highlighted the importance that researchers describe teachers’ contexts more fully (i.e. country, teacher background, school system, school curriculum, etc.) to enable deeper and more critical insight into each other’s research settings.

The paper by Kolovou & Kynigos differed from the two previous papers by focusing specifically on the learning processes of the designers of dynamic digital resources to foreground students’ and teachers’ mathematical creativity, which is fully described in their paper. By focusing on a ‘community of interest’ (which included teachers) that had been formed around the design of a particular creative book (c-book), the authors show how the participants’ learning was stimulated by the boundary objects (Fischer, 2005) in the design process.

A focus on pre-service teachers

The papers in this theme offered different approaches to pre-service teachers’ training and the different interpretations of their required knowledge about technology.

Prodromou investigated the usefulness of a flipped classroom approach in tertiary education in Australia. The theoretical frame was that of the ‘four pillars’ that define a flipped classroom, which take account of the flexible environment; a shift in the learning culture; intentional content; and the role of educators (Flipped Learning Network, 2014). The analysis of an experiment with pre-service teachers was presented with a particular focus on the role of the lecturer in a flipped classroom. The study by Herrelko tracked the implementation of technology in a mathematics methods course for pre-service teachers in the USA. The method, based on the Apple Classroom of Tomorrow framework (ACOT, Dwyer, Ringstaff, Haymore & Sandholtz, 1994), sought to describe the necessary conditions for the development of pre-service teachers who are knowledgeable about instructional technology. Baya’a et al. focused on pre-service teachers’ TPACK (Koehler & Mishra, 2005) and provided analyses of the impact of teacher preparation courses that had been shown to develop teachers’ TPACK.

The main questions that these presentations highlighted, and were discussed by the TWG, are linked to understanding the pre-service teachers’ perspectives in the design and implementation of

mathematics with technology. Leading on from this, there is a need for deeper understanding of the required technological, mathematical, pedagogical, and epistemological knowledge that is essential for future teachers in order to prepare them to use digital tools effectively in their teaching.

A focus on in-service teachers

This theme concerned in-service teachers, their professional engagement in the various activities related with teaching: planning lessons; using technologies; working in communities; orchestrating different devices in laboratory activities for students; balancing laboratory activities and more traditional teaching. The presenters of the papers showed various aspects of the ways that teachers work with technologies, both related to their teaching practices and to the design and management of educational materials.

Tamborg shared research on the use of a platform in Denmark, Meebook, for planning mathematics lessons in accordance with the teachers' pre-determined learning objectives, teaching approaches and curriculum. The framework used for the study is the 'instrumental approach' (Gueudet & Trouche, 2009), along with the 'documentational approach' (Gueudet et al., 2012) to describe the teachers' collective processes in their use of the platform to plan their lessons. Kayali's study centered on an investigation into the uses of mathematics education software by English secondary mathematics teachers, to understand why some software is used more or less than others, in which ways and for which reasons. Again, the instrumental and documentational approaches are the adopted frames alongside the 'teaching triad' (Jaworski, 1994), for the collection and analysis of data on teachers' considerations when implementing tasks in mathematics lessons. Zender and colleagues showed a motivating way to support students' learning with technologies outside of German classrooms using MathCityMap, a geo-located application for smartphones, which is used as an instrument for a range of situated mathematical tasks. The collaborative professional development of teachers in Lebanon on the use of GeoGebra in mathematics classes was the theme introduced and discussed by Kasti et al.. They based their research on the frame of Valsiner's three zones (Valsiner, 1997) and the TPACK theory (Koehler & Mishra, 2005), using questionnaires and interviews to investigate how GeoGebra is introduced in various mathematics activities.

Advancing theories on technology use in mathematics education

The papers by Gustafsson and Grønþæk et al. both focused on advancing theories on technology use in mathematics education. Gustafsson investigated the potential of Ruthven's Structuring Features of Classroom Practice framework (2009) as a tool to analyze empirical data to conceptualize and probe teachers' rationales for technology integration in the mathematics classroom. Gustafsson's results showed that, whilst the framework captures most aspects of their rationales, it did not fully encompass teachers' justifications with respect to their students' attitudes and behaviors. Hence, he suggested the addition of a new (sixth) structuring feature that relates to teachers' craft knowledge of the use of technology to manage different types of student behaviors or attitudes.

Grønþæk et al. suggested the addition of the concepts of *out-* and *in-sourcing* to Chevallard's (1985) Anthropological Theory of the Didactics. These concepts, taken from the field of business economics, are used as metaphors within the dialectics of tool and content in the planning of teaching. Their addition offers a 'production model' to support teachers' reflection on crucial choices between instrumented and non-instrumented praxeologies. This highlights the need for teachers to be able to

identify core activities (with potential for in-sourcing) and non-core activities (candidates for out-sourcing) based on the learning goals and the possible praxeologies when planning their use of technology in mathematics lessons.

TWG15 participants' reflections

During the final TWG session, the participants were invited to reflect upon, and record on paper, their own insights and learning during the conference. A textual analysis of the 31 responses highlighted the following aspects of knowledge exchange:

- **Broader appreciation of theoretical frames and their uses:** exposure to new theories; consideration of the limitations of theories; discussions of networking theories; and reflections on personal interpretations and applications of theories.
- **Deeper understanding of international contexts:** theoretical traditions; institutional constraints (i.e. curriculum and examinations); the rejection of the concept of an 'ordinary' teacher, which seems difficult to define or to establish a common meaning across countries.
- **Widening of knowledge on emerging themes:** opportunities afforded for large-scale teacher development through MOOCs and OOCs; the need for 'quality' criteria for digital resources to support their selection/uses by teachers; and the role of technology within formative and summative assessment that relates strongly to the mathematics curriculum, its values and traditions.

Some participants reflected upon the unintended consequences of the division of the technology group for the two different foci (teachers and students), highlighting that an opportunity was missed to explore 'How to develop a good framework to capture the interplay between mathematics content, technology (tools/resources), teachers *and* students/learners?'.

Finally, a few participants commented on the collegiality of TWG15, highlighting they had also been 'inspired' as they learned about the CERME spirit: 'humility - and how to express both orally and in writing with humility'.

Conclusions

The broad range of papers and posters presented at CERME 10 highlighted the diversity of research interests in the participating countries. However, many common concerns prevail. The design and implementation of programmes and courses for future and practicing teachers was one such challenge. Debate over the exact content of such courses in order to address the knowledge and skills to integrate technology into future mathematics teaching practice was paramount, alongside the modes of delivery and the integration of teachers' classroom experimentations. This highlighted the different views and perceptions of the simplicity and complexity for teaching mathematics with digital tools and the dilemma between technology appearing to make a teacher's role easier (e/g/ by automatically marking students' productions), whilst at the same time introducing new teaching challenges (e.g. by introducing new representational forms and related interactions). The TWG discussed ways to face such challenges through the development of research-informed teachers' collaborative professional development models that integrated coaching, face-to-face and online communities, often conducted or sustained over a time period of years, rather than months, drawing on the outcomes of the recent ICMI Survey on this theme (Robutti et al., 2016).

A common challenge is the scaling of such professional development models, for which (M)OOCs might offer some solution, although substantial research is needed to evaluate the ‘best designs’ to respond to the many different cultural contexts and requirements. The networking of theories proposed by Panero et al. and Taranto et al. provided some potential theoretical and methodological tools to this effect.

The topic of the automated assessment of students’ digital work led the group to question deeply the nature of mathematical activity (and mathematics itself) that warranted such assessment. This raised a general concern over the ease with which closed mathematical questions can be posed and digitally assessed and the much greater technical challenge to design automated assessment that privileged mathematical processes such as reasoning, justification and proof. The poster by Recio and the two papers by Olsher & Yerushalmy and Chenevotot-Quentin et al. respectively contributed greatly to this debate.

TWG 15 critiqued advancement in theories concerning teachers’ uses of technology in mathematics education. In particular, the notions of ‘tensions and disturbances’ as a theoretical construct to support analyses of teachers’ practices (Abboud & Rogalski), alongside extensions to Ruthven’s ‘Structuring Features of Classroom Practices’ (Bretscher & Gustafsson, and the poster by Simsek & Bretscher).

Looking ahead to CERME11, TWG15 concluded the following questions, which might inform individual and collaborative research efforts over the next two years:

- Which theoretical frames and methodological approaches focus on aspects of the collaborative work of researchers and teachers within the context of the use of technology for teaching mathematics?
- What approaches might be fruitful to raise teachers’ awareness of the mathematical-pedagogical decisions concerning the design and use of technology for learning and its assessment?
- How do we create opportunities and approaches that support teachers to appreciate and plan for the process of students’ instrumental genesis?
- In the design of technology-focused professional learning for mathematics teachers (pre- and in-service), what is the balance between professional needs across generic technologies and mathematics-specific needs? and how can this be achieved?

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