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O P E N P R O B L E M S

Partial Truth: An Open Problem

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The notion of partial truth has recently attracted the attention of philosophers and logicians. This note briefly presents a puzzle that an account of partial truth must solve and discusses some attempted solutions.

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1. A puzzle

The notion of partial truth has recently attracted the attention of philosophers and logicians. There are many reasons to be interested in the notion of partial truth.¹ One is that partial truth raises a puzzle. On the one hand, the notion of something being false but still partially true seems to make intuitive sense: expressions like “half truths” or “there is something true in it” are commonly used to describe hypotheses which strike us as partly right and partly wrong. On the other hand, any account of partial truth faces the following puzzle (Humberstone 2003, Fine 2013). Consider these assumptions:

1. If A is true, then A&B is partially true. (**Conjuncts**)
2. If A is partially true and B is logically equivalent to A, B is partially true. (**Congruentiality**)
3. Something is not partially true. (**Non-vacuity**)

If we want to make sense of the notion of partial truth, A1-A3 should strike us as highly plausible. A conjunction with a true conjunct is taken by many as the paradigmatic example of a partially true sentence: such a conjunction “has some truth in it” and should therefore count as partially true, as maintained by **Conjuncts**.²

It seems natural to assume that whether a sentence is partially true depends on the proposition it expresses and not on its syntactical form. If we adopt the intensional conception of propositions, the proposition expressed by A is just the set of possible worlds at which A is true. If A and B are logically equivalent, they are true at the same possible worlds, hence they express the

¹ On motivations to take partial truth seriously, see Yablo (2014, Ch. 1 and Ch. 5). See Humberstone (2003, fn. 1) for a list of some philosophers who believe that the notion of partial truth makes sense and of some who believe that it does not. On the notion of partial truth and partial content see (among many others) Yablo (2014), Humberstone (2003), Fine (MS) and Gemes (2007).

² It simplifies things if we let sentences that are true *simpliciter* count as partially true (see Yablo (2014) and Fine (MS)). Note that according to **Conjuncts** every contradiction is partially true (see Fine MS for discussion).

same proposition, hence we should have that either both are partially true or none of them is. This reasoning supports **Congruentiality**.³

Finally, not only some claims (like that $2+2=5$) strike us as completely false; the notion of partial truth would lose any interest if everything turned out to be partially true. This is the rationale for **Non-vacuity**.

The problem: A1-A3 are inconsistent (Fine 2013).

Proof: Choose A to be a completely false claim (in virtue of 3 there must be one) and B to be a true claim (let us assume there is one). $A \vee B$ is true, given that B is true. So, in virtue of 1, $A \& (A \vee B)$ is partly true; but A is logically equivalent to $A \& (A \vee B)$, so, in virtue of 2, A is partly true. But A is not partly true, in virtue of 3, so A is partly true and it is not partly true: contradiction. QED.

2. A Simple Solution

I will describe a simple solution to the puzzle presented in section 1. The solution is based on a model by Stephen Yablo (2012, 2014).⁴ After reviewing some virtues of that solution, I will point out some limitations of the model, which Yablo himself acknowledges: the model I am going to present should be taken as a toy model, a useful starting point (Yablo 2014, Ch.4).

Here is the model (Yablo 2012). Start with a propositional language L. A partial valuation of L is a function that assigns a truth value to some, but not necessarily all, the atomic letters of L. A partial valuation v verifies a sentence S of L if and only if every classical valuation that includes v (every *extension* of v) assigns the value true to S. A minimal model of S is a partial valuation v such that none of the proper subsets of v verifies S (a minimal countermodel of S is a minimal model of the negation of S).

For every sentence S, call the minimal models of S the *truthmakers* of S and the minimal countermodels of S the *falsemakers* of S. A *literal* is an atom

³ We are going to see that those who reject **Congruentiality** also reject the intensional conception of propositions.

⁴ To the best of my knowledge, Yablo does not discuss explicitly the puzzle from section 1. But he says something that suggests he would use minimal truthmakers to solve it (2014, p. 69).

or the negation of an atom. It is useful sometimes to think of finite partial valuations and minimal models in particular as conjunctions of *literals*: think of $\{(p, \text{true}), (q, \text{false})\}$, the truthmaker of $\sim(p \rightarrow q)$, as $p \& \sim q$. This provides one way to understand what it means for a minimal model to entail another: it means that the usual relation of logical consequence holds between the corresponding conjunctions of literals. The only truthmaker of $\sim(p \rightarrow q)$ entails the only truthmaker of p because $p \& \sim q \models p$.

Possible worlds are taken here to be classical valuations of L. Call the set of classical valuations that assign the value true to a sentence S the proposition expressed by S. Let A and B be sentences of L or, alternatively, the propositions expressed by such sentences.⁵

(P) We say that B is a part of A if and only if:

- a. A entails B
- b. Every truthmaker of B is entailed by a truthmaker of A
- c. Every falsemaker of B is entailed by a falsemaker of A.⁶

Here is one way to connect definition **(P)** with Yablo's work on the notion of subject matter. A subject matter or topic might be identified with a question. To take David Lewis' example, the topic *the number of stars* might be identified with the question *how many stars are there?* A question can be identified with the set of its possible answers and answers can be taken to be propositions, so that the question *how many stars are there?* might be conceived as the set of propositions {that there are no stars, that there is one star, that there are two stars, ...}. Question Q' contains question Q just in case every answer to question Q is entailed by an answer to Q': this is why the question *what did you do during the weekend* includes the question *what did you do on Sunday?*

Yablo (2014, p. 43) takes the set of A's truthmakers to be the *subject pro-matter* of A and the set of its falsemakers to be the *subject anti-matter* of A. The *subject pro-matter* of A is the set of reasons why A is true, i.e. the answers

⁵ Yablo chooses to "use the word "hypothesis" ambiguously for a sentence or its propositional content" (2014, p. 11).

⁶ Requirement c can be simplified to: every countermodel of B is a countermodel of A (see Yablo 2012).

to the question *how/why is A true?* and the *subject anti-matter* of A is the question *why/how is A false?*

Looking back at **(P)**, we can now see that B is part of A if and only if the inference from A to B is truth-value preserving (requirement a) and subject-matter preserving (requirements b and c).

We adopt the following definition of partial truth (Yablo 2014, p.11):

(PT) A is partially true \equiv_{df} for some A^* : A^* is a non-trivial part of A and A^* is true.⁷

PT seems to capture a very natural intuition, i.e. that what we say is partially true just when part of what we say is true.

Let me briefly mention an important feature of **PT**. The parthood relation defined by **PT** is a partial order, as can be verified (see Yablo 2012). This is not by chance. Yablo takes pretty seriously the use of the term “part” in PT. A^* should be a part of A in the same sense in which Tuscany is part of Italy, my arm is part of my body and Sunday is part of the week-end (Yablo 2014, Ch. 3 and Yablo 2016).

Note also that our definition of part **P** distinguishes between parts and consequences of a given hypothesis. Every A entails $A \vee B$. But $A \vee B$ is not in general part of A. For instance, $p \vee q$ is not part of p , because one of the truthmakers of $p \vee q$, i.e. q , is not entailed by the only truthmaker of p , i.e. p .

In the simple construction just sketched, logically equivalent sentences hold in the same valuations, hence they have the same minimal models and countermodels, hence they have the same truth(false)makers.⁸ This means that logically equivalent hypotheses have the same parts, which in turn yields that given logically equivalent A and B either A and B are both partially true or none of them is. This means that **Congruentiality** is vindicated: If A is partially true and B is logically equivalent to A, B is partially true.

But **Conjunction**, is not vindicated: A is not always part of $A \& B$.

⁷ Non-trivial parts must have at least one falsemaker (Yablo 2012).

⁸ *Proof:* Suppose $A \equiv B$. Let M be a minimal truthmaker of A: $M \models A$ and $A \models B$, so $M \models B$. Suppose for *reductio* that $M' \subset M$ and $M' \models B$; given that $B \models A$, it would follow that $M' \models A$, contrary to the assumption that M is minimal.

Take A to be pvq and B to be p . $A \& B = p \& (pvq)$, but pvq is not part of $p \& (pvq)$, for the same reason why pvq is not part of p : pvq has one truthmaker, i.e. q , that is not entailed by the only truthmaker of $p \& (pvq)$, i.e. p .

3. Some issues with the simple solution

The simple model described in section 2 gives up the idea that every conjunction with a true conjunct is partially true. According to such model, A is not always part of $A \& B$, so the fact that A is true is not enough to guarantee that $A \& B$ is partly true. In other words, the simple model rejects the principle **Conjunction** from section 1.

Fine (2013) takes the rejection of **Conjunction** to be a major weakness. It should be noted that rejection of **Conjunction** is justified in the simple model by the adoption of an attractive definition of partial truth (**PT**), which is connected to an interesting theory of subject matter.

Perhaps the principle that a conjunction with a true conjunct should count as partly true was our initial intuition about partial truth (see Humberstone 2003); but the simple model replaces this intuition with a *theory* of partial truth and to abandon an intuition in order to have a theory might be a reasonable price to pay. It is also worth noting that the simple model vindicates a restricted version of **Conjunction**: when A and B are atomic sentences, A and B are parts of $A \& B$. The fact that Conjunction holds for atomic sentences but fails for more complex ones means that the parthood relation (indicated by “>”) is not preserved under uniform substitution (Fine 2013): $p \& q > q$, but $p \& (pvq) \not> pvq$.⁹ But again, this might be seen as a feature of the model rather than a bug.

Despite this, the simple model *does* have some problems, beyond the obvious one of being confined to the propositional fragment of a language.¹⁰ A first non-obvious problem is that taking truthmakers to be minimal models yields the result that logically equivalents share the same verifiers and falsifiers (see above). This means, in particular, that all tautologies share the same

⁹ On failures of substitution see also Humberstone (2003, 616-7). In the simple model, when p is an atom, the partial truth of p entails the truth of p (because the only part of p is p itself). Using P as an operator of partial truth, this means that in the simple model $P(p)$ entails p, but $P(A)$ does not entail A.

¹⁰ See Yablo (2014, ch. 4) for some ideas on how to extend the simple model to a language containing quantifiers.

subject matter, which is an unwelcome result: one might argue that *I am 21 years old* or *I am not 21 years old* should be about my age and *I am Italian* or *I am not Italian* should be about my nationality. The truthmakers for $pv\neg p$ should be $\{(p, \text{true})\}$, $\{(p, \text{false})\}$ and those for $qv\neg q$ should be $\{(q, \text{true})\}$, $\{(q, \text{false})\}$. Yablo himself (2014, 2) acknowledges that his theory of subject matter should allow distinguishing hypotheses that are true in the same possible worlds (hypotheses should be individuated hyperintensionally).¹¹ The first problem for the simple model is that it fails to be hyperintensional.

A second problem has to do with the conception of truthmakers underlying the simple model, the so-called minimalist conception (Fine 2017c, Yablo 2014): minimal models of the same hypothesis cannot include each other, in virtue of the definition of minimal models. This means that in the simple model for no hypothesis A there are truthmakers t_1 and t_2 such that t_1 properly entails t_2 . But there seems to be cases where there need to be sequences of weaker and weaker truthmakers. One reason why it is true that *There are infinitely many Fs* should be that $Fx_0, Fx_1, Fx_2, Fx_3, \dots$. Another truthmaker should be that for all $n > 7$, Fx_n , yet another that for all $n > 10$, Fx_n and so on. *There are infinitely many Fs* has no minimal truthmaker.

These two problems suggest to move away from the simple model and the minimalist conception of truthmakers and try to solve the puzzle presented in section one by giving up the **Congruentiality** assumption, as suggested towards the end of Fine (2013). Giving up **Congruentiality** means accepting that for some A and B, A and B are true in the same possible worlds and yet only one of them is partially true.¹² This means that hypotheses should not be conceived merely as collections of possible worlds. Such a departure from the intensional conception of propositions is in line with the idea, endorsed by both Fine and Yablo, that two hypotheses might be true in the same possible worlds but for different reasons, which means that they have different truthmakers and hence different subject matters. Call a *directed proposition* a pair $\langle A, s \rangle$ composed by a set of possible world (those in which the proposition is true) and a subject matter (Yablo 2014): directed propositions might be different even though they

¹¹ Yablo (2014, Ch. 3) presents two accounts of truthmakers: one is based on the minimalist conception of truthmakers presented here; the other (inspired by early work from Van Frassen) is hyperintensional.

¹² Yablo (2014, 69) speaks against this move.

are true in the same possible worlds (see also Appendix A in Humberstone 2003).

Recent work by Kit Fine (2017a, 2017b, MS) explores one particularly attractive alternative to the simple solution. Fine's account solves our initial puzzle by giving up **Congruentiality**. I won't try to survey such a proposal here¹³. Rather, let me illustrate a simple model which satisfies **Conjuncts** and rejects **Congruentiality**.¹⁴

Given a formula A of the propositional language L , call the *issues touched by A* the atoms that appear in A . Say that B is part of A if and only if B is a consequence of A that does not touch any issue that is not touched by A . On this conception, the subject matter of A is the set of issues touched by A and the parts of A are the consequences of A whose subject matter is included in that of A , similarly as before.¹⁵ The definition of partial truth remains (**PT**) from section 2. **Conjuncts** is clearly satisfied in this model, because every atom that appears in A appears also in $A \& B$, but **Congruentiality** is not, because logically equivalent hypotheses do not share the same parts: $p \& (pvq) > pvq$, but $p \not> pvq$, because pvq touches on an issue not touched by p , namely q .

The model is horribly crude and syntactical, but Fine's approach might be presented as desyntacticized version of the crude model. The key idea is to obtain the subject matter of complex sentences as a result of composing (in some sense) the subject matters of the atoms they contain.

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¹³ See Fine (2017c) for a gentle introduction. We highly recommend the presentation of Fine's account contained in Hawke (2017). For a defense of some virtues of the simple model, see Yablo (2017).

¹⁴ In the terminology of Hawke (2017), the model presented here is the simplest form of an *atom-based* conception of subject matters.

¹⁵ The notion of part adopted by Fine (MS) is different from the one just cited: in his account, B is part of A only if the *positive* subject matter of B is contained in the *positive* subject matter of A .

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