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EDITORES:
José Díez, Manuel García-Carpintero, José Martínez y Sergi Oms

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Editado por:

José A. Díez Calzada
Manuel García-Carpintero Sánchez-Miguel
José Martínez Fernández
Sergi Oms Sardans

Diseño de cubierta: Cristina Zafra Vizcaíno

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The conception of numbers as preconceived objects*

Plebani, Matteo (Basilicata University)

1. Introducing preconceived objects

Consider the relation between our idea of Napoleon and Napoleon himself. Surely there is more to Napoleon than what he is required to be by our conception of him: we can say that Napoleon *surpasses* our idea of him (Yablo 2010, pp. 6-7). Consider now a fictional character like Hamlet: if Hamlet existed, many things would be true of him, which would not be entailed by our conception of him; so even Hamlet would surpass our conception of him, if he existed (the fact that he does *not* surpass our idea of him is therefore an indication that he does not exist).

Could there be objects radically different from both Napoleon and Hamlet, objects such that they don't have any properties beyond those entailed by our conception of them? Stephen Yablo (2010, pp. 6-7) calls such objects *preconceived objects*. Calling the job description of an object the way we characterize it (the kind of requirements it must satisfy to be that kind of object), we can say that objects are preconceived if they 'either should have feature F, given their job description, or [..] don't have feature F' (Yablo, 2010, p. 7). Yablo considers pure abstract objects as a paradigmatic case of preconceived objects. And it is indeed difficult to think of an (intrinsic) feature of the empty-set which is not a consequence of its job's description.

Yablo's characterization of abstract objects as preconceived objects accounts for their strange modal profile: abstract objects seem to have the same properties in all possible worlds.[1] This makes them very different from entities like you and me, whose properties vary considerably across different possible worlds and are largely accidental (Yablo 2002).

The idea, roughly put, is that the features of an abstract object are fixed by its job description, quite independently of whether it exists or not. The number zero is the number of the Fs such that there are no Fs purely in virtue of its job description; similarly, zero has no predecessor in virtue of its position in the natural number structure – and that position is a defining feature of its identity.

This is why it strikes us as correct to say that zero has no predecessor, quite independently of debates about the existence of numbers. *If something is the number zero, it has no predecessor* is true for a reason that has nothing to do with the (non) existence of such a number: it is a consequence of the job description of the number zero.

Consider how the situation is different with concrete objects. *Italy's Prime Minister is short* is not true as a consequence of the job description of the Italian Prime Minister. And surely, in the absence of an Italian Prime Minister, such a claim would strike us as neither true nor false.

2. Do preconceived objects exist?

The idea that the same properties could be correctly ascribed to preconceived objects whether they existed or not is what makes very hard, according to Yablo's view, to tell whether they do exist or not. The preconceived view is used by Yablo to argue that there is no fact of the matter about

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[1] We are talking about intrinsic properties of abstract objects and of relational properties between them. Whether the empty set = the set of dragons in a possible world obviously depends on whether there are dragons or not in such a possible world. This does not suggest that the empty set changes between worlds populated by dragons and worlds that lack dragons. One can still say that the relational properties between the Fs and the empty set supervene on the job description of the empty set and on facts about the Fs.

the existence of abstract mathematical objects (call this view non-factualism). This, in turn, should vindicate 'the feeling of mootness and pointlessness that some existence-questions arouse in us' (Yablo 1998, p. 119): if there is no fact of the matter about a question, it is pointless to debate it.

The key idea of Yablo's argument for non-factualism is that the correctness of a mathematical claim does not hinge upon the existence of mathematical objects.

Call α the asserted content of a typical utterance of a mathematical sentence S (I will focus on basic numerical sentences). We can identify α with what remains when we subtract from the full content |S| of S the operative presupposition π that numbers exist and fulfill their job description, so that $\alpha = |S| - \pi$. The minus sign here indicates the operation of logical subtraction (see Yablo 2014, ch. 8-9), by means of which one can scale down part of the content of a claim. Without going into details, one can get a sense of what logical subtraction amounts to by considering enthymematic arguments of the form:

$$\frac{\begin{array}{c}\pi\\ ?\end{array}}{S}$$

The asserted content of S can be defined as the "best candidate" to fill the gap between π and S.

Example: take π as the presupposition that numbers exist, and S as the claim that the number of planets in the solar system is nine:

$$\frac{\begin{array}{c}\text{Numbers exist}\\ ?\end{array}}{\#\ \text{Planets} = 9}$$

Assume as a conceptual truth that, *If there are numbers*, then #F = n if and only if there are n F. Easily, the best candidate to fill the gap is "There are nine planets in the solar system".[2] (For the question of how to choose the best candidate for filling the gap, see Yablo, 2006, 2014).

This generalizes: for any applied mathematical sentence S, |S| -π is the concrete content of S, ||S||, intuitively the condition of the physical world that would suffice to make S true, on the assumption that mathematical objects existed.

One can also define a notion of asserted content for a pure mathematical sentence S, where the asserted content is obtained as |S| -π (Yablo 2002, 2014, section 5.8).

In this case, the asserted content of S can be thought as the claim that *it is in the nature of numbers that S*. For instance, the asserted part of 0<1 is that 0 *and 1 are such that 0 precedes 1.* The asserted part of *There are infinitely many prime numbers is numbers are such that they include infinitely many prime numbers.*

Natural numbers include infinitely many primes is true for a reason similar to that which makes *Dragons are mighty* true (even if there are no Dragons): it is in the nature of dragons to be mighty and it is in the nature of numbers to include infinitely many primes.

The nature of the Italian Prime Minister does not decide between *If somebody is the Italian Prime Minister, (s)he is short* and *If somebody is the Italian Prime Minister, (s)he is not short*: in the absence of an Italian Prime Minister providing information about his/her height, both conditionals are true for the same reason, i.e. because the antecedent is false. The nature of numbers, on the other hand, is enough to choose between *if there numbers exist, there are infinitely many primes*, and *if numbers exist, there are not infinitely many primes.* In a piece of Yablovian terminology, say that a math-infused sentence is *fail-safe* if the truth value of its asserted content is the same whether the presupposition that numbers exist is true or not.

Now consider Yablo's argument for the mootness of existence questions about abstract mathematical entities. Yablo's premises are (Yablo, 2010, p.6):

[2]Nominalists concede that it is a conceptual truth that *if there are numbers*, then #F's =n if and only if there are n F's (see Field, 1989, p.169).

1) *Numbers are typically presupposed, they figure in π not in α.*

2) *The presupposition is fail-safe- α evaluates the same [has the same truth-value] whether π is true or not.*

   [...]

3) *Whether numerals refer is determined by their effects on truth-value [of the asserted content].*

4) *Numbers exist if and only if numerals refer.*

From these premises one can derive that (Yablo, 2010, p. 6):

5) *Numerals' effect on truth-value is the same whether they refer or not. (by 2)*

6) *There is nothing to determine whether numerals refer. (by 3 and 5)*

7) *There is no fact of the matter about whether numerals refer. (by 6)*

8) *There is no fact of the matter about whether numbers exist. (by 7 and 4)*

Premises 1 and 2 have already been discussed. Premise 4 is pretty uncontroversial, once correctly understood: it only claims that in the actual world, where there is a numeral '9', this expression refers if and only if the number 9 exists. Premise 3 is the Neo-Fregan idea that whether numerals refer is determined (in a metaphysical, not just in an epistemological sense) by their sentence-level effects, i.e. the distribution of truth-values among mathematical sentences (so called *Frege's Context Principle*). This is a controversial principle, but, remarkably, one that many platonists subscribe to (see Wright 1983).

3. Are identity questions fail-safe?

Yablo's argument is an interesting one, but also one that raises many questions. Daly (2014) wonders which mathematics-infused sentences are fail safe (= their asserted content has the same truth value independently of whether numbers exist or not). Surely, not all mathematics-infused sentences: "the number nine exists" not only presupposes that there are numbers, but asserts it as well; its asserted content is true on the assumption that numbers exist, false otherwise. So (2) is not true if taken unrestrictedly: not all mathematics-infused sentences are fail-safe.

But then, if (2) is not taken unrestrictedly, (3) becomes less plausible: it should be the distribution of truth-values among *all* mathematical sentences that determines whether numerals refer.

Yablo explicitly says that he is willing to consider only "typical mathematical statement[s]", "the kind encountered in the marketplace, not the philosophy room" (Yablo 2010, p. 6). There are also places where he considers only the truth value of applied mathematical statements as relevant for determining whether numerals refer (Yablo 2009, p. 313, note 22).

But this choice can be challenged in various ways. One could claim that pure mathematical sentences should be taken into account as well. This may not be a fatal problem (cfr. Yablo 2002, 2005, 2014, sect. 5.8), but it signals that the restriction to applied mathematical sentences' truth-values is somewhat arbitrary.

Turning to claims discussed in the philosophy room, it might sound unfair to consider claims like "The number 9 exists" as counterexamples to Yablo's thesis that mathematical sentences are fail-safe. These sentences are not fail-safe, but it is open to the non-factualist to reply that claims like *numbers do (not) exist* do not count as *proper* mathematical sentences.

Still, one could argue that there are mathematical sentences beyond those affirming the existence of numbers which would have a different truth values in case numbers existed and in case they did not. Cross identity-statements like $2 = \{\emptyset, \{\emptyset\}\}$are our case in point.

The suggestion that an abstract object's features are exhausted by those entailed by its job description seems to conflict with the fact that the job description of some mathematical objects seems to leave their identity indeterminate. Consider for instance the famous identification problem posed by Benacerraf (1965): there seems to be no way to decide, just by looking at the job description of number 2 whether 2 = {{ø}} or 2 = {ø, {ø}} or whether it is different from both sets. This can be seen as a problem for the conception of numbers as preconceived objects. One could argue that:

1. If numbers existed, there would be a fact of the matter about whether 2 = {{ø}} or not.
2. There is no fact of the matter about whether 2 = {{ø}} if we consider only the job description of number 2.

This would mean that:

3. If numbers existed, they would have features not entailed by their job description.

Which means, by the definition of 'preconceived entity' that:

4. Numbers are not preconceived entities.

Call this the cross-identity objection. I see two ways to resist the objection.

The first rejects premise (2) from the argument above: either 2 *should* be identical to {{ø}}, given their relative job descriptions, or they are different; so their job descriptions *do* settle the issue of their (non) identity. This kind of reply is in line with Yablo's definition of preconceived objects as those objects that have no other properties beyond those essential to them (see Plebani (forthcoming)).

The advantage of this reply is that it covers also the famous *Julius Cesar Objection* (cfr. Rosen and Yablo MS): given that the job description of the number two presumably does not entail that 2 = Cesar, then the number 2 is different from Julius Cesar.

Of course, more should be said about the notion of the job description of a mathematical object. First of all, as Yablo himself notes (2010, p.7), there might be no mechanical way to extract information about a preconceived object from its job description: numbers' job description, for instance, might be provided by a second-order theory, with a non-decidable consequence relation.

Moreover, there might be aspects of an abstract object's job description that are not immediately recognized. For instance, Steinhart (2002) argues that numbers' job description actually entails that they are identical to the Von Neumann numbers.

A second reply to the cross-identity objection rejects premise (1). Even if both 2 and {{ø}} existed, the question whether 2 = {{ø}} would still be indeterminate. How could this be the case? A model is offered by Balaguer (1998a,b): suppose that the reference of "2" is indeterminate between a number of candidate-referents: {ø,{ø}}, {{ø}}, ||, etc. Then, whether 2 = {{ø}} would be indeterminate for the same reason why *Aristotle = Jacqueline Kennedy's husband* is indeterminate (see Balaguer 1998a, p.58). "Aristotle" might denote both the Stagirite and Onassis in one case and "2" can denote different sets. Of course such indeterminacy is compatible with the truth of the claim that 2 exists: the claim that 2 exists turns out true in every admissible precisification given that "2" gets a referent in each of them.[3]

This connects with Yablo's idea that sometimes presuppositions fail non-catastrophically (such a failure does not deprive the asserted content of a truth value). For instance, sometimes we speak about *the* square root of -1 even knowing that there really are two roots, i and –i (see Shapiro 2008). One hypothesis about why our claims about the square root of -1 can be still perceived as true is that their asserted content (something like *some and all the square roots of -1 are such that...*) is still true. Yablo has argued that the existential presuppositions of our mathematical claims might not fail catastrophically. Perhaps also uniqueness presuppositions can fail non-catastrophically: *the Nobel laureate for Medicine is a famous scientist* strikes us as true even in the presence of multiple Medicine Nobel laureates (Yablo 2006 hints at this possibility).

[3]See also Putnam's model argument, according to which "it is very hard to see how the question of the size of the continuum could have an 'objectively correct' answer, *even if there is a single fixed universe of mathematical objects.*" (Field, 2001, p. 319). Even if there were sets, we could interpret our mathematical vocabulary so that CH turns out true, but we could also interpret it so that CH turns out false and "there is nothing in our use of set theoretic predicates that could make such an interpretation of the set theoretic vocabulary 'bizzare' or 'unintended'" (Field, 2001, p.319). This is the reply offered in Plebani (forthcoming) to the worries raised by Turner (2010).

4. Conclusions

Preconceived objects are such that they don't have any properties beyond those entailed by our conception of them. This, according to Yablo, accounts for our difficulties in adjudicating whether they exist: the (in)correctness of a number-sentence depends entirely on numbers' job descriptions, not on their existence. How is this possible, if numbers' job description does not settle whether $2 = \{\{\emptyset\}\}$? Two answers are available: perhaps the job description does settle the identity, or perhaps the identity is indeterminate due to an indeterminacy in numerals' reference and such indeterminacy would remain even if numbers existed.

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