Hypothetico-Deductive Confirmation.

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Abstract

Hypothetico-deductive (H-D) confirmation builds on the idea that confirming evidence consists of successful predictions that deductively follow from the hypothesis under test. This article reviews scope, history and recent development of the venerable H-D account: First, we motivate the approach and clarify its relationship to Bayesian confirmation theory. Second, we explain and discuss the tacking paradoxes which exploit the fact that H-D confirmation gives no account of evidential relevance. Third, we review several recent proposals that aim at a sounder and more comprehensive formulation of H-D confirmation. Finally, we conclude that the reputation of hypothetico-deductive confirmation as outdated and hopeless is undeserved: not only can the technical problems be addressed satisfactorily, the hypothetico-deductive method is also highly relevant for scientific practice.

Keywords: hypothetico-deductive confirmation, severe tests, Bayesianism, relevant logical entailment, tacking paradoxes.

1 Introduction

Scientific hypotheses need, if they are supposed to be of any use, to be calibrated with the empirical world. The details of this process are, however, the subject of considerable discussion: How do data contribute to the assessment of a hypothesis? When do they undermine a hypothesis, and when do they confirm it? Can we formalize an intuitive, precise and accessible confirmation relation between hypothesis and data?

Answering these questions leads to important philosophical insights: first, we develop a practically useful assessment tool for the impact of data on a hypothesis; second, we improve our understanding of how science works and proceeds; third, the conclusions affect more general questions, such as the problem of induction and learning from experience.

This article introduces a particular appraoch to assessing theories in the light of empirical data, namely the *hypothetico-deductive (H-D)* account of scientific confirmation.[1] H-D confirmation takes a particular logical structure to be characteristic of evidential support: We form a hypothesis on the basis of

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Light exhibits wavelike behavior.	(Hypothesis)
A beam of light passes through two slits in an opaque plate.	(Background assumption)
The light is recorded on a screen behind the plate.	(Background assumption)
When sent through two slits, waves exhibit interference patterns.	(Background assumption)
An interference pattern is displayed on the screen.	(Observation report)

Table 1: Young's double-slit experiment, interpreted as a case of H-D confirmation.

the available evidence. If empirical predictions that are deduced from that hypothesis turn out to be successful, it is confirmed. An early description of that approach was given by William Whewell:

> Our hypotheses ought to *foretel* phenomena which have not yet been observed... the truth and accuracy of these predictions were a proof that the hypothesis was valuable and, at least to a great extent, true. (Whewell 1847, 62-63)[2]

Science often seems to proceed that way: Einstein famously came up with the General Theory of Relativity (GTR) both for general theoretical reasons and for solving longstanding observational problems, such as the anomalies in the perihelion of Mercury. Besides explaining away those anomalies, his new theory also predicted that light would be bent by massive bodies like the sun. The vindication of Einstein's forecasts by Eddington during the 1919 eclipse contributed a lot to the general acceptance of GTR.

A more recent example: Our best theories about the atmospheric system suggest that emissions of greenhouse gases such as CO_2 and Methane lead to global warming. That hypothesis has been vindicated by its successful (qualitative) predictions, such as shrinking arctic ice sheets, increasing global temperatures, its ability to backtrack temperature variations in the past, etc. The hypothetico-deductive concept of confirmation explicates the common idea of these and similar examples by stating that evidence confirms a hypothesis if we can derive it from the tested hypothesis, together with suitable background assumptions. Another intuition that supports the H-D view contends that successful prediction is epistemically superior to successful accommodation of a hypothesis; we have reasons to prefer hypotheses that have been predictively successful over those that we fit *ad hoc* to the data (Worrall 1989, Hitchcock and Sober 2004).

A nice illustration of the H-D approach is Young's classical double-slit experiment (figure 1). A beam of light is shot at an opaque plate that has two open slits in it. Behind the plate, there is a white screen where the light that passes through the slits is recorded. If light is indeed a wave, we expect that wave fronts emerge from each slit, propagate in concentric circles, interfere with each other and yield an interference pattern that is characteristic of a wave. Indeed, when both slits are open, we see such an interference pattern – a pattern of alternating light and dark bands on the screen (see figure 1). Background assumptions and the hypothesis under test work together to yield predictions that, if vindicated, confirm the wave nature of light. See table 1.

The employed scheme of reasoning can then be written as

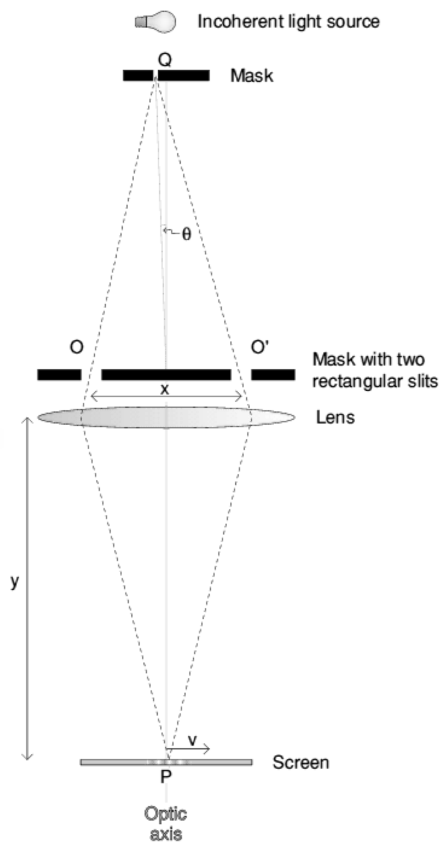

Figure 1: The setup of Young's famous doubleslit experiment.

Definition 1 *E H-D-confirms H relative to K if and only if*

- *$H.K$ is consistent,*
- *$H.K$ entails E ($H.K \models E$),*
- *K alone does not entail E.*

In this definition, the second condition reflects the basic H-D intuition (hypothesis and background knowledge entail the evidence) whereas the other conditions make additional, but evidently sensible requirements: the hypothesis under test must not contradict what we already know, and the evidence must be informative and not already be contained in the background knowledge.

The reader may have noted that the above definition is very close to a Popperian account of scientific corroboration/confirmation[3], and quite different from Bayesian accounts, which dominate the recent literature on confirmation. The next section will therefore contrast hypothetico-deductive and Bayesian confirmation and argue that the former is, although purely qualitative, not outdated in a time where statistical methods gain more and more terrain in the sciences. After that, we will review a series of objections to H-D confirmation before proceeding to solution proposals and a final evaluation.

2 Popper, Bayes and H-D Confirmation

Bayesianism is the main competitor to H-D confirmation, and generally perceived to be the most attractive account of confirmation in philosophy of science. It is a quantitative account of confirmation where degree of support is explicated as increase in degree of belief in the hypothesis under test. These degrees of belief are, on the pain of exposing the agent to a sure loss, supposed to conform to the probability calculus. This allows for a natural application of Bayesianism to statistical data analysis, as well as for transferring Bayesianism to the confirmation of non-probabilistic scientific theories. Assuming that scientists hold a certain degree of belief in the theory they would like to test. Then, the degree of support/confirmation corresponds to the credibility boost that the tested hypothesis experiences in the face of the evidence.

This is, of course, not the place for a detailed review of Bayesian inference, but a couple of things should be noted to be able to compare it to the H-D account. First of all, it is a decidedly subjective account where prior opinions on the hypotheses under test play a major role. Scientists may differ in their beliefs, and the theory does not provide grounds for rejecting apparently extreme prior opinions (and posterior conclusions) as irrational. Even strong evidence does not automatically necessitate the endorsement or rejection of a specific hypothesis. This problem is especially pressing in policy-related contexts (e.g., the global warming hypothesis) where objectivity is a major requirement and where the data are supposed to yield unambiguous policy advice.

Moreover, many scientific models are highly idealized, and the idea of having a degree of belief in the *truth* of a hypothesis might just not be applicable (Frame et al. 2007). This makes it hard to see which conclusions are justified by a Bayesian analysis, and how it bears on public policy. Overall, subjective theories of confirmation like Bayesianism fail, despite all their virtues, to explain why there is often consensus on the strength of a confirmatory argument even when the relevant degrees of belief are controversial. Rather than explicating the structure of confirmatory arguments, a Bayesian analysis measures their *effects*. In particular, evidence often seems to confirm a theory in virtue of certain objective, logical relations, whereas increase in degree of belief – the Bayesian explicatum of empirical confirmation – is only an *epiphenomenon* of that relation (Glymour 1980a). Thus, Bayesianism remains a useful tool for quantifying the strength of a support relation, but it does not deliver insights into the *structure* of evidential support. A subjective, Bayesian account of confirmation can, despite all its advantages, not be the whole story.

The above remarks motivate the search for a structural, 'objective' account of confirmation in science. The H-D account might be a suitable candidate since philosophers and scientists often see explanatory and predictive success, and severe testing of a hypothesis, as essential to evidential support. Even in the modern statistical literature, the aforementioned problems with applying Bayesian reasoning have prompted the question of whether a statistically refined hypothetico-deductive account might not be more appropriate for making evidential inferences (Gelman and Shalizi 2011): instead of assigning probabilities to hypotheses, we rather subject them to a battery of statistical tests in order to determine whether they are statistically adequate. This view is very close to the H-D account and makes it an attractive blueprint for those statisticians and philosophers of science who are, like Popper (1934/71) and Mayo (1996), skeptical about Bayesian updating as a model for scientific theory choice, and who believe in the importance of severe tests for grounding empirical support. Moreover, that fact distinguishes the H-D account among qualitative accounts of confirmation, e.g. vis-à-vis Hempel's (1945/65) satisfaction account.

Severe testing is, of course, also a key term in Karl Popper's philosophy of science. Indeed, the historic roots of the H-D method are close to those of the falsificationist methodology: hypotheses have to undergo severe tests, and if the predicted facts fail to obtain, we reject the hypothesis as falsified (Popper 1934/71). On the other hand, we also want to assess hypotheses which have *not* been refuted. Here, the concept of confirmation comes in. Popper denied that genuine confirmation consisted in raising the credibility of a hypothesis, as many of his colleagues in the Carnapian school did, but he affirmed that it made sense to evaluate hypotheses on an empirical basis, to talk of confirmation, or even degree of confirmation (Popper 1954, 1958), as the extent to which a hypothesis has survived severe tests. The degree of confirmation is determined by many details – number of successful predictions, variety of evidence, severity of tests, empirical content of the hypotheses –, but in any case, hypotheses have to be tested by deriving evidential consequences (Popper 1934/71, 212-213). In this sense, the hypothetico-deductive method can guide scientific research even in the realm of statistics. I will say a little bit more on this topic in the final section.

At this point, it should be clear that the H-D account is a serious alternative to the standard Bayesian approaches, and that it might be the best formal model for capturing the Popperian intuitions about scientific confirmation, progress and theory choice. The remaining parts of this paper explicate the hypothetico-deductive account in more detail and explore to what extent this Popperian account of confirmation can deal with major challenges such as the *tacking paradoxes*.

3 Objections to H-D Confirmation

The hypothetico-deductive scheme of reasoning described in definition 1 has more or less dominated the discussion of qualitative accounts of confirmation. For example, H-D confirmation can handle the notorious raven paradox much better than one of its main competitors, Hempel's satisfaction criterion. On the H-D account, the observation of a black raven ($Ba.Ra$) is not entailed by the raven hypothesis ($H = \forall x : Rx \rightarrow Bx$). But if we know that we are going to observe ravens and check their color ($K = Ra.Rb.Rc.\ldots$), then the observation that a particular raven is black ($E = Ra.Ba$) is implied by $H.K$ and thus confirms the hypothesis that all ravens are black, in line with our intuitions. Using the same scheme, we see that observing a white crow ($E' = \neg Ra.\neg Ba$) *can* confirm the raven hypothesis, if we condition on the appropriate background knowledge that the observed object is not black ($K' = \neg Ba$). On this reading, E' confirms the raven hypothesis relative to K', in line with Hempel's (1945/65) intuition that we are ruling out potential counterexamples to the raven hypothesis, namely grey or white birds that might turn out to be ravens instead of crows. Whereas Hempel's own criterion fails to reconstruct this intuition: even if a is *known* not to be a raven, the observation statement $\neg Ba.\neg Ra$ still Hempel-confirms that all ravens are black (Fitelson 2006; Fitelson and Hawthorne 2010).

However, H-D confirmation is troubled by a number of objections. We first mention an attempted *reductio ad absurdum* by Clark Glymour (1980b). Assume that evidence E and hypothesis H are contingent and consistent with each other, and that we believe evidence E to be true. Then, the (contingent) truth of E implies the (contingent) truth of the material conditional $H \rightarrow E$ for an arbitrary H, so we can add that formula – the logical consequence of a statement to believed true – to our background knowledge. But relative to $H \rightarrow E$, E can be derived from H, so E H-D-confirms H relative to $H \rightarrow E$. So an arbitrary H is H-D-confirmed by any true evidence E, leading to the famous conclusion that H-D confirmation is 'hopeless'. However, Glymour's objection misconstrues the relation between evidence and background knowledge: the latter is no *independent knowledge*, but *derived* from the evidence. Glymour doubly counts the evidence, by inferring from evidence E to the assumptions which serve as a background for evaluating whether the same E confirms the hypothesis. This kind of bootstrapping is, of course, not admissible, and we can safely reject Glymour's pessimistic conclusion.

Another classical objection concerns the vulnerability of H-D confirmation to the holistic challenge that was articulated by Duhem (1906) and Quine (1951). Since no single hypothesis can be tested in isolation, it seems that no single hypothesis ever entails an empirical prediction. Thus, it cannot be H-D confirmed. This seems to amount to a *reductio* of the hypothetico-deductive model. But actually, our distinction between hypothesis *under test* (H) and hypotheses *in use* (K) takes care of that problem: it is acknowledged that H can only be H-D confirmed relative to a set of auxiliary hypotheses K, but then, we can again test the claims in K against other parts of the theory to which it belongs, and thus pull ourselves up by our own bootstraps (Morrison 2010).[4]

The greatest challenge for H-D confirmation is certainly a family of objections anticipated by Hempel (1945/65), in his discussion of the Converse Consequence Condition: the *tacking paradoxes*. The idea is that *irrelevant conjunctions* are deliberately tacked to the hypothesis H while preserving the confirmation relation: If H is confirmed by a piece of evidence E (relative to any K), $H.X$ is confirmed by the same E for an arbitrary X that is consistent with H and K. We can easily check the three conditions for H-D confirmation: First, by assumption, $H.K.X$ is consistent. Second, if $H.K \models E$ then also $H.K.X \models E$ because logical implication is monotonous with regard to the antecedens. Third, K alone does not entail E because we already know that E H-D-confirms H relative to K. Thus, tacking an arbitrary irrelevant conjunct to a confirmed hypothesis preserves the confirmation relation. It is easy to see that this is highly unsatisfactory: Assume that the wave nature of light is confirmed by Young's double slit experiment. According to the H-D account of confirmation, this implies that the following hypothesis is confirmed: 'Light is an electromagnetic wave and Earth is a disc.' This sounds completely absurd. More generally, H-D confirmation needs an answer to why a piece of evidence does not confirm every theory that implies it.

The above problem has a counterpart on the side of the evidence. Tacking *irrelevant disjunctions* to the evidence E equally preserves the confirmation relation: If E confirms a hypothesis H, $E \vee E'$ H-D-confirms the same H for an arbitrary E' (unless K logically implies $E \vee E'$). The core of the problem is that if $H.K \models E$, then also $H.K \models E \vee E'$, preserving the deductive relationship between theory and evidence. Again, this tacking problem has unacceptable consequences (Gemes 1993, 1998; Moretti 2006). The hypothesis 'Light is an electromagnetic wave' is H-D-confirmed by the observations in the double-slit experiment (the interference pattern on the screen). Hence, it is also confirmed by Young's experimental observations *or* the observation that the Eiffel Tower is in Paris is 324 meter high. This is as absurd as the tacking of arbitrary conjunctions. Both objections exploit the fact that classical H-D confirmation gives no account of *evidential relevance*. That would, if unanswered, be lethal for H-D confirmation. The debate around these paradoxes has been going on for decades, with most contributions being published in the journal *Erkenntnis*. The next section presents solution proposals for this refractory paradox.

4 Solution Proposals for the Tacking Proposals

Most solution proposals try to rule out the tacking paradoxes by an account of relevant entailment in first-order predicate logic.[5] This simple language is suitable for developing solution ideas that can subsequently be transferred to more complicated languages. Early attempts have been made by Horwich (1982) and Grimes (1990); however, they fail to provide a general solution, as pointed out by Gemes (1993, 1998). These failures have even prompted the reaction that taking care of the tacking paradoxes is an unattainable goal (Moretti 2006). But that opinion is way too pessimistic. The first feasible proposal has been made by Gerhard Schurz (1991, 1994). His criterion is based on the *replaceability* of a well-formed formula in the consequens of a logical implication. Consider, for example, the logical implication $Fa \models Fa.Fb$. Obviously, we can replace Fb by Gb, $\neg Hb$ or any other formula without invalidating the logical implication. Thus, Fb is irrelevant for the inference in question whereas we cannot replace Fa in the consequens *salva validitate* by any other formula. This observation gives rise to a theory of *irrelevant conclusions* (Schurz 1991, 409):

Definition 2 *Assume $\Gamma \models \phi$. ϕ is a relevant conclusion of Γ if and only if no predicate in ϕ is replaceable on some of its occurrences by any other predicate of the same arity, salva validitate of $\Gamma \models \phi$. Otherwise, ϕ is an irrelevant conclusion of Γ.*

An analogous, yet more complicated definition can be made for irrelevant premises (Schurz 1991). Having achieved that, it is natural to require that in H-D confirmation, the crucial entailment $H.K \models E$ in the definition of H-D confirmation have neither irrelevant premises nor irrelevant conclusions:

Definition 3 *E H-D-confirms H relative to K according to Schurz if and only if*

- *the three conditions of the original definition of H-D definition are satisfied,*
- *the entailment $H.K \models E$ has neither irrelevant premises nor an irrelevant conclusion.*

In this definition, the tacking paradoxes vanish due to the replaceability salva veritate criterion, as the readers are invited to check themselves. So Schurz' account seems to deal well with the standard objections to hypothetico-deductive confirmation. A problem of that account is, though, the lack of invariance of this account of confirmation under logical equivalence. For instance, in $\forall x : Fx \models Fa$, the (relevant) conclusion Fa is logically equivalent to $Fa \vee Fa$ which is no relevant conclusion of $\forall x : Fx$. Confirmation relations should not depend on the way a theory (or an observation report) is formulated as long as the content remains the same. To cope with this problem, Schurz has proposed a number of technical modifications that I cannot review in detail, but they come at the expense of elegance and transparency. Moreover, there are a couple of minor objections (Gemes 1994b, 1998), such as the fact that $\forall x : Fx$ is not H-D-confirmed according to Schurz by Ga relative to the 'bridge law' $\forall x : (Fx \rightarrow Gx)$ due to premise irrelevancy. While Schurz gives the first sustainable solution in the literature, his revised account also leaves room for improvement.

An alternative tool is the idea of *relevant models*, i.e. models of the consequens that assign truth values to the 'relevant' atomic wffs only. So relevant models of $Fa \vee Fb$ would be those models that assign 'true' to both Fa and Fb, or 'true' to one formula and 'false' to the other, and no truth values to any other wffs (see appendix A for details). On that basis, Grimes (1990) proposes to amend hypothetico-deductive confirmation by the requirement that *at least one relevant model of the consequens be consistent with all models of the antecedens.*[6] This idea meets the tacking by disjunction paradox, but it runs, unfortunately, into straightforward counterexamples: $Fa \vee \neg Fb$ would, on that account, H-D-confirm the hypothesis $H = \forall x : Fx$ (Gemes 1993, footnote 4), but it is certainly not a relevant prediction of H, nor is it part of the proper *content* of H.

This idea of the content of a hypothesis is central to Ken Gemes' proposal. He modifies Grimes' suggestion by demanding that in a relevant logical entailment, *all* relevant models of the consequens (and not only one model) can be extended to relevant models of the antecedens:

Definition 4 *For two wffs α and β, β is a content part of α ($\alpha \models_{cp} \beta$) if and only if*

- *α and β are contingent,*
- *α logically entails β,*
- *every relevant model of β has an extension which is a relevant model of α.*

In other words, β is a content part of α if α logically implies β and if we can extend β to a model of the antecedens α by assigning truth values to further wffs. Indeed, the content part relation easily discerns irrelevant conclusions. For instance, $Fa \vee Ga$ is no content part of Fa because the model that assigns 'false' to Fa and 'true' to Ga is a relevant model of $Fa \vee Ga$ but no model of Fa. Such deductions are marked as irrelevant.

From the above examples, it is clear that replacing $H.K \models E$ by $H.K \models_{cp} E$ in the definition of H-D confirmation would resolve the tacking by disjunction paradox. But what about tacking by conjunction – the problem of irrelevant premises? The content part definition merely applies to one side of the problem. Therefore Gemes also introduces the notion of a natural axiomatization of a theory T (=the antecedens) where

> only those content parts of T that play a role in the derivation of E can be confirmed by E. In doing so it provides for the type of selective confirmation without which H-D would [...] be hopeless. (Gemes 1993, 483–484)

Gemes (1993) gives a technical definition and demonstrates that this approach takes care of the tacking by conjunction problem. Using both the notion of content part and of a natural axiomatization, he then achieves the following refined definition of H-D confirmation:

Definition 5 *E H-D-confirms axiom A of theory T relative to K according to Gemes if and only if*

- *E is a content part of $A.K$ ($A.K \models_{cp} E$),*
- *there is no natural axiomatization $N(T)$ of T so that for some set $\mathcal{S} \subset N(T)$, E is a content part of ($K. \bigwedge_{S \in \mathcal{S}} S$) and A is not a content part of ($K. \bigwedge_{S \in \mathcal{S}} S$).*[7]

This account of H-D confirmation works, on a whole, fine (Park 2004; Gemes 2005; Schurz 2005). But there are a couple of drawbacks. First, it is not clear which axiomatizations should count as 'natural' and which not. For instance, if A, B and C denote first-order sentences, the sentence $(A \rightarrow B).(B.C \rightarrow A)$ cannot be 'naturally' decomposed into its two conjuncts. Second, couldn't it make sense to test a scientific hypothesis even if there is no theory of which it is a natural axiom? Third, Definition 5 is rather complicated and hard to interpret intuitively, so much the more as the definition supervenes on the complex concept of natural axioms.

Keeping in mind Carnap's requirement that explications should be as simple and intuitive as possible, it is fair to say that we should continue our search. In the next section, we will supply a refined definition of H-D confirmation that explores how an entire theory can be H-D confirmed, without referring to a particular axiomatization.

5 Theory Confirmation

Sometimes, we want to confirm interrelated *collectives* of scientific hypotheses, such as Kepler's three laws of planetary motion, or Maxwell's laws of electrodynamics. There, relevant background assumptions that we use in testing a single hypothesis can be part of the overarching theory. The question is then: Can we define an account of H-D confirmation where such theories are confirmed as a whole, without reference to a particular axiomatization? This would not only be an attractive formal result, but also a interesting reply to the holistic challenge outlined in section three.

The first thing to do is to find a way to rule out tacking by conjunction without using natural axiomatizations. We propose to combine content parts and *modus tollens* in order to discern irrelevant conjunctions. The basic idea of H-D confirmation – that E is a prediction of H – can be expressed in two ways: $H \models E$ and $\neg E \models \neg H$. The latter formulation states that failure to observe a prediction refutes a hypothesis. This refers back to the falsificationist intuitions from which H-D confirmation emerged: the evidence has to put the hypothesis to a serious test. We now qualify this modus tollens entailment by means of content parts and restrict it to the *domain* of the evidence, i.e. the individual constants about which we make observational statements:

$$\neg E.K \models_{cp} \neg H_{|dom(E)}.K. \tag{1}$$

Here, $H_{|dom(E)}$ denotes the *development* of H to the domain of E, as defined by Hempel (1943, 1945/65). For instance, the domain of $Fa.Fb$ is $\{a, b\}$ whereas the domain of $Fa.Ga$ is $\{a\}$. More precisely, the *domain of a wff* α, denoted by $dom(\alpha)$, is the set of singular terms which occur in the atomic (!) well-formed formulas (wffs) of L that are relevant for α.

Then, the problem of irrelevant conjunctions becomes a variant of the problem of irrelevant disjunctions which the content part relation is able to resolve: if H is the compound of a 'relevant' and an 'irrelevant' hypothesis, then the content part relation will not hold between $\neg E.K$ and $\neg H_{|dom(E)}.K$. For example, if $E = Fa$, $H = (\forall x : Fx).(\forall x : Gx)$ and $K = \top$, then $H_{|dom(E)}.K = \neg Fa \vee \neg Ga$ is no content part of $\neg E.K = \neg Fa$.

That intuition can be generalized to the case of theory confirmation. We stipulate that evidence E H-D confirms a theory T if (i) we can derive that evidence from the theory (relative to background knowledge), and (ii) there is a decomposition of T into content parts $H_1, \ldots, H_n$ such that each H_i is potentially affected by predictive failure of the experiment. An experiment tests an entire theory if and only if content parts of the theory that jointly entail the whole theory are simultaneously subjected to a severe test. This reasoning can be condensed into the following definition:

Definition 6 *Evidence E H-D-confirms theory T relative to background knowledge K if and only if*

- *E is a content part of $T.K$ ($T.K \models_{cp} E$)*

- *There are wffs* $H_1, \ldots, H_n$ *such that* $H_1, \ldots, H_n \models T$ *and for all* $i \leq n$, $T \models_{cp} H_i$, *and there is a wff* E_i *such that*
  - $E \models_{cp} E_i$
  - $\neg(H_i)_{|dom(E_i)}.K$ *is a content part of* $\neg E_i.K$*, or in other words,* $\neg E_i.K \models_{cp} \neg(H_i)_{|dom(E_i)}.K$.

This final definition has a number of desirable implications (see Sprenger 2010 for details). It solves all tacking paradoxes, dispenses with natural axiomatizations, and gives an account of how entire theories can be confirmed in the H-D style using only a single concept: content part entailment, a refinement of deductive entailment. *A fortiori*, we can also apply it to the confirmation of single hypotheses. Thus, the proposal is considerably simpler, more general in scope than the rivalling suggestions of Gemes and Schurz, and arguably the best available model of H-D confirmation. Whether it will survive all objections is, however, a question that only the future can answer.

6 Final Discussion

This article has motivated H-D confirmation from its roots in the falsificationist philosophy of Karl R. Popper, and sketched its way through the history of the 20th and early 21th century. Outside the circles of confirmation theorists, hypothetico-deductivism is often considered to be an easy prey of various paradoxes and objections, or at least inferior to other accounts of confirmation such as Bayesianism.

The preceding discussion has shown that these judgments are ill-founded. Hypothetico-deductivism is, first of all, not hopeless: the alleged refutations can be rebutted, not without considerable efforts and sophistication, but still in an elegant and accessible way. In that context, it is notable that the falsificationist line of reasoning also plays an important role in the resolution of the tacking paradoxes. Limitations of that approach occur, of course, when unobservable quantities come into play. To obtain a hypothetico-deductive confirmation of purely theoretical claims, we need strong bridge hypotheses that connect unobservable quantities with their observable effects. On the other hand, this problem affects all accounts of confirmation.

Second, Bayesian reasoning may be a very successful approach to quantifying the degree of confirmation that a piece of evidence confers to a hypothesis, but it is usually silent on the reasons that make scientists change their degrees of beliefs. Hypothetico-deductive confirmation is linked much closer to epistemic virtues that may be confirmation-conducive, such as the generation of successful new predictions.

Third, the main idea of H-D, namely to check whether consequences of a model or a theory are compatible with reality, is even a cornerstone of modern statistical research where repeated *testing* of a data model is a main activity (Popper 1934/71, Fisher 1956). This can both mean classical significance testing in the social sciences and complex mathematical procedures that check, by means of a series of statistical tests, whether the data are really independent, normally distributed, have constant variance, etc. If all these predictions are vindicated, the model is deemed statistically adequate; otherwise, it is rejected.

Mayo (1996) has elaborated this line of reasoning into a general inductive philosophy and stressed that statistical models are tentatively confirmed if they fit the data reasonably, and if they have survived a (group of) severe test(s), in line with the basic H-D intuition.

All this demonstrates that in spite of all its limitations, hypothetico-deductivism has a wider scope than it is often believed, and stays alive and kicking.

A The Definition of a Relevant Model

This appendix presents an abridged definition of relevant models and content parts following Gemes 2006. See Gemes 1994a for an elaborate syntactic formulation, and Gemes 1997 for an elaborate semantic version.

Definition 7 *An atomic well-formed form (wff) β is relevant to a wff α if and only if there is some model M of α such that: if M' differs from M only in the value β is assigned, M' is not a model of α.*

Intuitively, β is relevant for α if at least in one model of α the truth value of β cannot be changed without making α false. In other words, the truth value of α is not fully independent of the truth value of β. Now we can define the notion of a relevant model which assigns truth values to only the relevant atomic wffs:

Definition 8 *A relevant model of a wff α is a model of α that assigns truth values to all and only those atomic wffs that are relevant to α.*

This account of a relevant model grounds the definition of content parts, and thus, also Gemes' and Sprenger's accounts of hypothetico-deductive confirmation.

Notes

[1]This endeavor must not be mistaken for spelling out the practical consequences of empirical (dis)confirmation, e.g. whether we should give up hypotheses with little empirical support, see Hempel 1945/65.

[2]Whewell's general account of confirmation is inductivist, not deductivist (Snyder 2006); thus, the above quote is, although a lucid statement of the H-D idea, not representative of his overall approach.

[3]Popper (1934/71, 198) states that he prefers the word corroboration to avoid confusion with an inductivist understanding of 'degree of confirmation' that was very popular following Carnap's (1950) book 'Logical Foundations of Probability'. I will, for the sake of simplicity, consistently use the term 'confirmation'.

[4]A variation of that idea will be used in section 5 for improving on standard formulations of H-D confirmation.

[5]By contrast, Bayesian confirmation theorists tend to bite to bullet: the evidence confirmas one of the conjoined hypotheses, and therefore also on the entire conjunction. However, they are eager to point out that the paradox is mitigated because for reasonable measures of confirmation, the irrelevant conjunction is (much) less confirmed than the relevant conjunct (Hawthorne and Fitelson 2004).

[6]Grimes actually used the syntactic criterion of decomposing the consequens into its disjunctive normal form, not a model-theoretic account, but due to the strong similarity to Gemes' semantic account, I rephrase it in those terms.

[7]Gemes (1993, 486) actually suggests a slightly different version in order to meet Glymour's (1980b) criticism, but I have already suggested a rebuttal of this criticism so that we can adopt a less strict formulation.

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