Recent Debates over the Existence of Abstract Objects: An Overview

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Recent debates over the existence of abstract objects: an overview

This volume is dedicated to recent debates over the existence of abstract objects. Three relevant questions for such debates are: 1) what is an abstract object? 2) Why is the debate over the existence of abstract objects important? 3) How should we conduct the debate? (See Burgess and Rosen 1997, p. 12 for a list of similar questions).

In this overview, we will survey traditional answers to those questions (readers already familiar with the debate can skim or skip this introductory piece).

1. What is an abstract object?

One way to answer this question is by simply giving a list of examples of abstract objects. Prototypical examples of abstract objects include mathematical objects (numbers, functions, sets…), universals (the property of being red, the property of being round…), propositions (conceived as the contents of sentences and the objects of belief), abstract types as opposed to concrete tokens (Marx's *Das Kapital* as opposed to Lenin's copy of *Das Kapital*, see Wetzel 2009).[1] Other candidates to the title of abstract object are fictional characters like Sherlock Holmes (Van Inwagen 1977, Thomasson 1999), scientific models (Contessa 2010), moral values (Mackie 1977, see also the list of candidates for abstractness in Liggins 2010), artworks (Mag Uidhir 2013) and perhaps institutional entities like universities and marriages (Burgess and Rosen 1997, p.15). The category of abstract objects is supposed to stand in contrast to the category of concrete objects; paradigmatic examples of concrete objects are human beings, animals, houses, stars and electrons.

When one tries to define the notion of abstract objects by giving a list of examples of abstract and non-abstract objects, the lists provided are not supposed to be exhaustive: the examples are offered to suggest that an abstract object is one like those in the list and one that differs in some important respects from the examples of concrete objects offered. However, isolating the features that distinguish abstract and concrete objects is a difficult task.

The popular way to approach the abstract/concrete distinction is to focus on some features that concrete objects possess but abstract objects lack (Lewis 1986, §1.7, calls this definition of abstractness "the way of negation". See also Rosen 2017). According to the standard definition, an

[1] It is standard to assume that no abstract object is concrete. However, some authors (Williamson 2013) hold that some objects are neither abstract nor concrete.

object is abstract if and only if it has no spatial location and has no causal relations (abstract objects do not make things happen and there is no way to act upon them).

The definition has initial plausibility, given that it does not seem to make sense to say that numbers exist here but not there and now but not then; and the opinion that numbers are a-causal seems to be confirmed by the fact that we cannot perceive them in any way and mathematicians do not study abstract objects by making them interact with other physical objects which possess causal powers (See Linnebo 2018, section 2.5 for a similar suggestion).

Despite its initial plausibility, there seems to be some counterexamples to the standard definition of abstract object (see Rosen 2017). As a counterexample to the idea that all abstract objects lack a spatial location, some argue that impure sets (sets whose members, or members of their members, or members of the members of … their members are concrete objects) share the location of the concrete elements they contain: on this account, {Obama} is located where Obama is (Lewis 1986). As counterexamples to the a-causality of abstract objects one could cite the case of what could be called *abstract artifacts*. The commonsensical view about an abstract object like the game of chess is that it did not exist until it was invented (Rosen 2017). Similarly, the natural view on fictional characters like Sherlock Holmes is that they have been created by the writers of certain novels (Thomasson 1999 and Rosen 2017). In this sense fictional characters and games like chess are artifacts, objects that exist thanks to the activity of some human beings.

Moreover, is not even clear that abstract objects like books (conceived as types) do not have causal powers: *Das Kapital* arguably had an impact on 20th century history. More generally, against the supposed a-causality of abstract objects, one might hold that the *relata* of causal relations are events and that abstract objects might be involved in some events that caused other events (Rosen 2017). One might hold that Pythagoras' theorem is part of the event in which I correctly remember Pythagoras' theorem and give the correct answer to a certain math question, thereby passing a math test.

In light of the difficulty of characterizing the notion of abstract object, some authors prefer to focus on particular examples of abstract objects. Field (1980), one of the most important recent defenses of a nominalist position, makes clear that its focus is on mathematical objects. Dorr (2008), Melia 2008) and Zsabo (2003) choose to discuss mathematical objects and universals, in order not "to be drawn into a pointless debate about how to define that technical term [abstract]" (Dorr 2008, p. 34).

The choice to focus just on some particular example of abstract objects instead of discussing in full generality the question of the existence of abstract objects seems legitimate. After all, the issue whether numbers and sets exist is interesting in itself and so is the question whether universals exist.

Moreover, there is an additional reason for those who want to deny the existence of some abstract objects not to formulate their position in too general terms. The problem with simply denying that there are any abstract objects is that apparent reference to abstract objects is ubiquitous in ordinary language, which makes the enterprise of denying the existence of every kind of abstract object we apparently refer to a formidable task. Some philosophers use such an apparent reference to abstract objects to make jokes about nominalism, the philosophical view that there are no abstract objects:

> if intellectual debts are abstract entities—and what else could they be? — then the fact that I have so many of them ought to serve by itself as an altogether convincing refutation of nominalism. (Rosen, preface to Burgess and Rosen 1997).

One could even detect an apparent reference to abstract objects in the standard definition of Nominalism as "the doctrine that there are no abstract entities" (Field 1980, 1): aren't doctrines abstract objects? (Cfr. Plebani MS).

Of course, no one sympathetic to the spirit of nominalism would be impressed by the aforementioned refutations of nominalism, which are better interpreted as jokes rather than serious arguments. But the fact remains that the list of potential candidates to the title of abstract object is very large. It makes little sense for an author interested in arguing against the existence of sets and numbers to discuss the question whether a nominalist account of doctrines or intellectual debts is available. This might justify the choice to focus on particular examples of abstract objects when discussing the question whether abstract objects exist.

Yet there is one reason to try to find a general account of abstract objects. The reason why many philosophers reject abstract objects is that they believe that our knowledge of abstract objects seems to be problematic in a way in which our knowledge of concrete objects is not (Burgess and Rosen 1997, 11-12). One naturally wonders which distinctive feature of abstract objects makes our knowledge of them so problematic. The following section explores this issue.

2. Why debating over the existence of abstract objects?

One reason to discuss the existence of abstract objects is that on the one hand, as we saw, apparent reference to abstract objects is ubiquitous in natural language; on the other hand, some philosophers deny the existence of abstract objects. It is worth discussing which one of the following project is more promising: (i) try to develop an account of our ordinary and scientific discourse that does not posit abstract objects or (ii) show that the arguments presented against the assumption that there are abstract objects are not satisfactory?

In this section, I will focus on the prospects for project (ii). I will start by reviewing the reasons that philosophers have offered to believe that there are no abstract objects. In their seminal paper, Goodmand and Quine (1947) claim that the rejection of abstract objects "is based on a philosophical intuition that cannot be justified by appeal to anything more fundamental" (Goodman and Quine 1947, p. 105). It might well be true that philosophers of a certain temperament tend to have antipathy towards abstract entities.[2] However, intuitions vary among philosophers. What Goodman and Quine call a primitive intuition other philosophers might well call a prejudice against abstract objects.

If one looks for a motivation for nominalism that goes beyond a mere appeal to intuition, one often finds arguments based on epistemological considerations. The term commonly used to refer to such considerations is "epistemological challenge against platonism", where platonism is the view that there are some abstract objects (see Liggins 2010). At the root of such arguments is the idea that the a-causal character of abstract objects makes it very hard for those who believe that abstract objects exist to explain the correlation between our beliefs about those objects and truths about them. The strongest formulation of the challenge is considered the one offered by Hartry Field (1989, Introduction), which is inspired by Benacerraf (1973). Field notes that platonists admit a phenomenon that he calls "mathematical reliability": most of the mathematical beliefs held by the mathematicians are correct and that is not by accident. Arguably, as Liggins (2010, 70) notes, even most of the mathematical beliefs of many non-mathematicians are non-accidentally true. Mathematical reliability is "so striking to demand explanation" (Field 1989, 26), but according to Field "the claims the platonist makes about mathematical entities appear to rule out any reasonable strategy for explaining the systematic correlation in question" (1989, 231).

[2] It is telling that even later in his life, when he changed his mind on the issue of nominalism and came to accept abstract objects, Quine still referred to abstract objects as *entia non grata* (see Quine 1960, chapter 50).

Field acknowledges that a first step towards explaining mathematical reliability is recognizing that in many cases mathematicians believe a certain theorem because they have derived it from the standard axioms. Assuming that we have an account of the mathematicians' logical competence, i.e. their ability to recognize when something logically follows from the axioms, this reduces the problem of explaining mathematical reliability to the problem of explaining the ability of the mathematicians to believe true axioms. Field challenges the platonist to explain this aspect of the phenomenon of mathematical reliability. Field also grants that it is possible to explain the mathematicians' ability to choose consistent sets of axioms; but he contends that this does not completely solve the problem of accounting for mathematical reliability because "there is a gap between the consistency of an axiomatic theory and its truth" (1989, 233)

A causal explanation of mathematical reliability seems to be ruled out by the a-causal nature of abstract objects. Platonists who accept the standard definition of abstract objects cannot claim that abstracts objects cause our beliefs about them, nor that beliefs about mathematical objects and truths about such objects have a common cause.

A non-causal explanation of mathematical reliability cannot be ruled out a-priori, Field admits, but he expresses some skepticism about the prospects of finding such a non-causal account of reliability. According to Field, part of the difficulty for such a non-causal explanation of mathematical reliability lies in the fact that mathematical objects are usually characterized by the platonists not only as a-causal, but also as mind-independent and language-independent. Field does not present his argument as a definite refutation of platonism, but rather as a challenge for the platonist, while at the same time offering some reasons to think that it is hard for the platonist to meet the challenge (see Liggins 2006, 2010). It should be noted that the challenge does not rely on any theory of knowledge. Field does not make any assumptions about the necessary or sufficient conditions for knowledge. His challenge is simply based on the idea that "we should view with suspicion any claim to know fact about a certain domain of objects if we believe it impossible in principle to explain the reliability of our beliefs about that domain" (1989, p. 233).

It should be noted that Fields' epistemological challenge is a challenge to traditional forms of mathematical platonism: in other words, it is a challenge to the assumption that there are mind-independent and language-independent abstract objects. It is not so clear whether the challenge is equally forceful against those abstract objects that in the previous section we called *abstract artifacts*, which can be described as mind-dependent (at least in some sense).[3]

An important to point to make is that there are two ways in which philosophers have tried to meet the epistemological challenge. One response can be roughly characterized as based on the idea that the way we know about abstract mathematical objects is different from the way we know about concrete objects. Kit Fine endorses this view in the course of defending an account of our knowledge of mathematical objects called *procedural postulationism*, according to which abstract objects might be introduced in the domain of discourse by an act of postulation, an order to expand the domain by including objects of a certain kind, as long as the order is consistent. As he explains:

> Each kind of object has its own way of being known. It is a peculiarity of perceptible objects that we may get to know of them through perception; it is a peculiarity of the theoretical entities of science that their existence is to be justified by way of inference to the best explanation; and it is a peculiarity of mathematical and other abstract objects that their existence is to be justified by way of postulation. […] If the present approach has any value, it lies in its making clear the distinctive way in which we may acquire our knowledge of mathematical objects, one that is not reducible to other, more familiar methods and is in keeping with the peculiarly a priori character of mathematical thought. (Fine 2005, p. 108)

One way to spell out the difference between the epistemology of abstract mathematical objects and concrete objects is to say that in the case of mathematical theories, i.e. theories about mathematical objects, there is no gap between consistency and truth: *every consistent mathematical theory is true of some mathematical objects* (call the principle in italics the consistency-truth link). The consistency-truth link goes back at least to Hilbert and is present in some form also in Fine's account: the only constraint on the use of an act of postulation to introduce some mathematical objects is that the act of postulation be a consistent order.

The consistency-truth link is also present in the so-called *plenitudinous platonism* (Balaguer 1995, 1998), according to which every possible mathematical object exists. According to plenitudous platonism, it is sufficient for a mathematical object to exist that the assumption of its existence is part of a consistent set of axioms. The same is not true of physical objects and physical theories: consistency is not the criterion for truth and existence in physics, as Balaguer himself acknowledges.

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[3] On the possibility of generalizing Field's challenge beyond the case of abstract mathematical objects, see Liggins (2010) and Enoch (2010). See Rosen (1994) for problems with the distinction between mind-dependent and mind-independent.

Similarly, in Fine's account only abstract objects might be postulated into existence, not concrete ones.

Zalta's theory of abstract objects (see Zalta 2016), arguably the most general account of abstract objects available, makes an important distinction between abstract and concrete objects, which has repercussions about the way we can know about the two different kinds of objects. Zalta distinguishes between two modes in which a property can be ascribed to an object: the notation Fa indicates that the object a exemplifies the property F, whereas the notation aF is used to indicate that the object a encodes the property F, in the sense that a is characterized as having property F. Flesh and blood detectives exemplify the property of being a detective, whereas Sherlock Holmes encodes the property of being a detective, in the sense of being characterized as possessing such property. In Zalta's account, concrete, i.e. spatio-temporally located, objects cannot encode properties; only abstract objects encode properties. On the other hand, a powerful comprehension principle guarantees that, for any group of properties, some abstract object encodes exactly those properties. Moreover, according to Zalta's theory, abstract and concrete objects have different identity criteria: concrete object x is identical to concrete object y if and only if x and y necessarily *exemplify* the same properties, whereas abstract object x is identical to abstract object y if and only if x and y necessarily *encode* the same properties. The reliability of our beliefs about mathematical objects, on this account, is explained by the fact we can recognize the properties that abstract objects encode, because they encode precisely those properties that are used to characterize them[4]:

> All one has to do to become so acquainted *de re* with an abstract object is to understand its descriptive, defining condition, for the properties that an abstract object encodes are precisely those expressed by their defining conditions (Linsky and Zalta 1995, p. 547).

In the case of the mathematical objects postulated in the mathematical theory T, such objects encode a certain property if and only if according to T they possess such a property, so that mathematical reliability can be explained by our capacity to identify the consequence of certain mathematical theories.

We have just seen that according to one reply to the epistemological challenge the explanation of the reliability of our mathematical beliefs has to be different from the explanation of the reliability of our

[4] Compare Zalta's view that an abstract object encodes exactly those properties that are used in its characterization with Yablo's idea (Yablo 2010, Introduction) that mathematical objects are *preconceived* objects, objects that "Either … *should* have feature F, given their job description, or … *don't* have feature F" (Yablo 2010, 7).

perceptual beliefs or of our beliefs about theoretical entities of physics. A different reply to the epistemological challenge is possible, which is based on the idea that:

> we have the same broadly inductive reason for believing in numbers as we have for believing in electrons: certain theories that entail that there are numbers are better, qua explanations of our evidence, than any theories that do not. (Dorr 2010, p. 133)

> [I]f our belief in electrons and neutrinos is justified by something like inference to the best explanation, isn't our belief in numbers and functions and other mathematical entities equally justified by the same methodology? After all, the theories that we use in explaining various facts about the physical world not only involve a commitment to electrons and neutrinos, they involve a commitment to numbers and functions and the like. (Field 1989, p. 16)

The consideration that our best scientific theories are heavily mathematized is at the core of the so-called indispensability argument (see next section) for the existence of abstract mathematical objects. This suggests one way in which numbers and electrons might be assimilated: both are posited by our best scientific explanations, including explanations of empirical phenomena (Baker 2005, see also next session).[5] Some go as far as to claim that the central role played by abstract mathematical objects in our empirical theories makes it hard to "clarify the distinctive way in which ordinary material bodies are causally active" and maintain that "abstracta […] are not causally active in that way" (Burgess and Rosen [1997], p. 23). This strategy might be presented as an attempt to call into question the assumption that abstract mathematical objects are a-causal, an assumption used by Field to rule out the possibility of a causal explanation of mathematical reliability. In connection with the issue of the a-causality of abstract objects it is worth noticing that according to one prominent account of universals (Armstrong 1978) universals, traditionally classified as abstract objects, are considered causally active.

More generally, as Burgess and Rosen (1997, 31) note, the difficulty of defining the notion of abstract object might be used to motivate an account that posits objects traditionally classified as abstract (like mathematical objects or universals), but avoids the claim that such objects belong to a category of objects which are radically different from ordinary concrete objects. In connection with the idea of blurring the abstract/concrete distinction, Alan Baker has raised the question whether

[5] "In recent times, many philosophers have been attracted to an 'assimilationist' model of mathematical knowledge; they have supposed that we know of mathematical objects in something like the way we know of other objects". Fine (2005, p. 108-9)

> Is it possible to clarify the distinctive way in which ordinary material bodies can play an explanatory role, and if so can it indeed be said that abstracta which are mentioned in the context of scientific explanations are not explanatory in that way (Baker 2009, p.632)[6]

It is worth noticing the connection between these two different replies to the epistemological challenge and the issue of whether it is possible to provide a satisfactory account of the abstract/concrete distinction. As we saw, one reply to the epistemological challenge is based on the idea that different kinds of objects are associated with different ways of knowing about them. This reply seems to presuppose that abstract and concrete objects are indeed different kinds of objects and that it is possible to trace the abstract/concrete divide in a satisfactory way.

The reply to the epistemological challenge based on the role of mathematics in our empirical theories, on the other hand, sits well with skepticism about the possibility of tracing the abstract/concrete divide in a satisfactory way. The two replies seem also to yield different criteria for the acceptability of a mathematical theory. According to the reply based on the consistency-truth link, the only criterion for the acceptability of a mathematical theory is its consistency, whereas according to the reply based on scientific confirmation the applicability of a theory to the study of the concrete world plays also an important role, so much so that Quine at a certain point considered the parts of set theory which have no connection to applied mathematics mere "mathematical recreation" (Quine 1981a, p. 400) and suggested not to accept the existential implications of those parts of set theory.

We have found a connection between the issue of defining the notion of abstract object and the question of how to reply to the epistemological challenge against platonism: one reply to the epistemological challenge is based on a clear distinction between the way we know about abstract objects and the way we know about concrete ones and seems to presuppose a sharp distinction between abstract and concrete objects; while another reply argues that we have the same grounds for believing in the existence of prototypical abstract objects like numbers as we have for believing in the existence of prototypical concrete objects as electrons. This second reply is also in harmony with the idea that there is no sharp boundary between abstract and concrete objects, a view that receives some support from the difficulty of characterizing the notion of abstract object discussed in the previous section.

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[6] See also Morrison (2007, 552, quoted by Knowles and Liggins (2015), 3406) on the property of spin: according to Morrison, spin is "perhaps best viewed as a curious hybrid of the mathematical and the physical"(Morrison (2007: 552) quoted by Knowles and Liggins (2015: 3406)).

3. How should we conduct the debate?

One might be tempted to think that philosophical debates like the one over the existence of abstract mathematical objects are unsolvable, in the sense that there is no evidence capable of convincing someone to change opinion about such questions. Quine was of a different advice:

> Existence statements in this philosophical vein do admit of evidence, in the sense that we can have reasons, and essentially scientific reasons, for including numbers [in our ontology] ... Numbers and classes are favoured by the power and facility which they contribute to theoretical physics and other systematic discourse about nature. (Quine 1969a, pp. 97-8)

A good part of the recent debate over the existence of abstract mathematical objects can be presented as an attempt to figure out whether the passage from Quine just quoted is right or not. Do we have "essentially scientific reasons" (Quine 1969a: 97-8) to believe that numbers exist? Those who answer *yes* usually subscribe to what is called "the Quine-Putnam indispensability argument".[7] At the core of the indispensability argument lies the remark that contemporary scientific theories are highly mathematized: such theories make apparent reference not only to concrete objects like neutrinos and magnetic fields, but also to abstract objects like functions and numbers. According to the argument, given that our best scientific theories entail the existence of abstract mathematical objects and we accept those theories, we should believe that abstract objects exist. The indispensability argument (henceforth IA) in its simplest form can be presented like this (see Liggins 2016: 532 and Plebani 2018: 255):

> (1) Mathematics is indispensable to science: our best scientific theories entail the existence of abstract mathematical objects.
> (2) If mathematics is indispensable to science, then there are mathematical objects.
> Therefore: there are mathematical objects.

If one does not want to call into question the validity of *modus ponens*, then to reject the argument one must reject one of its two premises. Field's defense of nominalism (Field 1980, 1989) is an attempt to reject premise (1) and provide a nominalistic versions of our best scientific theories that do not contain any reference to or quantification over any abstract mathematical objects like numbers or sets. Field's project is to prove that there can be a *Science without Numbers* and so that mathematics

[7] Liggins (2008) argues that neither Quine nor Putnam actually endorsed the Quine-Putnam indispensability argument. See also Putnam (2012).

is not indispensable to our best scientific theories. While Field's nominalistic version of Newtonian gravitational theory is considered an important accomplishment, there seems to be no clear way to extend the approach adopted in Field (1980) to provide a nominalistic reconstruction of contemporary physics. In light of this, the received wisdom is that at the moment Field's project has not been completed. In light of the difficulty posed by the task of reformulating our scientific theories in a nominalistically kosher way, the reply to IA based on a denial of premise 1 goes under the name of "hard road nominalism" (Colyvan 2010, Liggins 2016).

A different reply to IA rejects its second premise. The reply grants that we need to formulate our theories mathematically; but it contends that this does not imply that mathematical objects exist. There might be various ways in which "numbers and classes...contribute to theoretical physics and other systematic discourse about nature" (Quine 1969a, 97-8). Perhaps presupposing that numbers exist helps us to represent certain circumstances; this does not mean that numbers need to be there for those circumstances to obtain, or that we need to assume the existence of numbers to explain why those circumstances do obtain.[8] One way to reject premise two of IA is to argue that the role of mathematics in our scientific theories is merely that of a "representational aid" (Yablo 2005):

> mathematics appears in our empirical theories as a mere descriptive aid: by speaking in terms of the real number line, . . . or some other mathematical structure, we simply make it easier to say what we want to say about the physical world. (Balaguer 1998, p. 137)

> [T]he kind of theoretical utility that mathematics brings is not of the right kind [to justify belief in abstract mathematical objects] […] Using numbers to index quantities may enable us to say much more complicated things about relationships between the various quantities, but it is nothing more than a labelling device. (Melia 2008, p. 117. See also Melia 2000)

This style of response to IA is sometimes also called "easy road nominalism" (Colyvan 2010) because it does not require the nominalist to reformulate our scientific theories. Some friends of IA have tried to argue against this kind of reply by pointing out that in some cases mathematics does not merely play a representational role, but also an explanatory one (Colyvan 2002, Baker 2005, Baker and Colyvan 2011). What goes under the name of "enhanced indispensability argument" is the attempt to argue for the existence of abstract mathematical objects on the basis of the existence of mathematical explanations of empirical phenomena.

The debate on the enhanced indispensability argument is very lively and the literature on it is growing

[8] Yablo (2012, p. 1014) criticizes the argument "we cannot imagine-without-numbers a complex world" therefore "we cannot imagine a complex world lacking in numbers".

rapidly (Bangu 2017). One point that deserves to be stressed is that nominalists do not reject mathematics - they reject mathematical objects (as Azzouni (2012) and Yablo (2012) correctly note). What needs to be shown in order to establish the existence of abstract mathematical objects on the basis of the existence of explanations of empirical phenomena that appeal to some mathematical theorems is that the truth of those mathematical theorems entails the existence of mathematical objects. Baker (2005) shows that one explanation of some facts concerning the life cycles of cicadas appeal to some number theoretic results. But as Yablo (2012) points out, there are nominalistic interpretations of number theory: the platonist needs to argue that the number theoretic results, interpreted in a nominalistic-friendly way, loose their explanatory power. Until we understand better what it means for mathematics to play an explanatory role, there seems to be a gap between the recognition that mathematical results can have explanatory value (in some sense) and the conclusion that abstract mathematical objects exist (Saatsi 2017: 893). It should also be noted, in connection with the distinction between the representational and the explanatory role of abstract mathematical objects, that sometimes it might be difficult to trace the distinction between the elements of our representation of the world that are mere descriptive aids and those that are genuine elements of the situation that we want to represent. As Quine put it:

> The fundamental-seeming philosophical question, How much of our science is merely contributed by language and how much is a genuine reflection of reality?, is perhaps a spurious question which itself arises wholly from a certain particular type of language. (Quine 1953: 78)

Until now I have insisted on how the question of the existence of abstract objects can be approached following Quine's methodology. Let me end by pointing at different ways in which one can depart from Quine's approach.

Quine's focus on science, in particular theoretical physics, as the source of evidence capable of resolving the dispute over the existence of abstract objects has deeply influenced the contemporary debate and produced a lot of interesting results. But it should be noted that there might be different ways to approach the question whether abstract objects exist, which might be of particular interest for those who focus on abstract objects that are not mathematical objects.

It is true that one could try to make a case for the existence of abstract types (Wetzel 2009) based on their role in our scientific theorizing. And a case for the existence of scientific models conceived as abstract objects would probably be based on an analysis of the scientific practice. But there might be other abstract objects that do not play an essential role in any of the hard sciences. The case for the existence of fictional characters, conceived as abstract objects, is normally based on an analysis of areas of discourse such as literary criticism (Van Inwagen 1977). The case for admitting artworks conceived as abstract objects in our ontology is based on an analysis of art practices and art criticism (Mag Uidhir 2013). And it is natural to assume that the case for entities like propositions would probably come from disciplines like linguistics or the philosophy of language/mind, which are not usually included in the list of the hard sciences.

Apart from broadening the range of areas of discourse to be taken in account, one could depart from Quine's method in more radical ways. One could argue that the existence of abstract objects of various kinds is not a theoretical hypothesis that we should accept in light of its explanatory value, but simply a consequence of uncontroversial truths combined with the rules of usage for certain words. According to the approach defended in Thomasson (2015) ontology should be *easy*: the existence of numbers, for instance, should be acknowledged as a consequence of the fact that (a) I have two hands and (b) if I have two hands, then the number of my hands is two, where (a) is an uncontroversial truth and (b) is a rule that governs the use of the word "number" in English. Similarly, according to neo-Fregeans like Hale and Wright (2003), abstract objects might be introduced by so-called abstraction principles, bi-conditionals which work as implicit definitions of certain concepts: Hume's principle (the number of the Fs is identical to the number of the Gs if and only if there is a 1-1 relation between the Fs and the Gs), for instance, is presented by the neo-fregeans an implicit definition of the concept of number. Both the easy ontology approach and the one adopted by the neo-Fregeans bear some connections to Carnap's views on ontology (Carnap 1950), which have recently attracted the attention of several philosophers (see Blatti and Lapointe 2016).

Another aspect of Quine's way of setting the stage for the debate about abstract objects is the idea that we should posit abstract objects only when it is indispensable to do so. As we saw in the previous section, Zalta's theory of abstract objects posit a wealth of abstract objects (one for each group of properties, roughly speaking), including objects which hardly serve any deep theoretical purpose: not only the theory has a place for objects like Leibniz's monads, which arguably are not indispensable to theoretical physics, but also for rather bizarre abstract objects like the one encoding uniquely the property of being either Spanish or a prime number. Quine's view on abstract objects was that we should reluctantly accept them because reference to them is unavoidable. Zalta's view seems rather to be that we have an unproblematic way to know about abstract objects (essentially via the comprehension principle) so that there is no reason not to admit them in our ontology.

Yet another aspect of Quine's methodology that might be challenged is his conviction that it does not make sense to investigate the ontological consequences of our ordinary way of speaking: "ordinary language is slipshod […] a fenced ontology is just not implicit in ordinary language" (Quine 1981b, pp. 9-10). Also that assumption has been challenged. Part of the task of natural language ontology

(Moltmann 2013, 2017) is the description of the kinds of objects whose existence is presupposed by natural language, using the methods of contemporary linguistics. This analysis might reveal a tension by the opinions of philosophers about what there is and the kind of entities that are presupposed by natural language.

I reviewed some answers to the questions what is an abstract object? Why should we debate over the existence of abstract objects? How should we conduct such a debate? As we are going to see, the present volume contributes to the advancement of the debate on abstract objects by suggesting new ways to address these questions.

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