## First Observation of $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ and Measurement of Its Decay Dynamics

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(Received 26 March 2020; accepted 22 May 2020; published 9 June 2020)
By analyzing a data sample corresponding to an integrated luminosity of $2.93 \mathrm{fb}^{-1}$ collected at a center-of-mass energy of 3.773 GeV with the BESIII detector, we measure for the first time the absolute branching fraction of the $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ decay to be $\mathcal{B}_{D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}}=\left(10.4 \pm 1.0_{\text {stat }} \pm 0.5_{\text {syst }}\right) \times 10^{-4}$. Using the world averaged value of $\mathcal{B}_{D^{+} \rightarrow \eta e^{+} \nu_{e}}$, the ratio of the two branching fractions is determined to be $\mathcal{B}_{D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}} / \mathcal{B}_{D^{+} \rightarrow \eta e^{+} \nu_{e}}=0.91 \pm 0.13_{(\text {stat }+ \text { syst })}$, which agrees with the theoretical expectation of lepton flavor universality within uncertainty. By studying the differential decay rates in five four-momentum transfer intervals, we obtain the product of the hadronic form factor $f_{+}^{\eta}(0)$ and the $c \rightarrow d$ Cabibbo-Kobayashi-Maskawa matrix element $\left|V_{c d}\right|$ to be $f_{+}^{\eta}(0)\left|V_{c d}\right|=0.087 \pm 0.008_{\text {stat }} \pm 0.002_{\text {syst. }}$. Taking the input of $\left|V_{c d}\right|$ from the global fit in the standard model, we determine $f_{+}^{\eta}(0)=0.39 \pm 0.04_{\text {stat }} \pm 0.01_{\text {syst }}$. On the other hand, using the value of $f_{+}^{\eta}(0)$ calculated in theory, we find $\left|V_{c d}\right|=$ $0.242 \pm 0.022_{\text {stat }} \pm 0.006_{\text {syst }} \pm 0.033_{\text {theory }}$.

DOI: 10.1103/PhysRevLett.124.231801

In the standard model (SM), the couplings between three families of leptons and gauge bosons are independent of lepton flavors. This property is known as lepton flavor universality (LFU) [1-4]. Semileptonic (SL) decays of

[^0]pseudoscalar mesons, which are well understood in the SM, offer an ideal platform to test LFU and search for new physics effects. In the past decade, the $B A B A R$, Belle, and LHCb collaborations reported anomalies in LFU tests with various $\mathrm{SL} B$ decays. The measured branching fraction (BF) ratios $\mathcal{R}_{\tau / \ell}=\mathcal{B}_{B \rightarrow \bar{D}^{(*)} \tau^{+} \nu_{\tau}} / \mathcal{B}_{B \rightarrow \bar{D}^{(*)} \ell^{+} \nu_{\ell}}$ $(\ell=\mu, e)[5-11]$ deviate from the SM predictions by $3.1 \sigma$ [12]. Various models [2,13-17] were proposed to explain these differences. In view of this, scrutinizing the ratios of the semimuonic $D$ decay BFs over their corresponding semielectronic counterparts offers important
complementary tests of $e-\mu \mathrm{LFU}$. Recently, BESIII reported tests of LFU with the SL decays $D \rightarrow \bar{K} \ell^{+} \nu_{\ell}[18,19]$ and $D \rightarrow \pi \ell^{+} \nu_{\ell}$ [20]. Using the world averaged BFs [21], the difference between the BF ratio of $D \rightarrow \pi \ell^{+} \nu_{\ell}$ decays which are mediated via $c \rightarrow d \ell^{+} \nu_{\ell}\left(\mathcal{R}_{\mu / e}^{c \rightarrow d}=\right.$ $\mathcal{B}_{D \rightarrow \pi \mu^{+} \nu_{\mu}} / \mathcal{B}_{D \rightarrow \pi e^{+} \nu_{e}}$ ) and the SM prediction is below $2 \sigma$. Meanwhile, there is still no experimental confirmation of the $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ decay, although it was predicted in the quark model 30 years ago [22]. Verification of $\mathcal{R}_{\mu / e}^{c \rightarrow d}$ with $D^{+} \rightarrow \eta \ell^{+} \nu_{\ell}$ decays, which is also mediated via $c \rightarrow d \ell^{+} \nu_{\ell}$, is key to clarifying these situations. In this Letter, we report a complementary test of LFU with $D^{+} \rightarrow \eta \ell^{+} \nu_{\ell}$ decays based on the first measurement of the BF of $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$. Throughout this Letter, charge conjugate channels are always implied. The BF obtained will also be important for the determination of the $\eta-\eta^{\prime}$ mixing angle, which will benefit the understanding of nonperturbative quantum-chromodynamics (QCD) effects [23].

The investigation of $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ decay dynamics allows the determination of the $c \rightarrow d$ Cabibbo-Kobayashi-Maskawa (CKM) matrix element $\left|V_{c d}\right|$ and the hadronic form factor (FF) $f_{+}^{\eta}(0)$. The value of $f_{+}^{\eta}(0)$ has been calculated with various approaches, e.g., QCD light cone sum rules (LCSR) [24-26], light front quark model (LFQM) [27], covariant confined quark model (CCQM) [28,29], and relativistic quark model (RQM) [30]. The predicted values vary in a wide range from 0.36 to 0.71 . According to Refs. [31,32], the predicted FFs of the SL $D$ decays are expected to be insensitive to the spectator quark. Measurement of the hadronic FF in $D^{+} \rightarrow$ $\eta \mu^{+} \nu_{\mu}$ decay can be used to distinguish between these calculations. The predicted FF that passes the experimental test is useful to determine $\left|V_{c d}\right|$. Conversely, measurements of SL $D$ decay hadronic FFs help constrain lattice QCD calculations and lead to more reliable calculations of the hadronic FFs of SL $D$ and $B$ decays, which are crucial to accurately determine CKM parameters [31-34], and test the unitarity of the CKM matrix.

In this analysis, we use a data sample corresponding to an integrated luminosity of $2.93 \mathrm{fb}^{-1}$ [35] taken at the center-of-mass energy $\sqrt{s}=3.773 \mathrm{GeV}$ with the BESIII detector. Details about the design and performance of the BESIII detector are given in Ref. [36]. Simulated samples produced with the GEANT4-based [37] Monte Carlo (MC) package, which includes the geometric description of the BESIII detector and the detector response, are used to determine the detection efficiency and to estimate the backgrounds. The simulation includes the beam energy spread and initial state radiation in the $e^{+} e^{-}$annihilations modeled with the generator ккмс [38]. The inclusive MC samples consist of the production of $D^{0} \bar{D}^{0}, D^{+} D^{-}$, and non- $D \bar{D}$ decays of the $\psi(3770)$, the initial state radiation production of the $J / \psi$ and $\psi(3686)$ states, and the
continuum processes incorporated in KКМС [38]. The known decay modes are modeled with evtaen [39] using BFs taken from the particle data group [21], and the remaining unknown decays from the charmonium states with Lundcharm [40]. The final state radiation from charged particles is incorporated with the РНотоs package [41]. The vector hadronic FF of the SL decay $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ is simulated with the modified-pole model [42], where the parameter $\alpha$ of the vector hadronic FF is set to be that of $D^{+} \rightarrow \pi^{0} e^{+} \nu_{e}$ measured by BESIII [43], and the pole mass is set at the nominal $D^{*+}$ mass [21].

The analysis is performed with the double-tag (DT) method, benefiting from the advantage of $D^{+} D^{-}$pair production at $\sqrt{s}=3.773 \mathrm{GeV}$. If a $D^{-}$meson [called single-tag (ST) $D^{-}$meson] is fully reconstructed by the hadronic decays $D^{-} \rightarrow K^{+} \pi^{-} \pi^{-}, \quad K_{S}^{0} \pi^{-}, \quad K^{+} \pi^{-} \pi^{-} \pi^{0}$, $K_{S}^{0} \pi^{-} \pi^{0}, K_{S}^{0} \pi^{+} \pi^{-} \pi^{-}$, and $K^{+} K^{-} \pi^{-}$, the presence of a $D^{+}$meson is guaranteed. If the $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ decay can be found in the system recoiling against an ST $D^{-}$meson, the candidate event is called a DT event. The BF of the SL decay is determined by

$$
\begin{equation*}
\mathcal{B}_{D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}}=N_{\mathrm{DT}} /\left(N_{\mathrm{ST}}^{\mathrm{tot}} \cdot \varepsilon_{\mathrm{SL}} \cdot \mathcal{B}_{\eta \rightarrow \gamma \gamma}\right), \tag{1}
\end{equation*}
$$

where $N_{\mathrm{ST}}^{\mathrm{tot}}$ and $N_{\mathrm{DT}}$ are the yields of the ST and DT candidates in data, respectively, and $\mathcal{B}_{\eta \rightarrow \gamma \gamma}$ is the BF of the $\eta \rightarrow \gamma \gamma$ decay. The $\varepsilon_{\mathrm{SL}}=\Sigma_{i}\left[\left(\varepsilon_{\mathrm{DT}}^{i} N_{\mathrm{ST}}^{i}\right) /\left(\varepsilon_{\mathrm{ST}}^{i} N_{\mathrm{ST}}^{\mathrm{tot}}\right)\right]$ is the effective signal efficiency of finding $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ decay in the presence of the ST $D^{-}$meson. Here $i$ denotes the tag mode, and $\varepsilon_{\text {ST }}$ and $\varepsilon_{\text {DT }}$ are the efficiencies of selecting the ST and DT candidates, respectively.

This analysis uses the same $K^{ \pm}, \pi^{ \pm}, K_{S}^{0}, \gamma$, and $\pi^{0}$ selection criteria as those employed in Refs. [18,20,44-47]. The ST $D^{-}$mesons are distinguished from combinatorial backgrounds by using the energy difference $\Delta E \equiv$ $E_{D^{-}}-E_{\text {beam }}$ and the beam-constrained mass $M_{\mathrm{BC}} \equiv$ $\sqrt{E_{\text {beam }}^{2}-\left|\vec{p}_{D^{-}}\right|^{2}}$. Here, $E_{\text {beam }}$ is the beam energy, and $E_{D^{-}}$and $\vec{p}_{D^{-}}$are the total energy and momentum of the ST $D^{-}$candidate in the $e^{+} e^{-}$center-of-mass frame. If multiple combinations for an ST mode are present in an event, the combination with the smallest $|\Delta E|$ per tag mode per charge is retained for further analysis. The candidates are required to satisfy $\Delta E \in(-0.055,0.045) \mathrm{GeV}$ for the tags containing $\pi^{0}$ and $\Delta E \in(-0.025,0.025) \mathrm{GeV}$ for the other tags. For each tag mode, the yield of ST $D^{-}$mesons is determined from the maximum likelihood fit of the $M_{\mathrm{BC}}$ distribution of the accepted candidates. In the fit, the signal and background are described by an MC-simulated shape and an ARGUS function [48], respectively. To take into account the resolution difference between data and MC simulation, the MC-simulated signal shape is convolved with a double-Gaussian function. The widths and relative abundances of the Gaussian components are free parameters of the fit. The resulting fits of these $M_{\mathrm{BC}}$ distributions


FIG. 1. Fits to the $M_{\mathrm{BC}}$ distributions of the ST $D^{-}$candidates. Data are shown as dots with error bars. The blue solid and red dashed curves are the fit results and the fitted backgrounds, respectively. Pairs of red arrows show the $M_{\mathrm{BC}}$ windows.
are exhibited in Fig. 1. The candidates with $M_{\mathrm{BC}} \in$ $(1.863,1.877) \mathrm{GeV} / c^{2}$ are kept for further analysis. The total yield of ST $D^{-}$mesons is $N_{\mathrm{ST}}^{\text {tot }}=1522474 \pm 2125_{\text {stat }}$.

At $\sqrt{s}=3.773 \mathrm{GeV}$, the $D^{+}$and $D^{-}$mesons are in back-to-back direction. The $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ candidates are selected in the sides recoiling against the ST $D^{-}$mesons. It is required that there is only one charged track available for muon identification. The muon candidate is required to satisfy $\left|V_{x y}\right|<1 \mathrm{~cm}$ and $\left|V_{z}\right|<10 \mathrm{~cm}$, where $\left|V_{x y}\right|$ and $\left|V_{z}\right|$ are the distances of closest approach to the interaction point of the reconstructed track in the transverse plane and along the axis of the drift chamber, respectively. Its polar angle $(\theta)$ with respect to the axis of the drift chamber must be within $|\cos \theta|<0.93$.

Muon identification uses information from the time-offlight and the electromagnetic calorimeter, as well as the specific ionization energy loss measured in the main drift chamber. The combined confidence levels for various particle hypotheses $\left(\mathrm{CL}_{i}, i=e, \mu\right.$, and $\left.K\right)$ are calculated. Muon candidates are required to satisfy $\mathrm{CL}_{\mu}>0.001$, $\mathrm{CL}_{\mu}>\mathrm{CL}_{e}$, and $\mathrm{CL}_{\mu}>\mathrm{CL}_{K}$. To reduce misidentification between hadrons and muons, the deposited energy in the electromagnetic calorimeter of the muon candidate is required to be within $(0.105,0.275) \mathrm{GeV}$, since it is expected to concentrate around 0.2 GeV .

The $\eta$ candidates are reconstructed via the $\eta \rightarrow \gamma \gamma$ decay. The invariant mass of the $\gamma \gamma$ candidate is required to be within $(0.510,0.570) \mathrm{GeV} / c^{2}$. To improve momentum resolution, a one-constraint kinematic fit is done on the selected photon pair, whose invariant mass is constrained to the $\eta$ nominal mass $\left(m_{\eta}\right)$ [21].

Due to the misidentification of pions as muons, some hadronic $D^{+}$decays survive the above selection criteria. To suppress the peaking backgrounds from $D^{+} \rightarrow \eta \pi^{+}$decays, we require the $\eta \mu^{+}$invariant mass $\left(M_{\eta \mu^{+}}\right)$to be less than $1.74 \mathrm{GeV} / c^{2}$. To reject the backgrounds containing $\pi^{0}$, e.g., $D^{+} \rightarrow \eta \pi^{+} \pi^{0}$, we require that the maximum energy of
any extra photon $\left(E_{\text {extray }}^{\max }\right)$ is less than 0.30 GeV and there is no extra $\pi^{0}\left(N_{\text {extra } \pi^{0}}\right)$ in the candidate event. Here, the extra photon and $\pi^{0}$ denote the ones which have not been used in the DT selection.

The number of SL decays is determined using a kinematic quantity defined as $U_{\text {miss }} \equiv E_{\text {miss }}-\left|\vec{p}_{\text {miss }}\right|$, which is expected to peak around 0 for the correctly reconstructed signal events. Here, $E_{\text {miss }} \equiv E_{\text {beam }}-E_{\eta}-E_{\mu^{+}}$ and $\vec{p}_{\text {miss }} \equiv \vec{p}_{D^{+}}-\vec{p}_{\eta}-\vec{p}_{\mu^{+}}$are the missing energy and momentum of the DT event in the $e^{+} e^{-}$center-of-mass frame, in which $E_{\eta\left(\mu^{+}\right)}$and $\vec{p}_{\eta\left(\mu^{+}\right)}$are the energy and momentum of the $\eta(\mu)$ candidates. The $U_{\text {miss }}$ resolution is improved by constraining the $D^{+}$energy to the beam energy and $\vec{p}_{D^{+}} \equiv-\hat{p}_{D^{-}} \sqrt{E_{\text {beam }}^{2}-m_{D^{+}}^{2}}$, where $\hat{p}_{D^{-}}$is the unit vector in the momentum direction of the ST $D^{-}$and $m_{D^{+}}$is the $D^{+}$nominal mass [21].

Figure 2(a) shows the $U_{\text {miss }}$ distribution of the accepted DT events in data. The SL decay yield is obtained from an unbinned fit to the $U_{\text {miss }}$ distribution, where the SL signal, peaking backgrounds of $D^{+} \rightarrow \eta \pi^{+} \pi^{0}$, and nonpeaking backgrounds (including a small contribution from wrongly reconstructed ST candidates) are described by the corresponding MC-simulated shapes. The yields of the signal and nonpeaking backgrounds are free parameters of the fit, while the yield of the peaking background from $D^{+} \rightarrow$ $\eta \pi^{+} \pi^{0}$ decays is fixed based on MC simulation. The fit result is shown in Fig. 2(a). From the fit, we obtain the yield of DT events $N_{\text {DT }}=234 \pm 22_{\text {stat }}$. The statistical significance, calculated by $\sqrt{-2 \ln \left(\mathcal{L}_{0} / \mathcal{L}_{\text {max }}\right)}$, is found to be greater than $10 \sigma$. Here, $\mathcal{L}_{\text {max }}$ and $\mathcal{L}_{0}$ are the maximal likelihood of the nominal fit and that of the fit without signal component, respectively.

Table I of the Supplemental Material [49] shows tag dependent numbers of $N_{\mathrm{ST}}^{i}, \epsilon_{\mathrm{ST}}^{i}$, and $\epsilon_{\mathrm{DT}}^{i}$. The average


FIG. 2. Fits to the $U_{\text {miss }}$ distributions of the $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ candidate events. Data are shown as dots with error bars. In each figure, the blue solid curve is the fit result, the black dotted curve is the fitted nonpeaking background, and the difference between red dashed and black dotted curves is the peaking background of $D^{+} \rightarrow \eta \pi^{+} \pi^{0}$.
efficiency of detecting $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ decays is $\varepsilon_{\text {SL }}=$ $0.3752 \pm 0.0013$. Here, the efficiency does not include the BF of the $\eta \rightarrow \gamma \gamma$ decay. To verify the reliability of the efficiency determination, we have compared distributions of momenta and $\cos \theta$ of the $\eta$ and $\mu^{+}$of the selected $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ candidate events between data and MC simulation, and they are in good agreement.

Inserting $N_{\mathrm{DT}}, \varepsilon_{\mathrm{SL}}, N_{\mathrm{ST}}^{\mathrm{tot}}$, and the world average of $\mathcal{B}_{\eta \rightarrow \gamma \gamma}=0.3941 \pm 0.0020$ [21] into Eq. (1), we obtain

$$
\mathcal{B}_{D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}}=\left(10.4 \pm 1.0_{\mathrm{stat}} \pm 0.5_{\mathrm{syst}}\right) \times 10^{-4}
$$

In the BF measurement, the systematic uncertainties arise from the following sources. The uncertainty in the total yield of ST $D^{-}$mesons has been studied in Refs. [18,20,44], and is assigned as $0.5 \%$. The muon tracking (PID) efficiencies are studied by analyzing $e^{+} e^{-} \rightarrow \gamma \mu^{+} \mu^{-}$events, and the muon tracking (PID) efficiency uncertainty is taken as $0.2 \%(0.2 \%)$ per muon, where the data and MC differences of the two-dimensional (momentum and $\cos \theta$ ) distributions of the control samples have been reweighted by those of the $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ signal decays. The uncertainty of $\eta$ reconstruction is assumed to be $2.0 \%$, the same as $\pi^{0}$ reconstruction, which was studied with DT $D \bar{D}$ hadronic decays of $D^{0} \rightarrow K^{-} \pi^{+}, K^{-} \pi^{+} \pi^{+} \pi^{-}$ vs $\bar{D}^{0} \rightarrow K^{+} \pi^{-} \pi^{0}, K_{S}^{0} \pi^{0}[18,44]$. The uncertainties of the requirements of $E_{\text {extray }}^{\max }$ and $N_{\text {extra } \pi^{0}}$ are estimated to be $2.3 \%$ by analyzing the DT candidate events of $D^{+} \rightarrow \eta \pi^{+}$and $\pi^{0} e^{+} \nu_{e}$. The uncertainty due to the $M_{\eta \mu^{+}}$requirement is evaluated by replacing the nominal requirement with $M_{\eta \mu^{+}}<1.69 \mathrm{GeV} / c^{2}$ or $M_{\eta \mu^{+}}<1.79 \mathrm{GeV} / c^{2}$, and the associated uncertainty is found to be negligible. The uncertainty in the $U_{\text {miss }}$ fit is assigned to be $3.7 \%$, which is estimated with alternative signal and background shapes. For alternative background shapes, the uncertainty due to the peaking background of $D^{+} \rightarrow \eta \pi^{+} \pi^{0}$ has been taken into account via varying the quoted BF by $\pm 1 \sigma$. The uncertainty due to the limited MC statistics is $0.5 \%$. The uncertainty in the MC model, $0.3 \%$, is assigned as the difference between our nominal DT efficiency and the DT efficiency determined by reweighting the $q^{2}\left(q=p_{D^{+}}-p_{\eta}\right.$ is the total four momentum of $\mu^{+} \nu_{\mu}$ ) distribution of the signal MC events using the FF parameters obtained from data. Adding these
uncertainties quadratically yields the total systematic uncertainty to be $4.9 \%$.

To study the dynamics in $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ decay, the SL candidate events are divided into five $q^{2}$ intervals: $(0.0,0.25), \quad(0.25,0.5), \quad(0.5,0.75), \quad(0.75,1.0), \quad$ and $\left[1.0,\left(m_{D^{+}}-m_{\eta}\right)^{2}\right] \mathrm{GeV}^{2} / c^{4}$. The partial decay rate in the $i$ th $q^{2}$ interval, $\Delta \Gamma_{\text {measured }}^{i}$, is determined by

$$
\begin{equation*}
\Delta \Gamma_{\text {measured }}^{i}=N_{\text {produced }}^{i} /\left(\tau_{D^{+}} N_{\mathrm{ST}}^{\mathrm{tot}}\right) \tag{2}
\end{equation*}
$$

where $N_{\text {produced }}^{i}$ is the $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ signal yield produced in the $i$ th $q^{2}$ interval in data, $\tau_{D^{+}}$is the lifetime of $D^{+}, N_{\mathrm{ST}}^{\mathrm{tot}}$ is the total yield of ST $D^{-}$mesons, and

$$
\begin{equation*}
N_{\text {produced }}^{i}=\sum_{j}^{N_{\text {intervals }}}\left(\varepsilon^{-1}\right)_{i j} N_{\text {observed }}^{j}, \tag{3}
\end{equation*}
$$

where $N_{\text {observed }}^{j}$ is the $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ signal yield observed in the $j$ th $q^{2}$ interval and $\varepsilon$ is the efficiency matrix (Table II of the Supplemental Material [49]) given by

$$
\begin{equation*}
\varepsilon_{i j}=\sum_{k}\left[\left(N_{\text {reconstructed }}^{i j} N_{\mathrm{ST}}\right) /\left(N_{\text {generated }}^{j} \varepsilon_{\mathrm{ST}}\right)\right]_{k} / N_{\mathrm{ST}}^{\text {tot }} . \tag{4}
\end{equation*}
$$

Here, $N_{\text {reconstructed }}^{i j}$ is the $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ signal yield generated in the $j$ th $q^{2}$ interval and reconstructed in the $i$ th $q^{2}$ interval, $N_{\text {generated }}^{j}$ is the total signal yield generated in the $j$ th $q^{2}$ interval, and the index $k$ sums over all ST modes.
$N_{\text {observed }}^{i}$ is obtained from the fit to the $U_{\text {miss }}$ distribution of the $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ candidate events in the $i$ th $q^{2}$ interval. The fit results of the $U_{\text {miss }}$ distributions in various intervals are shown in Figs. 2(b)-2(f), and the partial decay rates obtained are shown in Fig. 3.

With the $\Delta \Gamma_{\text {measured }}^{i}$ obtained above and the partial decay rate $\Delta \Gamma_{\text {expected }}^{i}$ predicted by theory, the $\chi^{2}$ is constructed as

$$
\begin{align*}
\chi^{2}= & \sum_{i, j=1}^{5}\left(\Delta \Gamma_{\text {measured }}^{i}-\Delta \Gamma_{\text {expected }}^{i}\right) C_{i j}^{-1} \\
& \times\left(\Delta \Gamma_{\text {measured }}^{j}-\Delta \Gamma_{\text {expected }}^{j}\right), \tag{5}
\end{align*}
$$

where $C_{i j}=C_{i j}^{\mathrm{stat}}+C_{i j}^{\mathrm{syst}}$ is the covariance matrix of the measured partial decay rates among $q^{2}$ intervals, and

$$
\begin{equation*}
\Delta \Gamma_{\mathrm{expected}}^{i}=\int_{q_{\min (i)}^{2}}^{q_{\max (i)}^{2}}\left\{\frac{G_{F}^{2}\left|V_{c d}\right|^{2}}{24 \pi^{3}} \cdot \frac{\left(q^{2}-m_{\mu}^{2}\right)^{2} \sqrt{E_{\eta}^{2}-m_{\eta}^{2}}}{q^{4} m_{D}^{2}}\left[\left(1+\frac{m_{\mu}^{2}}{2 q^{2}}\right) m_{D}^{2}\left(E_{\eta}^{2}-m_{\eta}^{2}\right)\left|f_{+}^{\eta}\left(q^{2}\right)\right|^{2}\right]\right\} d q^{2} \tag{6}
\end{equation*}
$$



FIG. 3. (a) Fit to the partial decay rates and (b) projection to $f_{+}^{\eta}\left(q^{2}\right)$ for $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$. Dots with error bars are data, and the solid curves are the fit results.
where $G_{F}$ is the Fermi coupling constant; $m_{\mu}$ is the $\mu^{+}$ mass; $\left|\vec{p}_{\eta}\right|$ and $E_{\eta}$ are the momentum and energy of $\eta$ in the rest frame of $D^{+}$, respectively; the vector hadronic FF $f_{+}^{\eta}\left(q^{2}\right)$ is formulated following Ref. [50]. Here, the scalar hadronic FF $f_{0}^{\eta}\left(q^{2}\right)$ has been ignored due to negligible sensitivity with limited data.

The FF of $f_{+}^{\eta}\left(q^{2}\right)$ can be parametrized by the series expansion [51], which was widely used in previous analyses and verified to be consistent with constraints from QCD [52-54]. Due to limited data, we adopt the twoparameter series expansion form
$f_{+}^{\eta}\left(q^{2}\right)=\frac{f_{+}^{\eta}(0) P(0) \Phi\left(0, t_{0}\right)}{P\left(q^{2}\right) \Phi\left(q^{2}, t_{0}\right)} \cdot \frac{1+r_{1}\left(t_{0}\right) z\left(q^{2}, t_{0}\right)}{1+r_{1}\left(t_{0}\right) z\left(0, t_{0}\right)}$,
where $t_{0}=t_{+}\left(1-\sqrt{1-t_{-} / t_{+}}\right), t_{ \pm}=\left(m_{D^{+}} \pm m_{\eta}\right)^{2}$, and the functions $P\left(q^{2}\right), \Phi\left(q^{2}, t_{0}\right)$, and $z\left(q^{2}, t_{0}\right)$ are defined following Ref. [51].

The statistical covariance matrix is constructed as

$$
\begin{equation*}
C_{i j}^{\mathrm{stat}}=\left(\frac{1}{\tau_{D^{+}} N_{\mathrm{ST}}^{\mathrm{tot}}}\right)^{2} \sum_{n} \varepsilon_{\mathrm{in}}^{-1} \varepsilon_{j n}^{-1}\left[\sigma\left(N_{\mathrm{obs}}^{n}\right)\right]^{2}, \tag{8}
\end{equation*}
$$

where $n$ sums from 1 to 5 intervals. The systematic covariance matrix is obtained by summing over that of each systematic uncertainty source, which is taken as

$$
\begin{equation*}
C_{i j}^{\text {syst }}=\delta\left(\Delta \Gamma_{\text {measured }}^{i}\right) \delta\left(\Delta \Gamma_{\text {measured }}^{j}\right) \tag{9}
\end{equation*}
$$

where $\delta\left(\Delta \Gamma_{\text {measured }}^{i}\right)$ is the systematic uncertainty of the partial decay rate in the $i$ th $q^{2}$ interval. The systematic uncertainties arising from $N_{\mathrm{ST}}^{\mathrm{tot}}, \tau_{D^{+}}$, muon tracking and PID, $\eta$ reconstruction, as well as $E_{\text {extray }}^{\max }$ and $N_{\text {extra } \pi^{0}}$ requirements are taken to be common across all the $q^{2}$ intervals; while the others are determined separately in each $q^{2}$ interval as above.

Minimizing the $\chi^{2}$ in Eq. (5) gives the product of $f_{+}^{\eta}(0)\left|V_{c d}\right|$ and the first order coefficient $r_{1}$ to be $0.087 \pm$ $0.008_{\text {stat }} \pm 0.002_{\text {syst }}$ and $-0.9 \pm 2.7_{\text {stat }} \pm 0.2_{\text {syst }}$, respectively. The nominal fit parameters are taken from the fit with the combined statistical and systematic covariance matrix, and their statistical uncertainties are taken from the fit only with the statistical covariance matrix. For each parameter, the systematic uncertainty is obtained by calculating the quadratic difference of uncertainties between these two fits. The fit result is shown in Fig. 3 and the goodness of fit is $\chi^{2} / \mathrm{NDOF}=1.0 / 3$, where NDOF is the number of degrees of freedom. The $f_{+}^{\eta}(0)\left|V_{c d}\right|$ measured in this work is consistent with the measurements using $D^{+} \rightarrow \eta e^{+} \nu_{e}$ by CLEO [55] and BESIII [56].

Using our product of $f_{+}^{\eta}(0)\left|V_{c d}\right|$ and the $f_{+}^{\eta}(0)$ calculated in Ref. [29] leads to $\left|V_{c d}\right|=0.242 \pm 0.022_{\text {stat }} \pm$ $0.006_{\text {syst }} \pm 0.033_{\text {theory }}$. This result is consistent with our previous measurements of $\left|V_{c d}\right|$ via $D^{+} \rightarrow \ell^{+} \nu_{\ell}[57,58]$ and $D^{0(+)} \rightarrow \pi^{-(0)} e^{+} \nu_{e} \quad[43,54]$. Conversely, using our product of $f_{+}^{\eta}(0)\left|V_{c d}\right|$ and the $\left|V_{c d}\right|=0.22438 \pm 0.00044$ given by CKMFitter [21] yields $f_{+}^{\eta}(0)=0.39 \pm 0.04_{\text {stat }} \pm$ $0.01_{\text {syst }}$. Table I shows comparison of our BF and hadronic FF with various theoretical calculations for $D^{+} \rightarrow \eta \ell^{+} \nu_{\ell}$. Our BF result disfavors the prediction in Ref. [26] by $2.6 \sigma$ but agrees with the other predictions [24,25,28,30,59] within $1.5 \sigma$. Our result for $f_{+}^{\eta}(0)$ agrees well with the predictions in Refs. [25,29]. However, it clearly rules out the prediction in Ref. [27], and disfavors the predictions in Refs. [24,26,28,30] by about $2.4-3.8 \sigma$. Using our $f_{+}^{\eta}(0)$ and the world average of $f_{+}^{\pi}(0)=0.6351 \pm 0.0081$ [12], we determine the hadronic FF ratio to be $f_{+}^{\eta}(0) / f_{+}^{\pi}(0)=0.61 \pm 0.06_{\text {stat }} \pm 0.02_{\text {syst }}$. This provides a

TABLE I. Comparison of our BF (in $\times 10^{-4}$ ) and hadronic FF with various theoretical calculations for $D^{+} \rightarrow \eta \ell^{+} \nu_{\ell}$. The first and second uncertainties are statistical and systematic, respectively. Theoretical calculations listed in the table assume no gluon component for $\eta^{\prime}$. Numbers marked with ${ }^{*}$ denote that the predicted $\mathcal{B}_{D^{+} \rightarrow \eta e^{+} \nu_{e}}$ is listed due to no predictions for $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ in Refs. [25,26].

|  |  | LCSR | LCSR |  | CLFQM | CCQM | CCQM | RQM |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | This work | $[24]$ | $[25]$ | LCSR [26] | LFQM [27] | [59] | $[28]$ | $[29]$ | $[30]$ |
| $\mathcal{B}_{D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}}$ | $10.4 \pm 1.0 \pm 0.5$ | $8.4_{-1.4}^{+1.6}$ | $14 \pm 11^{*}$ | $24.5 \pm 5.3^{*}$ | $\ldots$ | $12 \pm 1$ | 9.12 | $\ldots$ | 12.1 |
| Difference $(\sigma)$ | $\ldots$ | 1.0 | 0.3 | 2.6 | $\ldots$ | 1.1 | 1.1 | $\ldots$ | 1.5 |
| $f_{+}^{\eta}(0)$ | $0.39 \pm 0.04 \pm 0.01$ | $0.56_{-0.05}^{+0.06}$ | $0.43_{-0.14}^{+0.17}$ | $0.55 \pm 0.05$ | $0.71 \pm 0.01$ | $\ldots$ | $0.67 \pm 0.11$ | $0.36 \pm 0.05$ | 0.547 |
| Difference $(\sigma)$ | $\ldots$ | 2.5 | 0.3 | 2.5 | 7.5 | $\ldots$ | 2.4 | 0.5 | 3.8 |

valuable constraint to improve the calculations of these hadronic FFs in lattice QCD.

In summary, the SL decay $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ has been observed by analyzing $2.93 \mathrm{fb}^{-1}$ of data collected at $\sqrt{s}=3.773 \mathrm{GeV}$. The absolute BF of this decay is determined for the first time to be $\mathcal{B}_{D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}}=$ $\left(10.4 \pm 1.0_{\text {stat }} \pm 0.5_{\text {syst }}\right) \times 10^{-4}$. Using the world averaged value of $\mathcal{B}_{D^{+} \rightarrow \eta e^{+} \nu_{e}}=(11.4 \pm 1.0) \times 10^{-4}$ gives the BF ratio $R_{\mu / e}=\mathcal{B}_{D^{+} \rightarrow \eta \mu^{+} \nu_{e}} / \mathcal{B}_{D^{+} \rightarrow \eta e^{+} \nu_{e}}=0.91 \pm 0.13$, where the uncertainty is the sum in quadrature of the statistical and systematic errors, but dominated by the statistical error. This result agrees with the SM predictions (0.97-1.00) [24,29,59], thereby implying no LFU violation within current sensitivity. The obtained BF can be used to determine the $\eta-\eta^{\prime}$ mixing angle once $\mathcal{B}_{D^{+} \rightarrow \eta^{\prime} \mu^{+} \nu_{e}}$ is measured with large data samples $[60,61]$ in the near future. By analyzing the dynamics of $D^{+} \rightarrow \eta \mu^{+} \nu_{\mu}$ decay, the product of $f_{+}^{\eta}(0)\left|V_{c d}\right|$ has been determined to be $0.087 \pm$ $0.008_{\text {stat }} \pm 0.002_{\text {syst }}$ Combining necessary inputs, we have obtained $f_{+}^{\eta}(0)=0.39 \pm 0.04_{\text {stat }} \pm 0.01_{\text {syst }}$ and $\left|V_{c d}\right|=$ $0.242 \pm 0.022_{\text {stat }} \pm 0.006_{\text {syst }} \pm 0.033_{\text {theory }}$. The obtained $f_{+}^{\eta}(0)$ provides important data to test various theoretical calculations, while the obtained $\left|V_{c d}\right|$ is valuable for the CKM matrix unitarity test.

The BESIII collaboration thanks the staff of BEPCII and the IHEP computing center for their strong support. This work is supported in part by National Key Basic Research Program of China under Contract No. 2015CB856700; National Natural Science Foundation of China (NSFC) under Contracts No. 11775230, No. 11475123, No. 11625523, No. 11635010, No. 11735014, No. 11822506, No. 11835012, No. 11935015, No. 11935016, No. 11935018, No. and 11961141012; the Chinese Academy of Sciences (CAS) Large-Scale Scientific Facility Program; Joint Large-Scale Scientific Facility Funds of the NSFC and CAS under Contracts No. U1532101, No. U1932102, No. U1732263, and No. U1832207; CAS Key Research Program of Frontier Sciences under Contracts No. QYZDJ-SSW-SLH003 and No. QYZDJ-SSW-SLH040; 100 Talents Program of CAS; INPAC and Shanghai Key Laboratory for Particle Physics and Cosmology; ERC under Contract No. 758462; German Research Foundation DFG under Contracts No. Collaborative Research Center CRC 1044 and No. FOR 2359; Istituto Nazionale di Fisica Nucleare, Italy; Ministry of Development of Turkey under Contract No. DPT2006K-120470; National Science and Technology fund; STFC (United Kingdom); The Knut and Alice Wallenberg Foundation (Sweden) under Contract No. 2016.0157; The Royal Society, UK under Contracts No. DH140054 and No. DH160214; The Swedish Research Council; U.S. Department of Energy under Contracts No. DE-FG02-05ER41374 and No. DE-SC0012069.
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