D3-Branes Wrapped on a Spindle

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We construct supersymmetric $AdS_3 \times \Sigma$ solutions of minimal gauged supergravity in $D = 5$, where Σ is a two-dimensional orbifold known as a spindle. Remarkably, these uplift on S^5, or more generally on any regular Sasaki-Einstein manifold, to smooth solutions of type IIB supergravity. The solutions are dual to $d = 2$, $\mathcal{N} = (0, 2)$ SCFTs and we show that the central charge for the gravity solution agrees with a field theory calculation associated with D3-branes wrapped on Σ.

Introduction.—Important insights into strongly coupled supersymmetric conformal field theories (SCFTs) can be obtained by realizing them as the renormalization group fixed points of compactifications of higher-dimensional field theories. Such SCFTs can be constructed by starting with the field theories arising on the world volumes of branes in string theory or M theory, wrapping them on a compact manifold Σ, and then flowing to the infrared (IR).

In favorable circumstances these configurations can be described within the AdS/CFT correspondence by brane solutions of $D = 10/11$ supergravity. Such solutions have a boundary of the form $AdS_{d+1} \times M$, where M is a compact manifold, which describes the ultraviolet (UV) of the SCFT in d spacetime dimensions. They also have near horizon geometries of the schematic form $AdS_{d+1-n} \times \Sigma \times M$, which describes the SCFT in $d - n$ dimensions arising in the IR, where Σ has dimension n. After a Kaluza-Klein reduction of the $D = 10/11$ supergravity theory on M, to obtain a gravity theory in $d + 1$ spacetime dimensions, these solutions may be viewed as black branes in AdS_{d+1} with horizon Σ.

Such solutions were first constructed in the foundational work [1] and describe M5-branes and D3-branes wrapping Riemann surfaces of constant curvature, with genus $g > 1$. Subsequently there have been many generalizations including wrapping manifolds of higher dimension, relaxing the constant curvature condition, allowing for punctures, etc. (e.g., Refs. [2–4]). In all these developments, supersymmetry is preserved by demanding that the field theory

arising on the brane world volume is "topologically twisted" [5,6]. This involves a specific coupling both to the metric on Σ and to external R-symmetry gauge fields. An important consequence of this twisting is that the Killing spinors preserved by the solution are independent of the coordinates of Σ.

Here we discuss a class of supersymmetric solutions which have fundamentally new features. We present $AdS_3 \times \Sigma$ solutions of $D = 5$ minimal gauged supergravity which we interpret as the near horizon limit of black brane solutions associated with D3-branes wrapped on Σ. The first new feature is that supersymmetry is *not* realized by a topological twist. The second is that Σ is not a compact manifold but an *orbifold* [7]. More specifically, we will consider the weighted projective space $\Sigma = \mathbb{WCP}^1_{[n_-,n_+]}$, also known as a spindle. This is topologically a two-sphere but with conical deficit angles $2\pi(1 - 1/n_\mp)$ at the poles, specified by two coprime positive integers $n_\mp$, with $n_- \neq n_+$ [8].

Remarkably, after uplifting the $AdS_3 \times \Sigma$ solutions on specific Sasaki-Einstein five-manifolds, SE_5, to obtain solutions of type IIB supergravity, they become completely smooth [9]. Moreover, the resulting $AdS_3 \times \mathcal{M}_7$ solutions are precisely those of Ref. [11]. Our construction suggests that the $d = 2$, $\mathcal{N} = (0, 2)$ SCFTs dual to the $AdS_3 \times \mathcal{M}_7$ solutions of Ref. [11] arise from compactifying $d = 4$, $\mathcal{N} = 1$ SCFTs on a spindle, where the $d = 4$ theories are dual to $AdS_5 \times SE_5$. This includes the case of $\mathcal{N} = 4$ SYM, where $SE_5 = S^5$. We make a precision test of this interpretation: we compute the central charge and superconformal R-symmetry of the $d = 2$ field theories using anomaly polynomials and c extremization [12] and find exact agreement with the gravity result [11].

$D = 5$ solutions.—The equations of motion for $D = 5$ minimal gauged supergravity [13] are given by

$$R_{\mu\nu} = -4g_{\mu\nu} + \frac{2}{3}F_{\mu\rho}F_{\nu}{}^{\rho} - \frac{1}{9}g_{\mu\nu}F_{\rho\sigma}F^{\rho\sigma},$$

$$d * F = -\frac{2}{3}F \wedge F, \tag{1}$$

where $F = dA$, with A the Abelian R-symmetry gauge field. A solution is supersymmetric if it admits a Killing spinor satisfying

$$\left[\nabla_\mu - \frac{i}{12}(\Gamma_\mu{}^{\nu\rho} - 4\delta_\mu^\nu\Gamma^\rho)F_{\nu\rho} - \frac{1}{2}\Gamma_\mu - iA_\mu\right]\epsilon = 0, \tag{2}$$

where ϵ is a Dirac spinor and $\{\Gamma_\mu, \Gamma_\nu\} = 2g_{\mu\nu}$.

The supersymmetric solution of interest is given by

$$ds_5^2 = \frac{4y}{9}ds_{\text{AdS}_3}^2 + ds_\Sigma^2, \qquad A = \frac{1}{4}\left(1 - \frac{a}{y}\right)dz. \tag{3}$$

Here $ds_{\text{AdS}_3}^2$ is a unit radius metric on AdS$_3$, while

$$ds_\Sigma^2 = \frac{y}{q(y)}dy^2 + \frac{q(y)}{36y^2}dz^2, \tag{4}$$

is the metric on the horizon Σ and

$$q(y) = 4y^3 - 9y^2 + 6ay - a^2, \tag{5}$$

with a a constant.

Assuming $a \in (0, 1)$ the three roots y_i of $q(y)$ are all real and positive. Defining $y_1 < y_2 < y_3$, we then take $y \in [y_1, y_2]$ to obtain a positive definite metric (4) on Σ. However, as y approaches y_1 and y_2 it is not possible to remove the conical deficit singularities at both roots by a single choice of period Δz for z, to obtain a smooth two-sphere. Instead we find that if

$$a = \frac{(n_- - n_+)^2(2n_- + n_+)^2(n_- + 2n_+)^2}{4(n_-^2 + n_-n_+ + n_+^2)^3},$$

$$\Delta z = \frac{2(n_-^2 + n_-n_+ + n_+^2)}{3n_-n_+(n_- + n_+)}2\pi, \tag{6}$$

then ds_Σ^2 is a smooth metric on the orbifold $\Sigma = \mathbb{WCP}^1_{[n_-,n_+]}$. Specifically, there are conical deficit angles $2\pi(1 - 1/n_\mp)$ at $y = y_1, y_2$, respectively, where $n_\pm$ are arbitrary coprime positive integers with $n_- > n_+$.

Note that there is magnetic flux through Σ:

$$\frac{1}{2\pi}\int_\Sigma F = \frac{n_- - n_+}{2n_-n_+}. \tag{7}$$

This may be contrasted with the Euler number

$$\chi(\Sigma) = \frac{1}{4\pi}\int_\Sigma R_\Sigma\text{vol}_\Sigma = \frac{n_- + n_+}{n_-n_+}, \tag{8}$$

where R_Σ is the Ricci scalar of Σ, and vol$_\Sigma$ is its volume form [14].

To solve Eq. (2) we write $\Gamma^a = \gamma^a \otimes \sigma^3$, for $a = 0, 1, 2$ with $\gamma^0 = -i\sigma^2$, $\gamma^1 = \sigma^1$, $\gamma^2 = \sigma^3$, and $\Gamma^3 = 1 \otimes \sigma^2$, $\Gamma^4 = 1 \otimes \sigma^1$, where σ^i are Pauli matrices. We then write $\epsilon = \vartheta \otimes \chi$ with ϑ a Killing spinor for AdS$_3$ satisfying $\nabla_a\vartheta = \frac{1}{2}\gamma_a\vartheta$. The two-component spinor χ on the spindle is given by

$$\chi = \left(\frac{\sqrt{q_1(y)}}{\sqrt{y}}, i\frac{\sqrt{q_2(y)}}{\sqrt{y}}\right), \tag{9}$$

where

$$q_1(y) = -a + 2y^{3/2} + 3y, \qquad q_2(y) = a + 2y^{3/2} - 3y, \tag{10}$$

which satisfy $q(y) = q_1(y)q_2(y)$. In contrast to the topological twist, this spinor depends on the coordinates of Σ. Moreover, as shown in Ref. [15], the spinor is in fact a section of a nontrivial bundle over Σ. Note that the gauge choice used in Eq. (3) has been fixed by requiring ϵ to be independent of z.

Uplift to IIB string theory.—Any supersymmetric solution to Eq. (1) uplifts (locally) to type IIB supergravity via [16]:

$$ds_{10}^2 = L^2\left[ds_5^2 + \left(\frac{1}{3}d\psi + \sigma + \frac{2}{3}A\right)^2 + ds_{KE_4}^2\right]$$

$$g_sF_5 = L^4\left[4\text{vol}_5 - \frac{2}{3}*_5 F \wedge J\right.$$

$$\left. + \left(2J \wedge J - \frac{2}{3}F \wedge J\right) \wedge \left(\frac{1}{3}d\psi + \sigma + \frac{2}{3}A\right)\right]. \tag{11}$$

Here F_5 is the self-dual five-form, g_s is the string coupling constant, and $L > 0$ is a length scale that is fixed by flux quantization. KE$_4$ is an arbitrary positively curved Kähler-Einstein four-manifold with Kähler form J, normalized so that the Ricci form is $\mathcal{R} = 6J$, and σ is a local one-form with $d\sigma = 2J$.

Substituting Eq. (3) into Eq. (11) we find that the $D = 10$ metric may be written as

$$ds_{10}^2 = \frac{4}{9}L^2y[ds_{\text{AdS}_3}^2 + ds_{\mathcal{M}_7}^2], \tag{12}$$

where $\mathcal{M}_7$ is a compact seven-manifold. This is the same solution of type IIB supergravity given in Ref. [11].

It was shown in Ref. [11] that $\mathcal{M}_7$ is the total space of a lens space $S^3/\mathbb{Z}_q$ fibration over the KE$_4$, where the twisting is parametrized by another positive integer p. The lens space fiber has coordinates y, z, ψ. In terms of our parameters $n_\pm$ we identify

$$p = kn_+, \qquad q = \frac{k}{I}(n_- - n_+), \qquad (13)$$

where $p, q \in \mathbb{N}$ are coprime. The Fano index I is the largest positive integer for which $\int_S c_1/I \in \mathbb{Z}$, for all two-cycles S in the KE$_4$, where $c_1 = [\mathcal{R}/2\pi] \in H^2(\text{KE}_4, \mathbb{Z})$. We have also defined

$$k = \text{hcf}(I, p), \qquad (14)$$

and identify ψ with period $\Delta\psi$ given by [17]

$$\Delta\psi = \frac{2\pi I}{k}. \qquad (15)$$

At a fixed point in the $D = 5$ spacetime, the internal five-dimensional metric $ds_{\text{SE}_5}^2 = (\frac{1}{3}d\psi + \sigma)^2 + ds_{\text{KE}_4}^2$ in Eq. (11) is then a *regular* Sasaki-Einstein manifold, which is simply connected when $k = 1$.

To obtain a string theory background one must also quantize the five-form flux F_5 through all five-cycles in $\mathcal{M}_7$. This was carried out in Ref. [11]. We define the integers

$$M = \int_{\text{KE}_4} c_1 \wedge c_1 = \frac{1}{4\pi^2} \int_{\text{KE}_4} \mathcal{R} \wedge \mathcal{R}, \qquad (16)$$

and $h = \text{hcf}(M/I^2, q)$. Then if we choose L to satisfy

$$\frac{L^4}{g_s \ell_s^4} = \frac{108\pi}{I^3 h} k^2 n_+ n_- n, \qquad (17)$$

where ℓ_s is the string length and $n \in \mathbb{N}$, then one finds that $1/(2\pi\ell_s)^4 \int_D F_5 \in \mathbb{Z}$, for all five-cycles D in $\mathcal{M}_7$.

There is a finite set of choices for the positively curved KE$_4$. If KE$_4 = \mathbb{CP}^2$ then $I = 3$, $M = 9$. For this case, $k = 1$ gives SE$_5 = S^5$ as the internal space while $k = 3$ gives $S^5/\mathbb{Z}_3$. If KE$_4 = S^2 \times S^2$ we have $I = 2$, $M = 8$. Now $k = 1$ gives SE$_5 = T^{1,1}$, while $k = 2$ gives SE$_5 = T^{1,1}/\mathbb{Z}_2$. Finally, for KE$_4 = dP_m$, $3 \leq m \leq 8$, where dP_m is a del Pezzo surface, we have $I = 1$, $M = 9 - m$.

A key observation is that $\mathcal{M}_7$ in the AdS$_3$ solutions of Ref. [11] may also be viewed as SE$_5$ fibrations over $\Sigma = \mathbb{WCP}^1_{[n_-, n_+]}$. We can begin with any weighted projective space, with weights $n_- > n_+$, and then define $p, q \in \mathbb{Z}$ via (13), where we also define

$$k = \frac{I}{\text{hcf}(I, n_- - n_+)}. \qquad (18)$$

With this definition, p and q are manifestly coprime, and one can check that Eq. (14) is equivalent to Eq. (18). With this perspective, we can calculate the flux of F_5 through the SE$_5$ fiber:

$$N \equiv \frac{1}{(2\pi\ell_s)^4} \int_{\text{SE}_5} F_5 = \frac{M}{I^2 h} k n_+ n_- n \in \mathbb{N}. \qquad (19)$$

Notice that for a given spindle, specified by $n_\pm$, and a given choice of KE$_4$ we only get a smooth type IIB solution for k as in Eq. (18) and hence a specific SE$_5$. E.g., if KE$_4 = \mathbb{CP}^2$ and $n_+ = 2$, then for $n_- = 3, 7, \ldots$ and $n_- = 5, 9, \ldots$ we can uplift on $S^5/\mathbb{Z}_3$ and S^5, respectively.

The central charge is given by $c = 3L/2G_{(3)}$, where $G_{(3)}$ is the Newton constant obtained by compactifying type IIB supergravity on $\mathcal{M}_7$ [18]. We can rewrite the result of Ref. [11] as

$$c = \frac{4(n_- - n_+)^3}{3n_- n_+(n_-^2 + n_- n_+ + n_+^2)} a_{4d}, \qquad (20)$$

where

$$a_{4d} \equiv \frac{\pi^2 N^2}{4\text{vol}(\text{SE}_5)}. \qquad (21)$$

In Ref. [11] the dual $d = 2$, $\mathcal{N} = (0, 2)$ SCFTs were not identified, but our $D = 5$ construction of the solutions, together with the flux condition (19), leads to a conjecture. Begin with the $d = 4$ SCFT dual to AdS$_5 \times$ SE$_5$, describing N D3-branes at the Calabi-Yau threefold singularity with conical metric $dr^2 + r^2 ds_{\text{SE}_5}^2$. The large N a-central charge of this theory is precisely given by a_{4d} in Eq. (21) [19]. One then compactifies that theory on $\Sigma = \mathbb{WCP}^1_{[n_-, n_+]}$, with a background R-symmetry gauge field with magnetic flux (7). The solutions we have described suggest the theory flows to a $d = 2$, $\mathcal{N} = (0, 2)$ SCFT in the IR, and we will give evidence for this below by computing the central charge via a field theory calculation.

The $U(1)_R$ symmetry of the (0,2) theory dual to the AdS$_3 \times \mathcal{M}_7$ solutions is realized by a Killing vector, R_{2d}, on $\mathcal{M}_7$. Using the results of Refs. [20,21] we deduce

$$R_{2d} = 2\partial_\psi + \frac{3n_- n_+(n_- + n_+)}{n_-^2 + n_- n_+ + n_+^2}\partial_\varphi. \qquad (22)$$

Here we have defined $\varphi = \frac{2\pi z}{\Delta z}$ so that $\Delta\varphi = 2\pi$, and ∂_φ generates the $U(1)$ isometry of the weighted projective space Σ, which we shall refer to as $U(1)_J$. Note that the Killing spinor on the SE$_5$ has unit charge under $R_{4d} = 2\partial_\psi$, which may therefore be identified with the superconformal $U(1)_R$ symmetry of the $d = 4$ SCFT before compactification on Σ. In other words, Eq. (22) states that the $d = 4$ R-symmetry mixes with $U(1)_J$ in flowing to the $d = 2$ R-symmetry in the IR. We shall also recover Eq. (22) from a field theory calculation in the next section.

$d = 4$ SCFTs on Σ.—We begin with a general $d = 4$ SCFT with anomaly polynomial given by the 6-form

$$\mathcal{A}_{4d} = \frac{\mathrm{tr}R^3}{6} c_1(R_{4d})^3 - \frac{\mathrm{tr}R}{24} c_1(R_{4d})p_1(TZ_6). \quad (23)$$

As is standard, in (even) dimension d the anomaly polynomial is a $(d+2)$-form on an abstract $(d+2)$-dimensional space, here called Z_6. $c_1(R_{4d})$ denotes the first Chern class of the $d = 4$ superconformal $U(1)_R$ symmetry bundle over Z_6, and p_1 denotes the first Pontryagin class. The trace is over Weyl fermions when the theory has a Lagrangian description, and in any case we may always write

$$\mathrm{tr}\, R^3 = \frac{16}{9}(5a_{4d} - 3c_{4d}), \qquad \mathrm{tr}\, R = 16(a_{4d} - c_{4d}), \quad (24)$$

in terms of the central charges a_{4d}, c_{4d}. We focus on the large N limit in which $a_{4d} = c_{4d}$ to leading order, and hence

$$\mathcal{A}_{4d} = \frac{16a_{4d}}{27} c_1(R_{4d})^3 \quad \text{(at large } N\text{)}. \quad (25)$$

We now compactify the $d = 4$ theory on $\Sigma = \mathbb{WCP}^1_{[n_-,n_+]}$, with magnetic flux (7) for the $d = 4$ R-symmetry gauge field. The resulting $d = 2$ anomaly polynomial will then capture the right-moving central charge c_r, but crucially we need to include the $U(1)_J$ global symmetry in $d = 2$ that comes from the isometry of Σ. Geometrically, this involves taking Z_6 to be the total space of a Σ fibration over a four-manifold Z_4 [22]. More precisely, we let J be a $U(1)$ bundle over Z_4, with connection corresponding to a background gauge field A_J for the $d = 2\, U(1)_J$ global symmetry, and then fiber Σ over Z_4 using the $U(1)_J$ action and connection A_J. In practice, this amounts to the replacement $d\varphi \mapsto d\varphi + A_J$.

Incorporating the magnetic flux (7) into this construction amounts to "gauging" the $U(1)$ gauge field A in Eq. (3), as just described. Thus, we define the following connection one-form on Z_6:

$$\mathcal{A} = \frac{1}{4}\left(1 - \frac{a}{y}\right)\frac{\Delta z}{2\pi}(d\varphi + A_J) \equiv \rho(y)(d\varphi + A_J), \quad (26)$$

where recall that the gauge choice we made is such that the Killing spinors are uncharged under the $U(1)_J$ symmetry generated by ∂_φ. This is necessary for the twisting to make sense. $\mathcal{A}$ defined by Eq. (26) is a gauge field on Z_6, which restricts to the supergravity gauge field A in Eq. (3) on each Σ fiber. We compute the curvature

$$\mathcal{F} = d\mathcal{A} = \rho'(y)dy \wedge (d\varphi + A_J) + \rho(y)F_J, \quad (27)$$

where $F_J = dA_J$. The one-form $d\varphi + A_J$ is precisely the global angular form for the $U(1)$ bundle, and so is globally defined on Z_6 away from the poles of $\Sigma = \mathbb{WCP}^1_{[n_-,n_+]}$ at

$y = y_1, y_2$. Moreover, one can verify that $\rho'(y)dy \wedge d\varphi$ vanishes smoothly at the poles, where the angular coordinate φ is not defined, implying that $\mathcal{F}$ is a globally defined closed two-form on Z_6. By construction, the integral of $\mathcal{F}$ over a fiber Σ of Z_6 satisfies Eq. (7). More generally, the integrals of wedge products over the fibers are given, for $s \in \mathbb{N}$, by

$$\int_\Sigma \left(\frac{\mathcal{F}}{2\pi}\right)^s = \frac{1}{2^s}\left(\frac{1}{n_+^s} - \frac{1}{n_-^s}\right)\left(-\frac{F_J}{2\pi}\right)^{s-1}. \quad (28)$$

The curvature form $\mathcal{F}$ defines a $U(1)$ bundle $\mathcal{L}$ over Z_6 by taking $c_1(\mathcal{L}) = [\mathcal{F}/2\pi] \in H^2(Z_6, \mathbb{R})$. This is different from Ref. [22], where the $U(1)$ bundle was taken to be the tangent bundle to the fibers $T_{\text{fibers}}Z_6$, which gives the Euler class (8), rather than Eq. (7). We note that at the poles we have $c_1(\mathcal{L})|_{y=y_1,y_2} = -\frac{1}{2n_\pm}c_1(J)$, where we have defined $c_1(J) = [F_J/2\pi] \in H^2(Z_4, \mathbb{Z})$. In the anomaly polynomial we then write

$$c_1(R_{4d}) = c_1(R_{2d}) + c_1(\mathcal{L}), \quad (29)$$

where R_{2d} is the pullback of a $U(1)$ bundle over Z_4. Notice that the twisting (29) will make sense globally only if the $d = 4$ R charges of fields satisfy appropriate quantization conditions, and for gauge-invariant operators this is equivalent to the global regularity and flux quantization conditions imposed on the supergravity solutions, cf. the discussion below Eq. (19).

The $d = 2$ anomaly polynomial is obtained by integrating $\mathcal{A}_{4d}$ in Eq. (25) over Σ. Using Eqs. (28) and (29) we compute

$$\mathcal{A}_{2d} = \frac{2a_{4d}}{27}\left[12\left(\frac{1}{n_+} - \frac{1}{n_-}\right)c_1(R_{2d})^2 \right.$$
$$\left. -6\left(\frac{1}{n_+^2} - \frac{1}{n_-^2}\right)c_1(R_{2d})c_1(J) + \left(\frac{1}{n_+^3} - \frac{1}{n_-^3}\right)c_1(J)^2\right].$$
$$(30)$$

The coefficient of $\frac{1}{2}c_1(L_i)c_1(L_j)$ in $\mathcal{A}_{2d}$ is $\mathrm{tr}\gamma^3 Q_i Q_j$, where the global symmetry Q_i is associated to the $U(1)$ bundle L_i over Z_4, and γ^3 is the $d = 2$ chirality operator. On the other hand, c extremization [12] implies that the $d = 2$ superconformal $U(1)_R$ extremizes

$$c_{\text{trial}} = 3\mathrm{tr}\,\gamma^3 R_{\text{trial}}^2, \quad (31)$$

over the space of possible R-symmetries. We set

$$R_{\text{trial}} = R_{2d} + \varepsilon J, \quad (32)$$

and extremize the quadratic function of ε one obtains from Eqs. (30) and (31). The extremal value is

$$\varepsilon_* = \frac{3n_- n_+ (n_- + n_+)}{n_-^2 + n_- n_+ + n_+^2}. \tag{33}$$

The right-moving central charge is given by Eq. (31) evaluated on the superconformal R-symmetry [12]. Substituting Eq. (33) into Eqs. (31), (32) we find

$$c_r = \frac{(n_- - n_+)^3}{n_- n_+ (n_-^2 + n_- n_+ + n_+^2)} \frac{4a_{4d}}{3}. \tag{34}$$

At leading order in N, $c_r = c_l \equiv c$ is the central charge of the SCFT, and we see that the field theory result (34) precisely matches the gravity result (20). Moreover, the R-symmetry (32), with $\varepsilon = \varepsilon_*$, precisely matches the supergravity R-symmetry (22).

Discussion.—Our solutions exhibit a number of new properties, raising several directions for future research. First, despite having orbifold singularities in $D = 5$, when uplifted to $D = 10$ the solutions are completely regular. What type of singularities are permitted in lower-dimensional supergravity theories that have this property? Second, it is often claimed that supersymmetry requires a topological twist when branes wrap a compact manifold, so that the "twisted spinors" are constant on the manifold. Our near horizon solutions are a counterexample and it would be interesting to understand this more systematically. Third, our results suggest there should exist black string solutions which approach AdS$_5$ in the UV and AdS$_3 \times \Sigma$ in the IR. Such solutions will reveal the precise deformations of the $d = 4$, $\mathcal{N} = 1$ SCFTs that can then flow to the $d = 2, \mathcal{N} = (0, 2)$ SCFTs dual to the AdS$_3 \times \mathcal{M}_7$ solutions of Ref. [11], including the way in which the D3-brane wrapping the spindle is preserving supersymmetry.

In Ref. [15] we present analogous supergravity solutions in $D = 4$, associated with M2-branes wrapped on a "spinning spindle." In that case the full black hole solution is known and it approaches AdS$_4$ in the UV and AdS$_2 \times \Sigma$ in the IR, with a spindle horizon Σ. The $D = 4$ black hole is accelerating and this leads to the conical deficit singularities on Σ. Once lifted to $D = 11$ these orbifold singularities are removed and the solutions become completely regular. Supersymmetry is again not realized by the topological twist for the AdS$_2 \times \Sigma$ solution, similar to our AdS$_3 \times \Sigma$ solutions. In the UV, the black holes at finite temperature have a conformal boundary consisting of a spindle. In the supersymmetric and extremal limit, however, the spindle degenerates into "two halves," each of which *is* associated with a topological twist, but with a different constant spinor on each component. It is another fascinating open question to determine how typical such a novel realization of supersymmetry is for branes wrapping spindles, as well as spaces of higher dimension.

Our results also suggest many questions on the field theory side. When defining a SCFT on a spindle what additional data should be specified at the orbifold points?

Considering weakly coupled $\mathcal{N} = 4$ SYM theory would be an important first step. Our holographic analysis shows that SCFTs dual to regular SE manifolds can only be placed on certain spindles and there is an apparent obstruction for those dual to irregular SE manifolds; why is this? The anomaly polynomial technique we employed is based on smooth manifolds and yet it gives a consistent result in the context of the orbifolds we studied, in the large N limit. It would be interesting to justify this more systematically (also see Ref. [23]) and determine subleading contributions.

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