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# Mathematics of a Mantra

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**Abstract:** This paper aims to examine the enunciation (*uccāraṇa-kāla*) time intervals for *śrīvidyā pañcadaśī*, a fifteen seed-syllable *mantra* (*bija-mantra*) related to the homonymous *śākta* school *Śrīvidyā* or *Traipuradarśana*. Following the indications provided in the *Yoginīhṛdaya*, with *Dīpikā* commentary by Amṛtānanda, and the *Varivasyāraharasya* by Bhāskararāya with *Prakāśa* auto-commentary, the research finds that these durations are not arbitrary at all but rather the result of a rigorous assessment. Moreover, the duration values suggest a specific conceptual goal that the mathematical rigor manifested by the authors seeks to fulfil: the progressive diminution of time intervals in order to achieve an atemporal dimension. The choice of the units of measurement itself is designed to meet this metaphysical and ritual need. By counting the intervals (both relative and overall) of *mantra* recitation, it is also possible to confirm the resonance nature of the sounds following nasalisations, sounds conceived by the authors as entirely independent of the reciter's phonatory activity.

**Keywords:** Mantra, mantra recitation, *Śrīvidyā*, Indian mathematics, Indian metrology

## 1 Introduction

*Śrīvidyā* is the name under which is commonly known a particular *śākta* tradition (*paramparā*) devoted to the deity Rājārājeśvarī Śrīlālītā Mahātripurasundarī – the ‘Beauty of the three cities’, ultimately recognised as pure consciousness (*cidrūpiṇī*).<sup>1</sup> Close to the Kashmir Śaivism theoretical

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<sup>1</sup> Goudriaan (1979: 6–7): “Śāktism is defined in different ways [...] Sometimes it is incorrectly identified with ‘the cult of female deities in general’ [...] It can be shortly characterized as the worship of Śakti [...] i. e. the universal and all-embracing dynamism which manifests itself in human experience as a female divinity. To this should be added that inseparably connected with her is an inactive male partner as whose power of action and movement the Śakti functions [...] Although Śāktism is often defined also by means of typical ritual practices, it is advisable to restrict the use of this term for a world view oriented towards Śakti, while Tantrism should be applied to a conglomerate of ritual and yogic practices and presuppositions”. For a general

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framework, Śrīvidyā fully accepts its *ṣaṭtriṃśat-tattva* cosmology (the thirty-six principles of reality) and its emphasis on the triadic structure of the real, such as in the case of the knower (*jñātṛ*), the knowledge (*jñāna*) and the known (*jñeya*).<sup>2</sup> Śrīvidyā adopts also the Kashmiri “sophisticated speculation regarding the nature of sound and the corresponding technical vocabulary of *mantra* theory and practice”<sup>3</sup>. In particular, cosmic manifestation (*śṛṣṭi*) itself is here conceived as essentially phonic in nature; thereby *mantra*, deity (i. e. consciousness) and world do appear as radically nondifferent (*abheda* or *abhinna*).<sup>4</sup>

Furthermore, “Śākta Tantric theology [...] espouses a peculiar form of *samuccayavāda* or ‘combination doctrine’. *Samuccayavāda* is the doctrine that final liberation (*mokṣa*) must involve a combination of knowledge (*jñāna*) and action (*karma*), including specialized forms of ritual performance”<sup>5</sup>. That explains why the *Yoginīhṛdaya* (YH)<sup>6</sup> and *Varivasyāraharasya*

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survey of *śākta* schools, Śrīvidyā school and the texts under consideration, see Goudriaan (1981), part I, *Hindu Tantric Literature in Sanskrit*; Brooks (1990); Brooks (1992). For a framing of *śākta* doctrine as part of the broader *śaiva* horizon, see Sanderson (1988); Sanderson (2014: 65–91). On the connections between Śrīvidyā, *advaita* and orthodox brahmanical milieu, cf. in particular Pellegrini (2013: 53–81).

2 Padoux (1994: 15): “The philosophical notions of YH are those of non-dualist Kashmiri śaivism. The supreme Reality is transcendent, without division (*niṣkala*), transcending space and time, pure light (*prakāśa*), consciousness (*saṃvit*). It is also the phoneme A, the ‘peerless one’ (*anuttara*). This absolute flashes forth, vibrates luminously (*sphuratta*, *ullāsa*). It expands as a luminous wave (*sphuradūmi*) by its own free will (*svecchayā*), and thus manifests the cosmos made up of the thirty-six *tattvas*, from Śiva to *pṛthivī*”.

3 Brooks (1990: 76).

4 Brooks (1990: 60): “Creation in its material manifestations is paralleled in sounds that are metalinguistic formations reflecting the original state of ontological unity. The Absolute Brahman in its most subtle, prearticulated form is ‘sound’ (*vāc*). Brahman-as-sound (*śabda-brahman*) [...] has gradually devolved from a state of transcendent unity into subtle and increasingly more mundane forms of sound and language. Mantras are the means by which the adept taps into this primordial resonance that pervades creation”.

5 Brooks (1990: 49).

6 Regarding the dating of YH, see Padoux (1994: 10): “It seems unlikely that the date of the YH can be earlier than the XI c. at most, and might be later”; “especially if we believe, with Sanderson, that some passages of the YH (śl. 1.156, for instance) reflect Kṣemarāja’s [fl. c. 1000–50] version of the *Pratyabhijñā*” (n. 5). Amṛtānanda — fl. c. 1325–75; cf. Sanderson (1988: 690); Sanderson (2014: 72) — claims that he is the first commentator of the text: YH-Dī (Amṛtānanda’s *Dīpikā* comment): *tad anekārthasandarbhānānāsaṃketasaṃkulaṃ | vivṛṇomy amṛtānandaḥ śivayor eva śāsanāt || 8. anyathā ’nādisaṃsāre kiṃ nedaṃ vyākṛtaṃ purā | tadā na santi santaḥ kiṃ kiṃ vā nātra prayojanam || 9. śivādīguruparyantaṃ pāramparyakramāgatam | etaḥ jñānaṃ mayā labdham akramāṇām agocaram || 10.*

“As ordered (*śāśana*) by the two benevolents (*śiva*; i. e. Śiva and Śakti), I, Amṛtānanda, will comment (*viṃṛ*) this collection of practices (*saṃketa*; lit. ‘agreement’, ‘meeting’). I follow here Padoux

(VVR)<sup>7</sup> thoroughly describe the structure, function and meaning of the *śrīvidyā pañcadaśī mantra* (ŚPM), the Śrīvidyā most sacred and meaningful *mantra*.

For the purposes of this article suffice it to note that ŚPM consists of fifteen (*pañcadaśa*) seed syllables (*bijākṣara*), the origins (*uddhāra*) of which, according to texts, can be traced from preceding authoritative sources.<sup>8</sup> YH and VVR diverge solely in the first three syllables of the *mantra*. The former represents the *hādi* form of the ŚPM, the first phoneme of which is *ha*; the latter, instead, proposes the *kādi* version of the ŚPM, the first phoneme of which is *ka*.

The ŚPM is divided into three sets or *kūṭa*<sup>9</sup>, appearing in the following succession: *vāg-bhāva-kūṭa*, composed of four members: *ha, sa, ka, la* (YH)<sup>10</sup> or *ka, e, ī, la* (VVR); *kāmā-rāja-kūṭa*, composed of five members: *ha, sa, ka, ha, la*; *śakti-kūṭa*, composed of three members: *sa, ka, la*. The phoneme *hṛīm̐*, called *hṛllekhā*, follows at the end of each set to comprise a sum of fifteen *bījas*. According to the description offered by the texts under consideration, the ŚPM appears as follow:

<i>vāgbhāvakūṭa</i>	<i>ha, sa, ka, la, hṛīm̐</i> (YH)
	<i>ka, e, ī, la, hṛīm̐</i> (VVR)
<i>kāmārājakūṭa</i>	<i>ha, sa, ka, ha, la, hṛīm̐</i>
<i>śaktikūṭa</i>	<i>sa, ka, la, hṛīm̐</i>

*Mantra* recitation (*japa*) is certainly part of a larger ritual, as clearly described in the three sections of YH.<sup>11</sup> Nevertheless, it seem to acquire a special relevance in personal, internal spiritual discipline (*sādhana*) — especially in Bhāskararāya’s work. Perfectly focused repetition, with perfect timing (VVR 49–51)<sup>12</sup> — along

(1994: 16, 96 n.9), who translates: ‘*prātiques*’) covering manifold topics (*artha*) This is a knowledge (*jñāna*) I acquired through the lineage (*pārampara*) from Śiva himself up to my *guru*, and which is inaccessible (*agocara*) out of this transmission. Otherwise, in the context of the beginningless *saṃsāra*, why would it not have been explained yet? Were here scholars or reasons perhaps missing?”

<sup>7</sup> Regarding the life and work of the Maharashtrian brahmin Bhāskararāya, flourishing in Tamil-Nadu in the first half of XVIII c. (1690–1785), cf. Sanderson (2014: 72); see also S. Śāstrī, *Introduction to Varivasyāraharasya*, (1941: XXIII–XLIII). *Prakāśa* auto-commentary, from here onwards referred to as VVR-Pr.

<sup>8</sup> VVR-Pr. 8–11, for instance, identifies it as deriving from *Tripurā Upaniṣad*, 8.

<sup>9</sup> YH-Dī, 2.64–65a., defines *kūṭa* as *akṣarapiṇḍa*, syllabic aggregate.

<sup>10</sup> YH-Dī, 2.17; quoting extensively from the *Saubhāgyasudhodaya*, Amṛtānanda’s commentary on *Nityāśoḍaśikāṃava*.

<sup>11</sup> *Cakra-saṃketa*, devoted to the description of the *śrīyantra* diagram; *Mantra-saṃketa*, describing the ŚPM; *Pūjasaṃketa*, discussing the ritual in its entirety.

<sup>12</sup> Here, a perfect timing is explicitly meant as the prime factor (*nidāna*) in realisation (*abhi-vyakti*) of *caitanya* (consciousness). VVR 49b: *tad idaṃ caitanyābhivyaktinidānaṃ [...]*.

with complex practices of visualizations (*bhāvanā*)<sup>13</sup> and projections (*nyāsa*)<sup>14</sup> — will assure the attainment of the ultimate goal of *mokṣa*.

It is also straightforwardly evident that ŚPM is not at all understandable in any natural language. In spite of that, both YH and VVR place the greatest emphasis on its manifold hidden meanings, respectively six and fifteen.<sup>15</sup> Bhāskararāya (for instance in VVR 54–55) takes in fact a clear stand in favour of the radical meaningfulness of Tantric *bīja mantras*: knowing the fifteen ŚPM meanings — which actually retrace all the main tenets of the Śrīvidyā doctrine — is an unavoidable condition for its own effectiveness.<sup>16</sup>

## 2 Analysis of *hṛllekhā*

Stanzas 12 and 13 of VVR and the commentary on them are focused on *hṛllekhā-svarūpa*, that is, the description, composition and duration of the ‘own form’ of *hṛllekhā*. Following the denomination suggested by *Prapañcasāra-tantra*<sup>17</sup>, Bhāskararāya systematically employs the term *hṛllekhā* to indicate the *bīja* ‘*hṛīm*’. This, he asserts in light of YH, must be conceived as a compound (*saṃhati*) of

<sup>13</sup> Padoux (2013: 182): “*bhāvanā*: intense creative and identifying meditation”.

<sup>14</sup> Brooks (1990: 59): “In sectarian Śākta traditions, ritual *nyāsa* is a process by which the pantheon of subdeities is systematically identified with aspects of the Supreme Śakti and with the individual’s various physical, verbal, and mental components. In the second stage, *nyāsa* is designated to dissolve the distinctions that separate the Tantric adept from the source of his or her ultimate being (*sat*), consciousness (*cit*), and bliss (*ānanda*). In contrast to the classical Sāṅkhya-Yoga model in which the yogi perfectly isolates the spirit (*puruṣa*) from matter (*prakṛti*), the Tantric yogi achieves a reunification of the material and efficient elements that constitute reality. In effect, the Tantric re-cognizes (*pratyabhijñā*) the ultimate nondistinction of creation and creator”.

<sup>15</sup> Cf. YH 2.15–76 and VVR 57–147.

<sup>16</sup> VVR 54–55: *nārthajñānavihinaṃ śabdasyoccāraṇaṃ phalati | bhasmani vahnivihine na prakṣiptaṃ havir jvalati || artham ajānānāṃ nānāvīdhaśabdāmātrapāṭhvatām | upameyaś cakrīvān malayajabhārasya voḍhaiva ||*. “Pronunciation without a knowledge of the true import bears little fruit, (even as) the offering thrown over ashes without fire does not burst into flame. To those who are merely conversant with the recital of the different sounds without a knowledge of their meaning may be compared the donkey with a load of sandalwood, grown in the Malaya mountains, on its back” (VVR: 35–36). On the debate about meaningfulness/meaninglessness of *mantras*, see Alper (1989).

<sup>17</sup> See *caturthaḥ paṭalaḥ* (Ch. IV); but it is also mentioned in Ch. II. The *Prapañcasāra-tantra* is an “anonymous digest of Mantraśāstra”. Ascribed to Śaṅkara, it is plausible that it “was written at an early date by a member of one of the monastic communities founded by him”; Goudriaan (1981: 131). *Hṛllekhā* also occurs in another text ascribed to Śaṅkara: *Ānandalahari*, 32c.

twelve elements: three main components plus nine subsequent sounds collectively called *nāda*, subtle sound or resonance.<sup>18</sup> According to this perspective, therefore, *ḥṛllekhā* is composed of: the aspiration *h*, spoken as *vyoman*; the semivowel *r*, called *agni*, and the long vowel *ī*, *vāmalocanā*. These three main components are followed by *nāda*, made up of *bindu* (the *anusvāra* or nasalization; i. e. the ‘*m̐*’ sound) plus eight additional sounds respectively termed *ardhacandra*, *rodhinī*, *nāda* (strictly speaking, not as a collective name for the entire nine-term *nāda* sequence), *nādānta*, *śakti*, *vyāpikā*, *samanā*, and *unmanī*.<sup>19</sup>

According to VVR 15–18, *ḥṛllekhā* devoid of *bindu* (i. e. the syllable *hṛī*) is estimated to equal three *mātrās* (time units or morae) in that it is formed of two consonantal sounds (each equaling half a *mātrā*) plus a long vowel (*dīrgha*, equal to two *mātrās*).<sup>20</sup> *Bindu*, by virtue of its consonantal nature (*vyañjanatva*), lasts half a mora<sup>21</sup>. The subsequent elements of *nāda* are held to be shorter and shorter by successive halving. Thus *ardhacandra* equates to half of a *bindu*, lasting a quarter of a *mātrā*; *rodhinī*, equating to half of *ardhacandra*, lasts an eighth of a *mātrā*, and so on. *Unmanī*, in contrast, exists outside of time: *unmanāyāstu nāsty eva kālaḥ*.<sup>22</sup> As such, it will be excluded from the reckoning of the ŚPM durations in this article (cf. Table 1).

Bhāskaraṛāya states that the total duration of *nāda* conceived in this way and including *bindu* amounts to one *mātrā* minus one *lava*. YH 1.29–34 lays out the same

**18** Padoux (1990: 96, n. 30): “Yoga and Tantrism [...], since these systems give a prominent place to cosmic or inner sound, [use the term] *nāda* to indicate a particularly subtle form of sound, a form that appears more akin to the resonance following a sound than to the sound itself; this is felt as too closely connected with the physical means wherefrom it arose to be considered as subtle. The term occurs in this sense, for instance, in Yoga Upaniṣad such as *Dhyānabindu* or the *Nādabindu*, where *nāda* is important in the meditation of *OM* associated with the arousal of the *kuṇḍalini*”.

**19** In this paper I will assume a basic difference between phonemes or clearly articulated sounds (*varṇa*) and sonic resonances or subtle sounds (*dhvani* or *nāda*). It will be argued that the sounds starting from the *anusvāra* of the *bija* ‘*hṛīm̐*’ (i. e. from ‘*m̐*’ onwards), corresponding to what YH and VVR term *nāda*, are *dhvani* and not *varṇa*. For a definition of *varṇa*, cf. n. 20. About *dhvani*, as subtle sound or sonic resonance comparable to the progressive lessening of a bell tolling (VVR-Pr 12–13; YH-Dī 2.37a, 3.169–170), cf. n. 18 (*nāda*).

**20** Allen (1953: 83–84): “From the point of view of syllabic structure the prosody of vowel-length is of great importance. The device adopted by the Indians for purposes of phonological description is that of the *mātrā* or ‘mora’. [...] The *mātrā* device has an evident utility in a system where the basic vowel-units are considered as members of quantitatively similar pairs each comprising a short and long member, and where the junction of two similar short vowels results in the corresponding long vowel (e. g. *divi* + *iva* = *diviiva*)”. Cf. also: *infra*, § 5 *The single components and overall durations*.

**21** *bindor api vyañjanatvād ardhāmātrā*, VVR-Pr 15–16.

**22** VVR 15–17a; or, as synthetically stated in YH-Dī 3.186, *kālātīva*.

**Table 1:** The ‘own form’ (*sva-rūpa*) of *hṛllekhā* according to VVR 12–13.

<i>hṛllekhā</i> : a 12-element set ( <i>saṃhatī</i> ) = 3 + 9				
3 <i>varṇas</i> (phonemes)		<i>nāda</i> (subtle sound), composed of 9 elements (1+7+1)		
<i>vyoman</i>	<i>h</i> ( <sup>1</sup> / <sub>2</sub> )	<i>bindu</i> ( <i>m̐</i> )	<i>ardhacandra, rodhinī, nāda, nādānta, śakti, vyāpikā, samanā</i>	<i>unmanī</i>
<i>agni</i>	<i>r</i> ( <sup>1</sup> / <sub>2</sub> )			
<i>vāmalocanā</i>	<i>ī</i> (2)			
$2(\frac{1}{2}) + 2 = 3$ <i>mātrā</i>		$\frac{1}{2}$ <i>mātrā</i>	lasting half the length of the previous element    atemporal	

succession and time sequence<sup>23</sup>: Amṛtānanda (YH-Dī 1.32–34) reckons the enunciation time of *bindu* amounts to half a mora, while the series from *bindu* (excluded) to *samanā* lasts half a mora minus (*paryantam*) one *lava*<sup>24</sup>: *bindvādisamanāntānām ardhāmātrā, tadardhādikrameṇa lavaparyantam uccāraṇakālah*.<sup>25</sup> In VVR 15–17a, the *mātrā-lava* ratio is explicated as one to 256 and both Amṛtānanda and Bhāskararāya affirm that *samanā* is equal to  $\frac{1}{256}$  of *mātrā*, that is to say, one *lava*.<sup>26</sup> According to the definition proposed by *Prapañcasāratantra*<sup>27</sup>, having taken in hand a

**23** Although the eleventh element, pronounced as *manonmanī*, seems to diverge, Amṛtānanda plainly states that it is *samanā* (YH-Dī 1.32–34).

**24** *Lava* is a pivotal concept in this paper, see *infra* for a detailed explanation. Böhtlingk (B) (1879–1889: V, 220) and Monier-William (MW) (1899: 898) define *lava* as the act of cutting; anything cut off, a section or fragment; a minute division of time; the numerator of a fraction; loss, destruction. According to *Vācaspatyam*, vol. VI, p. 4823, *lava* ‘in the meaning of: *leśe* (small part or portion, particle, atom; B: V, 23; MW: 903), *vināśe* (loss, annihilation, decay; B: VI, 99; MW: 968), *chedane* (cutting, splitting, breaking; section, part; B: II, 246; MW: 407), *kāla-parimāṇa-bhede* (time unit fraction). In *Śabda-kalpādruma*, vol. IV, 210: *vlū+ap* (to cut, sever, pierce, annihilate; B: V, 231; MW: 905), *leśa, chedana, kāla-bheda*.

**25** *Lit.*: half of a mora limited by a *lava*. Padoux (1994: 134), translates it differently: “de *bindu* à *samanā* il est d’une demi-more. Au-delà de cette [demi-more] il ne dépasse pas un *lava*”. Nevertheless, he raises the question in a note (134, n.167): “On vient pourtant d’expliquer que seul *bindu* dure une demi-more. [...] Ou faut-il comprendre que c’est tout ce qui suit *bindu*, jusqu’à *samanā*, qui, pris ensemble, ne fait qu’une demi-more?”. As I will show, it is possible to answer this question in the affirmative. Here the complete passage: *hākarādibindvantānām sthūlavaraṇānām uccāraṇakālo mātrā, bindvādisamanāntānām ardhāmātrā | tadardhādikrameṇa lavaparyantam uccāraṇakālah |*.

**26** *samanāyā eko lavaḥ* (VVR-Pr 15–17a; YH-Dī 1.32–34)

**27** *Prapañcasāratantra*, 1.29–30 (1935: 12): *naliniṇpatrasaṃhatyāṃ sūkṣmasūcyabhibhedane || 29. dale dale tu yaḥ kālah sa kālo lavavācakah |*. Passage quoted, with some minor variations, in YH-Dī 1.32–34 ([...] *abhivedhane* [...]) and in VVR 15–17a. ([...] *saṃhatyāḥ* [...] *abhivedhane* [...] *lavasaṃjīṭāḥ*).

well-stretched bundle of lotus leaves, a *lava* is the intervening time necessary to reach the second leaf after having pierced the first one with a thin needle.

At first glance, the assertions made by the two commentators appear to be completely arbitrary; at best, they would seem to allude to a duration that gradually diminishes until reaching its minimal terms, thus resulting in an atemporal dimension, *unmanī*. It might not straightforwardly appear that it is a question of a weighted figure.

### 3 Geometric progressions

*Nāda* — the nine-sound set starting from the nasalisation of the seed-syllable ‘*hṛīm̄*’ — is, as stated, a succession the terms of which occupy a constant ratio of  $1/2$ , that is to say, in which each term amounts to one half of the previous one. Therefore, the duration of the nine sounds takes the form of a geometrical progression. If this is indeed true, as with any ordinary progression it remains to be seen what the results might be.

If  $a$  is the first term of a progression,  $n$  the number of its terms and  $r$  its common ratio (i. e. the fixed value of the ratio of its terms), we have:

$$a = (a_1, a_2 = 1/2 a_1, a_3 = 1/2 a_2 = 1/2 \cdot 1/2 a_1, \dots a_8 = 1/2 a_7)$$

Applying the formula [1] for  $a_1 = 1/2$  (i. e. *bindu*), we promptly obtain the first significant result: the value of each term in the progression (see Table 2)

**Table 2:** Synopsis of *mātrā* fractions.

<i>varṇa &amp; dhvani</i>	<i>mātrā</i>	<i>lava</i>
<i>laghu akṣara</i>	1	256
<i>bindu</i>	1/2	128
<i>ardhacandra</i>	1/4	64
<i>rodhinī</i>	1/8	32
<i>nāda</i>	1/16	16
<i>nādānta</i>	1/32	8
<i>śakti</i>	1/64	4
<i>vyāpikā</i>	1/128	2
<i>samanā</i>	1/256	1
<i>unmanī</i>	atemporal	
tot. <i>bindu-samanā</i>	255/256	255

expressed in fractional notation with respect to the unit represented by the *mātrā*; specifically its eighth element, *samanā*.

$$\begin{aligned} a^k &= a_1 r^{k-1} \\ a_8 &= \frac{1}{2} \cdot \left(\frac{1}{2}\right)^7 = \left(\frac{1}{2}\right)^8 = \frac{1}{256} \end{aligned} \quad [1]$$

As the authors accurately claimed, *samanā* lasts  $\frac{1}{256}$  of a *mātrā*, i. e. one *lava*. The first conclusion we can draw in this case, therefore, is that the proposed value cannot be considered arbitrary; rather, it is the result of a careful mathematical ponderation.

The second datum Bhāskararāya proposes is the duration of the eight-sound set, from *bindu* to *samanā*: as we have seen, one *mātrā* minus one *lava*. Let us try to estimate this value. The formula [2] offers the subtotal of a convergent geometrical progression defined by its first eight terms, the first one being  $\frac{1}{2}$  (i. e. *bindu*), with  $r = \frac{1}{2}$ :

$$\begin{aligned} \sum_{k=1}^n r^k &= \frac{1 - r^{n+1}}{1 - r} - 1 \\ \sum_{k=1}^8 \left(\frac{1}{2}\right)^k &= \frac{1 - \left(\frac{1}{2}\right)^9}{1 - \frac{1}{2}} - 1 = 2 - 2 \cdot \left(\frac{1}{2}\right)^9 - 1 = 1 - \left(\frac{1}{2}\right)^8 = \frac{255}{256} \end{aligned} \quad [2]$$

Now, 1 minus  $\frac{1}{256}$  — that is, one *mātrā* minus the value obtained for one *lava* — is equal to  $\frac{255}{256}$ , precisely the value resulting from the subtotal estimated here above. It is evident then that the result would not be different even if we were to use the data provided by YH-Dī, the only difference being a seven-term sequence starting from  $\frac{1}{4}$  *mātrā* — that is, *nāda* devoid of *bindu*.<sup>28</sup>

Therefore, the second inference we can make is that Bhāskararāya and Amṛtānanda recognized not only the fractional value of one *lava* ( $\frac{1}{256}$  of a *mātrā*), but also the overall amount of the whole sequence, which is — as shown above — perfectly complementary to the *lava* in comprising the unit (cf. Table 2).

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**28** Let us try to subtract  $\frac{1}{2}$  (the *bindu* duration) from  $\frac{255}{256}$  (the progression overall duration):  $\frac{255}{256} - \frac{1}{2} = \frac{127}{256}$ . Let us now sum  $\frac{1}{256}$  of a *mātrā* (the value of one *lava*) to the obtained result:  $\frac{127}{256} + \frac{1}{256} = \frac{1}{2}$ ; QED. Cf. YH 1.29–34, YH-Dī 1.32–34; cf. also *supra*, n. 11.



## 4 The *Līlāvati* tools

As demonstrated in the previous paragraphs, the YH commentator and VVR author do not assert arbitrary duration values; on the contrary, they appear to have calculated these values with remarkable precision. Having proven the reliability of the figures offered by the texts, the question becomes one of trying to achieve them — intuitive though they may seem — through the mathematical tools likely available to the authors.<sup>29</sup>

One possible answer might be found in the most renowned and widespread handbook of mathematics in ancient India, the *Līlāvati* by Bhāskara II.<sup>30</sup> The text offers a set of formulas concerning progressions, *śreḍhī vyavahāra*<sup>31</sup>, both arithmetical and geometrical. Here, for instance, is *karaṇasūtra* number 127:

*viṣame gacche vyeke guṇakaḥ sthāpyaḥ same 'rdhite vargaḥ |*  
*gacchakṣayāntam antyād vyastam guṇavargajam phalam yattat |*  
*vyeḥkam vyeḥkaguṇoddhṛtam ādiguṇam syāt guṇottare gaṇitam ||.*<sup>32</sup>

Here is an operational translation of Bhāskarācārya's rule:

Subtract one (*vyeḥka*) if the number is odd (*viṣama*); this will function (*sthāpya*) as a multiplier (*guṇaka*). Otherwise, after having halved (*ardhita*) an even number (*sama*), the result will function as a square (*varga*). Calculate multiplications and squares starting from the last term of the sequence until 0 is reached (*kṣaya-antam antyād*). The result (*phala*) minus one, divided (*uddhṛta*) by the common ratio (*guṇa*) minus one, will be multiplied by the first term. That is the sum of the progression (*gaṇita*).<sup>33</sup>

<sup>29</sup> It goes without saying that in such geometric simple progressions, like the one proposed in YH and VVR, their overall sum will always equal the unit minus their last term. Nonetheless, at least a certain degree of expertise must be prior to this general statement. In order to avoid any gap, my aim in sketching out here *Līlāvati* 127 procedure — as an instance of a *śreḍhī* formula — is limited to show that a *śreḍhī* could be readily reachable even with a basic mathematical background (cf. n. 35) through the application of a simple formula, as plainly reported in a widely used handbook.

<sup>30</sup> For an initial, general survey of the history of Indian mathematics, see Burton (2011: 225–228). For biographical information about Bhāskara II or Bhāskarācārya (born 1114), see Plofker (2009: 318), Appendix B. Regarding Bhāskara II's "standard" texts, see: § 6.2, 182–207.

<sup>31</sup> Bhāskara II (1938: 42). See also Āryabhaṭa's *Āryabhaṭīya* (composed around 500 c.e.), Ch. 2, 19–22, regarding progression, successive sequences or series.

<sup>32</sup> Quoted from Bhāskara II (1938: 55). According to the Patwardhan et al. numbering, it is rule n. CXXXVI; according to Colebrookes' system, it is n. 127.

<sup>33</sup> Colebrookes' translation: "The period being an uneven number, subtract one, and note 'multiplier'; being an even one, halve it, and note 'square', until the period be exhausted. Then the produce arising from multiplication and squaring in the inverse order from the last, being lessened by one, the remainder divided by the common multiplier less one, and

What follows is an analytical description applied to the present case: the series of eight (plus one) *nāda* sounds. To make it easier to explicate, let us divide the rule application algorithm into three steps.

Step I.

1. Assume  $n$  is the number of terms in the series (in the case in question:  $n = 8$ ).
2. If  $n$  is even: calculate  $n/2$  and mark V (*varga*, square).
3. If  $n$  is odd: calculate  $n - 1$  and mark G (*guṇaka*, multiplier).
4. Repeat steps 2 and 3 until the result is 0.

Applying *Step I* to our case, for  $n = 8$ , the following *Instructions table* is generated<sup>34</sup>.

8	
4	V
2	V
1	V
0	

Step II:

1. Arrange an *Index* column, from 1 to 8. According to the *Līlāvātī* prescription, follow an order that is the reverse of the *Instructions table* and match the latter to the *Index column*, setting out an *Operation column*. Establish an empty *Output column*.
2. In the corresponding ‘*Output-Index 1*’ cell, mark  $r = 1/2$  (i. e. the ratio).
3. Follow the instruction specified in the *Operation column* left cell:  
if G, then multiply by  $r$   
if V, then square the number
4. Mark the result in the *Output* cell below.
5. Repeat steps 3 and 4 until reaching the last term of the *Index*.

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multiplied by the initial quantity, will be the sum of a progression increasing by a common multiplier”, Bhāskara II (1893: 71).

The significance of this rather abstruse passage may not be immediately apparent, on a first reading. While the second part appears quite similar to that prescribed by the formula [2], the first one would seem to be an alternative solution to avoid calculating  $r^{n+1}$ , preferring to instead break it up into a series of multiplications and squares.

<sup>34</sup> The idea of displaying the calculation procedure in tables is borrowed from Bhāskara II (2001: 110); the tables have been partly modified.

6. Name the value indicated in ‘*Output-Index 8*’ cell ‘*Step II Output*’ (marked in bold in the table).

Index	Operation	Output
1	V	1/2
2	V	1/4
4	V	1/16
8		<b>1/256</b>

Step III.

1. Subtract one from ‘*Step II Output*’; [(*Step II Output*) – 1 = x]
2. Divide the result by the progression’s common ratio minus one; [x/(r – 1) = y].
3. Multiply the results by the first term of the progression; [y·n<sub>1</sub>].
4. The value obtained is equal to the sum of the progression.

Therefore:

$$\frac{\frac{1}{256} - 1}{\frac{1}{2} - 1} \cdot \frac{1}{2} = \frac{255}{256}$$

The procedure set out in the *Lilāvati* allows us to achieve a result which corresponds perfectly to the one obtained by applying the formula [2] (i. e.  $\frac{255}{256}$  of a *mātrā*).<sup>35</sup>

## 5 The single components and overall durations

VVR 15–17a provides a careful analysis of the single ŚPM components on the basis of the most traditional criteria.<sup>36</sup> “One *mātrā* be short (*hrasva*), two *mātrās* long (*dīrga*), three *mātrās* protracted (*pluta*), but a consonant be assumed (*jñeya*) as half of a *mātrā*”<sup>37</sup>. Moreover: “one *mātrā* is equal to the duration of a light syllable (*laghu*); the

<sup>35</sup> The procedure adopted in rule 127 or CXXXVI implies the further application of the following rules: (according to the Patwardhan *et al.* numeration): addition and subtraction, XIII; multiplication, XV–XVII, division, XIX; methods of finding squares, XX, XXI; operations on fractions XXXI–XXXVIII; divisions of fractions, XLII; squares of fractions, XLIV.

<sup>36</sup> For a general introduction, cf. Allen (1953: 83–87), § 3.22, *Length and Duration*.

<sup>37</sup> *Śrutabhoda*, 3.

same, doubled (*dviguṇa*), shall be equal to a heavy syllable (*guru*)”<sup>38</sup>. The single sound durations comprising the ŚPM according to VVR 15–17a could thus be summarized as shown in Table 3.

**Table 3:** ŚPM component durations.

<i>varṇa</i> & <i>dhvani</i>	<i>mātrā</i>	Description
<i>bindu</i>	$\frac{1}{2}$	by virtue of its consonantal nature ( <i>vyañjanatva</i> )
<i>nāda samaṣṭī</i>	$1 - 1\text{ava}$	the eight-sound ( <i>nāda</i> ) set ( <i>samaṣṭī</i> ) following <i>bindu</i> ; each element’s duration is equal to half of the duration of the preceding term.
<i>ka, la, ha, sa</i>	$1 + \frac{1}{2}$	consonant = $\frac{1}{2}$ <i>mātrā</i> short vowel ( <i>hrasva svāra</i> ) = 1 <i>mātrā</i> $0.5 + 1 = 1.5$ <i>mātrā</i>
<i>ī, e</i>	2	long vowel ( <i>dīrgha svāra</i> ) = 2 <i>mātrā</i> diphthong = 2 <i>mātrā</i>
<i>ḥṛllekhā</i> devoid of <i>bindu</i> ( <i>hrī</i> )	3	$1 + 1$ consonant + 1 long vowel = $h + r + ī = 3$ <i>mātrā</i>

Bhāskaraṛāya considers the entire series of syllables up to *bindu* (see Tables 2 and 3) as a succession of *varṇa* (phonemes or sound-units)<sup>39</sup> in the strict sense of the term; the status of the nine sounds from *bindu* onwards is more complicated. They are termed peculiar (*viśeṣa*) *varṇa* — that is, *bindu* — and peculiar resonances (*dhvani*) — that is, the eight sounds following *bindu* — which have to be pronounced using subtle timing, more subtle timing and the most subtle timing.<sup>40</sup> These remaining eight sounds, which as I will show have a resonant

**38** VVR-Pr 15–17a: *mātrā laghvakṣarasya kālaḥ taddviguṇo gurvākṣarasya*.

**39** Cf. Allen (1953: 13–16): “In specialized, suffixal use [the term *varṇa*] may be adequately represented by ‘quality’ (‘i-quality’, &c.), it is its wider usage that presents some difficulty. Whilst it there has much in common with the modern term ‘phoneme’, no phonemic theory is implied by it, and it would be reading at once too much and too little into the term thus to translate it. The non committal ‘sound-unit’ [...] suffers from the disadvantage that, unlike *varṇa*, it is restricted to technical usage. A happier rendering, and one which would fit into the Latin terminological tradition, is ‘letter’ — letters after all come very near to being unself-conscious phonemes”.

**40** VVR-Pr 12–13: *bindvādayo navāpi sūkṣma-sūkṣmatara-sūkṣmatama-kālair uccāryā dhvaniviśeṣāḥ varṇaviśeṣāḥ vā*; “The nine [sounds], starting from *bindu*, are peculiar (*viśeṣa*) resonances (*dhvani*) or phonemes (*varṇa*) to be pronounced with subtle times, more subtle times, the most subtle times”.

(*dhvani*) nature, are apparently not produced directly by the reciter’s phonatory organs. VVR-Pr 12–13 states they should not be considered the same as the preceding syllables<sup>41</sup>: firstly, they are not clearly uttered (*anuccāryatva*); moreover, they are perceived as sounds (*svara*) analogous (*tulyatva*) to the resonance of the string (*tantri*) of a musical instrument, a cymbal (*kāṃsyatāla*)<sup>42</sup> or, according to YH, the progressive lessening of a bell tolling.<sup>43</sup> As for the uttering times of the sounds up to *bindu*, YH-Dī 32–34 defines them as ‘macroscopic phonemes’ (*sthūlavarṇa*). Although this point is not explicitly described in the passage, seeing as the sounds following after but not including *bindu* do not have the same property (*sthūlatva*) as the former sounds, they cannot but be ‘microscopic’ or ‘subtle’ in accordance with the description from VVR-Pr 12–13 cited here above.<sup>44</sup>

YH 2.64a restates this same concept: “in the three *kūṭa*, [as] *bīja*, *bindu* and *dhvani*, [the *vidyā*, i. e. ŚPM] has the nature of the planets”<sup>45</sup>. In this case the ŚPM is seen to be composed of three categories of sounds and, in this regard, the *vidyā* (ŚPM) is thus ninefold: (a) the *bīja* sequence up to *hrī* (*bindu* excluded) composed of *varṇa* in the strict sense of the term; (b) *bindu*, the *anusvāra* in *hrīm*, a peculiar *varṇa*; (c) the *dhvani* series, which is the eight sounds or resonances of *nāda* (devoid of *bindu*; from here onwards: *nāda\**); and this sequence reiterated in the three *kūṭa*:  $3 \times 3 = 9$  just like the planets, a metonym for the number nine.

In particular, *nādānta*, the ‘end of sound’, reabsorbed at the level of the fissure of Brahmā (*brahmarandhra*) into the *nāḍī* (which, in this context, cannot be anything other than *suṣumnā*)<sup>46</sup>, is a resonance that is no longer manifested

<sup>41</sup> *kakārādi*; lett. non assimilated “to the sounds beginning with *ka*”, i. e. the whole body of the *vidyā*. Cf. n. 42

<sup>42</sup> *na ca kakārādivat spaṣṭamanuccāryatvāt tantrīsvaratulyatvena śrīyamāṇatvāc ca* (VVR-Pr 12–13); [...] *abhighātād uttarottarakṣaṇeṣu kāṃsyatāladhvanivat | tasya śaktau layo bhāvya ityarthah* (VVR-Pr 47–48a).

<sup>43</sup> *nādo ’sau [...] ghaṅṭākvāṇa iva krameṇa virama* (YH-Dī 2.37a, 3.169–170).

<sup>44</sup> *hakārādibindvāntānām sthūlavarṇānām uccāraṇākālaḥ* (YH-Dī 32–34).

<sup>45</sup> *bījabindudhvaninām ca trikūṭeṣu grahātmikā*. According to Amṛtānanda (YH-Dī 2.64), in this case the genitive is to be understood as an instrumental. This passage is also quoted in VVR-Pr 12–13.

<sup>46</sup> *nādasyānto layo bhavati yatra brahmarandhre* (YH-Dī 1.30). According to a largely shared tradition, *nāḍīs* are subtle hidden channels that pervade the vertical axis of the body; along them *prāṇa* — power, subtle energy or subtle breath which needs to be controlled through yogic and tantric practice — flows upwards and downwards to animate all parts of the body (Flood 2006: 135, 159). The aim of the practice described in YH and VVR is to allot the *mantra* sounds (*varṇa*) and sonic resonances (*nāda*) along the central *nāḍī* (*suṣumnā*) inducing an upward movement of the ‘coiled’ *śakti* (*kuṇḍalīnī*) from the base of the spine, through the fissure of

(*avyakta-dhvani*)<sup>47</sup>; the same is true of the following four sounds. As noted above, the texts hold *unmanī* to be out of time or beyond time; it therefore seems to consist of a sort of atemporal resonance. Lacking a form of its own or the ability to be expressed, it is described by Amṛtānanda (YH-Dī 1.32–34) in the words of *Taittirīya Upaniṣad*: “Whence all speech and thought turn back without reaching it”<sup>48</sup>. Beyond *tattvas* (*tattvātīta*), exceeding words and thought (*vānmano ’tītagocara*), neither with nor without parts (*aniṣkala cāsakala*), formless (*nirākāra*), not uttered (*niruccāra*), undifferentiated (*nirvikalpa*), supreme unparalleled principle (*nirdvandvaṃ paraṃ tattvam*), supreme nondual reality, Śiva himself (*śiva eva*), in *unmanī* “anything is no longer uttered since it exceeds everything” (*nātrocyante, tasya viśvottīrnatvāt*)<sup>49</sup>. In the same way, whereas the three *bindus* denote three definite (*mita*) forms of the divinity, the threefold *nāda* refers instead to higher forms, unconditioned (*amita*)<sup>50</sup>, infinite (*ananta*), without any limitations (*aparicchinnarūpa*) on their being wholly absorbed into the “etheric principle of interiority” (*vyomatattvāntargatatvāt*)<sup>51</sup>, the pure space of consciousness.

To return to the temporal analysis conducted by Bhāskararāya, it makes sense to verify the claimed durations in order to investigate the nature of *nāda* itself. The first set, *vāgbhāva kūṭa*, is composed of five syllables out of the total of fifteen: *ka*, *e*, *ī*, *la*, and *hṛīm*. Nonetheless, underlines Bhāskararāya (VVR 14), these five syllables evidently break down into eighteen different sounds, *k*, *a*, *e*, *ī*, *l*, *a*, *h*, *r*, *ī*, plus the nine belonging to *nāda*. In keeping with the points outlined in VVR 15–17a (see Table 3), their overall span consists of eleven *mātrās* minus one *lava*.<sup>52</sup> Analogously, if the second *kūṭa* is composed of

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Brahmā (*brahmarandhra*), up to *dvādaśānta*, the point twelve fingers above the crown of the head (cf. VVR 21–21).

<sup>47</sup> *nādyāṃ brahmabile līnastvavyaktadhvanīlakṣaṇaḥ; Svachchanda Tantra*, 1234–39, vol. 5b, pp. 531–4; quoted in YH-Dī 1.30.

<sup>48</sup> *yato vāco nivartante aprāpya manasā saha (Taittirīya Upaniṣad, 2.4.1).*

<sup>49</sup> YH-Dī 1.32–34, with long quotations from *Svacchanda Tantra*. Amṛtānanda also cites *Vijñānabhairava Tantra* 42: by virtue of the ordered enunciation of the phonemes, aggregated in unity (*piṇḍarūpa*), one becomes Śiva, *ardhendu*, *bindu*, *nādānta* and void (*śūnya*). In YH-Dī 1.32–34, he suggests that the order between *ardhendu* (i. e. *ardhacandra*) and *bindu* be inverted, and the series then be completed with the missing sounds; he also uses *unmanī* to indicate the meaning of *śūnya* (*śūnyaśabdenonmanā ucyate*).

<sup>50</sup> *rudreśvarasadeśākhyā devatā mitavigrahāḥ || bindutrayerṇa kathitā amitāmitavigrahāḥ śāntīḥ śaktiś ca sambhuś ca nādatritayabodhanāḥ || YH 2.45b–46.*

<sup>51</sup> YH-Dī 2.45b–46.

<sup>52</sup> It could be noted that, following Table 3, the first *kūṭa* of *hādi ŚPM* (*ha*, *sa*, *ka*, *la*) would be slightly shorter than the *kādi* one, lasting six instead of seven *mātrā*:  $0.5 + 1 (s + a) + 0.5 + 1 (k + a) < 2 (e) + 2 (ī)$ .

twenty-two sounds, its duration consists of 11.5 *mātrās* minus one *lava*; while the third one, composed as it is of eighteen sounds, has a span of 8.5 *mātrās* minus one *lava*. The sum amounts to 31 *mātrās* minus three *lava*: the same duration as the ŚPM (*vidyāsvārūpasya kāla*; VVR-Pr 31) as obtained by merely calculating its elemental components.

Nevertheless, in VVR 31 Bhāskararāya introduces different values for recitation (*uccāraṇa-kāla*). The first *kūṭa*, losing one *mātrā* minus one *lava*, now lasts only ten *mātrās*; the second one also loses one *mātrā* minus one *lava*, and therefore now lasts 10.5; the third set remains unvaried, for an overall duration of the entire recited *mantra* of 29 *mātrās* minus one *lava* (cf. Table 4).

How might this discrepancy be resolved? Comparing the two overall durations side by side, it is immediately clear that the variation between them is equal to two *mātrās* minus two *lava*, that is, it is equal to ‘duration 1’ minus the two *nādas*\* of the first two *kūṭas* (‘duration 1’ – ‘duration 2’ = 2 *mātrās* – 2 *lavas* = 2 *nādas*\*).

Therefore, it is as if two *nādas*\* were simply not present or were not actually pronounced. And yet, not only they have not vanished, but according to the authors they actually assume the preeminent role. The only viable solution would thus seem to be an appeal to the resonant nature of *nāda*\* sounds.

The reciter (*jāpaka*) or practitioner (*sādhaka*), after having exhausted the first *kūṭa* sequence and pronounced the *anusvāra* (*ṛi*) of the first *hṛllekhā* (*hrīm*), must seamlessly engage his or her phonatory organs in reciting the first *akṣara* of the second *kūṭa*. The same process must repeat at the completion of the second *kūṭa*. The reciter could not reasonably be involved in the production of the *nāda*\* because – having pronounced *bindu* – he or she must immediately skip to the following syllabic set lest he or she fail to comply with the ‘duration 2’ prescription.<sup>53</sup> If the active production of the sounds by the reciter’s phonatory organs is excluded as physically (that is, temporally) impossible, then *nāda*\* cannot but have the nature of an independent resonance. Moreover, *nāda*\* must, necessarily and autonomously, resound together with the first syllable of the succeeding *kūṭa*, thereby validating the nature of its resonance as similar to the reverberation of a musical instrument string. Figure 1 shows the sequences overlapping.

53 It is here argued this is the sense of VVR 28 statement: *nādaḥ prāthamīkastu dvitīyakūṭena sākamuccāryaḥ | dvitīyīkaṃ nādaṃ tārtīyenoccareṇa na pṛthak ||*. “The *nāda* of the first *kūṭa* be pronounced jointly (*sākam*) with the second [*kūṭa*]. The *nāda* [related to] the second [*kūṭa*] be uttered not separately (*na pṛthak*) from the third [*kūṭa*]”.

**Table 4:** ŚPM component duration according to VVR.

<i>kūṭa</i>	<i>śrīvidyā</i> <i>bījākṣara</i>	<i>ḥṛllekhā (hrīm̄)</i>	<i>number of</i> <i>sounds</i> <i>VVR 14</i>	<i>duration 1</i> <i>VVR 17b-18</i>	<i>duration 2</i> <i>VVR 31</i>
<i>vāgbhāva</i>	k + a e ī l + a ka e ī la	12 elements ( <i>saṃhati</i> ): – <i>vyoman (h)</i> – <i>agni (r)</i> – <i>vāmalocanā (ī)</i> • <i>bindu (ṛi)</i> [• <i>nāda</i> ] • # <i>ardhacandra</i> [# <i>dhvani</i> ] • # <i>rodhini</i> • # <i>nāda</i> • #* <i>nādānta</i> [* not audible] • #* <i>śakti</i> • #* <i>vyāpikā</i> • #* <i>samanā</i> • #*o <i>unmanī</i> [° atemporal]	18 sounds k, a, e, ī, l, a = 6 6 + 12 ( <i>hrīm̄</i> ) = 18	11 <i>mātrā</i> – 1 <i>lava</i> k + a = 1.5 e = 2 ī = 2 l + a = 1.5 <i>hrīm̄</i> = 3 + 1' —— 1' = 1 <i>m</i> - 1 <i>lava</i>	10 <i>mātrā</i>
<i>kāmārāja</i>	h + a s + a k + a h + a l + a <i>ha sa ka</i> <i>ha la</i>	<i>idem</i>	22 sounds h, a, s, a, k, a, h, a, l, a = 10 10 + 12 = 22	11.5 <i>mātrā</i> – 1 <i>lava</i> 1.5 + 1.5 + 1.5 + 1.5 + 1.5 + 3 + 1' (cf. <i>supra</i> )	10.5 <i>mātrā</i>
<i>śakti</i>	s + a k + a l + a <i>sa ka la</i>	<i>idem</i>	18 sounds s, a, k, a, l, a = 6 6 + 12 = 18	8.5 <i>mātrā</i> – 1 <i>lava</i> 1.5 + 1.5 + 1.5 + 3 + 1' (cf. <i>supra</i> )	8.5 <i>mātrā</i> – 1 <i>lava</i>
<i>kadi śrīvidyā</i> complete form: 3 <i>kūṭa</i> + 3 <i>ḥṛllekhā</i>			58 sounds 18 + 22 + 18 = 58	31 <i>mātrā</i> – 3 <i>lava</i>	29 <i>mātrā</i> – 1 <i>lava</i> Δ 2 <i>m</i> -2 <i>l</i>

Notes to Table 4:

- *duration 1*: *vidyāsvarūpa-kāla*, according to VVR 17b-18
- *duration 2*: *uccāraṇa/japa-kāla*, according to VVR 31

## 6 What is a *lava*?

The *Līlavatī* addresses linear, quadratic and weight, etc. measurement units in stanzas 2–11, albeit very concisely. Time measurement units are taken for



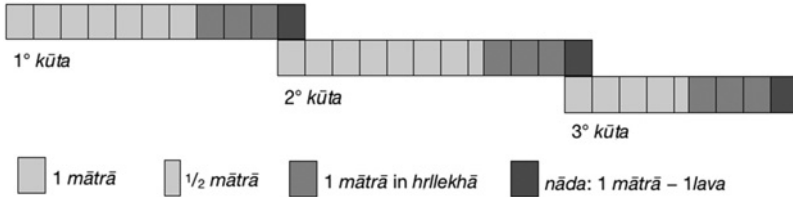


Figure 1: Sequences overlapping, according to VVR 31.

granted and left to the reader’s knowledge (*Lilavatī*, 8): *śeṣāḥ kālādīparibhāṣā lokataḥ prasiddhā jñeyāḥ*. Nevertheless, these units – beginning with the *lava* – are actually far from univocal. Hayashi (2017) and Gupta (2010) aptly note that some time intervals might have had the same names in different systems but differed widely in value.<sup>54</sup>

Drawing on *Ṛg-Jotiṣa-Vedāṅga* 16 and *Yajur-Jotiṣa-Vedāṅga* 38, we can establish the values of *muhūrta* (48 min.) and *ghaṭikā* (24 min.), the only two time spans which are constant across all sources. “Ten *kalās* and a twentieth is [a *ghaṭikā*], two *ghaṭikā* [are the equivalent] of a *muhūrta*; a day is thirty of those, or six hundred plus three *kalās*”<sup>55</sup>.

In *Siddhānta-śiromani* 16–17, Bhāskara II defines the time measurement units which are shorter than a day. Table 5 specifies the duration of each single unit in relation to 24 hours. *Lava* is not included, here.<sup>56</sup>

<sup>54</sup> Hayashi (2017): “My main interest lies in the great variety of the names of the units and of the conversion ratios between them”. Gupta (2010: §1.2.4, *Time measurements*, 3–9); see also his *Logarithmic time scale in ancient India* (2010: 9).

<sup>55</sup> Quoted in: Plofker (2009: 37). This passage also refers to *kalā*: a 60<sup>3rd</sup> of 24 hours, that is, 2.39 minutes. “The earliest known explicitly mathematical exposition of astronomy and calendrics [...] is found not in the Vedic corpus itself but in its associated *Vedāṅga*. [*Jotiṣa-Vedāṅga*] is the first available link between the ambiguous celestial and calendric utterances of the Vedas and the full-blown Sanskrit mathematical astronomy of the first millenium CE”, Plofker (2009: 35). On the occurrences of *muhūrta*, cf. Hayashi (2017).

<sup>56</sup> Bhāskara II (1981: 5–6): *yo ’kṣṇor nimeṣasya kharāmabhāgaḥ sa tatparastacchatabhāga uktā | truṭir nimeṣair dhṛtibhiś ca kāṣṭhā tattriṃṣatā sadgagaṇakaiḥ kaloktā || 16. triṃśatkalā ’rkṣī ghaṭikā kṣaṇaḥ syān nāḍivayaṃ tai khaḡuṇair diṇaṃ ca | gurvakṣaraiḥ khendumitair asustaiḥ ṣaḍbhiḥ palaṃ tairghaṭikā khaṣaḍbhiḥ || 17.*

Table 5: Siddhānta-śiromani 16–17.

24h	30 <i>kṣaṇas</i> <sup>a</sup>
1 <i>kṣaṇa</i> (48 min.)	2 <i>ghaṭikās</i>
1 <i>ghaṭikā</i> (24 min.)	30 <i>kalās</i>
1 <i>kalā</i> (48 sec.)	30 <i>kāṣṭhās</i>
1 <i>kāṣṭhā</i> (1.6 sec.)	18 <i>nimiṣas</i> or <i>nimeṣas</i> <sup>b</sup>
1 <i>nimeṣa</i> or <i>paṅkṣma-pāta</i> , an eye blink (0.0889 sec.) <sup>c</sup>	30 <i>tatparās</i>
1 <i>tatparā</i> (0.0029 sec.)	100 <i>truṭis</i>
1 <i>truṭi</i> (0.000029 sec.)	

<sup>a</sup> equal to the more common *muhūrta*; as openly stated in the autocommentary, *Vāsanābhāṣya* 16–17;

<sup>b</sup> or 15, according to other *śāstra*, *Vāsanābhāṣya*, 16–18;

<sup>c</sup> quite a plausible value, although it is slightly too brief; see n. 67.

In the autocommentary (*Vāsanābhāṣya* 16–17) Bhāskara II offers an alternative scale (cf. Table 6). On the basis of this measurement one *mātrā* or light syllable lasts 0.2 sec.: a highly credible value, equal to five light syllable per second.<sup>57</sup>

57 “Durational properties of the speech signal have been studied for a variety of languages [...]. Factors known to influence segment and word durations range from phonetic and phonological factors to syntactic and semantic factors [...]. Early studies typically focused on segmental duration in a linear model [...] (f.i., Klatt (1973, 1976). Recent developments in phonological theory have given research on timing a new perspective. Contrary to previous linear representations, current non-linear phonological approaches (e. g. CV theory: Clements/Keyser 1983; moraic phonology: Hyman 1985; Hayes 1989; etc.) [...] also suggest that, in addition to intrinsic segment duration, higher levels of the prosodic hierarchy, such as syllable, foot, and phrase, all jointly determine phonetic duration”; Jongman (1998: 207). “[...] It is relative duration that matters rather than absolute duration. The length of any vowel will be in some measure dependent on its quality and context, and there is no minimum length for a long vowel or maximum length for a short vowel. If two vowels contrast with each other in length, what matters most is their duration relative to each other in comparable context”, Clark/Yallop (1995: 33).

Regarding instrumental measurements of syllable and segment durations to supply at least an approximate order of magnitude in order to compare the values offered by the texts, see Turk et al. (2006) (for a discussion of acoustic segment duration criteria, a great deal of duration data and the introduction of a method based on identifying clearly recognizable acoustic landmarks, so-called oral consonantal constriction events). Here are three examples: A Scottish speaker saying “concord”: /k/ (0–0.2 sec.), /ɔŋk/ (0.2–0.8), /ɔɪ/ (0.8–1.3), /d/ (1.3–1.8), tot. 1.8 sec.; A Japanese speaker saying /aga/ (0.24 sec.); A Southern Standard British English speaker saying “tosh”: /t/ (0–0.08); /ɒ/ (0.08–0.38; with 0.05 sec. of final aspiration); /ʃ/ (0.38–0.6), tot. 0.6 sec. See also Fletcher/McVeigh (1993), Figure 1, *Durations of vowels (ms) in four different*

**Table 6:** Vāsanābhāṣya 16–17.

24h	60 <i>ghaṭīs</i>
1 <i>ghaṭī</i> (24 min.)	60 <i>palas</i>
1 <i>pala</i> (24 sec.)	6 <i>prāṇas</i>
1 <i>prāṇa</i> (4 sec.) or <i>asu</i> (a breath) <sup>a</sup>	10 <i>gurus</i> or long syllables (0.4 sec.)
1 <i>guru</i> (0.4 sec.)	

<sup>a</sup> *praśastendriyapuruṣasya śvāsocchavāsāntarvartī kāla ity arthaḥ*, the duration of the breath of a man in full possession of his faculties.

Nṛsiṃha Daivajña, in his *Vārttika* commentary to *Siddhānta-śiromani* 16–17, quotes a passage by Garga<sup>58</sup> mentioning *lava* (cf. Table 7). In this case, since one *lava* equals 0.16 sec. and corresponds to a 256th fraction of a *mātrā*, the latter is equal to 40.96 sec., a value that is unquestionably excessive.<sup>59</sup> Evidently, in this case *nimeṣa* is used in a different sense.

**Table 7:** Siddhānta-śiromani-vārttika 16–17.

1 <i>ahorātra</i> (24h)	30 <i>muhūrtas</i>
1 <i>muhūrta</i> (48 min.)	30 <i>kalās</i>
1 <i>kalā</i> (96 sec.)	30 <i>kāṣṭhās</i>
1 <i>kāṣṭhā</i> (3.2 sec.)	10 <i>kṣaṇas</i>
1 <i>kṣaṇa</i> (0.32 sec.)	2 <i>lavas</i>
1 <i>lava</i> (0.16 sec.)	2 <i>truṭīs</i>
1 <i>truṭī</i> (0.08 sec.)	2 <i>nimeṣas</i>
1 <i>nimeṣa</i> (0.04 sec.)	

*prosodic contexts, unstressed, stressed, pitch accented and phrase-final*: mean = 58.1 ms, 103.4 ms, 113 ms, 177.1 ms.; Figure 2 *Durations (ms) of unstressed, stressed, pitch accented and phrase-final syllables*: mean = 155.7 ms, 270.2 ms, 299.6 ms, 379 ms. In Duanmu (1994), “Average syllable durations were determined. It was found that the average syllable duration in Mandarin was 215 ms and that in Shanghai was 162 ms”, p. 1 (see also § 5, *Results*, 11–16). “The measured duration of English diphthongs ranges from about 150 to 400 ms”, Clark/Yallop (1995: 36). Consistent data in: Schweitzer/Möbius (2004). For the limited and preliminary purposes of this article, we can conclude that a *mātrā* could not, ostensibly, last less than 0.1 sec. and more than 0.5 sec.; i. e.  $10 < mātrā < 2$  per second.

**58** Regarding *Gargasamhitā* (I b.c.e. - I c.e.), see Pingree (1981: 69–71).

**59** Both here and in the following instances, the obtained *lava* value is multiplied to reconstitute the *mātrā*, for 256, according to the posited definition.

Even the renowned Jaina mathematician Mahāvīra (IX sec.), in *Gaṇita-sāra-saṅghraha* 32–34, defines the duration of *lava* according to the scale shown in Table 8.<sup>60</sup> It goes without saying that, in this instance, the order of magnitude of one *lava* appears wholly unrelated to the topic under discussion: based on these numbers, one *mātrā* would last more than two and a half hours.

**Table 8:** Gaṇita-sāra-saṅghraha 32–34.

24h	30 <i>muhūrtas</i>
1 <i>muhūrta</i> (48 min.)	2 <i>ghaṭis</i>
1 <i>ghaṭi</i> (24min.)	38.5 <i>lavas</i>
1 <i>lava</i> (37.4026 sec.)	7 <i>stokas</i>
1 <i>stoka</i> (5.3432 sec.)	7 <i>ucchvāsas</i>
1 <i>ucchvāsa</i> (0.7633 sec.)	n. <i>āvalis</i>
1 <i>āvali</i>	n. <i>samayas</i>
1 <i>samaya</i> = atomic movement	

Vaṭeśvara (XI-X sec.), in *Vaṭeśvara-siddhānta* 1.1.7, not only mentions *lava*, he also evokes the image of piercing a lotus leaf, here paralleling the duration of one *truṭi* (cf. Table 9).<sup>61</sup> In this case, one *mātrā* equals 0.228 sec., a reasonable value. This assessment is also internally consistent: for one *guru* lasting 0.4 sec. (see above), one *mātrā* shall be 0.2 sec. The two values are clearly congruent.

**60** Mahāvīra (1912: 4–5): *atha kālaparibhāṣā | anur aṅvantaraṃ kāle vyatikrām iti yāvati | sa kālas samayo ’saṅkhyais samayairāvalir bhavet || 32. saṅkhyātāvalir ucchvāsaḥ stokas tūcchvāsasaptakaḥ | stokās sapta lavas teṣāṃ sārdhāṣṭātriṃśatā ghaṭi || 33. ghaṭīdvayaṃ muhūrī ’tra muhūrtair strīṃśatā dinam | [...] 34.*

Terminology relating to [the measurement of] time. 32. The time in which an atom (moving) goes beyond another atom (immediately next to it) is a *samaya*; innumerable *samayas* make an *āvali*. 33. A measured number of *āvalis* makes an *ucchvāsa*; seven *ucchvāsas* make a *stoka*; seven *stokas* make one *lava*, and with thirty-eight and a half of this a *ghaṭi* is formed. 34. Two *ghaṭi* make one *muhūrta*; thirty *muhūrtas* make one day [...]. Plofker (2009: 163), comments: “The smallest of the given units in space and time are evidently infinitesimals [...]. The smallest possible amount of time, or instant, is that required for one atom to move past another, and the smallest finite time unit is defined as innumerable instants”.

**61** Vaṭeśvara (1985), II, 2: *kamaladalānatulyaḥ kāla uktas truṭis tacchatam iha lavasaṃjñas tacchatam syān nimeṣaḥ | sadalajaladhībhis tair gurv ihaivākṣaraṃ tatktaparimitakāṣṭātaccharārdhena cāsuḥ (1.1. 7)*

“The time taken [by a sharp needle] to pierce [a petal of] a lotus flower is called a *truṭi*; one hundred times that is called a *lava*; one hundred times that is a *nimeṣa*; four and a half times that is a long syllable; four times that is a *kāṣṭhā*; and one half of five times that is an *asu*”.

**Table 9:** Vaṭeśvara-siddhānta 1.1.7.

1 <i>ghaṭikā</i> (24 min.)	60 <i>palas</i>
1 <i>pala</i> ( <i>caṣaka</i> , <i>vināḍī</i> , <i>vighaṭkā</i> ) (24 sec.)	6 <i>asus</i> (breaths)
1 <i>asu</i> (4 sec.)	2.5 <i>kāṣṭhās</i>
1 <i>kāṣṭhā</i> (1.6 sec.)	4 <i>gurus</i> (long syllables)
1 <i>guru</i> (0.4 sec.) <sup>a</sup>	4.5 <i>nimeṣas</i>
1 <i>nimeṣa</i> (sec. 0.0889) <sup>b</sup>	100 <i>lavas</i>
1 <i>lava</i> (0.000889 sec.)	100 <i>truṭis</i>
<i>truṭi</i> (0.0000889 sec.)	piercing a lotus leaf

<sup>a</sup> thus, 1 *mātrā* = 0.2 sec.;

<sup>b</sup> the same value expressed in *Siddhānta-śiromani*, 16–17

**Table 10:** PST, 1.29–34.

24 h	30 <i>muhūrtas</i> (PST 34a)
1 <i>muhūrta</i> (48 min.)	2 <i>nādikās</i> (PST 33b)
1 <i>nādikā</i> (24 min.)	360 <i>śvāsas</i> (PST 33a)
1 <i>śvāsa</i> , a breath (4 sec.) = 1 <i>mātrā</i> <sup>a</sup>	8 <i>nimeṣas</i> (PST 31b.)
1 <i>nimeṣa</i> (0.5 sec.) <sup>b</sup>	30 <i>kāṣṭhās</i> (PST 31a)
1 <i>kāṣṭhā</i> (0.0166 sec.)	30 <i>kalās</i> (PST 30b)
1 <i>kalā</i> (0.00055 sec.)	30 <i>truṭis</i> (PST 30b)
1 <i>truṭi</i> (0.000019)	30 <i>lavas</i> (PST 29b–30a)
<i>lava</i> = 0.0000062 sec.	piercing a bundle of lotus leaves (see above)

<sup>a</sup> *mātrā sā tulyā svīyaikaśvāsamātrayā*, PST 32b. Clearly, *mātrā* is used in a different sense here than the prosodic (*chandās*) one;

<sup>b</sup> a reasonable value, see *infra*, n. 67

In *Prapañcasāraṇtra* (PST, 1.29–34) — the text quoted by YH and VVR — Nārāyana illustrates the most minute divisions of time for Brahmā, Viṣṇu and Rudra (cf Table 10).<sup>62</sup> On the basis of these units, a *lava* reaches the vertiginous

<sup>62</sup> PST (1935: 12): *lavādīpralayānto'yaṃ kālaḥ prastūyate hy aja | nalinīpatrasaṃhatyāṃ sūkṣmasūcyabhīhedane || 29. dale dale tu yaḥ kālaḥ sa kālo lavavācakaḥ | lavais truṭiḥ syāt triṃśadbhiḥ kalāṃ tāvat truṭiṃ viduḥ || 30.*

value of about  $6 \cdot 10^{-7}$  sec. or 0.6  $\mu$ s, while one *mātrā* lasts only 0.000159 of a sec., too brief by far.<sup>63</sup>

*Muhūrta* and its half, *ghaṭikā*, are univocal units of measurement that appear in all sources (cf. Table 11).<sup>64</sup> Essential for astronomical calculation, they are the results of accurate and repeated assessments in the history of Indian astronomy.<sup>65</sup> Mach's point about measurement being dependent on comparison, in defining the meaning of a concept, could thus only be applied to these units: "We do not measure mere space [or time]; we require a material standard of measurement, and with this the whole system of manifold sensations is brought back again. It is only intuitional sense-presentations that can lead to the formulation of the equations of physics, and it is precisely in such presentations that the interpretation of these equations consists"<sup>66</sup>.

In contrast, the minor time units appear to be mere fractions of *muhūrta*. Their remarkable variability in value and designation depends, reasonably enough, on the fact that there are no reliable measuring protocols for such small intervals. In his autocommentary *Vāsanābhāṣya* (16–18) Bhāskara II, accustomed to the rigor of astronomical measurement, first expresses the fractional value of *nimeṣa* and then, significantly, notes that its equivalence to the blink of an eye (*paṅkṣma-pāta*) is but a metaphorical indication: *sa yāvātā kālena niṣpadhyate tāvān kālo 'pi nimeṣaśabdenocyate upacārāt*. As mentioned above, the value proposed for this metaphorical indication is slightly too short, less than one-tenth of a second for a blink; nevertheless, although it is not completely congruent with modern instrumental measurement, it is a surprisingly reliable approximation.<sup>67</sup> Following Bhāskara II, there is all the more reason to

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<sup>63</sup> Gupta (2010: 9), collecting data from different sources in the historical section of his book on metrology, states that *truṭi* reaches the value of  $10^{-7}$  sec. (0,1  $\mu$ s) while one *lava*, its multiple, corresponds to  $10^{-3}$  sec. (0.001 sec.). In this case  $1 \text{ mātrā} = 0.256 \text{ sec.}$ , a plausible value.

<sup>64</sup> MW (1899: 898, 2) reports  $1/4000$  of *muhūrta*,  $1/5400$  or  $1/20250$  as possible values of *lava*. Be one *muhūrta*, as has seen, 48 min., one *lava* shall equate 0.72 sec., 0.53333 sec. or 0.112592 sec.; i. e. about one second, half a second, one tenth of a second. In this case, one *mātrā* would last 28,823552 sec. at best. That is about half a minute to pronounce a short syllable. Alternatively, MW states one *lava* parallels  $1/60$  of *nimeṣa*, i. e. the sixtieth part of an eye blink, whose value in this system is unknown. Too much in this case too, for one *mātrā* would endure 4.2667 eye blinks ( $256 \times 1/60$ ).

<sup>65</sup> On *muhūrta* and water-clocks, cf. in particular: Falk (2000) § 3.2, *The water-clock in India*. Cf. also Sarma (1991) and Sarma (2008: 125–175).

<sup>66</sup> Mach (1984: 343). Pushing even further the concept: "We mean by any concept nothing more than a set of operations; the concept is synonymous with the corresponding set of operations", Bridgman (1927: 5).

<sup>67</sup> On average, the duration of a single blink =  $1-4 \text{ ds}$  (0.1–0.4 s), see H.R. Schiffman (2001); also quoted in: D. Ramot, Harvard University "BIONUMB3R5, *The Database of useful Biological*

suppose that the indication describing one *lava* as parallel to the length of time necessary to reach the second leaf after having pierced a first lotus leaf with a thin needle could be conceived as metaphorical in the same manner.

In this case, the authors are dealing with a metaphorical — as Bhāskara II notes — definition of the meaning of a word in term of perceptual experience, which clearly cannot serve as a standard-setting reference in the same way as *muhūrta* does. They are addressing an ostensive definition<sup>68</sup> of its meaning by means of a vivid expression (*upacāra*) at the edge of perception and for communicative purposes. “Under ostensive definition I include [...] the formal instruction by which a person is explicitly taught the meaning of a word by being presented simultaneously with the word and with examples of what it applies to [...]”<sup>69</sup> It is not a question, in this case, of objectively measuring physical quantities and consequently describing natural phenomena; instead, this likely represents an attempt to render a subjective experience communicable. Although piercing a lotus leaf is merely a perceptual image, it does nonetheless show the intention of providing a common empirical criterion.

Bhāskara II, Vaṭeśvara and the PST 31b compiler suggest reliable values for *asu* (*śvāsa*, *prāṇa*) and *guru* durations: respectively 4 sec. for a complete breath taken by a healthy man and 0.04 sec. for a long vowel (see Table 11). In this way, they reveal the attempt to establish a fractional system that is as congruent as possible with observational phenomena and major units of measurement, such as *muhūrta*. Minimal fractions developed to describe extremely

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*numbers*” webpage (<http://bionumbers.hms.harvard.edu/bionumber.aspx?id=100706&ver=4>); and in E.H. Chudler, *Brain Facts and Figures*, § *Sensory Apparatus-Vision*, Washington University webpage (<http://faculty.washington.edu/chudler/facts.html>); both retrieved November, 2018.

**68** See Wittgenstein (2009: § 43): “[...] and the *meaning* of a name is sometimes explained by pointing to its *bearer*”. On Wittgenstein’s thesis, cf.: Hacker (2001: ch. 9); Id. (1975: 267–287). For a discussion of Hacker’s interpretation in favour to a full meaningfulness of ostensive definitions in Wittgenstein, cf.: Cheung (2014: 350–362). On ostensive definitions, cf. also: Russell (1948: 242): “All nominal definitions, if pushed back far enough, must lead ultimately to terms having only ostensive definitions, and in the case of an empirical science the empirical terms must depend upon terms of which the ostensive definition is given in perception”.

**69** Whiteley (1956: 332). “In order to use the expression subsequently with understanding, I must be able to recognize that to which it applies, which I can only do if it is presented to my observation. Now the only entities so presented are my private data”. Similarly, all the referents must be public, “for whatever can be *shown* by A to B must be something which they can both be aware of, not something private either to A or to B” (333). Thus, ostensively defined expressions “refer to private objects taken as public”: my private data correlated with the expression referring to the public object (334).

Table 11: Time metrology.

<i>Prapañcasāra Tantra</i> 1.29–33	<i>Vaṭeśvara-</i> <i>siddhānta</i> 1.1.7	<i>Siddhānta-</i> <i>śiromani</i> 16–17	<i>Siddhānta-</i> <i>śiromani</i> 16–17; <i>Vārttika</i>	<i>Gapita-sāra-</i> <i>saṅghraha</i> 32–34
<i>muhūrta</i> (48 min.)	<i>muhūrta</i> (48 min.)	<i>kṣaṇa</i> (48 min.)	<i>muhūrta</i> (48 min.)	<i>muhūrta</i> (48 min)
<i>nādikā</i> (24 min.)	<i>ghaṭikā</i> (24 min.)	<i>ghaṭikā</i> (24 min.)		<i>ghaṭi</i> (24min)
	<i>pala</i> (24 sec.)	<i>kalā</i> (48 sec.)	<i>kalā</i> (96 sec.)	
		<i>pala</i> (24 sec.)		<i>lava</i> (37.4026 sec.)
<i>śvāsa</i> (4 sec.)	<i>asu</i> (4 sec.)	<i>prāṇa /asu</i> (4 sec.)	<i>kāṣṭhā</i> (3.2 sec.)	<i>stoka</i> (5.3432 sec.)
	<i>kāṣṭhā</i> (1.6 sec.)	<i>kāṣṭhā</i> (1.6 sec.)	<i>kṣaṇa</i> (0.32 sec.)	
	<i>guru</i> (0.4 sec.)	<i>guru</i> (0.4 sec.)	<i>lava</i> (0.16 sec.)	<i>ucchvāsa</i> (0.763 s.)
<i>nimeṣa</i> (0.5 sec.)	<i>nimeṣa</i> (0.0889 sec.)	<i>nimeṣa</i> (0.0889 sec.)	<i>truṭi</i> (0.08 sec.)	
<i>kāṣṭhā</i> (0.0166 sec.)		<i>tatparā</i> (0.0029 sec.)	<i>nimeṣa</i> (0.04 sec.)	
<i>kalā</i> (0.00055 sec.)	<i>lava</i> (0.000889 sec.)			
<i>truṭi</i> (0.000019 sec.)	<i>truṭi</i> (0.0000889 s.)	<i>truṭi</i> (0.000029 sec.)		<i>āvali</i> ( <i>n samaya</i> )
	piercing a lotus leaf			
<i>lava</i> (0.00000062 s.)				<i>samaya</i> (atomic movement)
piercing a bundle of lotus leafs				

brief time spans, however, exceed the limits of empirical appraisal and would need instrumental assessment. Once the fractional values of minimal fractions (in relation to the *muhūrta* value) have been resolved, it appears without a shadow of a doubt that the authors were not aware either of the order of magnitude of the units they propose or of their comparative examples (e. g. piercing a lotus leaf). The high degree of variability in the suggested fractional values seems to prove this point. What remains are fractional systems which are internally rigorous yet independent, given the general impossibility of verifying them instrumentally.

As I have shown, Amṛtānanda and Bhāskaraṛāya appear to follow the system of measurement proposed by *Prapañcasāra Tantra*. In mentioning the



*lava* they advanced, at the edge of perception<sup>70</sup>, the minimal time span available to them: an ‘atom of time’, *paramāṇu*<sup>71</sup>. Among many time units, the authors of YH and VVR selected the one that is the most minute in fractional terms ( $1/4,665,600,000$  of a *muhūrta*) and they were arguably aware of this fact: beyond *lava* there is nothing but void (*śūnyaiva*), because no more subtle time span (*adhikasūkṣmatara*) exists (YH-Dī 1. 32–34).

On the contrary, they were likely not in a position to fully appreciate the actual value of one *lava* (i. e. what today is reckoned by equating  $6 \cdot 10^{-7}$  sec., see *supra*), the value of the time span involved in piercing a lotus leaf and, lastly, whether or not these two time spans equal the duration they aim to describe – namely,  $1/256$  of a *mātrā*. A *guru* unit (i. e. two *mātrās*) does not appear in the PST system, thus making it impossible to conduct a direct comparison, internal to the system itself, between *guru* and *lava*. However, this act of rendering the values explicit has nonetheless proven that one *lava* – conceived as  $1/4,665,600,000$  of a *muhūrta* – does not, apparently, correspond to  $1/256$  of a *mātrā* (see *supra*) in that it is excessively brief. If Amṛtānanda and Bhāskararāya had instead chosen a larger unit of measurement – such as the *lava* according to Vaṭeśvara (equal to  $1/3,240,000$  of a *muhūrta*,  $1/450$  of a *guru* and so  $1/225$  of a *hṛasva*) or the *kalā* in the PST system (equal to  $1/5,184,000$  of a *muhūrta*) – they would have obtained a much more adequate value. It is therefore reasonable to conclude that, having been unaware of the actual value of *lava*, they were primarily motivated by the concern to establish the smallest time fraction.

## 7 Ten thousand *truṭis*

In a different context, discussing the seven *viṣuvas*<sup>72</sup>, a specific aspect of *japa* practice, both YH and VVR propose an additional time measurement in relation to *nāda*. Quoting the same passage from an unnamed *tantra*<sup>73</sup>, YH-Dī and VVR-Pr

70 VVR-Pr 15–17a: *ataḥ sūkṣmatamaḥ kālo nopalabhyaḥ*; as with the subtlest (*śūkṣmatama*) time span, *lava* cannot be clearly perceived (*na upalabhya*).

71 VVR-Pr 15–17a: *kālaparamāṇur lava ity ucyate*. On the contrary, in *Vājasaneyi Pratiśākhya* (1934), *paramāṇu* indicates one eighth of a *mātrā*. See Allen (1953: 84): *Vājasaneyi Pratiśākhya or Kātyāyanīya Pratiśākhya with the commentaries of Uvaṭa and Anantabaṭṭa*, V.V. Sharma (ed.), Madras 1934.

72 A discussion of the nature and meaning of the seven *viṣuvas* (lit. *equalisations*) is clearly beyond the scope of this paper. Suffice it to say that they are aspects of *mantra-japa* practice that refer in particular to Tantric subtle physiology. For a discussion of *viṣuvas*, see YH 3.181b–187 and VVR 43–51.

73 *iti tantrāntarokta rītyā*; YH-Dī 3.187a, VVR-Pr 49b–51.

note that, for a healthy man seated in a symmetrical position, a 30th of a blink (*nimeṣa*) is equal to a *tatpara* and a 100th of the latter to a *truṭi*.<sup>74</sup> The duration of a *nāda* is thus equal to three and a half *nimeṣa* plus 317 *truṭis*, according to Bhāskaraṛāya, which comes to 10,817 *truṭis*.<sup>75</sup> After having evoked the *nimeṣa/tuṭi* ratio (1/3000), the compiler of the YH straightforwardly defines the duration of *nāda* in *tuṭi* (i. e. *truṭi*): *municandrāṣṭadaśabhis tuṭibhir nādavedanam* (YH 3.187a). YH-Dī clarifies: there are seven *munis*, there is only one moon (*candra*), the remaining numbers are clear: eight and ten. Therefore, *nāda* is to be understood as lasting 10,817 *truṭis* (i. e. *truṭis*).<sup>76</sup>

Amṛtānanda and Bhāskaraṛāya thus seem to adopt the same metrology Bhāskara II uses in *Siddhānta-śiromani* 16–17 (see *supra*). The point is that this system is not compatible with the one adopted previously, from *Prapañcasāraṇtra*, either in its fractional values or in its durations as calculated with respect to the *muhūrta* (see Table 12). Since some data are missing, it is not possible to conduct a full-fledged assessment of this system. Nonetheless, the measurements derived from this unnamed *tantra* are unexpectedly quite accurate. If the duration of *nāda*, equal to 10,817 *truṭis*, is calculated from the derived duration of the latter in Bhāskara II's system, the result is a time span of

**Table 12:** Nimeṣa fractions & values.

<i>Prapañcasāraṇtra</i> 1.29–33	<i>Siddhānta-śiromani</i> 16–17	YH-Dī 3.167a; VVR-Pr 49b–51
<i>nimeṣa</i> (0.5 sec.)	<i>nimeṣa</i> (0.0889 sec.)	<i>nimeṣa</i>
<i>kāṣṭhā</i> (0.0166sec.)	<i>tatparā</i> (0.0029 sec.)	1 <i>nimeṣa</i> = 30 <i>tatparās</i>
1 <i>nimeṣa</i> = 30 <i>kāṣṭhās</i>	1 <i>nimeṣa</i> = 30 <i>tatparās</i>	
<i>kalā</i> (0.00055 sec.)		
1 <i>kāṣṭhā</i> = 30 <i>kalās</i>		
<i>truṭi</i> (0.000019 sec.)	<i>truṭi</i> (0.000029 sec.)	1 <i>tatparā</i> = 100 <i>truṭis</i>
1 <i>kalā</i> = 30 <i>truṭis</i>	1 <i>tatparā</i> = 100 <i>truṭis</i>	1 <i>nimeṣa</i> = 3000 <i>truṭis</i>
1 <i>nimeṣa</i> = 30 <sup>3</sup> = 27000 <i>truṭis</i>	1 <i>nimeṣa</i> = 3000 <i>truṭis</i>	

<sup>74</sup> *svasthe nare samāsine yāvat spandati locanam | tasya triṃśattamo bhāgatatparaḥ parikīrtitaḥ || tatparasya śatāmśastu tuṭirityabhidhiyate |*. Bhāskaraṛāya comments: *nimeṣa locana-spanda-kālah | tasya trisahasratamo 'mśastruṭiḥ*, VVR-Pr 49b–51; “*nimeṣa* [is equal to] the duration of an eye blink; its 3000th fraction [is equal to] one *truṭi*”. Simply put: one *nimeṣa* = 30 *tatparās*; one *tatparā* = 100 *truṭis*; thus, one *nimeṣa* = 3000 *truṭis*.

<sup>75</sup> *adyuṣṭa-nimeṣottara-sapta-daśa-adhika-śatatraya-truṭibhiḥ || uccarite nāde sati tasyānte tattvavedanam bhavati*, VVR 50b–51a. *ayutottara-aṣṭa-śatottara-sapta-daśa-truṭi-paryantam*, VVR-Pr 49b–51.

<sup>76</sup> *sapta-daśa-adhika-aṣṭa-śatottara-daśa-sahasra-tuṭibhir-uccāre*, YH-Dī 3.186a.

0.3205 sec. As noted above, a *nāda* is defined as one *mātrā* less one *lava*, so in this case one *mātrā* would last about 0.3 sec., a value that is highly reliable (see n. 57) and, within the system, reasonably consistent with the duration of *guru* (i. e. 2 *mātrās* and equal to 0.4 sec.; see Table 11). This once again attests to a solid internal consistency characterizing metrology systems and, last but not least, a surprising degree of accuracy in describing phenomena which are still clearly observable such as *guru* (and consequently *mātrā*), *asu* or *nimeṣa* (cf. Table 12).

## 8 A non-infinite series

The fact that the progression is suddenly interrupted at its eighth term also appears to be significant. One might wonder, in the first instance, why the two authors did not choose to extend the sequence into infinity — a more sophisticated option, at first blush. Indian thought, as noted earlier, does not appear to shy away from vertiginous time units. Instead, the choice in this case appears to be surprisingly modest. It could not be a case of mere calculation problems, either: Bhāskara II names decimal places up to  $10^{17}$  (*parārdha*)<sup>77</sup>, while the authors of YH and VVR stop at  $10^{-2}$  in relation to the unit represented by the *mātrā*. Moreover, the Indian mathematical tradition has repeatedly courted the idea of infinite series and infinitesimal values: “there should be no need nowadays to point out that [...] Mādhava’s power series for trigonometric functions predates by centuries Newton’s and Leibnitz’s versions of them”<sup>78</sup>. It has been also noted that the *Siddhānta-śiromani*, in the parts concerning the problem of *tātkālika* (‘at-that-time’ motion of planets), shows similarities with Bhāskara II’s ideas of motion and concepts in differential calculus<sup>79</sup>; and “perhaps these ratios of small quantities are what he was referring to in his commentary on *Līlavatī* 47, when he spoke of calculations with factors of 0/0 being ‘useful in astronomy’”<sup>80</sup>.

<sup>77</sup> *Līlavatī*, 11; see also Plofker (2009: 184). For the Āryabhaṭa’s alphanumeric system, see: 73–74. Mahāvīra goes as far as  $10^{23}$  (*mahākṣobha*), stanza 68: 163.

<sup>78</sup> Plofker (2009: 4). For a discussion of the Mādhava school’s methods in infinite series and early modern European infinitesimal calculus techniques, see: 252.

<sup>79</sup> See Rao (2004).

<sup>80</sup> Plofker (2009: 198): “This analogy should not be stretched too far: for one thing, Bhāskara II is dealing with particular increments of particular trigonometric quantities, not with general functions or rates of change in the abstract. But it does bring out the conceptual boldness of the idea of an instantaneous speed, and of its derivation by means of ratios of small increments”.

Let us proceed mathematically once more. It goes without saying that a convergent geometrical series approaches zero — that is to say, when the absolute value of the common ratio ( $r$ ) is less than one and the number of terms in the series ( $n$ ) approaches infinity. In other words: if ( $|r| < 1$ ) and ( $n \rightarrow \infty$ ), then  $r^n \rightarrow 0$ . Conversely, its sum at infinity recomposes the unit, as shown by formula [3].<sup>81</sup> Its application to the present case — with  $r = 1/2$  minus one, since  $k = 1$  and not  $k = 0$ , the sequence starting from *bindu* and not *mātrā* — follows this formula:

$$\begin{aligned} \sum_{k=1}^{\infty} r^k &= \left( \frac{1}{1-r} \right) - 1 \\ \sum_{k=1}^{\infty} \left( \frac{1}{2} \right)^k &= \left( \frac{1}{1-\frac{1}{2}} \right) - 1 = 1 \end{aligned} \quad [3]$$

Therefore, in hypothesizing an infinite series of progressively halved sounds, it is clearly understood that *nāda* would last for one *mātrā*, whilst its  $n$ -ary term would tend to zero.

## 9 Conclusion

Amṛtānanda and Bhāskaraṛāya, having shown themselves no strangers to mathematical knowledge and particularly wary about choosing the tiniest units of measurement, thus do not appear to be interested in describing either a physical phenomenon or the infinitesimal approach to zero. It could be argued that they were aware that an infinite series would have asymptotically pulverized the resonance of its  $n$ -ary term ( $r^n \rightarrow 0$ ). Even without actually calculating the sum of the series, its result can easily be estimated. Following an infinite series approaching zero would, however, entail addressing phenomena which are no longer perceptible; indeed, as in the case of for instance Mahāvīra's *samaya* and *āvali*, these phenomena are significantly not defined even in relative, that is, fractional, terms ( $n$  *samayas* equal one *āvali*, whose ratio with *ucchvāsa* remains undefined; see Table 11). Amṛtānanda and Bhāskaraṛāya seem, on the contrary, more interested in describing the process of reaching definite perceptions from a *sādhaka*'s point of view, thereby identifying the steps that make up meditative experience. The analogy between *nāda* and the lessening of the resonance of a

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<sup>81</sup> Kudryavtsev (2002).

musical instrument applies in so far as it indicates the process of sound progressively thinning up to and beyond the perceptual threshold, not with respect to its vanishing in an actual physical sense. Here, the resonances of *samanā* (the eighth term of the sequence, equal to  $1/256$  of a *mātrā*) and then the atemporal *unmanī* are conditions achieved by the *sādhaka*; the apparently arbitrary interruption of the series seems to suggest this achievement, clearly incompatible with the description of any process beyond the empirical threshold. The ostensive definition of *lava* itself – i. e. piercing a lotus leaf – suggests the intent to remain in the realm of the *sādhaka*'s awareness. The experience of the unconditioned, of Śiva, of the highest peaks of nondual achievements – beyond names (*nāma*), forms (*rūpa*) and principles of reality (*tattva*) – cannot, by definition, be considered impossible to achieve. In other words, the authors assume that this condition might be accessible by the *sādhaka* precisely because their texts set out to show how to achieve it.

*Lava*, as shown above, is the smallest time fraction in the reciter's sphere of experience and piecing a lotus leaf is its ostensive, perceptible, image. *Lava*, conceived thusly, meets the authors' doctrinal needs not only because it is the tiniest clearly defined time span in its rigorous (and non-infinitesimal) fractional value, but also because it is situated at the juncture of the phenomenal and metaphysical dimensions, at once both perceivable and not perceivable, extraordinarily evanescent or resting at the very edge of perception, like piercing a lotus leaf.

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