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Rotating black holes in the FI-gauged $N = 2$, $D = 4$ $\overline{\mathbb{C}\mathrm{P}}^{\mathrm{n}}$ model

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Abstract: We construct supersymmetric black holes with rotation or NUT charge for the $\overline{\mathbb{C}\mathrm{P}}^n$ — and the t^3 model of $N = 2$, $D = 4$ U(1) FI-gauged supergravity. The solutions preserve 2 real supercharges, which are doubled for their near-horizon geometry. For the $\overline{\mathbb{C}\mathrm{P}}^n$ model we also present a generalization to the nonextremal case, which turns out to be characterized by a Carter-Plebański-type metric, and has $n + 3$ independent parameters, corresponding to mass, angular momentum as well as $n + 1$ magnetic charges. We discuss the thermodynamics of these solutions, obtain a Christodoulou-Ruffini mass formula, and shew that they obey a first law of thermodynamics and that the product of horizon areas depends on the angular momentum and the magnetic charges only. At least some of the BPS black holes that we obtain may become instrumental for future microscopic entropy computations involving a supersymmetric index.

Keywords: AdS-CFT Correspondence, Black Holes, Classical Theories of Gravity, Supergravity Models

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Contents

1 Introduction and summary of results

Black holes in gauged supergravity theories provide an important testground to address fundamental questions of gravity, both at the classical and quantum level. In particular, one may be interested in uniqueness — or no hair theorems, the final state of black hole evolution, or the problem of black hole microstates. In gauged supergravity, the solutions often (but not always; this depends essentially on whether the scalar potential has critical points) have AdS asymptotics, and one can then try to study these issues guided by the AdS/CFT correspondence. A nice example for this is the recent microscopic entropy calculation [1–4] for the black hole solutions to $N = 2$, $D = 4$ Fayet-Iliopoulos (FI)-gauged supergravity constructed in [5]. These preserve two real supercharges, and are dual to a topologically twisted ABJM theory, whose partition function can be computed exactly using supersymmetric localization techniques. This partition function can also be interpreted as the Witten index of the superconformal quantum mechanics resulting from dimensionally reducing the ABJM theory on a Riemann surface. The results of [1–4] represent the first exact black hole microstate counting that uses AdS/CFT and that does not involve an AdS_3 factor[1] with a corresponding two-dimensional CFT, whose asymptotic level density is evaluated with the Cardy formula. Subsequently, this matching was extended to many other examples and in various directions, see e.g. [7–13] and references therein.

On the other hand, black hole solutions to gauged supergravity are also relevant for a number of recent developments in high energy — and in condensed matter physics, since they provide the dual description of the quark-gluon plasma [14] as well as of certain condensed matter systems at finite temperature (cf. [15] for a review) and quantum phase transitions [16]. Of particular importance in this context are models that contain Einstein gravity coupled to U(1) gauge fields and neutral scalars, which have been instrumental to study transitions from Fermi-liquid to non-Fermi-liquid behaviour, cf. [17, 18] and references therein. Notice that the necessity of a bulk U(1) gauge field arises, because a basic ingredient of realistic condensed matter systems is the presence of a finite density of charge carriers. Such models are provided by matter-coupled gauged supergravity. Especially we shall be interested in the $N = 2$ U(1) FI-gauged theory in four dimensions, which contains indeed neutral scalars as well as abelian gauge fields.

There are thus a number of reasons to extend the spectrum of known black hole solutions to gauged supergravity. Since there exists by now a rather large amount of literature on this subject, in the following we will give an overview on existing solutions, which may be useful for the reader in its own right. In order to avoid escalation we shall thereby restrict our attention to the $N = 2$, $D = 4$ U(1) FI-gauged theory only. To the best of our knowledge, the first paper on this subject was [19], where nonextremal black holes in the stu model were constructed. These carry four charges, which are either all electric or magnetic. Ref. [20] derives electrically charged 1/2 BPS solutions for arbitrary prepotential, which unfortunately are naked singularities as soon as the gauge coupling constant is nonvanishing. In [5] (using the classification scheme of [21]), the first examples of genuine supersymmetric black holes in AdS_4 with nonconstant scalar fields were presented for the t^3 and the stu model. Typically these are magnetically charged and represent also the prime instance of static BPS black holes in AdS_4 with spherical symmetry. [22, 23] elaborate further on the solutions of [5], while [24–27] and [28–31] generalize them to other prepotentials (with dyonic gaugings) and finite temperature respectively. Rotation was added in [32] (BPS case, $-iX^0X^1$-model, only magnetic charges), [33, 34] (same model, but nonextremal and dyonic) and very recently in [35] (BPS, cubic prepotential and dyonic gauging). NUT-charged supersymmetric black holes were constructed in [36] for the $-iX^0X^1$-model and in [37] for a cubic prepotential with dyonic gauging. It is worth noting that there exists also a strange class of black holes whose horizon is noncompact but nevertheless has finite area [34, 38]. These may provide an interesting testground to address fundamental questions related to black hole physics or holography.

Many further hitherto unknown solutions might exist, but are very probably difficult to construct by trying to solve the coupled Einstein-Maxwell-scalar equations. However, the

[1]Or geometries related to AdS_3, like those appearing in the Kerr/CFT correspondence [6].

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supersymmetric subclass of them (if it exists) satisfies first order equations, which should facilitate their discovery and explicit construction.

In this paper we shall consider the $\overline{\mathbb{C}\mathrm{P}}^n$ — and the t^3 model, characterized by a quadratic and cubic prepotential respectively. We start in section 2 with a brief review of $N = 2$, $D = 4$ FI-gauged supergravity as well as a summary of some results of [21, 39], where the one quarter and one half supersymmetric backgrounds of the theory were classified. In section 3 we apply the recipe of [21] to construct rotating extremal BPS black holes in the $\overline{\mathbb{C}\mathrm{P}}^n$ model, which preserve two real supercharges. It is shown that the latter are doubled for the near-horizon geometry. Moreover, we also obtain BPS black holes with NUT charge in the same model. The following section is dedicated to the prepotential $F = -(X^1)^3/X^0$, for which we present first a supersymmetric near-horizon solution, which is subsequently extended to a full black hole geometry. Finally, 5 contains a generalization of the solutions in section 3 to the nonextremal case, which turns out to be characterized by a Carter-Plebański-type metric, and has $n + 3$ independent parameters, corresponding to mass, angular momentum as well as $n+1$ magnetic charges. We also discuss the thermodynamics of these solutions, obtain a Christodoulou-Ruffini mass formula, and shew that they obey a first law of thermodynamics and that the product of horizon areas depends on the angular momentum and the magnetic charges only.

We believe that at least some of the black holes constructed in this paper may become instrumental for future microscopic entropy computations involving a supersymmetric index, along the lines of [1–4].

An appendix contains the equations of motion of the theory under consideration.

2 $N = 2$, $D = 4$ FI-gauged supergravity

2.1 The theory and BPS equations

We consider $N = 2$, $D = 4$ gauged supergravity coupled to n abelian vector multiplets [40].[2] Apart from the vierbein e^a_μ, the bosonic field content includes the vectors A^I_μ enumerated by $I = 0, \ldots, n$, and the complex scalars z^α where $\alpha = 1, \ldots, n$. These scalars parametrize a special Kähler manifold, i.e., an n-dimensional Hodge-Kähler manifold that is the base of a symplectic bundle, with the covariantly holomorphic sections

$$\mathcal{V} = \begin{pmatrix} X^I \\ F_I \end{pmatrix}, \qquad \mathcal{D}_{\bar\alpha}\mathcal{V} = \partial_{\bar\alpha}\mathcal{V} - \frac{1}{2}(\partial_{\bar\alpha}\mathcal{K})\mathcal{V} = 0\,, \tag{2.1}$$

where $\mathcal{K}$ is the Kähler potential and $\mathcal{D}$ denotes the Kähler-covariant derivative. $\mathcal{V}$ obeys the symplectic constraint

$$\langle \mathcal{V}, \bar{\mathcal{V}} \rangle = X^I \bar F_I - F_I \bar X^I = i\,. \tag{2.2}$$

To solve this condition, one defines

$$\mathcal{V} = e^{\mathcal{K}(z,\bar z)/2} v(z)\,, \tag{2.3}$$

[2]Throughout this paper, we use the notations and conventions of [41].

where $v(z)$ is a holomorphic symplectic vector,

$$v(z) = \begin{pmatrix} Z^I(z) \\ \frac{\partial}{\partial Z^I} F(Z) \end{pmatrix} . \tag{2.4}$$

F is a homogeneous function of degree two, called the prepotential, whose existence is assumed to obtain the last expression. The Kähler potential is then

$$e^{-\mathcal{K}(z,\bar{z})} = -i\langle v, \bar{v}\rangle . \tag{2.5}$$

The matrix $\mathcal{N}_{IJ}$ determining the coupling between the scalars z^α and the vectors A^I_μ is defined by the relations

$$F_I = \mathcal{N}_{IJ} X^J , \qquad \mathcal{D}_{\bar\alpha}\bar{F}_I = \mathcal{N}_{IJ}\mathcal{D}_{\bar\alpha}\bar{X}^J . \tag{2.6}$$

The bosonic action reads

$$\begin{aligned} e^{-1}\mathcal{L}_{\rm bos} = {} & \frac{1}{2}R + \frac{1}{4}(\mathrm{Im}\,\mathcal{N})_{IJ} F^I_{\mu\nu}F^{J\mu\nu} - \frac{1}{8}(\mathrm{Re}\,\mathcal{N})_{IJ}\, e^{-1}\epsilon^{\mu\nu\rho\sigma}F^I_{\mu\nu}F^J_{\rho\sigma} \\ & - g_{\alpha\bar\beta}\partial_\mu z^\alpha \partial^\mu \bar{z}^{\bar\beta} - V , \end{aligned} \tag{2.7}$$

with the scalar potential

$$V = -2g^2\xi_I\xi_J[(\mathrm{Im}\,\mathcal{N})^{-1|IJ} + 8\bar{X}^I X^J] , \tag{2.8}$$

that results from U(1) Fayet-Iliopoulos gauging. Here, g denotes the gauge coupling and the ξ_I are FI constants. In what follows, we define $g_I \equiv g\xi_I$.

The most general timelike supersymmetric background of the theory described above was constructed in [21], and is given by

$$ds^2 = -4|b|^2(dt+\sigma)^2 + |b|^{-2}(dz^2 + e^{2\Phi}dw d\bar{w}) , \tag{2.9}$$

where the complex function $b(z,w,\bar{w})$, the real function $\Phi(z,w,\bar{w})$ and the one-form $\sigma = \sigma_w dw + \sigma_{\bar{w}} d\bar{w}$, together with the symplectic section 2.1[3] are determined by the equations

$$\partial_z\Phi = 2ig_I\left(\frac{\bar{X}^I}{b} - \frac{X^I}{\bar{b}}\right) , \tag{2.10}$$

$$\begin{aligned} & 4\partial\bar\partial\left(\frac{X^I}{\bar{b}} - \frac{\bar{X}^I}{b}\right) + \partial_z\left[e^{2\Phi}\partial_z\left(\frac{X^I}{\bar{b}} - \frac{\bar{X}^I}{b}\right)\right] \\ & \quad - 2ig_J\partial_z\left\{e^{2\Phi}\left[|b|^{-2}(\mathrm{Im}\,\mathcal{N})^{-1|IJ} + 2\left(\frac{X^I}{\bar{b}} + \frac{\bar{X}^I}{b}\right)\left(\frac{X^J}{\bar{b}} + \frac{\bar{X}^J}{b}\right)\right]\right\} = 0 , \end{aligned} \tag{2.11}$$

$$\begin{aligned} & 4\partial\bar\partial\left(\frac{F_I}{\bar{b}} - \frac{\bar{F}_I}{b}\right) + \partial_z\left[e^{2\Phi}\partial_z\left(\frac{F_I}{\bar{b}} - \frac{\bar{F}_I}{b}\right)\right] \\ & \quad - 2ig_J\partial_z\left\{e^{2\Phi}\left[|b|^{-2}\mathrm{Re}\,\mathcal{N}_{IL}(\mathrm{Im}\,\mathcal{N})^{-1|JL} + 2\left(\frac{F_I}{\bar{b}} + \frac{\bar{F}_I}{b}\right)\left(\frac{X^J}{\bar{b}} + \frac{\bar{X}^J}{b}\right)\right]\right\} \\ & \quad - 8ig_I e^{2\Phi}\left[\langle\mathcal{I}, \partial_z\mathcal{I}\rangle - \frac{g_J}{|b|^2}\left(\frac{X^J}{\bar{b}} + \frac{\bar{X}^J}{b}\right)\right] = 0 , \end{aligned} \tag{2.12}$$

[3] Note that also σ and $\mathcal{V}$ are independent of t.

$$2\partial\bar{\partial}\Phi = e^{2\Phi}\left[ig_I\partial_z\left(\frac{X^I}{\bar{b}} - \frac{\bar{X}^I}{b}\right) + \frac{2}{|b|^2}g_Ig_J(\mathrm{Im}\mathcal{N})^{-1|IJ} + 4\left(\frac{g_IX^I}{\bar{b}} + \frac{g_I\bar{X}^I}{b}\right)^2\right], \quad (2.13)$$

$$d\sigma + 2\star^{(3)}\langle\mathcal{I}\,, d\mathcal{I}\rangle - \frac{i}{|b|^2}g_I\left(\frac{\bar{X}^I}{b} + \frac{X^I}{\bar{b}}\right)e^{2\Phi}dw\wedge d\bar{w} = 0\,. \quad (2.14)$$

Here $\star^{(3)}$ is the Hodge star on the three-dimensional base with metric[4]

$$ds_3^2 = dz^2 + e^{2\Phi}dwd\bar{w}\,, \quad (2.15)$$

and we defined $\partial = \partial_w$, $\bar{\partial} = \partial_{\bar{w}}$, as well as

$$\mathcal{I} = \mathrm{Im}\left(\mathcal{V}/\bar{b}\right)\,, \qquad \mathcal{R} = \mathrm{Re}\left(\mathcal{V}/\bar{b}\right)\,. \quad (2.16)$$

Note that the eqs. (2.10)–(2.13) can be written compactly in the symplectically covariant form

$$\partial_z\Phi = 4\langle\mathcal{I},\mathcal{G}\rangle\,, \quad (2.17)$$

$$\Delta\mathcal{I} + 2e^{-2\Phi}\partial_z\left\{e^{2\Phi}\left[\langle\mathcal{R},\mathcal{I}\rangle\Omega\mathcal{M}\mathcal{G} - 4\mathcal{R}\langle\mathcal{R},\mathcal{G}\rangle\right]\right\} - 4\mathcal{G}\left[\langle\mathcal{I},\partial_z\mathcal{I}\rangle + 4\langle\mathcal{R},\mathcal{I}\rangle\langle\mathcal{R},\mathcal{G}\rangle\right] = 0\,, \quad (2.18)$$

$$\Delta\Phi = -8\langle\mathcal{R},\mathcal{I}\rangle\left[\mathcal{G}^t\mathcal{M}\mathcal{G} + 8|\mathcal{L}|^2\right] = 4\langle\mathcal{R},\mathcal{I}\rangle V\,, \quad (2.19)$$

where $\mathcal{G} = (g^I, g_I)^t$ represents the symplectic vector of gauge couplings,[5] $\mathcal{L} = \langle\mathcal{V},\mathcal{G}\rangle$, Δ denotes the covariant Laplacian associated to the base space metric (2.15), and V in (2.19) is the scalar potential (2.8). Moreover,

$$\Omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\,, \qquad \mathcal{M} = \begin{pmatrix} \mathrm{Im}\mathcal{N} + \mathrm{Re}\mathcal{N}(\mathrm{Im}\mathcal{N})^{-1}\mathrm{Re}\mathcal{N} & -\mathrm{Re}\mathcal{N}(\mathrm{Im}\mathcal{N})^{-1} \\ -(\mathrm{Im}\mathcal{N})^{-1}\mathrm{Re}\mathcal{N} & (\mathrm{Im}\mathcal{N})^{-1} \end{pmatrix}\,. \quad (2.20)$$

Finally, (2.14) can be rewritten as

$$d\sigma + \star_h\left(d\Sigma - A + \frac{1}{2}\nu\Sigma\right) = 0\,, \quad (2.21)$$

where the function Σ and the one-form ν are respectively given by

$$\Sigma = \frac{i}{2}\ln\frac{\bar{b}}{b}\,, \qquad \nu = \frac{8}{\Sigma}\langle\mathcal{G},\mathcal{R}\rangle dz\,, \quad (2.22)$$

A is the gauge field of the Kähler U(1),

$$A_\mu = -\frac{i}{2}(\partial_\alpha\mathcal{K}\partial_\mu z^\alpha - \partial_{\bar{\alpha}}\mathcal{K}\partial_\mu\bar{z}^{\bar{\alpha}})\,, \quad (2.23)$$

and $\star_h$ denotes the Hodge star on the Weyl-rescaled base space metric

$$h_{ij}dx^idx^j = \frac{1}{|b|^4}(dz^2 + e^{2\Phi}dwd\bar{w})\,. \quad (2.24)$$

[4] Whereas in the ungauged case, this base space is flat and thus has trivial holonomy, here we have U(1) holonomy with torsion [21].

[5] In the case considered here with electric gaugings only, one has $g^I = 0$.

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(2.21) is the generalized monopole equation [42], or more precisely a Kähler-covariant generalization thereof, due to the presence of the one-form A. In order to cast (2.14) into the form (2.21), one has to use the special Kähler identities

$$\langle \mathcal{D}_\alpha \mathcal{V}, \mathcal{V}\rangle = \langle \mathcal{D}_\alpha \mathcal{V}, \bar{\mathcal{V}}\rangle = 0\,. \tag{2.25}$$

Note that (2.21) is invariant under Weyl rescaling, accompanied by a gauge transformation of ν,

$$h_{mn}\mathrm{d}x^m\mathrm{d}x^n \mapsto e^{2\psi}h_{mn}\mathrm{d}x^m\mathrm{d}x^n\,, \quad \Sigma \mapsto e^{-\psi}\Sigma\,, \quad \nu \mapsto \nu + 2\mathrm{d}\psi\,, \quad A \mapsto e^{-\psi}A\,. \tag{2.26}$$

It would be very interesting to better understand the deeper origin of the conformal invariance of (2.21) in the present context.

The integrability condition for (2.21) reads

$$D_i[h^{ij}\sqrt{h}(D_j - A_j)\Sigma] = 0\,, \tag{2.27}$$

with the Weyl-covariant derivative

$$D_i = \partial_i - \frac{m}{2}\nu_i\,, \tag{2.28}$$

where m denotes the Weyl weight of the corresponding field.[6] It is straightforward to show that (2.27) is equivalent to

$$\langle \mathcal{I}, \Delta\mathcal{I}\rangle + 4e^{-2\Phi}\partial_z\left(e^{2\Phi}\langle\mathcal{I},\mathcal{R}\rangle\langle\mathcal{G},\mathcal{R}\rangle\right) = 0\,, \tag{2.29}$$

which follows from (2.18) by taking the symplectic product with $\mathcal{I}$. To shew this, one has to use

$$\frac{1}{2}(\mathcal{M} + i\Omega) = \Omega\bar{\mathcal{V}}\mathcal{V}\Omega + \Omega\mathcal{D}_\alpha\mathcal{V}g^{\alpha\bar{\beta}}\mathcal{D}_{\bar{\beta}}\bar{\mathcal{V}}\Omega, \tag{2.30}$$

$$\langle \mathcal{D}_\alpha\mathcal{V}, \mathcal{D}_\beta\mathcal{V}\rangle = 0\,, \qquad \langle \mathcal{D}_\alpha\mathcal{V}, \mathcal{D}_{\bar{\beta}}\bar{\mathcal{V}}\rangle = -ig_{\alpha\bar{\beta}}\,, \tag{2.31}$$

as well as (2.17) and (2.25).

Given b, Φ, σ and $\mathcal{V}$, the fluxes read

$$\begin{aligned} F^I = {} & 2(dt+\sigma)\wedge d\left[bX^I + \bar{b}\bar{X}^I\right] + |b|^{-2}dz\wedge d\bar{w}\left[\bar{X}^I(\bar{\partial}\bar{b} + iA_{\bar{w}}\bar{b}) + (\mathcal{D}_\alpha X^I)b\bar{\partial}z^\alpha - \right. \\ & \left. X^I(\bar{\partial}b - iA_{\bar{w}}b) - (\mathcal{D}_{\bar{\alpha}}\bar{X}^I)\bar{b}\bar{\partial}\bar{z}^{\bar{\alpha}}\right] - |b|^{-2}dz\wedge dw\left[\bar{X}^I(\partial\bar{b} + iA_w\bar{b}) + \right. \\ & \left. (\mathcal{D}_\alpha X^I)b\partial z^\alpha - X^I(\partial b - iA_w b) - (\mathcal{D}_{\bar{\alpha}}\bar{X}^I)\bar{b}\partial\bar{z}^{\bar{\alpha}}\right] - \\ & \frac{1}{2}|b|^{-2}e^{2\Phi}dw\wedge d\bar{w}\left[\bar{X}^I(\partial_z\bar{b} + iA_z\bar{b}) + (\mathcal{D}_\alpha X^I)b\partial_z z^\alpha - X^I(\partial_z b - iA_z b) - \right. \\ & \left. (\mathcal{D}_{\bar{\alpha}}\bar{X}^I)\bar{b}\partial_z\bar{z}^{\bar{\alpha}} - 2ig_J(\mathrm{Im}\,\mathcal{N})^{-1|IJ}\right]\,. \end{aligned} \tag{2.32}$$

[6] A field Γ with Weyl weight m transforms as $\Gamma \mapsto e^{m\psi}\Gamma$ under a Weyl rescaling.

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2.2 1/2 BPS near-horizon geometries

An interesting class of half-supersymmetric backgrounds was obtained in [39]. It includes the near-horizon geometry of extremal rotating black holes. The metric and the fluxes read respectively

$$ds^2 = 4e^{-\xi}\left(-r^2dt^2 + \frac{dr^2}{r^2}\right) + 4(e^{-\xi} - Ke^{\xi})(d\phi + rdt)^2 + \frac{4e^{-2\xi}d\xi^2}{Y^2(e^{-\xi} - Ke^{\xi})}\,, \tag{2.33}$$

$$\begin{aligned} F^I &= 16i\sqrt{K}\left(\frac{\bar{X}X^I}{1-iY} - \frac{X\bar{X}^I}{1+iY}\right)dt\wedge dr \\ &\quad + \frac{8\sqrt{K}}{Y}\left[\frac{2\bar{X}X^I}{1-iY} + \frac{2X\bar{X}^I}{1+iY} + (\operatorname{Im}\mathcal{N})^{-1|IJ}g_J\right](d\phi + rdt)\wedge d\xi\,, \end{aligned} \tag{2.34}$$

where $X \equiv g_I X^I$, $K > 0$ is a real integration constant, and Y is defined by

$$Y^2 = 64e^{-\xi}|X|^2 - 1\,. \tag{2.35}$$

The moduli fields z^α depend on the horizon coordinate ξ only, and obey the flow equation[7]

$$\frac{dz^\alpha}{d\xi} = \frac{i}{2\bar{X}Y}(1-iY)g^{\alpha\bar{\beta}}\mathcal{D}_{\bar{\beta}}\bar{X}\,. \tag{2.36}$$

(2.33) is of the form (3.3) of [43], and describes the near-horizon geometry of extremal rotating black holes,[8] with isometry group $\mathrm{SL}(2,\mathbb{R})\times\mathrm{U}(1)$. From (2.36) it is clear that the scalar fields have a nontrivial dependence on the horizon coordinate ξ unless $g_I\mathcal{D}_\alpha X^I = 0$. As was shown in [39], the solution with constant scalars is the near-horizon limit of the supersymmetric rotating hyperbolic black holes in minimal gauged supergravity [45].

Using Y in place of ξ as a new variable, (2.36) becomes

$$\frac{dz^\alpha}{dY} = \frac{Xg^{\alpha\bar{\beta}}\mathcal{D}_{\bar{\beta}}\bar{X}}{(Y-i)\left[-\bar{X}X + \mathcal{D}_\gamma X g^{\gamma\bar{\delta}}\mathcal{D}_{\bar{\delta}}\bar{X}\right]}\,. \tag{2.37}$$

This can also be rewritten in a Kähler-covariant form, as a differential equation for the symplectic section $\mathcal{V}$,

$$D_Y\mathcal{V} = \frac{X\mathcal{D}_\alpha\mathcal{V}g^{\alpha\bar{\beta}}\mathcal{D}_{\bar{\beta}}\bar{X}}{(Y-i)\left[-\bar{X}X + \mathcal{D}_\gamma X g^{\gamma\bar{\delta}}\mathcal{D}_{\bar{\delta}}\bar{X}\right]}\,, \tag{2.38}$$

where

$$D_Y \equiv \frac{d}{dY} + iA_Y \tag{2.39}$$

denotes the Kähler-covariant derivative.

[7] Note that this is not a radial flow, but a flow along the horizon.

[8] Metrics of the type (2.33) were discussed for the first time in [44] in the context of the extremal Kerr throat geometry.

2.3 The $\overline{\mathbb{C}\mathrm{P}}^n$ model

We shall now give an explicit example of a near-horizon geometry with varying scalars, taking the $\overline{\mathbb{C}\mathrm{P}}^n = \mathrm{SU}(1,n)/(\mathrm{SU}(n)\times\mathrm{U}(1))$ model, defined by the quadratic prepotential

$$F = \frac{i}{4}X^I\eta_{IJ}X^J\,, \qquad \eta_{IJ} \equiv \mathrm{diag}(-1,1,\ldots,1)\,. \tag{2.40}$$

This yields

$$F_I = \frac{\partial F}{\partial X^I} = \frac{i}{2}\eta_{IJ}X^J\,. \tag{2.41}$$

If we choose homogeneous coordinates by $Z^0 = 1$, $Z^\alpha = z^\alpha$, the holomorphic symplectic section and the Kähler potential read respectively

$$v = \left(1, z^\alpha, -\frac{i}{2}, \frac{i}{2}z^\alpha\right)^t\,, \qquad e^{-\mathcal{K}} = 1 - \sum_{\alpha=1}^{n}|z^\alpha|^2\,, \tag{2.42}$$

which implies that the complex scalars are constrained to the region $0 \le \sum_\alpha |z^\alpha|^2 < 1$. The special Kähler metric and its inverse are given by

$$g_{\alpha\bar\beta} = e^{\mathcal{K}}\delta_{\alpha\beta} + e^{2\mathcal{K}}\bar z^{\bar\alpha}z^\beta\,, \qquad g^{\alpha\bar\beta} = e^{-\mathcal{K}}(\delta^{\alpha\beta} - z^\alpha\bar z^{\bar\beta})\,, \tag{2.43}$$

while the period matrix is

$$\mathcal{N}_{IJ} = -\frac{i}{2}\eta_{IJ} + i\frac{Z_IZ_J}{Z_KZ^K}\,, \qquad \mathrm{Im}\,\mathcal{N}_{IJ} = -\frac{1}{2}\eta_{IJ} + \frac{1}{2}\left(\frac{Z_IZ_J}{Z_KZ^K} + \text{c.c.}\right)\,, \tag{2.44}$$

$$(\mathrm{Im}\,\mathcal{N})^{-1|IJ} = 2\left[-\eta^{IJ} + \left(\frac{Z^I\bar Z^J}{Z^K\bar Z_K} + \text{c.c.}\right)\right]\,, \tag{2.45}$$

where we defined $Z_I \equiv \eta_{IJ}Z^J$. The scalar potential (2.8) reads

$$V = 4g^2 - 8\frac{|g_0 + \sum_\alpha g_\alpha z^\alpha|^2}{1 - \sum_\beta |z^\beta|^2}\,, \tag{2.46}$$

with $g^2 \equiv \eta^{IJ}g_Ig_J$ from now on. V has an extremum at $z^\alpha = -g_\alpha/g_0$, where $V = 12g^2$. For $z^\alpha = -g_\alpha/g_0$ to lie in the allowed region, the vector of gauge couplings g_I must be timelike, i.e., $g^2 < 0$. The extremum corresponds then to a supersymmetric AdS vacuum. In addition, it is easy to see that the potential has flat directions given by $g_0 + \sum_\alpha g_\alpha z^\alpha = 0$, where $V = 4g^2$. For $n = 1$, the flat directions degenerate to the point $z^1 = -g_0/g_1$, which lies in the allowed region for $g^2 > 0$. In this case one has thus a critical point corresponding to a supersymmetry-breaking de Sitter vacuum. If there is more than one vector multiplet, the situation is of course more complicated.

2.4 The t^3 model

Cubic models are of special interest. In the ungauged theory, these can be embedded in higher dimensional supergravity theories describing the low energy limit of some string theory. This appealing property is also displayed after gauging the theory at least for some of the cubic models. This is the case for the FI-gauged *stu* model, which contains $n = 3$ vector multiplets, and represents the best known example. It can be obtained as a consistent truncation of eleven-dimensional supergravity compactified on S^7 [46]. Moreover, if the three vector multiplets are identified, one gets the so-called t^3 model, which we will consider in this work. The bosonic content of the theory contains the metric $g_{\mu\nu}$, two gauge fields A^I_μ and one complex scalar τ. The theory is defined by the prepotential

$$F = -\frac{(X^1)^3}{X^0}\,. \tag{2.47}$$

In this case we have

$$F_I = X^0\left(\tau^3, -3\tau^2\right)\,, \tag{2.48}$$

where we use homogeneous coordinates in the scalar manifold, with $\tau \equiv X^1/X^0$. The Kähler potential and scalar metric are then

$$e^{-\mathcal{K}} = 8\,(\mathrm{Im}\tau)^3\,, \qquad\qquad g_{\tau\bar{\tau}} = \frac{3}{4(\mathrm{Im}\tau)^2}\,, \tag{2.49}$$

which implies $\mathrm{Im}\tau > 0$. The scalar potential is then

$$V = -\frac{4g_1^2}{3(\mathrm{Im}\tau)}\,, \tag{2.50}$$

which has no critical point, so the theory does not admit AdS_4 vacua with constant moduli. Still, we will be able to construct a nontrivial family of black hole solutions, which of course do not asymptote to AdS_4.

3 Supersymmetric rotating black holes in the $\overline{\mathbb{C}\mathrm{P}}^n$ model

In this section we obtain a generalization of the asymptotically AdS black holes found in [32, 36] to include an arbitrary number of vector multiplets n. To do so, we shall use some ansätze which are inspired by those articles. We begin constructing in detail a rotating black hole specified in terms of $n + 2$ parameters — see (3.20), (3.23) and (3.25). Moreover, in section 3.4 we present a solution with NUT charge.

3.1 Solving the BPS equations

A natural generalization of the successful ansatz used in [32] for the $\overline{\mathbb{C}\mathrm{P}}^1$ model is given by

$$\frac{\bar{X}^I}{b} = \frac{f^I(z) + \eta^I(w,\bar{w})}{\gamma(z)}\,, \qquad e^{2\Phi} = h(z)\ell(w,\bar{w})\,, \tag{3.1}$$

where $f^I(z)$ is a purely imaginary function, while $\gamma(z)$, $\eta^I(w,\bar{w})$, $h(z)$ and $\ell(w,\bar{w})$ are real. With these assumptions, the BPS equations, although remaining nonlinear, become separable and can be solved. The first of them, (2.10), boils down to

$$\partial_z \ln h = -\frac{8g_I \mathrm{Im} f^I}{\gamma}. \tag{3.2}$$

One can see that the symplectic constraint (2.2) implies that $X^I \eta_{IJ} \bar{X}^J = -1$, which in turn gives

$$|b|^{-2} = \frac{1}{\gamma^2} \eta_{IJ} \left(f^I f^J - \eta^I \eta^J \right) . \tag{3.3}$$

Using these expressions, equation (2.13) reduces to

$$\frac{\partial \bar{\partial} \ln \ell}{\ell} = h \left[-\frac{1}{4} \partial_z^2 \ln h + \frac{4}{\gamma^2} g_I g_J \left(\eta^{IJ} \eta_{LK} (\eta^L \eta^K - f^L f^K) + 2(f^I f^J + \eta^I \eta^J) \right) \right] . \tag{3.4}$$

Now we observe that if we take $h/\gamma^2 = \text{const.} \equiv c_1 > 0$, this differential equation is separable, and one can define a constant c_2 such that

$$-\frac{h}{4} \partial_z^2 \ln h + \frac{4h}{\gamma^2} g_I g_J \left(-\eta^{IJ} \eta_{LK} f^L f^K + 2 f^I f^J \right) = c_1 c_2 , \tag{3.5}$$

$$\frac{\partial \bar{\partial} \ln \ell}{\ell} - 4 c_1 g^2 \eta_K \eta^K - 8 c_1 \eta^2 = c_1 c_2 , \tag{3.6}$$

where $\eta \equiv g_I \eta^I$ and capital indices are lowered with η_{IJ}. Equations (3.2) and (3.5) can be solved by using the polynomial ansatz

$$\gamma = c + a z^2 , \qquad h = c_1 (c + a z^2)^2 , \qquad f^I = i (\alpha^I z + \beta^I) , \tag{3.7}$$

for some real constants a, c, α^I, β^I, which are constrained by

$$g_I \alpha^I = -\frac{a}{2} , \quad g_I \beta^I = 0 , \quad \alpha^I \eta_{IJ} \beta^J = 0 , \quad -ac + 4 g^2 \beta^2 = c_2 , \quad a^2 = 4 g^2 \alpha^2 , \tag{3.8}$$

where $\alpha^2 \equiv \eta_{IJ} \alpha^I \alpha^J$ and $\beta^2 \equiv \eta_{IJ} \beta^I \beta^J$.

The Bianchi identities (2.11) are then easily solved, and lead to

$$\alpha^I = -\frac{2 \eta^{IJ} g_J \alpha^2}{a} . \tag{3.9}$$

Observe that the set of constraints obtained so far completely fixes α^I and c_2 in terms of a, c and β^α, while c_1 remains free. As we will see, some of these degrees of freedom can be eliminated by a coordinate transformation.

After some computation, Maxwell's equations (2.12) reduce to

$$\partial \bar{\partial} \eta_I - 4 \ell c_1 \eta g_I \left(\eta_K \eta^K + \beta^2 - \frac{\alpha^2 c}{a} \right) = 0 . \tag{3.10}$$

Together with (3.6), they define a system of $n+1$ second order, nonlinear differential equations, and looking for the general solution might seem a hopeless endeavour. Remarkably, the system can be solved using the ansatz of the type considered in [32],

$$\ell = \frac{1+\delta}{\cosh^4(k\tilde{x})} , \quad \eta^I = \hat{\eta}^I \tanh(k\tilde{x}) , \quad \delta = A \cosh^4(k\tilde{x}) , \quad \frac{dx}{d\tilde{x}} = \frac{\cosh^2(k\tilde{x})}{1+\delta} , \tag{3.11}$$

where $A, k, \hat{\eta}^I$ are some constants and $x \equiv (w + \bar{w})/2$. Defining $\hat{\eta} \equiv g_I \hat{\eta}^I$, equation (3.6) becomes

$$k^2 + c_1 c_2 + \sinh^2(k\tilde{x}) \left(-2k^2 + c_1 c_2 + 4 c_1 g^2 \hat{\eta}_K \hat{\eta}^K + 8 c_1 \hat{\eta}^2 \right) = 0 , \tag{3.12}$$

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which is solved provided

$$k^2 = -c_1 c_2\,, \qquad 3k^2 = 4c_1 g^2 \hat{\eta}_K \hat{\eta}^K + 8c_1 \hat{\eta}^2\,. \tag{3.13}$$

On the other hand, Maxwell's equations (3.10) simplify to

$$k^2 \hat{\eta}_I + \sinh^2(k\tilde{x}) 4c_1 \hat{\eta} g_I \left(\hat{\eta}_K \hat{\eta}^K + \beta^2 - \frac{\alpha^2 c}{a}\right) + 4c_1 \hat{\eta} g_I \left(\beta^2 - \frac{\alpha^2 c}{a}\right) = 0\,, \tag{3.14}$$

which are satisfied if

$$\hat{\eta}^K \hat{\eta}_K + \beta^2 - \frac{\alpha^2 c}{a} = 0\,, \qquad k^2 \hat{\eta}_I + 4c_1 \hat{\eta} g_I \left(\beta^2 - \frac{\alpha^2 c}{a}\right) = 0\,. \tag{3.15}$$

In summary, we can combine (3.8), (3.13) and (3.15) to find

$$k^2 = 4c_1 \hat{\eta}^2\,, \qquad g^2 \hat{\eta}_I = \hat{\eta} g_I\,. \tag{3.16}$$

This implies that the only independent parameter in (3.11) is A.

Finally, to completely specify the solution we have to integrate (2.14). To this end we use $(\tilde{x}, y, z)$ as coordinates, where $y = (w - \bar{w})/2i$. The relevant Hodge duals on the metric (2.15) are

$$\star^{(3)} d\tilde{x} = \frac{1+\delta}{\cosh^2(k\tilde{x})} dy \wedge dz\,, \qquad \star^{(3)} dz = \frac{e^{2\Phi}\cosh^2(k\tilde{x})}{1+\delta} d\tilde{x} \wedge dy\,,$$

and thus (2.14) takes the form

$$\begin{aligned}\partial_{\tilde{x}}\sigma_y d\tilde{x}\wedge dy - \partial_z \sigma_y dy \wedge dz - \frac{k}{\gamma^2}\left(\alpha^I \hat{\eta}_I z\right) \frac{1+\delta}{\cosh^4(k\tilde{x})} dy\wedge dz - \frac{2\hat{\eta}\alpha^2 c_1}{a} \frac{\tanh(k\tilde{x})}{\cosh^2(k\tilde{x})} d\tilde{x}\wedge dy \\ + 4c_1 \hat{\eta}\left(\alpha^2 z^2 + \beta^2 + \hat{\eta}_K \hat{\eta}^K \tanh^2(k\tilde{x})\right) \frac{\tanh(k\tilde{x})}{\gamma \cosh^2(k\tilde{x})} d\tilde{x}\wedge dy = 0\,,\end{aligned} \tag{3.17}$$

which can be easily integrated to give

$$\sigma_y = \frac{\hat{\eta}}{4g^2 k}\left[\frac{a c_1}{\cosh^2(k\tilde{x})} - \frac{k^2}{\gamma}\left(A + \frac{1}{\cosh^4(k\tilde{x})}\right)\right]\,. \tag{3.18}$$

3.2 The fields

The metric (2.9) of the solution obtained here can be simplified by the coordinate transformation

$$\begin{pmatrix} t \\ y \end{pmatrix} \mapsto \sqrt{\frac{\mathsf{E}}{-2A}} \begin{pmatrix} 0 & -\frac{a A \mathsf{E} L^3}{8\hat{\eta}} \\ -\frac{1}{kL} & \frac{\mathsf{E}L}{2k} \end{pmatrix} \begin{pmatrix} t \\ y \end{pmatrix}\,, \qquad p = B\tanh(k\tilde{x})\,, \qquad q = Dz\,, \tag{3.19}$$

where $B = \sqrt{\frac{\mathsf{E}}{-8g^2}}$, $D = \sqrt{\frac{a^2 c_1 \mathsf{E}}{-8g^2 k^2}}$ and E is a positive constant, so A must be negative. In these coordinates the metric takes the Carter-Plebański [47, 48] form

$$\begin{aligned} ds^2 = \frac{p^2+q^2-\Delta^2}{P} dp^2 + \frac{P}{p^2+q^2-\Delta^2}\left(dt + (q^2-\Delta^2)dy\right)^2 + \\ + \frac{p^2+q^2-\Delta^2}{Q} dq^2 - \frac{Q}{p^2+q^2-\Delta^2}\left(dt - p^2 dy\right)^2\,, \end{aligned} \tag{3.20}$$

where

$$P=(1+A)\frac{\mathsf{E}^2L^2}{4}-\mathsf{E}p^2+\frac{p^4}{L^2}\,,\qquad Q=\frac{1}{L^2}\left(q^2+\frac{\mathsf{E}L^2}{2}-\Delta^2\right)^2\,,\tag{3.21}$$

and L^2 and Δ^2 are two positive constants defined by

$$\Delta^2=\frac{\mathsf{E}\beta^2}{8\hat{\eta}^2}\,,\qquad L^2=-\frac{1}{4g^2}\,.\tag{3.22}$$

The scalar fields z^α read

$$z^\alpha=-\frac{g_\alpha}{g_0}\frac{p^2+q^2+i\Delta_1p-\Delta_1q}{p^2+q_1^2}-i\Delta_1\frac{\beta^\alpha}{\beta^0}\frac{p-iq_1}{p^2+q_1^2}\,,$$

or equivalently, in a neater fashion

$$z^\alpha=\frac{1}{p+iq_1}\left(-\frac{g_\alpha}{g_0}(p+iq)-i\Delta_1\frac{\beta^\alpha}{\beta^0}\right),\tag{3.23}$$

with

$$q_1=q-\Delta_1\,,\qquad \Delta_1=\frac{\beta^0}{g_0}\sqrt{\frac{-g^2}{\beta^2}}\Delta\,.\tag{3.24}$$

If $\Delta_1=0$ (or equivalently $\Delta=0$), the scalars are constant and they assume the value $-g_\alpha/g_0$ for which the potential (2.46) is extremized.

To complete the solution we need an expression for the gauge potentials, which are found by integrating (2.32). This leads to

$$A^I=2\eta^{IJ}g_J\mathsf{E}L^2\sqrt{-A}\frac{p}{p^2+q^2-\Delta^2}\left(dt+(q^2-\Delta^2)dy\right).\tag{3.25}$$

The solution is thus specified by $n+2$ free real parameters, and therefore represents a generalization of the black holes with $n=1$ constructed in [32]. The parameters can be taken to be A, E, Δ and β^α/β^0, subject to the constraint $g_I\beta^I=0$, cf. (3.8).

A particular, interesting choice is given by

$$\sqrt{-A}=\frac{L^2+j^2}{L^2-j^2}\,,\qquad \mathsf{E}=\frac{j^2}{L^2}-1\,.\tag{3.26}$$

Then, after the change of coordinates

$$p=j\cosh\theta\,,\qquad y=-\frac{\phi}{j\Xi}\,,\qquad t=\frac{T-j\phi}{\Xi}\,,\qquad \Xi\equiv1+\frac{j^2}{L^2}\,,\tag{3.27}$$

and defining the functions

$$\rho^2=q^2+j^2\cosh^2\theta,\quad \Delta_q=\frac{1}{L^2}\left(q^2+\frac{j^2-L^2}{2}-\Delta^2\right)^2,\quad \Delta_\theta=1+\frac{j^2}{L^2}\cosh^2\theta,\tag{3.28}$$

the metric (3.20), the scalars (3.23) and the gauge potentials (3.25) become respectively

$$\begin{aligned} ds^2 &= \frac{\rho^2-\Delta^2}{\Delta_q}dq^2+\frac{\rho^2-\Delta^2}{\Delta_\theta}d\theta^2+\frac{\Delta_\theta\sinh^2\!\theta}{(\rho^2-\Delta^2)\Xi^2}\left(jdT-(q^2+j^2-\Delta^2)d\phi\right)^2 \\ &\quad -\frac{\Delta_q}{(\rho^2-\Delta^2)\Xi^2}\left(dT+j\sinh^2\!\theta d\phi\right)^2, \end{aligned} \tag{3.29}$$

$$\begin{aligned} z^\alpha &= \frac{1}{j^2\cosh^2\theta+q_1^2}\left[-i\Delta_1\left(\frac{g_\alpha}{g_0}(j\cosh\theta+iq)+\frac{\beta^\alpha}{\beta^0}(j\cosh\theta-iq)\right)\right. \\ &\quad \left.-\frac{g_\alpha}{g_0}\rho^2+\frac{\beta^\alpha}{\beta^0}\Delta_1^2\right], \end{aligned} \tag{3.30}$$

$$A^I = 2\eta^{IJ}g_J L^2\frac{\cosh\theta}{\rho^2-\Delta^2}\left(jdT-\left(q^2+j^2-\Delta^2\right)d\phi\right). \tag{3.31}$$

The metric depends only on the two constants Δ and j, that can be interpreted respectively as scalar hair and rotation parameters. Note that for $j=0$ the scalars are real, whereas in the rotating case there is a nontrivial axion.

3.3 Near-horizon limit

The metric (3.29) has an event horizon at $\Delta_q=0$, i.e., for $q=q_h$ with

$$q_{\rm h}^2=\Delta^2+\frac{1}{2}\left(L^2-j^2\right). \tag{3.32}$$

To obtain the near-horizon geometry, we set

$$q=q_{\rm h}+\epsilon q_0 z\,, \qquad T=\frac{\hat{t}q_0}{\epsilon}\,, \qquad \phi=\hat{\phi}+\Omega\frac{\hat{t}q_0}{\epsilon}\,, \tag{3.33}$$

and then zoom in by taking the limit $\epsilon\to 0$. The parameter $\Omega=j/(q_{\rm h}^2+j^2-\Delta^2)$ represents the angular velocity of the horizon, while $q_0\equiv\frac{L^2\Xi}{2\sqrt{2}q_{\rm h}}$. In this limit the metric boils down to

$$\begin{aligned} ds^2 &= \frac{\rho_{\rm h}^2-\Delta^2}{4q_{\rm h}^2z^2}L^2dz^2+\frac{\rho_{\rm h}^2-\Delta^2}{\Delta_\theta}d\theta^2+\frac{L^4\Delta_\theta\sinh^2\!\theta}{4(\rho_{\rm h}^2-\Delta^2)}\left(d\hat{\phi}+\frac{j}{q_{\rm h}}zd\hat{t}\right)^2 \\ &\quad -\frac{\rho_{\rm h}^2-\Delta^2}{4q_{\rm h}^2}L^2z^2d\hat{t}^2, \end{aligned} \tag{3.34}$$

where $\rho_{\rm h}^2\equiv q_{\rm h}^2+j^2\cosh^2\!\theta$.

The final coordinate transformation

$$e^{-\xi}=L^2\frac{q_{\rm h}^2+j^2\cosh^2\!\theta-\Delta^2}{16q_{\rm h}^2}\,, \qquad x=\frac{q_{\rm h}}{j}\hat{\phi}\,, \tag{3.35}$$

casts the metric into the form (2.33), namely

$$ds^2=4e^{-\xi}\left(-z^2d\hat{t}^2+\frac{dz^2}{z^2}\right)+4\left(e^{-\xi}-Ke^{\xi}\right)\left(dx+zd\hat{t}\right)^2+\frac{4e^{-2\xi}d\xi^2}{Y^2(e^{-\xi}-Ke^{\xi})}\,, \tag{3.36}$$

where $K\equiv L^8\Xi^2/1024q_{\rm h}^4$.

There is thus a supersymmetry enhancement for the near-horizon geometry, which preserves half of the 8 supercharges of the theory. Exploiting this fact, there is an alternative way to arrive at this solution that goes as follows. In the $\overline{\mathbb{C}\mathrm{P}}^n$ model, the flow equation (2.37) becomes

$$\frac{dz^\alpha}{dY} = \frac{(g_\alpha + z^\alpha g_0)(g_0 + \sum_\beta g_\beta z^\beta)}{g^2(Y-i)}\,, \tag{3.37}$$

which is solved by

$$z^\alpha = \frac{\mu_\alpha g_0 - g_\alpha(Y-i)}{g_0(Y-i-\mu_0)}\,. \tag{3.38}$$

Here, $\mu_I = (\mu_0, \mu_\alpha) \in \mathbb{C}^{n+1}$ is a constant vector orthogonal to the gauge coupling, $\mu_I \eta^{IJ} g_J = 0$. One can now compute $|X|^2$ as a function of Y, with the result

$$|X|^2 = -\frac{g^4(Y^2+1)}{g^2(Y^2+1) + g_0^2 \mu\cdot\bar{\mu}}\,, \qquad \text{where} \quad \mu\cdot\bar{\mu} = \mu_I \eta^{IJ}\bar{\mu}_J\,. \tag{3.39}$$

Plugging this into (2.35) gives then $\xi(Y)$, and the metric (2.33) becomes

$$\begin{aligned} ds^2 \;=\; & \frac{-g^2(Y^2+1) - g_0^2\mu\cdot\bar{\mu}}{16g^4}\left(-r^2dt^2 + \frac{dr^2}{r^2}\right) + \frac{P(Y)(d\phi + r dt)^2}{16g^4[-g^2(Y^2+1) - g_0^2\mu\cdot\bar{\mu}]} \\ & + \frac{[-g^2(Y^2+1) - g_0^2\mu\cdot\bar{\mu}]dY^2}{4P(Y)}\,, \end{aligned} \tag{3.40}$$

where we defined the quartic polynomial

$$P(Y) = [g^2(Y^2+1) + g_0^2\mu\cdot\bar{\mu}]^2 - K(64g^4)^2\,. \tag{3.41}$$

Using $Y = -j\cosh\theta/q_{\mathrm{h}}$, one finds that the modulus of the parameters μ_I is related to Δ by

$$\mu\cdot\bar{\mu} = \frac{\Delta^2}{4L^2 g_0^2 q_{\mathrm{h}}^2}\,.$$

The expression for the vector μ_I can be found requiring that the scalar fields (3.30) coincide in the near-horizon limit with the expression (3.38), yielding

$$\mu_0 = -i\frac{\Delta_1}{q_{\mathrm{h}}}\,, \qquad \mu_\alpha = i\frac{\Delta_1\beta^\alpha}{q_{\mathrm{h}}\beta^0}\,. \tag{3.42}$$

Since the static supersymmetric black holes in the $\overline{\mathbb{C}\mathrm{P}}^n$ model constructed in [5] have necessarily hyperbolic horizons, one may ask whether spherical rotating horizons are possible. As was discussed in detail in [49], this question is intimately related to the behaviour of $P(Y)$. Namely, for spherical horizons to be feasible $P(Y)$ must have four distinct roots, and then Y is restricted to the region between the two central roots where $P(Y)$ is positive. The latter condition, together with $-g^2(Y^2+1) - g_0^2\mu\cdot\bar{\mu} > 0$, is necessary in order for the metric to have the correct signature.[9] Imposing $P(Y) = 0$ yields

$$-g^2(Y^2+1) - g_0^2\mu\cdot\bar{\mu} = 64g^4\sqrt{K}\,. \tag{3.43}$$

[9]Note that there is a curvature singularity for $-g^2(Y^2+1) - g_0^2\mu\cdot\bar{\mu} = 0$.

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There are thus only two roots $\pm Y_0$ (with $Y_0 > 0$), and spherical horizons are therefore excluded in the rotating solution as well. One can show that, in the static limit, the near-horizon geometry of the black holes constructed in [5] is recovered.

3.4 NUT-charged black holes

In this section we construct supersymmetric NUT-charged black holes. To do so it is sufficient to mimic what was done in [36], where the theory with only one vector multiplet was considered. Since the BPS equations can be solved following the same steps of that paper, we will just briefly summarize the process here and refer to [36] for further details. We assume that both the scalars and the function b depend on the coordinate z only, and use the ansatz

$$\frac{X^I}{\bar{b}} = \frac{\alpha^I z + \beta^I}{z^2 + iDz + C}\,, \qquad \Phi = \psi(z) + \gamma(w, \bar{w})\,, \tag{3.44}$$

where α^I, β^I, C are complex constants and D is a real constant. The dependence of the solution on the coordinates $w, \bar{w}$ is obtained from (2.13), which reduces to

$$-4\partial\bar{\partial}\gamma = \kappa e^{2\gamma}\,, \tag{3.45}$$

where κ is a constant whose value will be fixed later. This is the Liouville equation for the metric $e^{2\gamma}dwd\bar{w}$, which consequently has constant curvature κ. We will take as a particular solution

$$e^{2\gamma} = \left(1 + \frac{\kappa}{4}w\bar{w}\right)^{-2}. \tag{3.46}$$

From (2.10) one gets

$$\psi(z) = \mathrm{Im}\alpha\left(\ln[z^4 + z^2(2\mathrm{Re}C + D^2) + 2Dz\mathrm{Im}C + |C|^2]\right)\,, \tag{3.47}$$

provided the following constraints are satisfied

$$\mathrm{Im}\beta = D\mathrm{Re}\alpha\,, \quad -2[\mathrm{Im}(\bar{\alpha}C) + D\mathrm{Re}\beta] = \mathrm{Im}\alpha(2\mathrm{Re}C + D^2)\,, \quad -2\mathrm{Im}(\bar{\beta}C) = D\mathrm{Im}\alpha\mathrm{Im}C\,, \tag{3.48}$$

where $\alpha \equiv g_I\alpha^I$ and $\beta \equiv g_I\beta^I$.

The expressions displayed so far coincide with those found for the case with one vector multiplet; the explicit form of the prepotential has not been used to solve (2.10) and (2.13). In order to solve the remaining BPS equations we choose $\mathrm{Im}\alpha = 1/2$, since in this way they assume a polynomial form. Then the Bianchi identities fix α^I and κ to

$$\alpha^I = \frac{i}{2g^2}\eta^{IJ}g_J\,, \qquad \kappa = 2\mathrm{Re}C - 8g^2\beta^K\bar{\beta}_K\,. \tag{3.49}$$

On the other hand, Maxwell's equations are automatically satisfied provided these relations hold. Finally, integration of (2.14) gives

$$\sigma = i\frac{D}{32g^2}\frac{\bar{w}dw - wd\bar{w}}{1 + \frac{\kappa}{4}w\bar{w}}\,, \tag{3.50}$$

from which it is evident that the parameter D is related to the NUT charge of the solution.

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The warp factor of the metric is in this case

$$|b|^{-2} = -\frac{z^2 + 4g^2\beta^K\bar{\beta}_K}{4g^2|z^2 + iDz + C|^2}\,, \tag{3.51}$$

where we recall that in this model $g^2 < 0$. The solution will have an event horizon at $z = z_{\rm h}$ if $b(z_{\rm h})$ vanishes, which happens for

$$z_{\rm h}^2 = -{\rm Re}C\,, \qquad Dz_{\rm h} = -{\rm Im}C\,. \tag{3.52}$$

This is possible if $({\rm Im}C)^2 = -D^2{\rm Re}C$ and ${\rm Re}C < 0$. There is a curvature singularity at $z^2 + 4g^2\beta^K\bar{\beta}_K = 0$, which is hidden behind the horizon if

$${\rm Re}C < 4g^2\beta^K\bar{\beta}_K. \tag{3.53}$$

Then, from (3.49) we see that $\kappa < 0$ and therefore the horizon is always hyperbolic.

The solution is in principle specified by $2n+2$ real parameters, which can be taken as β^I, D and ${\rm Re}C$ with the constraint $\beta = -D/4$, which follows from (3.48). If (3.53) holds, the metric describes a regular black hole. Notice that we can use the scaling symmetry $(t, z, w, C, D, \beta^I, \kappa) \mapsto (t/\lambda, \lambda z, w/\lambda, \lambda^2 C, \lambda D, \lambda\beta^I, \lambda^2\kappa)$ to set $\kappa = -1$ without loss of generality, which reduces the number of independent parameters to $2n+1$.

The fluxes can be computed by plugging the results found so far into (2.32). A long but straightforward calculation yields

$$\begin{aligned}
F^I &= 4(dt+\sigma)\wedge dz\frac{1}{(z^2+4g^2\beta^K\bar{\beta}_K)^2}\Big[4g^2\left(2{\rm Im}C{\rm Im}\beta^I - {\rm Re}\beta^I\right)z \\
&\quad - 2\eta^{IJ}g_J Dz(1+2{\rm Re}C) + \left(-1-2{\rm Re}C+2z^2\right)\left(2g^2D{\rm Im}\beta^I + \eta^{IJ}g_J{\rm Im}C\right)\Big] \\
&\quad - \frac{1}{2}e^{2\gamma}dw\wedge d\bar{w}\frac{i}{4g^2(z^2+4g^2\beta^K\bar{\beta}_K)}\Big[\eta^{IJ}g_J\left(-1-2({\rm Re}C+z^2)\right) \\
&\quad + 4D\eta^{IJ}g_J z\left(Dz+{\rm Im}C\right) + 8g^2D\left({\rm Re}\beta^I z^2 + D{\rm Im}\beta^I z + {\rm Re}(\bar{C}\beta^I)\right)\Big].
\end{aligned} \tag{3.54}$$

The magnetic and electric charges of the solution are given by

$$P^I = \frac{1}{4\pi}\int_{\Sigma_\infty} F^I\,, \qquad Q_I = \frac{1}{4\pi}\int_{\Sigma_\infty} G_I\,, \tag{3.55}$$

where Σ_∞ denotes a surface of constant t and z for $z \to \infty$, and G_I is obtained from the action as $G_I = \delta S/\delta F^I$. This leads to

$$\frac{P^I}{V} = \frac{1-2D^2}{8\pi g^2}\eta^{IJ}g_J - \frac{D{\rm Re}\beta^I}{2\pi}\,, \qquad \frac{Q_I}{V} = -\frac{g_I{\rm Im}C}{8\pi g^2} + \frac{\eta_{IJ}{\rm Im}\beta^J D}{4\pi}\,, \tag{3.56}$$

where V is defined by

$$V = \frac{i}{2}\int e^{2\gamma}dw\wedge d\bar{w}\,. \tag{3.57}$$

Finally, the scalars read

$$z^\alpha = \frac{2g^2\beta^\alpha + ig_\alpha z}{2g^2\beta^0 - ig_0 z}\,. \tag{3.58}$$

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4 Supersymmetric rotating black holes in the t^3 model

4.1 A near-horizon solution

Before starting, we notice that when looking for solutions of the $\overline{\mathbb{C}\mathrm{P}}^n$ model, it proved useful to work with a factorized ansatz for the real and imaginary components of $\bar{X}^I/b$. If a similar decomposition is performed in the case at hand, the equations of motion do not factorize unless we assume that the real or imaginary part of $\bar{X}^0/b$ vanishes. We will only explore here the latter possibility, as in the former we just found trivial solutions. Since in homogeneous coordinates X^0 is purely real, one can see that this is equivalent to setting $\bar{b} = b$. We will thus use the ansatz

$$\frac{\bar{X}^0}{b} = \frac{\eta^0(w,\bar{w})}{\gamma(z)}\,, \qquad \frac{\bar{X}^1}{b} = \frac{f^1(z) + \eta^1(w,\bar{w})}{\gamma(z)}\,, \qquad e^{2\Phi} = h(z)\ell(w,\bar{w}) \tag{4.1}$$

in the system of BPS equations (2.10)–(2.14). From (2.10) and (2.13) we get

$$\partial_z \ln h = 8i\frac{g_1 f^1}{\gamma}\,, \qquad \frac{\partial\bar{\partial}\ln\ell}{\ell} = -\frac{1}{4}\partial_z^2 h - \frac{32}{3}\frac{h}{\gamma^2}(g_1 f^1)^2\,. \tag{4.2}$$

Using the first equation, we find that the second is separable and boils down to

$$\frac{\partial\bar{\partial}\ln\ell}{\ell} = \frac{C_1}{4}\,, \qquad \partial_z^2 h - \frac{2}{3}\frac{(\partial_z h)^2}{h} = -C_1\,, \tag{4.3}$$

for some constant C_1. (4.3) determines the dependence on $w, \bar{w}$ and z of the three-dimensional base space. For $C_1 \neq 0$,[10] the solution for h reads[11]

$$h(z) = \frac{3}{2}C_1\left(z + \frac{c}{a}\right)^2\,, \tag{4.4}$$

which implies

$$\frac{f^1}{\gamma} = -\frac{i}{4g_1\left(z + \frac{c}{a}\right)}\,. \tag{4.5}$$

The first of (4.3) is just Liouville's equation, and thus the explicit form of $l(w,\bar{w})$ depends on the choice of a meromorphic function. In order to make further progress, from now on we shall consider a particular case that has been proven successful for our purpose, i.e.,

$$l(w,\bar{w}) = \frac{2}{C_1\sinh^2\left(\frac{w+\bar{w}}{2}\right)}\,. \tag{4.6}$$

Then the Bianchi identities (2.11) are automatically solved, so that Maxwell's equations (2.12) represent the last obstacle. Setting, like in the $\overline{\mathbb{C}\mathrm{P}}^n$ case, $h(z)/\gamma(z)^2$ to a constant, the latter assume a simple form. The value of this constant is totally arbitrary,

[10] The case $C_1 = 0$ belongs to a qualitatively different family of solutions to (4.3), which however does not seem to be well-suited for solving the remaining differential equations of the system.

[11] a and c are integration constants. Although h and f^1/γ depend only on the ratio c/a, we prefer to keep them both, for reasons that become clear further below.

but with a redefinition of a, and thus of c in order to keep c/a unchanged, we can always bring it to $\frac{3C_1}{2a^2}$, in which case the Maxwell equations become

$$\partial\bar{\partial}\left[\frac{1}{\eta^{0\,2}} - 48\frac{g_1^2}{a^2}R^2\right] = 0\,, \qquad 2\partial\bar{\partial}R - \frac{R}{\sinh^2\left(\frac{w+\bar{w}}{2}\right)} = 0\,, \tag{4.7}$$

where

$$R(w,\bar{w}) \equiv \frac{g_I\eta^I}{g_1\eta^0}\,. \tag{4.8}$$

The second equation of (4.7) can be readily solved,

$$R(x) = \Xi_1 \coth x + \Xi_2\left[x\coth x - 1\right]\,, \tag{4.9}$$

where $\Xi_{1,2}$ are integration constants and $x \equiv (w+\bar{w})/2$. The first of (4.7) implies

$$\frac{1}{\eta^{0\,2}} = 48\frac{g_1^2}{a^2}R^2 + \mathrm{Re}F(w)\,, \tag{4.10}$$

for some arbitrary function $F(w)$ that in a first step we will simply set to 0.

The equation (2.14) for the shift vector σ boils down to

$$\partial_z\sigma_w = -\frac{3i\partial R}{4g_1^2\left(z+\frac{c}{a}\right)^2}\,, \qquad \partial_z\sigma_{\bar{w}} = \frac{3i\bar{\partial}R}{4g_1^2\left(z+\frac{c}{a}\right)^2}\,, \qquad \partial\sigma_{\bar{w}} - \bar{\partial}\sigma_w = -\frac{3i\partial\bar{\partial}R}{2g_1^2\left(z+\frac{c}{a}\right)}\,,$$

which is solved by

$$\sigma = \frac{3i}{4g_1^2\left(z+\frac{c}{a}\right)}(\partial R dw - \bar{\partial}R d\bar{w})\,. \tag{4.11}$$

Defining $y \equiv (w-\bar{w})/(2i)$, the metric (2.9) becomes

$$ds^2 = -\frac{8g_1^2}{\sqrt{3}R}\left[\left(z+\frac{c}{a}\right)dt - \frac{3}{4g_1^2}\partial_x R dy\right]^2 + \frac{\sqrt{3}R}{2g_1^2}\left[\frac{dz^2}{\left(z+\frac{c}{a}\right)^2} + \frac{3(dx^2+dy^2)}{\sinh^2 x}\right]\,, \tag{4.12}$$

while the scalar field is given by

$$\tau = -\frac{g_0}{g_1} + R(x) + i\sqrt{3}R(x) = -\frac{g_0}{g_1} + 2e^{i\pi/3}R(x)\,. \tag{4.13}$$

For $\Xi_2 = 0$ one can readily identify this solution as belonging to the class of half-supersymmetric near-horizon backgrounds presented in section 2.2. Performing the change of coordinates

$$e^{-\xi} = \sqrt{K}\coth x\,, \qquad r = z + \frac{c}{a}\,, \qquad \phi = -\sqrt{3}y\,, \qquad T = \frac{t}{2\sqrt{K}}\,, \tag{4.14}$$

where $\sqrt{K} = \frac{\sqrt{3}\Xi_1}{8g_1^2}$, the metric is brought to the form (2.33) with $Y^2 = 1/3$, namely

$$ds^2 = 4e^{-\xi}\left(-r^2dT^2 + \frac{dr^2}{r^2}\right) + 4(e^{-\xi} - Ke^{\xi})(d\phi + rdT)^2 + \frac{12e^{-2\xi}d\xi^2}{e^{-\xi} - Ke^{\xi}}\,. \tag{4.15}$$

In the same way, one can check that the scalar (4.13) satisfies the flow equation (2.36).

4.2 Black hole extension

We will now construct a black hole whose near-horizon geometry is given by the solution found in the previous subsection. This is achieved with a slight generalization of the ansatz (4.1). We maintain the factorization form of $e^{2\Phi}$ and $\mathrm{Im}(\bar{X}^I/b)$, but leave $\mathrm{Re}(\bar{X}^I/b)$ as arbitrary functions of the three spatial coordinates.

The first steps of subsection 4.1 that determine the functions $h(z)$, $l(w,\bar{w})$ and $\mathrm{Im}(\bar{X}^I/b)$ remain identical. The difference appears in the first of Maxwell's equations, which now read

$$-\frac{r^2}{\sinh^2\left(\frac{w+\bar{w}}{2}\right)}\partial_r^2\left[\frac{1}{r^2\mathrm{Re}^2\left(\bar{X}^0/b\right)}\right]-\frac{4}{3}\partial\bar{\partial}\left[\frac{1}{r^2\mathrm{Re}^2\left(\bar{X}^0/b\right)}\right]+64g_1^2\partial\bar{\partial}\left(R^2\right)=0\,, \tag{4.16}$$

$$2\partial\bar{\partial}R-\frac{R}{\sinh^2\left(\frac{w+\bar{w}}{2}\right)}=0\,, \tag{4.17}$$

where $r=z+c/a$. A simple solution to (4.16) is

$$\mathrm{Re}\left(\bar{X}^0/b\right)=\frac{1}{4\sqrt{3}g_1r\sqrt{\alpha r+\beta+R(w,\bar{w})^2}}\,, \tag{4.18}$$

while (4.17) is solved by (4.9). Here, α and β denote integration constants. Then, the scalar, metric and gauge potentials read respectively

$$\tau=-\frac{g_0}{g_1}+R+i\sqrt{3}\sqrt{\alpha r+\beta+R^2}\,, \tag{4.19}$$

$$\begin{aligned}ds^2=&-\frac{8g_1^2}{\sqrt{3}\sqrt{\alpha r+\beta+R^2}}\left[r\,dt+\frac{3}{4g_1^2}\partial_xR\,dy\right]^2+\\&+\frac{\sqrt{3}}{2g_1^2}\sqrt{\alpha r+\beta+R^2}\left[\frac{dr^2}{r^2}+\frac{3(dx^2+dy^2)}{\sinh^2x}\right]\,,\end{aligned} \tag{4.20}$$

$$\begin{aligned}A^0&=-\frac{2g_1}{3(\alpha r+\beta+R^2)}\left(rdt+\frac{3}{4g_1^2}\partial_xRdy\right)\,,\\A^1&=-\frac{2g_1}{3(\alpha r+\beta+R^2)}\left(R-\frac{g_0}{g_1}\right)\left(rdt+\frac{3}{4g_1^2}\partial_xRdy\right)-\frac{\coth x}{2g_1}dy\,.\end{aligned} \tag{4.21}$$

Now the scalar depends on the radial coordinate as well, and we recover the near-horizon geometry discussed above by rescaling $r\mapsto\epsilon r$, $t\mapsto t/\epsilon$ and taking the limit $\epsilon\to0$.

As we already mentioned, the asymptotic limit of this solution cannot be AdS_4 since the scalar potential has no critical points. For large values of r, the metric behaves as

$$ds^2=d\rho^2+\frac{3}{16}\rho^2\left[-\frac{g_1^8}{108\alpha^2}\rho^4\,dt^2+\frac{8g_1^2\Xi_1}{3\alpha}\sinh^2\theta\,dtdy+\sinh^2\theta\,dy^2+d\theta^2\right]\,, \tag{4.22}$$

where we defined ρ and θ by $r\equiv\frac{g_1^4\rho^4}{192\alpha}$, $\coth x\equiv\cosh\theta$, and chose $\Xi_2=0$.

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5 Nonextremal rotating black holes in the $\overline{\mathbb{CP}}^n$ model

In this section we shall construct a nonextremal deformation of the one quarter BPS solution presented in section 3. To this end we shall take a Carter-Plebański-type ansatz for the metric similar to (3.20), where $Q(q)$ and $P(p)$ are quartic polynomials in q and p respectively,

$$ds^2 = -\frac{Q}{W}\left(dt - p^2 dy\right)^2 + \frac{P}{W}\left(dt + (q^2 - \Delta^2)dy\right)^2 + W\left(\frac{dq^2}{Q} + \frac{dp^2}{P}\right), \tag{5.1}$$

$$Q = \sum_{n=0}^{4} a_n q^n, \qquad P = \sum_{n=0}^{4} b_n p^n, \qquad W = p^2 + q^2 - \Delta^2, \tag{5.2}$$

where a_n, b_n and Δ are real constants. The ansatz for the scalars and the gauge potentials is inspired by (3.23) and (3.25),

$$z^\alpha = \frac{1}{p + i(q - \tilde{\Delta})}\left(-\frac{g_\alpha}{g_0}(p + iq) + ic^\alpha\right), \tag{5.3}$$

$$A^I = \mathsf{P}^I \frac{p}{W}\left[dt + (q^2 - \Delta^2)dy\right], \tag{5.4}$$

with P^I real constants related to the magnetic charges, $\tilde{\Delta}$ real and c^α complex constants. Plugging these expressions into the equations of motion (A.1)–(A.3) gives a set of constraints for the constants. It then turns out that at a certain point one has to choose whether Δ vanishes or not. In what follows we shall assume $\Delta \neq 0$, while the case $\Delta = 0$ is postponed to section 5.3.

For P^I not proportional to the coupling constants g_I one class of solutions is obtained by taking

$$\begin{aligned}
&a_0 = b_0 + b_2\Delta^2 - 4g^2\Delta^4 - \frac{(g_I\mathsf{P}^I)^2}{2g^2} + \frac{\mathsf{P}^2}{4}, && a_1 = \frac{(g_I\mathsf{P}^I)\sqrt{(g_I\mathsf{P}^I)^2 - g^2\mathsf{P}^2}}{2g^2\Delta}, \\
&a_2 = -b_2 + 8g^2\Delta^2, \qquad a_3 = 0, && a_4 = b_4 = -4g^2, \qquad b_1 = b_3 = 0, \\
&\tilde{\Delta} = \Delta\frac{(g_I\mathsf{P}^I)g_0 + g^2\mathsf{P}^0}{g_0\sqrt{(g_I\mathsf{P}^I)^2 - g^2\mathsf{P}^2}}, && c^\alpha = \Delta\frac{(g_I\mathsf{P}^I)g_\alpha - g^2\mathsf{P}^\alpha}{g_0\sqrt{(g_I\mathsf{P}^I)^2 - g^2\mathsf{P}^2}}.
\end{aligned} \tag{5.5}$$

Here we defined $\mathsf{P}^2 \equiv \eta_{IJ}\mathsf{P}^I\mathsf{P}^J$. Fixing the Fayet-Iliopoulos constants g_I the solution depends on $n+4$ parameters b_0, b_2, Δ and P^I. However, our ansatz is left invariant under the scale transformation

$$\begin{aligned}
&p \to \lambda p, \qquad && q \to \lambda q, \qquad && t \to t/\lambda, \qquad && y \to y/\lambda^3, \\
&\Delta \to \lambda\Delta, && a_n \to \lambda^{4-n}a_n, && b_n \to \lambda^{4-n}b_n,
\end{aligned} \tag{5.6}$$

which reduces the number of independent parameters to $n+3$.

With a few lines of computation it is possible to show that this solution contains the one presented in [34] for the prepotential $F = -i\tilde{X}^0\tilde{X}^1$ (a tilde is introduced in order to distuinguish between the two solutions). In order to do so, we must consider the case of just

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one vector multiplet ($n = 1$) and perform a symplectic rotation. In particular, introducing the symplectic vectors

$$\mathcal{G} = \begin{pmatrix} 0 \\ g_I \end{pmatrix}, \qquad \mathcal{Q} = \begin{pmatrix} \mathsf{P}^I \\ 0 \end{pmatrix}, \tag{5.7}$$

and the symplectic matrix

$$T = \left(\begin{array}{cc|cc} 1 & 1 & \multicolumn{2}{c}{\multirow{2}{*}{0}} \\ 1 & -1 & & \\ \hline \multicolumn{2}{c|}{\multirow{2}{*}{0}} & \frac{1}{2} & \frac{1}{2} \\ & & \frac{1}{2} & -\frac{1}{2} \end{array}\right), \tag{5.8}$$

the solution for the rotated $F = -i\tilde{X}^0\tilde{X}^1$ prepotential can be obtained from the same metric and gauge fields in (5.1) and (5.4), but with the charges and gauge couplings replaced by their rotated counterparts according to $\mathcal{Q} = T\tilde{\mathcal{Q}}$ and $\mathcal{G} = T\tilde{\mathcal{G}}$. On the other hand, the scalar field is

$$\tilde{\tau} = \frac{\tilde{X}^0}{\tilde{X}^1} = \frac{1 - z^1}{1 + z^1}, \tag{5.9}$$

where $\tilde{X}^I$ belongs to the new symplectic section $\tilde{\mathcal{V}} = T^{-1}\mathcal{V}$.

In the supersymmetric, extremal limit we recover the solution presented in section 3. To this end, the charge parameters need to be chosen proportional to the gauge couplings, $\mathsf{P}^I = \lambda\,\eta^{IJ}g_J$, with $\lambda \in \mathbb{R}$, hence the relations presented above simplify to

$$\begin{aligned} &a_0 = b_0 + b_2\Delta^2 - 4g^2\Delta^4 - \frac{\lambda^2 g^2}{4}, && a_1 = 0, && a_2 = -b_2 + 8g^2\Delta^2, \\ &a_3 = 0, && a_4 = b_4 = -4g^2, && b_1 = b_3 = 0, \end{aligned} \tag{5.10}$$

while on the other hand we are no more able to derive an explicit expression for c^α, but we can only assert that they must satisfy the conditions

$$-g_0 + c^\alpha g_\alpha = 0, \qquad c^\alpha c^\alpha = c^\alpha \bar{c}^\alpha = \tilde{\Delta}^2 - \frac{\Delta^2 g^2}{g_0^2}, \tag{5.11}$$

where summation over α is understood. Then, we see that the BPS solution (3.20), (3.23) and (3.25) is recovered for

$$L^2 = -\frac{1}{4g^2}, \qquad b_0 = (1 + A)\frac{\mathsf{E}^2 L^2}{4}, \qquad b_2 = -\mathsf{E}, \qquad \lambda = 2\,\mathsf{E}L^2\sqrt{-A}, \tag{5.12}$$

and by identifying $\tilde{\Delta} = \Delta_1$ and $c^\alpha = -\Delta_1\beta^\alpha/\beta^0$.

5.1 Properties of the compact horizon case

Since P is an even polynomial we may assume it has two distinct pairs of roots $\pm p_a$ and $\pm p_b$, where $0 < p_a < p_b$. We then consider solutions with p in the range $|p| \leq p_a$ by setting $p = p_a \cos\theta$, where $0 \leq \theta \leq \pi$, to obtain black holes with a compact horizon. We now use the scaling symmetry (5.6) to set $p_b = L$ without loss of generality, where $L^{-2} = -4g^2$. Defining the rotation parameter j by $p_a^2 = j^2$, this means

$$b_0 = j^2 \,, \qquad b_2 = -1 - \frac{j^2}{L^2} \,. \tag{5.13}$$

Then, after the coordinate transformation,

$$t \mapsto t + \frac{j\phi}{\Xi} \,, \qquad y \mapsto \frac{\phi}{j\Xi} \,, \tag{5.14}$$

with $\Xi = 1 - \frac{j^2}{L^2}$, the metric (5.1) becomes

$$\begin{aligned} ds^2 = &-\frac{Q}{W}\left(dt + \frac{j\sin^2\theta}{\Xi}d\phi\right)^2 + \frac{\Delta_\theta \sin^2\theta}{W}\left(jdt + \frac{q^2 - \Delta^2 + j^2}{\Xi}d\phi\right)^2 + \\ &+ W\left(\frac{dq^2}{Q} + \frac{d\theta^2}{\Delta_\theta}\right) \,, \end{aligned} \tag{5.15}$$

where

$$W = q^2 - \Delta^2 + j^2\cos^2\theta \,, \qquad \Delta_\theta = 1 - \frac{j^2}{L^2}\cos^2\theta \,.$$

We notice that for zero rotation parameter, $j = 0$, (5.15) boils down to the static nonextremal black holes with running scalar constructed in [28], after the $n = 1$ truncation and the symplectic rotation (5.8) are performed.

(5.15) has an event horizon at $q = q_{\mathrm{h}}$, where q_{h} is the largest root of Q. The Bekenstein-Hawking entropy of the black hole is given by

$$S = \frac{\pi}{\Xi G}\left(q_{\mathrm{h}}^2 - \Delta^2 + j^2\right) \,, \tag{5.16}$$

where G denotes Newton's constant. In order to compute the temperature and angular velocity it is convenient to write the metric in the ADM form

$$ds^2 = -N^2dt^2 + \sigma(d\phi - \omega dt)^2 + W\left(\frac{dq^2}{Q} + \frac{d\theta^2}{\Delta_\theta}\right) \,, \tag{5.17}$$

with

$$N^2 = \frac{Q\Delta_\theta W}{\Sigma^2} \,, \qquad \sigma = \frac{\Sigma^2\sin^2\theta}{W\Xi^2} \,, \qquad \omega = \frac{j\Xi}{\Sigma^2}[Q - \Delta_\theta(q^2 - \Delta^2 + j^2)] \,,$$

where

$$\Sigma^2 = \Delta_\theta(q^2 - \Delta^2 + j^2)^2 - Qj^2\sin^2\theta \,.$$

The angular velocity at the horizon and at infinity are thus

$$\omega_{\mathrm{h}} = -\frac{j\Xi}{q_{\mathrm{h}}^2 - \Delta^2 + j^2} \,, \qquad \omega_\infty = \frac{j}{L^2} \,. \tag{5.18}$$

The angular momentum may be computed as a Komar integral, which leads to

$$J = \frac{a_1 j}{2\Xi^2 G} \,. \tag{5.19}$$

To get the mass of the solution we use the Ashtekar-Magnon-Das (AMD) formalism [50, 51], applied to the conformally rescaled metric $\bar{g}_{\mu\nu} = (L/q)^2 g_{\mu\nu}$. This gives

$$M = -\frac{a_1}{2\Xi^2 G} \, . \tag{5.20}$$

Notice that the 'ground state' $a_1 = 0$ represents a naked singularity. This can be seen as follows. The curvature singularity at $W = 0$,[12] is shielded by a horizon if $q_{\rm h}^2 - \Delta^2 + j^2 \cos^2\theta > 0$, and thus $q_{\rm h}^2 > \Delta^2$, which is equivalent to

$$a_2^2 - \frac{4a_0}{L^2} > \left(1 + \frac{j^2}{L^2}\right)^2 \, ,$$

where we used the expression for $q_{\rm h}$. This relation is easily shown to be violated for $a_1 = 0$ by using (5.5).

The magnetic charges p^I are given by

$$\mathsf{p}^I = \frac{1}{4\pi} \oint_{\mathrm{S}^2_\infty} F^I = -\frac{\mathsf{P}^I}{\Xi} \, . \tag{5.21}$$

The product of the horizon areas reads

$$\prod_{\Lambda=1}^{4} A_\Lambda = \frac{(4\pi)^4}{\Xi^4} \prod_{\Lambda=1}^{4} (q^2|_{\mathrm{h}_\Lambda} - \Delta^2 + j^2) = (4\pi)^4 L^4 \left[\frac{(\mathsf{p}^2)^2}{16} + 4G^2 J^2\right] \, , \tag{5.22}$$

where $\mathsf{p}^2 \equiv \eta_{IJ}\mathsf{p}^I\mathsf{p}^J$. In the second step we followed what has been done in [34] and the procedure explained in [29]. The charge-dependent term on the r.h.s. of (5.22) is directly related to the prepotential; a fact that was first noticed in [29] for static black holes.

Now that we have computed the physical quantities of our solution, we see that one may choose the $n+3$ free parameters as P^I, Δ, j, or alternatively p^I, M, J. Our black holes are therefore labeled by the values of $n+1$ independent magnetic charges, the mass and the angular momentum.

5.2 Thermodynamics and extremality

Imposing regularity of the Wick-rotated metric it is straightforward to compute the Hawking temperature, with the result

$$T = \frac{Q'_{\rm h}}{4\pi(q_{\rm h}^2 - \Delta^2 + j^2)} \, , \tag{5.23}$$

where $Q'_{\rm h}$ denotes the derivative of Q evaluated at the horizon.

Using the extensive quantities S, M, J and p^I computed above, it is possible to obtain the Christodoulou-Ruffini-type mass formula

$$\begin{aligned} M^2 = \frac{S}{4\pi G} + \frac{\pi J^2}{SG} + \frac{\pi}{4SG^3}\frac{(\mathsf{p}^2)^2}{16} + \left(\frac{L^2}{G^2} + \frac{S}{\pi G}\right)\left((g_I \mathsf{p}^I)^2 + \frac{\mathsf{p}^2}{8L^2}\right) + \\ + \frac{J^2}{L^2} + \frac{S^2}{2\pi^2 L^2} + \frac{S^3 G}{4\pi^3 L^4} \, . \end{aligned} \tag{5.24}$$

[12] Note also that for $W < 0$, the real part of the scalar field becomes negative, so that ghost modes appear.

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Since S, J and p^I form a complete set of extensive parameters, (5.24) gives the thermodynamic fundamental relation $M = M(S, J, \mathsf{p}^I)$. The intensive quantities conjugate to S, J and p^I are the temperature

$$\begin{aligned} T = \left.\frac{\partial M}{\partial S}\right|_{J,\mathsf{p}^I} = \frac{1}{8\pi GM}\Bigg[1 - \frac{4\pi^2 J^2}{S^2} - \frac{\pi^2}{S^2G^2}\frac{(\mathsf{p}^2)^2}{16} + 4\left((g_I\mathsf{p}^I)^2 + \frac{\mathsf{p}^2}{8L^2}\right) + \\ + \frac{4SG}{\pi L^2} + \frac{3S^2G^2}{\pi^2 L^4}\Bigg] , \end{aligned} \tag{5.25}$$

the angular velocity

$$\Omega = \left.\frac{\partial M}{\partial J}\right|_{S,\mathsf{p}^I} = \frac{\pi J}{MGS}\left[1 + \frac{SG}{\pi L^2}\right] , \tag{5.26}$$

and the magnetic potentials

$$\begin{aligned} \Phi_I = \left.\frac{\partial M}{\partial \mathsf{p}^I}\right|_{S,J,\mathsf{p}^{K\neq I}} = \frac{1}{MG}\Bigg[\frac{\pi}{4SG^2}\frac{\mathsf{p}^2}{16}\eta_{IK}\mathsf{p}^K + \\ + \left(\frac{L^2}{G} + \frac{S}{\pi}\right)\left((g_K\mathsf{p}^K)g_I + \frac{1}{16L^2}\eta_{IK}\mathsf{p}^K\right)\Bigg] . \end{aligned} \tag{5.27}$$

These quantities satisfy the first law of thermodynamics

$$dM = T\,dS + \Omega\,dJ + \Phi_I\,d\mathsf{p}^I . \tag{5.28}$$

It is straightforward to verify that expression (5.25) for the temperature agrees with (5.23), while from (5.26) we observe that

$$\Omega = \omega_{\mathrm{h}} - \omega_\infty , \tag{5.29}$$

with ω_{h} and ω_∞ given by (5.18). Thus, what enters the first law is the difference between the angular velocities at the horizon and at infinity.

Extremal black holes have vanishing Hawking temperature (5.23), which happens when q_{h} is at least a double root of Q. The structure function Q can then be written as

$$Q = (q - q_{\mathrm{h}})^2\left[\frac{q^2}{L^2} + \frac{2q_{\mathrm{h}}}{L^2}q + a_2 + \frac{3q_{\mathrm{h}}^2}{L^2}\right] ,$$

so we must have

$$a_0 = a_2 q_{\mathrm{h}}^2 + \frac{3q_{\mathrm{h}}^4}{L^2} , \qquad a_1 = -2a_2 q_{\mathrm{h}} - \frac{4q_{\mathrm{h}}^3}{L^2} . \tag{5.30}$$

It is straightforward to check that these relations are satisfied in the supersymmetric limit $P^I = \lambda\eta^{IJ}g_J$ previously described. On the other hand, it may happen that the free parameters are chosen such that (5.5) is compatible with (5.30), even if the charges are not proportional to the gauge couplings. In that case we would obtain an extremal, non-supersymmetric black hole.

To obtain the near-horizon geometry of the extremal black holes, we define new (dimensionless) coordinates $z, \hat{t}, \hat{\phi}$ by

$$q = q_{\mathrm{h}} + \epsilon q_0 z , \qquad t = \frac{q_0}{\Xi\epsilon}\hat{t} , \qquad \phi = \hat{\phi} + \frac{\omega_{\mathrm{h}} q_0}{\Xi\epsilon}\hat{t} , \tag{5.31}$$

with

$$q_0^2 \equiv \frac{\Xi(q_{\rm h}^2 - \Delta^2 + j^2)}{C}\,, \qquad C = \frac{6q_{\rm h}^2}{L^2} + a_2\,,$$

and take $\epsilon \to 0$ keeping $z, \hat{t}, \hat{\phi}$ fixed. This leads to

$$\begin{aligned} ds^2 = \frac{q_{\rm h}^2 - \Delta^2 + j^2 \cos^2\theta}{C} \left(-z^2 d\hat{t}^2 + \frac{dz^2}{z^2} + C\,\frac{d\theta^2}{\Delta_\theta}\right) + \\ + \frac{\Delta_\theta (q_{\rm h}^2 - \Delta^2 + j^2)^2 \sin^2\theta}{\Xi^2 (q_{\rm h}^2 - \Delta^2 + j^2\cos^2\theta)} \left(d\hat{\phi} + \frac{2q_{\rm h}\omega_{\rm h}}{C} z\, d\hat{t}\right)^2 , \end{aligned} \tag{5.32}$$

where the constant C is explicitly given by

$$C = \left[\frac{(L^2-\Delta^2)^2}{L^4} + \frac{(j^2-\Delta^2)^2}{L^4} + 14\,\frac{(L^2-\Delta^2)(j^2-\Delta^2)}{L^4} + 24(g_I \mathsf{P}^I)^2 + \frac{3\mathsf{P}^2}{L^2}\right]^{1/2}.$$

Note that in the extremal limit it is manifest that the entropy is a function of the charges J and p^I by solving (5.25) (for $T = 0$) in terms of S.

5.3 Case $\boldsymbol{\Delta = 0}$

Solving the equations of motion with the Carter-Plebański-like ansatz (5.1) and the assumption $\Delta = 0$ leads to the relations

$$\begin{aligned} &a_0 = b_0 - \frac{\mathsf{P}^2}{4}\,, \quad a_2 = -b_2\,, && a_3 = 0\,, && a_4 = b_4 = -4g^2\,, \\ &b_1 = b_3 = 0\,, \quad \tilde{\Delta} = \frac{(g_I\mathsf{P}^I)g_0 + g^2\mathsf{P}^0}{2g_0 g^2 a_1} g_I \mathsf{P}^I\,, && c^\alpha = \frac{(g_I\mathsf{P}^I)g_\alpha - g^2\mathsf{P}^\alpha}{2g_0 g^2 a_1} g_I\mathsf{P}^I\,. \end{aligned} \tag{5.33}$$

Notice that in this case a_1 is not fixed by any condition, and remains thus a free parameter. Moreover the equations of motion yield an additional condition on the charges,

$$(g_I \mathsf{P}^I)^2 = g^2 \mathsf{P}^2\,. \tag{5.34}$$

This implies that the charges are proportional to the gauge couplings.[13] Nevertheless, notice that the solution is only supersymmetric if the free parameter a_1 is set to zero and the relations (5.12) hold. If $a_1 \neq 0$, the solution generalizes the Kerr-Newman-AdS black hole with n magnetic charges and constant scalars. In order to shew this, one has to take b_0 and b_2 in the form (5.13) and identify $a_1 = -2m$, where m and j are the mass and angular momentum of the Kerr-Newman-AdS solution.

The mass, angular momentum and magnetic charges may be computed as in the case $\Delta \neq 0$, which leads to the same expressions. The Christodoulou-Ruffini formula (5.24) is still valid, but with a simplification due to (5.34),

$$M^2 = \frac{S}{4\pi G} + \frac{\pi J^2}{SG} + \frac{\pi}{4SG^3}\frac{(\mathsf{p}^2)^2}{16} - \left(\frac{L^2}{G^2} + \frac{S}{\pi G}\right)\frac{\mathsf{p}^2}{8L^2} + \frac{J^2}{L^2} + \frac{S^2}{2\pi^2 L^2} + \frac{S^3 G}{4\pi^3 L^4}\,. \tag{5.35}$$

This relation reduces correctly to equation (43) of [52] in the KNAdS case if we identify $\mathsf{p}^2 = -4Q^2$.

[13] To see this, choose in $(n+1)$-dimensional Minkowski space with metric η_{IJ} a basis in which the only nonvanishing component of g_I is g_0 (note that g_I is timelike). Then (5.34) boils down to $\mathsf{P}^\alpha = 0$.

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A Equations of motion

The equations of motion following from (2.7) are given by

$$R_{\mu\nu} = -(\mathrm{Im}\,\mathcal{N})_{IJ} F^I_{\mu\lambda} F^{J\ \lambda}_{\ \nu} + \frac{1}{4} g_{\mu\nu} (\mathrm{Im}\,\mathcal{N})_{IJ} F^I_{\rho\sigma} F^{J\rho\sigma} + 2 g_{\alpha\bar{\beta}} \partial_{(\mu} z^\alpha \partial_{\nu)} \bar{z}^{\bar{\beta}} + g_{\mu\nu} V \,, \tag{A.1}$$

$$\nabla_\mu \left[(\mathrm{Im}\,\mathcal{N})_{IJ} F^{J\mu\nu} - \frac{1}{2} (\mathrm{Re}\,\mathcal{N})_{IJ}\, e^{-1} \epsilon^{\mu\nu\rho\sigma} F^J_{\rho\sigma} \right] = 0 \,, \tag{A.2}$$

$$\begin{aligned} &\frac{1}{4} \frac{\delta (\mathrm{Im}\,\mathcal{N})_{IJ}}{\delta z^\alpha} F^I_{\mu\nu} F^{J\mu\nu} - \frac{1}{8} \frac{\delta (\mathrm{Re}\,\mathcal{N})_{IJ}}{\delta z^\alpha}\, e^{-1} \epsilon^{\mu\nu\rho\sigma} F^I_{\mu\nu} F^J_{\rho\sigma} + \frac{\delta g_{\alpha\bar{\beta}}}{\delta \bar{z}^{\bar{\gamma}}} \partial_\lambda \bar{z}^{\bar{\gamma}} \partial^\lambda \bar{z}^{\bar{\beta}} \\ &\qquad + g_{\alpha\bar{\beta}} \nabla_\lambda \nabla^\lambda \bar{z}^{\bar{\beta}} - \frac{\delta V}{\delta z^\alpha} = 0 \,, \end{aligned} \tag{A.3}$$

which hold for any prepotential F.

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