



# Nothing But Gold. Complexities in Terms of Non-difference and Identity

## Part 1. Coreferential Puzzles

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**Abstract** Beginning from some passages by Vācaspati Mīśra and Bhāskaraṛāya Makhin discussing the relationship between a crown and the gold of which it is made, this paper investigates the complex underlying connections among difference, non-difference, coreferentiality, and qualification qua relations. Methodologically, philological care is paired with formal logical analysis on the basis of ‘Navya-Nyāya Formal Language’ premises and an axiomatic set theory-based approach. This study is intended as the first step of a broader investigation dedicated to analysing causation and transformation in non-difference.

**Keywords** Non-difference · Coreferentiality · Qualification · Relation

### Abbreviations

$a$	Primitive term (lowercase italics)
$\_t$	Abstraction functor, expressing the Sanskrit suffix <i>-tva</i> or <i>-tā</i> (e.g., $a_t = a$ -hood)
$A$	Set $A$ (capital)
$ a_t $	Extension of an abstract; $ a_t  = A$
$R$	Relation $R$ (capital italics)
$\mathbf{R}$	Relational abstract (bold capital italics)
$R^{(R')}$	Relation $R'$ interpreted as $R$ , <i>salva veritate</i>
$R[A]$	The relation $R$ set of destination; for $R: A \mapsto B$ , $\text{dom}R \subseteq A$ , $\text{ran}R \subseteq B$ , and $R[A] = B$
$\lrcorner$	<i>Avacchedaka</i> operator; identifying the <i>limitor</i> of a relational abstract
$\llcorner$	<i>Nirūpaka</i> operator; identifying the <i>conditioner</i> of a relational abstract
$\cdot$	<i>Niṣṭha</i> operator; connecting an abstract to a primitive term

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- TvN *Tadvattva-Nyāya* ('Axiom of Possession')
- SVN *Samānādhikaraṇa-Viśiṣṭatva-Nyāya*  
(‘Principle of Coreferential Qualification’)
- \* $\phi$  ‘It is false that  $\phi$ ’
- (<sup>1</sup>) ‘...’ *Tāparya* (purport of an expression)

## **Pariṇāma-vāda and atyanta-abheda**

The Maharashtrian encyclopedic writer Bhāskararāya Makhin, flourishing in Tamil-Nadu in the first half of XVIII c., was a prominent figure of the Tantrik *śākta* school commonly known as Śrīvidyā. Highly learned and prolific, calling himself genuinely Tantrik (*tāntrika*) and fully Vedic (*vaidika*) at once, Bhāskararāya relentlessly pursued his effort to harmonize the main tenets of the Śrīvidyā school with the most authoritative sources of the brahmanical milieu—the Vedic (*śruti*) and post-Vedic (*smṛti*) corpora, first and foremost, as well as his repeatedly declared affiliation with the *advaita-vedānta* tradition. In *Varivasyārāhasya* 3 (VVR 3), along with his *Prakāśa* auto-commentary (VVR-P 3), Bhāskararāya describes the nature of *mahat-prakāśa*, the great radiance.<sup>1</sup> Once that light (*jyotis*)—which is *brahman* and unobstructed (*anāvṛta*) *ātman*—is known, every other thing is known as well.<sup>2</sup> This alone is the true fundament (*adhiṣṭhāna*). That ultimate (*carama-vṛtti*), indeterminate (*nirvikalpa*), transformation of cognitive motion will definitely obliterate (*nāśyatva*) all further content. The phenomenal object (*dr̥śya*)—

<sup>1</sup> VVR 3 (1976, pp. 4–6): *sa jayati mahān prakāśo yasmin dr̥ṣṭe na dr̥śyate kim api | katham iva tasmīn jñāte vijñātam ucyate vede ||* VVR-P 3: *sa sarveṣāṃ āmatvena prasiddhaḥ | mahān deśakālādyanavacchinnmah parāprakāśaḥ, prakāśaḥ sarvadā anāvṛtātmasvarūpajyotiḥ |*; “That Great Radiance triumphs, at the sight of which nought else is seen. How then is it said in the Veda-s that all is known on Its being known!”. About Bhāskararāya (1690–1785), see Sanderson (2014, p. 72) and Sāstrī in VVR (1941: XXIII–XLIII, *Introduction*). Concerning the *śaiva-śākta* school termed *Śrīvidyā* (‘Auspicious wisdom’) Bhāskara belongs to, cf. Goudriaan (1981: part I); Brooks (1992, 1998). About *śākta* schools: Goudriaan (1979, pp. 6–7); Sanderson (1988); Sanderson (2014, pp. 65–91). Padoux (YH 1994, p. 15): “The philosophical notions of *Yoginīhṛdaya* [and VVR, consequently] are those of non-dualist Kashmir Śaivism. The supreme Reality is transcendent, without division (*niṣkala*), transcending space and time, pure light (*prakāśa*), consciousness (*saṃvit*). It is also the phoneme *A*, the ‘peerless one’ (*amuttara*). This absolute flashes forth, vibrates luminously (*sphuratta, ullāsa*). It expands as a luminous wave (*sphuradūrmi*) by its own free will (*svecchayā*), and thus manifests the cosmos made up of the thirty-six *tattvas*, from *Śiva* to *pṛthivī*”. Regarding VVR’s genealogical dependence on *Yoginīhṛdaya*, cf. also Anrò (2019). Nonetheless, by his own admission, Bh. considers his account to be not in contrast, if not in complete compliance, with Śāṅkara’s theses; cf. VVR-P 3 (1976, p. 6). On the connections between Śrīvidyā, *advaitavedānta* and orthodox brahmanical milieu, cf. in particular Pellegrini (2013, pp. 53–81); regarding the self-understanding and cultural impact of *śākta* intellectuals of South India, see Fisher (2012).

<sup>2</sup> VVR-P 3: *yasminn adhiṣṭhāne dr̥ṣṭe nirvikalpātmakacaramavṛttivijayīkṛte sati kim api dr̥śyam na dr̥śyate, adhiṣṭhānajñānanāśyatvāt |* Cf.: (1) *Bṛhadāraṇyaka Upaniṣad* 2.4.5: *ātmā vā are draṣṭavyaḥ śrotavyo mantavyo nididhyāsītavyo maitreyi | ātmano vā are darśanena śravaṇena matyā vijñānenedaṃ sarvaṃ viditām ||*; (2) *Muṇḍaka Upaniṣad* 1.3: *śaunako ha vai mahāśālo ṅgirasam vidhivad upasannaḥ | kasmin nu bhagavo vijñāte sarvaṃ idaṃ vijñātaṃ bhātīti ||*; (3) *Chāndogya Upaniṣad* 6.1.3: *śvetaketo yan nu somyedaṃ mahāmanā anūcānamānī stabdho’si | uta tam ādeśam aprākṣyaḥ yenāśrutam śrutam bhavaty amatam matam avijñātam vijñātam iti | kathaṃ nu bhagavaḥ sa ādeśo bhavātīti ||*

denotable (*gamyā*) by means of the demonstrative pronoun ‘this’, *idam*—is but a real transformation (*pariṇāma*) of ‘that [sole reality]’ (*tat=ahaṃtā-rūpa-śakti-viśiṣṭa-brahman*).<sup>3</sup> That is, between ‘power’ (*śakti*) and ‘power-owner’ (*śaktimat*) and between the material cause (*upādāna*) and what is caused (*upādeya*), there is absolute non-difference (*atyanta-abheda*).<sup>4</sup> Bhāskarārāya highlights that all Vedic passages (*śruti*)—the peak (*mūrdhanyā*) of all means of true cognition (*pramāṇa*)—concerning non-duality and every Tantra consistent with the former agree on this issue (*abhiprāya*).<sup>5</sup> “Brahman, you see, is this whole world” (*Chāndogya Up.* 3.14.1: 209); syntactical homogeneity (*sāmānādhikaraṇya*), claims Bhāskara, expresses non-difference (*abheda*) and the absence of any contradiction (*bādha*).<sup>6</sup> While the two ‘differential counterparts’ (*bhedāṃśa*) effect (*kārya*) and cause (*kāraṇa*) manifest as constructs (*kalpita*), the entire phenomenal extension (*prapañca*) does not.<sup>7</sup> Negation concerns only the facet (*aṃśa*) of difference. Passages such as “there is nothing diverse at all here!” (*Bṛhadāraṇyaka Up.*, 4.4.19: 125; *Kaṭha Up.*, 4.11: 395) or “one only, without a second one” (*Chāndogya Up.*, 6.2.1: 247) refer to the absence (*abhāva*) of that which possesses difference (*bhedavat*) by reason of the lack (*abhāva*) of any qualifier (*viśeṣaṇa*).<sup>8</sup> The features of Bhāskara’s thesis are therefore: full agreement with the *śruti*, absolute non-difference (linguistically expressed by syntactic homogeneity), radical negation of any difference but not of what is differentiated (that is, phenomenal manifestation), and the absence of any qualifier to differentiate real transformation. Bhāskara supports his thesis by referring to the section of Vācaspati Mīśra’s *Bhāmatī* (IX-X c.; hereafter VM-B) dedicated to the ‘golden crown’ (*hātaka-makuṭa*).<sup>9</sup> “The property of possessing a [degree of] reality (*sattākatva*) that is [ontologically]

<sup>3</sup> VVR-P 3: *parāhaṃtām antareṇedaṃtāyā asaṃsphuraṇād ahamidamoḥ saṃbandhikatvād idaṃ padagaṃyasya dṛśyasyāhaṃtārūpaśaktiṃ tadviśiṣṭabrahmaṇā vā janyatvam | tac ca dṛśyaṃ tatpariṇāma eva |*

<sup>4</sup> VVR-P 3: *śaktiśaktimatoḥ upādānopādeyayoḥ atyantam abhedaḥ |*

<sup>5</sup> VVR-P 3: *advaitaśrūtayoḥ sarvā api etadabhiprāyikā evāvīruddhāḥ | sarvapramāṇamūrdhanyayā śrūtyā tadanusāritantrais cādvaite kathite |*

<sup>6</sup> VVR-P 3: *ata eva “sarvaṃ khalv idaṃ brahma” iti sāmānādhikaraṇyam abhede, na punar bādhyām |* Cf., *Chāndogya Upaniṣad*, 3.14.1: *sarvaṃ khalv idaṃ brahma tajjalān iti śānta upāsita | atha khalu kratumayaḥ puruṣo yathākratur asmiṃl loke puruṣo bhavati tathetaḥ pretya bhavati | sa kratuṃ kurvīta ||*

<sup>7</sup> VVR-P 3: *tadviruddhatvena bhāsamāṇāḥ kāryakāraṇayoḥ abhedāṃśa eva kalpita āstām na punaḥ sarvo’pi prapañcaḥ |*

<sup>8</sup> VVR-P 3: *“neha nānāsti kiṃcana” ityādi śrūtiṣv api bhedāṃśasyaiva niṣedho na prapañcasya | “ekam evādvīṭyam” ityādaḥ śrūyamāṇo bhedavatprapañcābhāvo’pi viśeṣaṇābhāvavaprayukta eva |* Cf.: *Bṛhadāraṇyaka Upaniṣad*, 4.4.19: *manasaivānudraṣṭavyaṃ neha nānāsti kiṃcana | mṛtyoḥ sa mṛtyum āpnoti ya iha nāneva paśyati ||*; *Kaṭha Upaniṣad*, 4.11: *manasaivedam āptavyaṃ neha nānāsti kiṃcana | mṛtyoḥ sa mṛtyuṃ gacchati ya iha nāneva paśyati ||*; *Chāndogya Upaniṣad*, 6.2.1: *sad eva somyedam agra āsīd ekam evādvīṭyam | tad dhaika āhur asad evedam agra āsīd ekam evādvīṭyam | tasmād asataḥ saj jāyata ||*

<sup>9</sup> Vācaspati Mīśra, *Bhāmatī* (VM-B, pp. 72–73); section named by Bhāskara as ‘*hātaka-makuṭa-grantha*’, ‘the golden crown section’ or ‘section concerning gold and crown’. Comm. to *Brahmasūtra* 1.1.4 (*samanvayādhikaraṇa*, 4): *tat tu samanvayāt*. Chakraborty (1967, p. 42): “*Bhāmatī* also is an important commentary of Śāṅkara bhāṣya [the Śāṅkara’s commentary on *Brahma-sūtra*]. [...] The *Bhāmatī* school originates out of *Bhāmatī*. [...] *Bhāmatī* plays an important role in the interpretation of Śāṅkara Vedānta. Of course, the orthodox thinkers think that Vivaraṇa [the mainstream line of interpretation based on

inferior (*nyūna*) to gold is said solely of difference, not of the crown; since [what results from] transformation (*pariṇāma*) possesses by necessity (*avaśyakatva*) the very same (*samāna*) [degree of] reality as that which has transformed (*pariṇāmin*)”.<sup>10</sup>

Needless to say, “the main concern of the Advaitin is to establish non-duality (*advaitasiddhi*). Of course *advaitabrahman* is always a self-established reality (*svatahsiddha*), auto-luminous (*svaprakāśa*), pure consciousness (*śuddhacaitanya*), so no proof is necessary to establish it. As consciousness requires no proof, [usually it is first and foremost] the falsity of the world alone [that] is to be established. Once established the falsity of the world [and this is the keystone of the argument], the non-duality becomes automatically established” (Pellegrini 2014, pp. 3–4).<sup>11</sup> The strategy advanced by Bhāskara appears instead to be symmetrically opposite. Here, the aim is to harmonise, in strictly non-dual terms, the *brahman-ātman* auto-evidence with the full reality of manifestation. In other words, the *advaitin*’s first move usually consists in proving the falsity (*mithyātva*) of the world (*prapañca*, or *jagat*) in order to validate the reality and unity of *brahman*: where I saw a snake, there is but a rope. So, where is the difference between snake and rope, if there is no snake? In this case, however, Bhāskara draws on Vācaspati to overturn the question. It is no longer a matter of denying the reality of the world, but rather of denying only the reality of difference—and this becomes the keystone of his argument. How to fully conceive the difference, if there is one, between a crown and the gold of

Footnote 9 continued

Padmapāda’s *Pañcapādīkā*) represents Śaṅkara more faithfully than *Bhāmātī*”. Potter (1981, p. 17): “Vācaspati Mīśra is said to have continued Maṇḍana’s brand of Advaita in a commentary now lost, on the *Brahmasiddhi* and in his *Bhāmātī* on Śaṅkara’s *Brahmasūtrabhāṣya*”. Potter (2002, pp. 172–173): “One model, known as ‘limitationism’ (*avacchedakavāda*), derives from the fact that there are entities that we ordinarily assume not to break into parts even when they are limited by other entities. For example, the general property of blueness remains general even though at this moment it is instantiated in the cover of the book at my elbow; the particular spatio-temporal location of it is merely accidental and does not affect the essence, blueness. [...] A second model [is] known as ‘reflectionism’ (*pratibimbavāda*) [...]. The tradition is that the *Bhāmātī* school propounds limitationism and the Vivaraṇa school reflectionism [...].” For a detailed analysis of Vācaspati’s main philosophical tenets, cf. also Ram-Prasad (2002, pp. 95–132). Regarding contrasting models within Advaita, cf. also: Timalina (2006, pp. 21–24). For a general survey on Śaṅkara and his *Brahmasūtrabhāṣya*, cf. Potter (1981, pp. 115–179). For a well-documented survey of the complex mutual interchange between the Advaita and Nyāya schools, cf. Phillips (1997).

<sup>10</sup> VVR-P 3: *ata eva bhāmātyāṃ hāṭakamakūṭaḡranthe bhedasyaiva hāṭakanyūnasattākatvaṃ na makūṭasyoktam, pariṇāmasya pariṇāmisamānasattākatvāvaśyakatvāt | “māyāmātram idaṃ dvaitam” (Gauḍapādīya-kārikā, 1.17) ity atrāpi dvaitaśabdena bhedasyaiva mithyātvam ucyate, na punar bhedavataḡ |*

<sup>11</sup> A quote analysing a passage of Madhusūdana Sarasvatī’s *Advaitasiddhi* (cf. AS 1997, p. 8; *tatrādvaitasiddher dvaitamithyātvāsiddhipūrvakatvād dvaitamithyātvam eva prathamam upapādanīyam*); additions in square brackets are mine. Cf. also Chakraborty (1967, p. 41): “Śaṅkara interprets *Brahma Sūtra* in his own way and shows that the consistency of the *upaniṣadic* texts can alone be maintained on the admission of the sole reality of consciousness and falsity of the world”. On this point, see also Timalina (2009, p. 85): “There are two ways Advaita can be established: by confirming the existence of a singular reality, or by rejecting the existence of duality”.

which it is made? If there is no difference, where is the difference between *jagat* and *brahman*?<sup>12</sup>

If “Advaitins [undoubtedly] place the stream of arguments that refute difference at the core of their logical investigation”, in so doing, “they first utilize the categorical analysis found in Nyāya” (Timalsina 2009, p. 86; cf. also Ganeri 2011, pp. 223–236), just as this paper is methodologically proposing to do. Indeed, the highly refined language and techniques of Navya-Nyāya—along with the formalistic methodology derived therefrom, named ‘Navya-Nyāya Formal Language’ (NL; cf. *infra*)—will be here programmatically adopted in order to describe in detail a non-dualistic argumentative architecture. Clearly, this does not imply that the Naiyāyikas’ account, conceived in its own prerogatives, will be considered interchangeable or confusedly intermingled with the Advaitins’ one. On the contrary, the philosophical claims of NL qua hermeneutical device methodologically stop just before being committed to the various and different theoretical frameworks NL purposes to analyse (cf. Anrò, [forthcoming](#)). This therefore means that, despite the respective deep structural differences, the Nyāya machinery—envisioned, in accordance with a well-established tradition, as a ‘*lingua franca* for intellectual exchange’ (Ganeri 2011, p. 223)—will be here methodologically put at the service of Vācaspati’s reasoning, in its turn viewed through the lens of the issue Bhāskara raised.

## Syntactic Homogeneity and Coreferentiality

What is this golden bracelet? Undoubtedly, it is gold. It is, in this perspective, non-distinct (*a-bhinna*) from its cause (*kāraṇa*) because, as stated above, the bracelet is golden. Nevertheless, it is also a bracelet, and not another ornament such as an earring or crown. Indeed, the bracelet is distinct as an effect (*kārya*) exactly because

<sup>12</sup> Dasgupta (1933, pp. 161–162): “If, however, it is contended that this view of real transformation is only from a relative point of view, then there must be at least one *sūtra* where the absolute point of view is given; but no such *sūtra* has been discovered even by Śaṅkara himself. If experience always shows the casual transformation to be real, then how is one to know that the ultimate point of view of all effects are false and unreal? If, however, it is contended that there is a real transformation of the *māyā* stuff, whereas Brahman always remains unchanged, and if *māyā* is regarded as the power of Brahman, how then can the power of Brahman as well as its transformation be regarded as unreal and false, while the possessor of the power is regarded as real and absolute? [...] The world is identical with Brahman, inasmuch as it has been and is identical with being, and different from it, inasmuch as it has its characteristics of materiality and change”. Dasgupta (1941, p. 333): “But in passages like those found in Śaṅkara’s *bhāṣya* on the *Brahmasūtra*, I.1.2, it might appear as if the world-phenomena are no mere appearance but are real, inasmuch as they are not merely grounded in the real but are also the emanations from the real—the Brahman. But strictly speaking Brahman is not alone the *upādāna* or the material cause of the world but Brahman-with-*avidyā* is the material cause of the world and such a world is grounded in Brahman and is absorbed in Him; and Vācaspati in his *Bhāmatī* on Śaṅkara’s *bhāṣya* on the same *sūtra* (B.S. I. 1.2) makes the same remark. [fn. 1] *avidyā-sahita-brahmo-pādānaṃ jagat brahmaṇy evāsti tatraiva ca līyate. Bhāmatī*, I.1.2.”. For a preliminary survey of Vācaspati’s tenets about *māyā*, cf. Potter (2002, pp. 168–171); on causal chain and causal models in Indian philosophical systems, cf. Potter (2002, pp. 102–115).

it is a bracelet and not an earring. It seems, thereby, to appear as simultaneously distinct and non-distinct.<sup>13</sup>

The notion (*pratyaya*) of *sāmānādhikarāṇya* indicates syntactical homogeneity on the linguistic and grammatical level, and coreferentiality on the ontological one, both at the same time. Using this notion to express the relationship between earring and gold clearly exhibits the simultaneous occurrence of difference and non-difference (*bhedābheda*).<sup>14</sup> In Vācaspati's view, the *sāmānādhikarāṇya* relation acts as the *ratio cognoscendi* with respect to the *a-bheda* relation, for its part the *ratio essendi* of the former. This relationship of *sāmānādhikarāṇya* between two terms in a Sanskrit sentence—terms which share the same grammatical ending (say, nominative or first ending) and the same referent, here generically named A and B—can be expressed in the following manner<sup>15</sup>:

[1] A<sup>1</sup>B<sup>1</sup>. 'x is y' (e.g. *kuṇḍalam suvarṇam*, 'The earring is gold').

Vācaspati points out that sentences such as [1] are not in any way reducible to the *substratum-superstratum* relation (*ādhāra-ādheya-bhāva*): if A<sup>1</sup>B<sup>1</sup> ('B on/in A'), e.g. *kuṇḍe badaram* ('A jujube in a bowl'), this does not imply that the fruit is the bowl (*na hi bhavati kuṇḍam badaram iti*); or to the relation of 'residing in one *locus*'

<sup>13</sup> VM-B: 73: *bhinnābhinnam tad iti cet; tathā hi tad eva kāraṇātmanābhinnam, bhinnam ca kāryātmanā, kaṭakādaya ivābhinnā hāṭakātmā bhinnāś ca kaṭakādyātmanā*. And also: *athānuvṛttivyāvṛttivyavasthā ca hemni jñāte kuṇḍalādijijñāsā ca katham? na khalv abhede aikāntike'naikāntike caitad ubhayam upapadyate ity uktam*; "Then, how is that [particular] state of affairs (*vyavasthā*) between continuity (*anuvṛtti*) and discontinuity (*vyāvṛtti*) [possible]? And [how is it possible that] once having known gold, the necessity of knowing (*jijñāsā*) earrings, etc., still remains? Indeed (*na khalu*), in absolute and non-absolute (*an-aikāntika*) non-difference, it has been said they are both possible (*upapad*)". The issue is also addressed in the same terms by Vṛṣabhadeva in his *Paddhati* to Bhartṛhari's *Vākyapadīya* 1.59 (1966, p. 117). There, the chosen example is a golden ring (*suvarṇa-aṅgulīyaka*) and it is a case of *vyapadeśivadbhāva*. *Vyapadeśivadbhāva* is defined an extension of a specific designation (*vyapadeśa*): "fait que (tel mot est traité) comme s'il avait reçu une désignation speciale", Renou (1957, pp. 295–296). The canonical example concerns the mythological episode of Rahu's head: if only his head is left, what could be meant by the expression 'Rahu's head' (*rahoḥ śīrah*)? Indeed, a distinction is still suggested even where there is no more duality. In this passage, Vṛṣabhadeva combines the Rahu example with the 'golden ring' case of *vyapadeśivadbhāva*: that is, a specific designation in *abheda* to define the specific nature of the matter, where there is but a single object. Highlighting the same topic, cf. Candotti (2005, p. 337).

<sup>14</sup> VM-B: 72: '*kuṇḍalam idaṃ suvarṇam*' *iti sāmānādhikarāṇyapratyaye vyaktaṃ bhedābhedau cakāstah; tathā hi āyantike'bhede'nyatarasya dviravabhāsaprasaṅgaḥ; bhede cāyantike na sāmānādhikarāṇyaṃ gavāśvavat; ādhārādheyabhāve ekāśrayatve vā na sāmānādhikarāṇyam; na hi bhavati kuṇḍam badaram iti; nāpy ekāśanasthayoś cāitramaitrayoś cāitro maitra iti* l. Cf. Filliozat (1988, p. 69): "Le rapport du mot et de ce à quoi il réfère dans la réalité objective est donné comme un rapport de location: l'objet signifié est la location (*adhikarāṇa*) ou le support (*ādhāra*) du mot. [...] On dit que les mots sont *sāmānādhikarāṇa* littéralement 'ont la même location', s'ils réfèrent au même objet. L'identité de location est l'identité de référence dans la réalité objective". Cf. also Staal (1988, p. 62), who sets off from Jagadīśa Bhaṭṭācārya's definition of *tatpuruṣa*, quoted by Nyāyakośa (cf. *infra*), and clearly formalises "sameness of locus" as:  $K(p) = {}_d\{C_k(t, p) \wedge [axB(x, t) = axB(x, p)]\}$ .

<sup>15</sup> Capital letters, such as A and B, stand for terms while, according to a longstanding tradition (cf. *kaumudī corpus*, i.e. grammatical commentaries), superscripts stand for the grammatical endings (*vibhakti*) in which the terms appear. So, e.g. 'B<sup>1</sup>' will mean 'the term B in the first case or nominative' (*prathamā vibhaktiḥ*); and 'A<sup>7</sup>' will mean 'the term A in the seventh case or locative' (*saptamī vibhaktiḥ*). For this updated usage, cf. Ganeri (2006, p. 36 ff).

(*ekāśrayatva*):  $C^7A^1B^1$  ('A and B on/in C'). If 'Caitra and Maitra [are dwelling] on the same seat' (*ekāsane caitramaitrau*) it does not follow that 'Caitra is Maitra' (cf. fn. 14).

Two possible interpretations of [1] are then formulated in Vācaspati's analysis. [a] The relation of *sāmānādhikarānya* can point to an absolute non-difference (*ātyantika-abheda*) according to which in [1]  $A = B$ ; e.g. 'The earring is gold', that is, 'earring = gold'. However, based on this premise, what will occur is the undesired outcome (*prasaṅga*) of a double occurrence (*dviravabhāsa*) of the term itself: if  $A = B$ , then  $A = A$  or  $B = B$ . Thereby, if 'earring = gold', then: 'earring = earring'; or 'gold = gold'.<sup>16</sup> [b] In the case where, in order to avoid the double occurrence at point [a], the total difference (*ātyantika-bheda*) between the two terms in [1] is stressed, then  $A \neq B$ , with the likewise undesired consequence that any form of *sāmānādhikarānya* relation would be then denied—as in the case of *go ≠ aśva* (cow ≠ horse). Thus, if  $A \neq B$ , then: earring ≠ gold ≠ horse.

Still claiming [1], it is therefore not possible to conclude either that the erring is gold, without falling into [a], nor that the earring is not gold, without falling into [b]. The relation of *sāmānādhikarānya*—while being unobstructed (*abādhita*), indubitable (*asaṃdigdha*) and universal (*sarvajanīna*)—ends up determining (*vyavasthā*) both the difference and the non-difference between the effect (earring) and its cause (the gold of which it is made), simultaneously.<sup>17</sup>

## Cognition as a Relation

If the relation of difference (*bheda* or *dvaita*;  $A \neq B$ , e.g. *go ≠ aśva*) does not seem to present any difficulty, what kind of a relation is there between the two terms of a non-difference? Since non-difference (hereafter expressed by the strikethrough

<sup>16</sup> The logical fallacy of *āmāśraya* ('self-foundation') is a vicious circle or self-dependency (*sva-apekṣā*) in cognition. Cf. Jha (2001, p. 84). NK, p. 121. *āmāśraya*—*tarkaḥ* [ka] *svasya svāpekṣāpādakaḥ prasaṅgaḥ* (Jagadīśa Bhaṭṭācārya, *Jagadīśī*, Id., *Tarkāmṛtam*) | *yathā kāryatvāvachchinmakāryatānirūpitakāraṇatvam sādḥāraṇakāraṇatvam ity ādau* | *atra ekaḥ kāryatvam avacchedakam* | *aparaḥ tv avacchedyam* | *tathā cāvachchedakajñānaḥ vinā avacchedyajñānaḥ na bhavati* | *avacchedyajñānaḥ vināpy avacchedakajñānaḥ ca na bhavati* | *parasparasāpekṣatvād ity āmāśraya iti bodhyam* l. "[The term] 'self-foundation' (*āmāśraya*) must be understood as a circular (*paraspara*) [vicious] self-dependency (*sva-apekṣā*). Like in cases such as 'generic causeness' (*sādḥāraṇa-kāraṇatva*), where the property of being a cause is both conditioned by effect-ness (*kāryatva*) and limited by [the same] effect-ness (*kāryatā*). Here effect-ness is the limitor; nonetheless, [yet] it is [also] the limited too. But, a knowledge of the limited without the knowledge of limitor cannot be, and vice versa." With the term *sādḥāraṇa* ('generic') is indicated the fallacy of *anaikāntika* ('inconclusive' [probans]). Cf. NK, pp. 998–999: *sādḥāraṇaḥ—hetvābhāsaḥ duṣṭahetuḥ* | [ka] *sapaḥsavipakṣavṛttir hetuḥ* (Viśvanātha Pañcānana Bhaṭṭācārya, *Bhāṣāpariccheda*, 2, śl. 74) | *yathā parvato vahnimān prameyatvāt ity ādau prameyatvam hetuḥ sādḥāraṇaḥ* l; "it is 'generic' an apparent or fallacious probans; it consists in being a probans which occurs both in similar and dissimilar instances. In examples such as 'the mountain possesses fire by virtue of the probandum', it is 'generic' the probandum as probans". Cf. also, NK, pp. 36–37: *anaikāntika—hetvābhāsaḥ* [...] *anaikāntikaḥ savyabhicārasābheda vyavahriyate* l; 'It is 'inconclusive' a fallacy concerning probans; it is commonly referred to by the term *deviation* (or *discrepancy*). Perfectly analogous is the definition of *savyabhicāra*: cf. NK, pp. 979–980.

<sup>17</sup> VM-B: 72: *so 'yam abādhito 'saṃdigdhaḥ sarvajanīnaḥ sāmānādhikarānyapratyaya eva kāryakāraṇayor bhedābhedaḥ vyavasthāpayati*. It has been said that Vācaspati asserts, moreover, that the issue cannot be solved by appealing to the relation of 'substratum-superstratum' (*ādḥāra-ādheya-bhāva*) nor to the property 'residing in one locus' (*eka-āśraya-tva*): in these cases there is no coreferentiality at all, which is, on the contrary, the starting point of the analysis.

cypher ‘2’, i.e. *advaita*, lit. ‘non-two’) cannot be reduced to an equality or identity relation ( $A = B$ ), how can these two terms be simultaneously *equal* and *different*, as explicitly claimed by Vācaspati?

Some formal tools are required to perform the analysis in NL (Navya-Nyāya Formal Language).<sup>18</sup> Let the notation ‘ $_I$ ’ be here the abstraction functor, capable of expressing the Sanskrit suffix *-tva* or *-tā*.<sup>19</sup> So, for instance, if the primitive term *g* (small italics) is a single pot (*ghaṭa*),<sup>20</sup> then ‘ $g_I$ ’ = ‘the property of being *g*’ or ‘*g*-hood’ (i.e. *ghaṭatva*, ‘pot-hood’) whose extension corresponds to the set ‘pots’  $G$  (capital), to obtain  $|g_I| = G$ . According to what could be called the Axiom of Possession or *Tadvattva-Nyāya* (TvN), the element *g* is said to belong to the set  $G$  because  $g_I$ -possessing (viz., qualified by the property *g*-hood = *pot-hood*). Thus *ghaṭo ghaṭatvavān*, ‘a pot [is a pot because it is] in possession of pot-hood’, lest it be not the pot it is. More generally, TvN: *tadvattvam* (in extended form, *taddharmavattvam* or *tattvavattvam*) *tad eva*, ‘What possesses the property of being that, is that’.<sup>21</sup>

That premised, the crown (*m*) is surely gold (*h*). What would be left, indeed, if the gold of which the crown is made were subtracted? Thus,  $m=h$ . Nevertheless,  $m \neq h$  because the crown is not only gold, it is a crown as well. Saying that ‘the crown is gold’ implies two distinct properties: the abstract properties ‘gold-ness’, *hāṭakatva* ( $h_I$ ), and ‘crown-hood’, *mukūṭatva* ( $m_I$ ), in reference to the two distinct sets  $M$  (the set Crowns; for  $m \in M$  and  $|m_I| = M$ ) and  $H$  (the set Gold; for  $h \in H$  and  $|h_I| = H$ ).

<sup>18</sup> For a discussion of NL technicalities, scope, and aims and the strengths and weaknesses of such an extensional set-theoretic approach, cf. Anrò (forthcoming). Regarding the strategy of using a set-theoretic ontology, cf. Ganeri (2008, p. 112 ff.). For a general survey of axiomatic set theory, see Jech (2006, pp. 3–13). NL is directly inspired by Ganeri (2008 I & II).

<sup>19</sup> Ganeri (2008, p. 113): on “abstraction functor”. The abstraction functor ‘ $_I$ ’ here plays the role—including in a graphic sense—of Ingalls’ subscript convention ‘*locus<sub>I</sub>*’ for ‘locus-hood’ (e.g. Ingalls 1951, p. 46); ‘*t*’ clearly stands for ‘*tva*’ or ‘*tā*’.

<sup>20</sup> A primitive term—expressed in NL by a small letter in italic (e.g. ‘*g*’), that is, by a “simple symbol”—refers to an “individual or a particular”, cf. Russell (1919, pp. 141–143, 173): “a ‘simple’ symbol is one which has no parts that are symbols”.

<sup>21</sup> Alternatively, *tadvattā* (that is *taddharmavattā* or *tattāvattā*) *tad eva*; cf. Anrò (forthcoming). Similarly, in sentences such as ‘*ghaṭe ghaṭatvam*’, the locative of residence (*vṛttitva*) denotes the relation ‘ $\in$ ’;  $g \in (G = |g_I|)$ . Cf. Ganeri 2008, pp. (128, 131–132); and Matilal (1998, p. 29): “ $x + vat + tva = x$ ”. Cf. Pāṇini (1999, p. 569) *sū.* 5.2.94: “*tad asyāsty asminn iti matup* [...] A *taddhita* affix, namely *matUP*, occurs to denote the sense of *ṣaṣṭhī* ‘genitive’ (*asya*), or of *saptamī* locative (*asmin*), after a syntactically-related nominal stem which ends in *prathamā* ‘nominative’, provided it is qualified with the denotatum of *asti* ‘existence’”. According to Shaw’s terminology, TvN expresses an “atomic cognition”, while a property (P) may be defined as “(□*x*) (*x* is a locus of P)” (1989: 382 *et passim*). Cf. also Staal (1988, p. 63); in his notation:  $ax B(x, t) = t$ .



Clearly, these two properties could be structured around three possibilities: golden crowns, golden bracelets and iron crowns could exist.<sup>22</sup> In current notation:

[a]  $(\exists x) (Mx \wedge Hx) — h\ddot{a}taka\dot{m} muku\dot{t}am$  ( $A^1B^1$ ; ‘A golden crown’); in case both qualificans and qualificandum are present: *ubhaya-bh\ddot{a}va-prayuktavi\ddot{s}i\ddot{s}ta-bh\ddot{a}va*.

[b]  $(\exists x) (Mx \wedge Hx) — h\ddot{a}taka\dot{m} na muku\dot{t}am$  or *h\ddot{a}taka\dot{m} maku\ddot{t}\ddot{a}nyatvam* ( $A^1B^1$ ; ‘Gold which is not a crown’); in case the qualificandum is absent and qualificans present: *vi\ddot{s}e\ddot{s}y\ddot{a}bh\ddot{a}va-prayuktavi\ddot{s}i\ddot{s}t\ddot{a}bh\ddot{a}va*, *vi\ddot{s}e\ddot{s}anabh\ddot{a}va-prayuktavi\ddot{s}i\ddot{s}ta-bh\ddot{a}va*.<sup>23</sup>

[c]  $(\exists x) (Mx \wedge Hx) — ah\ddot{a}taka\dot{m} muku\dot{t}am$  ( $A^1B^1$ ; ‘A non-golden crown’); in case the qualificandum is present and qualificans absent: *vi\ddot{s}e\ddot{s}ya-bh\ddot{a}va-prayuktavi\ddot{s}i\ddot{s}ta-bh\ddot{a}va*, *vi\ddot{s}e\ddot{s}an\ddot{a}bh\ddot{a}va-prayuktavi\ddot{s}i\ddot{s}t\ddot{a}bh\ddot{a}va*.

Nevertheless, the conjunction expressed in standard notation for assertion [a] cannot be considered fully proper from a Naiy\ddot{a}yika’s perspective. “Ny\ddot{a}ya develops a language which can perhaps be given the appellation of a ‘property-location language’ [...]. The model sentence of such a language contains the introduction of general concepts and ‘the indication of their incidence’. Under this interpretation, the qualifier can be viewed as the *feature-universal* [...], and the qualificand can be viewed as the *locus* where the qualifier is said to occur” (Matilal 1968: 16).<sup>24</sup> For a

<sup>22</sup> I ignore as being superfluous to my argument here the fourth possibility in which both properties are negated: *ubhaya\ddot{a}bh\ddot{a}va-prayuktavi\ddot{s}i\ddot{s}t\ddot{a}bh\ddot{a}va*, ‘absence qualified by the absence of both qualificandum and qualificans’. Being not crowns or gold, there would be, for instance, gingerbread or whatever else. I refer here to the inferential taxonomy displayed by Vi\ddot{s}van\ddot{a}tha’s *Mukt\ddot{a}val\ddot{i}* when discussing the “governing factors” (*prayojaka*) for “qualified negation” (*vi\ddot{s}i\ddot{s}t\ddot{a}bh\ddot{a}va*)” as conditions for inferential subjectness (*pak\ddot{s}at\ddot{a}*). I add here a fourth possibility (case [a]) not present in the original text: *ubhaya-bh\ddot{a}va*, or double presence. Cf. NSM (1988: 496–506) and Rai (1995, pp. 6–7), also quoted by Pellegrini (2014, p. 12).

<sup>23</sup> Instead of the perhaps more common ‘¬’, I will use the *tilde operator* ( $\sim$ ) as negation functor here to avoid any confusion with the NL operator *top left corner* ( $\neg$ ); cf. *infra* fn. 51–52.

<sup>24</sup> Cf. also Matilal (1968, p. 32): Naiy\ddot{a}yikas “tended [...] to speak in terms of *dharma* (property) and *dharmin* (property possessor or locus of property)”; and Matilal (2001, p. 202): “A simple qualificative cognitive state is one where the cognizer cognizes something (or some place or some locus, as we will have to call it) as qualified by a property or qualifier. It is claimed by most Sanskrit writers that to say that something or some place is qualified by a qualifier is equivalent to saying that it is a locus of some property or locatable”. The concept of ‘property’ is used in this paper according to a principle of ontological parsimony, which *naiy\ddot{a}yikas* name *l\ddot{a}ghava*: “It seems that Ny\ddot{a}ya tries in the main to avoid disputes about ontology, and develops a theoretical language which can be used even by those who do not share its ontological dispositions (cf. [16, p. 66], [3, p. 201])”, Ganeri (2008, p. 112), quoting B.K. Matilal and S. Bhattacharyya. Cf. also Matilal (2001, pp. 208–209): “Suppose by ‘property’ we mean non-universal, abstract features, or even tropes”; the thought experiment of ‘ability to swim’ and ‘water’; and Matilal (2001, pp. 209–210) on the difficulties in translating *dharma*: “[...] the word *dharma* has a wider extension than the word ‘property’ [...]. *Dharma* sometimes means not only abstract properties or universals but also concrete features, that is, the particular features of some object or locus. *Dharma* and *dharmin* constitute a pair in Sanskrit that is the equivalent to the pair ‘locatee’ (or the locatable) and ‘locus’ (location, which may be a place or a time or even an abstract object)”.

Naiyāyika, a golden crown is a ‘qualified entity’ and, bizarre though it may seem at first glance, ‘A golden crown is a crown’ just as ‘A blue pot is a pot’.<sup>25</sup>

In compliance with assertion [a], the statement ‘A blue pot’ can be plainly described, in standard notation and according to a predicative account, through the linear string  $(\exists x)(Gx \wedge Nx)$ , true iff ‘There does exist a variable  $x$ ’, ‘This variable is a pot’ ( $Gx$ ), and ‘This variable is blue’ ( $Nx$ ).<sup>26</sup> According to the Nyāya-property-location language (implying TvN), the attribution of these properties would be better described not by the coordination of a double predication, but by a relational structure whose fulcrum is a primitive term and not an *existing* variable. In dealing with such a sentence, ‘A blue pot’, it must first be noted that the element under discussion here is relations, not predications.<sup>27</sup> In general terms, this case of coreference could be seen as a *viśeṣaṇa-viśeṣya-bhāva*, i.e. a qualifier-qualified relation, conceived as a form of determined cognition (*savikalpa* or *viśiṣṭa jñāna*).<sup>28</sup>

<sup>25</sup> *Nīlatva-viśiṣṭa-nīla-rūpa-vān ghaṭatva-viśiṣṭah ghaṭah*, ‘a pot, possessing blue color in turn qualified by blue-ness, is qualified by pot-ness’; cf. Matilal (1968, p. 15), translation mine. The puzzle ‘a pot = a blue pot’ ‘[...] express a truism, viz., a thing is identical with itself no matter whether you refer to it in a general way (*sāmānyena*) by calling it a ‘pot’ or in a special way (*viśeṣeṇa*) by calling it ‘a blue pot’.’; Matilal (1968, p. 48). Cf. also Ingalls (1951, p. 69): ‘That which is expressed by ‘pot’ alone is the type of framework that subsists in all knowledges of pots. [...] If one does not accept [that ‘blue pot = pot’], one must admit that a blue pot is not a pot (cf. Raghunātha, *Dīdhiti*, 19–20; in Ingalls 1951, pp. 160–161)’. Thus, *viśiṣṭa* (qualified) as ‘accompanied by’: ‘*vaiśiṣṭyaṃ ca sāhityaṃ sāmānādhikaranyam vā jñeyam*’, according to the NK definition (cf. NK, p. 779 and Ingalls 1951, p. 69, fn. 137). In this case I follow Matilal (1968, p. 48) regarding Ingalls’ positions (Id. 1951, P. 69); cf. also the reviews of Ingalls (1951) in Potter (1954) and Staal (1960).

<sup>26</sup> Quine (1981, p. 27): ‘To say that a city or a word has a given property, e.g. populousness or disyllabism, we attach the appropriate predicate to a name of the city or word in question’. Cf. also Quine (1981: § 22. Class and Member, 119–123): ‘In such context [‘Paris is a city’] ‘is’ expresses rather possession of a property, or membership in a class: Paris belongs to the class of cities [...]. It is this sense of ‘is’ that is rendered symbolically with ‘ $\in$ ’: ‘Paris  $\in$  city’ [...]. Or, to put it in its simplest terms: “Traditional [western] grammar tell us that the simplest sentences are composed of a *subject* and a *predicate*. [...] [The subject] tell us what the sentence is about. The rest is the predicate: this tell us what is said about it”; Priest (2000, pp. 17–18). For a survey of “*subject-predicate discourse*” in the context of the problem of universals and realist-antirealist debate, cf. Loux (2006, pp. 21–27).

<sup>27</sup> Regarding the primacy of proposition and predication as conceived in Western logic, and its differences with respect to the Nyāya qualifier-qualified approach, cf. Matilal (2001, pp. 201–205): ‘Now, in the Indian context the basic combination is not called a proposition. It is a structured whole that is grasped by an atomic cognitive event. We call it an atomic qualificative (*viśiṣṭa*) cognition. [...] A qualifier and a predicate-property may not always be the same, such that we can say there is only a terminological variation’. Regarding the “*basic combination* of predication” in propositions as the “focal point in current logic”, Matilal quotes Quine (1960, p. 96) and Strawson (1974, p. 4). See also Shaw (1976, 1989, 2010) and Staal (1988, p. 63): ‘Western thought is inclined to analyse a close relationship in terms of subject and attribute whereas Indian thought considers the relation to the *adhikarāna*’.

<sup>28</sup> Interpreting coreference as qualification will be discussed in Part 2. On “determinate and indeterminate knowledge”, cf. Ingalls (1951, pp. 39–40). Cf. also Matilal (1968, p. 13): ‘The content of a qualificative cognition, then, taken as a whole, is articulated in such a way that a certain feature or features of it will be emphasized as features of, or occurrent in, or related to, the remaining portion or portions of the content. [...] Thus a qualificative cognition may be said to be an answer to question of the form: ‘What is this?’, ‘What property does it possess?’, ‘When or where does it occur?’; and Id. (1968: § 3.7). See also Dalai (1992, pp. 10–13). Regarding *jñāna*—fundamentally as “cognition or psyche-dependent awareness”—see also Bilimoria (1985, p. 75): ‘we may note how *jñāna* is used [...] sometimes to indicate ‘knowing’ in the sense of ‘propositional attitude’ [Matilal (1968, pp. 8–9)] towards beliefs, or towards what one is actually believing and judging at some time, as would occur, say, in reflective and introspective states, where there

The Nyāya relation-based analysis cannot therefore be directly reduced to predication, and any attempt to force the Nyāya account into this grid seems doomed to failure. If the first inaccuracy is thinking in terms of predication, the second is confusing the connective ‘and’ ( $\wedge$ ); which in the theory of sets corresponds to intersection, ( $\cap$ ), with the qualifier-qualified relation.<sup>29</sup>

The abstract property  $g_i$  (*ghaṭatva*, pot-hood; cf. *supra* Gx) has as its *locus* the primitive term  $g$ —that is, an actual pot—while the further abstract property  $n_i$  (*nīlatva*, blue-ness; cf. *supra* Nx) occurs in an instance of blue ( $n$ ), which is in turn *located* in ‘a pot *locus* of pot-hood’.<sup>30</sup> If the property  $g_i$  (whose reference set is G) is referred to its *locus*  $g$  (*ghaṭa-niṣṭha-ghaṭatva*), then this property will be the *prakāra* or *mukhya-viśeṣaṇa* (chief or root qualifier) and the primitive term  $g$  the *mukhya-viśeṣya* (chief or root qualificand).<sup>31</sup> Yet, the root-property  $g_i$  is in turn the *locus* of a collocated (*samānādhikaraṇa*) second-order property  $n_i$ . In other words,  $n_i$  (blue-ness) occurs in  $g_i$  (pot-ness), referred to the primitive term  $g$  (an actual pot). Thereby, the collocated second-order property  $n_i$  turns out to be dependent on the first-order property  $g_i$ , the *mukhya-viśeṣaṇa*. Blue pots are thereby pots because blue-ness is

Footnote 28 continued

is affirmation of particular cognitive contents, as for example, when one becomes aware of ‘table contentness’ in his consciousness as his eyes fall on the large ‘object’ (this something) in the kitchen. The judgement is not about the ‘object’ as such, but it is an affirmation of his mental mode in relation to the object. However, often, too swift a move is made [...; thereby] when a reflective judgement is taken to be an assertion of the truth-value of a cognition, *jñāna* is rendered as *knowledge*, implying that it is a judgement with a truth-value [...].” In any case, the “significance”—in an “epistemic sense”—of a *jñāna* is “having contentness: *viśayatā*”; Bilimoria (1985, pp. 76–77). Regarding *viśeṣya-viśeṣaṇa-bhāva-sambandha* cf., of the many possible sources, Gadādhara (1990, pp. 125–126): “*saṃsargatayā ca samam prakāratāyā viśeṣyatāyāś ca nirūpyanirūpakabhāvākhyah sambandhaviśeṣo bhyupagantavyah | sa ca sambandhah kāryatvakāraṇatvādheyatvādharatvapratīyogitvānuyogitvādīnām mithas tādṛśasambandha iva svarūpaviśeṣah padārthāntaram eva vā, anyathā tatra tena sambandhena tat prakārakam ity etad arthasya durvacatvāt | Translation: It is also to be admitted that there is a relation of determiner-determined-ness between relationness, on the one hand, and modeness as well as qualificandness, on the other. And that relation is either a particular self-linking relation or a separate ontological reality, just like the relation with cause-ness and effect-ness, superstratum-ness and substratum-ness, successor-ness and predecessor-ness. Otherwise the meaning of ‘having that as a mode by that relation’ cannot be explained”. Regarding *prakāratā*, cf. also NK, p. 515. 1) *prakāratā—viśayatā* | [ka] *viśeṣaṇatvāparanāmā vilakṣaṇaviśayatāviśeṣah*.*

<sup>29</sup> Halmos (1960, p. 12): “If A and B are sets, the intersections of A and B is the set  $A \cap B$  defined by  $A \cap B = \{x \in A: x \in B\}$ ”. Jech (2006, p. 8): “One consequence of the Separation Axioms is that the intersection and the difference of two sets is a set, and so we can define the operation  $X \cap Y = \{u \in X: u \in Y\}$  and  $X - Y = \{u \in X: u \notin Y\}$ ”; Cf. also Enderton (1977, p. 21). For a plain explanation about the connections between basic operations on sets, Boolean operations, and Venn diagrams: Moschovakis (2006, pp. 2–4). Cf. also Levy (2002, pp. 244–246) on Boolean algebra; and Quine (1981, pp. 11–12) for an introduction to connective ‘and’.

<sup>30</sup> It must be noted here the absence of quantifiers, variables and operators, such as ‘ $\wedge$ ’ (cf.  $\exists x: m_i(x) \wedge h_i(x)$ ). The NN logic syllabus thus basically consists of primitive terms, abstract properties, relational abstracts, and the two operators ‘limitor’ and ‘conditioner’; cf. Anrò (forthcoming). Regarding the fact that “Indian logic has no *variables*” and the “strange doctrine of *repeated abstraction*” without quantifiers, cf. also Bochenski (1956, pp. 149–150).

<sup>31</sup> Cf. Ingalls (1951, p. 43) and Matilal (1968, p. 15).

dependent on pot-ness—which sounds quite striking if not wholly false. How could such a claim be justified? More generally, how could such a relation be conceived?

“Relation (*sambandha*) is what, though distinct (*bhinnatva*) from the relata (*sambandhin*), in them occurs (*āsrita*). [...] So, [for instance] *contact* (*saṃyoga*) [is the relation between] pot and ground; and *direct contact* (*saṃnikarṣa*), in the case of perception, between sense organ and the [perceived] object” (NK, p. 920).<sup>32</sup> Similarly, in set theory, a “pairing function”<sup>33</sup> or “relation is a set of ordered pairs” without any further restrictions: “any set of ordered pairs is some relation, even if a peculiar one” (Enderton 1977, p. 40).<sup>34</sup> To put it another way, given two generic sets or classes A and B, for  $x \in A$  and  $y \in B$ , the relation  $R$  is their Cartesian product ( $A \times B$ )—written  $xRy$  or  $\langle x, y \rangle \in R$ , in which  $x$  stands in the relation  $R$  to  $y$ . Conversely, any subset of ordered pairs, an element of the power set  $\mathcal{P}(A \times B)$ , is some sort of relation.<sup>35</sup> “The domain of  $R$  ( $\text{dom}R$ ), the range of  $R$  ( $\text{ran}R$ ), and the field of  $R$  ( $\text{fld}R$ ) [are defined] by:  $(x \in \text{dom}R) \leftrightarrow (\exists y) (\langle x, y \rangle \in R)$  [i.e.,  $x$  belongs to the domain of  $R$  iff there exists at least an  $y$ , such that  $x$  stands in relation  $R$  with  $y$ ],  $(x \in \text{ran}R) \leftrightarrow (\exists t) (\langle t, x \rangle \in R)$ , and  $\text{fld}R = (\text{dom}R \cup \text{ran}R)$  [i.e., the union of the two]” (Enderton 1977, p. 40). Consequently,  $R$  is a relation from A (set of departure) to B (the set of destination) iff:  $R$  is a relation,  $\text{dom}R \subseteq A$ , and  $\text{ran}R \subseteq B$ . In other words,  $R$  maps the image set of the

<sup>32</sup> NK, p. 920. *sambandham*—[ka] *sambandhibhinnatve sati sambandhyāsritaḥ* | [kha] [...] *yathā ghaṭabhūtalayoḥ saṃyogaḥ | yathā vā pratyakṣasthale indryārthasaṃnikarṣaḥ* |. Regarding the six kinds of “intercourse” (*saṃnikarṣa* or *sannikarṣa*; *ṣaṭsaṃnikarṣa*) in perception, cf. Sihna (1934, pp. 75–85): “Perception depends upon some sort of intercourse (*sannikarṣa*) or dynamic communion between its object and a particular sense-organ”. Contact or *saṃyoga* is the first *sannikarṣa*, given the case of a substance (*dravya*; say, a pot) in union with the visual organ. See also Shaw (1989, p. 383; 2010, p. 626).

<sup>33</sup> Bourbaki II.2.1-2 (1968, pp. 72–74) on “The axiom of ordered pair”. Levy (2002, p. 24): “A very useful notion of set theory is the notion of a *pairing function*. A pairing function is a function  $\langle x, y \rangle$  (with the arguments  $x$  and  $y$ ) such that: [...] 6.1 Definition (Wiener 1914, Kuratowski 1921)  $\langle x, y \rangle = \{\{x\}, \{x, y\}\}$ . A set  $z$  is said to be an ordered pair if for some  $x$  and  $y$ ,  $z = \langle x, y \rangle$ . 6.2 Proposition  $\langle x, y \rangle = \langle u, v \rangle \leftrightarrow x = u \wedge y = v$ ”. Cf. also: Levy (2002, p. 25): “A class  $S$  is said to be a (binary) relation if every member  $x$  of  $S$  is an ordered pair. We shall write  $y S z$  for  $\langle y, z \rangle \in S$ . For example, if  $<$  is the natural order relation on the natural numbers (i.e.,  $\langle x, y \rangle \in <$  if and only if  $x$  is less than  $y$ ), then we write  $x < y$  for  $\langle x, y \rangle \in <$ . [...] *Historical Remark*. This way of representing relations is essentially due to Hausdorff 1914 who represented ordered relations in a way similar to this one”.

<sup>34</sup> Cf. also Halmos (1960, pp. 26–27).

<sup>35</sup> Enderton (1977, p. 40); Smullyan (1996, p. 23). For a definition of class, cf. Russell (1919, p. 193). For an intuitive distinction between sets and classes, cf. Halmos (1960, pp. 1, 11): “A pack of wolves, a bunch of grapes, or a flock of pigeons are all examples of sets of things. [...] We shall sometimes say *collection* instead of *set*. [...] In some approaches to set theory ‘class’ has a special technical meaning. [...] Some sets are not really sets and even their names must never be mentioned. Some approaches to set theory try to soften the blow by making systematic use of such illegal sets but just not calling them sets; the customary word is ‘class’. [...] Roughly speaking, a class may be identified with a condition (sentence), or, rather, with the ‘extension’ of a condition”; and Enderton (1977, p. 6): “Any collection of sets will be a *class*. Some collection of sets [...] will be sets. But some collections of sets (such as the collections of all sets not members of themselves) will be too large to allow as sets. These oversize collections will be called *proper classes*”. Or, more formally, Moschovakis (2006, p. 27): “For every unary, definite condition  $P$  there exists a class  $A = \{x \mid P(x)\}$  (3.7), such that for every object  $x$ ,  $x \in A \leftrightarrow P(x)$  (3.8). [...] Every set will be a class, but because of the Russell Paradox [cf. 3.5], there must be classes which are not sets, else (3.8) leads immediately to the Russell Paradox in case  $P(x) \leftrightarrow \text{Set}(x) \ \& \ x \notin x$ . [...] By definition, a class is either a set or a unary definite condition which is not coextensive with a set”. Cf. also Levy (2004, pp. 7–11) and Russell (1919, pp. 42–51).

domain in  $A$  into  $B$  ( $R: A \rightarrow B$ ), since the image set of the domain is equal to or a subset of the set of destination.<sup>36</sup>

Now, what could possibly be meant by the *qualifier-qualified relation*? “A qualifier (*viśeṣaṇa*) [is known as such because it is] in possession of the property qualifier-ness (*viśeṣanatā*). [...] In the case of [a cognition such as] ‘A blue pot’, etc., the property qualifier-ness [finds his limitor] in the property blue-ness. [...] The limitor (*avacchedaka*) of the qualifier-ness in the qualifier is the qualifier itself. Accordingly, in the example ‘A man with a staff’, the property staff-hood [operates] as the limitor of [this] qualifier-ness” (NK, pp. 788–789).<sup>37</sup> In parallel, “it is said qualified (*viśiṣṭa*) a qualificandum (*viśeṣya*) possessing a qualifier (*viśeṣaṇa*). Therefore, a substance (*dravya*) [e.g., a pot] possessing a quality (*guṇa*) [e.g., blueness] is a substance qualified (*viśiṣṭa*) by that quality” (NK, p. 779).<sup>38</sup> Linking the previous two notions, it could be stated that “a qualified-qualifier cognition (*viśiṣṭa-viśeṣaṇaka-jñāna*) has as its content (*viśaya*) a property (*vaiśiṣṭya*) [occurring] in a subject (*dharmīn*); [in particular, it is a cognition] of a qualificand in possession of a qualificans. So, [e.g.] it is the cognition [concerning] ‘A man with a staff’. [...] [In the same way], it becomes evident that the qualifier [i.e. the staff] of a certain qualified [e.g. the man] is [in turn] qualified by another qualifier (*viśeṣaṇāntara*) [i.e. the staff-hood]. In such a cognition, by virtue of the property qualifier-ness (*viśeṣanatā*), the staff appears as the qualifier on the man’s side, and the property staff-hood as the qualifier of the staff. In such a cognition, on the man’s side

<sup>36</sup> Enderton (1977, p. 40): “For example, let  $\mathbb{R}$  be the set of all real numbers [...] and suppose that  $R \subseteq \mathbb{R} \times \mathbb{R}$ . Then  $R$  is a subset of the coordinate plane. The projection of  $R$  onto the horizontal plane axis is  $\text{dom}(R)$ , and the projection onto the vertical axis is  $\text{ran}(R)$ ”. Smullyan (1996, p. 23): “By the domain,  $\text{dom}(R)$ , of a relation  $R$  is meant the class of all  $x$  such that  $\langle x, y \rangle \in R$  for at least one  $y$ . By the range of  $R$ ,  $\text{ran}(R)$ , is meant the class of all  $y$  such that  $\langle x, y \rangle \in R$  for at least one  $x$ . [...] We note that  $R \subseteq (\text{dom}(R) \times \text{ran}(R))$ . We say that a relation  $R$  is on a class  $A$  if  $\text{dom}(R)$  and  $\text{ran}(R)$  are both subclasses of  $A$ . (This is equivalent to saying that  $R$  is a subclass of the Cartesian product  $A \times A$ )”. Cf. also Levy (2002, p. 26). Cf. also: Halmos (1960, p. 27): “If  $R$  is the relation of marriage, so that  $xRy$  means that  $x$  is a man,  $y$  is a woman, and  $x$  and  $y$  are married to one another, then  $\text{dom}(R)$  is the set of married man and  $\text{ran}(R)$  is the set of married women”. In referring to relations here, I use a lexicon commonly proper only to functions (mapping, image, etc.) on the account which defines a binary relation as a *multi-valued function*: “This term [*multi-valued function*] is generic; it indicates that we are not solely concerned with ‘single-valued’ functions. In fact, convention forces us to use different terms, following the preoccupations of different authors: we speak of a *multi-valued mapping* whenever we study properties concerned with linearity or continuity; we speak of a *binary relation* whenever we study certain structural properties (order, equivalence, etc.); we speak of an oriented *graph* whenever we study combinatorial properties”; Berge (1963, p. v). Cf. also Berge (1963: ch. II ‘Mapping one set into another’, § 1 ‘Single-valued, semi-single-valued and multi-valued mappings’, 20–22): “Let  $X$  and  $Y$  be two sets. If with each element  $x$  of  $X$  we associate a subset  $\Gamma(x)$  of  $Y$ , we say that the correspondence  $x \rightarrow \Gamma(x)$  is a mapping of  $X$  into  $Y$ ; the set  $\Gamma(x)$  is called the image of  $x$  under the mapping  $\Gamma$ ”.

<sup>37</sup> NK, pp. 788–789: *viśeṣaṇam*—[1] *viśeṣaṇatāvāt* | [...] *nilo ghaṭa ity ādau nilatve viśeṣaṇatā* | [...] *viśeṣaṇatāvachchedakam tu viśeṣaṇe yad viśeṣaṇam tat* | *yathā daṇḍavān puruṣa ity atra daṇḍatvam viśeṣaṇatāvachchedakam iti* |. Regarding self-linking relations (*svatūpasambandha*) and *viśeṣaṇatā*, cf. Matilal (1968, pp. 40–44): “an absence of something [e.g.] is looked upon as the qualifier of the locus [...]. Nyāya calls such relations relations of qualifier-ness. This is a merely stylistic method Nyāya adopts to describe such a ‘supposed’ relation without committing itself to the reality of such a relation as a separate entity over and beyond the data”. Along the same lines, cf. also Matilal (1968, pp. 69–70, 142).

<sup>38</sup> NK, p. 779: *viśiṣṭam*—[1] *viśeṣaṇavad viśeṣyam* | *yathā dravyam guṇavad ity ādau dravyam guṇaviśiṣṭam* |.

(*āṃśa*), [the qualifier is] the staff, [but] on the staff side what appears is the staff-ness, by virtue of the relational abstract qualifier-ness: staff-hood must not be conceived on the man's side indeed, because it [only operates] as the limiter (*avacchedaka*) of qualifier-ness. It must be understood, in this regard, that a distinct (*viśrñkhala*) object of cognition (*upasthit*) is the eliciting factor (*prayojikā*)” (NK, p. 780).<sup>39</sup> Indeed, man-hood qualifying men is completely independent from staff-hood qualifying staffs. Nevertheless, in the context of this particular qualified-qualifier cognition, staff-hood is the limiter of qualifier-ness, occurring in this particular staff qualifying this man.

And again in NK, “the qualifier-qualified relation (*viśeṣaṇa-viśeṣya-bhāva*) is a specific (*viśeṣa*) objectivity (*viśayatā*). Consequently, in the verbal cognition (*śābdabodha*) of [the expression] ‘staff holder’, the relation qualifier-qualified [itself is the very object of cognition, and that conceived] between staff and man. [...] The qualifier-ness and the qualified-ness, both stand (*āpanna*) in a conditioned-conditioner (or restricted-restrictor) relation (*nirūpya-nirūpaka-bhāva*)” (NK, p. 789).<sup>40</sup> Although this last sentence may appear straightforward, it deserves a glossa. On the surface—in an initial broad sense which ignores the word-order asymmetry in the text—this could generically refer to the relata mutual dependence within the given relation: which is certainly true, but not very informative. The latter definition (*nirūpya-nirūpaka*) should thus be taken as a mere rephrasing of the former (*viśeṣaṇa-viśeṣya*): the qualifier (*viśeṣaṇa*) is the conditioner or restrictor (*nirūpaka*) and the qualified (*viśeṣya*) is what is conditioned or restricted (*nirūpya*). Taking more seriously the inversion of the word-order symmetry in NK text (*nirūpya-nirūpaka* vs. *viśeṣaṇa-viśeṣya*), however, the extended copulative structure (*ca*) and abstracting forms (*-tva*), there is also a potential second sense: both qualifier-ness and qualified-ness could equally and complementary acquire the status of conditioner or conditioned. The first case has already been discussed: the qualifier is the conditioner and the qualified is the conditioned. The second appears much more striking, however: the qualifier would be the conditioned and the qualified the conditioner.

<sup>39</sup> NK, p. 780: *viśiṣṭa-viśeṣaṇaka-jñānam*—[1] *viśeṣaṇavadviśeṣyasya dharmiṇi vaiśiṣṭyaviśayakam jñānam | yathā daṇḍavān puruṣaḥ iti jñānam | [...]* [2] *kvacit viśeṣye yad viśeṣaṇam tatrāpi viśeṣaṇāntaram iti rītyā jāyamānam jñānam bhavati | yathā daṇḍavān puruṣaḥ iti jñānam | atra jñāne puruṣāṃse daṇḍaḥ daṇḍe ca daṇḍatvaṃ viśeṣaṇatayā bhāsate na tu daṇḍatvaṃ puruṣāṃse viśeṣaṇatāvachchedakatayā bhāsate | atra viśrñkhalopasthitih prayojikā iti vijñeyam |* Cf. NK, p. 784. *viśrñkhalatvam—pārthakyam*; ‘severalty’. Cf. also NK, p. 175. *upasthitih*—[1] *budhivadasyārthonusamdehayaḥ | [2] smṛtiḥ [...]*; ‘The object of the cognitive discourse. Recollection’.

<sup>40</sup> NK, p. 789: *viśeṣaṇa-viśeṣya-bhāvaḥ*—[...] [2] *viśayatāviśeṣaḥ | yathā daṇḍi puruṣaḥ iti śābdabodhe daṇḍapurūṣayor viśeṣaṇaviśeṣyabhāvaḥ | atra vighrahaḥ viśeṣaṇam ca viśeṣyam ca viśeṣaṇaviśeṣye | tayor bhāvaḥ iti* (‘The analysis [of the compound] qualifier-qualified [here reads]: qualifier and qualified [i.e., it is a *dvandva*, a copulative compound]’. [This is] their relation’) | *nirūpyanirūpakabhāvāpannam viśeṣaṇatvaṃ viśeṣyatvaṃ cety arthaḥ |* Cf. Gadādhara (1990: II, 78): “XIV. *viśeṣyatāprakāratāvat saṃsargatvasyāpi viśayatāviśeṣātmakatvāt, saṃsargasyāpi viśiṣṭadhīviśayatvam ||* Translation: Like qualificandumness [*viśayatā*] and qualifierness [*prakāratā*] the qualificationness [*saṃsargatva*; i.e. the property of being a relation of qualification], too, is a special kind of objectivity; hence, the qualification [*saṃsarga*], too, has the objectivity of qualified cognition. Explanation: The object of qualified cognition is a relational complex having three elements—a qualificandum, a qualifier and a relation between them. Now, since the entire relational complex is what is cognized, and, according to Nyāya, the relational complex is not an ontological entity over and above the three elements, all three of the elements have to be accorded a different type of objectivity”. Square brackets are mine. Cf. also, NK, p. 935. *saṃsargaḥ*—[1] [*ka sambandhaḥ*] | Regarding the notion of ‘restriction’ cf. Anrò (forthcoming: § 3.2).

Thereby, in the context of a qualifier-qualified relation, the qualifier could be conceived as what is conditioned, thereby becoming a *conditioned qualifier*; and the qualified as the conditioner or restrictor, acting as a *qualified conditioner* (or *conditioning qualified*)—paradoxical though it may sound (cf., end of § 4.).<sup>41</sup>

## Relations in NL & the Colocated Qualification Principle (SVN)

We can now return to the case of the golden crown. Following the NL formalisation method, let crown-hood (*mukūṭatva*,  $m_t$ ) be the root-property (*mukhya-viśeṣaṇa*; cf. *supra*), for  $|m_t| = M$  and  $m \in M$ ; and gold-ness (*hāṭakatva*,  $h_t$ ) a second-level colocated property, for  $|h_t| = H$  and  $h \in H$ . Furthermore, let  $\mathbf{N}$  (italic bold capital) be the relational abstract ‘coreferentiality’ (*sāmānādhikaraṇyatā*) referring to the binary relation  $\mathbf{N}$  (italic capital; *sāmānādhikaraṇya*, ‘coreference’ or ‘syntactic homogeneity’).<sup>42</sup> In parallel, be  $\mathbf{V}$  (*viśeṣya-viśeṣaṇa-saṃsargatā*, or *viśeṣaṇatā*) the relational abstract of relation  $\mathbf{V}$  (*viśeṣya-viśeṣaṇa-bhāva-sambandha*), the relation qualifier-qualified as *viśiṣṭa-jñāna* (cf. *supra*). Let ‘ $\top$ ’ (top left corner) be the *avacchedaka* operator, so that ‘ $b + \text{top left corner} + \text{relational abstract}$ ’ (i.e.,  $b \top \mathbf{R}$ ) would mean ‘ $b$  operates as the *avacchedaka* of the relational abstract  $\mathbf{R}$ ’ (for  $\langle a, b \rangle \in R$ ). In parallel, be ‘ $\perp$ ’ (bottom right corner) the *nirūpaka* operator, so that ‘relational abstract + bottom right corner +  $a$ ’ (i.e.,  $\mathbf{R} \perp a$ ) would mean ‘ $a$  is the *nirūpaka* of  $\mathbf{R}$ ’. A basic relation would thus appear in NL as:  $b \top \mathbf{R} \perp a$ , ‘The relation  $R$  is conditioned by  $a$  (the relational adjunct, or *pratyogin*) and limited by  $b$  (the relational subjunct, or *anuyogin*)’. We are now in a condition to analyse the assertion ‘*mukūṭam hāṭakam*’ (‘A golden crown’)<sup>43</sup> in NL as:

<sup>41</sup> Cf. NK, p. 432: *nirūpyatvam—nirūpitatvam* |. NK, p. 432. *nirūpitatvam—svarūpasambandhaviśeṣaḥ | yathā rājñāḥ puruṣa ity adau puruṣaṇiṣṭhasvatve rājaniṣṭhasvāmītvānirūpitatvam | siṣṭam tu nirūpakatvaśabde draṣṭavyam* |. ‘The property of *being conditioned* (or *restricted*) is a peculiar self-linking relation. In sentences such as ‘The servant of the king’, the property of *being conditioned* by the ownership (*svāmītvā*) occurring in the king [must be sought] in the possess-ness (*svatva*) occurring in the man. What remains must be seen *sub voce nirūpakatva* (*being a conditioner*)’. In this example, the qualifier (the ownership occurring in the king) is also the conditioner, precisely because the property of *being conditioned* (i.e. the qualified-ness) of the qualified (the possess-ness occurring in the man) is in question here. However, the relation could easily be reversed: the qualifier-ness occurring in the qualifier (the ownership occurring in the king) could be conditioned (*nirūpita*) by the qualified-ness occurring in the qualified (the possess-ness occurring in the man), which consequently becomes the conditioner. QED.

<sup>42</sup> For a discussion of ordered pairs formalisation (albeit limited to *vṛtti-niyāmaka* relations only), cf. also Staal (1973, p. 152 ff). In his plain notation:  $A(x, y)$ . Cf. also Bhattacharyya (1987, p. 174): “To distinguish this sense of ‘property’ [...], we shall write ‘property (N–N)’”.

<sup>43</sup> For the purposes of this analysis, it is not paramount whether the description is definite or indefinite; let us assume here that it is indefinite and non-generic: ‘A golden crown’, expressed by  $m$  as a primitive term. For these particular examples, cf. Ganeri (2006, pp. 10–11). Cf. also Matilal (1968, § 9.7, 78–79); Ingalls (1951, p. 50): “Navya-Nyāya regularly expresses its universal statements and knowledges not by quantification but by means of abstract properties”; Ganeri (2008, pp. 110, 118): “The Nyāya authors themselves do not [...] show much interest in the problems of scope ambiguity [...]. And often the language is used in only a semiformal way, especially when used by non-Nyāya authors”. Russell (1919, pp. 167–180): “An indefinite description is a phrase of the form ‘a so-and-so’, and a definite description is a phrase of the form ‘the so-and-so’ (in the singular)”. For a complete introductory survey of generic (or definite) and generic (or non-generic) descriptions, cf. Ludlow (2018).

[2]  $h. N \ulcorner m$

*yā sāmānādhikaraṇyatā hāṭaka-niṣṭhā sā mukuṭa-nirūpitā*; ‘The relational abstract of coreferentiality or syntactic homogeneity, conditioned (*nirūpita*) by a crown (*m*), occurs (*niṣṭha*) in an instance of gold (*h*; viz., it refers to this gold as its *locus*); iff  $h \in (|h_t| = H)$  (‘Being an instance of the property *gold-ness*, a specimen of gold belongs to the set What is golden, that is, the set Gold’),  $m \in (|m_t| = M)$  (‘Being an instance of the property *crown-ness*, a crown belongs to the set Crowns’),  $(h \in |N \ulcorner m|)$  (‘A specimen of gold belongs to the set What is coreferential with a crown’), that is,  $\langle m, h \rangle \in N$  (‘A crown and an instance of gold are an ordered couple belonging to the relation *x* is coreferential/syntactically homogeneous to *y*’). In standard notation:  $(\exists x) (Hx \wedge Mx)$  (‘There do exist an *x* which is gold *and* a crown’), for  $H \cap M \neq \emptyset$  (‘The intersection of the set Gold and the set Crown is not empty’).

Be noted here the *niṣṭha* operator (‘.’; a dot instead of ‘ $\ulcorner$ ’), connecting a property with a primitive term conceived as its *locus*.<sup>44</sup> The relation [2] can then be further specified, for TvN, as:

[2<sub>a</sub>]  $(h \cdot h_t) \ulcorner N \ulcorner (m \cdot m_t)$

*yā sāmānādhikaraṇyatā hāṭaka-niṣṭha-hāṭakatvāvacchinnā sā mukuṭa-niṣṭha-mukuṭatva-nirūpitā*; ‘The relational abstract of coreferentiality, conditioned by the property *crown-hood* referring to a crown, is limited (*avacchinna*)<sup>45</sup> by the property *gold-ness* occurring in an instance of gold’—the purport (*tātparya*,

<sup>44</sup> Regarding the *niṣṭha* operator, cf. Anrò (forthcoming). A primitive term is always on the operator’s left side, while a property is always on its right. Thereby, for a generic primitive term ‘*a*’ and a generic property ‘*a*<sub>*t*</sub>’, the expression ‘*a. a*<sub>*t*</sub>’ will mean ‘*a*-hood occurring in *a*’.

<sup>45</sup> Ganeri (2008, pp. 109, 115): “So a conditioner maps to an existential quantifier, whose domain is restricted to the class assigned to the conditioner, and which binds the second place of a dyadic predicate. Similarly, a delimiter maps to a universal quantifier, whose domain is restricted to the class assigned to it, and which binds the first place of a dyadic predicate. [...] The universal quantifier corresponding to the limiter always has wider scope than the existential quantifier corresponding to the conditioner”. Cf. also Ingalls (1951, p. 48): “The relational abstracts [...] are limited by the qualifiers of the entities in which they reside. Technically these abstract are said to be limited through a relation of residency (*niṣṭhatva-sambandhāvacchinna*; cf. Śiv. Miśra, 22.8)”. Ingalls (1951, p. 49): “No one method can be followed for reducing expressions employing ‘limited’ to the terms of Western logic”.



henceforward <sup>(t)</sup> of which is <sup>(t)</sup>‘Gold-ness in a specimen of gold occurring in a crown qua instance of crown-hood’.

The relation [2<sub>a</sub>] can now be interpreted and rephrased in terms of the qualifier-qualified relation ( $V$ ). The crown is ( $N$ ) gold because it is qualified ( $V$ ) by gold:

$$[3] (h . h_i) \neg V^{(N)} \neg (m . m_i)$$

*yā viśeṣaṇatā hāṭaka-niṣṭha-hāṭakatvāvachinnā sā mukuṭa-niṣṭha-mukuṭatva-nirūpitā*; ‘The relational abstract *qualifier-ness*, conditioned by the property *crown-hood*, referring to a crown, is limited by the property *gold-ness* occurring in an instance of gold’. Iff  $h \in (|h_i|=H)$ ;  $m \in (|m_i|=M)$ ;  $h \in |V^{(N)} \neg m|$  (‘A specimen of gold belongs to the set [Coreferential] Qualifiers of a crown’).<sup>46</sup> Note here  $V^{(N)}$ , that is, ‘ $N$  interpreted as  $V$ , *salva veritate*’.

The root property crown-hood (*mukuṭatva*,  $m_i$ )—adjunct of the relational abstract  $V^{(N)}$  ( $N$  as  $V$ )—binds the dominion of the relation to  $M$  (the set Crowns), thereby effectively ruling out the complement set of  $M$  (i.e.  $\bar{M}$ , the set Everything which is not a crown). Consequently, if coordination [a] is true for  $M \cap H \neq \emptyset$ , relation [3] from the set of departure  $M$  *de facto* excludes possibilities [b] and [c]. In other words, it obliterates  $\bar{M}$ —consequently,  $H \cap \bar{M}$  as well, viz. ‘Everything which is gold but not a crown’—and it is true for  $H^{\text{sub}[3]} \subseteq V^{(N)}[M]$  (i.e., given [3], we are dealing only with gold coreferential to crowns; for  $\text{dom } V^{(N)} \subseteq M$  and  $\text{ran } V^{(N)} \subseteq V^{(N)}[M]$ ). Stemming from the fact that we are talking about the properties of a crown, *gold-ness* ends up being *gold-ness in crowns* and thereby included in the set Coreferential properties of crown-ness ( $V^{(N)}[M]$ ).<sup>47</sup>

<sup>46</sup> Obviously, different formulations would have been possible. For instance, in terms of ‘specification’ (or distinguishing property): *vaiśiṣṭya*, whose relational abstract will be *vaiśiṣṭyatva*. Or in terms of ‘qualified-ness’ (*viśeṣyatā*), conceived as the converse of the relation qualifier-ness (*viśeṣaṇatā*): thereby,  $V^{-1}(\text{viśeṣaṇatā}^{-1}) = \text{viśeṣyatā}$ . The formula [3] would, in that case, be reversed accordingly: [3<sub>b</sub>]  $(m . m_i) \neg V^{(N)} \neg (h . h_i)$ ; *yā mukuṭa-niṣṭha-mukuṭatvāvachinnā sā hāṭaka-niṣṭha-hāṭakatva-nirūpita-viśeṣyatā*, ‘Qualified-ness, with respect to gold-ness in an instance of gold, is referred to crown-ness occurring in a crown’. Cf. NK, p. 812; *vaiśiṣṭyaṃ*—[ka] *sambandhaḥ | yathā bhūtaḥ ghaṭaviśiṣṭaḥ ity ādau ghaṭabhūtalayoḥ saṃyoganāmā sambandho vaiśiṣṭyaḥ* |; ‘Specification is a relation, according to which, in [expressions] such as ‘The ground qualified by a pot’, the relation named as ‘contact’, between pot and ground, is implied”. Cf. also: Staal (1988, p. 62). Regarding the converse: Schmidt (2011, p. 39): “Definition 4.2. Given a relation  $R: X \rightarrow Y$ , its converse (or transpose)  $R^T: Y \rightarrow X$  is that relation in the opposite direction in which for all  $x, y$  containment  $\langle y, x \rangle R^T$  holds precisely when  $\langle x, y \rangle R$ ”. Regarding the transpose, inverse, or converse, see, among others: Bourbaki II.3.2 (1968, p. 78); Jech (2006, p. 11); Berg (1997, p. 24); Enderton (1977, pp. 44–46); Russell (1919, pp. 16, 32, 42–49). In relation to converse in NL, cf. Anró (forthcoming).

<sup>47</sup> Cf. Ingalls (1951, p. 50): *nirūpitatva-sambandhenāvachinna*, an abstract “limited through a relation of described-ness”. Since the specific case of the qualified-qualifier binary relation ( $V^{(N)}$ ) has as a rule of assignment ‘The crown ( $m$ ) is qualified by gold ( $h$ )’ or  $\langle m, h \rangle V^{(N)}$ , it is worth underlining the inversion occurring in [2] and [3] between the two relata: the abstract property  $V^{(N)}$ , or the property to qualify the crown, is in gold not in crown. Consequently, it is the crown that is qualified.

Since it concerns a pair of coreferential (*samānādhikaraṇa*) locatees occurring in the very same locus, relation [3] is describable by what I will hereafter call *Samānādhikaraṇa-Viśiṣṭatva-Nyāya* (SVN, ‘Principle of Coreferential Qualification’). In case of coreference, SVN, following a strictly relational logic, can bind all further properties to a chief or root one (*mukhya-viśeṣaṇa*). According to SVN, the qualifier (*viśeṣaṇa*) corresponds—under the condition of relation  $V^{(N)}$ —to the image of the qualificandum (*viśeṣya*); this is in turn already qualified (i.e. it is a crown and not a bucket) and alone defines, as the root-property, the relational dominion. Thereby, *gold-ness*<sup>sub[3]</sup> ends up being a subset of properties of crowns because, having considered the *viśeṣya* primarily as a crown, no further cognition can avoid this basic qualification any longer. The qualificans *gold-ness*, occurring in the qualifier and referring to a crown, corresponds to the image of crown-hood under relation  $V^{(N)}$ , which consequently has as its elements the instances of gold-ness solely in crowns because it is conditioned by crown-ness ( $h_i \neg V^{(N)}$ , *hātakatvāvacchinnaviśeṣanatā*, ‘Gold-ness as qualifier’—as a consequence, we are not primarily talking about gold, which is *only* what qualifies something else;  $V^{(N)} \perp m_i$ , *mukūṭatvanirūpita-viśeṣanatā*, ‘Crown-hood as qualified’, that is, what we are talking about). It goes without saying that SVN applies only in coreference cases (i.e.  $N$  as  $V$ ). If a blue pot is a pot ( $N$  as  $V$ ), a man with a stick is not a stick ( $V$  only)—even though the man is qualified by his stick.

A relation can be grasped more effectively if topologically displayed in a Cartesian coordinate system. Ordered pairs on the plane make pictorially evident the fact that the first and foremost concern of Nyāya account is relations. To provide a first example, be given a general relation different from  $V$ . Let  $L$  be the relation ‘locus of’ and  $L$  its relational abstract ‘locus-hood’ (*āśrayatā*). An instance of smoke ( $d$ , *dhūma*) on a mountain ( $p$ , *parvata*) could be thus expressed in NL as:  $p. L \perp d$ , *yā āśrayatā parvata-niṣṭhā sā dhūma-nirūpitā*, true for  $p \in |L \perp d|$ , viz. ‘A mountain belongs to the set Loci of an instance of smoke’. Because the relation is  $\langle p, d \rangle \in L$  (or ‘ $p$  is the locus of  $d$ ’), it follows that on the Cartesian plane  $L$  identifies the ordered pair ‘smoke’ (in abscissa) and ‘mountain’ (in ordinate). This latter is a member of the set ‘Loci of a smoke’ along with e.g. ‘a portion of space’, ‘a fire’, etc. Mountain and smoke are obviously distinct objects, with different qualifying properties (for TvN) and different reference sets. Nevertheless, bound by the relation ‘locus of’ under the condition ‘smoke’, this mountain ends up belonging to the set ‘Loci of a certain smoke’. This implies that the main element of interest is neither the mountain nor the smoke. As topologically made evident in the Cartesian plane, what is at stake here is the property locus-hood with respect to smoke; a property occurring in this mountain along with others that are completely different in nature (e.g. ‘a fire’). Clearly, SVN cannot apply.

Let us now focus on the specific relation  $N$  as  $V$ . So, let be in abscissa the set ‘Triangles’ (T) and in ordinates the set ‘Coreferential properties of triangles’ ( $V^{(N)}[T]$ ). This latter includes all the properties referable to triangles, such as ‘having

the sum of internal angles equal to 180°' ( $p_1$ ), 'possessing a right angle' ( $p_2$ ), 'possessing equal sides' ( $p_3$ ), etc. (i.e.,  $p_1, \dots, p_n$ ). If  $p_1$  is a property possessed by all instances of triangles,  $p_3$  (itself a subset of the set in ordinates) it will on the contrary be referable only to a subset of T in abscissa: by definition, referable only to equilateral triangles. Thereby the relation  $\langle t, p_3 \rangle \in V^{(N)}$  (' $t$  is qualified by  $p_3$ ', for  $V^{(N)} \subseteq T \ V^{(N)}[T]$ ) will define the portion of the plane identifying equilateral triangles. The dominion of the relation plainly claims that only triangles are under discussion here: an equilateral triangle—qualified via  $\langle t, p_3 \rangle \in V^{(N)}$ —is but a triangle, for:  $(\text{dom } V^{(N)} \subseteq T) \wedge (|p_3^{\text{sub-dom } V^{(N)}}| \subseteq V^{(N)}[T])$ .<sup>48</sup> However,  $N$  as  $V$  by definition imposes that  $V^{(N)}[T]$  refer to T; consequently, both  $\text{dom } V^{(N)}$  and  $\text{ran } V^{(N)}$  are equal to or a subset of T, for  $V^{(N)}: T \mapsto V^{(N)}[T]$  and  $V^{(N)}[T] \subseteq T$ . In general, "for a relation  $R$ , a class  $A$  is said to be  $R$ -closed, or closed under  $R$ , if whenever  $x \in A$  and  $xRy$  then also  $y \in A$  (i.e.,  $R[A] \subseteq A$ )" (Levy 1979, p. 61). Therefore, the relation  $N$  as  $V$  under examination is revealed to be an instance of closure: the set Coreferential properties of triangles is T-closed under the relation  $N$  as  $V$ .<sup>49</sup>

The same applies to the case of golden crowns and blue pots. Indeed, the relation is presented as ordered pairs with crowns or pots in abscissa (for M, the set Crowns; and G, the set Pots), and Properties of crowns or Properties of pots in ordinate. It follows that in [3]:  $(h \in (|h_t|=H)) \in |V^{(N)} \cup m|$ , i.e. an instance of the property *gold-ness* belongs to the set What qualifies a crown (or Properties of a crown)—along with many others, such as *heaviness*, *brightness*, etc. The set  $H^{\text{sub}[3]}$  ( $\text{qua } H^{\text{sub}[3]} \subseteq V^{(N)}[M]$ ) is thus M-closed under the formula [3], for  $V^{(N)}[M] \subseteq M$ . The relation  $N$  as  $V$  is in fact a mapping of M (for  $\text{dom } V^{(N)} \subseteq M$ , the set Crowns as set of departure) onto the set Properties of crowns (for  $\text{ran } V^{(N)} \subseteq V^{(N)}[M]$ , the set of destination); that is,  $V^{(N)}: M \mapsto V^{(N)}[M]$ . In other words, the relation  $N$  as  $V$  defines the image of Crowns in Properties of crowns through the medium of a particular property, here *gold-ness*; for this reason, the property *gold-ness*<sup>sub[3]</sup> is but a sub-set of Properties of crowns. Clearly, the properties involved—*gold-ness* and *crown-ness*—are reciprocally unrelated (*viśṛīkhalā*) (cf. fn. 39) because the former is certainly not a subset of the latter; at most, the intersection of their two domains might be non-empty. However, here *hāṭakatva* plays the role of coreferential *viśeṣaṇa* (qualifier) of a particular *viśeṣya* (qualified), in turn qualified by the property *mukūṭatva*—and this root-qualification cannot simply be dismissed. A golden crown is a crown because the *viśeṣya* itself (the crown) in relation  $N$  as  $V$  is already qualified by *crown-hood*:

<sup>48</sup> For  $|p_3^{\text{sub-dom } V^{(N)}}| =$  'The extension of the property *being equilateral* under the condition imposed by the relational domain of  $V^{(N)}$  relation (in this case, Triangles)'. It goes without saying that all squares, equilateral pentagons, etc., are immediately ruled out by definition.

<sup>49</sup> Cf. Smullyan (1996, p. 132): "The Henkin closure condition. Given a subset B of A and a formula  $\varphi$  whose constants are all in B, we shall say that B is *A-closed with respect to  $\varphi$*  if for every subformula of  $\varphi$  the form  $(\exists x)(x, y_1, \dots, y_n)$  (where the free variables of  $\varphi$  are  $x, y_1, \dots, y_n$ ) and for all element of  $b_1, \dots, b_n$  of B, if there is some element  $a$  of A such that the sentence  $(a, b_1, \dots, b_n)$  is true over A, then there is some  $b$  in B such that  $\psi(b_1, \dots, b_n)$  is true over A". Cf. also Bourbaki III.1 ex. 13 (1968, pp. 216–217); Enderton (1977, p. 78); Berge (1997, p. 12); Schmidt (2010, pp. 169–170). It goes without saying that the first example—regarding the *loci* of smoke ( $|L \cup d_i| = L[D]$ )—implying the relation  $L$  is not a case of closure. Because, if  $L: D \mapsto L[D]$  and therefore a mountain belongs to the set *Loci* of smoke,  $p \in |L \cup d_i|$  or  $p \in L[D]$ , still,  $L[D] \subseteq D$ ; i.e. the *Loci* of smoke are not a subset of Smoke, unlike Blue pots which are clearly a subset of Pots.

SVN in [3] identifies *gold-ness* as a property occurring in a crown—precisely, a golden one—and not the set of all golden things. For the same reason ‘A blue pot is a pot’.

Let us now proceed by adopting a different approach to demonstrate SVN in terms of limiting properties only (*avacchedaka*). It has been shown (cf. fn. 47) that the  $V$ -relational subjunct (*viśeṣaṇatā-saṃsargīya-anuyogin*,  $a$ ) or limitor ( $v \dashv V$ , *viśeṣaṇāvacchinna-viśeṣaṇatā*) is always the  $V$ -qualifier (*viśeṣaṇa*,  $v$ ; thus:  $v = a$ ) because it is what expresses the quality (*viśeṣa*). In the example, *gold-ness* in gold is the  $V$ -limitor. It should be recalled that the relational abstract  $V$  reverses the terms of relation  $V$  (i.e.  $\langle viśeṣya, viśeṣaṇa \rangle \in V$ ; or in short and for  $v^{-1} = viśeṣya$ :  $\langle v^{-1}, v \rangle \in V$ ), making explicit the fact that  $V$  refers to the *viśeṣaṇa* only under the condition of the *viśeṣya*.<sup>50</sup> In the case of a golden crown ( $N$  as  $V$ ), *gold-ness* in gold is the qualifier ( $v$ ) of a crown ( $v^{-1}$ ): *mukūṭa-viśeṣaṇaṃ hāṭaka-niṣṭha-hāṭakatvam*. Thereby, *hāṭakatvāvacchinna-viśeṣaṇatā*, ‘The qualifier-ness ( $V^{(N)}$ ) is limited by gold-ness’ (cf.  $v \dashv V$ ). In general:

[4]  $v \dashv V^{(N)} \dashv v^{-1}$   
*yā samānādhikaraṇa-viśeṣaṇatā viśeṣaṇāvacchinṇā sā viśeṣya-nirūpitā*;  
 ‘Coreferential ( $N$ ) qualifier-hood ( $V^{(N)}$ ), conditioned by the qualified ( $v^{-1}$ ), is limited by the qualifier ( $v$ )’.

As a general scheme, ‘The relational abstract *subjunct-ness* ( $A$ ), limited by the relational subjunct ( $a$ ), is limited by the relational abstract *coreferential qualifier-ness* ( $V^{(N)}$ ) limited by the qualifier ( $v$ ), expressing the ascribed quality (*viśeṣa*; e.g. *gold-ness* in gold)’, that is:

<sup>50</sup> Let us recall that what we are dealing with here is the sentence *hāṭakam mukūṭam*, ‘The crown (subject qualified) is gold (qualifying property, or qualifier)’. What we are talking about are thus crowns, and we are attributing to them a particular quality (e.g. gold-ness). Therefore, the property ‘being a qualifying property’ (or ‘being a qualifier’, viz. ‘qualifier-ness’) obviously lies in gold-ness, and it is precisely here the inversion occurs: because now, shifting from crowns to gold, we are dealing with gold-ness and its qualifying power. This pattern can be found in every instance of qualification: *daṇḍī puruṣaḥ*, ‘Staff holder’; *vahnivān parvataḥ* (and all its permutation: e.g. *savahnir parvataḥ*, etc.), ‘A mountain with fire’; *meghadūtaḥ*, ‘Cloud messenger’; etc. Syntactical permutations often found in common expressions—e.g. *parvate vahniḥ*, ‘Fire on a mountain’—merely confirm the schema: in this case fire is the qualified, while mountain is the qualifying item. Therefore, the property qualifier-ness here refers to mountain. The superscript ( $x^{-1}$ ) points out the transpose of the base ( $x$ ); cf. *supra* fn. 46.

$$[5] v \neg V^{(N)} \neg a \neg A$$

*viśeṣaṇa-avacchinna-samānādhikaraṇa-viśeṣaṇatā-avacchedaka-avacchinna-anuyogy-avacchinna-saṃsargīyānuyogitā*. More straightforwardly: <sup>(9)</sup>The qualifier ( $v$ ) is always the relational subjunct ( $a$ ) in  $V^{(N)}$ ,<sup>51</sup>

Conversely, the relational *pratiyogin* ( $a^{-1}$ ; i.e. the qualified, *viśeṣya*,  $v^{-1}$ ) operates in  $V^{(N)}$  as a dominion conditioner (*nirūpaka*): *mukūṭatva-nirūpitam hāṭaka-niṣṭha-hāṭakatvam*, ‘Gold-ness in gold conditioned (i.e., under the dominion restriction imposed) by crown-ness’. At the same time, the crown is the qualified which is qualified by gold-ness: *hāṭakatvena viśiṣṭam viśeṣyaṃ mukūṭam*. And yet the *viśeṣya* ( $v^{-1}$ )—being that which is qualified, as well as the adjunct (*pratiyogin*) and conditioner (*nirūpaka*) in  $V^{(N)}$  (cf. [4])—is in  $N$  as  $V$  (cf. *supra*:  $\langle v^{-1}, v \rangle V^{(N)}$ ) the limitor (*avacchedaka*) of the attributed property (*viśeṣa*,  $s$ ). Indeed, the property occurs in what is qualified:  $v^{-1} \neg s$ , *viśeṣyāvacchinna-viśeṣaḥ* (‘The quality limited by the qualified’). What does gold-ness refer to? The only viable answer is obviously the crown. Thus:  $m_i \neg h_i$ , *mukūṭatvāvacchinna-hāṭakatvam*. In general, substituting [4] and [5] in  $\langle v^{-1}, v \rangle V^{(N)}$ :

$$[6] \langle (v^{-1} \neg V^{(N)-I} \neg a^{-1} \neg A^{-I}), (v \neg V^{(N)} \neg a \neg A) \rangle \in V^{(N)}$$

*viśeṣaṇa-avacchinna-samānādhikaraṇa-viśeṣaṇatā-avacchedaka-avacchinna-anuyogy-avacchinna-saṃsargīyānuyogitā-viśiṣṭa-viśeṣya-avacchinna-samānādhikaraṇa-viśeṣyatā-avacchedaka-avacchinna-pratiyogy-avacchinna-saṃsargīya-pratiyogitā*; ‘The relational abstract *adjunct-ness* ( $A^{-I}$ ), limited by the relational adjunct ( $a^{-1}$ ), is limited by the relational abstract *coreferential qualified-ness* ( $V^{(N)-I}$ ) limited by what is qualified ( $v^{-1}$ ); this compound is in turn qualified (*viśiṣṭa*; in bold) by relational abstract *subjunct-ness* ( $A$ ), limited by the subjunct ( $a$ ), limited by the relational abstract *coreferential qualifier-ness* ( $V^{(N)}$ ) limited by the qualifier ( $v$ )’.<sup>52</sup>

<sup>51</sup> Formula [5] speaks about a specific kind of *relatum*: that *relatum* which: (1) is a limitor (*anuyogin*) and (2) is involved in a qualified-qualifier relation. In this sense, the generic relational abstract subjunct-ness (*saṃsargīyānuyogitā*,  $A$ ) is limited (*avacchinna*) by qualifier-ness (*viśeṣaṇatā*,  $V$ ), thereby becoming a specific subjunct-ness: the subjunct-ness concerning qualifier-ness. If the formula had been inverted (i.e.  $a \neg A \neg v \neg V$ ), it would have been about the ‘property of being a qualifier’ occurring in an *anuyogin*. That makes perfect sense, but it is not the ‘property of being an *anuyogin*’ occurring in the qualifier, which is instead the case at stake here. See Shaw (1989, p. 383): “Since  $a$  is the first term of the relation  $R$  and  $b$  is the second term,  $a$  has the property of being the first term and  $b$  has the property of being the second term. Hence corresponding to every relation the Nyāya recognises two relational abstract properties such that one of them resides in the first term and the other one resides in the second term”; and also Shaw (2010, p. 627).

<sup>52</sup> It is worth noting that formulas such as [2], [3], or [4] could be called ‘NL relational formulas’ (NL-RF, or simply RF). In these cases, a well formed formula contains a relational abstract in its central position, between the operators *nirūpaka* (‘ $\neg$ ’, on right side; consequently: *avacchinna*, ‘limited’) and *avacchedaka* (‘ $\neg$ ’, on right side): as suggested above, if  $\langle a, b \rangle R$ , then  $b \neg R \neg a$ . Here, the relational abstracts *pratiyogitā* and *anuyogitā* do not have to be—or, better, must not be—expressed because they are already embedded in the formula’s positional order. On the contrary, expressions such as [7] are not relations, but descriptions—which could be called ‘NL *avacchinna-avacchedaka descriptions*’ (NL-AAD, or simply AAD). A well-formed AAD reads no central relation but rather a string of limitors and limited. In these cases, the relational abstracts *pratiyogitā* and *anuyogitā*—referring to the corresponding RF—could be made explicit. For a discussion of well-formed formulas, RF, and AAD, cf. Anrò (forthcoming).

Roughly speaking, if ‘ $x$  is qualified by  $y$ ’ ( $\langle x, y \rangle \in V^{(N)}$ ), what is  $x$ ? The *pratiyogitā* in the *pratiyogin* occurring in the *qualified-ness* in the qualified. And what is  $y$ ? The *anuyogitā* in the *anuyogin* occurring in the *qualifier-ness* in the qualifier. However, it has been shown that the qualifying property ( $\delta$ ) occurs in the qualified ( $v^{-1} \neg \delta$ ) and it goes without saying that the qualifying property is nothing but the qualifier ( $v = \delta$ ); thus, in composing the above partial formulas, we can bring together [4] and [6] in a description such as [7]:

$$[7] v^{-1} \neg V^{(N)-I} \neg a^{-1} \neg A^{-I} \neg v \neg V^{(N)} \neg a \neg A$$

*viśeṣya-avacchinna-samānādhikaraṇa-viśeṣyatā-avacchedaka-avacchinna-pratiyogy-avacchinna-saṃsargīya-pratiyogitā-avacchinna-viśeṣaṇa-avacchinna-samānādhikaraṇa-viśeṣaṇatā-avacchedaka-avacchinna-anuyogy-avacchinna-saṃsargīyānuyogitā*; ‘The relational abstract *subjunct-ness* ( $A$ ), limited by the subjunct ( $a$ ), limited by the relational abstract *coreferential qualifier-ness* ( $V^{(N)}$ ) limited by the qualifier ( $v$ ), whose limitor is the relational abstract *adjunct-ness* ( $A^{-I}$ ), limited by the relational adjunct ( $a^{-1}$ ), limited by the relational abstract *coreferential qualified-ness* ( $V^{(N)-I}$ ) limited by what is qualified ( $v^{-1}$ )’. Roughly speaking, <sup>(i)</sup>‘That which is the *anuyogin* in  $V^{(N)}$  (i.e. the qualifier) occurs in the *pratiyogin* (i.e. the qualified)’.

In light of the above, however, given [3]  $h_i h_t \neg V^{(N)} \neg m_i m_t$  (implying  $m_i = v^{-1}$  and  $h_t = v$ ), then:

$$[7_a] m_i m_t \neg V^{(N)-I} \neg a^{-1} \neg A^{-I} \neg h_t \neg V^{(N)} \neg a \neg A$$

*mukūṭa-niṣṭha-mukūṭatva-avacchinna-samānādhikaraṇa-viśeṣyatā-avacchedaka-avacchinna-pratiyogy-avacchinna-saṃsargīya-pratiyogitā-avacchedaka-avacchinna-hāṭakatva-avacchedaka-avacchinna-samānādhikaraṇa-viśeṣaṇatā-avacchedaka-avacchinna-anuyogy-avacchinna-saṃsargīyānuyogitā*; ‘The relational abstract *subjunct-ness* ( $A$ ), limited by the subjunct ( $a$ ), limited by the relational abstract *coreferential qualifier-ness* ( $V^{(N)}$ ) limited by *gold-ness* ( $h_t$ ), whose limitor is the relational abstract *adjunct-ness* ( $A^{-I}$ ), limited by the relational adjunct ( $a^{-1}$ ), limited by the relational abstract *coreferential qualified-ness* ( $V^{(N)-I}$ ) limited by *crown-hood* ( $m_i$ ) in a crown’.

It is thus confirmed that, if [3], then  $m_i m_t \neg h_t$ , or *mukūṭa-niṣṭha-mukūṭatvāvacchinna-hāṭakatvam* (‘*Gold-ness* in *crown-ness* in a crown’). Indeed, if there is a colocated *viśeṣaṇa*, there must be a *viśeṣya* on which the former is dependent, lest it not be the qualifier it is. Therefore, *gold-ness* is revealed to be a colocated *conditioned qualifier* by virtue of its being conditioned by the domain it qualifies; and *crown-ness* is a *qualified conditioner* (or *conditioning qualified*), imposing the relational reference domain on the colocated qualifier that qualifies it.

SVN can conclude that, in cases of coreferentiality interpreted as a qualified-qualifier relation ( $N$  as  $V$ ), whatever further colocated qualification (*viśeṣa*) be attributed to whatever target of qualification (*viśeṣya*), the former must be considered as already bound to the root-property of the latter, the relation reference domain. In other words, since  $N$  as  $V$  is an instance of closure, its range must be acknowledged

as a subset of the dominion. Golden crowns are crowns because the relational domain is rooted in the set Crowns. Or rather, if there are golden crowns it is because there is gold-ness in crowns. In other terms,  $N$  as  $V$  is a mapping of the domain of the qualified (*viśeṣya*) onto the range of collocated qualifiers (*viśeṣaṇa*) and, in so doing, defining a subset of the range which is in turn equal to or a subset of the domain. Consequently, setting aside predication and connective ‘and’ (‘ $\wedge$ ’), in Nyāya relational account a golden crown is a crown because the set Crowns is the starting and arrival point—a set which stands alone, along with its image under the condition ‘gold-property’ as a subset of itself. In relation  $N$  as  $V$  in [3], when talking about gold-ness we are talking about nothing but crowns. The same holds for blue pots qua pots.

At this juncture, a preliminary account of the notion of coreferentiality has been provided here, relying on the unforeseen and to some extent counterintuitive output of SVN. If that is the case, then it is clear that—being the very same being—a crown and the gold of which it is made cannot actually be said to be *different* tout court, e.g. the way a crown and a chair are. Nonetheless, it still remains unanswered the question regarding the relational nature of non-difference, and in particular whether this latter might be considered, or rather reduced, to simple cases of equivalence, equality, or identity. The second part of this investigation will be devoted to this issue.

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### Compliance with ethical standards

**Conflict of interest** The author states that there is no conflict of interest.

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