MOOCs-UniTo: Theoretical Framework and Research Lines on Teachers and Researchers

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Abstract. Il presente lavoro riporta le idee principali discusse durante la plenaria, tenuta dagli autori del contributo, in occasione della giornata di studio "e-learning e matematica nella formazione universitaria e post-universitaria: da buone pratiche a linee di ricerca". L'intervento si è focalizzato sulle esperienze di formazione online rivolte ad insegnanti di matematica, in seno al progetto Math MOOC UniTo, del Dipartimento di Matematica "G. Peano" dell'Università di Torino. Tre momenti di questa esperienza formativa online sono esaminati: (i) l'elaborazione di un modello di apprendimento a seguito dell'analisi delle varie esperienze MOOC progettate ed erogate in seno al progetto; (ii) il ruolo dei ricercatori nell'esperienza dei MOOC; (iii) come la formazione online ricevuta dagli insegnanti di matematica si riversa in classe sui loro studenti. La parte finale illustra il significato di questa ricerca nel quadro di un'educazione matematica adeguata alle nuove tecnologie e traccia le linee per il proseguimento della ricerca.

1. Introduction

On the occasion of the study day "e-learning and mathematics in university and post-graduate education: from good practices to lines of research", we thought to share our experience, now five years, of designing and delivering MOOCs (Massive Open Online Courses) for mathematics teacher education. Our experience is part of the Math MOOC UniTO project of the Department of Mathematics "G. Peano" of the University of Turin. The project started in 2015, the year in which Eugenia Taranto started her PhD, which had as its object of study and research the MOOCs for mathematics teacher education.

At that time, MOOCs were an emerging phenomenon in Italy. The first MOOCs had spread in 2008 in the USA. The American press defined 2012 as the year of MOOCs for the USA: it was in that year that some of the most popular platforms for the delivery of MOOCs (Udacity, Coursera, edX) were born. The MOOCs arrived in Italy in 2013, the year from which we see the growth and multiplication of platforms (POK, Eduopen, Federica.EU, ...) able both to offer this type of online courses, and especially to manage a large number of students, who are mainly university students. In addition to traditional teaching materials, such as readings and problem sets, many MOOCs provide educational videos and interactive forums to support community interactions between students, professors and tutors (Taranto et al., 2017a). Therefore, "a MOOC can be considered as a digital resource with many other digital resources inside" (Taranto et al. 2018, p. 167).

The Math MOOC UniTo project, details of which will be discussed in the next section, was the first to launch a MOOC for mathematics teacher education, in Italy. Since 2015, five MOOCs have been delivered, one for each year until 2020. In the light of our experience, it is therefore worth considering the following research question: *What theoretical and practical results have the MOOCs-UniTo achieved as online training experiences for mathematics teachers?*

To articulate an answer to this question, we will divide it into three points:

1. We will start by considering the switch from the analysis of the MOOC experience to the elaboration of the learning model (of teachers) in MOOC as an ecosystem through the hybridization of three theoretical frameworks (Meta-Didactical Transposition; Instrumental Approach; Connectivism).
2. We will move on to an analysis of the role of researchers involved in the MOOC experience, illustrating new research perspectives.
3. We will conclude with a cascading look, i.e. from the MOOC student teachers to the students of their classes (but not only), outlining an open question about a possible new picture of the didactical framework.

2. Research context

The Math MOOC UniTo project, started in 2015, is focused on researchers' design and deliver of MOOCs for mathematics teachers, mainly from secondary schools.
For the description of MOOCs from a structural point of view, we use the framework introduced in the ICME survey (Robutti et al., 2016), and we focus on the theme that describes "*Contexts and characteristics of collaborative work between mathematics teachers*". In fact, the teachers involved in these MOOCs collaborate, although they never meet in person, as the MOOC is delivered through a platform. They collaborate depending on their professional needs, role, skills, following the structure of the MOOC and using the technological resources made available. Working together they learn, as they are involved in various types of knowledge (content, pedagogical, technological, and institutional).
The theme is expressed in 4 dimensions which are briefly illustrated below.

1) *The initiation, foci and aims of collaborations*: The aim of Math MOOC UniTo project was to increase teachers' professional competencies and improve their classroom practices. To date, five MOOCs have been designed and delivered. The first four were based on the main topics in the Italian National Guidelines (MIUR, 2010), in the following order: geometry, arithmetic and algebra, change and relations, uncertainty and data. The last had modelling as its central theme. These MOOCs are open, free, and available online for teachers through the DI.FI.MA platform (https://difima.i-learn.unito.it/). Each 10-week MOOC is subdivided into modules lasting one or two weeks.
2) *The scale of collaborations (numbers of teachers and time-line)*: The graph (Figure 1) relates the number of teachers involved in collaborative work to the duration of the collaboration. Our experience of MOOC is in the upper left quadrant: numerous teachers, short periods of work (our MOOCs usually last 10 weeks). The MOOCs are different in each year, beginning in 2015[1], and touching each year a mathematical content from a didactic-methodological perspective. Table 1 gives some numerical data about the teachers' involvement and the number of teachers who completed all the activities in the different years.

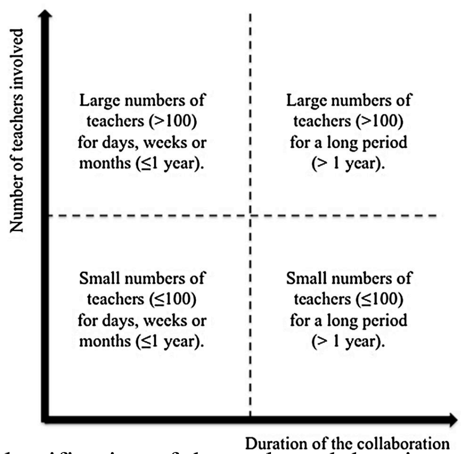

Figure 1: A classification of the scale and duration of collaborations (taken from Robutti et al., 2016, p. 665)

Table 1: Teachers enrolled in MOOCs and completion rate

	# enrolled teachers	completion rate[2]
MOOC Geometria	424	36%
MOOC Numeri	278	42%
MOOC Relazioni e Funzioni	358	39%
MOOC Dati e Previsioni	450	40%
MOOC Modelli	271	38%

[1] MOOC Geometria (on geometry contents, from October 2015 to January 2016); MOOC Numeri (on arithmetic and algebra contents, from November 2016 to February 2017); MOOC Relazioni e Funzioni (on changes and relations concepts, from January 2018 to April 2018); MOOC Dati e Previsioni (on uncertainty and data concepts, from January 2019 to April 2019); and MOOC Modelli (on modelling, from concepts, from January 2020 to April 2020).

[2] In the literature, the completion rate of mathematics MOOC is 8% (http://www.katyjordan.com/MOOCproject.html), of mathematics MOOC for teacher training is 12% (Panero et al., 2017).

3) *The composition of collaborative groups and the roles of the participants*: Within MOOCs for teacher education, two communities can be distinguished, the mathematics teacher educators (MTEs) and the participating MOOC-teachers. Concerning the MTEs, this community is made of the fruitful intersection of different agencies:
   - a research team, made of the three authors;
   - a design team, made of the three components of the research team, plus a group of teacher educators, who are three in-service teachers with a second-level master as teachers' trainers;
   - an implementation team, made of the previous team, plus a group of in-service teachers helping the educators in following the trainees' works in the platform and giving them feedback or help.

   The first team is involved in a research project on teachers' development through MOOCs (inside UniTo); the second team is engaged thanks to the grant of a national project of professional development (Piano Lauree Scientifiche - PLS); the third team is as well in PLS project.
   All of them are involved in the design, delivery and monitoring of the course. All of them have been involved in the creation of the activities delivered in the MOOCs. Some of these activities have been adapted from national/international projects, others designed by MTEs based on classroom experiments. In addition, they all assisted the MOOC participants with tutorials, weekly emails and monitoring actions.
   Concerning the participating MOOC-teachers, each week they are involved in various activities: viewing videos where experts introduce the topics of the module; consulting activities based on a mathematics laboratory (and the option to do teaching experiments with these in their classroom with their students); using the communication message boards (CMBs) to express opinions about the content of the course, exchange experiences with colleagues, and benefit from other participants' ways of thinking.
4) *Collaborative ways of working and their conception*: In all MOOCs there was a collaborative climate and, surprisingly (for the MTEs), some of the participants started voluntarily sharing material they created and used in their lessons. As MTEs we have chosen to limit our interventions in the CMBs to a minimum, in order to support the birth of a MOOC-teachers-only online community. We were more active within the webinars: educational online events for trainees. Each of our MOOCs has two final productions: designing a teaching activity (PW) using specific software and reviewing (PR) a project designed by a colleague. For all those who took part in all MOOC stages a participation certificate was issued by the Mathematics Department of the University of Turin. The collaboration between MOOC-teachers (both small groups and the whole group) was possible through CMBs. They have engaged in discussions on didactics, activities, classroom experiments, and formative assessment (the PR that each trainee has to do). On the other hand, the collaboration among MTEs takes place both during the design and during the monitoring stages of the MOOC. The experience of monitoring the MOOC-teachers' discussion on the CMB and the feedback received through questionnaires - that we administered during the delivery - have been taken into account for the evolutions of MOOC from one edition to another.

In the following we will address the three points made explicit in the introduction.

3. Development of the teacher learning model in MOOC

The emergence and use of MOOCs for teacher education is still uncommon, especially in mathematics. In fact, although there is a wide choice of many different topics, when looking specifically for a MOOC aimed at mathematics teacher education the range is limited (Aldon et al. 2017; Borba et al. 2017). Nevertheless, there is a growing interest in MOOCs involving mathematics teachers as participants (see http://www.icme13.org/files/tsg/TSG_44.pdf). However, the specific intersection of MOOCs and teacher education is poorly researched yet. Our initiative to provide MOOCs for mathematics teacher education has been the first example in Italy. Our MOOCs have been designed for teacher education and are studied as objects of research. The language in which the courses were delivered is Italian, so they have been followed only by Italian teachers (and a few people abroad).
In mathematics education there are some theories on how teachers can develop their professional learning in face-to-face environments and on the relationship between training and technology. A framework used precisely to analyze professional development programs in face-to-face is the Meta-Didactical Transposition, MDT, (Arzarello et al., 2014). This framework makes use of the notion of praxeology, in the frame of didactical transposition Chevallard (1999), which describes the transformation of knowledge from university to

school level. In the school context, the didactical transposition examines the praxeologies that teachers use to transpose knowledge. In the context of teachers' professional development, the meta-didactical transposition considers the praxeologies of teachers working and learning in communities with colleagues and under the guidance of researchers. Since these praxeologies refer to teachers in their professional learning, they are called "meta-didactical". The meta-didactical level means that there is a process of professional learning in teachers and at the same time reflection on the teaching praxeologies to be used in the classroom (Robutti, 2018). At both levels (didactical and meta-didactical) a praxeology is composed of the following components: task, technique, technology and theory. The components can be considered internal or external to a community (or to an individual): internal if used by members of a community (or by an individual), external if not used. The goal of a teachers' professional learning programme is to transform praxeological components that are initially external to the teachers' community into internal ones (e.g., tasks and techniques around a use of technology for learning, theoretical results from research on teaching, …). If one or more components of a praxeology shift from external to internal, then the community of teachers can evolve towards a sharing of this component among the community members withal the researchers (considered in the role of teacher educators).

The authors of the paper, as researchers involved in the Math MOOC UniTo project, had a long period of experience with face-to-face courses for teacher training and developed suitable lenses to analyze them (such as MDT). So it was very natural to ask ourselves what lenses to consider when analysing teacher education experiences in these new types of courses totally online. The MDT, which allows face-to-face course analysis, gives a frame that can be used in MOOC analysis, but integrated with other elements. With a mathematical metaphor, we can say that:

face-to-face teacher education : MDT = MOOC for teacher education : X

The investigation carried out in Taranto's PhD thesis (Taranto, 2018) was oriented to elaborate an integration of MDT: the result has been the new frame, called *MOOC-MDT* (Taranto & Arzarello, 2019).

This theoretical framework emerged through the use of strategies to connect theories. Specifically, MOOC-MDT arises from a process of hybridization (Arzarello, 2016). Briefly, this process consists of considering a particular component of a theory. This is "implanted" in another theory that, for this reason, will be hybridized: the old theoretical framework is so enriched and the language as well. Let us describe the MOOC-MDT below.

We could say that a MOOC is another small world in which the protagonists live virtually. And this world nourishes, influences, "expands" those who live in it. In return, those who live in the MOOC also feed, influence and expand it; and so on. This relationship, involving subjects and objects, recalls a process known in mathematics education, the *instrumental genesis* (Verillon & Rabardel, 1995), which describes the passage from an artifact into an instrument, thanks to the presence of a subject. We can therefore formalize this relationship by using this theoretical fragment, operating a first hybridization on the MDT.

We can say that a MOOC can be understood as an artifact, and we will indicate it as *MOOC-artifact*, when it is inert, i.e. when it is just an online platform where educators have uploaded mathematical activities and technological resources. Once the MOOC is open to participants, they start to explore it, select some resources, interact with others. It becomes an instrument for each of them, i.e. a *MOOC-instrument*. Individuals are not only influenced by the MOOC-artifact, but also by how many other individuals are interacting with it at the same time. The goal is to describe a process that is certainly bidirectional, but also iterated and intertwined in a very dynamic way. The instrumental approach seems not to capture these facets. There is therefore the need to add another fragment of another theory: the *network of knowledge*, which comes from Connectivism (Siemens, 2005). Personal knowledge is a particular type of network where a node is any entity that can be connected (information, data, images, ideas...), and an arc is a connection/relation/connection between two nodes. Personal knowledge can be understood as an evolving network: the knowledge possessed at a given moment corresponds to the precise conformation of the network, while the act of learning, of increasing knowledge, corresponds to the process through which the structure and complexity of the network expands. Learning is therefore a continuous process of construction, development, self-organization of knowledge - understood as a network (Downes, 2012).

By implanted these components (the instrumental genesis and network of knowledge) in the MDT, the MOOC-MDT was obtained (Figure 1).

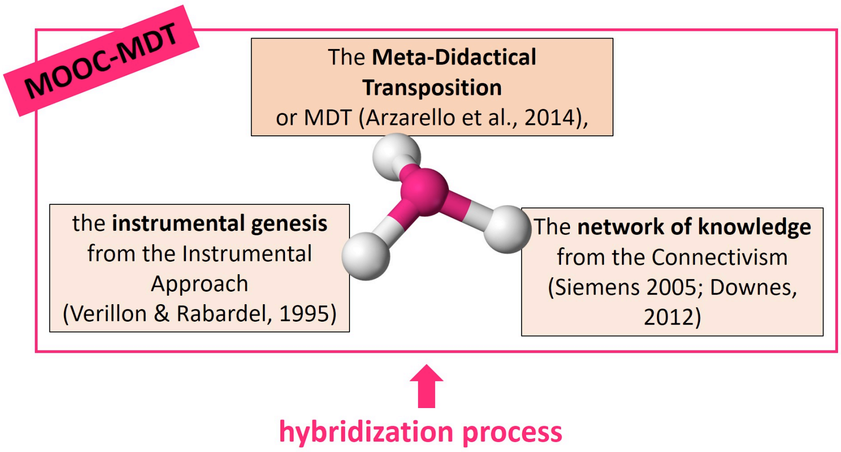

Figure 2: Hybridization process on MDT to obtain MOOC-MDT

We said that with the hybridisation "the old theoretical framework is so enriched and the language as well". We, in fact, coined a new term: *MOOC-ecosystem*, meaning "all the connections (exchange of materials, experiences and personal ideas/points of view) put in place by participants of an online community thanks to the technological tools through which they interact with each other, establishing connections within the given context" (Taranto et al. 2017b, p. 2481). The network-knowledge of the MOOC-ecosystem is dynamic: it evolves as MOOC-artifact's network, thanks to the participants' contribution. Also the network-knowledge of individuals evolves as a personal self-organization of the ecosystem. The MOOC-ecosystem is an instrument that belongs to each single participant. The process of transformation from artifact to instrument is here replaced by an evolution from artifact to ecosystem-instrument, called *double learning process* (Figure 3) (Taranto and Arzarello, 2019).

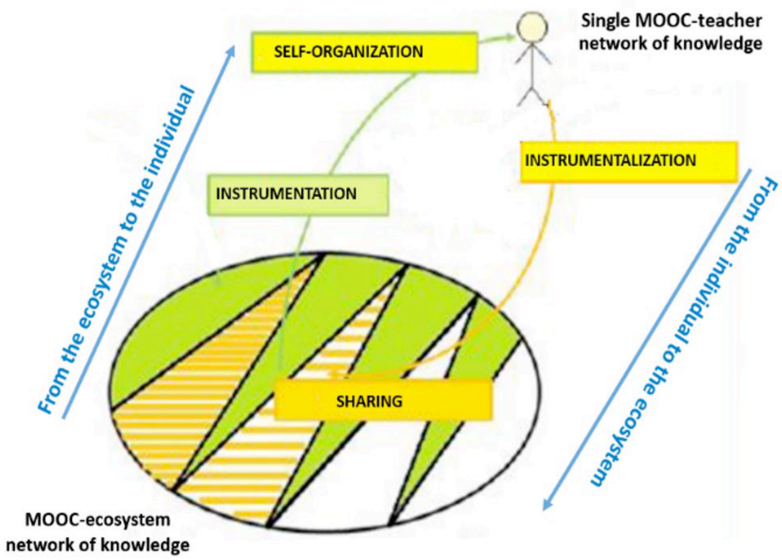

Figure 3: The double learning process
(taken from Taranto & Arzarello, 2019)

It is a dynamic process with the following intertwined components:
(i) *Instrumentation/Self-organization* (from the ecosystem to the individual): process by which the MOOC-ecosystem network of knowledge expands the individual's network of knowledge. In particular, the instru-

mentation is the phase by which the chaos of the ecosystem network reaches the individual. The many novelties of views and experiences ensure that the individual compares herself with new utilization schemes. A phase of self-organization of the MOOC information follows: when the individual selects which utilization schemes proposed by the MOOC are valuable and which are not.
(ii) *Instrumentalization/Sharing* (from the individual to the ecosystem): process by which the individual's network of knowledge expands the MOOC-ecosystem network of knowledge. The instrumentalization is the phase by which the individual, with her renewed network of knowledge, independently builds new connections. The individual is stimulated by a task requested by the MOOC and she caters to the ecosystem to turn it according to her own (new) utilization schemes. The individual wants to integrate it with her own cognitive structures. Sharing is the phase by which the MOOC welcomes the contribution of the individual and makes it available to all: information goes towards (is available to) all members.
The frame described above, along with a recent enlargement (Taranto, Robutti & Arzarello, in press), allowed us to distinguish two different type of teachers learning. We said that each teacher has their own network of knowledge and once they enter the MOOC virtual world, it can evolve. On the one hand, the content of MOOC can be perceived as new and this contributes to the expansion of the network by adding new nodes. On the other hand, the teacher has the opportunity to interact remotely with other participating colleagues and this can affect the creation of new and/or different connections than he had before participating in MOOC. In particular, teachers' online actions and interactions give researchers the opportunity to follow the evolution of teachers' praxeologies during MOOC activities. This evolution is intertwined with the network of knowledge that teachers create in MOOC. In fact, we observe that:

- adding a new node to their knowledge network means that a meta-didactical component of a praxeology moves from the outside to the inside;
- connecting the old nodes of one's network of knowledge in a new way means that one reflects on one's own meta-didactical praxeologies and possibly modifies the didactical ones.

We have categorized these learning in a more or less explosive way. In fact, we realized that if the mathematical object of discussion is known to the community, a whirlwind of discussions is generated that goes beyond the praxeologies that we want to transpose, more connections in the network of knowledge are fed. So, we are witnessing an explosive phenomenon. If, on the other hand, the topic is new, explosive phenomena are not generated, but a fuse is lit so to speak: a new node is inserted in the knowledge network.

4. Researchers involved in the MOOC

In this section we will consider MTEs, with particular regard to researchers. In the following, therefore, we will talk about researchers and teacher educators (second and third team described in section 2).
Researchers in the MOOC are involved in three phases: in developing the investigation; in designing the structure, the different activities, and the resources for teachers' development; and in implementing the distance course (less than the teachers' educators). In our research project, the design is seen as a dynamic and evolutionary process, to produce the MOOC materials as the final product. The agents in the design team are researchers and teachers, and they decided to work together in a collaborative and cooperative way, in the sense that:

- the structure of the MOOC has been designed in a collaborative way by all members of the team, during meetings in face-to-face and in distance work;
- the different activities for teachers and corresponding resources have been prepared after having decided structure, level of difficulty, target, and so on, in a cooperative way, as a puzzle of which every piece was in charge of one/two people.

During the design meetings, researchers and educators took together decision on structure and activities, tasks and credits, assessment and duration, in a process of about 6-8 months per each MOOC (May/June-December/January more or less), in which praxeologies of design have evolved, to reach common choices discussed and taken in a shared way. We refer to these as shared praxeologies of design (Figure 4).

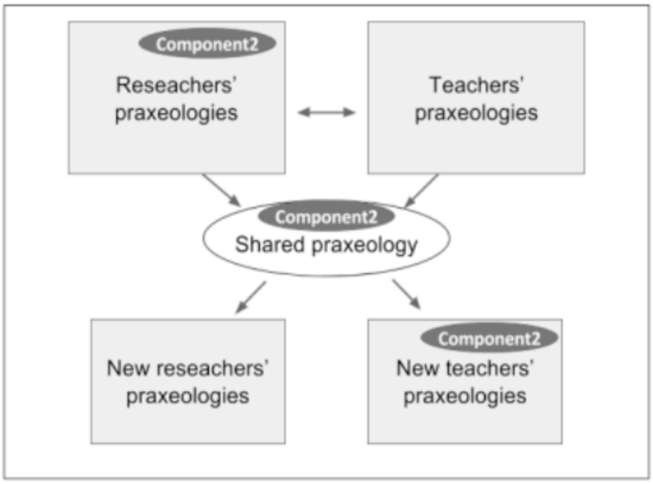

Figure 4: Meta-Didactical Transposition

The praxeologies components, given in terms of: tasks, technique, technology and theory, may be different at the beginning if refer to educators or researchers, and after the convergence to shared praxeologies. As a result, both the communities, of researchers and teachers, may have acquired new praxeologies. This is due to the experience of working together in the design of the MOOC.

Over the years we collected several data. In the following we list the ones that are related to the researchers.

A. In the first two MOOCs edition (Geometria and Numeri):

- Resources: contents, tasks, references (institutional, of research, didactical), suggestions for teaching practices, activities;
- Interviews to researchers and educators;
- Feedback by MOOC-teachers.

B. After 4 MOOCs:

- Interviews to researchers and educators.

C. In the design of the 5th MOOC:

- Recording of the meetings of design;
- Final interviews to researchers and educators.

To describe researchers' praxeologies, we identify some variables to which they are referred: for example, the target (in a broad sense), the theme, the interactions with MOOC-teachers (Taranto, 2018). Following the data referred to these variables, it is possible to trace an evolution in researchers' praxeologies (from the first to the last MOOC). This kind of investigation gives information about the evolution of praxeologies (or their components): from the first to the last MOOC, researchers evolved because they learnt from each experience, and the MOOC correspondently evolved, insofar it played the role of a boundary object (Star & Griesemer, 1989) between the two communities of researchers and educators in the phase of design.

As an example, we can give some results on the target variable:

- The task for researchers is to refer to the theme of a specific MOOC, to a precise school level, to teachers in their school with various experiences, but generally far from research in mathematics education, to their institutional references in terms of national curriculum and assessment.
- The technique is to use other activities of national plans (as m@t.abel: http://www.scuolavalore.indire.it/superguida/matabel/) to design the structure and parts of the MOOC activities for MOOC-teachers, that include the activities for their students: didactical aims, concepts, times, activities, teaching practices, example of assessment.
- The technology and theory components of the praxeology consist in some theoretical lenses of mathematics education, as: post-Vygotskijan context, the mathematics laboratory, the instrumental transposition and the instrumental orchestration, the meta-didactical transposition.

What we observed from the first to the last MOOC, following this variable target across the years as treated by researchers, is that an evolution in their praxeologies occurred. Specifically:

- they passed from considering only secondary school teachers to including primary and middle school teachers in the target of the MOOC;
- from designing activities separately one from the other, to planning them considering altogether in a common stream, from the beginning to the end of a MOOC;
- from giving MOOC-teachers only activities, to explicating them didactical approaches and teaching practices to implement them into the classes;
- from introducing in the MOOC too many resources, to diminishing their number;
- from giving MOOC-teachers short time, to give them more time for accomplish the task(s) required in the MOOC modules.

5. Discussion and aims of future research

The connectivistic frame used for grasping the rationale of teachers' changes within the MOOCs environment has pointed out some features of this new way of teaching/learning, and its specificities have been carefully described through the MDT lens. Teachers' knowledge has been depicted as an evolving network: its increasing and organization corresponds to the process through which the structure and complexity of the network expands. Our investigation wants to picture the evolution of teachers' praxeologies (or of their components) and the parallel evolution of researchers' ones from the first to the last MOOC, as illustrated in Figure 4 and discussed in the previous section. Also MOOCs, as boundary objects between the two communities of researchers and educators, correspondently evolved. The use of the MOOCs' technology with its tools, like the CMBs, has allowed asynchronous effective ways of interactions between the teachers as productive reactions to the proposals uploaded in the platform by the design team of researchers.

The resulting picture of learning represents a fresh way to look at the processes that trigger and support the acquisition of new knowledge: this happens precisely because of the affordances and constraints of the MOOCs' technological environment.

The picture is particularly consonant with the way knowledge is structured in nowadays society, which is brilliantly described through Dennet's model of the 'mind's unloading' (Dennet, 1997). According to Dennet, the progress of human mental powers, the rise of human intelligence and mental capacity, does not consist only in the production, assimilation, and retention of knowledge, but also and above all by the unburdening of brains through the expedient of storing information in technological artifacts, from the most primitive stone tools up to the most capacious servers and the worldwide web.

Bauman underlines the cognitive relief illustrated by Dennet's model:

> "Dennett's model implies that human intelligence is improved, and the human brain's potential is better used, for the vacation of the brain's contents and the squeezing-out of information away from the 'natural' warehouse made of brain cells. Having dislodged knowledge that otherwise would clog it and severely constrain its processing powers […] the human brain needs to retain only a relatively small set of 'indices' and 'clues'; this would be enough to allow humans access to the virtually unlimited amounts of information lodged away from the brain in the artifices scattered all over the human-made world. With the help of indices and clues, small and manageable samples of information, appropriate to the current problem which the mind is aiming to tackle, can be time and again retrieved—only to be returned to the external storage devices once the problem in question has been solved, thereby freeing the brain's capacity again for another batch of information required by the next problem or task." (Bauman, 2005, p. 315).

This is exactly the picture of knowledge given by the connectivistic model and by the way it is generated and improved through the dynamics of the double learning process, as discussed in Section 3 and illustrated by Figure 3. The joint events of the internet revolution and of economic globalization is accelerating the explosive nature of this model and poses with dramatic evidence the contradiction between this new scattered structure for knowledge and the way, according to which education is organized. This contradiction represents a challenge for school curricula, mainly still organized according to a 'pre-connectivistic' approach.

This state of things is acutely described by Z. Bauman within his frame of 'liquid society' (Bauman, 2000) in the following way:

> "All this militates against the very essence of school-centred education, known for its predilection for a stiff curriculum and predetermined succession of learning. In a liquid-modern setting, centres of teaching and learning are subjected to a 'de-institutionalizing' pressure and prompted to surrender their loyalty to 'canons of knowledge' (whose very existence, not to mention utility, is increasingly cast in doubt), thus putting the value of flexibility above the surmised inner logic of scholarly disciplines."
>
> (Bauman, 2005, p. 316)

Beyond the speculations of philosophers and sociologists, considerations like these pose an important bulk of questions, related to the requests of fresh educational programs necessary for the century into which we have just entered. More precisely:

- How to teach within the more and more connected world of the globalized society?
- What mathematics to teach within the new environment?

These questions are still more pressing within the new circumstances created by the Covid-19 pandemic all over the world. The findings of our research can give important suggestions for the proper directions towards which new investigations can be developed in order to answer the questions above. We shall so shortly discuss here some ideas about them, especially in order to avoid some possible dramatic mistakes that could appear on the horizon.

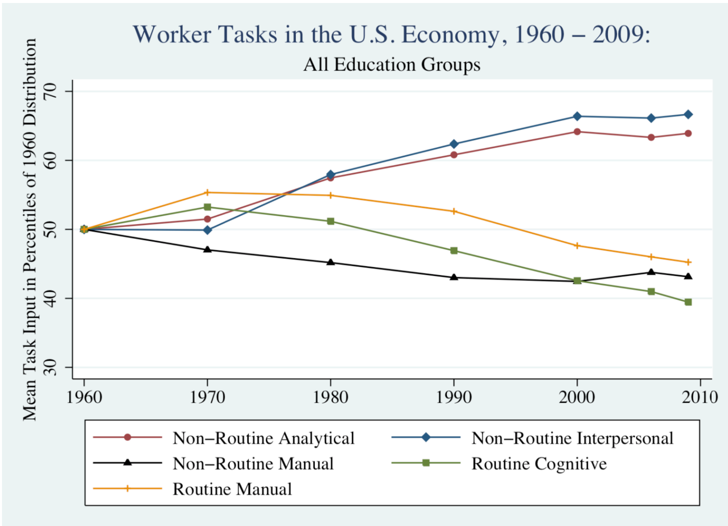

Figure 5 (from Autor & Price, 2013)

The first issue concerns the new needs of nowadays society. It is well known that the skills required on the job-place have changed and will still more changing in the years: see Figure 5 (Autor & Price, 2013) as one of the many examples that widely are circulating since a few years. They underline the dramatic bifurcation between the non-routine and routine or non-routine manual tasks in the job places. Other researches moreover point out the fact the 65% of kids who are currently entering primary school will be employed (if employed) in jobs that nowadays do not yet exist (Romano, 2017).

These stances require to look at learning/teaching processes in a fresh and creative way. OECD is going in this direction. For example, it has deeply changed its framework introducing a new structure for mathematical literacy, as illustrated by a report by Peggy G. Carr (2018), Vice-Chair of PISA Governing Board: see Figure 6.

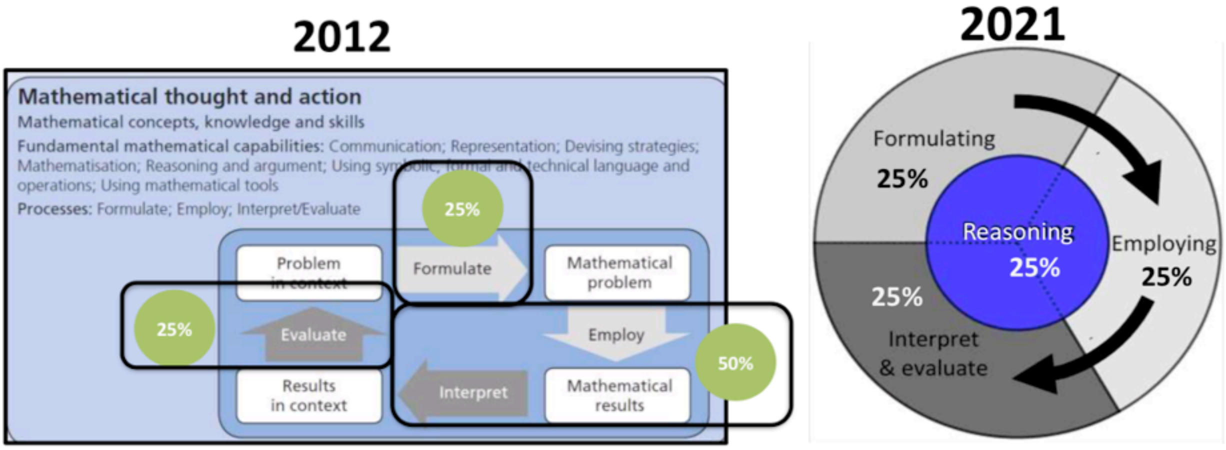

Figure 6 (from Carr, 2018)

The addition of Reasoning in the 2021 PISA frame with respect to the 2012 one specifically concerns the following topics pursued within personal, occupational, societal, and scientific contexts: Number systems and algebraic properties; Mathematics as a system based on abstraction and symbolic representation; Structure of mathematics and its regularities; Functional relationships between quantities; Mathematical modeling as a lens to the real world; Variance at the heart of statistics.

The connectivistic framework can be the right environment where such reasoning skills can be properly pursued. Our framework, developed within programs for teachers' professional development, can inspire further research, where connectivism can be a key for designing programs also for students' mathematical learning.

In fact, with unlimited, fast computer power and easy access to masses of data, learning can be achieved by working on meaningful, real problems with real data, where teaching activities can be structured around programs centered on the way the data and know-how necessary for solving the problems can be achieved within their networked structure. Focus of teaching/learning should be on the skills necessary to get such data in a smart and useful way more than on the content, which should not be any longer the primary issue. This approach will require project-based learning within teams of people, who solve problems connecting to suitable sources within a structured evolving network.

This approach will allow to properly design mathematics teaching transposition (Chevallard, 1999) so that cultural roots can be primarily taken into account. Connectivism, properly interpreted, can be a powerful tool, through which such instances can find the right place in the way mathematical ideas will be concretely learnt and developed. In a sense, it will be through the great flexibility, which such phenomena like the double learning processes allow, that a really customized learning, tailored according the personal and cultural instances of the learners will be effectively developed.

Our future research program consists in extending our investigations to properly deepen such questions.

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