Growth point and gestures: looking inside mathematical meanings

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32 Abstract

The literature on gestures describes how they often comprise iconic, deictic and metaphoric dimensions, but the interplay between these dimensions can be very subtle and nuanced. Due to the abstract nature of the subject, the use of gestures in the learning of mathematics means that the metaphoric dimension is often prominent. However, iconic and deictic gestures also play their part, and it has not been clear how the transition from gestures that have primarily iconic and deictic dimensions to those that are primarily metaphoric arises. In this paper, we consider three cases in an attempt to identify the emergence of the metaphoric dimension of gesture. The vignettes are analysed from semiotic and cognitive perspectives as we attempt to explain elements of the evolution by describing it in terms of McNeill's concept of a growth point. In each example, the results highlight the evolution from a grounded to a more abstract blending following the particular point at which a switch from an emphasis on the iconic/deictic to the metaphoric dimension occurs.

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Growth point and gestures: looking inside mathematical meanings

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Abstract The literature on gestures describes how they often comprise iconic, deictic and metaphoric dimensions, but the interplay between these dimensions can be very subtle and nuanced. Due to the abstract nature of the subject, the use of gestures in the learning of mathematics means that the metaphoric dimension is often prominent. However, iconic and deictic gestures also play their part, and it has not been clear how the transition from gestures that have primarily iconic and deictic dimensions to those that are primarily metaphoric arises. In this paper, we consider three cases in an attempt to identify the emergence of the metaphoric dimension of gesture. The vignettes are analysed from semiotic and cognitive perspectives as we attempt to explain elements of the evolution by describing it in terms of McNeill's concept of a growth point. In each example, the results highlight the evolution from a grounded to a more abstract blending following the particular point at which a switch from an emphasis on the iconic/deictic to the metaphoric dimension occurs.

Keywords Gestures · Metaphor · Semiotics · Growth point · Catchment · Conceptual blending

> "Beyond the traditional psychological concentration on mental structures and functions 'inside' an individual it [the semiotic framework] considers the personal appropriation of signs by persons within their social contexts of learning and signing. Beyond behavioural performance this viewpoint also concerns patterns of sign use and production, including individual creativity in sign use, and the underlying social rules, meanings and contexts of sign use as internalized and deployed by individuals. Thus a semiotic approach draws together the individual and social dimensions of mathematical activity

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as well as the private and public dimensions. These dichotomous pairs of ideas are understood as mutually dependent and constitutive aspects of the teaching and learning of mathematics, rather than as standing in relations of mutual exclusion and opposition." (Ernest, 2006, p. 68)

1 Introduction

In recent years, there has been a growing interest in analysis of gestures and how gestures matter, not only in communication (organising a discourse) but also in construction of mathematical meanings and concepts. This paper addresses the interesting issue of how the metaphoric dimension of gestures may evolve through mathematical discourse.

Mathematics has a special status in analysing gestures, because it seems to be the most abstract and conceptual of all fields, ostensibly having little necessary connection to physical experience. For gesture theorists, it is interesting to investigate how such abstract concepts are made present through the physicality of gestures. Recent work in cognitive science and mathematics education has begun to show that mathematics, far from being disembodied and wholly cerebral, has both roots and expression in bodily knowing (Arzarello, 2007; Arzarello & Edwards, 2005; Edwards, 2009; Lakoff & Núñez, 2000; Nemirovsky & Borba, 2003; Nemirovsky, Tierney, & Wright, 1998; Núñez, 2000, 2004; Núñez, Edwards, & Matos, 1999; Radford, 2009; Robutti, 2006). One reason why researchers are interested in working with gestures is that they may reveal aspects of concept formation and links with embodied metaphors that underlie mathematical abstractions and artefacts designed to foster the development of these concepts (Arzarello, Robutti, Sabena, & Paola, 2009; Edwards, 2009; Radford, 2009; Yoon, Thomas, & Dreyfus, 2009, 2011, 2014). Gestures produced by mathematics teachers and learners provide a rich source of data, comparable in scope to that provided by language, which can be read in terms of bodily metaphors, object construction and the formation of mathematical concepts, and the relationships among mathematical concepts.

Besides specifying their inherent differences, research in psychology informs us about an intimate unity between speech and gesture, with some evidence of a common semantic system involving both gesture and speech (Goldin-Meadow, 2003). Empirical evidence shows in fact that they are semantically and pragmatically co-expressive, they are essentially synchronous in time and meaning and they develop together in children. In quite expressive terms, McNeill claims that gestures have an active constitutive role in thought: "gestures do not just reflect thought but *have an impact on thought*. Gestures, together with language, *help constitute thought*". (McNeill, 1992, p. 245, emphasis in the original)

2 Semiotic aspects in connection with gestures

Gestures can be analysed in order to gain an insight into cognitive processes, as well as language or actions with tools and media. So, gestures are particularly interesting in mathematics education, as a means of understanding better students' (or teachers') ways of reasoning. When a gesture, an action and a word are used together in synchronicity, we have a "semiotic node", that is a piece of the students' semiotic activity to achieve knowledge objectification (Radford, Demers, Guzmán, & Cerulli, 2003). From a semiotic-cultural viewpoint, this mobilisation of more than one semiotic resource, produced synchronically, with a coordination of gesture, gaze and words, is particularly important, because it marks a new moment in the conceptualisation, namely a new element, which was not present before, enters the scene with a meaning. Often, it consists of a gesture along with locative words and time-related expressions, to achieve a coordination of time, space and movement. In this way, researchers can locate specific points in students' semiotic activity where gestures and words achieve a coordination of time, space and movement leading to the social objectification of abstract mathematical spatial-temporal relationships. Since knowledge objectification is the process of becoming aware of certain conceptual states of affairs, semiotic nodes are associated with the progressive course of becoming conscious of something.

Students become more and more conscious of mathematical meanings by working together and by mutual interaction with materials and tools. In introducing semiotic nodes, which mark their awareness of mathematical meanings, they make use of gestures, mediated actions and words all together, at particular points that are simultaneously sensual and conceptual (Radford, Cerulli, Demers, & Guzmán, 2004).

In his work, McNeill (1992) classified gestures using a number of dimensions. Of these *iconic*, *deictic* and *metaphoric* (rather than *beat* and *cohesive*[1]) are the ones of most interest in mathematics. He made it clear that these should be seen as overlapping dimensions, rather than discrete categories, stating that "Most gestures are multifaceted—iconicity is combined with deixis, deixis is combined with metaphoricity, and so forth. Rather than categories we should think in terms of dimensions." (McNeill, 1992, p. 38) Thus, a gesture may be, for example, simultaneously iconic, deictic and metaphoric. The iconic dimension relates to how the form of a gesture visually resembles the entity it is intended to describe, or as McNeill puts it, they are "gestures in which the form of the gesture and/or its manner of execution embodies picturable aspects of semantic content". (McNeill, 1992, p. 39) The deictic dimension of a gesture reflects the way it points, to either a physical place or an abstract idea represented at a specific point in a space. The metaphoric dimension involves the manner in which gestures convey abstract rather than concrete concepts. The intertwining of these aspects is particularly relevant when a mathematical discourse is analysed: in fact, mathematical inscriptions like graphs or formulas are deeply intermingled with the semantic content of mathematical concepts, and gestures can deeply reflect that (Lim et al., 2009). Moreover, a gesture is often not introduced alone, but, especially in mathematics, it is often aligned synchronically with words or other signs. This is described by Radford (2003) in terms of the semiotic node and by Arzarello (2006) as a semiotic bundle. More precisely, iconic gestures that mimic inscriptions (e.g. of an increasing function) can also assume a metaphorical aspect while the discourse is developing. Gestures can thus "enact symbols and provide grounding of novel and abstract ideas and representations" (Nathan, 2008, p. 377); we will deepen this point below in our examples 2 and 3.

3 Growth points and catchments

The constitutive role of gestures in thinking depends upon imagery, as a "form that directly embodies meaning". (McNeill, 2005, p. 56) In fact, "imagery […] is embodied in gestures that universally and automatically occur with speech. Such gestures are a necessary component of

[1] We do not consider gestures of this kind here because they are not related to our focus.

speaking and thinking". (*ibid.*, p. 15) McNeill proposes an explanatory model based on a mechanism of language-gesture integration: the "growth point".[2] His model combines in a dialectical way both the static and dynamic dimensions of language. In the static dimension (after de Saussure, 1916), which has been the focus in traditional linguistics, language is regarded as an object, trying to uncover the organisation of the *langue* system. In the dynamic dimension, after Vygotsky (1934/86), language is considered as a process, and gestures are a special route through which to access this language. The reason that gestures provide this special route is that in their unity with speech, they contribute dynamically to focusing the attention of the subject. As pointed out by MacNeill (2006, p. 2), this was observed at the beginning of the last century, by Saussure himself and by Wundt:

> "…language requires two simultaneous modes of thought—what Saussure, in recently discovered notes composed around 1910 [11 – Saussure, F. de, 2002], termed the 'double essence' of language (although he expressed this without reference to gestures). Wundt, 1970, writing at almost exactly the same time, had a similar insight in this famous passage:
>
> "From a psychological point of view, the sentence is both a simultaneous and a sequential structure. It is simultaneous because at each moment it is present in consciousness as a totality even though the individual subordinate elements may occasionally disappear from it. It is sequential because the configuration changes from moment to moment in its cognitive condition as individual constituents move into the focus of attention and out again one after another". (Wundt, 1970, p. 21)"

Following Vygotsky, McNeill introduces the idea of a growth point (GP), as the starting point for the emergence of noteworthy information prior to its full articulation. A growth point combines both imagery and linguistic components in a dialectical way: "A GP contains opposite semiotic modes of meaning capture—instantaneous, global, nonhierarchical imagery with temporally sequential, segmented, and hierarchical language". (McNeill, 2005, p. 18) In a growth point, the two modes are simultaneously active in the mental experience of the speaker, creating a dialectic, and, therefore, a sort of instability. The process ends when the growth point "is unpacked into an increasingly well-formed, hence increasingly stable, structure on the static dimension". (McNeill, 2005, p. 18) The unpacking of the growth point provides a resolution of the dialectic; this resolution is shown by a linguistic form, often accompanied by gesture. Of course, the gesture component may be more or less present, but this aspect concerns only the materiality in which the expressive act is accomplished, not its underlying base: "Images vary materially from no apparent gesture at all to elaborate multidimensional displays; but, hypothetically, imagery is ever present. What varies is the amount of materialization." (McNeill, 2005, p. 18)

The construct of a growth point has been evoked in science education (Pozzer-Ardenghi & Roth, 2008) to study communicative meaning units and, in particular, to identify the moments when new ideas are brought into conversations. In our research, we include the growth point for two main reasons. The first is that with its dynamic nature, the growth point constitutes a psychological account of the genesis of new knowledge in the learner. The second concerns

[2] Within different frameworks, in the field of psycholinguistics, there are well-sustained theories claiming gesture-speech integration: see the Information Packaging Hypothesis sustained by Kita (Alibali, Kita, & Young, 2000; Kita, 2000). In contrast to some earlier theories, such as the Lexical Retrieval Hypothesis (Krauss, 1998), both McNeill's and Kita's views assign a prominent role to gestures in thinking processes.

the fact that it frames context as an inherent part of the model. In fact, in contrast to typical approaches in psycholinguistics, growth point analysis incorporates context as an integral part of the process of forming and unpacking meaning, rather than as an external input. As McNeill (2005) points out, "a growth point is always connected to the discourse context, including any social interactive aspects". (p. 82) Again, this aspect is developed from an original idea from Vygotsky.[3] Context is incorporated into the dialectic model automatically, because each growth point constitutes what Vygotsky termed a psychological predicate—a point of differentiation of noteworthy content from a background; the background, or context, is an integral part of the growth point, without which it does not exist.

Context is formalised in the growth point model as a field of opposition, and empirically recovered via catchments. A catchment is defined as a "thematic discourse unit realized in an observable thread of recurring gestural imagery". (McNeill, 2005, p. 18) Thus, a catchment is recognisable when some gestural features are seen to recur in at least two (not necessarily consecutive) gestures. According to McNeill, a catchment indicates discourse cohesion, and it is due to the recurrence of consistent visuospatial imagery in speaker's thinking. Catchments may therefore be of great importance for research such as ours, since they provide us with information about the underlying meanings in a discourse, and about their dynamics:

> By discovering the catchments created by a given speaker, we can see what this speaker is combining into larger discourse units – what meanings are being regarded as similar or related and grouped together, and what meanings are being put into different catchments or are being isolated, and thus are seen by the speaker as having distinct or less related meanings. (McNeill et al., 2001, p. 10)

Examples of catchments have been widely discussed by scholars who study gesticulation: one of the most known is given by McNeill, who uses catchments to analyse the gesture-speech unity in people who describe a cartoon of Sylvester (a cat) and Tweety (a bird) immediately after they have seen it (McNeill, 2005). For example, when relating the scene where a bowling ball is dropped down a drainpipe towards Sylvester, McNeill describes a catchment where the narrator uses two-handed symmetrical gestures in a description of the bowling ball as an antagonist, the dominant force, and to highlight the shape of the bowling ball or its motion, using "an iconicity appropriate for its antagonist role". (McNeill, 2002, p. 18) Catchments have been observed not only in everyday conversation (Quek, 2004) but also in discourses produced in science and mathematical classrooms. In such cases, their function is generally stressed as "ways that instructors attempt to provide continuity of meaning across representations". (Nathan, 2008, p. 379) In one of Nathan's (*ibid.*) examples, a mathematics teacher used an arbitrary gesture to index elements in an equation with a student's speech. She exhibited a catchment by repeating this same gesture for corresponding elements of the solution in a second equation to establish their relationship across the representations. Another example, presented by Pozzer-Ardenghi and Roth (2008), involves a science teacher using a gesture with both hands, arms folded at the elbows, hands initially apart performing a squeezing movement that represents the contraction of the cardiac muscle. In our examples below, we will illustrate how catchments are produced by students while grappling with abstraction of a new mathematical concept.

[3] In chapter 3 of his book, McNeill (2005) discusses Vygotsky's ideas of psychological predicate, inner speech (Vygotsky, 1934/86) and relates them to the GP theory, arguing that they contain the seeds for its main ideas.

3.1 Conceptual blending

One way to analyse the complex relationship between the semiotic role of gestures and the construction of mathematical ideas is through the theory of *conceptual blending* (Fauconnier & Turner, 2002). This theoretical framework describes how two or more *mental spaces*, "small conceptual packets constructed as we think and talk, for the purposes of local understanding and action" (Fauconnier & Turner, 2002, p. 40), are integrated into a new, blended mental space. Crucially, the inputs are not just transferred wholesale into the blend, but a combination of the processes of composition of projections from the inputs, completion and elaboration (or 'running the blend') "develops emergent structure that is not in the inputs [i.e. the mental spaces]." (Fauconnier & Turner, 2002, p. 42) In mathematics, the two input spaces are often both abstract conceptual spaces, comprising knowledge of two domains that may be related or unrelated. An example from mathematics, given by Edwards (2009, p. 128), is the manner in which the notion of a number line draws on "two input spaces: our knowledge of numbers (initially whole numbers, and later, rational and real numbers), and our imagery and knowledge of the geometric entity called a 'line'." A second mathematical example, taken from Fauconnier and Turner (2002, p. 25) describes complex numbers as a blend of knowledge of real numbers and geometric knowledge of the properties of vectors in 2-space. Researchers such as Lakoff and Núñez (2000) have described how many important ideas in mathematics comprise conceptual blends, and that these convey powerful cognitive benefits, such as compression of ideas, "where the blended objects are given their own identity and the inputs are 'relegated behind the scenes'". (Alexander, 2008, p. 13; see also Tall, 2008) The production of gestures draws on knowledge of one's immediate physical environment (Edwards, 2009), with the mental space representing this referred to by Liddell (1996) as "real space". Thus, one of the input spaces behind gestures can be considered to be real space, with Liddell (1998) calling such a conceptual blend a *grounded blend*, and Parrill and Sweetser (2004) using the term *blending model* for describing the possible connections between iconic and metaphoric aspects of gestures involved in conceptualisation of meaning.

This framework of *conceptual blending* has previously proved a useful means for analysing gestures (Edwards, 2009; Yoon et al., 2011). We now present several examples, along with analyses from several perspectives, in an attempt to use the notion of grounded blends to describe what may be involved in the evolution in emphasis from the iconic to the metaphoric as the dominant dimension of a gesture. Our aim is to provide evidence for the hypothesis that an evolution from the prominence of an iconic to a metaphoric dimension of a gesture, part of a sequence of catchments, may indicate a process of construction of a mathematical concept. As defined by McNeill (2005), the dominant dimension of a gesture is a function of the input space employed: when this includes knowledge of the real world or comes from an embodied experience, then the gesture is primarily iconic; when it is knowledge from an abstract context (here mathematical), then the gesture is primarily metaphoric. Our proposal is that during the catchment, as the gesture recurs, the input spaces in the grounded blend may alter. Furthermore, we suggest that language has a key role in changing the blend through movement from one mental space to another.

For example, if one gestures the shape seen in the representation in Fig. 1a, in the context of a discussion about the outdoors, it may be understood primarily as iconic, referring to a hill or mountain. However, if the surrounding discussion is mathematical and the word "graph" is used, then, while the gesture may draw from both concrete and abstract input spaces, it may be seen as primarily metaphoric, picturing a parabola. Accessing the mathematical knowledge about parabolas may then give further information on related properties of quadratic functions. Q4

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Q1

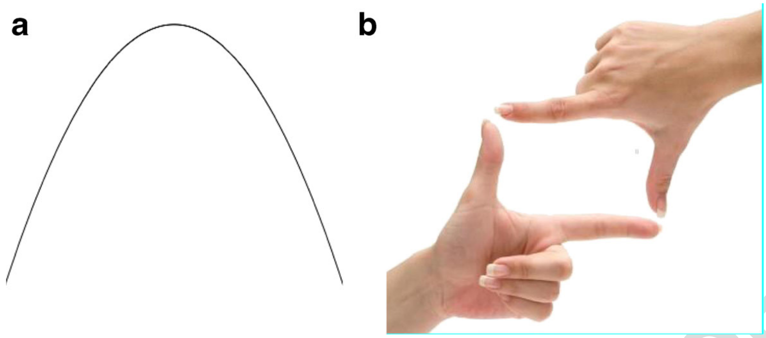

Fig. 1 The role of language in establishing mental spaces

Similarly, if we meet the gesture in Fig. 1b in a discussion about movies or television programmes, we may well focus primarily on its iconic and deictic dimensions, referring or pointing to a movie or television screen. However, if the context of the discussion is mathematical and uses language about two-dimensional shapes, then the dominant dimension may be metaphoric and we "see" it as representing a geometric figure, a rectangle, opening access to the corresponding mathematical properties that our cognitive structures contain about rectangle.

3.2 The case of Alessio: building numbers with gestures

In the context of the fairy tale "The three little pigs", 3-year-old kindergarten children are involved in counting three objects. The activity is directed not only at memorising the counting sequence but also at synchronising gesture and speech in the act of counting, which means showing one object a time, pronouncing the numeral corresponding to the object and finally recognising that the last numeral said is also the quantity counted. To take one thing each time and to count the things have a background in the metaphor "Arithmetic is Object Collection". (Lakoff & Núñez, 2000) The teacher pays attention to the use that the children make of gestures along with words, and in doing this, she supports the use not only of the deictic gestural dimension—pointing to objects while counting—but also the iconic and metaphoric dimensions—representing the cardinality of the set of objects examined.

In the excerpts below, the teacher guides the children in counting their set of three things, while they are sitting down on the floor in a circle.

11:02 Teacher: Then, children, let's try to count how many things we have, shall we count together?
11:05 Children: one, two, …
11:08 Teacher: let's put up the finger, then we count them.
11:11 Altogether: one, two, three (the teacher and the children count showing the objects with their index finger).
11:15 Teacher: how many are they?
11:17 Someone: three.
11:22 Teacher: how many are they?
11:23 Someone: like that.
11:24 Teacher: *like that*, *good*, *or like that*, (she shows the children which kind of gesture: with three fingers as in Fig. 2c, or without the thumb) *or also that is ok*. (While

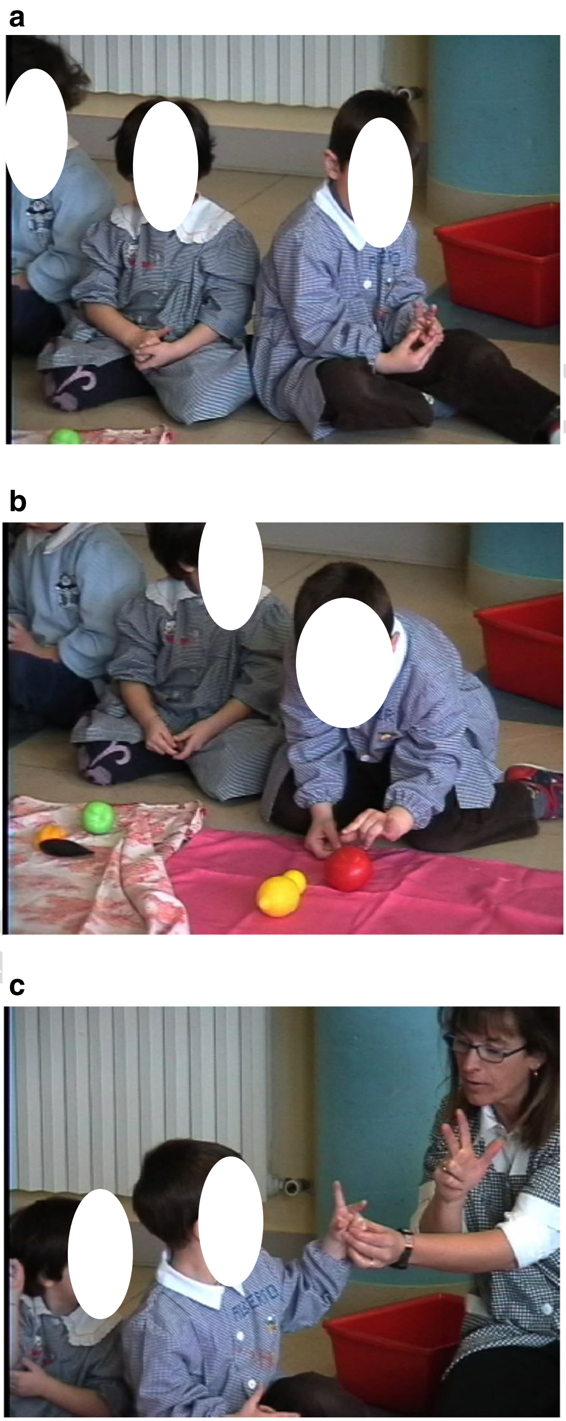

Fig. 2 **a** Alessio opens his hand after trying to keep the thumb and little finger closed. **b** Alessio opens first his thumb and then his index finger. **c** The teacher helps Alessio to open his fingers

Q1

> the teacher is following some children, Alessio is trying to associate his three fingers – index, medium and ring finger – with the three objects. In doing this, he opens his hand after trying to keep his thumb and little finger closed – Fig. 2a – so he is no longer able to repeat the gesture for correspondence between his three fingers and the three objects. After that, he tries again, starting with his hand completely closed and he opens first the thumb, then the index finger - Fig. 2b). *You must add one more finger to the gun*, *here you are*, *excellent*! *Three*, *excellent*! (Fig. 2c).

Research into the existence of a correspondence between fingers and numbers is necessary to be sure of both the representation and its correctness (Fig. 2b), and Alessio is especially involved in looking for that correspondence, in that he wants to be sure that his gesture and the numbers of his fingers exactly fit the quantity of objects he has with him. It is a matter not only of a match between the number of fingers and the number of objects, but also of the representation of "three" with his fingers. So, his gesture, which has an iconic dimension because of the fingers corresponding to the objects, evolves and acquires both a metaphoric dimension and an abstract meaning, representing the cardinality of a set. The iconic and metaphoric dimensions of the gestures co-exist, and their important meaning appears in the sequence of Alessio's movements. His concentration on remembering the correct gesture that means "three" in an abstract and symbolic way is transferred simultaneously in the process of finger-object matching. In doing this, the child is not only satisfied he can remember the gesture as a sign for "three", but he also tries to reconstruct his own meaning, searching for a reason for the gesture. The gesture is the same (index, medium and ring finger open, and thumb and little finger closed), but has two dimensions: iconic and metaphoric, representing the concept of "three" as a cardinal number. The sequence of catchments, comprising gestures that all involve open and closed fingers, marks the evolution in Alessio's imagery and representation of the number and his awareness of counting as a process with a result, in terms of cardinality, that corresponds exactly with the number of objects, and represents it. Alessio's gesture of three open fingers is the result of the growth point to generalise the number "three" (like a gun with one more finger—Fig. 2c), although without an explicit linguistic component other than the teacher's. Alessio approaches abstraction with the help of the teacher, who supports him in moving his fingers to a new representation of the number "three". Alessio's grounded blend is changing, and now, with the help of the teacher, it includes a mathematical context: if at the beginning the child was looking for an embodied, physical correspondence between fingers and objects, he now detaches his thinking from the physical world to approach an abstract concept of number, representing it with his fingers. This new grounded blend essentially includes an abstract, mathematical input space, while the previous one was based on a correspondence with reality (objects). And the gesture of showing "three" has changed its primary emphasis, from Fig. 2b—iconic—to Fig. 2c—metaphoric. This evolution takes place with the help of a word ("three"), which supports a changing in the blend space from physical to more abstract.

3.3 The case of Cyril: building the gradient with gestures

The second case we present involves Cyril, a student attending the third year of secondary school (11th grade, 16–17 years old): the case is widely discussed from another standpoint in Arzarello and Robutti (2008). Cyril's class had been introduced to the fundamental concepts of calculus at the beginning of high school (in the 9th grade, see Arzarello, Pezzi, & Robutti, 2007). Thus, the students had gradually become acquainted with the concept of functions and of their variations (first in a qualitative and then in a quantitative way). Moreover, they had developed the habit of using different types of software (Excel, Derive, Cabri-Géomètre, TI-interactive, Graphic Calculus) to represent functions, using both their Cartesian graphs and algebraic representations. The approach to the derivative concept is now pursued using Graphic Calculus, see Fig. 3.

We will comment on some excerpts from the activity of a group of three students, who interact with the graphs described in Fig. 3 (they are on the screen of their computer while they are working with the function f: $x \to 0.5x^3 – 5x^2 + 3$) and with their teacher. They are answering the following specific question—asked by the teacher while discussing with them—Imagine that you do not have the red curve (the one with a minimum), but you are seeing the tangent moving. Do you have any information on the concavity of the slope function?

The students have had previous experience with sketching functions, know the concepts of increasing/decreasing functions and have solved problems using first and second finite differences of functions, but they do not yet know the notion of derivative. Moreover, while they are good at using the Graphic Calculus program, they do know that the graph of the "quasi" tangent, constructed as a line joining two points very close together on the curve, is not the real tangent, because of the inherent approximation.

Initially, Cyril is working with a computerised graphical mathematical environment (Fig. 4). We observed the semiotic resources introduced and used by students and teacher,

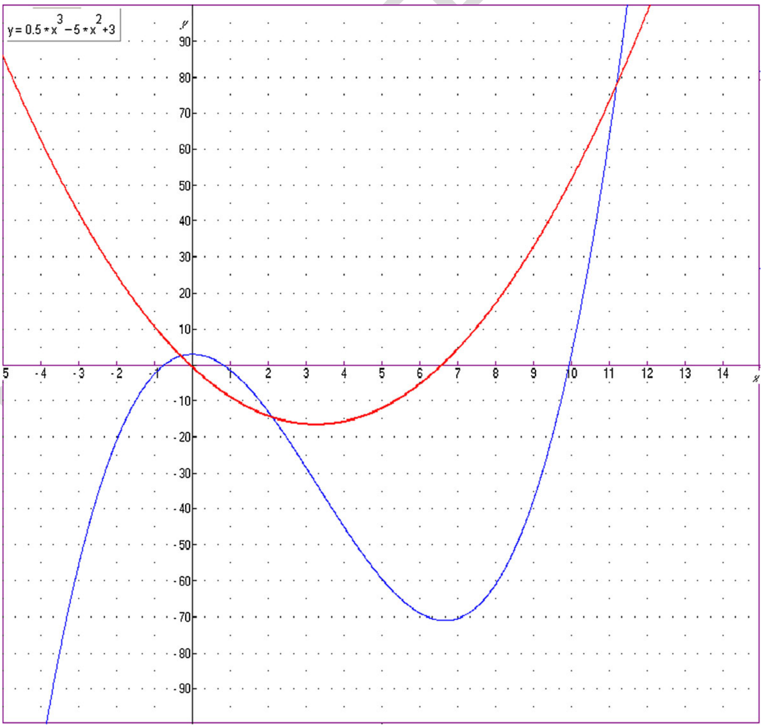

Fig. 3 The *blue graph* (with a maximum and a minimum) is that of the function $f : x \to 0.5x^3 – 5x^2 + 3$; the *red graph* (with a minimum) is the graph of the slope of the "quasi" tangent (namely a secant between two very near points on the blue curve) to the function; software Graphic Calculus allows one to see the genesis of the two graphs through a point *P* that moves in time tracing the graph of the function *f*, the corresponding moving "quasi" tangent and the graph of its slope, which is drawn in real time while *P* moves

Q1

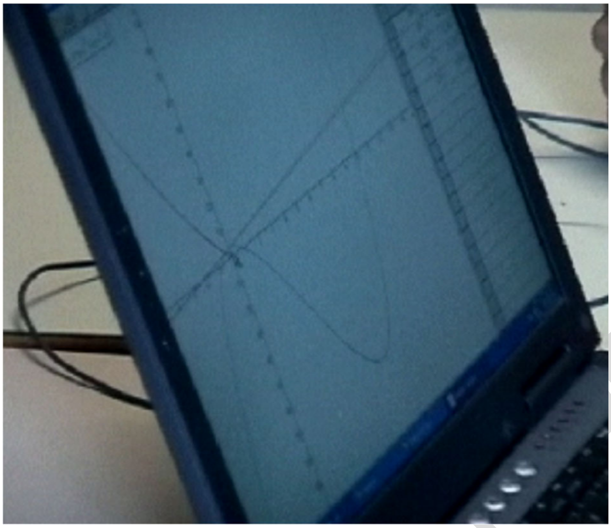

Fig. 4 The graphs and tangent line on the computer screen

which are as follows: the inscriptions on the screen (the two dynamic graphs and the moving tangent line in Fig. 4), Cyril's gestures and words and teacher's words (Arzarello, 2007). Cyril explains how the tangent is varying. Initially, the distances on the screen representing small sections of graph are represented by Cyril's gestures (see Figs. 4 and 5), which are primarily iconic.

The dominant iconic dimension of gestures is supported by explanations in words, which are embodied and iconic (Tall, 2008), and relates the sizes of the two distances he has captured by gestures:

Cyril: The *x*-interval is the same.
Teacher: The *x*-interval is the same. [i.e., you are right] delta *x* [Δ*x*] is the same.

This moment is crucial, insofar as it marks the growth point, which generates the sequence of catchments and the corresponding evolution of gesture's dimensions (iconic and metaphoric). For this reason, we call the gestures in Fig. 5 the basic gestures. This evolution is also

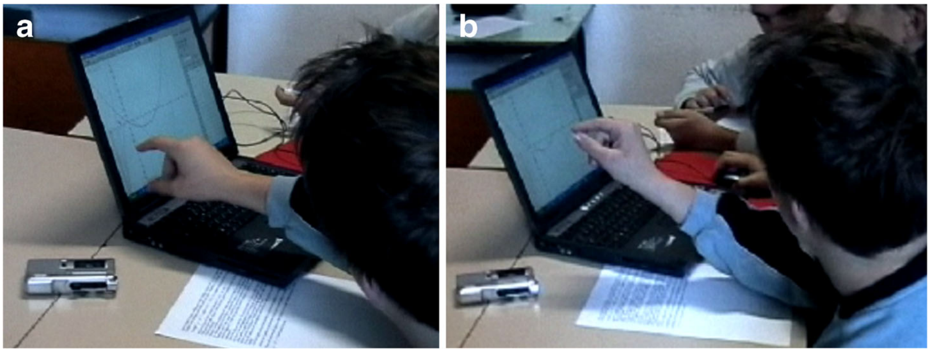

Fig. 5 **a**–**b** Two of Cyril's gestures

marked by the intervention of the teacher, who plays a version of the semiotic game (Arzarello & Paola, 2007; Arzarello & Robutti, 2008; Robutti, 2009), namely he reflects the gesture back, but with correct terminology attached, naming the concept delta x [Δx]. Initially, Cyril does not pick up on the Δx, but persists with his iconic perspective, and the iconic idea that points is "further apart".

> 34.56 Cyril: To explain it you can say that… this line must join two points that on the y interval are further apart.

However, the sequence of successive catchments (Fig. 6a–c, f), comprising fresh gestures (Fig. 6d, e, g) and simultaneous utterances, signals a key change in his thinking. Actually, the repetition of the basic gesture of Fig. 5a in these catchments is accompanied by a shift in the terminology and in the gesture, from the simple iconicity of the distance to the richer meaning of the delta x [Δx] itself. The gestures in Fig. 6d, e, g—along with the words—make the co-variation of the Δy versus Δx and its relationship with the steepness of the tangent explicit:

> 35.03 Cyril: Delta…eh, indeed, however there are some points where… to explain it … one can say that this straight line (35.10) [Fig. 6a] must join two points (35.12) [Fig. 6b] on the y-axis (35.14), which are farther from each other,… [Fig. 6c] hence it is steeper towards (35.16)… [Fig. 6d]
> 35.18 Student G: Yes
> 35.19 Cyril: Let us say towards this side. When, here, …when …however it must join two points that are farther…(35.24) [Fig. 6e]… that is there is less … less distance (35.26) [Fig. 6f]
> 35.27 Teacher: More or less far?]
> 35.28 Cyril: Less… less far [he corrects what he said before]
> 35.29 Teacher: Eh?
> 35.30 Cyril: On the y-axis I am saying

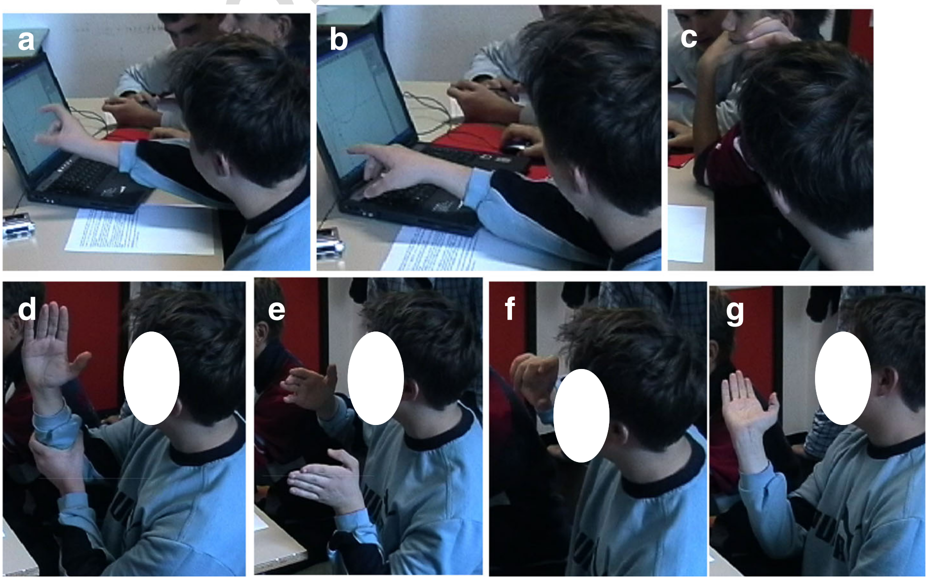

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Fig. 6 **a**–**g** Cyril's catchments

35.31 Teacher: Yes

35.33 Cyril: It slopes gently from this side. In fact here is the point [Fig. 6g].

The teacher's use of the expression Δ*x* has helped Cyril's grounded blend evolve, so that he is now accessing a mental space related to differentiation and employing it to form a grounded blend. Now, he has access to the use of Δ*x* and Δ*y* in representing gradients. In this way, the gestures are now not only simply more iconic but also increasingly metaphoric in dimension, pointing him towards abstract mathematics. Hence, he begins to express, with both gesture and speech, the fact that for equal and small Δ*x*, the corresponding Δ*y* is different (e.g. bigger and bigger when the slope increases). While he is still using nonmathematical expressions —such as "less distance"—there is an indication that he is beginning to think in terms of the abstract notion of gradient, referred to by the terms "steeper" and "slants". The basic gesture of Fig. 5 has evolved: the catchments of Fig. 6 and the intertwined fresh gestures make this evolution palpable. Cyril has thus conceptualised a mathematical meaning, first showing an idea with a gesture that is mostly iconic and then generalising it with the same gesture shown in catchments, which then also evolves to acquire a metaphoric dimension in itself: from the source domain represented by the dynamic figure on the screen (Figs. 5a, b, and 6a, b) to the target of steepness (Fig. 6d) represented as a relationship between the vertical (Fig. 6f) and the horizontal displacement (Fig. 6e).

3.4 The case of Ava and Noa: building an antiderivative with gestures

It is not always easy to see a clear evolution from an emphasis on the iconic dimension to the metaphoric one. The interplay may take place over some time, with language and other aspects of the semiotic resources all playing their part. In this example, two teachers Ava and Noa are working on a task where they have to construct a graph of a tramping track given the graph of its gradient (see Fig. 7). While graphical derivatives are considered, this kind of graphical

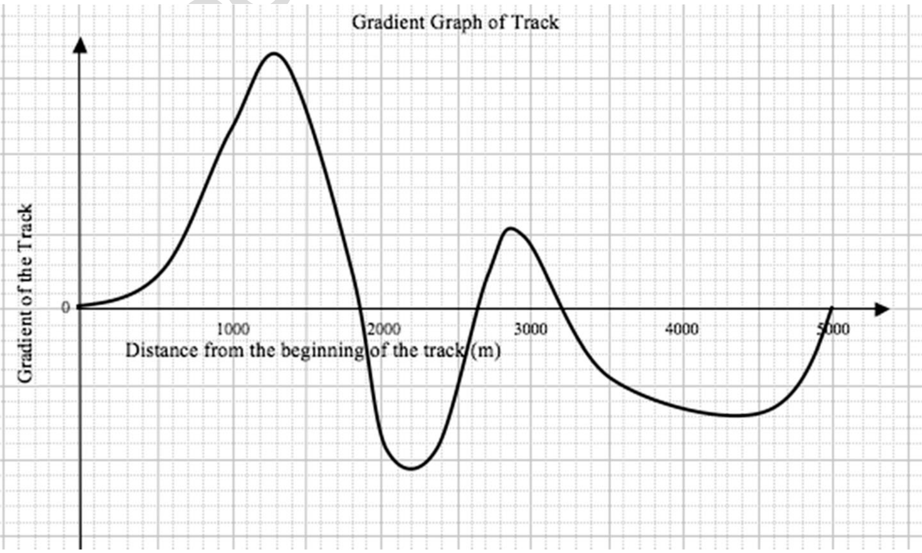

Fig. 7 The graph of the gradient of the tramping track

antiderivative task is often not covered in the local secondary school curriculum and so would be unfamiliar to students, as it proved to be to these teachers. The teachers worked on the task in a semi-clinical setting, with a researcher present. Their work was audio and video taped and later transcribed (see Yoon et al., 2011 for full details).

Following a "warm-up" task, and 1 min and 30 s after receiving the problem statement, Noa starts to gesture, finally relating gestures to the graphs 2 min and 23 s into the task.

118a 10.28 Noa Let's do positives and negatives, so uphill, downhill. [pointing with index fingers to sections of the graph in Fig. 7, above and below the distance axis]
118b 10:37 Noa Uphill, downhill [places her hands flat across the sections of the graph above and below the distance axis–see Fig. 8a]
121 10:45 Ava This [points to the gradient graph in Fig. 7] is a gradient graph and it's going uphill downhill and we're at the top. That make sense?
122 10:59 Noa Yes.
123 11:00 Ava So the top is going to be here?
124a 11:05 Noa Probably.. [pauses for 6 seconds to think]
124b 11:11 Noa Yes, that's the same shape as there. [pauses and thinks for 12 seconds]
124c 11:23 Noa But just generally on the graph, the gradient (..) plus gradient is up [see Fig. 4b] and negative gradient is down isn't it? [see Fig. 8c] So this would be all uphill [places her hand flat on the section of the graph above the distance axis]

The gestures here mark a sequence of catchments where the hand, with fingers open or closed, is used to gesture the direction of the graph. However, Noa's comment that "… generally on the graph, the gradient… plus gradient is up and negative gradient is down isn't it?" accompanied by her two gestures (see Fig. 8) seems to indicate that she is thinking about the gradient of graphs in general, and hence, her gestures likely comprise a grounded blend involving conceptual structures related to gradients of graphs. Thus, it appears that these two gestures are removed from the tramping context and have a primarily metaphoric dimension. However, a short time later when Ava and Noa refer to gradient, they are interpreting the gradient on the given graph and so appear to be thinking firmly in a physical tramping context, translating it with meanings such as "very steep" and "difficult walking".

140 12:29 Noa And we are going along the gradient is quite small so not very steep, but then it gets steeper.
141 12:40 Ava But the gradient gets – here the gradient is very steep, this is hard. This is difficult.
142 12:46 Noa Yes that's right. That's the hardest place.
143 12:48 Ava This is difficult walking.

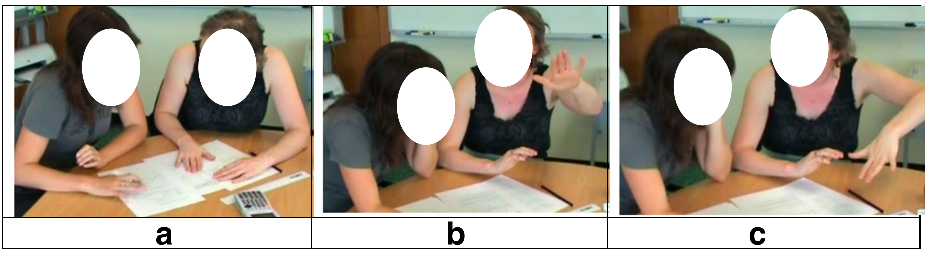

Fig. 8 **a**–**c** The sequence of gestures for lines 118–124

This gesture (Fig. 8b) is again a basic gesture for our story; the speech and gestures in Fig. 8 form the growth point of all the subsequent evolution of catchments in Ava and Noa's semiotic productions.

Hence, when they next gesture to indicate the slope of the track (see Fig. 7), they begin by referring to the context rather than a mathematical graph. They still employ terms such as "steep hill" and "going up". Their grounded blend appears to have a primarily iconic, rather than metaphoric, dimension, although at some point there seems to be a crossover to the latter in their thinking. The whole of this discussion is accompanied by continuous gesturing.

152 13:24 Noa Steep hill [gesture as in Fig. 9a] and still going up but not as sharply. So it's really, really steep section.
153 13:34 Ava Really, really steep hill.
154 13:36 Noa And then the point of inflection [gesture in Fig. 9b] and then starting to get not so steep up to the summit.
155 13:45 Ava Yes.
156 13:48 Noa And then we are starting to go downhill, negative gradient. [gesture by both Ava and Noa in Fig. 9c].
157 13:52 Ava Yip, yip. Downhill.
158 13:54 Noa And it's quite gentle gradient, getting steeper gradient to the point where it is the hardest gradient [gesture in Fig. 9d] and then it starts levelling off and getting easier again until you get to like a bottom. And then we start going back up, gentle to a harder point.
159 13:21 Ava But not as hard as it was over there.
160 13:23 Noa But not as hard as it was over there. And then gradient is getting easier – we're still going up but we're flattening off and then we reach another point where we've got to the top.

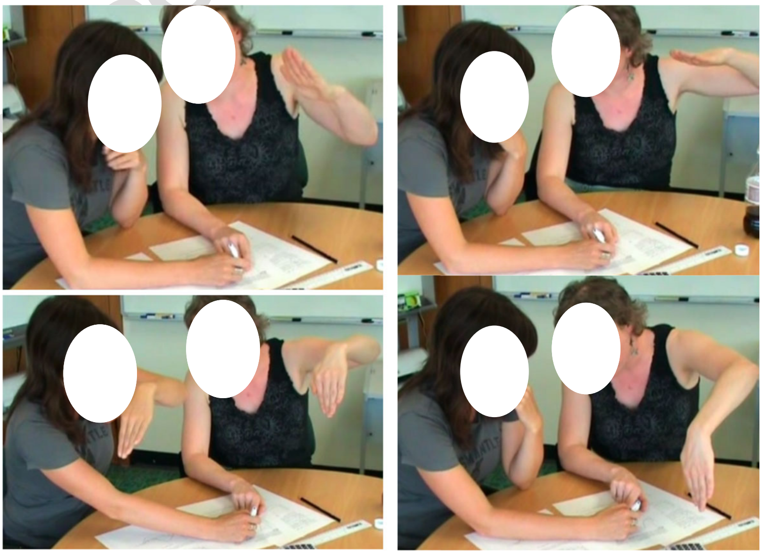

Fig. 9 The sequence of gestures for lines 152–158

Initially, they both begin with the similar embodied, physical context ideas of “going up” and “really steep”, but the use of the term “point of inflection” by Noa appears to mark a crucial point. Although she expresses the term in an interrogative manner, suggesting that she is very uncertain of its correctness, it may be that it brought a graphing schema from calculus to the forefront of her thinking. This enabled a change in her blend at that point to include knowledge of mathematical graphs. Hence, her very next comment, 12 s later, now includes both the embodied physical idea of “starting to go downhill” and the mathematical concept of “negative gradient”. Her following comments comprise a mixture of terms, but now, the gestures relate to “gentle gradient”, “steeper gradient” and “hardest gradient”, and finally, she talks about “the gradient is getting easier”. Thus, her grounded blend seems to have employed the mathematical concept of gradient to assist in the transition from the graph in Fig. 7 to an antiderivative constructed through gestures. At the same time, the gestures have evolved from a primarily iconic dimension to a mostly metaphoric one, moving from representing physical “steepness” to the abstract mathematical idea of instantaneous gradient at a point on a graph as measured by a tangent line. This evolution is again marked by a starting growth point, namely when a hand gesture is used to represent, not the physical steepness of a hill, but the mathematical gradient of a tangent line to a graph, and by a sequence of catchments, which mark the evolution of the gestures from mostly iconic to include a more abstract, metaphoric dimension.

4 Discussion

McNeill describes *catchments* as a “thread of recurring gestural imagery” (McNeill, 2005, p. 19): as such, they show how language and imagery can contribute to making sense of mathematical concepts through their dialectic. In our examples, we have shown that through the blending of imagistic and discursive aspects, catchments can contribute to making apparent the new concepts of the following: the number three, incremental ratio and the instantaneous gradient of a function. The three examples are characterised by a production of catchments completely linked to the context. And framing the abstraction of mathematics in a context is crucial for having a growth point.

The growth point, introduced by McNeill as a mark of the beginning of information prior to its full articulation, is made of imagery and linguistic components, articulated in a context. In our three examples, the presence of a growth point is essential to the construction and development of mathematical meanings and is independent of the age of the subjects involved. Both very young and older students show similar use of a growth point: the dialectic between words and gestures determines a sort of instability and a process involving catchments, until the production of a final and static sign. The analysis of this process provides us with an interpretation of the cognitive activity of students, marked by subsequent steps where the interaction with the teacher, or between students, is an essential part. The cognitive activity is particularly evident when students are using multiple gestures, as in the examples discussed above, because we can observe the process of catchment development and the evolution of meaning. When a gesture is initially introduced with an iconic dimension, it is not sufficient to provide information about the cognitive activity, but if at a certain point there is a switch in emphasis from the iconic to the metaphoric dimension, it means that there is a cognitive evolution from a grounded to a more abstract blending. All three of the examples above show this evolution.

In describing and analysing data, we have considered at least three types of intertwined semiotic resources: speech, gestures and inscriptions. We have considered the multimodal semiotic resources used by students and teachers in a systemic way, taking into account the signs introduced in the social interaction, their evolution and their mutual relationships. They constitute a unit in our analysis, which is inherently composed of different modalities, in view of the dynamic evolution. This aspect is parallelled in gesture studies by McNeill's growth point analysis, in which the gesture-speech unity is the basis for the analysis. Both approaches are inspired by Vygotsky's intuition that to understand a complex phenomenon/process, it is important to consider a minimal psychological unit that retains the essential properties of being a whole. In the case of Vygotsky, the minimal unit is the "word meaning". In the case of McNeill and gesture studies, the minimal unit includes gesture and speech. However, in our analysis, the minimal unit entails all the activated semiotic resources, such as speech, gesture, inscriptions and so on, as well as signs coming from technological devices (such as a graph). In our last two examples, inscriptions on the screen or on the paper are crucial in the formation of a grounded blend from which the successive productions of catchments evolved. In the first example, the experience with concrete objects is essential in finding a correspondence between object and finger and hence in supporting the grounded blend that gives rise to the production of catchments.

Thus, we maintain the data which support our hypothesis that the evolution from the prominence of an iconic to a metaphoric dimension of a gesture, part of a sequence of catchments, can indicate a process of construction of a mathematical concept, independent of the age of students and the difficulty of the task. Further, this passage is also characterised by the use of language and of other signs coming from the context where the students are working together. This complex minimal unit of analysis (called a semiotic bundle in Arzarello, 2006) features the mathematical discourse, stressing its similarities to, and differences from, discourses produced in everyday conversation.

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