Between Berlin and Cambridge: classical conceptions of the general economic equilibrium in the late 1920s

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Classical Conceptions of the General Economic Equilibrium in the late 1920s

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1. Introduction

In the years between the two world wars, the General Economic Equilibrium (GEE) theory was one of the most important topics of inquiry. Historians of economics traditionally present Vienna as the cradle of the neo-Walrasian GEE theory (for example Weintraub, 1983, Ingrao-Israel, 1987, Punzo, 1991). However, the concentration on Viennese contributions represents a narrow perspective, not only because it neglects important aspects of the development itself of a neoclassical GEE in Vienna, i.e. the Paretian path of research (Marchionatti and Mornati, 2016), but also because it neglects different contributions to a debate that was not reducible solely to the Walrasian perspective: these can be defined as representing the classical approach to GEE developed by the Italian-born economist Piero Sraffa in Cambridge, and by the German mathematician Robert Remak, and by the economist of Russian origin Wassily Leontief in Berlin, at the end of the 1920s.

The contemporary literature on, above all, Sraffa and Leontief (Remak, who died in Auschwitz in 1942, was quite totally neglected until recently) does not consider those 1920s contributions, if not marginally, as a sort of prelude of the successive works of these authors. In fact, in the 1960s the Leontief-Sraffa connection proved to be an interesting point of discussion, but in an intellectual context where the debate and the contributions of the 1920s were largely unknown. While Remak was remembered simply as a marginal 'precursor' of neo-walrasian theory (Baumol and Goldfeld, 1968), Leontief's work in the 1940s and 1950s and

Sraffa's 1960 book were considered major events in the economic science of those years. Not by chance, Paul Samuelson used the expression "this age of Leontief and Sraffa" (Samuelson 1971, p. 400, see also Levine, 1974, and Samuelson 1987) – emphasizing the common programme of superseding Marshallian partial equilibrium by constructing linear systems of simultaneous equations that describe the general interdependence among the different sectors of the economy,[1] but denying the non-neoclassical nature of the approach of the two authors – in his attempt, quoting what Garegnani (2007) wrote about Samuelson on Sraffa, "to translate [them] into the terms he is familiar with" (p. 183) and interpreting them as different versions of GEE theory. On the contrary, the neo-Ricardian literature launched by Sraffa's book has considered Leontief and Sraffa's contributions as a return to the classics – partly involving also Von Neumann (1937). However, the thesis of a return to the classics has been questioned in particular as far as Leontief is concerned by William Baumol (2000, and 2009 with Raa).[2] Indeed, the meaning of their contributions to economics (above all in the case of Sraffa's[3]) has remained a matter of extensive debate in which disagreement has prevailed.

This paper focuses on the 1920s contributions of Sraffa, Remak and Leontief. It considers the intellectual environment where the ideas emerged. In particular, the paper investigates the relationship between these contributions and a previous phase of the debate in economics in Germany, between the 1890s and the first decade of the twentieth century, involving classical economists, Marx and Walras, as well as the introduction of mathematical reasoning into economics: an original synthesis of these connections emerged in the contributions of the German-Russian Mathematical School. The paper is divided into three parts. Section 2 is devoted to presentation of Sraffa's contribution in Cambridge and Remak's and Leontief's contributions in Berlin. Section 3 deals with the cradle of what we may call the classical conceptions of GEE: the German debate on the Classics and Marx and on Walras, which gave origin to the synthesis attempted by the first neo-Ricardians, Vladimir K. Dmitriev and

Ladislaus von Bortkiewicz – without forgetting Georg von Charasoff. Section 4 considers the relationship between neo-Ricardian and von Neumann contributions in the 1930s. The final section concludes.

2. Classical conceptions of GEE in the late 1920s

2.1. The neo-Ricardian perspective in Cambridge: Sraffa's contribution

2.1.1. Premise: seeking a classical foundation for a general equilibrium analysis

Sraffa received his training as an economist in Italy at Turin University at the school of Luigi Einaudi (Marchionatti et. al., 2013), and then in the United Kingdom at LSE and Cambridge, where he lived by 1927. Sraffa (1925 and 1926) showed that the Marshallian-Pigouvian theory of value under perfect competition was logically weak due to the inadequacies of the treatment of variable costs and the difficulties of co-ordinating the laws of returns under competitive conditions in a partial equilibrium context. The routes available to overcome those difficulties were, according to Sraffa, (a) the simultaneous equilibrium of all industries (Pareto's route in Sraffa's terms), or (b) the abandonment of the assumption of perfect competition (Cournot's route). However, according to him, the Paretian conception was not fruitful because of its complexity. Therefore Cournot's route seemed to be the only viable direction for inquiry. Indeed, it was followed by most Marshallians in the 1930s and permitted the foundation of the theory of imperfect competition. But this solution was considered unsatisfactory by Sraffa. He maintained that a foundation of the theory of normal value under competitive conditions had to refer to the classical approach that assumed the invariability of costs. Initially, Sraffa made the assumption of constant returns, but he abandoned it in 1928

when he recognized, in a draft of the "central propositions" of *Production of Commodities by means of commodities*, that prices and distribution could be determined even if no assumptions on returns were made. This standpoint was essentially different from the marginal approach, which "requires attention to be focused on change" (Sraffa 1960, p. v). On the basis of his unpublished notes it is possible to reconstruct Sraffa's early work (see Kurz and Salvadori, 2001; Gehrke and Kurz, 2006; Sinha, 2016; Naldi 2018). From the end of 1927 onwards Sraffa elaborated several sets of homogeneous linear equations in terms of physical quantities. These equations represent the roots of the final version of the corresponding equations in *Production of Commodities*. Sraffa (1960) suggests that they correspond approximately to the equations in the first two chapters of his book, whereas other particular points "such as the standard commodity, joint products and fixed capital, were worked out in the thirties and early forties" (ibid., p. v).

2.1.2. Sraffa's "central propositions"

Sraffa's model represents the economic system as a circular process with n industries, n commodities, and $n \times n$ relations between them (or "methods of production"), that is, the products are also the means of production and there are no other means of production apart from the products. Sraffa distinguishes between a system in a strict self-replacing state (production for subsistence) and not in a strict self-replacing state, in which the economy produces more than the minimum necessary for replacement (production with a surplus). The general system of n equations for $(n + 2)$ unknowns, p_i, r, w is the following:

$$A_{11}pi + \ldots + A_{n1}p_n)\,(1 + r) + L_1w = A_1p_1$$

.....

$$A_{1n}pi + \ldots + A_{nn}p_n)\,(1 + r) + L_nw = A_np_n$$

with the constraints:

$$\sum_{i=1}^{n} A_{ij} \leq A_i \quad (i = 1, \ldots, n)$$

$$\Sigma L_i = 1$$

where:

A_i	(i = 1,..., n)	=	quantities of commodities annually produced ($A_i > 0$)
A_{ij}	(i , j = 1, ..., n)	=	technical coefficients of production
p_i (i = 1, ..., n)		=	"exchange values" of commodities.

To solve this system Sraffa assumes as a standard a "composite commodity", that is, the set of commodities of which the national income is composed. The system becomes of (n + 1) equations with (n + 2) unknowns, p_i, w and r.

In the sixth chapter of his book Sraffa presents an approach different from that of simultaneous equations: "the reduction to dated quantities of labour" – "a second way of approach", as he called it in 1931 (Sraffa Papers, D3/12/7) – by which prices are considered "from their cost of production aspect, and the way in which they resolve themselves into wages and profits" (Sraffa 1960, p. 34). Taking the equation which represents the production of a certain quantity of a commodity A_i

$$A_i p_i = L_i w + (1 + r) \sum_{j=1}^{n} a_{ij} p_j$$

Sraffa points out that $(1 + r) \sum_{j=1}^{n} a_{ij} p_j$, i.e. the value of the means of production used to produce A_i, is itself a commodity, produced a year earlier and multiplied by a profit factor (1 + r) at a compound rate for the appropriate period; thus it can be replaced with its means of

production and the quantities of labour employed in its production. This procedure can be repeated, backwards through the years, by replacing the means of production with their own means of production and quantities of labour, and applying to the formula a profit factor for the appropriate period. In the end, we obtain the following "reduction equation" for the product *i*, "in the form of an infinite series" (ibid., p. 35):

$$A_i p_i = L_i w + L_{i\,1} w (1 + r) + L_{i\,2} w (1 + r)^2 + \ldots + L_{i\,n} w (1 + r)^n + \ldots$$

where $L_{i\,j}$ (j = 0,1, …, n) are the quantities of labour employed in the production of the commodity *i* in the year $t - j$, *t* being the current year. The general term of the series is $L_{i\,n} w (1 + r)^n$ in which the quantities of labour are "dated", i.e. a different weight is applied to them depending on the time of utilization.

2.2. The classical perspective in Berlin: Remak's and Leontief's models

2.2.1. Premise: theoretical reflections in Bortkiewicz's circle

In the 1920s Berlin was an important centre for mathematical economics. This was essentially due to Ladislaus von Bortkiewicz, economist and statistician of Polish-Russian origin, professor of political economy at the University of Berlin since 1901. He was a follower of two great scholars: the German statistician and economist Wilhelm Lexis who was his PhD in statistics advisor in Göttingen in 1892, and Léon Walras, with whom he was an important correspondent highly valued by the French economist. He was deeply interested in mathematical economics and in Ricardo's and Marx's economics. Although an effective group of followers was never formed in Germany, Bortkiewicz's home in Berlin was a sort of intellectual open house (Anderson, 1932; Gumbel, 1968). Leontief and Remak had a close relationship with Bortkiewicz.

Wassily Leontief, after obtaining a degree in economics at the University of St. Petersburg in

1925, went to the University of Berlin to continue his studies with the sociologist Werner Sombart – a leading European social scientist and leader of the Youngest historical school – and Bortkiewicz himself (on Leontief in Germany see Bjerkholt, 2016). There, in 1928, he obtained his Ph.D. in economics with a dissertation entitled "Wirtschaft als Kreislauf" (with Bortkiewicz and Sombart as advisors), later translated as "The economy as a circular flow". In his dissertation he presented the economy as a system of circular interrelationships – "a long path describing a wide circle and ending up again at its starting point" (Leontief 1928, p. 182) or "a closed casual chain" (ibid., p. 184) – based on an objective technological framework, and investigated which set of relative prices support this system.

Robert Remak (Hagemann and Punzo, 2007; Parys, 2014) studied under the mathematicians Ferdinand Georg Frobenius and Herman A. Schwarz at Friedrich-Wilhelms University in Berlin, where he was *Privatdozent* from 1929 to 1933. [4] His contributions to mathematics range over a broad area – principally group and number theories -, but he was also interested in economics. In 1929 Remak carried out a study on the determination of "rational prices" for a centrally planned economy, presented in a seminar at the Berlin Institute of Mathematics and published with the title "Kann die Volkswirtschaftslehre eine exacte Wissenschaft werden?" (Can economics become an exact science ?) In starting his research on this subject his socialist interests played certainly an important role (as Hagemann and Punzo, 2007, maintained), as also Bortkiewicz's suggestions on the issue of the theory of different socio-economic systems (see Wittmann, 1967, and Remak, 1933).[5] Remak's paper attracted the interest of economists and mathematicians, and Leontief himself in particular.[6,7]

In their works both Remak and Leontief adopted a concept of value referring to the exchange relations deduced from the relation of production (having nothing to do with consumer's judgements and tastes) and in their description of the economic process both authors used the classical idea of circular flow. There were numerous sources of this idea: reading of the

classics – Leontief remembers that, when he was a student, he read the works of economists of the 17th and 18th centuries, François Quesnay included (Kaliadina, 2006); the German debate on classics, Marx and Walras in which Bortkiewicz was involved; last but not least also the debate of the 1920s on planning in the USSR. In fact, at the beginning of the 1920s, the Soviet planners P. I. Popov and L. N. Litosenko referred to both Quesnay's *Tableau* and Marx's schemas of reproduction in their development of a sort of primitive input-output matrix of the Soviet economy as a whole. Leontief was marginally involved in that debate, writing an article in 1925, when he was a student in Berlin, on "the balance of the national economy of USSR", a critical commentary on the Popov report (Leontief 1966).[8]

2.2.2. Leontief's and Remak's models and their relation with Sraffa's model

<u>*Leontief's model*</u>. In an 'excursus' at the end of the first part of his Ph.D. dissertation, Leontief outlined a model to solve the problem of the "general conditions which must be fulfilled within the framework of a circular flow" (Leontief 1928, p. 193). Leontief assumes: the production process is circular; the system is a simple exchange economy without profits; the total quantities of each product and their distribution between the various uses in the circular flow are known; the production is single-product; constant return to scale. Finally, "in order to avoid unnecessarily complicated calculations" (ibid., p. 197), Leontief examines only "the familiar elementary case of a two-branch circular flow system" (ibid.). In this pattern, starting from the basic production relationship

$$aA + bB \rightarrow A$$

$$(1 - a)A + (1 - b)B \rightarrow B$$

(where A and B are the quantities produced of two commodities and a and b the shares of these commodities used in the production as means of production and consumption)

we obtain two “general exchange formulae”:

$$Ap_1 = k\,(aAp_1 + bBp_2) + m\,[(1 - a)Ap_1 + (1- b)Bp_2]$$

$$Bp_2 = l\,(aAp_1 + bBp_2) + r\,[(1 - a)Ap_1 + (1- b)Bp_2]$$

where the unknown productive coefficients are subject to the following constraints:

$$k + l = 1$$

$$m + r = 1$$

If we set one of the two prices, for example p_2, to 1, “we are presented with four equations and five unknowns, and no clear resolution of this problem is possible” (ibid., p. 194). In order to reach a solution, Leontief applies the “value principle”, i.e. “the tendency to attribute the same value to cost items … and the output items produced from them” (ibid., p. 196). Through a series of steps Leontief presents the following reduced system:

$$Ap_1 = kAp_1 + mBp_2$$

$$Bp_2 = (1 - k)\,Ap_1 + (1 - m)\,Bp_2$$

And the equation system becomes:

$$Ap_1 = k\,(aAp_1 + bBp_2) + m\,[(1 - a)Ap_1 + (1- b)Bp_2]$$

$$Bp_2 = (1 - k)\,(aAp_1 + bBp_2) + (1 - m)\,[(1 - a)Ap_1 + (1- b)Bp_2]$$

$$Ap_1 = kAp_1 + mBp_2$$

$$Bp_2 = (1 - k)\,Ap_1 + (1 - m)\,Bp_2$$

or, reducing it to two equations with some algebraic calculations,

$$Ap_1 = aAp_1 + bBp_2$$

$$Bp_2 = (1 - a)\,Ap_1 + (1 - b)\,Bp_2$$

Remak's model. Remak (1929) considers a closed economy without wage and profits in which the quantities of the various commodities produced and consumed are known. The problem for such a system is the determination of a set of prices which would "provide the basis for a financially viable economy" (Remak 1929, p. 271).

Under the usual assumptions – the production process is circular, the total quantities of each product and the productive technology are given; the period of production is the year; the system is in a stationary state – the problem to solve is the following: given the technical coefficients of production, $a_{ij} \geq 0$ (i, j = 1, … , n), i.e. the unit of each commodity the industry *i* furnishes to the industry *j* to produce a unit of the commodity *j* (which could be positive or zero in the case in which the industry *i* does not supply anything to the industry *j*), to determine the prices of the commodities y_i (i = 1, … , n) so that each industry's income from the supplied commodities is equal to its expenditure of the received goods. This system is called a 'superimposed price system' or 'rational price system' i.e. a system of prices which satisfy the condition that each price covers the costs of the goods used in production and the productive consumption. In an *addendum* to the 1933 article, Remak emphasises that these prices do not depend on the law of supply and demand.

Mathematically: $\sum_{j=1}^{n} a_{ij} y_j = y_i \sum_{k=1}^{n} a_{ki}$ (i = 1, …, n)

or $\sum_{j=1}^{n} a'_{ij} y_j = 0$ (i = 1,…, n)

with $a'_{ij} = a_{ij}$ $(i \neq j)$

$a'_{ii} = a_{ii} - \sum_{k=1}^{n} a_{ki} = -\sum_{k=1}^{n} a_{ki}$

As the determinant of the system of equations becomes zero,

$$\sum_{j=1}^{n} a'_{ij} = \sum_{j=1}^{n} a_{ij} - \sum_{j=1}^{n} a'_{ij} = 0$$

equations are linearly dependent and the system has at least one solution different from the trivial one. In the rest of the article, Remak demonstrates the existence of an economically relevant solution, i.e. where prices are ≥ 0, unique up to a factor of proportionality.

Sraffa's, Leontief's and Remak's models. Many contributions have emphasized the common features of the contributions of Sraffa, Leontief and Remak (for example: Kurz and Salvadori, 2006; Parys, 2014). Indeed, the analytical comparison of the three models show that Remak's and Leontief's models formally correspond to Sraffa's production for subsistence model, the only difference being that while Sraffa's and Remak's models consider *n* commodities, Leontief's model is limited to the case of two commodities.[9]

3. The intellectual origin of the classical conceptions of GEE

3.1 The 'Bortkiewicz connection'

Since the mid-1970s, in relation to the increasing interest in Sraffa's work, some scholars have emphasized the existence of a connection between Sraffa's *Production of Commodities* and the works of some Russian-German mathematical economists from the beginning of the century – principally Bortkiewicz and Dmitriev – who demonstrated the logical consistency of the classic approach (Nuti, 1974; Marchionatti, 1981; Gilibert, 1990; Samuelson, 1991; Marchionatti and Fiorini, 2000). This link, essentially *via* Bortkiewicz, can be considered to exist also in the case of Leontief and Remak. These scholars do not suggest that our authors explicitly took as a starting point for their works, or found the idea for them, in those mathematical economists. As regards Sraffa and Leontief no such evidence exists anywhere in their published or unpublished papers, and scarce evidence exists in the case of Remak.[10] What I suggest here is that their work – the theoretical genesis of their work – is totally

understandable only on considering the intellectual background of the German world in which the Russian-German mathematical economists made their contributions: a context characterized by rich debates on the relationship between economics and mathematics, on Ricardo and Walras, and on Marx and the transformation problem which informed the successive theoretical reflections. In that context the figure of Bortkiewicz was fundamental, because of the importance of his theoretical work, and of the role that he played as a connection between, and an end point of, different paths of research.

3.2. The introduction of mathematical reasoning into economics and the German debate on Walras, the classics and Marx

The introduction of mathematical reasoning into economics between the 1870s and 1890s represented a crucial change and was at the centre of a lively discussion. For economists like Walras and Pareto mathematics was a tool necessary to understand general relationships between variables and to make rigorous deductions through the representation of an economy by systems of simultaneous equations; and the GEE was the field of application for mathematics *par excellence* (Marchionatti, 2004a). In the gradual establishment of a mathematical school in Russia and in the German area, Walras was an essential reference, as Bortkiewicz, at that time a young student interested in the new economics, wrote in 1887 to the French economist (letter to Walras of 24th October 1887, in Jaffé 1965, vol. II).[11]

Lexis (1885) was the first in Germany to review Walras constructively, when the German translation of Walras' *Théorie mathematique de la richesse sociale* (1877) was published. The main point considered by Lexis was indeed the mathematical mode of exposition: it makes it possible to present theories with formal rigour and, in Walras' system of simultaneous equations, with great generality. However Walras, Lexis continued, took the abstract method

to its extreme, revealing both its merits and its limitations: Lexis emphasized that the abstract conception of perfect competition was of great value in giving direction to the formulation of economic principles; but, on the other hand, this conception neglected too many factors of decisive importance in the real world for it to serve as a guide to policy – it recalled Edgeworth's idea of excess of abstraction regarding the Walrasian system (Marchionatti, 2007).[12] Some years later, Bortkiewicz (1906-7 and 1921) maintained that Walrasian GEE analysis could be interpreted as a wider setting within which to insert the cost equations determined in the Ricardian model. Analogously, Dmitriev (1904) had promised an organic synthesis of the labour theory of value and the theory of marginal utility. However, the distinctive feature of the works of our authors is that, in their examination of the theories of prices, they always used a 'classical' approach: in fact, the common subject of their inquiry was the defence and mathematical reformulation of Ricardo's and Marx's classical theory of prices against the criticisms of Walras (on Ricardo) and Böhm-Bawerk (on Marx).

In fact, the classics (from Quesnay to Ricardo) and Marx are protagonists of that intellectual debate. Their importance is strictly connected with the issue of the use of mathematical method in economics and its application to the solution of Marx's so-called problem of transformation of values into prices of production. In the German economic debate from the 1880s to the end of the 1890s, Marx's work on the occasion of the publication of the second and third volumes of *Das Kapital* respectively in 1885 and 1894, played a major role. Their publication generated an extensive debate among economists and philosophers from several European countries (Marchionatti 1998). Between 1894 and 1898 the centre of this debate was in the German world. The best-known work was Bohm-Bawerk's 1896 essay, from then on considered *the* marginalist critique of Marx. There were also other non-marginalist contributions of worth and significance, in particular those of Lexis, Sombart and Conrad Schmidt (a Marxist scholar much appreciated by Engels). Rooted in the German cultural

context, influenced by positivism and neo-Kantianism, the latter contributions, which are all critical of the marginalist school, offered a new reading of Marx. Compared to the initial ideas of Marx's economics, they represented an epistemological discontinuity, in that for the first time they located the basic theoretical opposition between Marx (and the classics) and the Marginalists within the framework of objectivist-subjectivist positions.

At the centre of the discussion was the issue of the transformation of values into prices of production. It had already been raised by several authors in the 1880s within a debate begun by Friedrich Engels in 1885, on the occasion of publication of the second book of *Das Kapital*. Engels launched what Bohm-Bawerk termed 'a regular prize-essay competition' on the average rate of profit and its relation to the law of value; as a result, a number of authors devoted themselves to the task. In a long review of Marx's work, Lexis (1895) related Marx's ideas to those of Ricardo and Quesnay. With regard to Ricardo, Lexis maintained that Marx resembled Ricardo 'in method and in cast of mind', and that Ricardo supplied Marx with the point of departure for a system which could be essentially considered a development of Ricardo's own. With regard to Quesnay, Lexis considered Marx and Quesnay comparable 'in their mode of conceiving economic phenomena'.[13] As regards the transformation problem, Lexis argued that Marx's solution was simple and obvious. He remarked that from a mathematical point of view it was simple to determine a general average rate of profit, but "how this is carried out in the actual world Marx explains in a manner far from satisfactory" (Lexis 1895, p. 10). A major point discussed by Lexis was the concept of value: it is conceived as "a purely theoretical conception" (ibidem, p. 11). Lexis's interpretation of value as a theoretical concept without any real existence, though useful to give order to the analysis, was sustained by Sombart (1894) and Schmidt (1895). From an epistemological point of view, the Lexis-Sombart-Schmidt thesis that value is a fiction constitutes a break in the discussion of the Marxist theory of value. However, in its original formulation it seemed incomplete: did the

purported abstraction admit empirical verification or did it represent a way of avoiding it? Sombart apparently subscribed to the second interpretation, while Schmidt and Lexis showed interest in actually testing the hypothesis. After Bohm-Bawerk's 1896 critique, this would become a crucial issue. The challenge that he launched to the 'Marxists' was to show that an objective theory of prices was possible: this was the task taken up by Bortkiewicz and some Russian mathematical economists.

3.3. Bortkiewicz and the German-Russian mathematical economists

3.3.1. Premise

Bortkiewicz may be considered the most eminent representative of that small group of scholars who, between the second half of the 1890s and the first decades of the 1900s, established an original research programme characterized by the application of mathematical method to the theory of prices in a classical political economy framework (see Marchionatti and Fiorini 2000 and the references there quoted). Together with Bortkiewicz we must mention Dmitriev and Georg von Charasoff.

Vladimir K. Dmitriev, a scholar well known in his days not only in the Russian and German world (Bortkiewicz, 1931; Zauberman-Nove 1961, Nuti, 1974 and 1987), wrote three essays entitled *Ekonomicheskie Ocherki* (*Economic Essays on Value, Competition and Utility),* published in 1904, the first of them, on "The theory of value of D. Ricardo", already published in 1898. The Russian mathematician Georg von Charasoff (Egidi and Gilibert, 1989; Stamatis, 1999; Gehrke ,2013; Parys, 2014) was the only 'Marxist' of this group of scholars – "not a Marxist in the conventional sense of the term", as he wrote to Kautsky in 1909 (quoted in Gehrke 2013, p. 22). As far as we know, he worked principally on his own, starting from the study of the classical economists and Marx, the works of Menger, Bohm-Bawerk and Walras,

as well as his contemporary literature, in particular Bortkiewicz's articles. He published two books in 1909 and 1910 which should have formed part of a trilogy devoted to the study of Marxist economic theory. His work received limited attention in Germany - it was indeed strongly criticized by the Marxist Otto Bauer (1911) and remained unknown abroad. *Ex post* these mathematical economists, as a whole, deserve the name of the Russian-German mathematical "school" of political economy. The use of mathematical method and a classical approach to the theories of prices were the essential elements unifying their contributions.

3.3.2. Bortkiewicz's, Dmitriev's and Charasoff's contributions

Starting from Lexis' criticism, Bortkiewicz (1906 and 1907) tried to answer the question: was Marx's transformation correct from a mathematical point of view? He attempted to formulate a logically-unassailable objectivist approach to the classical and Marxist theory of value and distribution.[14] Like Lexis, Bortkiewicz assumed value and price to be "a purely theoretical structure" (Bortkiewicz, 1907a, p. 6), where the price (the natural price of classical economists) represented a higher degree of approximation to reality than the value. Bortkiewicz considered Marx's attempt to calculate value into prices "as a failure" (ibid., p. 13); however, he thought that "the idea of such a double calculation should not be dismissed out of hand" (ibid.). He adopted the algebraic method used by Dmitriev in his work on Ricardo. In his first essay on Ricardo, Dmitriev had rejected Walras' criticism: Walras had accused Ricardo of trying to make 'one equation determine two unknowns' by suggesting that price is determined by the cost of production, consisting of profit plus wages and profit, calculated as the difference between aggregate prices and wages. Using the mathematical method, Dmitriev was the first to demonstrate rigorously that Ricardo's theory was immune to Walras' criticism; he was also the first to define the mathematical core of classical thought on value and to define Ricardo's inquiry as dominated by his theory of profit – a thesis then sustained by Bortkiewicz

and later by Sraffa. Adopting Dmitriev's approach Bortkiewicz demonstrated that the price could be determined simply by objective factors (the condition of production) independently of subjective considerations. We may refer to a single model Dimitriev-Bortkiewicz because, even if the purposes of these economists are different – the defence of Ricardo's theory of value from Walras' criticism in the case of Dmitriev, and the solution of Marx's problem of transformation in the case of Bortkiewicz, from the mathematical point of view their models are the same (see the exposition of the model in Marchionatti and Fiorini, 2000): in contemporary terminology, Bortkiewicz-Dmitriev's model is a system of commodity production by means of dated labour.

Charasoff (1909 and 1910) took up the transformation problem with the aim of rigorously reformulating the theoretical foundations of the critique of political economy. He mantained, *à la* Bortkiewicz, that the transformation procedure followed by Marx was not mistaken but only incomplete, as it could be interpreted as the first step in an iterative transformation process capable of approaching the solution of production prices. Von Charasoff introduced two basic concepts: the "series of production" and the "original capital". The series of production is similar to the "reduction to dated quantities of labour" approach, with the difference that Charasoff's series of production represents a reduction to original capital. The original capital is, for Charasoff, the 'real' capital in the specific case of a competitive economy where, given the necessaries for the workers, profits are completely invested. In this situation, the proportions of the various industries are equal to the proportions of original capital, so that the rate of growth of the economy is equal to the rate of profits.

Charasoff's schema present analytical similarities with Sraffa's standard system (one of the theoretical developments of the late 1930s and early 1940s) and Von Neumann's model (1937). This makes interesting to inquiry the possible relationship between the classical contributions of the Twenties and von Neumann's model.

4. On the relationship between the classical conception of GEE and von Neumann's model

The neo-walrasian programme was developed in Vienna in the *Mathematisches Kolloquium* founded in 1928 by the mathematician Karl Menger in the context of the conception of the scientific discourse prevailing between the *Kolloquium*'s members. i.e. Hilbert's mathematical formalism. Starting from the Walras-Cassel model (Cassel, 1899), Karl Schlesinger and Abraham Wald reformulated the system in terms of inequalities and Wald demonstrated the existence of an equilibrium for the system. Von Neumann's circular model of an economy expanding at a uniform rate (1937) offered an advanced mathematical formalization of the problem of the existence of an equilibrium, where equilibrium is defined as the state where the whole economy expands without change of structure.

Von Neumann published his paper in the proceedings of the *Kolloquium* in 1937 on the invitation of Karl Menger, but the paper had been already presented in 1932 at the *Princeton Mathematical Society* - he emigrated to the United States in 1930 upon invitation by the University of Princeton after obtaining an outstanding reputation as a mathematician in his European years. After studying at the University of Budapest, Berlin, Zurich, Göttingen (with Hilbert) – at the same time establishing intellectual relationship with the most important scientific and philosophical centres of Central Europe like the *Wiener Kreis* and the *Mathematisches Kolloquium* in Vienna - he had obtained his habilitation (necessary to become a full professor at German universities) in mathematics at the Friedrich Wilhelm University in Berlin in 1927 and then started his lectures as a *privatdozent* at the same University in 1928. The intellectual origin of his model just seems to derive from the Berlin debates at the end of the 1920s in which he was involved. Nicholas Kaldor, who in the academic year 1925-26

studied economics in Berlin, in his reminiscences of that period remembers that von Neumann told him that "they [Walras's equations] provided no genuine solutions, since the equations can result in negative prices (or quantities) just as well as positive ones" (Kaldor 1989, p. viii). These thoughts originated in his Berlin years when von Neumann attended the seminars organized by the Hungarian physicist Leo Szilard and the Berlin Mathematical Society seminars. Szilard organized an informal study group to hear lectures on the role of mathematics in other disciplines. In this group, probably in 1928, the Russian-born economist Jacob Marschak, who had attended Bortkiewicz's lectures in 1919, was asked to give a lecture about economics and he talked about the Walras-Cassel equations of GEE. Weintraub (1983, p. 74, note 16) reports a story by Axel and Earlene Leijonhufvud: one of the mathematicians who attended the seminar "became extremely agitated and began a stream of interruptions, arguing that the equilibrium relationships should be described by inequalities instead of equations. That mathematician was von Neumann". [15] Weintraub concludes that "this story suggests that the genesis of von Neumann's *Ergebnisse* paper was quite specific and roughly contemporary with von Neumann's paper on game theory. The min-max idea, the duality ideas, and the strategy of proof to be used later for the fixed-point theorem are found in each paper. The papers appear, then, to be naturally related not only by content, but also by place of origin". This statement can be shared, but requires to be completed with another important fact (see Wittmann, 1967; Kurz and Salvadori, 1993; Leonard 1995): von Neumann was also influenced by the price model formulated by Remak (in particular by his circular representation of production process) who was at the University of Berlin in the same period as when Von Neumann was there (Ulam, 1958; Rowe, 1998) and presented it in a Mathematical Society seminar.[16]

Therefore, the intellectual origin of Von Neumann model seems to be found in the Berlin environment, in the context of a wide discussion involving Walrasian issues as well as classical

references, in the same years when Remak and Leontief were elaborating their models in the Bortkiewicz's circle. In fact, in the literature many authors have recognized that von Neumann's model shows similarities with the classical approaches (Champernowne, 1945-46; Napoleoni, 1965; Schefold, 1980; Kurz and Salvadori, 1993 and 2001). The objective affinity between von Neumann's view of the economy and the classical economists' approach was emphasized for the first time by Champernowne in his paper accompanying the English publication of von Neumann's paper. In general, in addition to the similarities from the analytical point of view pointed out in this literature, there are two fundamental points of similarity traditionally emphasized: a) the adoption of the same representation of the production process: the classical conception of the productive process as a circular flow; 2) the fact that distribution is not conceived as a particular aspect of the price formation, as in Walrasian model, but the distributive variables are treated in an asymmetrical way: given one of the variables (the wage for example) the other (profit or interest) is determined residually. Despite of these similarities, we have to stress what may be considered the crucial difference between von Neumann model and the classical approaches: the conception of the relationship between economics and mathematics.

The adoption of the Hilbertian mathematical formalism in economics by the Viennese theoreticians of GEE, and Von Neumann in particular - who, in his 1937 paper, was the first to apply the axiomatic approach in economics in a totally coherent way, in the sense that the concern for the economic interpretation of the model disappears - reflected in a peculiar conception of economics as mathematical science. On the contrary, Sraffa, Leontief and Remak refer mainly to a pre-axiomatic mathematical tradition typical of the nineteenth century, "that was of a 'concrete, constructive, algorithmic character'" as Velupillai (2008, p. 339) writes about Sraffa's mathematics, where the conception of the mathematical model is always dominated by the problem of the relation between the model and the real world. Remak and

Leontief, emphasizing the theme of computability of the model strongly express this perspective. Leontief highlighted the aversion of the economists for systematical empirical inquiry and criticized that dominating formalist line of thought admonishing his colleagues for overemphasizing mathematical and theoretical elegance at the expenses of empirical verification (Leontief 1982). Sraffa certainly shared this opinion. The mathematical language and the demonstrative techniques adopted in *Production of Commodities*, as it is well known, raised perplexities among scholars used to the neo-Walrasian GEE theory of the 1960s. Sraffa, in contrast to the convention among contemporary mathematical writers of reducing explanations to a minimum and stating assumptions as concisely as possible, preferred to give examples and descriptions of his argument, in order not only to ease the reader's task of comprehension but also to maintain some references to empirical economic processes. We can say that the purpose of our authors may have been to show that the language of rigour in economics does not necessarily imply the adoption of a language reduced to a manipulation of symbolic strings.

5. Conclusions

Traditional reconstructions of the history of the GEE theory between the two world wars neglect some contributions which tried to offer a version of the theory in a classical perspective, those of Sraffa in Cambridge, and those of Leontief and Remak in Berlin. This paper has addressed this gap and has showed that the intellectual origin of those contributions is rooted in the German cultural environment of the late nineteenth/early twentieth centuries, where the approaches of the classical and Marxist economists were debated and compared with the Walrasian mathematical approach, flowing into the original contributions of the German-Russian mathematical economists. In their works the classical revival appears to have

been an attempt to re-elaborate the theory of the cost of production using the mathematical method to show that price can be reduced to elements independent of the subjective aspect of economic calculus. This was their legacy to the successive theoretical reflections on the 1920s. Renouncing what they considered to be the metaphysical investigation of the 'absolute value' problem, which had occupied Ricardo and Marx, they extracted the mathematical core of the classical theory of the cost of production, developing a scheme of production of commodities by means of commodities (or dated labour), in which the old theory of labour value represented only a particular case. According to Bortkiewicz and Dimitriev the mathematical, in particular the algebraic, method of exposition appeared to be the satisfactory expression for their "superior standpoint". Their position did not express a conflicting but a complementary view of the relationship between classics and Walras (considered the leader of the mathematical school). Undoubtedly, this theoretical attitude remained in Leontief, and this seemed to contemporaneous authors, used to a conflicting view of the relationship between marginalist and classical approaches, a sort of ambiguity. A conflicting view was certainly characterizing Sraffa's research. In his intellectual process of moving from a Marshallian theoretical context to a new foundation of the theory of value based on classical economics, Sraffa adopted a research programme along the lines of the Russian-German mathematical school of political economy, but with a crucial difference. The aim of the Italian economist was not simply to prove the logical consistency of the classical approach, but rather to demonstrate that it could be a *general* theory, *alternative* to marginal theory. From this point of view, the issue of the returns to scale was crucial. Sraffa, taking in consideration "no changes in output", adopted a theoretical position very different from those of Dmitriev and Bortkiewicz, whose models assumed constant returns of scale. In this theoretical contest it becomes clear that those interpretations which consider the schema of *Production of Commodities* to be a Walrasian semi-general economic equilibrium in which prices are independent of demand, as

follows from the application of the non-Substitution Theorem, is unable to grasp the purpose of Sraffa's research programme and his theoretical originality and meaning. In actual fact, within the rigorous limits of the assumptions on which *Production of Commodities* is based, Sraffa – we can call him the purist of neo-ricardian theory just as Wicksteed, as Sraffa himself had written in 1925, was *the* purist of marginal theory - gave an anti-marginalist significance to the neo-Ricardian models of the Russian-German mathematical economists.

As the paper has shown, the intellectual context in which Sraffa's, Remak's and Leontief's contributions emerged was the same in which the reflection on the Cassel-Walras model was developed. However, the consequences of the neo-walrasian reflection were different and divergent from those of the classical conception of GEE. The crucial difference deals with the relationship between economics and mathematics and the deriving conception of the nature of economics. The prevalence of the neo-walrasian perspective, in which abstract theory dominates empirical evidence, in the post-war period has deeply affected the economic thought shaping the epistemological context of the contemporary mainstream economics. Contemporary debate in methodological and theoretical fields of economics seems to question this dominant epistemological perspective. In this reconsideration, the richness of the past debates in economic theory, as our story of the theoretical reflection on the GEE between the wars has shown, may be of some help in making economists aware of the potential different paths of research available in pursuing rigour *and* relevance in their models.

Notes

1. Samuelson was disappointed that Leontief and Sraffa never cited each other, and seemed to pay no attention to the other's work. On the basis of new archival evidence, Parys (2016) makes significant corrections to the widely held views about zero personal contacts and zero attention to each other's work.

2. Leontief himself had an ambiguous position in regard of the characterization of his input-output model as Walrasian.

3. In a remarkable review of the book, Newman (1962) ascribed the differences of opinion to the extreme difficulty of the work, maintaining that the "main trouble" lies in wrenching oneself out of the more usual Walrasian approach to general equilibrium, and in substituting a Ricardian viewpoint.

4. Although his dissertation was first submitted in 1911, Remak obtained habilitation in 1929, on his third attempt after the first two had failed, essentially due to his strange behaviour which provoked strong objections from some of his professors. The positive change of opinion of Richard von Mises (the brother of Ludwig) who was at that time the director of the Institute of Applied Mathematics, enabled Remak to obtain habilitation. His thesis referees Issai Schur and Erhard Schmidt were the same as those who assessed von Neumann's habilitation thesis in December 1927 (I am indebted to one of the referees for this information).

5. This was a problem which Bortkiewicz had found interesting since the end of the 1880s. In a letter to Walras (23rd November 1887, in Jaffé, 1965)), the young Bortkiewicz stated the following problem, which according to him was essential in defining the relevance of political economy: economists assume the present societal organization as a given, he wrote, but, is it possible to have a theory of wealth in different socio-political regimes?

6. Dorfman (1973, p. 431) recalls "Leontief's gleeful excitement when he came across the work of Remak" and we know from Leontief's correspondence that he wrote in June 1931 to

Remak that "you turned your mind, from the mathematical point of view, to the same problems of mathematical political economy that I have arrived at from the economic point of view" (letter of June 11, 1931, Wassily Leontief Papers, Harvard University Archives, translated in English by W. Parys, 2014, pp. 1006-7).

7. Hagemann and Punzo (2007) maintain that his contributions "find their appropriate intellectual place if set against two contemporary events: the birth of the modern axiomatic approach in economics and the invention of the input-output analysis. With the former, Remak's slim work in economics shares the emphasis on the need for firm mathematical foundations to render economics an exact science; he corresponds with the latter in stressing the empirical nature of economic data and the policy orientation of modelling" (p. 1-2).

8. As far as we know, Leontief did not know the work of Father Maurice Potron (1872-1942), a French Jesuit mathematician who laid the foundations of input-output analysis in various papers published between 1911 and 1914 (see Abraham-Frois and Lendjel, 2006 and Bidard, Erreygers and Parys, 2009). Potron used matrices to describe the economic interdependency between products and branches, and he was the first to apply Perron-Frobenius theorems to demonstrate the possibility of the existence of an economic equilibrium.

9. Wittmann (1967) and Hagemann and Punzo (2007) discuss the similarity of the concept of price in Remak and Leontief (1941). While Wittmann maintains that they are conceptually identical, Hagemann and Punzo maintain that they coincide only if constant returns to scale are assumed, an hypothesis not assumed by Remak.

10. Regarding Sraffa in particular, this hypothesis has been sometimes suggested. Kept among Sraffa's unpublished papers, is a notebook on Bortkiewicz, written in the 1940s, by which time the "central propositions" of *Production of Commodities* had by then been completed (Gehrke and Kurz 2006). The possibility cannot be ruled out that Sraffa, in his Italian years, had heard about the Russian-German mathematical school of political economy, because

Dmitriev and Bortkiewicz were known in the intellectual circles of Italian economists at the beginning of the century. On this point Gehrke's and Kurz's (2006) opinion that Sraffa first became aware of Dmitriev's contributions via Bortkiewicz may be controversial, but there can be no doubt, on the basis of the evidence available, that Sraffa elaborated his theoretical position autonomously.

Regarding Leontief, Bortkiewicz himself, in a confidential appraisal which accompanied Leontief's application for the Ph.D. degree, wrote that "in developing his – in my opinion very doubtful – theoretical constructs the candidate received no guidance whatsoever from his academic teachers. He arrived at his present position quite independently, one might say, despite them. It is very likely that he will maintain this scientific point of view also in the future" (quoted in Leontief 1991 [1928], p. 179). (Bortkiewicz's evaluation of the dissertation is on file at Humboldt University and is partially presented in English translation in O. Bjerkholt (2016). We have to say that if Leontief developed his theoretical position "quite independently", the intellectual role played by Bortkiewicz probably influenced his brilliant doctoral student in the elaboration of his work.

Regarding Remak, on the other hand, Bortkiewicz's role was probably important and explicit, since the problem discussed by the young German mathematician had interested Bortkiewicz in previous years. Moreover, as Hagemann and Punzo (2007) maintain, superposed prices remind us of many passages in Dmitriev, and Wittmann (1967) supposed that Remak was probably stimulated by the work of Dmitriev.

11. On the reception of Walras' ideas in Russia see Allisson 2009.

12. Among the other points made by Lexis there is the important consideration that Walras' equation system might not have any real positive solutions or any solutions at all.

13. This resemblance between Marx and Quesnay had been emphasised in the same years by the Russian economist M.I. Tugan-Baranowskij who made use of the Marx's schema of

reproduction in his analysis of the question of the breakdown of capitalism (On Tugan see Allisson, 2015).

14. In their rational reconstruction of the long history of input-output analysis and Sraffa's theory of production, neo-Ricardians tend to consider Marx as belonging to the same tradition as Bortkiewicz and other economists, starting from Petty and Quesnay. Consequently, Marx's errors are considered not to stem from any substantive theoretical differences but are essentially technical ones caused by Marx's lack of the mathematical tools necessary to solve the problem that he addressed. However, as well argued in Kliman (2014) - and as we have emphasized in our intellectual reconstruction of the German debate - there is a profound difference between Bortkiewicz's and Marx's methods, of which Bortkiewicz seemed to be well aware.

15. The argument that the equality between the number of equations and the number of unknows does not necessary mean that the system possesses positive solutions in prices, as propounded by Von Neumann during the seminar, was at the centre of the discussion of the Walras-Cassel system in the German world in the late 1920s and early 1930s. Three papers by Hans Neisser (1932), Frederik Zeuthen (1932) and Heirich von Stackelberg (1933) almost simuitaneously pointed out in different ways the problem of existence of meaningful solutions. The problem was then adressed by Karl Schlesinger at Menger's *Kolloquium* in Vienna opening the way for Abraham Wald's work published in Menger's *Ergebnisse*, where von Neumann was invited to publish his 1932 paper. According to Read (2012, p. 12), "Neisser had been instrumental in the formation of … Vienna Kolloquium".

16. Moreover, according to Thomson (1989), von Neumann could have also read Leontief's PhD thesis.

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