De-alloying Kinetics of an Au-based Amorphous Alloy

Published version:

DOI:10.1016/j.jallcom.2011.11.087

The coordinated movements of collaborative mathematical tasks: The role of affect in transindividual sympathy

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This article examines collaborative mathematical tasks that entail sympathetic coordinated movements. We discuss the affective bonds that form when students participate in such tasks. Using Maxine Sheets-Johnstone's term "affectivity" to characterize the responsive nature of bodies, we analyse data from a teaching experiment where students collaboratively explore the dynamic aspects of mathematical figures. We work with the ancient Greek concept of 'sympathy' to study the complex ways that multi-body assemblages actively coordinate their movements in the midst of a mathematical task. We include here diverse kinds of often imperceptible body movement (gesture, face, eye, foot, etc), and discuss how mathematical concepts are assembled through such movements. Our analysis bridges three scales: (1) the micro-phenomenological scale of the pre-individual affect, (2) the individual scale of human movement, and (3) the transindividual scale of collective endeavours.

Keywords: mathematical tasks, movement, affect, emotion, sympathy, collaborative learning

1. Introduction

In this paper we argue that sympathy sheds light on the complex process of collaboration that is involved in particular kinds of mathematical tasks, when students explore mathematical concepts through coordinated but diverse movements. We include here diverse kinds of often imperceptible movement (gesture, face, eye, foot, etc). Our aim is to show how particular collaborative mathematical practices can contribute to a kind of transindividual sympathy that can fan out across the classroom. We are studying *sympathy* as it operates across bodies often unconsciously, whereby students 'feel' for each other and follow each other often without explicit or rational choice, but in ways that involve mutual care. The teaching experiment discussed in this paper was deliberately designed to demand coordinated movements from participating students. In other words, we designed the classroom tasks so that students might develop skills at exploring mathematical concepts through responding to each other, and so that an embodied shared *response-ability* might emerge. As Massumi (2015) suggests, one must prime or condition the environment in order to create opportunities for transindividual sympathy to emerge. Notably, our mathematical tasks focus on the dynamic and genetic nature of mathematical concepts.[1] We believe that this allows us to show how *distinct* mathematical concepts are implicated in particular affective bonds, explored as they are through bodily interaction. Different concepts entail different movements, and thus different kinds of affectivity.

Our approach focuses on affect as part of an event, rather than on 'emotion' as a human trait or expressive behaviour that is ultimately at source individual. Thus we push past a theory of emotion that rests on individualism, using ideas from contemporary theorists who have shaped the affective turn in the post-Humanities

[1] Concepts can be conceived in at least two ways: by genesis (how one might create it) and by property (how one might recognize it).

This is the accepted version of the original published on ZDM: The International Journal on Mathematics Education, 51(2): 305-318. DOI: 10.1007/s11858-018-1007-4.

since the 2000s. We frame our approach in terms of the work on affect by Maxine Sheets-Johnstone (2009a; 2009b; 2012) and Brian Massumi (2002; 2015), in order to (1) explicate the relation between affect, e/motion and coordinated movement, and (2) reveal how the dynamic intensity of mathematical concepts can be tapped pedagogically, in such a way as to contribute to sympathetic collaborative classroom efforts.

In the last sections of the paper, we analyse a case study to show how pre-individual micro-scale affects contribute to group endeavours. We focus on the coordinated movements of two girls in a grade nine classroom, Barbara and Lucrezia, as they participate in the teaching experiment. We track the way that the task brought forth opportunities for these two girls to develop new forms of relationality in their shared achievement. Moreover, we suggest that through the embodied activity, the students begin to grasp how their own coordinated movements are linked to the complex set of differentials and gradients that comprise the circle concept. Learning thus involves coordinating movements of all kinds, bringing bodies together with concepts in a sympathetic arrangement. This perspective follows the inclusive materialism of de Freitas and Sinclair (2014) who highlight the genetic and dynamic nature of mathematical concepts—numeric, geometric, algebraic, etc.—in teaching and learning and doing mathematics.

2. The affective turn

Research on the role of affect in mathematics education has been discussed at least since the 1980s, typically focusing on the relationship between beliefs, attitudes and emotions (Mcleod and Adams 1989; McLeod 1992; Zan et al. 2006). Motivation, mood and interest have also received attention (Hannula 2006) as well as emotion as a coping mechanism (Hannula 2002). As part of the socio-cultural turn, emotions have been conceptualised as socially organised phenomena that are constituted in discourse and shaped by relations of power (e.g. Evans et al. 2006; Op't Eynde et al. 2006; Radford 2015). Most socio-cultural studies of the emotional dimension of mathematics continue to assign particular emotions to particular students, who show frustration or anxiety or joy, as they encounter the socio-cultural rituals of school mathematics (e.g. Yackel and Cobb 1996; Goldin 2000; Roth 2007).

More recent attempts to move from beliefs to "affective systems" show promise in their attempt to study *ensembles* of emotions, feelings, attitudes, beliefs, and conceptions (Philippou and Christou 2002), and in their recognition that affect is dynamic and variable in intensity (Pepin and Roesken-Winter 2015). And yet we find the research therein continues to methodologically emphasize expressions of belief and value, without actually operationalizing key ideas from systems theory (Varela and Depraz 2005) and without tapping the extensive work outside of psychology on affective networks (Delanda 2011; Deleuze and Guattari 1987; Sheets-Johnstone 2009a; 2009b). Moreover, attending to the dynamics of emotional or motivational states in a classroom or other learning community are still rare (Hannula 2012). The time is now to pursue theoretical approaches that better help us follow the *movement* of affect across learning events with multiple and diverse participants. This involves delving deeper into the affective nature of mathematical practices which are lived in and through material bodies. For that purpose, we turn to recent work on affect in the humanities.

Since the 2000s, scholars across the humanities have pursued what is known as the *affective turn* (Clough and Haley 2007; Gregg and Seigworth 2010). Shifting away from psychological approaches that focus on affect as individual judgements of value (like, dislike, happy, unhappy), this new approach aims to study the *collectively dispersed nature of affect across a material ecology* (Gregg and Seigworth 2010). In particular we follow Massumi (2002; 2015) and Sheets-Johnstone (2009a; 2009b) in studying affect and emotion less as that which is produced and possessed by a psychological subject, and more as an impersonal intensive flow across relational and provisional learning assemblages. We caution to add that the word 'impersonal' is used to emphasize the affective *system*, as a generalized group experience. Different students feel affect differently (and "personally") because affect circulates and contracts with different intensities.

The affective turn is significant for how it moves away from the individualistic theories of cognitive psychology towards a renewed interest in (1) the somatic and embodied expressions of affect, as bodily organic forces rather than ideational enactments of interior states and (2) the transindividual collective nature of circulating affect. Attending more carefully to the flow of affect during a learning event breaks with the individualism of other approaches. Circulating affects traverse the individual somewhat indifferently; in other words, the flow of affect contracts and expands across an event, recruiting our bodies and participation to varying degrees. Of course there are different ways of partaking in this flow of affect, and different degrees of conscious involvement. The notion of "degree" is crucial here, as it underscores how affect can be intensely concentrated or contracted in one body and not another with varying intensity.

3. Affect or emotion?

The words emotion and affect are commonly used together, not always with too much care for their different meanings. Here, we draw on the work of Maxine Sheets-Johnstone (2009a; 2009b; 2012), to help distinguish these terms, and to build a theory of affectivity that we will use in analysing our case study. Sheets-Johnstone (2009a) describes *affectivity* as the fundamental "responsivity" of life, drawing on a long line of phenomenology. Affectivity characterizes the way bodily activity is implicated in collective feelings. Affectivity thus characterizes the responsive nature of bodies, how they turn away or lean in, and at the same time how they join with other bodies in coordinated movements. Animate forms of life enjoy (for good or bad) a congruency between affect and bodily motion, precisely because affect is lived through bodily movement. The dynamics of feelings (of comfort, agony, excitement, …) coincide with micro-facial expression, minute changes in bodily posture, foot-tapping rhythms, changes in heart rate, etc.

Sheets-Johnstone (2009a) posits that "the affective and the kinetic are clearly dynamically congruent; emotion and movement coincide" (p. 377). Through attention to the micro-movements and pre-individual affective-kinetic dynamics of everyday life we can begin to understand the emergence of what we typically call emotion. For Sheets-Johnstone, emotions are not enacted, but *emerge in* *movement*. Enactivism, she suggests, falls short of recognizing this powerful "spatio-temporal-energic" dynamism that saturates all activity. Moreover, it fails to grasp the dynamically congruent relationship between affect, movement, and concept. She is at pains to show how emotions are not only "coping mechanisms" that evaluate or appraise or cope with the sudden break-down of rational discernment. She critiques the early systems theorists such as Varela (Varela 1999; Varela and Depraz 2005) who treat emotions as such when they study them only as responses to something not working or to surprises.

In avoiding the term "enactment", we too want to resist the tendency to define emotion as "a movement outward". This way of thinking about emotion has perhaps fuelled theories of embodiment that treat bodily movement as the *externalization* of inner immaterial feelings. Contesting this approach, Sheets-Johnstone (2009a; 2009b) points out that etymologically the word 'emotion' first signified the migration of peoples and geological transformations, and only in the 18th century took on the psychological flavour of "agitations or stirrings of mind, feeling, passion" (OED). She emphasizes the earlier meanings to argue that emotions *are themselves motion*, and do not connote motion in some indirect fashion, where one represents the other. She puts it concisely: "emotions move through the body at the same time that they move us to move" (Sheets-Johnstone 2009a, p. 379). They do not 'motivate' motion, as though some distinct interior force, but they do *inform* motion "every step, turn, gesture, clenching or quivering of the way" (ibid., p. 379).

The shuddering, trembling, quaking, constriction and heaviness that we feel at certain times are the thoroughly corporeal happenings of anger, fear, joy, anticipation, etc. Emotions are thus not states, but are *moving phenomena*, because of a "natural binding of affective and tactile-kinesthetic bodies" (Sheets-Johnstone 2012, p. 399). Accordingly, she suggests that feelings of fear are "dynamically congruent" to kinaesthetic feelings of running away, while feelings of joy would be dynamically congruent to kinaesthetic feelings of moving towards. Despite the insights of such an approach, one has to be careful to consider the distinctive geopolitical coordinates of different bodies, and attend to the multiplicity of agencies in any event. We cannot and should not claim that any movement is felt the same way by all people or on all occasions. When Sheets-Johnstone (2009a) describes the affective kinetic dynamics of *joy* as that which "spatially expand the body outward and infuse it in a lightness and buoyancy that are spatially and temporally open-ended" (p. 395), we are left to wonder how she addresses the fact that such an expansive movement is joyful in certain cultures and not others. This problem needs to be addressed, in that events are always populated by multiple bodies with varying agencies. Delight, grief, remorse, all move the body in different ways. How many different congruent movements inform these feelings as they circulate across an event? Our approach aims to attend to the important tensions and indeed incongruencies sustained in *shared endeavours*. In the next section we will turn to the concept of sympathy to help deal with this particular issue in the work of Sheets-Johnstone.

4. What is sympathy?

The word sympathy comes from ancient Greek (sumpátheia) and refers to the state of *feeling together*, derived from a composite of *fellow* and *feeling*

(Schliesser 2015). Sympathy is a complex concept with a complex history, but always seems to reference the way that two or more bodies can creatively and actively share or coordinate their affect. Over the centuries, the notion of sympathy has been used to describe all sorts of activity—everything from contagious yawn catching to cosmological harmony. Sympathy was an important part of Stoic natural philosophy, and was used to explain natural phenomena like magnetism and the molecular bonding of certain materials, but also the joining of mind and body. Sympathy explained the coaffection of mind and body, and also the forces of attraction that operated in the physical world (Brouwer 2015).

It was in the 18th century that sympathy was used to describe both the corporeal "mechanical communication of passions and feelings" as well as processes of identification (with others) using the imagination and reason (Hanley 2015, p. 172). One can identify the complex ideas at work here—the *contagion* of feeling, the *common sense* or shared sensibility, and the *compassion* for the other. Within physiology, sympathy was studied as an 'extension of sensibility'. Scientists studied the "action of sensation, the coordination of organs in the body, and the 'social principle' that allows 'fellow-feeling' to emerge in a society." (Forget 2003, pp. 291-292).

The emergent ethics of sympathy during this period can be characterized in terms of: "its foundations in epistemic associationism, its role as an action-motivating sentiment, and its relationship to self-interest and self-love." (Hanley 2015, p. 174). Sympathy thus involves an association achieved through imagination and reason, as well as an ethical or perhaps normative action to modify one's own actions so as to *feel with* the other. There is no uni-directional sympathy—there is always at least two different agencies engaged. Sympathy is a kind of *agreement* between bodies when they are mutually affected by each other.

It's important to distinguish sympathy from empathy, because the latter concept—first named by the American psychologist Edward Titchener in 1909—performs a kind of erasure of otherness (Debes 2015). Empathy claims to feel what another feels, and often involves misrecognition and even appropriation. Emerging out of the problem of "other minds", and how to feel what they feel, theories of empathy tend to erase the differences between the agents. In contrast, we start very differently, arguing that sympathetic bonds are the bedrock of learning and the condition for the emergence of independent minds.

Sympathy is thus "something to be reckoned with, a bodily struggle" because it is not a matter of identification or 'putting oneself in the other's shoes' but a matter of modulating related movements —a process of *becoming other that does not erase the other* (Deleuze and Parnet 2007, p. 53). A sympathetic coordination is not a bland alignment, nor an identification amongst parts, nor the creation of a unified homogeneous assemblage, but rather describes the assembling of heterogeneous agencies and powers. For instance, a symbiotic relationship entails a *sympathetic* agreement between two very different bodies that form together a productive assemblage without erasing their distinctness (i.e. the orchid and the wasp; see Deleuze and Guattari 1987).

The challenge we have taken on in this paper is to show how the concepts of affectivity and sympathy can shed light on mathematics teaching and learning. In the remaining sections of this paper we focus on data from an Italian high school classroom where students work collaboratively with Wii graphing technology. The students are tasked with using the Wii graphing technology to create figures—such as the rectangle, rhombus, and circle. We examine episodes in the classroom when the teacher, a researcher, and a pair of girls pursued this aim. The task explicitly demands coordinated movement, exploring the various motions that engender mathematical figures. Our analysis points to how diverse movements at micro-scales are also coordinated. We show how the participants *learn* to respond to each others' movements, increasing their ability to respond (they become response-able) while they make and explore mathematical concepts. This speaks to the ethics of the mathematical task, and the degree of care and co-affectability between participating students.

5. The teaching experiment

5.1 Wii graphing technology and context

This article centres on a teaching experiment using WiiGraph in a classroom, where the graphical output was displayed on an interactive whiteboard. WiiGraph is an interactive software application that uses Wii remotes' multiple features to detect and graphically display the location of two users as they move along life-size number lines (Nemirovsky et al. 2012[2]). The experiment took place in a secondary school in Northern Italy, as part of a wider study carried out during regular mathematics lessons. The study involved a class of 30 grade 9 students (aged 15-16) participating in activities aimed at introducing the concept of function through a graphical approach using digital technology. The mathematics teacher considered the activities as partially overlapping with the expected grade 9 curriculum, and the students knew that they would have to face a written test in the end. The classroom was composed by a majority of male students (20 male, 10 females). The teacher regularly divided the class into heterogeneous groups composed each of 1 female and 2 males.

WiiGraph produces real-time graphs corresponding to the movement of the Wii remotes. As the player moves her remote, the graph is depicted in real time on a single plane and captures instant by instant the movement of the corresponding controller (see Fig. 1). The graph on the screen documents the distance of the remote from a sensor bar, which is positioned in the interaction space. Two players can play at the same time, and two different graphs can be shown on the screen (see Fig. 1b). WiiGraph can also be used to assemble the independent movements of the two players so that a single graph is produced. These kinds of graphs lend themselves to two-person collaborative tasks involving two spatial variables, for instance activities of creating a figure where one player controls the x-coordinate and another controls the y-coordinate.

[2] WiiGraph has been developed by R. Nemirovsky, B. Rhodehamel and C. Bryant at the Center of Research in Mathematics and Science Education, San Diego State University.

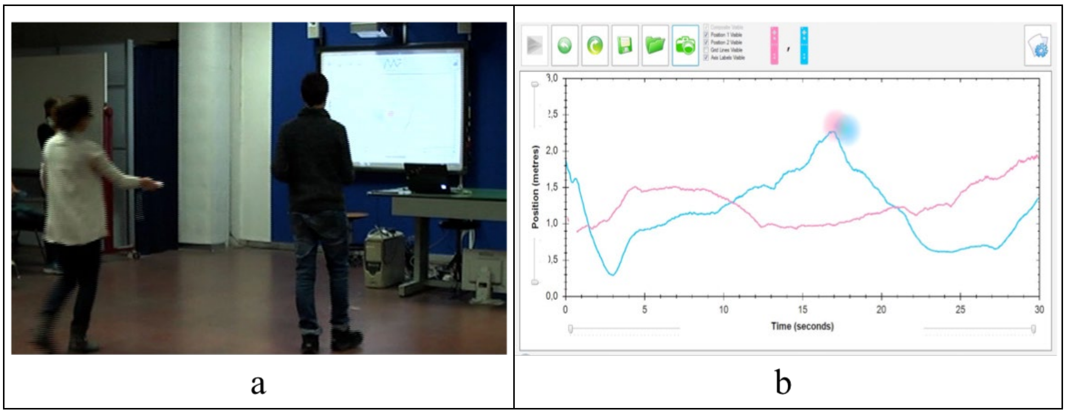


Fig. 1 (a) Two players pointing the controllers; (b) two separate graphs

In this article we focus on two-person collaborative tasks that involve making a single graph from the two independent movements of two students. The axes of the graph are *a* and *b*, and the graph has coordinates ($b(t)$, $a(t)$), for each *t* of the interval under consideration. These kinds of tasks offer the students the opportunity of working together to collaborate and coordinate with each other for reaching a common goal. In Figure 2a, that common goal was to create a rhombus when one student controlled the vertical variable and the other the horizontal. In general, such a task entails tapping into time or duration in challenging ways, combining individual heterogeneous rhythms to achieve a third kind of movement. This third movement is then expressed as the target shape—which could be anything (similar tasks are discussed in Noble et al. 2006).

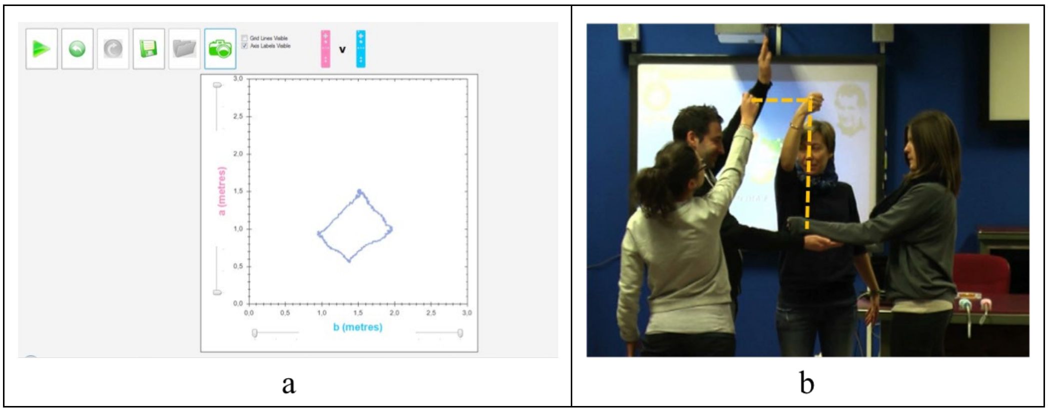


Fig. 2 (a) Image of jointly made rhombus; (b) miming the orthogonal components

Prior to making the rhombus (Fig. 2a), students were asked to imagine and gesture the orthogonal components of the motion required in making a rhombus without holding the Wii remotes (Fig. 2b). The researcher together with two students stood in front of the class and gestured as though drawing an imaginary rhombus in space, while the two students had to move their right hands miming simultaneously the movements of the two components (see Fig. 3). In so doing, the researcher's movement dictates the timing of the students' hand movements and the way in which they have to be assembled so that the rhombus is the combined effect of such movements. We mention this here, to show how students were acculturated to the practice of miming the necessary movements.

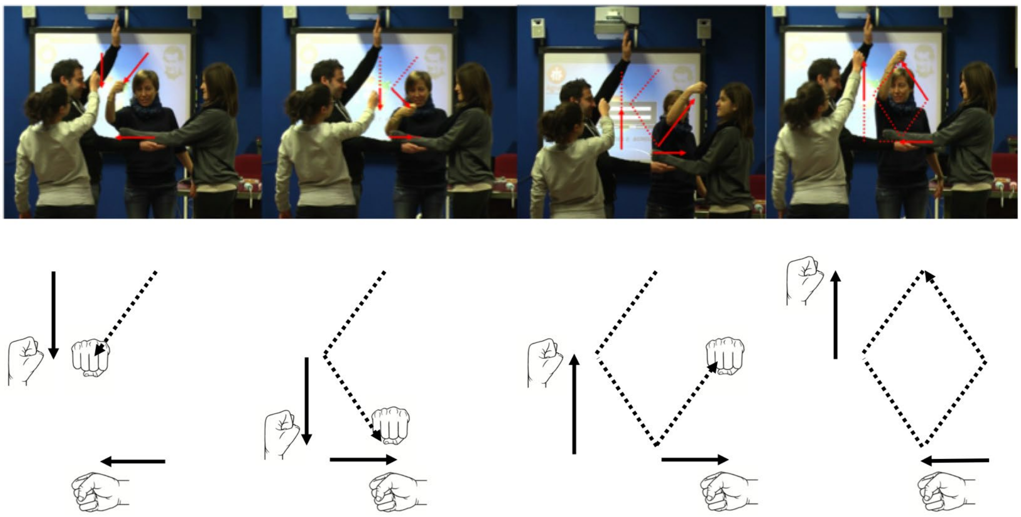


Fig. 3 Gestural movements for the rhombus

Students were then asked if it were possible to use the technology to create a graphical circle. Compared to the rhombus and other straightedge figures, the circle introduces an additional complexity because of its curvature. In particular, the related rates of each of the movements must change at different points in the trajectory. The students were conscious of the difficulty of creating "curved" parts based on what they had done previously. In response Emanuele suggested: "One should do a little similar to the rhombus but [...] one shouldn't stop to make the vertices, one should be always moving at a constant speed... for the whole circle". Lucrezia suggested the vertical and horizontal diameters and the extremes of these diameters were connected by pieces of curved lines. Barbara added: "In this case, we don't have four reference points, but many, therefore none, so it's difficult […]". Davide put forward the need for coordinating movements to reach a circular shape: "[…] Here, collaboration between the two people, who move the controllers, also comes into play, cuz in the circle, after a while the line will be steeper, it will be almost vertical to complete a quarter of the circle, it will be almost vertical: it means that one, one of the two, slows down and the other accelerates […] The two of them have to be sufficiently coordinated with each other to understand when they have closed the curve". These comments focus on the lack of vertices or pivot points, and the question of speed. Davide's comment raised the important idea that two constant speeds were not adequate, and that two *differently* accelerated speeds might be needed. Class ended after this brief discussion, with only these few ideas put forward.

5.2 Coordinated circular movement

In this section, we focus on excerpts from the video data collected during the next class meeting when the possibility of making a circle was further discussed. In order to prime the situation, one of the researchers goes to the front of the class and begins moving her fist in space around a circular trajectory, again and again. She looks quietly out at the class while this rhythmic circular movement is repeated, a continuous movement of circles drawn in space. Then she asks "who will come up?" and two students, Lucrezia and Barbara, came to join her in front of the class, to combine and assemble with the gestures of the researcher, each moving along orthogonal directions (Fig. 4). Of course the students are familiar with this kind of activity, having done something similar in the case of the rhombus.

Fig. 4 Gestural movements for the circle

The teacher also joins the group, next to the right side of the researcher, stretching out his arms to embody the orthogonal Cartesian axes. The researcher speeds up many times her hand in a wide circular trajectory. Lucrezia and Barbara (L and B in Fig. 4) begin coordinating their movements accordingly. Lucrezia moves her fist up and down (vertical displacement), with her head and gaze following the circle drawn in the air. Barbara moves her arm in a back and forth horizontal movement that is slightly elliptical. This shared activity continues for about one minute. The three different movements are sustained while the teacher stands relatively still, his two arms outstretched in the form of two orthogonal axes.

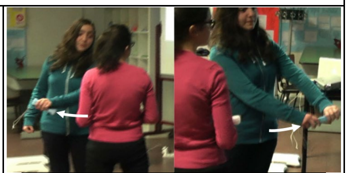
The two girls are focused on the researcher's circular trajectory, ignoring each other. At some point, the researcher asks the girls to continue and then steps away, having primed the situation. Almost immediately, however, the girls' two movements become uncoordinated and giggles and inaudible sounds from the other watching students begin to emerge. The researcher then re-enters and draws new circles in the air. This again primes the situation so that the students are able to better coordinate. She changes her speed, making circles faster in the air, and they follow suit. We can see how this arrangement captures a complex interdependency amongst the four participants. The teacher's body is used as a frame of reference to determine the appropriate direction of the students' movements. The researcher's continuous cranking of an invisible wheel sets the tempo and pace, while the students to-and-fro gestures are clearly chained to it. These to-and-fro gestures decompose the perceived circular movement into two tacit linear parameters. The coordination of these two gestures, however, is mediated through the researcher, and so when she steps away, they are not adequately assembled with each other and fall apart. In other words, they are not yet coordinated from their shared energy. The researcher steps away again, and although this time the two remaining heterogeneous movements remain hitched a little longer, again they fall apart.

There is great intensity throughout this one-minute scene. The researcher plays the dominant role here, setting the rhythm and the limits of possible movement. This entails a pedagogical demand whereby the students are asked to *follow her in their own way* (not to imitate her exactly, but to follow her using their own kind of movement). Thus she becomes the engine of the learning assemblage, the force that sustains the collaborative transindividual effort. And, accordingly, it falls apart when she leaves. At a micro-scale, the two girls' different movements are strongly affected by the researcher's movement, and they must follow her lead somewhat passively. The students' gestures are primarily submissive, and isolated from each other (as they mostly ignore each other) and thus not yet *affectively* coordinated. In other words, they have yet to join their heterogeneous movements in a sympathetic bond so as to generate a circle. *Decomposing* the circle into different movements in this activity involves active learning about the dynamic concept of circle, but it's not yet *composing* a circle. The affective difference between the decomposing task and the composing task is significant, as we shall see. Still, the students take forward into the next activity the embodied learning during this miming task.

Next, the two students are asked to produce (compose) a circle with WiiGraph, using the remotes. It's important to mention a few technical matters before describing the data. In the case of a rectangle graph (with sides parallel to the Cartesian axes), each side is created by having one remote remain still while the other moves at a constant speed. At each vertex, they switch. In the case of the rhombus, the two movements again entail constant speeds, but there is also the fact that *the ratio* between speeds is always constant. Finally, in the case of the circle graph, the coordination between the remotes is much more difficult. The need to achieve a non-linear curve makes the task different from all the previous ones. As Davide intimated in the class discussion, the circle will entail different kinds of coordinated accelerated movements —while one user is at maximum speed, the other is at minimum speed, in the opposite direction; when one is accelerating, the other needs to decelerate, and vice versa. In other words, the ratio between the speeds now is not constant, requiring the users to modulate their accelerations. This makes coordination that much harder. Moreover, WiiGraph technology requires the two movements of the x and y coordinates (b and a, respectively) be performed literally *in parallel*. In other words, the vertical up and down movement and the horizontal to-and-fro movement must now both be performed on a horizontal plane. Such change in the corporeal activity adds to the difficulty of translating the learning from the miming activity to the use of the Wii technology.

5.3 Shared achievement

Lucrezia and Barbara start discussing how to move their hands to produce a circle. They stand in front of the whole class, holding the remotes, discussing what they plan to do before the software is turned on (see Table 1; R marks the researcher; bold text describes some movements not visible in images; the grey parts mark segments that are further analysed in the diagrams of Figure 6 below).

L B	So, we start at the same distance, while you go like this, I arrive like this (***L steps forward towards the sensor, facing the sensor, then looks at B***) (**1**) So, you, you place there, in front of me? (***B keeps her gaze down, points in front of L, entering her space, and causes L to shift slightly***) (**2**)	 **1** **2**
L B L	Yes, but we need to point to the sensor (***L shares her doubt, points the remote to the sensor, keeps looking at B***) Yes, ok. Place kind of like this. In front of me (***B raises voice with excitement, holds the remote with both hands and makes little jumps. Then B twists her wrist and flaps her hand to mimic how L should turn to her. She glances rapidly at L and the remote. L turns towards B so that now the two girls face each other***) okay, like this, and then we move, we move the controller like this Ok (***L nods minimally, as though half-convinced, looks away from B for a moment)***	
B B	So, you start kind of, from the limit here (**3**), I start (***holds the remote with both hands***) (**4**) from here and then I do like this (***B rocks back and forth, her arms moving like a pendulum, and purses her lips and then grins at L, now looking directly into her face***) (**5**) We have to move like (***swings to and fro like a clock, performing a measured rhythm, stares into the empty space beyond the remote***) (**6**) …more or less like a clock, it's just that we are two different clocks (***B glances at L and gesticulates to capture an inclusive "we"***), aren't we?	 **3** **4**  **5** **6**
L	Yes, yes, yes (***looks rapidly at B, then again at the screen***) ***B also looks at the screen***	
L stares at the screen for 2 seconds, detached from B, while B seems to realize that L is not listening to her or not convinced. There is a disconnect. B juggles her remote.		
B L	(***B suddenly begins swinging left to right, her tone appealing for attention***) (**7-8**) Like this, like this, like I'm doing. (***But L looks again away from B, towards the screen)*** (***L suddenly holds the remote with both hands, straightens herself, and looks at B***) Wait, go	 **7** **8**
	L and B now face each other, and look to their hands/remotes: B moves again, in silence. L also begins moving her remote trying to coordinate with Barbara with jerky movements (**9-10**) (see Fig. 6a: a notation for their coordinate movement)	 **9** **10**
B L B L	You have to be behind me (***B seems beseeching, requesting not demanding, rocking like a pendulum***) (**11**) (***looks at L***) Eh (***L responds with a sound that is neither affirming nor negating (perhaps annoyed?), while she moves her arms in a jerky to-fro***) ***L and B keep moving*** (**12**) Hm hm, more, more behind (***B looks at L and smiles, as though a query***) ***The girls both stop at the farthest position from the sensor*** You go first (***nods repeatedly, and gently takes command***)	 **11** **12**
	For a few seconds, B goes on moving and L tries again to coordinate with B, both in silence (**13-14**) (see Fig. 6b).	 **13** **14**

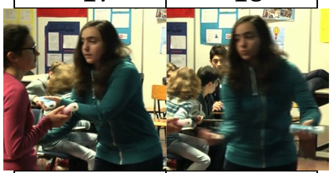
B B	You have to reach it even more… ***L slows down, looks at B*** Like this (***nods encouragingly, gazes at L's hands***)	
L B L B L	More or less like this We get a thing of this kind, maybe (***B tilts her head, raises her eyebrow as she raises her hand, twists her torso and smiles***) (**15**) ***L and B both look at the screen*** For me, no… (***L giggles***) Let's try … cuz, when you were here, I was here (***by crossing arms, points to the two extremes***) (**16**) (***L emphasizes their difference, then slouches and shrugs a little***)	 **15** **16**
B L B	Hm hm, a little more. You've to be here, like this, *pock* (***B questions L's account, and further models for L, now using her two separate hands to mimic both her and L's movements. "Pock" marks the point when the second hand reaches the maximum distance***) (**17-18**) But if you go fast (***L raises pitch, as though sceptical, but with humour. Then shakes her head, and offers mocking smile***) Well, fast, it's up to us (***B shrugs a little, slows slightly, but continues to move both hands to-fro***) (**19-20**)	 **17** **18**  **19** **20**
R	Can you tell me (***the two girls both turn towards R***), excuse me, please, tell us what you've decided to do, what you're deciding to do	
B L	We're thinking that, because she's in front of me, we stand like this, kind of, if I start here, she starts (***B points with the other hand to a middle position. She uses confident voice and L nods approvingly***) (**21-22**), I start here, she starts like this, when I will arrive here, she will follow me (***performs again a back and forth movement with L***) (**23**), a little, she will be there when I will be here (**24**) While she goes backward (***L interjects, and nods, looking at R***)	 **21** **22**  **23** **24**
R L B L B	Will the speed at which you move matter? Yes, yes (***L confidently nods repeatedly***) We have to move at the same distance, at the same speed, and at a constant speed (***L begins nodding in agreement, but interjects as though to correct B***) With the same rhythm, but she starts before and I follow her (***L reaches her hand out and flaps the air between the two girls as she speaks***) We decide the speed, but we have to move at the same speed and at a constant speed between us (***B is incorrect in this claim***) (***L begins by nodding, then shakes her head a little, indicating some mixture of dis/agreement, but says nothing***)	

Table 1. B and L's discussion about how to make a circle

The transcript reveals the delicate way in which the two girls negotiate a plan of action, beginning with the incorrect suggestion that "we start at the same distance" and exploring different scenarios in which their two movements are assembled. Barbara embodies the rocking and rhythmic motion of a pendulum clock, her two hands on the remote swinging to and fro, and indeed she says that they "have to move more or less like a clock", and "it's just that we are two different clocks". Then she uses the sound "pock" to characterize the point when the combined movements must change direction. She grins as she rocks, and looks Lucrezia directly in the face. And she continues throughout, encouraging Lucrezia to follow her, “like this, like I’m doing”. Lucrezia says “yes, but we need to point to the sensor”. The two girls engage with the “yes, but …” refrain throughout, as a means of negotiating their agency and power. It’s evident that the two girls are struggling to find a way to coordinate their movements. Barbara tries to convince Lucrezia, who appears to not be listening at times. These tensions are productive. Lucrezia is hesitant, and uses jerky movements that seem to chase Barbara. She says “for me, no” as though to emphasize the difference of her own movement, in contrast to Barbara. Lucrezia points in one direction with one hand, and then in the other direction with the other hand, saying “cuz when you were here, I was here”. In these gestures she communicates the layering of different directions simultaneously. Barbara responds and uses her two hands as well, but keeps them moving simultaneously, as though they were different eddies in a stream. When they seemed to have reached an agreement, Lucrezia then raises a concern almost half-joking, “but if you go fast” and Barbara replies “well, fast, it’s up to us”. The challenge is to achieve that “us” while also achieving the circle graph. The researcher then intervenes and asks what they are planning, and the matter of speed is discussed as a relevant one. This issue is exactly what Davide had discussed earlier in the previous class, when the circle task was introduced. We note that Lucrezia disagrees slightly with Barbara’s response, but the two girls are ready to use the software despite these disagreements.

The software is then turned on, and Lucrezia and Barbara start moving the Wiimotes with the aim of producing a circle on the screen. Figure 5a shows the two girls while they are moving the remotes trying to be synchronized both in rhythm and speed. Note that Barbara stares down at the remotes, while Lucrezia stares at the screen where the figure is unfolding. Figure 5b shows the new circular movement that they are able to obtain on the screen (with some trials).

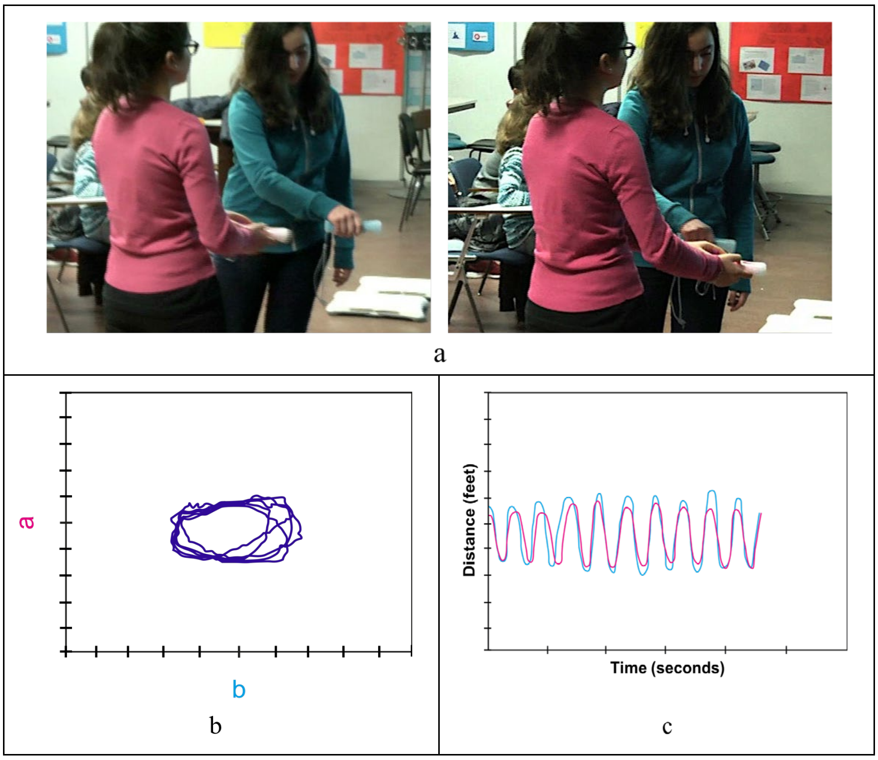


Fig. 5 (a) B and L’s coordinated movement; (b) the circle; (c) the periodic functions

Finally, after the girls have achieved the kind of synchronized rhythms required to jointly form a circle, the researchers switch the software modality so that the screen shows the periodic functions that are associated with the circular trajectory (Fig. 5c). The girls continue to move the Wii remotes, while the researcher repeats again and again "Continue, continue" and the class watches as the periodic functions unfold according to their movements. The whole class now witnesses that the *same* two coordinated movements (performed in real time by the girls) create both the circle graph and the periodic graphs (Fig. 5). We mention this as it underscores the rich variety of movements that are uncovered and coordinated in this task.

The scenarios in Figure 6 help us track how the coordination of the girls' movement emerges, as they stand facing each other in the video. Each captures the degree of coordination at different stages of the task. The first two columns represent the girls' movements as they plan, and the last column captures their movement as they successfully graph the circle. In column one, Lucrezia follows Barbara tentatively, and in column two, she stops moving, as Barbara continues her sweeping gesture. The final column shows how Barbara's movement has become significantly more constrained by the movement of Lucrezia who she now follows. The diagram captures the micro-adjustments in the three scenarios, which last together about four minutes. The arrows capture the nature (a swing or a straight line) and the direction of arm movement (to the right/left of L's body; similarly, to the left/right of B's body, as if we watched from the top). The numbers on the right are time stamps when the leading student reaches an extreme or the central position, completing the gesture represented. Length corresponds to the distance travelled in the specific time interval; a longer arrow means faster while a shorter one means slower. Turning arrows indicate changes in direction during specific time intervals.

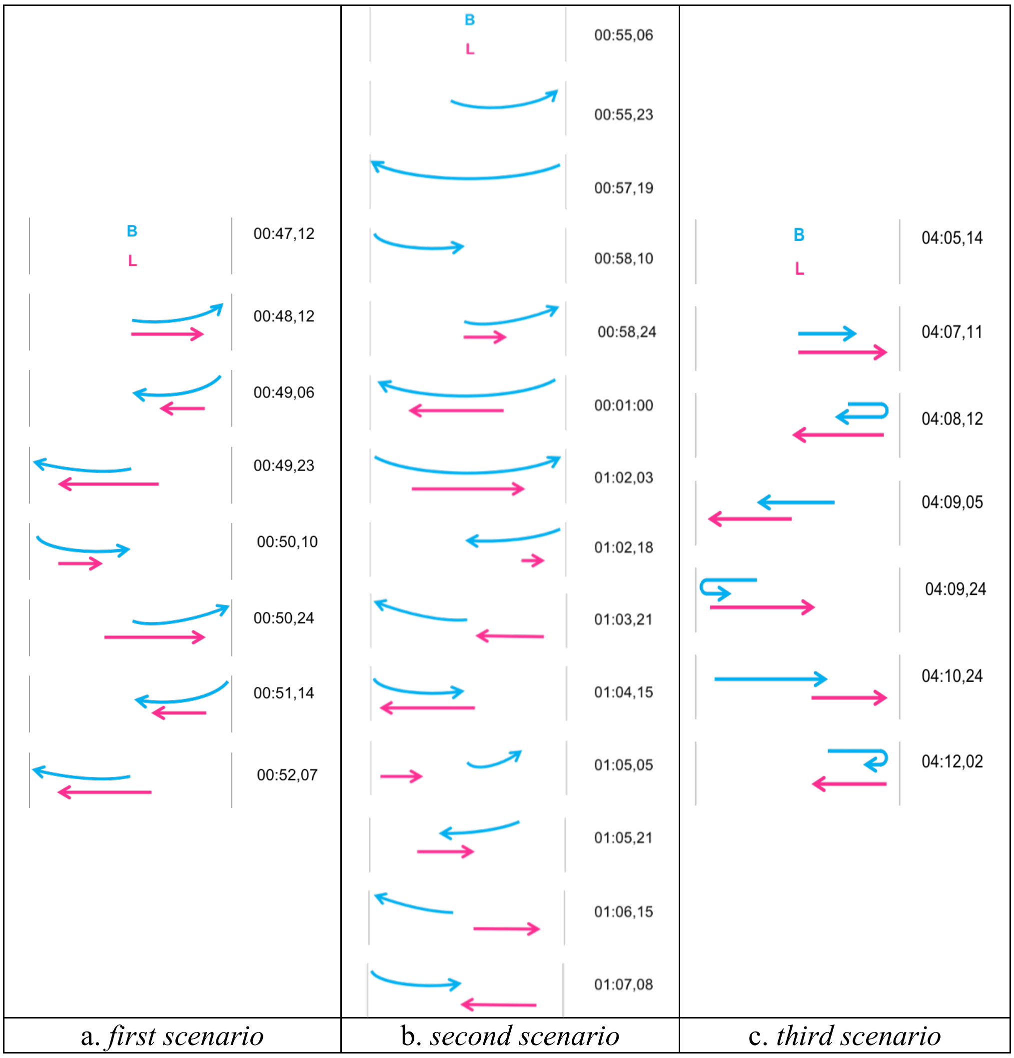


Fig. 6 (a) The first, (b) the second, and (c) the third scenario of assembling movements

6. Discussion

The repeated attempts of the girls throughout the tasks show how sympathies proliferate in everyday minute interactions, and are assembled into larger overt coordinated responses between bodies. These movements are lived in and as affective bonds, at the micro-scale beneath consciousness, and lived as well at the individual scale of conscious speech (see Table 1). Minute sympathetic movements contribute to passionate attachments, so that the affective investment in such shared activities becomes pronounced. We see in the video data how affect circulates across minute movements as the two girls coordinate their activity. There is ample evidence of disagreement in the planning session, and indeed these tensions are the important friction that sustains a sympathetic coordination. We see the learning assemblage evolve through these tensions, when sympathy becomes “something to be reckoned with, a bodily struggle” (Deleuze and Parnet 2007, p. 53). The girls do not identify with each other or ‘put oneself in the other’s shoes’, but they assemble with each other, with the Wii, and with the concept of circle, and thereby enter a process of *becoming mathematical* that does not erase the other. A relationship of response-ability emerges through the sympathetic coordination of movement.

Notably, the particular task creates an opportunity for that relationship to emerge. The researcher primes the event for sympathy, so that coordinated movements emerge, and action-paths form. But these action-paths too can become staid or fall apart. The researcher steps away during the first task of miming the circle movement, and the girls' actions become uncoordinated. The event must then be further primed, introducing the cranking circular movement once more, the researcher heightens the intensity of the moment, and the students are recharged and attend to the collective endeavour, pursuing the necessary "fellow-feeling".

In the beginning of the project, Barbara was reluctant to take part in group work: she expressed herself in long meandering statements that often confused her class mates (and the researchers and the teacher). In the process of the teaching experiment, we noticed a serious change in Barbara's position and relationality within the class, although some students had 'learned' to dismiss her contributions. Lucrezia, in contrast, was initially silent and timid in class, but successful on tests. She also experienced a change in her way of engaging in collective discussions, becoming more willing to intervene and express her opinion. She was enthusiastic to participate, and challenged by the task. The two girls came forward to join the collaborative effort of creating a circle, despite their very different ways of being in the class.

The productive intensity of this task comes from the various contrasts or tensions that are entailed—there are two girls, each with their own life history; two orthogonal directions to be performed; two very different movements to produce the one graph. Sympathy is the coming together of these contrasts, not so one obliterates the other, but instead as an onto-creative act in which *commotion* becomes coordinated and brings forth new joint learning. This is a task that demands all three components of a sympathetic relation: (1) there is a circulation of feeling as minute facial expressions and changes in bodily posture occur, the two girls leaning in and out, attending to the micro-scale corporeal signals that circulate beneath consciousness; (2) there is a common sense or shared sensibility in the shared obligation to follow each other and work with a shared objective; (3) there is the compassion for the other, and the care of ensuring that others (with different objectives) are coming along, moderating the tensions that sustain any learning assemblage.

When Lucrezia and Barbara begin discussing how to achieve a circle using the Wii remotes, they are conscious of their different public faces. Lucrezia typically thinks quietly to herself, as she contemplates tasks, while Barbara is outspoken and tends to share her ideas as they are formed. These two very different personalities are both individually eager to achieve the circle, but all too aware that this achievement depends entirely on coordinating with the independent movements of the other. We note that during the discussion Lucrezia physically follows Barbara's lead, while her words indicate some hesitation regarding Barbara's suggestions. Then, during the actual producing of the circle, Lucrezia stares hard at the unfolding diagram on the screen while repeatedly moving her arm back and forth, and it is Barbara, with her back to the screen, who watches and follows Lucrezia's changing cadence and rhythm. This unscripted change in how the bodies affect each other points to the dynamic flow of affect that sustains their endeavour. The two girls are together determined to make a circle, and there is a shared intensity while the power to lead is taken by Lucrezia. And yet such moving-together and power-switching is successful precisely because the two girls are coordinating at the pre-individual scale of micro gestures and petites perceptions. The task itself has created an opportunity for shared affect and transindividual sympathy.

The circle is a truly collaborative effect, *a doing done through the individuals (rather than by the individuals)*. The circle is made through Lucrezia and Barbara, an achievement that emerges between the cooperating agencies. This shared affect and “bare activity” - that rumbles across the affective environment - is what sustains the transindividual sympathy. As the two begin to work together, their achievement is not entirely about rational choices, because it rests fundamentally on somatic and unconscious ways of moving together, and the immediate communication of affections. Sympathy is the seed of learning because it affords new action-paths across the event, and furnishes opportunities for collaborative inventive practices. Concepts emerge and settle in such an environment as a function of sympathy, without a master who legislates the nature of the sense-making. We emphasize this point, because it helps open up discussion of how achievements in classrooms are truly collective and impersonal insofar as they are *done through us*.

When the graphs of the periodic functions are shown (Fig. 5c), the class “oohs” and “aahs” and someone says “beautiful” and another says “humps!”. Listening carefully to the affective tone of these responses, we can track the rippling effect across the class, as the emotion fans out. The flow of affect recruits other student bodies by varying degrees, as they shift in their seats, lean in and squint, and perhaps unconsciously wiggle their toes as evidence of a sympathetic investment in the collective endeavour. As Massumi (2015) claims, sympathy “can reverberate across a relational field, faster than the field of conscious calculation” (p. 84).

He goes on to suggest that this is how the micro ethnographic scale reverberates out to other scales. In our case, we believe the micro-gestures that occur at the pre-individual level, are amplified by the individual movements, and then fan out and percolate across the transindividual classroom:

> it is a defining characteristic of complex environments that the extremes of scale are sensitive to each other, attuned to each other’s modulations. This is what makes them oscillatory. They can perturb each other. (Massumi 2015, p.10).

In our teaching experiment, the individual students Lucrezia and Barbara are clearly key players, but rather than treat them as mediators, we suggest that they act as *amplifiers of affect*. In other words, we believe that the individual human body amplifies so that sympathy can scale up to the transindividual. Affectivity can “channel” through the individual body, reverberating out to the larger scales. In other words, the micro-ethnographic scale plugs into the trans-individual scale of the classroom so that the other students become implicated in “a doing done through us” (Massumi 2015, p. 20). The *infra-scale* of affect can be studied for how it fuels an enveloping social-emotional space in the classroom.

The fact that the task focuses on the dynamic movement buried in the mathematical figures is significant. This teaching experiment helps the students grasp the many different ways in which related movements are at work in the apparently fixed and familiar figure of the circle, deepening their understanding of the geometric figure, and enhancing their embodied understanding of the mathematics involved. The task binds affect with mathematical concept. Lucrezia and Barbara's instantaneous speeds are captured as the two derivatives db/dt and da/dt that constitute the speed of movement around the circle. WiiGraph assembles the girls' collective movement as the partial derivatives of the circle – tracking the slope of the tangent line point by point. Their timed accelerated movements *are* the gradients that are imperceptible in the graph. The assemblage of graph-concept-student is achieved through these gradients. Their speeds must be different but coordinated for the combined effect. Each hand movement has its own rhythmic pattern, and each hand must move at a different speed, and indeed at related rates of changed speed, in order to achieve the effect. Thus the two bodies are moving together but apart, and the coupling of these movements forms a third movement that belongs to neither of the original bodies. The various motions inherent to the concept of circle are experienced in the affective bonds that the girls form. It is not that a particular affect is associated with the circle, but rather that a particular experience of affectivity (dynamic coordinated movements at various scales) is associated with the circle. The tension and contrast found in the many micro-movements of gesture, facial expression, and words (see Table 1), when Lucrezia and Barbara negotiate the circle concept, underscores the complex nature of this affectivity. They achieve the circle graph and a transindividual sympathy precisely because they are obliged to moderate these tensions and contrasts.

7. Concluding comments

Affect belongs to the environment or ecology, which can thus be primed, activated or modulated by designing tasks that allow for transindividual sympathy to emerge. These can be tasks that help us study the affective modulation of mathematical experiences. This approach to affect points to new ways of understanding collaborative achievement in mathematics classrooms. We believe this study of corporeal coordination shows how affect is moderated at the micro-scale, and that small everyday tasks in classrooms can engender transindividual sympathy. In such classrooms the teacher becomes an activist of a very powerful kind, but not in the typical way – the teacher becomes someone who seeks to catalyse affective communication, to condition and modulate the affective tonality of perception, and to trigger a coordinated endeavour whereby students enter into the dynamic intensity of a mathematical concept or relationship.

Sympathy takes on a pivotal role in any learning assemblage. Such sympathetic transindividuality is not a matter of identification but of *coordinating* with the other—this coordinating allows for radically diverse forms of heterogeneous movement, and is not a matter of compliance or becoming the same. It is rather an attempt to think about how we form assemblages of radically heterogeneous movements in ways that are productive of learning and ethical relationships. In this case, the bodily agreement or coordination produces rich mathematical thinking—an assembling of gradients and directions that speaks directly to the shape of the periodic functions and circular movement. The learning assemblage that we have analysed here is a complex entanglement of affect and concept, demonstrating how innovative technologies add to our understanding of fundamental aspects of mathematics learning.

We are not proposing that particular movements or mathematical concepts have transcendent emotions attached to them, independent of place or context. We are arguing that affective engagement entails participation in *distinctive* kinds of activity, and thus research must attend to the specificity of that activity. This paper treats affect as immanent to material practices and processes - affective conditions are always singular, always inhering in particular material activity. Affects do not transcend the relations of a learning assemblage, because they are precisely what sustains or animates or innervates that assemblage. Affect nourishes sympathetic relations of dis/agreement between bodies whereby they are mutually affected by each other.

Of course contractions of affect become recognizable in particular cultural-historical contexts (as sad or happy, for instance), but such contractions include the conceptual matter as well as the cultural and the organic. *The concepts matter*. For instance, the amorphous concept of circle is implicated in mathematical activity in different ways, distinctively inflected by the flow of affect between Barbara and Lucrezia. Similarly, other mathematical concepts, if considered as dynamic and variable, are embodied in different material practices (de Freitas and Ferrara 2015). Rather than reduce all experiences of mathematics to the same emotional note, our approach attends to the nuanced or tonal differences between one experience and another. Our aim is to attend to the specific and dynamic configuration of affect that *is* mathematics in all its multiplicity. We hope to trigger more research on the complex affective dimension of the material labour of mathematics, and to spur on studies that show how such affective labour is distinct from other kinds of labour. We believe this teaching experiment opens up new areas for research on the nuanced ways that affect circulates across different kinds of mathematical activity, revealing how mathematical concepts are entailed. And finally, we hope this work leads to more experiments with pedagogical tasks that afford opportunities for a transindividual sympathy to emerge.

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