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(Article begins on next page)
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Abstract

In humans, as in many other animals, preferences between options can be reversed when other irrelevant options are added to the choice set. Heuristics theories view such puzzling departure from economic rationality as evidence that decisions rely on simple rules that make a context-dependent use of the available information. The computational mechanisms underlying these rules, however, remain largely unresolved. Using a “sequential-sampling” model of decision making, I show that irrational decisions may arise when an information-processing mechanism that works optimally in one-choice tasks is co-opted in multiple-choice contexts. The model supports the assumption that different heuristics may sometimes be the elusive expression of a single general mechanism and that natural selection, rather than promoting the evolution of different mechanisms and rules, may favour the parsimonious use of bounded computational resources.

KEYWORDS

Decision-making, Decoy effect, Evo-mecho, Heuristics, Irrational behaviour, Rule-of-thumb, Sequential sampling models.


**Introduction**

Humans, as many other animals, make decisions by integrating different sources of information about the expected benefits and costs of the available options. Economic theories of consumer behaviour assume that decisions maximize the chooser's perceived utility (Simon 1955). Evolutionary theory predicts a positive association between the perceived utility of an option and its expected consequences on the Darwinian fitness of the chooser (Parker, Maynard Smith 1990). While the option may vary along multiple dimensions, the perceived utility is a one-dimensional representation of its economic value (Kacelnik 2006). Decisions that optimize economic utilities obey the two axioms of economic rationality: transitivity (if A is preferred to B and B to C, then A is preferred to C) and regularity (the probability of choosing A over B is the same, independent of the presence or the absence of a third option, C) (Kacelnik 2006).

Humans, and other animals as well, however, often show cognitive biases, which cause systematic departures from economic rationality (Tversky and Kahneman 1974). A well-known example is the “decoy effect”, whereby the preference between two options depends on the presence of a third option (Huber, Payne and Puto 1982, Lea and Ryan 2015). According to some authors, these violations, though economically irrational and sub-optimal, could be rational and optimal from an evolutionary point of view (Kacelnik 2006). For example, models of optimal foraging show that intransitive choices and decoy effects can be favoured by natural selection, if the fitness benefits they provide depend on either the spatial (Houston et al. 2007, Trimmer 2013) or the temporal patterns (McNamara et al. 2014) of the
available options. According to this view, cognitive biases are evidence that decisions do not rely on complex and accurate representations of perceived utilities, but on “fast and frugal” heuristics (or rules of thumb), which may not perform optimally in all situations, but satisfactory in the ecological contexts where decisions are actually made (Gigerenzer and Goldstein 1996).

Both optimality and heuristic models focus on the decision rules. The optimality models first define the decision problem and the currency that the decision maker is expected to maximize, then, they search for the rule that maximizes that currency, often, under the assumption that decision makers are omniscient, fully-rational agents (Stephens and Krebs 1986). The heuristic models, in contrast, relax the rational assumption and search for rules that can make “satisficing” decisions, with a parsimonious use of time, information, and computation. Heuristics are not optimal, but approximate optimality. They are not strictly rational, but ecologically rational, because they exploit the environmental structure to yield simple, efficient rules, which could be plausibly implemented by rationally-bounded agents. Despite these relevant differences, both optimality and heuristic models explicitly ignore the computational mechanisms of decision rules and this neglect of mechanisms has been argued to have impoverished the functional analyses of behaviours (McNamara and Houston 2009, Kacelnik 2012). For example, Kacelnik et al. (2011) show that when European starlings are given a choice between two foraging options, they adopt the same computational mechanism independent of whether the alternatives are presented sequentially or simultaneously. These results not only conflict with the “toolbox” hypothesis of heuristics as narrow-domain decision devices (Gigerenzer, Todd 1999), but they also suggest that mechanisms are important constraints in the evolution of optimal decision rules and, thus, that they should
play also an important role in the functional studies of behavioural flexibility (McNamara and Houston 2009).

Here I present a race model of decision making, which describes decisions as the bounded accumulation of noisy sensory information (Bogacz 2006, Ratcliff et al. 2016, Vickers 1970). Following Kacelnik et al. (2011), I also assume the choice mechanism to work in both the sequential and the comparative contexts. Most race models assume that all types of supporting evidence for an option are integrated into a single accumulator, which thus provides a one-dimension, dynamic representation of the option value (Ratcliff et al. 2016). My model relaxes this assumption and, consistently with some recent neurophysiological studies of action choice (Cisek and Kalaska 2010, Cisek 2012, Engel et al. 2013), it assumes the parallel, multi-dimension processing of the option attributes. The decision mechanism is equivalent to a race in a multi-dimension plane, which terminates as soon as the accumulating evidence enters either a rejection or an acceptance consensus area. This general mechanism can explain decisions in both one-, two-, and multiple-choice contexts. However, while in one- and two-choice contexts, the mechanism predicts economically rational decisions, in multi-choice contexts, it introduces systematic errors and predicts irrational choices, such as those described by the “decoy effect”.

The model

The model considers the behaviour of a hypothetical insect, which is hovering above the grass, collecting nectar from flowers. The insect spots a bunch of brightly coloured flowers just a few meters on the left and it decides to turn left. While approaching the bunch, its attention is captured by a yellow daisy on the left and a purple poppy on the right. The insect
decides to turn right and to land on the poppy. The behaviour of this foraging insect is the result of two types of choice. The first choice depends only on the quality of the bunch of flowers being inspected, which is considered to be worthy of further inspection. I call this “sequential choice” (Kacelnik et al. 2011), because it is the type of decisions that an animal makes when it encounters options sequentially. In contrast, the second type of decision depends on the relative quality of the purple poppy, which is considered more worthy than the yellow daisy. I call it “simultaneous choice”. The model assumes that the sequential and the simultaneous choice are not the expression of different heuristics, but of the same cognitive machinery designed by natural selection to make both types of decision effectively. This cognitive machinery processes sensory information and provides an internal representation of the benefits and costs of either exploiting or rejecting the prospective resource.

Benefits and costs may be viewed as the two axes of a Cartesian information plane (Fig. 1): the y-axis describes the perceived benefits; the x-axis describes the perceived costs. Benefits and costs are a function (either a linear, monotonic or unimodal) of the characteristics of the prospective resource. For example, the benefits depend on the quantity and quality of the nectar provided by the flower, whereas the costs depend on the time required to collect the nectar, which depends on the type and the location of the flower and on the presence of other competitors. Benefits and costs, however, should not be viewed as the internal representation of the economic value of the resource, but as the supporting evidence for either exploiting or rejecting it (see also Appendix 1).

During the decision process, the insect accumulates over time the noisy sensory information about the costs and benefits of an action, which is executed only when the accumulating
evidence reaches a given threshold. From a mathematical point of view, the dynamic of decision making can be described as a random-walk process in the two-dimensional informational plane, with transition probabilities that depend only on the type of stimulus processed (Forstmann, Ratcliff and Wagenmakers 2016, Ratcliff et al. 2016). Depending on the choice context, the insect can assess a single option at a time (sequential choice) or process simultaneously and in a parallel fashion several options (simultaneous choice).

First, I consider the dynamic of the decision making in a sequential-choice context, for example, when the insect spots the brightly coloured flowers on its left and decides to visit them. During the evaluation of action $A$ (to visit the flowers), the insect is assumed to collect a sequence of noisy information about the amount of energy it is expected to obtain from the flowers $(Q^A(1), Q^A(2), \ldots Q^A(n))$ and the amount of time it is expected to spend for handling them $(H^A(1), H^A(2), \ldots H^A(n))$. Since information is noisy, $Q$ and $H$ are assumed to be two normally-distributed stochastic variables with parametric means $q^A$ and $h^A$ and covariance matrix $G = \begin{bmatrix} \sigma^2_Q & 0 \\ 0 & \sigma^2_H \end{bmatrix}$. By transforming and integrating over time each piece of sensory information, the insect obtains a sequence of accumulating evidence for the benefits and the costs of the evaluated action. After $n$ samples, the evaluated action will be represented in the informational plane by a point with coordinates $[B^A(n), C^A(n)]$, where

$$B^A(n) = B^A(n-1) + b(Q^A(n))$$

and

$$C^A(n) = C^A(n-1) + c(H^A(n))$$

represents the amount of evidence for rejecting it. In Appendix 1, I provide a formal description of the functions $b(Q)$ and $c(H)$, that is, of how sensory information is converted into the perceived benefits and the perceived costs of an action.
Further and most important, the model assumes that the benefits and the costs of an action interact additively in determining its overall value (see Appendix 1). Suppose that, under certain conditions, there exist two optimal decision thresholds, $T^*$ and $L^*$ (with both $T > 0$ and $L > 0$), so that the insect would maximize its long term rate of energy intake by exploiting the resource if the amount of supporting evidence is larger than $T^*$ or by rejecting the resource if it is lower than $-L^*$. One decision mechanism could be simply to compute the difference between benefits and costs and to use this new variable as the decision variable: the insect would exploit the resource if $B^A(n) - C^A(n) \geq T^*$, it would reject the resource if $B^A(n) - C^A(n) \leq -L^*$, or it would continue the assessment if none of the two previous conditions is satisfied. From a mathematical point of view, the decision mechanism is a 1-dimensional random walk (1-DRW), because for each time step of equal length, the decision variable can only increase (if the perceived benefits overcome the costs) or decrease (if the perceived costs overcome the benefits).

There exists, however, an alternative and functionally equivalent mechanism that saves the computational costs of computing the difference between benefits and costs. Mathematically speaking, the alternative mechanism is a 2-Dimension Random Walk (2-DRW). Each dimension has a lower ($\vartheta^B_L(n), \vartheta^C_L(n)$) and an upper ($\vartheta^B_U(n), \vartheta^C_U(n)$) threshold, which is dynamically adjusted during the evaluation of an option according to the following rules:

\[
\begin{align*}
\vartheta^B_L(n) &= T + C^A(n) \\
\vartheta^B_U(n) &= -T + B^A(n) \\
\vartheta^C_L(n) &= -L + C^A(n) \\
\vartheta^C_U(n) &= L + B^A(n)
\end{align*}
\]

Eq. 1
The insect would execute the action when BOTH $B^A(n) \geq \theta_B^u(n)$ and $C^A(n) \leq \theta_C^l(n)$, that is, as soon as the random walk reaches the threshold line $b = T + c$. The insect would reject the resource when BOTH $B^A(n) \leq \theta_B^l(n)$ and $C^A(n) \geq \theta_C^u(n)$ (Fig. 1a, see also the Jupyter notebook in the supplementary materials). The two parameters $(T, L)$ of Eq. 1 describe the insect’s decision strategy and directly affect its fitness, by influencing both the response times and the error probabilities.

Now, let us consider the decision process when the insect has to choose between two ($k = 2$) or more ($k > 2$) alternative actions. For example, suppose the insect has to decide whether to land on the yellow daisy or on the purple poppy (Fig. 3a and the Jupyter Notebook in the supplementary materials). In this case, the 1-DRW assumes, first, that the insect computes, for each option, the difference between benefits and costs; then, that it integrates over time these differences into two decision variables; and, finally, that it chooses the resource, whose decision variable first reaches the acceptance threshold $T$. Unlike the 1-DRW, the 2-DRW assumes that evidence for either the D-action (approach the daisy) or the P-action (approach the poppy) is accumulated in two decision vectors, $[B^D, C^D]$ and $[B^P, C^P]$, and that the decision depends on which of them first enters the “consensus” area, where $B \geq \theta_B^u$ and $C \leq \theta_C^l$. While in one-choice context, the thresholds defining the consensus area are a function of the accumulating benefits and costs of the single assessed resource (Eqn. 1), in two- ($k = 2$) and in multiple-choice contexts ($k > 2$), the thresholds are assumed to be a function of the average benefits and costs of all the assessed resources. Specifically, the insect is assumed to dynamically adjust the four thresholds according to the following set of equations:
\[ \theta_B^u = T + \tilde{C}(n) \]
\[ \theta_C^l = -T + \tilde{B}(n) \]
\[ \theta_C^l = -L + \tilde{C}(n) \]
\[ \theta_B^u = L + \tilde{B}(n) \]

Eq. 2

Where \( \tilde{B}(n) = \frac{1}{k} \sum_i B_i(n) \) and \( \tilde{C}(n) = \frac{1}{k} \sum_i C_i(n) \) are the average benefits and costs after having processed \( n \) samples of the \( k \) resources.

**Results**

In one-choice decision tasks, the 1-DRW and 2-DRW are mathematically equivalent, because a 2-dimensional random-walk described by two vectors \( X \) and \( Y \), with threshold line \( y = T + x \) can be reduced to a 1-dimensional random walk, described by the vector \( Z = (X - Y) / \sqrt{2} \), with threshold \( T' = T / \sqrt{2} \). From a computational point of view, this means that 1-DRW and DCM are equivalent both in terms of error probabilities and of average number of observations needed to make a decision (Fig. 1b).

In contrast, in two-choice tasks, the 2-DRW is less efficient than the 1-DRW, because, for a given level of accuracy, it takes longer to make a decision. Fig. 2 shows results of two series of simulations, which use the 1-DRW and the 2-DRW to choose between a high- and a low-quality resource. Both the 1-DRW and the 2-DRW simulations predict preferences for the high-quality option to increase asymptotically with the increase of \( T \) and, consequently, with the increase of the decision time. However, the 1-DRW performs better than the 2-DRW, because at any decision time, it predicts stronger preferences and, thus, more accurate decisions. Furthermore, in two-choice contexts, the 2-DRW introduces a weak bias in the pattern of choice, as shown by the divergent pattern of the iso-probability lines of Figure 3b.
When a focal option (i.e. the poppy, represented by the star marker of Fig. 3b, with Benefits \(b(q^p) = 5\) and Costs \(c(h^p) = -1\)) is compared against an alternative of equal overall value (i.e. the daisy, with \(b(q^d) - c(h^d) = 6\)), the 2-DRW algorithm predicts always random choice (i.e. the slope of the 0.5 iso-probability line of Fig. 3b is 1). However, when the alternatives show different overall values (\(b(q^d) - c(h^d) \neq 6\)), preferences depend on the \(b(q^d)\) and \(c(h^d)\) and not just on their sum. For example, in the simulations of Fig. 3b, when the alternative option has \(b(q^d) = 0\) and \(c(h^d) = -4\), the probability that it is chosen is 0.13, but it is twice as large (preference = 0.26) when it has \(b(q^d) = 4\) and \(c(h^d) = 0\).

The performance of 2-DRW decreases even more when the number of items to choose among increases. Fig. 4a, b show what happens when a third option, a lily (\(L\)), is added to the choice set. In this example, the poppy and the daisy differ in both benefits and costs, but have the same economic value (i.e. the same difference between benefits and costs): \(b(q^p) = -c(h^p) = 5\) and \(b(q^d) = -c(h^d) = 1\). For this reason, in a dyadic choice, the insect does not show any preference either for the poppy or for the daisy. To simulate the decoy effect, I further assume the lily to be asymmetrically dominated by the daisy, that is, \(c(h^L) = c(h^d) - \delta\), with \(0 < \delta \ll c(h^d)\), and \(b(q^L) = b(q^d)\).

When the lily is introduced in the set of the available options, the positions of both the decision thresholds and the consensus area change. In the two-choice test, the accumulating evidence for either the daisy or the poppy causes the benefit threshold to decrease and the cost threshold to increase at a similar rate. As a consequence, during the process, the minimum distance between the consensus area and either the poppy or the daisy was similar, and the two options had the same probability of winning the race. With the lily, however, the two thresholds no longer change at similar rates and the consensus area no longer maintains.
the same minimum distance from the options (see also the Jupyter notebook in the supplementary materials). In fact, since the lily has very low costs and moderate energetic benefits, during the assessment, the benefit threshold ($\theta_{BB}$) decreases faster than the cost threshold ($\theta_{CC}$), because $T + \bar{C}(n) < T - \bar{B}(n)$. For this reason, with the lily, the option with the lowest costs (the daisy) is closer to and more likely to enter the consensus area than the option with the highest benefits and costs (the poppy) (Fig. 4a).

**Discussion**

This model follows the long tradition of sequential-sampling models in theoretical psychology (Busemeyer and Townsend 1993, Ratcliff et al. 2016, Vickers 1970), in that it views decision making as a process of noisy accumulation of evidence over time. According to these models, decision makers use sensory information of the available options to build a dynamic internal representation of their perceived value (i.e. economic “utility”). They make options to compete against each other and choose the option whose perceived value first reaches a strategically-set decision threshold. The competition can be in the form of either a “race” (if options are valued independently of the others) or a “tug-of-war” (if options are valued relatively to the others) (Kacelnik et al. 2011). In both types of models, the perceived values are assumed to be one-dimensional and the decision rules to be context-independent, in that they are strategically adjusted before the process, but kept fixed throughout. The one-dimension assumption is a sufficient (but not necessary) condition for economic rational choices and for the two axioms of rationality, transitivity and regularity. Since humans and other animals (both vertebrates and invertebrates) often choose irrationally, some authors have proposed modified versions of the “tug-of-war” model, with ad-hoc (non-linear)
computational mechanisms that could reconcile the one-dimension assumption with the reversal effects in multi-alternative, multi-attribute choice (review in Tsetsos et al. 2010).

Although my model shares some similarities with the “race” models, in that it assumes the perceived values to be context independent, it shows two important differences. First, the decision rules are context-dependent, because the thresholds are dynamically adjusted during the decision process. Second, the internal representation is multi-dimensional, because decision makers are not assumed to integrate the option attributes into a one-dimension representation of its value, but to accumulate different kinds of evidence along different levels of representations. For these reasons, the decision process might be no longer viewed as a one-direction race against a fixed finish line, but as a multi-direction race towards a consensus area, whose boundaries vary dynamically during the process. This view is consistent with some recent findings in the neurophysiology of decision making: for example, Cisek (2012) reviews neurophysiological evidence that sensorimotor neurons are not simply the executors of a command, but the main actors of the deliberative process of decision making. According to this view, alternative actions are simultaneously represented in the sensorimotor regions of the brain and compete against each other for execution (Gallivan et al. 2015). The competition occurs at multiple levels and involves the parallel processing of different types of information, such as the subjective benefits and costs of the options, the biomechanical costs of actions, and the geometric relationships between the options (Cisek and Pastor-Bernier 2014). The competition terminates when the different levels of representations (i.e. different axes of the decision plane) reach a consensus.

A distributed-consensus mechanism of decision-making requires thresholds to be dynamically adjusted during the decision process. In one-choice decision tasks, where the
options are assessed in terms of their intrinsic perceived benefits and costs, the optimal
calibration can be achieved by simple inhibitory mechanisms (see eq. 1), which guarantee
that decision is made as soon as the difference between benefits and costs is greater than a
given (optimal) threshold. This independent dynamic adjustment of thresholds makes the 2-
DRW functionally equivalent to the 1-DRW model, because it requires the same amount of
evidence to make a decision. From a computational point of view, however, the two models
are not equivalent, because the 2-DRW model is fully described by two variables ($B$, benefit,
and $C$, costs), whereas the 1-DRW requires a third variable, which is the difference of the
other two. One may be tempted to speculate that the 2-DRW algorithm might have been
favoured by natural selection over the 1-DRW alternative, because of its greater
computational (and neuro-anatomical) parsimony: in fact, the 2-DRW can works as
efficiently as the 1-DRW, but with fewer computational units (i.e. neurons).

In this paper, I have explored the hypothesis that the 2-DRW model, which performs
optimally in one-choice decision tasks, might have been co-opted in two-choice contexts,
where it still works satisfactorily, and in multiple-choice contexts, where it sometimes causes
irrational behaviour. According to this hypothesis, thus, there exists a single, general
mechanism of decision making. Sequential and simultaneous choice are not the expression of
different heuristics, but of the same flexible mechanism, used under different contexts. This
mechanism is not merely computational, but psychological, because it describes the process
of decision making. If we are to understand the functional role of flexible behaviour, in fact,
we need to study the evolution of the psychological mechanisms that control behavioural
flexibility (McNamara and Houston 2009, Castellano 2015, Castellano and Cermelli 2011,
2015). I argue that heuristic theories are limited in this respect, because they neglect the
mechanisms underlying the decision process. Although heuristics have inspired much
theoretical and empirical work on the rules that govern behavioural flexibility, it is time to
move on. Heuristics describe the decision rules, but not their underlying computational
mechanisms. They are in the mind of the beholder, not in the mind of the decision maker. In
the recent years, our understanding of the neurophysiology of decision making has advanced
to the point where evolutionary theories can be extended beyond the normative models of
optimal behaviours (Barron et al. 2015, Mobbs et al. 2018). We are now at the point where
we can address questions about the optimal mechanisms of information processing and, in the
process, achieve a much deeper understanding of the evolution of behavioural flexibility.

Competing interests

I have no competing interests.

References

Decision making and action selection in insects: inspiration from vertebrate-based


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Appendix 1

The insect decides to exploit a resource if the perceived benefits are greater than the perceived costs. Here I explain how sensory information about a flower is converted into the perceived benefits and costs of an action (i.e., exploiting or avoiding the resource).

Within the inspecting patch, the predation risk is assumed to be independent of the types of flower. It follows, thus, that the costs of a resource are only due to the loss of opportunities.

Suppose that the insect can assess the energetic value of a resource \( q_i \) and its handling time \( h_i \) and that it knows which is the average quality \( \bar{q} \), the average handling time \( \bar{h} \), and the average searching time \( \bar{s} \) of the resources within the patch. Under these assumptions, the insect is assumed to exploit a resource \( i \) if its rate of energy intake \( \frac{q_i h_i}{s + h} \) is greater than that expected if it decided to reject it and to search for the next resource.

\[
\frac{q_i}{h_i} > \frac{\bar{q} - \gamma \bar{s}}{\bar{s} + \bar{h}} \quad \text{Eq. A1}
\]

The right hand side of the inequality represents the opportunity costs: in the numerator, there is the expected quality of the \((i + 1)\) resource (that is, \( \bar{q} \), the average quality of the patch), discounted by the average cost of searching \( (\gamma \bar{s}) \) (where \( \gamma \), a constant, is the cost of searching per unit time). In the denominator, there is the expected time for exploiting the \((i + 1)\) resource, which is the sum of the average searching \( (\bar{s}) \) and handling \( (\bar{h}) \) times.

From Eq. A1, it derives that the expected value (in units of energy) of the \( i \) resource is the difference between what the insect is expected to gain and to lose if it decided to exploit the resource:

\[
e_i = q_i - \frac{(\bar{q} - \gamma \bar{s})}{\bar{s} + \bar{h}} h_i \quad \text{Eq. A2}
\]
Eqn. A2 can be rescaled in order to obtain an a-dimensional representation of the perceived economic value of the $i$ resource:

$$u_i = \frac{e_i}{(q - \gamma s)} = \frac{q_i - (q - \gamma s) \bar{q} - (q - \gamma s) \gamma s}{s + \bar{h}}.$$  \hspace{1cm} \text{Eq. A3}$$

The 1-DRW and the 2-DRW assume that the noisy sensory information about the energetic quality and the handling time of a resource is converted into the perceived benefits and costs by two linear filters, $b(Q)$ and $c(H)$:

$$b(Q) = \frac{Q - (\bar{q} - \gamma s)}{(q - \gamma s)}, \hspace{1cm} \text{Eq. A4a}$$

$$c(H) = \frac{H - (\bar{s} + \bar{h})}{s + \bar{h}}, \hspace{1cm} \text{Eq. A4b}$$

where $Q$ and $H$ are the two normally-distributed stochastic variables with expectations $q$ and $h$, and standard deviation $\sigma$ (see main text). Notice that from Eqn. A1 it derives that $b$ and $c$ are linear functions of the incoming sensory information, It might be emphasized, however, that linearity between sensation and perception is by no means a necessary condition of the model (see for example Castellano 2015) The $b$ and the $c$ functions convert different types of sensory information into the same currency (patterns of neural stimulation) and it is on this common ground that different features of one, or more than one, option can be compared.
**FIGURE CAPTIONS**

**Figure 1.** (a) Graphical representations of the two-dimension random-walk process of decision making (2-DRW) under a one-choice decision task. Noisy information about the expected benefits and the costs of an action is accumulated over time into two separate counters, which describe the position of the action in the informational plane. The action is executed when it enters the green zone and discarded when it enters the red zone. During the assessment, the thresholds that define the acceptance ($\theta^u_C, \theta^u_B$) and the rejection ($\theta^l_C, \theta^l_B$) areas are dynamically adjusted, as described by Eqn. 1 (see main text). The plot shows the acceptance/rejection areas in three moments of the decision process, which terminates with the execution of the action (when the random walk enter the green area). The dynamic adjustment of the two thresholds minimizes the response time, because it forces the process to stop as soon as the blue path touches the black threshold. (b) A contour plot of the preferences for a resource as a function of its costs and the benefits. Costs and benefits interact additively on the preferences. If the benefits are greater than the costs, the option would be positively selected ($P > 0.5$). If costs are greater than benefits, the option would be negatively selected ($P < 0.5$). The strength of selection is predicted to depend on only the difference between benefits and costs and to be independent of their absolute values.

**Figure 2.** A comparison of the speed-accuracy trade-offs in the one- (1-DRW, filled circles) and in the two-dimension random walk models (2-DRW, open circles), during a series of two-choice discrimination tests between a high-quality ($c(h_{\text{high}}) = 0.5, b(q_{\text{high}}) = 0.5$) and a low-quality resource ($c(h_{\text{low}}) = 0.0, b(q_{\text{low}}) = 0.0$), in which the acceptance ($T = \theta^u_B - \theta^u_C$) and the rejection ($L = \theta^l_B - \theta^l_C = -T$) decision thresholds were let to vary between 0.1 and 3 by
steps of 0.05. Each point is associated to a threshold value and shows the average decision latency and the preference for the high-quality option after running 10,000 random-walk simulations. Preferences for the high-quality option increase with the increasing latencies (that is, with the increasing $T$), but, at any decision latency, 1-DRW performs better than 2-DRW.

**Figure 3.** (a) A graphical representation of the bi-dimension random-walk process of decision making in a two-choice decision task. The process is the same as in Fig. 1, but now it is run in parallel on the star and the black-circle options. The black-circle option (D, the daisy) has lower expected costs and lower perceived benefits ($[c(h^D) = -5, b(q^D) = 1]$) than the star option (P, the poppy) ($[c(h^P) = -1, b(q^P) = 5]$), so that their overall values (the difference between benefits and costs) are the same. Since the acceptance thresholds along the cost and benefit axes increase with the increase, respectively, of the average costs ($\bar{C}(n)$) and the average benefits ($\bar{B}(n)$), they oscillate midway between the optimal values of the two conflicting options. As a consequence, the 2-dimension random walk needs a larger amount of evidence and much more time to enter the consensus area. (b) Contour plot of the two-choice preference for an option, when compared against the poppy (star). The 2-DRW introduces a slight bias in the preference pattern. In fact, the slope of the iso-probability lines is one only when the difference between the benefits and costs of an option equals that of the poppy, as shown by the daisy (black circle).

**Figure 4.** (a) A bi-dimension random-walk process in a three-choice decision task. The third option (the lily, grey circle, $c(h^L) = -4.8, b(q^L) = 1$) is asymmetrically dominated by the
daisy, (the black-circle option, \(c(h^L) = -5, b(q^L) = 1\)). Its introduction modifies the position of the consensus areas, biasing the preferences to the advantage of those items that have the lowest costs. While in two-choice test, the daisy is as likely as the poppy to be chosen by the insect, in the three-choice context, the daisy is strongly preferred over the poppy. (b) Contour plot of the preferences in three-choice discrimination tests. The iso-probability lines show the preferences for a given option (for example, the daisy, represented by the black circle) with respect to a fixed alternative (the poppy, represented by the star marker), when a third alternative (the decoy represented by the grey circle) is also present. Due to the characteristics of the decoy (very low costs and moderate benefits), small changes along the cost dimension have stronger effects on the preferences than changes of similar magnitude along the benefit dimension.