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Yangian symmetry of string theory on $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ with mixed 3-form flux

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Abstract

We find the Yangian symmetry underlying the integrability of type IIB superstrings on $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ with mixed Ramond–Ramond and Neveu–Schwarz–Neveu–Schwarz flux. The abstract commutation relations of the Yangian are formulated via RTT realisation, while its matrix realisation is in an evaluation representation depending on the quantised coefficient of the Wess–Zumino term. The construction naturally encodes a secret symmetry of the worldsheet scattering matrix whose generators map different Yangian levels to each other. We show that in the large effective string tension limit the Yangian becomes a deformation of a unitary loop algebra and we derive its universal classical r-matrix.

1. Introduction

1.1. Yangian symmetry and exact solvability

There are more symmetries in a theory than those displayed by its Lagrangian. This is particularly evident in four dimensional $\mathcal{N} = 4$ Super Yang–Mills theory, whose planar on-shell scattering amplitudes [1] and Wilson loops [2] are invariant under an infinite tower of non-local conserved charges. The very same symmetry, identified with the *Yangian* $\mathcal{Y}[\mathfrak{psu}(2, 2|4)]$, was found in type IIB superstrings on $\mathrm{AdS}_5 \times S^5$ by studying the coset structure of the background [3]. Generally, if $\mathfrak{g}$ is a graded Lie algebra, the corresponding Yangian $\mathcal{Y}[\mathfrak{g}]$ is a particular

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quantum deformation of $\mathcal{U}(\mathfrak{g}[u])$, the universal enveloping algebra of $\mathfrak{g}$-valued polynomials in the spectral parameter $u \in \mathbb{C}$. In terms of *Drinfeld first realisation* [4–6], $\mathcal{Y}[\mathfrak{g}]$ is formulated as

$$\mathcal{Y}[\mathfrak{g}] = \bigcup_{n\in\mathbb{N}} \mathcal{Y}_n[\mathfrak{g}], \quad \mathcal{Y}_n[\mathfrak{g}] = \text{span}\, \mathfrak{J}^A_{(n)},$$
$$\left[\mathfrak{J}^A_{(m)}, \mathfrak{J}^A_{(n)}\right\} = f^{AB}{}_C\, \mathfrak{J}^C_{(m+n)}, \quad m+n = 0, 1; \tag{1.1}$$

where $f^{AB}{}_C$ are the structure constant of $\mathfrak{g}$, $[X, Y\} = XY - (-)^{|X||Y|} YX$ is the graded commutator of X and Y and $|\cdot|$ the Graßmann grading. In particular, the Lie algebra $\mathfrak{g}$ coincides with $\mathcal{Y}_0[\mathfrak{g}]$. Moreover, $\mathcal{Y}[\mathfrak{g}]$ carries a Hopf algebra structure and its generators satisfy generalised Jacobi identities called *Serre relations*. All $\mathcal{Y}_n[\mathfrak{g}]$ with $n > 1$ can be reconstructed out of lower levels via (1.1): the whole $\mathcal{Y}[\mathfrak{g}]$ is then determined by $\mathcal{Y}_0[\mathfrak{g}]$ and $\mathcal{Y}_1[\mathfrak{g}]$. Eventually, $\mathcal{Y}[\mathfrak{g}]$ admits an *evaluation representation* if there exists a spectral parameter dependent map ρ_u such that

$$\rho_u\, \mathfrak{J}^A_{(n)} = u^n\, \mathfrak{J}^A_{(0)}, \qquad J^A_{(0)} \in \mathfrak{g}, \qquad n \in \mathbb{N}. \tag{1.2}$$

Yangian symmetry typically appears in integrable quantum field theories: indeed, both $\mathcal{N} = 4$ super Yang–Mills theory [7–9] and type IIB superstrings on $\mathrm{AdS}_5 \times S^5$ [10–12] are dual to an integrable spin chain system. This duality is even more powerful than the AdS/CFT correspondence [13–15] as the integrability picture allows for computing the conformal data of $\mathcal{N} = 4$ super Yang–Mills theory at all orders in the gauge coupling constant [16–22]. Technically, it is therefore very useful to have the control on the dual integrable picture through its symmetries, as the latter are able to strictly constrain or even determine gauge theory or gravity observables. Conceptually, Yangians are fascinating because they link to each other completely different models such as gauge, gravity and condensed matter systems via a rich mathematical structure. Yangians have been found in quite a few instances of the AdS/CFT correspondence [23–26]. In this paper we will focus on type IIB superstrings on $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ with mixed Ramond–Ramond (RR) and Neveu–Schwarz–Neveu–Schwarz (NSNS) flux. We shall now review the main features of the model.

1.2. *Superstrings on* $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ *with mixed 3-form flux*

The near-horizon geometry of two M5 branes intersecting an M2 brane is $\mathrm{AdS}_3 \times S^3 \times S^3 \times T^2$, whose dimensional reduction along S^1 gives type IIA superstrings on $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ with 16 supersymmetries. By performing T-duality along the S^1 one obtains the corresponding type IIB model [27–32]. $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ supports both RR and NSNS fluxes and displays $\mathrm{D}(2,1;\alpha) \times \mathrm{D}(2,1;\alpha) \times \mathrm{U}(1)$ isometry, with the AdS and sphere radii being parametrised by α:

$$\alpha = R^2_{\mathrm{AdS}}/R^2_{S^3_+}, \qquad 1-\alpha = R^2_{\mathrm{AdS}}/R^2_{S^3_-}, \tag{1.3}$$

where the spheres are labelled by $\pm$. If the spheres have equal radii, $\alpha = 1/2$ and the exceptional superalgebra $\mathfrak{d}(2,1;\alpha)$ becomes $\mathfrak{osp}(4|2)$. Instead, if $R_{\mathrm{AdS}} = R_{S^3_+}$ (respectively, $R_{\mathrm{AdS}} = R_{S^3_-}$), then $\alpha = 1$ ($\alpha = 0$), $\mathfrak{d}(2,1;\alpha)$ reduces to $\mathfrak{psu}(2|2)$ and $S^3_- \cong \mathbb{R}^3$ ($S^3_+ \cong \mathbb{R}^3$) can be compactified into a T^3 giving the $\mathrm{AdS}_3 \times S^3 \times T^4$ background, whose complete worldsheet S-matrix was found in [33]. The massive sector of the worldsheet scattering matrix of type IIB strings on $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ with RR flux was first derived in [34], while its integrability properties were studied in [35] exploiting the coset structure of the background,[1]

$$\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1 = \frac{\mathrm{D}(2,1;\alpha) \times \mathrm{D}(2,1;\alpha)}{\mathrm{SO}(1,2) \times \mathrm{SO}(3) \times \mathrm{SO}(3)}. \tag{1.4}$$

The investigation was generalised by including both RR and NSNS fluxes in [39] directly working on the Green–Schwarz action of the theory, ignoring the coset formulation. In light-cone gauge, the worldsheet scattering is encoded in an $\mathfrak{su}_c(1|1)^2$ invariant R-matrix, which was derived assuming the integrability of the quantum theory. As in the $\mathrm{AdS}_5 \times S^5$ case, to such an integrability shall correspond a Yangian symmetry restricting the observables of the conformal field theory dual, first tackled in [40,41] and deeply studied in [42–44]. The goal of this paper is indeed to extract the Yangian of $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ superstrings with mixed flux. Especially, we will employ the scattering matrix found in [39] to define its own symmetry algebra. The procedure we will use is called RTT realisation [45–47]: we introduce it in the next subsection.

1.3. *Integrable scattering matrices from Hopf algebras*

Integrable quantum field theories enjoy an infinite number of conserved charges. Such a symmetry, which we denote by $\mathcal{A}$, is so constraining that it completely fixes the scattering matrix of the system. This situation is rigorously described if $\mathcal{A}$ has got a Hopf algebra structure. Indeed, the coproduct map $\Delta : \mathcal{A} \to \mathcal{A} \otimes \mathcal{A}$ naturally provides a multiparticle representation of the conserved charges: conventionally, $\mathfrak{J} \in \mathcal{A}$ acts on in-states via $\Delta(\mathfrak{J})$ and on out-states via $\Delta^{\mathrm{op}}(\mathfrak{J}) = \mathcal{P} \circ \Delta(\mathfrak{J})$, where $\mathcal{P}(X \otimes Y) = (-1)^{|X||Y|} Y \otimes X$ is the graded permutation operator. The symmetry acts on antiparticle states through the antipode $\Sigma : \mathcal{A} \to \mathcal{A}$, which is a $\mathbb{C}$-linear anti-homomorphism.

This provided, the scattering matrix S acting on the Hilbert space $\mathscr{H}$ can be expressed as $S = \mathcal{P} \circ R$, with $R : \mathbb{C} \times \mathbb{C} \to \mathrm{End}(\mathscr{H} \otimes \mathscr{H})$ intertwining Δ and Δ^{op}. The R-matrix depends on two spectral parameters $u_1, u_2 \in \mathbb{C}$ related to the momenta of the scattering excitations and is fully determined by the *quasi co-commutativity condition*

$$\Delta^{\mathrm{op}}(\mathfrak{J})\, R = R\, \Delta(\mathfrak{J}), \qquad \forall\, \mathfrak{J} \in \mathcal{A}, \tag{1.5}$$

as well as by the *crossing equations*

$$(\Sigma \otimes \mathbb{1})\, R = (\mathbb{1} \otimes \Sigma)\, R = R^{-1}. \tag{1.6}$$

Specifically, (1.6) fixes the *dressing phases* left unconstrained by (1.5), once the analyticity properties of such phase factors are specified.[2] Then, the S-matrix immediately follows from $S = \mathcal{P} \circ R$. Importantly, (1.5) implies that, if $\mathfrak{C} \in \mathcal{A}$ is central, it must be co-commutative: $\Delta^{\mathrm{op}}(\mathfrak{C}) = \Delta(\mathfrak{C})$. We shall also require that the underlying Hopf algebra $\mathcal{A}$ is *almost quasi triangular*, meaning that the *fusion relations*[3]

$$(\Delta \otimes \mathbb{1})\, R_{12} = R_{13}\, R_{23}, \qquad (\mathbb{1} \otimes \Delta)\, R_{12} = R_{13}\, R_{12} \tag{1.7}$$

[1] Such a coset structure was also used to prove the T-self duality of $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ in [36]. T-self duality is related to dual superconformal symmetry, which is a subsector of the full Yangian algebra [37,38].

[2] The dressing phase for $\mathrm{AdS}_5/\mathrm{CFT}_4$ was studied in [48–51], while the $\mathrm{AdS}_3/\mathrm{CFT}_2$ case was investigated in [26,52].

[3] We denote by t_{ij} a tensor acting upon the i-th and j-th components of the multiple tensor product of representations $V_1 \otimes \cdots \otimes V_i \otimes \cdots \otimes V_j \otimes \cdots$.

hold. This is not restrictive as (1.7) and (1.5) imply the *quantum Yang–Baxter equation* (QYBE) [53]

$$R_{12}\,R_{13}\,R_{23} = R_{23}\,R_{13}\,R_{12}, \tag{1.8}$$

which is a necessary and characterising condition for R-matrices of integrable systems.

1.4. From R-matrices to Hopf algebras

The above discussion can be read backwards; indeed, an R-matrix satisfying the QYBE defines its own Hopf algebra structure by the *RTT relations*

$$R_{12}(u_1,u_2)\,\mathcal{T}_1(u_1)\,\mathcal{T}_2(u_2) = \mathcal{T}_2(u_2)\,\mathcal{T}_1(u_1)\,R_{12}(u_1,u_2), \tag{1.9}$$

where the monodromy matrix $\mathcal{T}:\mathbb{C}\to \mathrm{End}(\mathscr{H})\otimes\mathcal{A}$ generates the conserved charges. If we focus on the $\mathrm{AdS}_3\times S^3\times S^3\times S^1$ worldsheet scattering, $\mathrm{End}(\mathscr{H})=\mathfrak{su}_c(1|1)^2\subset\mathfrak{gl}(2|2)$ and we can express the R-matrix and the monodromy matrix in the standard basis[4] $\{e^a{}_b\}$ of the $\mathfrak{gl}(2|2)$ superalgebra. This embedding will be particularly convenient as it allows for dealing with both copies of $\mathfrak{su}_c(1|1)$ at the same time. Then, assuming that $\mathcal{T}(u)$ is holomorphic in a neighbourhood of $u=\infty$,

$$\mathcal{T}(u) = \sum_{n\in\mathbb{N}}\sum_{a,b=1}^{4} u^{-n}(-1)^{|b|}e^b{}_a\otimes\mathbb{T}^{a}_{(n-1)\,b}, \tag{1.10}$$

with $\mathbb{T}^a_{(n)\,b}$ being the abstract (representation independent) generators of $\mathcal{A}$. Similarly, the R-matrix takes the form

$$R = \sum_{a,b,c,d=1}^{4}(-1)^{|b|+|c|}R^{bd}_{ac}(u_1,u_2)\,e^a{}_b\otimes e^c{}_d. \tag{1.11}$$

Substituting (1.10) and (1.11) in (1.9) yields

$$\begin{aligned}\sum_{n_1\in\mathbb{N}}\sum_{n_2\in\mathbb{N}}\sum_{e=1}^{4}\sum_{f=1}^{4}u_1^{-n_1}u_2^{-n_2}\Big[&(-1)^{(|c|+|e|)(|b|+|d|)+|f|}\,R^{ef}_{ab}\,\mathbb{T}^{c}_{(n_1-1)\,e}\mathbb{T}^{d}_{(n_2-1)\,f}+\\ &-(-1)^{(|b|+|d|)(|d|+|f|)+|b|}\,R^{cd}_{ef}\,\mathbb{T}^{f}_{(n_2-1)\,b}\mathbb{T}^{e}_{(n_1-1)\,a}\Big]=0.\end{aligned} \tag{1.12}$$

The abstract commutation relations of $\mathcal{A}$ are therefore recovered by expanding this constraint around $u_1,u_2=\infty$: for instance, (1.12) implies $\mathbb{T}^a_{(-1)\,b}=\mathbb{U}\,\delta^a{}_b$, where $\mathbb{U}$ is a central element. Furthermore, by comparing the RTT relations (1.9) with the QYBE (1.8), one observes that R and $\mathcal{T}(u)$ can be connected via a representation map ρ_u depending on the spectral parameter u: $R(u_1,u_2)=\big(\mathbb{1}\otimes\rho_{u_2}\big)\,\mathcal{T}(u_1)$. As a result, the formula

$$\sum_{c=1}^{4}\sum_{d=1}^{4}(-1)^{|c|}R^{ac}_{bd}(u_1,u_2)\,e^d{}_c = T^a_{(-1)\,b}+\frac{1}{u_1}T^a_{(0)\,b}+\frac{1}{u_1^2}T^a_{(+1)\,b}+\dots \tag{1.13}$$

[4] Such a basis comprehends 16 matrices with components $\left(e^a{}_b\right)^i{}_j=(-1)^{|b|}\delta^{ia}\,\delta_{bj}$ and $a,b,i,j=1,\dots,4$. We choose the grading of the indices to be $|1|=|2|=-|3|=-|4|=+1$, meaning that $1,2$ are bosonic indices while $3,4$ are fermionic ones.

gives access to the representations $T^{a}_{(n)\,b} = \rho_u \mathbb{T}^{a}_{(n)\,b}$. As for the Hopf algebra structure, the fusion relations (1.7) induce the coproducts

$$\Delta\left(\mathbb{U}\right) = \mathbb{U}\otimes\mathbb{U}, \qquad \Delta\left(\mathbb{T}^{a}_{(0)\,b}\right) = \mathbb{T}^{a}_{(0)\,b}\otimes\mathbb{U}^{|b|} + \mathbb{U}^{|a|}\otimes\mathbb{T}^{a}_{(0)\,b},$$
$$\Delta\left(\mathbb{T}^{a}_{(1)\,b}\right) = \mathbb{T}^{a}_{(1)\,b}\otimes\mathbb{U}^{|b|} + \mathbb{U}^{|a|}\otimes\mathbb{T}^{a}_{(1)\,b} + \sum_{c=1}^{4}\mathbb{T}^{a}_{(0)\,c}\otimes\mathbb{T}^{c}_{(0)\,b}, \tag{1.14}$$

while the antipodes descend from (1.6):

$$\Sigma\left[\mathbb{T}^{a}_{(-1)\,b}\right] = \mathbb{U}^{-|b|}\delta^{a}{}_{b}, \qquad \Sigma\left[\mathbb{T}^{a}_{(0)\,b}\right] = -\mathbb{U}^{-|a|-|b|}\mathbb{T}^{a}_{(0)\,b},$$
$$\Sigma\left[\mathbb{T}^{a}_{(1)\,b}\right] = -\mathbb{U}^{-|a|-|b|}\mathbb{T}^{a}_{(1)\,b} + \sum_{c=1}^{4}\mathbb{U}^{-|a|-|b|-|c|}\mathbb{T}^{a}_{(0)\,c}\mathbb{T}^{c}_{(0)\,b}. \tag{1.15}$$

If the Hopf algebra of the system is a Yangian over a graded Lie algebra $\mathfrak{g}$, $\mathcal{Y}[\mathfrak{g}]$, it is useful to define the objects

$$\mathbb{J}^{a}_{(0)\,b} := -\frac{ih}{2m}\,\mathbb{U}^{-|b|}\,\mathbb{T}^{a}_{(0)\,b},$$
$$\mathbb{J}^{a}_{(1)\,b} := -\frac{ih}{2m}\,\mathbb{U}^{-|b|}\mathbb{T}^{a}_{(1)\,b} - \frac{im}{h}\sum_{c=1}^{4}(-1)^{(|a|+|c|)(|b|+|c|)}\mathbb{J}^{c}_{(0)\,b}\mathbb{J}^{a}_{(0)\,c}, \tag{1.16}$$

where the constants h, m appear for future convenience. These $\mathbb{J}$s satisfy

$$\Delta\left(\mathbb{J}^{a}_{(0)\,b}\right) = \mathbb{J}^{a}_{(0)\,b}\otimes\mathbb{1} + \mathbb{U}^{|a|-|b|}\otimes\mathbb{J}^{a}_{(0)\,b},$$
$$\Delta\left(\mathbb{J}^{a}_{(1)\,b}\right) = \mathbb{J}^{a}_{(1)\,b}\otimes\mathbb{1} + \mathbb{U}^{|a|-|b|}\otimes\mathbb{J}^{a}_{(1)\,b} + \frac{im}{h}\sum_{c=1}^{4}\mathbb{U}^{|c|-|b|}\,\mathbb{J}^{a}_{(0)\,c}\otimes\mathbb{J}^{c}_{(0)\,b} +$$
$$-\frac{im}{h}\sum_{c=1}^{4}(-1)^{(|a|+|c|)(|c|+|b|)}\mathbb{U}^{|a|-|c|}\,\mathbb{J}^{c}_{(0)\,b}\otimes\mathbb{J}^{a}_{(0)\,c}, \tag{1.17}$$

which are the coproducts of $\mathcal{Y}[\mathfrak{g}]$ in Drinfeld first realisation. Moreover,

$$\Sigma\left[\mathbb{J}^{a}_{(0)\,b}\right] = -\mathbb{U}^{|b|-|a|}\mathbb{J}^{a}_{(0)\,b},$$
$$\Sigma\left[\mathbb{J}^{a}_{(1)\,b}\right] = -\mathbb{U}^{|b|-|a|}\left(\mathbb{J}^{a}_{(1)\,b} - \frac{im}{h}\sum_{c=1}^{4}\left[\mathbb{J}^{a}_{(0)\,c}, \mathbb{J}^{c}_{(0)\,b}\right\}\right). \tag{1.18}$$

A great advantage of the RTT realisation is that it automatically provides the consistency conditions of the Hopf algebra, for instance [54,55]

$$\mu\circ(\Sigma\otimes\mathbb{1})\circ\Delta\left(\mathfrak{J}\right) = \mu\circ(\mathbb{1}\otimes\Sigma)\circ\Delta\left(\mathfrak{J}\right) = \epsilon\left(\mathfrak{J}\right)\mathbb{1}, \qquad \mathfrak{J}\in\mathcal{A}, \tag{1.19}$$

where μ and ϵ are the Hopf algebra multiplication and counit respectively.

1.5. Outline

To make the paper self-contained, in Section 2 we re-derive the $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ superstring worldsheet scattering matrix, originally found in [39]. In Section 3 we use the RTT realisation to extract its Yangian symmetry. We also derive the Yangian evaluation representation and show its dependence on the quantised coefficient of the Wess–Zumino term appearing in the $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ superstring action. In Section 4 we perform the large effective string tension limit and demonstrate that the resulting classical r-matrix can be written in a universal, representation independent form as a tensor product of $\mathfrak{u}(1|1)[u, u^{-1}]^2$ loop algebra generators.

1.6. Note

While completing this paper I became aware of [56], which has some overlap with this work. I am very grateful to the authors for sharing their draft before publication.

2. $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ R-matrix from Hopf algebra

2.1. Centrally extended $\mathfrak{su}_c(1|1)^2$ algebra and its representations

As we already mentioned, the fundamental symmetry of the worldsheet theory in light-cone gauge is $\mathfrak{su}_c(1|1)^2$, which is made of two copies of $\mathfrak{su}(1|1)$ and two central charges entangling them,

$$\mathfrak{su}_c(1|1)^2 = [\mathfrak{su}(1|1)_L \oplus \mathfrak{su}(1|1)_R] \ltimes \mathbb{R}^2, \tag{2.1}$$

where we labelled each copy by left (L) and right (R). Each $\mathfrak{su}(1|1)_A$ is a three-dimensional superalgebra generated by two supercharges Q_A, S_A and a central charge $H_A = \{Q_A, S_A\}$. Consequently, the algebra $\mathfrak{su}_c(1|1)^2$ has got 4 fermionic generators Q_L, S_L, Q_R, S_R and 4 central charges H_L, H_R, P, K whose commutation rules are

$$\{Q_A, S_B\} = \delta_{AB}\, H_B, \qquad \{Q_L, Q_R\} = P, \qquad \{S_L, S_R\} = K; \qquad A, B = L, R. \tag{2.2}$$

The elementary excitations of the $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ worldsheet scattering are represented by two bosonic states $|\phi_A\rangle$ and two fermionic ones $|\psi_A\rangle$ with $A = L, R$. The action of Q_A, S_A on the left module $\nu_L = \{|\phi_L\rangle, |\psi_L\rangle\}$ is

$$\begin{aligned} &Q_L\,|\phi_L\rangle = a_L\,|\psi_L\rangle\,, \qquad S_L\,|\psi_L\rangle = c_L\,|\phi_L\rangle\,, \qquad S_R\,|\phi_L\rangle = d_L\,|\psi_L\rangle\,, \\ &Q_R\,|\psi_L\rangle = b_L\,|\phi_L\rangle\,, \end{aligned} \tag{2.3}$$

while the value of the central charges on ν_L is

$$H_L = a_L c_L, \qquad H_R = b_L d_L, \qquad P = a_L b_L, \qquad K = c_L d_L. \tag{2.4}$$

The action on the right module $\nu_R = \{|\phi_R\rangle, |\psi_L\rangle\}$ follows from LR symmetry; namely, it is obtained by substituting L with R in (2.3) and (2.4): for instance, $Q_R\,|\phi_R\rangle = a_R\,|\psi_R\rangle$ and $S_L\,|\phi_R\rangle = d_R\,|\psi_R\rangle$. Such a representation satisfies the shortening condition $(H_L + H_R)^2 = (H_L - H_R)^2 + 4\,PK$ and is conveniently written in terms of Zhukovski variables $\left\{x_L^{\pm}, x_R^{\pm}\right\}$. These are kinematical variables depending on the momentum p and the mass m of the corresponding scattering excitation, as well as on the effective string tension h and the quantised coefficient κ of the Wess–Zumino term in the $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ superstring action. Specifically, $\{x_L^\pm, x_R^\pm\}$ are defined by the constraint

$$x_A^+ + \frac{1}{x_A^+} - x_A^- - \frac{1}{x_A^-} - \frac{2\kappa_A}{h} \log\left(\frac{x_A^+}{x_A^-}\right) = \frac{2\mathrm{i}m}{h}, \qquad A = L, R; \tag{2.5}$$

where $\kappa_L = -\kappa_R = |\kappa|$ and we chose the branch cut such that $\log e^{2\pi\mathrm{i}} = 0$. The link with the particle momentum is given by $x_L^+/x_L^- = x_R^+/x_R^- = e^{\mathrm{i}p}$. To simplify the R-matrix entries, we define

$$U := \sqrt{x_L^+/x_L^-} = \sqrt{x_R^+/x_R^-} = e^{\mathrm{i}p/2}, \qquad \eta_A := \left(\frac{x_A^+}{x_A^-}\right)^{1/4} \sqrt{\frac{\mathrm{i}h}{2}(x_A^- - x_A^+)}, \tag{2.6}$$

whose inverse relations read

$$x_A^+ = \frac{2\mathrm{i}\, U\, \eta_A^2}{h\left(U^2-1\right)}, \qquad x_A^- = \frac{2\mathrm{i}\, \eta_A^2}{h\, U\left(U^2-1\right)}, \qquad A = L, R. \tag{2.7}$$

In this setting, the representation coefficients assume the compact form

$$a_A = \eta_A, \qquad b_A = -\frac{\eta_A}{U x_A^-}, \qquad c_A = \frac{\eta_A}{U}, \qquad d_A = -\frac{\eta_A}{x_A^+}; \qquad A = L, R. \tag{2.8}$$

The Lie algebra $\mathfrak{su}_c(1|1)^2$ is enhanced to a Hopf algebra by introducing[5]

$$\Delta(Q_A) = Q_A \otimes \mathbb{1} + U \otimes Q_A, \qquad \Delta(S_A) = S_A \otimes \mathbb{1} + U^{-1} \otimes S_A, \tag{2.9}$$

which are the coproducts of the supercharges in *Drinfeld first realization*. Acting with $\mathcal{P}$ provides the opposite coproducts

$$\Delta^{\mathrm{op}}(Q_A) = Q_A \otimes U + \mathbb{1} \otimes Q_A, \qquad \Delta^{\mathrm{op}}(S_A) = S_A \otimes U^{-1} + \mathbb{1} \otimes S_A. \tag{2.10}$$

The coproduct is an algebra homomorphism, therefore $\Delta(H_A), \Delta(P), \Delta(K)$ are obtained by anticommuting (2.9): for example,

$$\Delta(H_A) = H_A \otimes \mathbb{1} + \mathbb{1} \otimes H_A, \qquad \Delta(P) = P \otimes \mathbb{1} + U^2 \otimes P. \tag{2.11}$$

Using (2.4) it is straightforward to check that the central charges are co-commutative.

The coproducts provide the two-particle representation of $\mathfrak{su}_c(1|1)^2$, which is sufficient to solve the scattering problem as the system is assumed integrable and n-body processes factorise into 2-body ones, as allowed by the QYBE.

2.2. $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ *R-matrix*

Using the basis $(\phi_L, \phi_R, \psi_L, \psi_R)$, the R-matrix for the worldsheet scattering of strings on $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ with mixed flux assumes the form

$$R\,|\phi_{L\,1}\,\phi_{L\,2}\rangle = \alpha_{LL}\,|\phi_{L\,1}\,\phi_{L\,2}\rangle\,,$$
$$R\,|\phi_{L\,1}\,\psi_{L\,2}\rangle = \beta_{LL}\,|\phi_{L\,1}\,\psi_{L\,2}\rangle + \gamma_{LL}\,|\psi_{L\,1}\,\phi_{L\,2}\rangle\,,$$

[5] With a slight abuse of notation we identify U with $U \times \mathbb{1}$.

$$
\begin{aligned}
&R\,|\psi_{L\,1}\,\phi_{L\,2}\rangle = \mu_{LL}\;|\psi_{L\,1}\phi_{L\,2}\rangle + \nu_{LL}\,|\phi_{L\,1}\,\psi_{L\,2}\rangle\,,\\
&R\,|\psi_{L\,1}\,\psi_{L\,2}\rangle = \rho_{LL}\,|\psi_{L\,1}\,\psi_{L\,2}\rangle\,,
\end{aligned}
\tag{2.12}
$$

in the LL sector and

$$
\begin{aligned}
&R\,|\phi_{L\,1}\,\phi_{R\,2}\rangle = \alpha_{LR}\,|\phi_{L\,1}\,\phi_{R\,2}\rangle + \beta_{LR}\,|\psi_{L\,1}\,\psi_{R\,2}\rangle\,,\\
&R\,|\phi_{L\,1}\,\psi_{R\,2}\rangle = \gamma_{LR}\,|\phi_{L\,1}\,\psi_{R\,2}\rangle\,,\\
&R\,|\psi_{L\,1}\,\phi_{R\,2}\rangle = \mu_{LR}\,|\psi_{L\,1}\,\phi_{R\,2}\rangle\,,\\
&R\,|\psi_{L\,1}\,\psi_{R\,2}\rangle = \nu_{LR}\,|\psi_{L\,1}\,\psi_{R\,2}\rangle + \rho_{LR}\,|\phi_{L\,1}\,\phi_{R\,2}\rangle\,,
\end{aligned}
\tag{2.13}
$$

in the LR sector. According to LR symmetry, the RL and RR sectors are found by just swapping L and R in the above expressions. Then, (1.5) fixes the LL and LR scattering elements:

$$
\begin{aligned}
&\alpha_{LL} = \sigma_{LL}, \qquad \beta_{LL} = \frac{\sigma_{LL}}{U_1}\,\frac{x^+_{L\,2}-x^+_{L\,1}}{x^+_{L\,2}-x^-_{L\,1}}, \qquad \gamma_{LL} = \sigma_{LL}\,\frac{\eta_{L\,2}\left(x^+_{L\,1}-x^-_{L\,1}\right)}{\eta_{L\,1}\left(x^+_{L\,2}-x^-_{L\,1}\right)},\\
&\mu_{LL} = \sigma_{LL}\,\frac{U_2\left(x^-_{L\,2}-x^-_{L\,1}\right)}{x^+_{L\,2}-x^-_{L\,1}}, \qquad \nu_{LL} = \gamma_{LL}, \qquad \rho_{LL} = \sigma_{LL}\,\frac{U_2\left(x^+_{L\,1}-x^-_{L\,2}\right)}{U_1\left(x^+_{L\,2}-x^-_{L\,1}\right)},\\
&\alpha_{LR} = \sigma_{LR}, \qquad \beta_{LR} = \sigma_{LR}\,\frac{\eta_{L\,1}\left(x^+_{R\,2}-x^-_{R\,2}\right)}{\eta_{R\,2}\left(x^+_{L\,1}x^-_{R\,2}-1\right)}, \qquad \gamma_{LR} = \sigma_{LR}\,\frac{U_1\left(x^-_{L\,1}x^-_{R\,2}-1\right)}{x^+_{L\,1}x^-_{R\,2}-1},\\
&\mu_{LR} = \frac{\sigma_{LR}}{U_2}\,\frac{x^+_{L\,1}x^+_{R\,2}-1}{x^+_{L\,1}x^-_{R\,2}-1}, \qquad \nu_{LR} = \sigma_{LR}\,\frac{U_1\left(x^-_{L\,1}x^+_{R\,2}-1\right)}{U_2\left(x^+_{L\,1}x^-_{R\,2}-1\right)}, \qquad \rho_{LR} = \beta_{LR},
\end{aligned}
\tag{2.14}
$$

where $x^\pm_{A\,j}$ are the kinematic variables referring to the state $|\Phi_{A\,j}\rangle$, with $\Phi = \phi, \psi$; $A = L, R$ and $j = 1, 2$. For simplicity, we will consider scattering particles with equal masses m, leaving m unspecified. Accordingly, $\alpha_{RL}, \ldots, \rho_{RR}$ are obtained by exchanging L and R in (2.14). As a result, the R-matrix is fixed up to four scalar factors $\sigma_{LL}, \sigma_{LR}, \sigma_{RL}, \sigma_{RR}$: these are the dressing phases determined by the crossing equations (1.6). Such scalars encode important physical information (*e.g.* the bound states of the system); however, they do not affect the Hopf algebra structure and specifying their exact form will be unnecessary.

3. RTT realization of the Yangian

3.1. Representations of the RTT generators

In the form (1.11), the R-matrix entries (2.14) are distributed as

$$
\begin{aligned}
&R^{11}_{11} = \alpha^{LL}, && R^{12}_{12} = \alpha^{LR}, && R^{13}_{13} = \beta^{LL}, && R^{14}_{14} = \gamma^{LR},\\
&R^{21}_{21} = \alpha^{RL}, && R^{22}_{22} = \alpha^{RR}, && R^{23}_{23} = \gamma^{RL}, && R^{24}_{24} = \beta^{RR},\\
&R^{31}_{31} = \mu^{LL}, && R^{32}_{32} = \mu^{LR}, && R^{33}_{33} = \rho^{LL}, && R^{34}_{34} = \nu^{LR},\\
&R^{41}_{41} = \mu^{RL}, && R^{42}_{42} = \mu^{RR}, && R^{43}_{43} = \nu^{RL}, && R^{44}_{44} = \rho^{RR},\\
&R^{12}_{34} = -\beta^{LR}, && R^{13}_{31} = -\gamma^{LL}, && R^{21}_{43} = -\beta^{RL}, && R^{24}_{42} = -\gamma^{RR},\\
&R^{31}_{13} = \nu^{LL}, && R^{34}_{12} = \rho^{LR}, && R^{42}_{24} = \nu^{RR}, && R^{43}_{21} = \rho^{RL}.
\end{aligned}
$$

In a neighbourhood of $u_j = \infty$, the $x^{\pm}_{A\,j}$ given by

$$x^{\pm}_{A\,j} = u_j + \frac{\pm \mathrm{i}m + 2\kappa_A}{h} + \frac{-h^2 \pm 2\mathrm{i}m\,\kappa_A}{h^2\,u_j} + \frac{\pm \mathrm{i}m + 2\kappa_A}{h\,u_j^2} + \ldots, \qquad A = L, R \tag{3.1}$$

satisfy (2.5). The spectral parameters u_j expressed in terms of $x^{\pm}_{A\,j}$ are then

$$\frac{1}{2}\left(x^+_{A\,j} + \frac{1}{x^+_{A\,j}} + x^-_{A\,j} + \frac{1}{x^-_{A\,j}}\right) = u_j + \frac{2\,\kappa_A}{h}, \qquad A = L, R. \tag{3.2}$$

Thanks to (3.1) one can expand the R-matrix and read off the algebra commutation relations from (1.12) and the representations of their generators from (1.13). To begin with, we find

$$T^{a}_{(-1)\,b} = \delta^a{}_b\, U_2^{|b|}, \tag{3.3}$$

namely $T^{1}_{(-1)\,1} = \mathbb{1}$, $T^{1}_{(-1)\,2} = 0$, $T^{3}_{(-1)\,3} = U_2\,\mathbb{1}$ and so forth. As anticipated, the leading term in the asymptotic expansion of $\mathcal{T}(u)$ turns out to be central. Since we are embedding $\mathfrak{su}_c(1|1)^2$ in $\mathfrak{gl}(2|2)$, only 8 of the 16 $T^{a}_{(0)\,b}$ are non-vanishing. The same happens for the $T^{a}_{(n)\,b}$ with $n \geq 0$: these are $T^{a}_{(n)\,a}, T^{a+2}_{(n)\,a+2}, T^{a}_{(n)\,a+2}, T^{a+2}_{(n)\,a}$ with $a = 1, 2$. A sample of these are

$$\begin{aligned}
T^{1}_{(0)\,1} &= \frac{\mathrm{i}m}{h}\left(e^2{}_2 - e^3{}_3\right), \qquad & T^{1}_{(0)\,3} &= -\frac{2\mathrm{i}\sqrt{m}\,\eta_{L\,2}}{h}\,e^1{}_3 + \frac{\sqrt{m}\left(1-U_2^2\right)}{\eta_{R\,2}}\,e^4{}_2, \\
T^{2}_{(0)\,2} &= \frac{\mathrm{i}m}{h}\left(e^1{}_1 - e^4{}_4\right), \qquad & T^{2}_{(0)\,4} &= -\frac{2\mathrm{i}\sqrt{m}\,\eta_{R\,2}}{h}\,e^2{}_4 + \frac{\sqrt{m}\left(1-U_2^2\right)}{\eta_{L\,2}}\,e^3{}_1.
\end{aligned} \tag{3.4}$$

The expressions of the higher generators are rather lengthy, *e.g.*

$$\begin{aligned}
T^{1}_{(+1)\,1} = &\frac{m}{2h^2\,\eta_{R\,2}{}^2}\left[-(m-4\mathrm{i}\kappa)\,\eta_{R\,2}{}^2 + 2h^2\,U_2\left(-1+U_2^2\right)\right]e^2{}_2 + \\
&- \frac{m\left(m - 4\mathrm{i}\kappa - U_2\left(8\,\eta_{L\,2}{}^2 + (m-4\mathrm{i}\kappa)U_2\right)\right)}{2h^2\left(-1+U_2^2\right)}\,e^3{}_3.
\end{aligned} \tag{3.5}$$

3.2. Abstract commutation relations

Expanding (1.9) around ∞ with respect to both u_1 and u_2 gives the abstract commutation relations for the $\mathbb{T}$s. A sample of the latter is

$$\begin{aligned}
&\left\{\mathbb{T}^{4}_{(1)\,2}, \mathbb{T}^{3}_{(0)\,1}\right\} = \frac{\mathrm{i}m}{h}\,\mathbb{T}^{3}_{(0)\,1}\mathbb{T}^{4}_{(0)\,2} + \frac{2\mathrm{i}m}{h}\left(\mathbb{T}^{2}_{(0)\,2} - \mathbb{U}\,\mathbb{T}^{4}_{(0)\,4}\right) + \frac{m^2}{h^2}\left(\frac{2\mathrm{i}\kappa}{m} + 1\right)\left(\mathbb{1} - \mathbb{U}^2\right), \\
&\left[\mathbb{T}^{3}_{(0)\,1}, \mathbb{T}^{2}_{(0)\,2}\right] = \left[\mathbb{T}^{3}_{(0)\,1}, \mathbb{T}^{4}_{(0)\,4}\right] = \frac{\mathrm{i}m}{h}\,\mathbb{T}^{3}_{(0)\,1}, \\
&\left\{\mathbb{T}^{4}_{(0)\,2}, \mathbb{T}^{3}_{(0)\,1}\right\} = \frac{2\mathrm{i}m}{h}\left(\mathbb{1} - \mathbb{U}^2\right).
\end{aligned} \tag{3.6}$$

Notice that m, h and κ combine to structure constants. The commutation rules of $\mathbb{T}$s provide those of the $\mathbb{J}$s: for instance, the relations (3.6) imply

$$\left\{\mathbb{J}^{4}_{(1)\,2}, \mathbb{J}^{3}_{(0)\,1}\right\} = -\frac{\mathrm{i}\kappa}{2m}\left(\mathbb{1} - \mathbb{U}^2\right) + \frac{1}{2}\left(\mathbb{1} + \mathbb{U}^2\right)\left(\mathbb{J}^{2}_{(0)\,2} - \mathbb{J}^{4}_{(0)\,4}\right). \tag{3.7}$$

3.3. Lie superalgebra from RTT

We now reconstruct the whole Hopf algebra $\mathcal{A}$. By choosing

$$\begin{aligned}
&\mathbb{Q}_L = \sqrt{m}\,\mathbb{J}^{3}_{(0)\,1}, \qquad \mathbb{S}_L = \sqrt{m}\,\mathbb{J}^{1}_{(0)\,3},\\
&\mathbb{H}_L = m\left(\mathbb{J}^{1}_{(0)\,1} - \mathbb{J}^{3}_{(0)\,3}\right), \qquad \mathbb{P} = \frac{ih}{2}\left(\mathbb{U}^2 - 1\right),\\
&\mathbb{Q}_R = \sqrt{m}\,\mathbb{J}^{4}_{(0)\,2}, \qquad \mathbb{S}_R = \sqrt{m}\,\mathbb{J}^{2}_{(0)\,4},\\
&\mathbb{H}_R = m\left(\mathbb{J}^{2}_{(0)\,2} - \mathbb{J}^{4}_{(0)\,4}\right), \qquad \mathbb{K} = \frac{ih}{2}\left(1 - \mathbb{U}^{-2}\right),
\end{aligned} \tag{3.8}$$

we exactly reproduce the $\mathfrak{su}_c(1|1)^2$ graded commutation relations:

$$\{\mathbb{Q}_A, \mathbb{S}_B\} = \delta_{AB}\,\mathbb{H}_B, \qquad \{\mathbb{Q}_L, \mathbb{Q}_R\} = \mathbb{P}, \qquad \{\mathbb{S}_L, \mathbb{S}_R\} = \mathbb{K}; \qquad A, B = L, R. \tag{3.9}$$

The coproducts (2.9) are recovered by applying Δ to (3.8) by means of (1.17). As for the antipodes, the equations (1.18) provide

$$\begin{aligned}
&\Sigma\,[\mathbb{Q}_A] = -\mathbb{U}^{-1}\,\mathbb{Q}_A, \qquad \Sigma\,[\mathbb{S}_A] = -\mathbb{U}\,\mathbb{S}_A, \qquad \Sigma\,[\mathbb{H}_A] = -\mathbb{H}_A,\\
&\Sigma\,[\mathbb{P}] = -\mathbb{U}^{-2}\,\mathbb{P}, \qquad \Sigma\,[\mathbb{K}] = -\mathbb{U}^{2}\,\mathbb{K}, \qquad A = L, R,
\end{aligned} \tag{3.10}$$

which preserve the algebra commutation relations. Notice that the antipode is involutive when evaluated on $\mathfrak{su}_c(1|1)^2$.

3.4. Outer generators

As in [23,25], the sum

$$\mathbb{A} = \mathbb{J}^{1}_{(0)\,1} + \mathbb{J}^{2}_{(0)\,2} + \mathbb{J}^{3}_{(0)\,3} + \mathbb{J}^{4}_{(0)\,4} \tag{3.11}$$

is an outer central generator of $\mathfrak{su}_c(1|1)^2$. Since $\mathbb{A}$ never appears on either the left or the right hand side of the non-trivial $\mathcal{A}$'s commutation relations, it can be modded out of the algebra. In other words, if $\mathbb{B}_L$ and $\mathbb{B}_R$ are the bosonic generators

$$\mathbb{B}_L = \mathbb{J}^{1}_{(0)\,1} + \mathbb{J}^{3}_{(0)\,3}, \qquad \mathbb{B}_R = \mathbb{J}^{2}_{(0)\,2} + \mathbb{J}^{4}_{(0)\,4}, \qquad \mathbb{B}_L + \mathbb{B}_R = \mathbb{A}, \tag{3.12}$$

whose action on the supercharges reads[6]

$$[\mathbb{B}_L, \mathbb{Q}_L] = \mathbb{Q}_L, \qquad [\mathbb{B}_L, \mathbb{Q}_R] = -\mathbb{Q}_R, \qquad [\mathbb{B}_L, \mathbb{S}_L] = -\mathbb{S}_L, \qquad [\mathbb{B}_L, \mathbb{S}_R] = \mathbb{S}_R, \tag{3.13}$$

one realises that $[\mathbb{B}_L, \cdot]$ and $[\mathbb{B}_R, \cdot]$ are not independent: in fact, they add up to $[\mathbb{A}, \cdot] \equiv 0$. Then, the Lie symmetry algebra of the theory is not the full $\mathfrak{gl}(1|1)^2$ but, rather, $\mathfrak{gl}(1|1)^2/\mathbb{A}$.

6 LR symmetry provides the other commutation relations, *e.g.* $[\mathbb{B}_R, \mathbb{Q}_L]$ and $[\mathbb{B}_R, \mathbb{Q}_R]$.

3.5. $\mathcal{Y}[\mathfrak{su}_c(1|1)^2]$ Yangian from RTT

Setting

$$\begin{aligned}
\hat{\mathbb{Q}}_L &= \sqrt{m}\,\mathbb{J}^3_{(+1)\,1} + \frac{\kappa}{h}\,\mathbb{Q}_L + \frac{1}{2}\left(1+\mathbb{U}^2\right)\mathbb{S}_R,\\
\hat{\mathbb{S}}_L &= \sqrt{m}\,\mathbb{J}^1_{(+1)\,3} + \frac{\kappa}{h}\,\mathbb{S}_L + \frac{1}{2}\left(1+\mathbb{U}^{-2}\right)\mathbb{Q}_R,\\
\hat{\mathbb{H}}_L &= m\left(\mathbb{J}^1_{(+1)\,1} - \mathbb{J}^3_{(+1)\,3}\right) + \frac{2\kappa}{h}\,\mathbb{H}_L + \frac{\mathrm{i}h}{4}\left(\mathbb{U}^2-\mathbb{U}^{-2}\right),\\
\hat{\mathbb{Q}}_R &= \sqrt{m}\,\mathbb{J}^4_{(+1)\,2} - \frac{\kappa}{h}\,\mathbb{Q}_R + \frac{1}{2}\left(1+\mathbb{U}^2\right)\mathbb{S}_L,\\
\hat{\mathbb{S}}_R &= \sqrt{m}\,\mathbb{J}^2_{(+1)\,4} - \frac{\kappa}{h}\,\mathbb{S}_R + \frac{1}{2}\left(1+\mathbb{U}^{-2}\right)\mathbb{Q}_L,\\
\hat{\mathbb{H}}_R &= m\left(\mathbb{J}^2_{(+1)\,2} - \mathbb{J}^4_{(+1)\,4}\right) - \frac{2\kappa}{h}\,\mathbb{H}_R + \frac{\mathrm{i}h}{4}\left(\mathbb{U}^2-\mathbb{U}^{-2}\right),
\end{aligned} \tag{3.14}$$

and

$$\hat{\mathbb{P}} = \frac{1}{2}\left(1+\mathbb{U}^2\right)(\mathbb{H}_L+\mathbb{H}_R), \qquad \hat{\mathbb{K}} = \frac{1}{2}\left(1+\mathbb{U}^{-2}\right)(\mathbb{H}_L+\mathbb{H}_R) \tag{3.15}$$

we obtain the commutation relations for the level one of the $\mathcal{Y}[\mathfrak{su}_c(1|1)^2]$ Yangian:

$$\begin{aligned}
&\left\{\hat{\mathbb{Q}}_A, \mathbb{S}_B\right\} = \left\{\mathbb{Q}_A, \hat{\mathbb{S}}_B\right\} = \delta_{AB}\,\hat{\mathbb{H}}_B, \qquad A, B = L, R,\\
&\left\{\hat{\mathbb{Q}}_L, \mathbb{Q}_R\right\} = \left\{\mathbb{Q}_L, \hat{\mathbb{Q}}_R\right\} = \hat{\mathbb{P}}, \qquad \left\{\hat{\mathbb{S}}_L, \mathbb{S}_R\right\} = \left\{\mathbb{S}_L, \hat{\mathbb{S}}_R\right\} = \hat{\mathbb{K}}.
\end{aligned} \tag{3.16}$$

The corresponding coproducts are

$$\begin{aligned}
\Delta(\hat{\mathbb{Q}}_L) &= \hat{\mathbb{Q}}_L \otimes \mathbb{1} + \mathbb{U} \otimes \hat{\mathbb{Q}}_L + \frac{\mathrm{i}}{h}(\mathbb{Q}_L \otimes \mathbb{H}_L - \mathbb{U}\,\mathbb{H}_L \otimes \mathbb{Q}_L +\\
&\quad + \frac{\mathbb{P}}{\mathbb{U}} \otimes \mathbb{S}_R - \mathbb{U}^2\,\mathbb{S}_R \otimes \mathbb{P}),\\
\Delta(\hat{\mathbb{S}}_L) &= \hat{\mathbb{S}}_L \otimes \mathbb{1} + \mathbb{U}^{-1} \otimes \hat{\mathbb{S}}_L + \frac{\mathrm{i}}{h}(\mathbb{U}^{-1}\,\mathbb{H}_L \otimes \mathbb{S}_L - \mathbb{S}_L \otimes \mathbb{H}_L +\\
&\quad + \frac{\mathbb{Q}_R}{\mathbb{U}^2} \otimes \mathbb{K} - \mathbb{U}\,\mathbb{K} \otimes \mathbb{Q}_R),\\
\Delta(\hat{\mathbb{H}}_L) &= \hat{\mathbb{H}}_L \otimes \mathbb{1} + \mathbb{1} \otimes \hat{\mathbb{H}}_L + \frac{\mathrm{i}}{h}\left(\mathbb{U}^{-2}\,\mathbb{P} \otimes \mathbb{K} - \mathbb{U}^2\,\mathbb{K} \otimes \mathbb{P}\right),
\end{aligned} \tag{3.17}$$

and $\Delta(\mathfrak{J}_R) = \Delta(\mathfrak{J}_L)_{L\to R}$ for the R generators. Finally, Δ acts on the Yangian central charges as

$$\begin{aligned}
\Delta(\hat{\mathbb{P}}) &= \hat{\mathbb{P}} \otimes \mathbb{1} + \mathbb{U}^2 \otimes \hat{\mathbb{P}} + \frac{\mathrm{i}}{h}\left(\mathbb{P} \otimes \mathbb{H} - \mathbb{U}^2\,\mathbb{H} \otimes \mathbb{P}\right),\\
\Delta(\hat{\mathbb{K}}) &= \hat{\mathbb{K}} \otimes \mathbb{1} + \mathbb{U}^{-2} \otimes \hat{\mathbb{K}} + \frac{\mathrm{i}}{h}\left(\mathbb{U}^{-2}\,\mathbb{H} \otimes \mathbb{K} - \mathbb{K} \otimes \mathbb{H}\right).
\end{aligned} \tag{3.18}$$

Ultimately, the antipodes are

$$\begin{aligned}
&\Sigma[\hat{\mathbb{Q}}_A] = -\mathbb{U}^{-1}\,\hat{\mathbb{Q}}_A, \qquad \Sigma[\hat{\mathbb{S}}_A] = -\mathbb{U}\,\hat{\mathbb{S}}_A, \qquad \Sigma[\hat{\mathbb{H}}_A] = -\hat{\mathbb{H}}_A,\\
&\Sigma[\hat{\mathbb{P}}] = -\mathbb{U}^{-2}\,\hat{\mathbb{P}}, \qquad \Sigma[\hat{\mathbb{K}}] = -\mathbb{U}^2\,\hat{\mathbb{K}}, \qquad A = L, R.
\end{aligned} \tag{3.19}$$

The antipode Σ acts as an involution on the level one partners of the $\mathfrak{su}_c(1|1)^2$ generators. This will no longer be true for the secret symmetry, as we shall see in the next subsection.

3.6. Secret symmetry

The outer generator

$$\text{ß} = \mathbb{J}^{1}_{(+1)\,1} + \mathbb{J}^{2}_{(+1)\,2} + \mathbb{J}^{3}_{(+1)\,3} + \mathbb{J}^{4}_{(+1)\,4} \tag{3.20}$$

is not central and cannot be quotiented out as it was previously done for $\mathbb{A}$. As a result, the bosonic charges

$$\text{ß}_L = \mathbb{J}^{1}_{(+1)\,1} + \mathbb{J}^{3}_{(+1)\,3}, \qquad \text{ß}_R = \mathbb{J}^{2}_{(+1)\,2} + \mathbb{J}^{4}_{(+1)\,4} \tag{3.21}$$

are independent. Their commutation relations read

$$\begin{aligned} &[\text{ß}_L, \mathbb{Q}_L] = 2\,\hat{\mathbb{Q}}_L - \left(\mathbb{1} + \mathbb{U}^2\right)\mathbb{S}_R, \qquad [\text{ß}_L, \mathbb{Q}_R] = -\left(\mathbb{1} + \mathbb{U}^2\right)\mathbb{S}_L, \\ &[\text{ß}_L, \mathbb{S}_L] = -2\,\hat{\mathbb{S}}_L + \left(\mathbb{1} + \mathbb{U}^{-2}\right)\mathbb{Q}_R, \qquad [\text{ß}_L, \mathbb{S}_R] = \left(\mathbb{1} + \mathbb{U}^{-2}\right)\mathbb{Q}_L, \end{aligned} \tag{3.22}$$

where the others are given by the swapping L $\leftrightarrow$ R. The coproducts are

$$\Delta(\text{ß}_L) = \text{ß}_L \otimes \mathbb{1} + \mathbb{1} \otimes \text{ß}_L + \frac{2i}{h}\left(\mathbb{U}\mathbb{S}_L \otimes \mathbb{Q}_L + \mathbb{U}^{-1}\mathbb{Q}_L \otimes \mathbb{S}_L\right) \tag{3.23}$$

and $\Delta(\text{ß}_R) = \Delta(\text{ß}_L)_{L\to R}$, while the antipodes read

$$\Sigma[\text{ß}_L] = -\text{ß}_L + (2i/h)\,\mathbb{H}_L, \qquad \Sigma[\text{ß}_R] = \Sigma[\text{ß}_L]_{L\to R}. \tag{3.24}$$

The existence of these additional generators implies that the symmetry of the theory is not just $\mathcal{Y}[\mathfrak{su}_c(1|1)^2]$, but rather $\mathcal{A} = \mathcal{Y}[\mathfrak{gl}_c(1|1)^2/\mathbb{A}]$. These $\text{ß}_L, \text{ß}_R$ generate the $\text{AdS}_3 \times S^3 \times S^3 \times S^1$ version of the *secret symmetry* first found in the $\text{AdS}_5 \times S^5$ case [57–59] and then discovered also in the worldsheet scattering on $\text{AdS}_3 \times S^3 \times \mathcal{M}^4$ with RR flux [24] as well as in the massive sector of the $\text{AdS}_2 \times S^2$ superstring [25]. On the field theory side, the secret symmetry corresponds to a helicity operator acting on scattering amplitudes [60] as well as on Wilson loops [61–63]. On the string theory side, the existence of secret symmetries was demonstrated by means of the pure spinor formalism [64]. As anticipated, the antipode is not involutive on ß_A:

$$\Sigma^2(\text{ß}_A) = \text{ß}_A - (4i/h)\,\mathbb{H}_A, \qquad A = L, R. \tag{3.25}$$

This signals that $\mathcal{A}$ possesses a non-trivial *Liouville contraction* $\mathcal{Z}(u)$ [23,54,55] shifting the double antipode of diagonal elements, *e.g.* $\mathbb{J}^{a}_{(+1)\,a}, \mathbb{J}^{a}_{(+2)\,a}$. More precisely, $\mathcal{Z}(u)$ is a central element of the algebra satisfying $\mathbb{1} \otimes \mathcal{Z}(u) = \mathcal{T}(u)\Sigma\,[\mathcal{T}(u)]$. In our case, by direct computation we find

$$\mathcal{Z}_A(u) = \exp\left\{(2i/h)\,u^{-2}\,\mathbb{H}_A + \ldots\right\}. \tag{3.26}$$

The order u^{-2} deviation from $\mathcal{Z}_A(u) = \mathbb{1}$ is proportional to the shift in $\Sigma^2(\text{ß}_A)$, while higher orders provide $\Sigma^2(\mathbb{J}^{a}_{(+2)\,a})$, $\Sigma^2(\mathbb{J}^{a}_{(+3)\,a})$ and so forth.

3.7. Evaluation representation

Acting with the function ρ_u on the Yangian charges one finds that the latter are in evaluation representation. For instance, if $Q_L = \rho_u \mathbb{Q}_L$ and $\hat{Q}_L = \rho_u \hat{\mathbb{Q}}_L$, then $\hat{Q}_L |\psi_R\rangle = [u - (2\kappa/h)]\, Q_L |\psi_R\rangle$ holds. In general, if $\hat{\mathfrak{J}}$ is the level one counterpart of $\mathfrak{J}$, one obtains

$$\hat{\mathfrak{J}}\,|\Phi_A\rangle = \left(u + \frac{2\kappa_A}{h}\right)\mathfrak{J}\,|\Phi_A\rangle\,, \qquad \Phi = \phi, \psi, \qquad A = L, R. \tag{3.27}$$

Then, the quantised coefficient of the Wess–Zumino term κ splits the evaluation representation in two branches: the left one, with effective spectral parameter $u_L = u + (2\kappa/h)$; and the right one, with $u_R = u - (2\kappa/h)$. In the limit $\kappa \to 0$, corresponding to RR flux only, the two branches collapse and become one, with a single spectral parameter u. This was the situation analysed in [24,65].

4. Loop algebra and universal classical r-matrix

4.1. Loop algebra

In this section we study the *classical limit* of the $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ R-matrix and its Yangian by taking the effective string tension h to be very large, namely[7] $h \to \infty$. To this aim, we parametrise the Zhukovsky variables $x^{\pm}_{L,R}$ in terms of the constants h, m and of a new variable z [66]:

$$x^{\pm}_L = x^{\pm}_R = z\left(\sqrt{1 - \frac{(m/h)^2}{(z - z^{-1})^2}} \pm \frac{\mathrm{i}(m/h)}{z - z^{-1}}\right). \tag{4.1}$$

Consequently, we expand with respect to h^{-1}. Such $x^{\pm}_{L,R}$ satisfy the constraint (2.5) because κ-corrections are of order h^{-2}. In this regime, the spectral parameter u reads $u = z + z^{-1}$ and the representations of the Yangian generators become

$$\begin{aligned}
&\frac{\mathfrak{Q}_L}{\sqrt{m}} = -\frac{e^2{}_4 + z\, e^3{}_1}{\sqrt{z^2-1}}, \qquad \frac{\mathfrak{S}_L}{\sqrt{m}} = -\frac{z\, e^1{}_3 + e^4{}_2}{\sqrt{z^2-1}},\\
&\frac{\mathfrak{H}_L}{m} = \frac{z^2\left(e^1{}_1 - e^3{}_3\right) + \left(e^2{}_2 - e^4{}_4\right)}{1 - z^2},\\
&\frac{\mathfrak{Q}_R}{\sqrt{m}} = -\frac{e^1{}_3 + z\, e^4{}_2}{\sqrt{z^2-1}}, \qquad \frac{\mathfrak{S}_R}{\sqrt{m}} = -\frac{z\, e^2{}_4 + e^3{}_1}{\sqrt{z^2-1}},\\
&\frac{\mathfrak{H}_R}{m} = \frac{\left(e^1{}_1 - e^3{}_3\right) + z^2\left(e^2{}_2 - e^4{}_4\right)}{1 - z^2},\\
&\mathfrak{B}_L = u^{-1}\,\text{ß}_L = \left(z^4 - 1\right)^{-1}\left[z^4 e^1{}_1 + e^2{}_2 - z^2\left(3z^2 - 2\right) e^3{}_3 + \left(2z^2 - 3\right) e^4{}_4\right],\\
&\mathfrak{B}_R = u^{-1}\,\text{ß}_R = \left(z^4 - 1\right)^{-1}\left[e^1{}_1 + z^4 e^2{}_2 + \left(2z^2 - 3\right) e^3{}_3 - z^2\left(3z^2 - 2\right) e^4{}_4\right].
\end{aligned} \tag{4.2}$$

[7] By AdS/CFT, this limit corresponds to the strong coupling limit on the gauge theory side.

Let $\mathfrak{G}$ be one of the generators reported above. Then, we can uplift $\mathfrak{G}$ to its level n counterpart via evaluation representation: $\mathfrak{G}^{(n)} = u^n\, \mathfrak{G}$. The algebra formed by (4.2) is then

$$\begin{aligned}
&\left\{\mathfrak{Q}_L^{(n_1)}, \mathfrak{S}_L^{(n_2)}\right\} = \mathfrak{H}_L^{(n_1+n_2)},\\
&\left\{\mathfrak{Q}_L^{(n_1)}, \mathfrak{Q}_R^{(n_2)}\right\} = \left\{\mathfrak{S}_L^{(n_1)}, \mathfrak{S}_R^{(n_2)}\right\} = \mathfrak{H}_L^{(n_1+n_2-1)} + \mathfrak{H}_R^{(n_1+n_2-1)},\\
&\left[\mathfrak{B}_L^{(n_1)}, \mathfrak{Q}_L^{(n_2)}\right] = 2\,\mathfrak{Q}_L^{(n_1+n_2)} - 2\,\mathfrak{S}_R^{(n_1+n_2-1)},\\
&\left[\mathfrak{B}_L^{(n_1)}, \mathfrak{S}_L^{(n_2)}\right] = -2\,\mathfrak{S}_L^{(n_1+n_2)} + 2\,\mathfrak{Q}_R^{(n_1+n_2-1)},\\
&\left[\mathfrak{B}_L^{(n_1)}, \mathfrak{Q}_R^{(n_2)}\right] = -2\,\mathfrak{S}_L^{(n_1+n_2-1)},\\
&\left[\mathfrak{B}_L^{(n_1)}, \mathfrak{S}_R^{(n_2)}\right] = 2\,\mathfrak{Q}_L^{(n_1+n_2-1)}, \qquad n_1, n_2 \in \mathbb{Z}.
\end{aligned} \tag{4.3}$$

The remaining commutation rules descend from LR symmetry. The relations reported in (4.3) can be identified as a deformation of the $\mathfrak{u}(1|1)[u,u^{-1}] \oplus \mathfrak{u}(1|1)[u,u^{-1}]$ loop algebra [56]. Let us now take the classical limit of the R-matrix and check that not only it enjoys the symmetry algebra (4.3), but it can also be written as a tensor product of $\mathfrak{u}(1|1)[u,u^{-1}]^2$ generators.

4.2. *Classical r-matrix*

Using (4.1), the R-matrix expands as

$$R = \mathbb{1} \otimes \mathbb{1} + h^{-1} r + \mathcal{O}(h^{-2}), \tag{4.4}$$

where r is the *classical r-matrix* of the system satisfying the *classical* Yang–Baxter equation

$$[r_{12}, r_{13}] + [r_{12}, r_{23}] + [r_{13}, r_{23}] = 0, \tag{4.5}$$

representing the QYBE (1.8) in the $h \to \infty$ regime. Since

$$\Delta(\mathfrak{J}) = \Delta_0(\mathfrak{J}) + h^{-1}\Delta_1(\mathfrak{J}) + \ldots, \qquad \Delta^{\text{op}}(\mathfrak{J}) = \Delta_0(\mathfrak{J}) + h^{-1}\Delta_1^{\text{op}}(\mathfrak{J}) + \ldots, \tag{4.6}$$

quasi co-commutativity (1.5) becomes

$$[\Delta_0(\mathfrak{J}), r] = \Delta_1(\mathfrak{J}) - \Delta_1^{\text{op}}(\mathfrak{J}), \qquad \forall\, \mathfrak{J} \in \mathcal{A}. \tag{4.7}$$

Therefore, the left-hand side of (4.7), also known as the *cobracket* of $\mathfrak{J}$, has to match the difference between $\Delta_1(\mathfrak{J})$ and $\Delta_1^{\text{op}}(\mathfrak{J})$. The classical r-matrix obtained from (2.14) reads

$$\begin{aligned}
r = \frac{2\mathrm{i}}{u_1 - u_2}\Big[&\mathfrak{Q}_L \otimes \mathfrak{S}_L - \mathfrak{S}_L \otimes \mathfrak{Q}_L + \mathfrak{Q}_R \otimes \mathfrak{S}_R - \mathfrak{S}_R \otimes \mathfrak{Q}_R +\\
&- \frac{1}{4u_1}(\mathfrak{B}_L - \mathfrak{B}_R) \otimes (\mathfrak{H}_L - \mathfrak{H}_R) - \frac{1}{4u_2}(\mathfrak{H}_L - \mathfrak{H}_R) \otimes (\mathfrak{B}_L - \mathfrak{B}_R) +\\
&- \frac{1}{4u_2}(\mathfrak{B}_L + \mathfrak{B}_R) \otimes (\mathfrak{H}_L + \mathfrak{H}_R) - \frac{1}{4u_1}(\mathfrak{H}_L + \mathfrak{H}_R) \otimes (\mathfrak{B}_L + \mathfrak{B}_R)\Big].
\end{aligned} \tag{4.8}$$

The generators $\mathfrak{Q}_L, \mathfrak{S}_L, \ldots, \mathfrak{B}_L, \mathfrak{B}_R$ are genuine symmetries of the r-matrix: for instance,

$$[\Delta_0(\mathfrak{B}_L), r] = 4\mathrm{i}\,(\mathfrak{Q}_L \otimes \mathfrak{S}_L + \mathfrak{S}_L \otimes \mathfrak{Q}_L) = \Delta_1(\mathfrak{B}_L) - \Delta_1^{\text{op}}(\mathfrak{B}_L), \tag{4.9}$$

which agrees with (4.7). In the region $|u_2| < |u_1|$, the r-matrix can be rewritten as[8]

$$
\begin{aligned}
r = 2\mathrm{i}\Big[&\mathfrak{Q}_L^{(-1-n)} \otimes \mathfrak{S}_L^{(n)} - \mathfrak{S}_L^{(-1-n)} \otimes \mathfrak{Q}_L^{(n)} + \mathfrak{Q}_R^{(-1-n)} \otimes \mathfrak{S}_R^{(n)} - \mathfrak{S}_R^{(-1-n)} \otimes \mathfrak{Q}_R^{(n)} + \\
&-\tilde{\mathfrak{L}}^{(-1-n)} \otimes \mathfrak{M}^{(n)} - \mathfrak{M}^{(-1-n)} \otimes \tilde{\mathfrak{L}}^{(n)} - \tilde{\mathfrak{B}}^{(-n)} \otimes \mathfrak{H}^{(n-1)} - \mathfrak{H}^{(-2-n)} \otimes \tilde{\mathfrak{B}}^{(n+1)}\Big],
\end{aligned}
\tag{4.10}
$$

where the redefinitions

$$
\begin{aligned}
&\mathfrak{H}^{(0)} = (\mathfrak{H}_L + \mathfrak{H}_R)/2, \qquad \mathfrak{B}^{(1)} = (\text{ß}_L + \text{ß}_R)/2, \qquad \tilde{\mathfrak{B}}^{(1)} = \mathfrak{B}^{(1)} + \mathfrak{H}^{(0)} - 2\mathfrak{H}^{(-1)}, \\
&\mathfrak{M}^{(0)} = (\mathfrak{H}_L - \mathfrak{H}_R)/2, \qquad \mathfrak{L}^{(1)} = (\text{ß}_L - \text{ß}_R)/2, \qquad \tilde{\mathfrak{L}}^{(1)} = \mathfrak{L}^{(1)} + \mathfrak{M}^{(0)},
\end{aligned}
\tag{4.11}
$$

were used. The object in (4.10) is representation independent; therefore, it is a well-posed candidate for the universal r-matrix of the loop algebra (4.3). In the light of (4.10), (4.8) is in fact in evaluation representation. Finally, notice that the identifications

$$
\begin{aligned}
&\mathfrak{Q}_A^{(n)} = -\mathbf{Q}_{A,n}, \qquad \mathfrak{S}_A^{(n)} = \bar{\mathbf{Q}}_{A,n}, \qquad 2\,\mathfrak{H}^{(n)} = -\mathbf{H}_n, \\
&2\,\mathfrak{M}^{(n)} = -\mathbf{M}_n, \qquad \mathfrak{B}^{(n)} = -\mathbf{B}_n, \qquad 2\,\mathfrak{L}^{(n)} = -\mathbf{b}_n,
\end{aligned}
\tag{4.12}
$$

map the r-matrix (4.10) to the one found in [56] up to central shifts.

5. Conclusions

5.1. Discussion

In this work we have investigated the integrability of type IIB superstrings on $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ background with mixed RR and NSNS flux from an algebraic viewpoint. Our analysis has shown that the worldsheet scattering of the theory enjoys an infinite dimensional symmetry $\mathcal{A}$ endowed with a Hopf algebra structure and spanned by an endless tower of generators. Specifically, we have used the R-matrix to write down the RTT relations and expanded these with respect to the spectral parameter, obtaining the abstract commutation relations of $\mathcal{A}$. Furthermore, expanding the R-matrix itself has provided the representations of the $\mathcal{A}$ generators. We have observed that the presence of both RR and NSNS fluxes deforms not only the algebra representations but also the abstract commutation relations of the RTT generators.

The infinite dimensional algebra $\mathcal{A}$ is organised in levels: the bottom level, $\mathcal{A}_{-1}$, only contains the identity and a braiding factor $\mathbb{U}$. The latter is a central element deforming commutation relations, coproducts, antipodes and so forth. The next level, $\mathcal{A}_0$, coincides with the building block of the light-cone off-shell symmetry algebra of the model, $\mathfrak{su}_c(1|1)^2$, whose central charges depend on the braiding factor $\mathbb{U}$ and vanish in the limit $\mathbb{U} \to \mathbb{1}$. The level zero of $\mathcal{A}$ also includes two outer generators of $\mathfrak{su}_c(1|1)^2$, $\mathbb{B}_L, \mathbb{B}_R$. These add up to a central generator $\mathbb{A}$ that never appears in the commutation relations of $\mathcal{A}$, implying that $\mathbb{B}_L, \mathbb{B}_R$ actually represent one single generator. We have summarised this situation by referring to $\mathcal{A}_0$ as $\mathfrak{gl}_c(1|1)^2/\mathbb{A}$. Subsequently, we have demonstrated that the level one of $\mathcal{A}$, $\mathcal{A}_1$, contains level one charges of the Yangian $\mathcal{Y}[\mathfrak{su}_c(1|1)^2]$. Such charges are in evaluation representation, where each copy of $\mathfrak{su}_c(1|1)$ presents its own spectral parameter depending on the quantised coefficient of the Wess–Zumino term. We have also found that at level one there are two bonus generators, $\text{ß}_L, \text{ß}_R$, mapping each level of $\mathcal{A}$ to the next one. In contrast to $\mathbb{B}_L$ and $\mathbb{B}_R$, ß_L and ß_R are algebraically independent and span the

[8] The r-matrix for $|u_1| < |u_2|$ is obtained by making the replacement $\mathfrak{G}_1^{(-1-n)} \otimes \mathfrak{G}_2^{(n)} \to -\,\mathfrak{G}_1^{(n)} \otimes \mathfrak{G}_2^{(-1-n)}$ in (4.10).

$\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ counterpart of the secret symmetry found in AdS_5 and AdS_2, as well as in another AdS_3 case. We have checked that the antipode fails to be involutive when evaluated on $ß_L, ß_R$, and displayed that the failure is quantitatively related to the non-trivial centre of $\mathcal{A}$. By taking into account this secret symmetry, we have concluded that the worldsheet scattering matrix of superstrings on $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ with mixed flux is invariant under the action of the $\mathcal{Y}[\mathfrak{gl}_c(1|1)^2/\mathbb{A}]$ Yangian, with evaluation representation depending on the quantised coefficient of the Wess–Zumino term.

Finally, taking the large effective string tension limit, we have showed that $\mathcal{Y}[\mathfrak{gl}_c(1|1)^2/\mathbb{A}]$ reduces to a deformation of the $\mathfrak{u}(1|1)[u, u^{-1}]^2$ loop algebra. Then, we have formulated a candidate for the universal, representation independent $\mathfrak{u}(1|1)[u, u^{-1}]^2$ r-matrix, linked to the classical limit of the R-matrix by means of the evaluation representation. We have explicitly verified that such a universal r-matrix, in analogy to those found in [25,67,68], is classically co-commutative with respect to the $\mathfrak{u}(1|1)[u, u^{-1}]^2$ generators regardless the representation.

5.2. Outlook

It would be very interesting to find a universal R-matrix for the worldsheet scattering on $\mathrm{AdS}_3 \times S^3 \times S^3 \times S^1$ with both RR and NSNS fluxes in terms of a Drinfeld quantum double construction [69,70]. Furthermore, it would be fascinating to see how q-Poincaré symmetry [56,71,72] behaves in presence of mixed flux. Another intriguing direction would be translating the Yangian symmetry derived in this paper into constraints applicable on observables of the conformal field theory dual [41–44], reverse-engineering what it was done in the context of $\mathcal{N} = 4$ super Yang–Mills theory [2,73,74]. Finally, it would be exciting to extend the analysis of this paper to other supergravity backgrounds; for instance, by extracting the Yangian corresponding to the $\mathrm{AdS}_2 \times S^2 \times S^2 \times T^4$ superstring [75,76].

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