Proceedings of the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)

Paul Drijvers, Csaba Csapodi, Hanna Palmér, Katalin Gosztonyi, Eszter Herendiné-Kónya

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Editors: Paul Drijvers, Csaba Csapodi, Hanna Palmér, Katalin Gosztonyi and Eszter Kónya

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Preface: CERME13 in Budapest, and beyond

Carl Winsløw[1], ERME President

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The 13th Congress of the European Society for Research in Mathematics Education – CERME13, for short – was a memorable event in many ways. It was the first CERME held "on site" since CERME 11 in Utrecht in February 2019, almost 4½ years before. After the outbreak of the COVID pandemic in early 2020, CERME12 was postponed to 2022 and was finally held online from Bozen-Bolzano in February 2022, as explained in the preface to the proceedings of that conference. At that time, it was still not clear whether the pandemic was nearing the end – although, as we now know, it was – and it was decided to defer the dates of CERME13 from February 2023 to July 2023. This way, we avoided the risks of new virus-related lockdowns in early 2023, which in fact did not materialize – and also real inconveniences related to the war in Ukraine, such as the energy crisis in the winter 2022-2023. In addition to these reasons, the new dates of CERME13, almost exactly between CERME12 and CERME14 (to be held in February 2025), allowed for a more realistic timeframe for organising CERME13 and CERME14 (1½ year for each). From February 2025, we will finally be back to the normal rhythm of biannual congresses in February of odd years, after CERME12 and CERME13 were exceptionally held in somewhat modified modalities and dates.

CERME13 took place in Budapest, the beautiful capital of Hungary. The many ways the event was memorable include, besides the reunion of our community after the pandemic, also the first and maybe only CERME ever held in the summer. It cannot be denied that central Europe is much warmer in this season than in the winter, and the hospitality of our Hungarian colleagues was equally warm and efficient. To the local organising committee, led by Csaba Csapodi, Katalin Gosztonyi and Ödön Vancsó, and to the institutions and staff who supported the committee, we extend our warmest thanks on behalf of all participants, for preparing and realising the complex organisation of the congress in a way that fully matched the quality of its former editions. The Lágymányosi campus, with the Faculty of Science buildings of the Eötvos Loránd University, provided the extensive facilities required for both plenary events and the sessions of the thematic working groups. The traditional gala dinner was held in the impressive halls of the Museum of Fine Arts, and added another memorable and very distinguished dimension to the social aspects of the congress.

The scientific programme is evidently the raison d'être of any CERME, and the contributions of the participants – papers and posters – form the core material. Unlike many other research conferences, CERME is not focused on presentations, but on in-depth discussions of the contributions. This, indeed, is the secret behind the constantly increasing attraction and success of CERME. Since the very first CERME, the purpose of promoting *communication, cooperation and collaboration* among mathematics education researchers has been implemented by organising the work of the congress in *thematic working groups*. In these groups, the main activity is in-depth discussion of the submitted contributions, with only very brief "reminder presentations" of the texts, which participants in each group are expected to read before attending the congress. At CERME13, no less than 28 such groups were organised – both new groups, reflecting new specialized areas of research, and groups which have already convened at several previous editions. To organise these groups, teams of coleaders are invited by the international programme committee. Both to every member of these coleader teams, and to the programme committee – led this year by Paul Drijvers and Hanna Palmér – the ERME community is deeply indebted and thankful. We also thank our plenary speakers, László Lovász and Berta Barquero, as well as the plenary panel led by João-Pedro da Ponte, for providing deep and state-of-the-art accounts of research themes that can inspire the community as a whole.

CERME13 received, as almost all previous CERMEs, an unprecedented high number of participants (941), from more nations than ever before (54), see Table 1.

Table 1: The success of CERME13 in numbers: 941 participants from 54 countries

Armenia	1	Germany	165	Romania	1
Australia	7	Greece	20	Serbia	3
Austria	27	Hong Kong	4	Slovakia	14
Belgium	5	Hungary	49	South Africa	2
Brazil	9	Iceland	3	Spain	51
Canada	19	Ireland	13	Sweden	46
Chile	11	Israel	27	Switzerland	14
China	7	Italy	66	Taiwan	2
Costa Rica	1	Japan	9	Thailand	1
Croatia	5	Lithuania	2	Tunisia	3
Cyprus	2	Malta	1	Turkey	46
Czech Republic	17	Mexico	17	Ukraine	1
Denmark	19	Netherlands	32	UAE	1
Egypt	1	New Zealand	5	UK	43
Estonia	4	Norway	66	USA	35
Faroe Islands	1	Philippines	1	Uruguay	1
Finland	6	Poland	1	Vietnam	1
France	24	Portugal	28		

In fact, the number of contributions to CERME congresses has increased almost linearly throughout the history of CERME, to reach a whopping 834 at CERME13. Of these, 757 were finally accepted and presented. Given the capacity of academic venues – roughly 900-1000 participants – and more subtly, of the organisation of a CERME, which depends entirely on voluntary committees and coleader teams – it has become ever clearer that this linear growth cannot continue, even if only one participant for each contribution were to participate. At CERME13, it was only to a small degree possible for co-authors to register, even for experienced scholars who attended all or most of the previous CERME congresses. These dilemmas were discussed at CERME13, and will continue to be worked on in the committees of ERME. Both CERME, and the other venues and supports that ERME offers to scientific exchange and creation by researchers in Europe and beyond, need to be continuously maintained and developed. The enthusiasm and quality of CERME13 promises a bright future for our research field and for the ERME community. See you at CERME14 in Bozen-Bolzano!

Introduction to the proceedings of the thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)

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About CERME13

The thirteenth Congress of European Research in Mathematics Education (CERME13) took place in Budapest, Hungary, from 10 to 14 July 2023. It was organised by the Alfréd Rényi Institute of Mathematics, hosted by the Eötvös Loránd University, and attracted 941 participants. Paul Drijvers and Hanna Palmér were chair and co-chair of the International Programme Committee, which included Federica Ferretti, Eirini Geraniou, Katalin Gosztonyi, Thomas Hausberger (from August 2022), Jeremy Hodgen, Sibel Kazak, Esther Levenson, Bozena Maj-Tatsis, Simon Modeste (until August 2022), Juuso Henrik Nieminen, and Florian Schacht. Csaba Csapodi chaired the Local Organizing Committee, which was co-chaired by Katalin Gosztonyi and Ödön Vancsó.

CERME13 hosted 27 Thematic Working Groups, listed in Table 1. New TWGs were TWG28 on Collaborative Settings in Mathematics Teacher Education and TWG29 on Embodied and Material Studies of Mathematical Behaviour. TWG7 on Mathematics for work, society and personal development: lifelong learning was suppressed this edition, but will reappear at CERME14. TWG11 on algorithms merged with TWG16 on Learning mathematics with technology and other resources. In total, CERME13 had 143 TWG (co-)leaders, who did an extremely valuable job which is key to CERME's success. The conference was preceded by the YERME-day for young researchers in ERME, that attracted 150 participants. Altogether, CERME13 was a success both in academic terms and in social atmosphere, which the European mathematics education research community enjoyed.

Table 1: Overview of the TWG leadership teams

Thematic Working Group	Leader	Co-leaders
TWG1: Argumentation and Proof	Andreas Moutsios-Rentzos	Orly Buchbinder, Nadia Azrou, Fiene Bredow, Dimitrios Deslis, Viviane Durand-Guerrier, David A. Reid, Mei Yang
TWG2: Arithmetic and Number Systems	Pernille Bødtker Sunde	Elisabeth Rathgeb-Schnierer, Renata Carvalho, Ioannis Papadopoulos
TWG3: Algebraic Thinking	Maria Chimoni	Cecilia Kilhamn, Jorunn Reinhardtsen, Luis Radford

TWG4: Geometry Teaching and Learning	Alik Palatnik	Lina Brunheira, Taro Fujita, Chrysi Papadaki, Petra Surynková
TWG5: Probability and Statistics Education	Caterina Primi	Daniel Frischemeier, Orlando Gonzalez, Sibel Kazak, Aisling Leavy, Martin Andre
TWG6: Applications and Modelling	Jonas Bergman Ärlebäck	Susana Carreira, Britta Eyrich Jessen, Gilbert Greefrath, Yana Lacek, Katrin Vorhölter
TWG8: Affect and the Teaching and Learning of Mathematics	Çiğdem Haser	Inés M Gómez-Chacón, Chiara Andrà, Janina Krawitz, Hanna Viitala
TWG9: Mathematics and Language	Jenni Ingram	Kirstin Erath, Alexander Schüler-Meyer, Máire Ní Ríordáin, Ingólfur Gíslason
TWG10: Diversity and Mathematics Education: Social, Cultural and Political Challenges	Laura Black	Anette Bagger, Sabrina Salazar, Timo Dexel, Juuso Nieminenn
TWG12: History in Mathematics Education	Antonio M. Oller-Marcén	Jenneke Krüger, Tanja Hamann, Bjørn Smestad, Olivera Đokić
TWG13: Early Years Mathematics	Bozena Maj-Tatsis	Camilla Björklund, Dorota Lembrér, Esther Levenson, Andrea Maffia, Marianna Tzekaki
TWG14: University Mathematics Education	Irene Biza	Olov Viirman, Matija Bašić, Ignasi Florensa, Ghislaine Gueudet, Mathilde Hitier, Igor' Kontorovich, Athina Thoma, Megan Wawro
TWG15: Teaching Mathematics with Technology and Other Resources	Ornella Robutti	Bärbel Barzel, Melih Turgut, Gülay Bozkurt, Daniel Thurm
TWG16: Learning Mathematics with Technology and Other Resources	Osama Swidan	Rogier Bos, Eleonora Faggiano, Seçil Yemen Karpuzcu, Simon Modeste, Florian Schacht, Jana Trgalová
TWG17: Theoretical Perspectives and Approaches in Mathematics Education Research	Heather Johnson	Mariam Haspekian, Abdel Seidouvy, Felix Lensing, Cecilie Carlsen Bach
TWG18: Mathematics Teacher Education and Professional Development	Marita Friesen	Tracy Helliwell, Raffaele Casi, Andreas Ebbelind, Janne Fauskanger, Pere Ivars, Macarena Larrain, Libuše Samková
TWG19: Mathematics Teaching and Teacher Practice(s)	Reidar Mosvold	Mark Hoover, Helena Grundén, Siún Nic Mhuirí, Chrysoula Choutou
TWG20: Mathematics Teacher Knowledge, Beliefs and Identity	Francesca Martignone	Miguel Montes, Federica Ferretti, Veronika Hubeňáková, Jimmy Karlsson, Nadia Kennedy, Miguel Ribeiro
TWG21: Assessment in Mathematics Education	Francesca Morselli	Christina Drüke-Noe, Gözde Kaplan-Can, Chiara Giberti, Johanna Rämö
TWG22: Curricular Resources and Task Design in Mathematics Education	Shai Olsher	Annalisa Cusi, Dubravka Glasnović Gracin, Hendrik Van Steenbrugge
TWG23: Implementation of Research Findings in Mathematics Education	Mario Sánchez Aguilar	Boris Koichu, Morten Misfeldt, Linda Marie Ahl

TWG24: Representations in Mathematics Teaching and Learning	Carla Finesilver	Anna Baccaglini-Frank, Elisa Miragliotta, Kate C. O'Brien
TWG25: Inclusive Mathematics Education – Challenges for Students with Special Needs	Petra Scherer	Michael Gaidoschik, Hana Moraová, Helena Roos, Andreas Ulovec
TWG26: Mathematics in the Context of STEM Education	Behiye Ubuz	Michelle Stephan, Clelia Cascella, Nelleke Den Braber
TWG27: The Professional Practices, Preparation and Support of Mathematics Teacher Educators	Ronnie Karsenty	Hilda Borko, Alf Coles, Birte Friedrich-Pöhler, Bettina Rösken-Winter, Stefan Zehetmeier
TWG28: Collaborative Settings in Mathematics Teacher Education	Stéphane Clivaz	Geoff Wake, Paula Gomes, Pernilla Mårtensson, Aoibhinn Ní Shúilleabháin
TWG29: Embodied and material studies of mathematical behaviour	Anna Shvarts	Elizabeth de Freitas, Ricardo Nemirovsky, Christina Krause, Giulia Ferrari

Editorial information

These proceedings are available as a complete volume on the ERME website. Each individual contribution is also available through the HAL open archive, where it can be found through keywords, title, or author name. This has been the practice since CERME9, to increase the visibility of the huge work done in CERME conferences.

After the preface by the president of ERME, Carl Winsløw, and this brief introduction by the proceedings editors, this volume continues with texts on two plenary activities of CERME13: the plenary lecture by Berta Barquero entitled "Mathematical modelling as a research field: Transposition challenges and future directions," and the report on the Panel on "Bridging the research-practice gap," chaired by João Pedro da Ponte, with contributions by Mario Sánchez Aguilar, Naďa Vondrová, Stefan Zehetmeier, Sarah Seleznyov, and Jorryt van Bommel.

The biggest part of these proceedings consists of the sections corresponding to the TWGs. These sections start with an introduction by the TWG leadership team. According to the team's choice, this is a 4-page introduction presenting the contributions, or an extended 8-page one, which proposes an additional analysis of the current research on the theme of the TWG, and perspectives for the future. Next, the section consists of the paper and poster contributions – in alphabetical order by first author's name.

These CERME13 proceedings result from a collaboration, involving the CERME13 IPC and LOC, the TWG leaders and co-leaders. Particular thanks are due to Nathalie Kuijpers for her editorial work. We warmly thank all these people for their involvement, and hope that this volume will contribute to the development of mathematics education research in Europe and beyond.

Higher order thinking skills through automatic formative assessment: Generating examples in ordinary differential equations to actively engage students in developing conceptual understanding

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The application of digital technologies empowers students to use their creativity and develop their critical thinking skills. This is why at the University of Turin, within the first-year module "Mathematics and biostatistics with computer science applications" in the study program in Biotechnologies students are engaged in multiple ways with hybrid ways of delivery mathematical contents, making use of digital interactive resources and automatic assessment empowered with formative practices. In particular, in this paper we are going to analyse students' group and individual activities in which they were asked to provide specific examples of Mathematical objects under specific restrictions, and to reflect on the tasks they performed. We analysed the way students think while performing example eliciting tasks, with a focus on Ordinary Differential Equations.

Keywords: Automatic formative assessment, example eliciting tasks, higher education, higher order thinking, ordinary differential equation.

Introduction

Mathematics education is an important component of the background for many sciences. As some authors highlighted, research in STEM education is increasing in importance internationally and the identity of STEM education journals is becoming clearer over time (Li et al., 2020). The use of automatic assessment combined with formative practices and boosted by interactive components that allows students exploration of concepts can have positive effects on the achievement of learning objectives. Moreover, as a general trend in Mathematics education, there is a growing number of universities and academies that use automatic assessment systems to evaluate their students. The combination of automatic assessment with formative practices helps the students in developing mathematical construction and conceptual understanding.

Li and colleagues (2020) found that research collaboration (publications with multiple authors) has been very common in STEM education research. Indeed, this research was developed thanks to an international collaboration between two universities, the University of Turin and Karlstad University, Sweden (Barana, et al., in press). In this joint project, similar tasks were proposed to Italian and Swedish students in various topics in mathematics, even though the ones described in this paper were only proposed to Italian students.

This paper reports the result of a study performed with first-year students in a Mathematics module of the bachelor's degree in biotechnology during the first semester of the academic year 2022/2023. The context of the study is represented by group activities which students could freely decide to join: we had 121 subscribers. Here we analyse the answers provided by 88 students to the group activity dedicated to Ordinary Differential Equations (ODEs), a topic that made students struggle a lot in previous years of the university module. Recently, ODEs have also been included to the topics that, according to the Italian National Guidelines, students should study at the end of Scientific Lyceum, so we expect that it is not a new topic for many of the students. In the following sections of the paper, we will explore the Theoretical background in performing the kind of Example Eliciting Tasks (EETs) that we require of students, together with formative practices, since those tasks were performed by students with their peers and with the purpose of consolidating mathematical knowledge. Next, we will provide the research methodology and discuss the results and the comments of students quantitatively and qualitatively. Conclusions and discussion will provide the main findings.

Theoretical background

In literature there are numerous examples of collaborative learning environments designed to let students perform explorative tasks. There are examples on the topic of functions (Brunström & Fahlgren, 2015; Jaworski & Matthews, 2011). This may be boosted by the mediation of technologies, to redesign tasks that are very difficult to assess with other means. In fact, today many instructors choose to utilise Automatic Assessment Systems (AAS) in mathematics modules in higher education (Rønning, 2017). Students' performances can be improved by reducing the cognitive load. Students were helped by the possibility of using dynamic mathematics software (DMS) environments (Sangwin, 2013), combining a dynamical exploration and automatic assessment to promote mathematical understanding (Joubert, 2017). An example of this kind of system is given by Möbius Assessment. There are studies that started investigating the integration of these two types of technology (Luz & Yerushalmy, 2019), but anyway further research is needed to better understand students' learning behaviour. This can be further enhanced by implementing also formative assessment (FA) (Black & Wiliam, 2009). Black and Wiliam definition entails the collection of data about students' understanding, and the use of such data to change the learning path. In previous works, we adopted automatic formative assessment (AFA) in a Digital Learning Environment, thus with the automatic elaboration of responses and provision of feedback (Barana et al., 2021). We have developed and experimented a model for the design of activities with AFA (Barana & Marchisio, 2019), based on Möbius Assessment, whose computational engine is empowered by advanced mathematical capabilities. By making use of programming languages or mathematical packages, it is possible to build interactive tasks based on algorithms where answers, feedback and values are calculated over random parameters and can be shown with different mathematical and graphical representations. Thus, new solutions for computer-based items can be conceived, including dynamic explorations, animations, and symbolic manipulations, which offer students experiences of mathematical construction and conceptual understanding (Sangwin, 2015). According to our model, AFA activities should:

A. be always available in a Digital Learning Environment, without limitations in data, time and number of attempts;
B. be algorithm-based, so that random values, parameters, formulas and graphs make questions, and their answers change at every attempt;
C. be open-ended or almost open-ended, the AAS's mathematical engine assures that open mathematical answers are graded independently of the form in which they are provided;
D. provide students with immediate feedback while they are focused on the task;

E. provide students with interactive feedback just after giving an incorrect answer. It has the form of a step-by-step guided resolution that interactively shows a possible solving process;
F. be contextualised in real-life, thus contributing to the creation of meanings through the association of abstract concepts to concrete experience.

Example eliciting tasks

There is a growing literature on Example Eliciting Tasks (EETs). As a teaching and learning tool in mathematics, EETs are a type of problem-solving activity where students have to provide examples of mathematical entities under certain conditions, rather than simply calculating solutions with standard procedures. This is a way to engage students actively in their mathematical understanding, using also different registers and representation of mathematical entities (Watson & Mason, 2002). This idea has been integrated into automatic assessment (Sangwin, 2003). Studies have shown that EETs can have a positive impact on students' understanding of mathematical concepts (Brunström et al., 2022). For example, research has found that students who participated in EETs had a deeper understanding of mathematical principles and a stronger ability to apply these principles in real-life situations. Additionally, EETs have been shown to improve students' problem-solving skills and ability to think critically. To further challenge students' thinking, some authors suggested asking students for several different examples to extend students' example space (Watson & Mason, 2002) and to support problem solving and to mirror the reasoning processes (Yerushalmy et al., 2017). EETs have also been shown to be effective for students of all abilities, including those who struggle with mathematics, since they allow students to work at their own pace and focus on their strengths and weaknesses. In didactical practices, EETs can be used as a teaching tool alone or as part of a larger lesson plan or activities. They can be used in the classroom, as well as in online or remote learning environments. Additionally, EETs can be used with a wide range of mathematical concepts, from basic arithmetic to advanced calculus. This approach is used to help students understand concepts by actively engaging with the material and making connections to real-life situations.

Mathematics and biostatistics with computer science applications

The context of our research lies in a first-year module of a study program in biotechnology called "Matematica e Biostatistica con Applicazioni Informatiche" (Mathematics and Biostatistics with Computer Science Applications). The module aims both at providing competences in basic mathematics that is useful for a future scientist and at improving several soft and transversal skills, such as digital competences and problem-solving skills. Students also participated to lab sessions in a room equipped with their own PCs (Bring Your Own Device mode), to work with an Advanced Computing Environment (ACE) to explore mathematical concepts and learn how to deal with high order mathematics through the use of technologies, and to solve problems contextualised in the applied sciences constituting the core of their curriculum of studies, such as Life Sciences and Chemistry, as well as Biotechnologies itself. We created an online course available from everywhere and at all times with a dedicated section for most of the course topics, in which ACE worksheets with interactive components and AAS tests flank more traditional resources such as static files. According to our model, tests are designed to allow multiple attempts, and random parameters make questions change at each attempt. This allows students to find repeatable exercises and activities inside our course, removing at least from a strict point of view the need for looking outside in order to train themselves better. Furthermore, we provided questions with an immediate and interactive feedback, by subdividing them in several parts, with the path proposed to the student dependent on his or her performance. Likewise, we also designed exercises that bore a similar construction with respect to those which constitute the exam, in order to let students become confident of the ways they would have been assessed and graded. The tasks that we are going to consider for this research possess most of the features that have been highlighted in the Theoretical background, with two main differences:

- The tasks are not based on real life applications of Mathematics since they aimed at empowering students' theoretical understanding and ability to generate examples.
- The feedback could be viewed by students only after performing the full test (which was composed by four questions) for the first time during the group activity. This choice has been made since students were performing the tasks in groups, and we tried to avoid one student suggesting the others the correct solutions while performing the tasks. Many activities with feedback in the course make the students read it directly after the task. Students could perform the test more than one time to recap the topics.

Research question and methods

This research has been mainly motivated by the need to find an answer to the following research question: *How students generate examples about ODEs when they have some degree of freedom in their construction?* To answer to this question, we will examine student responses to two tasks concerning ODEs. The collected data are student responses to the tasks typed inside Möbius Assessment AAS. The 88 students worked in groups, autonomously and independently from the lessons, and questions contained group tasks and individual tasks. We considered one student from each of the 35 groups, in order to highlight differences between the various teams. We considered the first attempt of those students. We analysed the two following group tasks:

Task 1

This question asks you to provide an example of a function that satisfies a differential equation.

a) Write two different functions that are solutions of the following differential equation:

$$\frac{d}{dx}\, y(x) \; + y(x)^2 = 1$$

b) Describe how you found the expressions of the functions.

For this task, the tricks that students had to pay attention were:

- There was only one possible way to solve this problem through usually-taught resolution methods, separating the variables.
- The application of the usually-taught resolution methods was more difficult than the standard ones, the inversion of the integrated hand sides to find a suitable function for *x* requires to be skilled with these aspects of Calculus.
- The main aim of the task was finding examples, so students are encouraged to find the shortest way to the solution, in this case the easiest ones were the constant functions -1 and 1.

In light of this, we classified the groups in some categories, according to the kinds of functions they provided as examples, and if they were actually solutions of the ODE or not. This quantitative aspect is then related with the qualitative description of how they proceeded in order to give the answers to the task, by focusing for example on why they followed a certain line of reasoning.

Task 2

This question is individually answered but can be investigated in a group. The question asks you to provide two examples of differential equations that have the function f(x) as a solution.

a) Write two different differential equations that have the following function as their solution:

$$f(x)= a_1 \cdot exp(a_2 \cdot x)+b_1 \cdot exp(b_2 \cdot x)+c_1 \cdot x+c_2$$

(where the parameters are random integers between -5 and 5, with a_2, b_2, c_2 nonzero)

b) Describe how you came up with the function expressions.

In this task, the challenge for students was to recognize that the usual ways of obtaining a solution of linear ordinary differential equations (no matter the order of the ODE), and looking for proper coefficients of the homogeneous solution and the particular solution, although resolutive, were not necessarily the best methods to find the examples with the most possible simplicity. Indeed, it would have been quite simpler to differentiate n times the function $f(x)$ and equating it to the n-th derivative of y, for every n (in order to answering the task, $n=1$ and $n=2$ are sufficient). Moreover, this task swaps the role of "solving an equation" into "finding an equation".

Again, we devised a proper classification for the groups, by considering on the one side who actually answered $y' = f'(x)$ or $y'' = f''(x)$ (or both of them), and on the other side who on the contrary constructed more complicated ODEs (either of first or second order), by still paying attention also to the correctness of the answers. We put here too in relation the quantitative submissions with the qualitative explanations, allowing us to understand what they thought to do.

Results and discussion

Let us start with the analysis of Task 1. Among the 35 groups of students, we were able to collect 29 groups' answers, since six of them did not respond. Our classification reads as follows:

- 28% of groups of students answered with two constant functions: they all submitted -1 and 1, thus correctly answering to the task;
- 14% of groups answered with a constant and a non-constant function: all of them but one by correctly providing both examples, while a group alone made an error on the non-constant;
- 55% of groups answered with two non-constant functions, but only just over half of them provided correct answers, with the other ones answering two erroneous functions;
- one group answered with a single non-constant erroneous functions, by leaving the second field blank.

By relating these findings with the qualitative description, we were able to observe that:

- Students calculated the full solution just to input constant functions: for instance, one of the groups answering with a constant and a non-constant function, declared to having substituted two values for C in the general integral. They probably substituted the easiest values for C in the general integral, but they forgot to look for the easiest way to write the answer. In fact, it is worth to note that, while teaching how to solve an ODE by separating the variables, the fact that constant functions are to be found (if present) otherwise explicitly emerges; nonetheless, most students appeared to skip that part. Relating this evidence to the research question, we can state that students tend to follow rigid patterns, also when they are not asked to do so, by having the possibility of being more flexible. A possible motivation about the skipping of finding constants can lie in the fact that these students could be used to follow rigid algebraic processes without exploring the problem from an analytic perspective or looking for simpler solutions. This could be due to the way they learned Mathematics at secondary level. Since the steps required to obtain the solutions to an ODE are more algebraic than analytic, students likely tend to see the constant solutions as less related with the differential problem, even if they actually provide other solutions to the ODE.

- two groups said to have looked firstly for solving the ODE by separating the variables, and only secondly for reasoning and finding the constant functions, when they found the standard resolution method too difficult. In a certain sense, answering with the constants has been a makeshift solution for them;
- generally speaking, most groups separated the variables, but this resulted in wrong answers in almost half of the cases, and furthermore they underwent a needlessly more challenging procedure, with respect to what really necessary in order to answer correctly.

By considering now the analysis of Task 2, we collected 27 groups' answers. The main findings were:

- only a group answered in the simpler way: $y' = f'(x)$ and $y'' = f''(x)$;
- 26% of groups gave as one of the answers $y' = f'(x)$, but apart from the just mentioned group, only one other group correctly gave the other answer;
- 74% of groups gave as at least one answer an ODE having order 1 or 2 more articulated than $y' = f'(x)$ and $y'' = f''(x)$, which resulted to be wrong in almost half of the cases: sometimes due to minor errors (like sign), but other times completely, even with some situations where in the description they explained the procedure with the right steps;
- less frequent errors concerned for example integrating $f(x)$ rather than differentiating it, or writing expressions in x without equating them to an expression in y and its derivatives.

Again, the majority of groups followed a procedure which was not the simplest one and often brought errors, essentially confirming the answer to the research question inferred by Task 1: students tend to generate examples by "answering using rigid procedures", even if they are not required to do so.

It has to be noted also how 19% of groups provided only an example rather than two, a percentage considerably higher than in Task 1 (where only one group did so). The fact that this problem is the inverse of what usually asked, i.e. equations have to be *found* rather than *solved*, could have constituted a difficulty also in framing it.

Future work

Our next steps will consider that, apart from the descriptive answers in the two tasks, we asked students also to keep track of their actions by redacting some logs, in which they were questioned to report what they learned through lab activities that they were unable to learn during the lectures; to explain how working in group supported or not their comprehension of mathematical concepts relative to the task subjects, by giving also an example; and to state what they perceived as simpler and what was more difficult, explaining also why.

By analysing the answers, we will have a deeper insight of the students' behaviour when facing tasks which require conceptual understanding and high-order thinking. It would be interesting to compare the findings with other countries (such as Sweden) to investigate if these results are the product of a national school system or if they are more generalizable. Moreover, we will be able to detect to what extent these activities help students' learning with respect to a more traditional course not considering anything of comparable, how teamwork plays a role in better understanding what they work on, and to target difficulties. This would suggest how to reinforce with proper teachers' actions where students have to put more effort to achieve their goals, such as in commitments comparable to Tasks 1 and 2, where most of them did not answer optimally. This would allow us to devise subsequent actions aimed at improving the benefits arising from this approach, which can be in turn studied in order to determine how they constitute progress, and if other further actions are still appropriate. In future work we are going to provide further feedback to the students to extend their example space and make them reflect on the example-generation process.

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