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Proceedings of the Thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)

Paul Drijvers, Csaba Csapodi, Hanna Palmér, Katalin Gosztonyi, Eszter
Herendiné-Kónya

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EDUCATION**

Editors: Paul Drijvers, Csaba Csapodi, Hanna Palmér, Katalin Gosztonyi and Eszter Kónya

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TABLE OF CONTENTS

Preface: CERME13 in Budapest, and beyond <i>Carl Winsløw</i>	1
Introduction to the proceedings of the thirteenth Congress of the European Society for Research in Mathematics Education (CERME13) <i>Paul Drijvers, Csaba Csapodi, Hanna Palmér, Katalin Gosztonyi and Eszter Kónya</i>	3
Plenary lecture and panel discussion	
Mathematical modelling as a research field: Transposition challenges and future directions <i>Berta Barquero</i>	6
Bridging the research-practice gap – a panel report <i>João Pedro da Ponte, Mario Sánchez Aguilar, Nad'a Vondrová, Stefan Zehetmeier, Sarah Seleznyov and Jorryt van Bommel</i>	31
TWG1: Argumentation and proof	51
An introduction to TWG1: Argumentation and proof <i>Andreas Moutsios-Rentzos, Orly Buchbinder, Nadia Azrou, Fiene Bredow, Dimitrios Deslis, Viviane Durand-Guerrier, David A. Reid and Mei Yang</i>	52
An epistemic-logical model for analysis of students' argumentation in mathematics education research <i>Miglena Asenova</i>	56
Views of teachers and students about teaching and learning proof at the university level <i>Nadia Azrou</i>	64
Analysis of proving in the Habermas' rationality perspective: The meta-knowledge – knowledge interplay <i>Paolo Boero and Nadia Azrou</i>	72
Teacher actions framing argumentation in the mathematics class <i>Fiene Bredow and Christine Knipping</i>	80
How novice teachers recontextualize the teaching of mathematics via reasoning proving – a dual case study <i>Orly Buchbinder</i>	88
Study of the potential of problems to practice a research activity in mathematics at elementary school in French-speaking Switzerland <i>Mickaël Da Ronch, Marie-Line Gardes and Ismaïl Mili</i>	96
Primary school teachers' mathematical knowledge and views about Lakatos-style proving activity: a latent profile analysis <i>Dimitrios Deslis, Andreas J. Stylianides and Mateja Jamnik</i>	104
Strategies in mathematical justification settings of students in the primary school education bachelor's program of Free University Berlin <i>Lisa Eggerichs</i>	112
Explanatory proofs in mathematics classroom practices: An investigation in empirical philosophy of mathematics <i>Joachim Frans and Karen François</i>	120
Understanding of genericity of preservice primary school teachers in the discussion of example-based visual proof attempts <i>Lara Gayer</i>	128
Towards a model of problem networks for studying problem solving in mathematics education <i>Katalin Gosztonyi and Simon Modeste</i>	136
Aspects of representing in proof-based teaching of fraction multiplication <i>Trond Stølen Gustavsen and Andrea Hofmann</i>	144

Opportunities for proving the rule for fraction multiplication – episodes from a task-based interview <i>Andrea Hofmann and Trond Stølen Gustavsen</i>	152
Some thoughts on an enculturation function of mathematical proof <i>Leander Kempen and Eva Müller-Hill</i>	160
Pre-service teachers’ use of the word <i>explain</i> when working on mathematical reasoning and proving <i>Marit Buset Langfeldt, Anita Valenta and Torkel Haugan Hansen</i>	168
Conceptualising the regular pentagon in collaborative argumentation <i>Maria Alessandra Mariotti, Eszter Kónya and Zoltán Kovács</i>	176
Mathematics and debate: a dialectical approach <i>Amedeo Matteucci and Francesco Saverio Tortoriello</i>	184
Pre-service teachers’ conceptions and beliefs of reasoning and proof in school mathematics <i>Jakub Michal</i>	192
Structural characteristics of generic arguments using (counter)examples produced by primary students: A Toulmin analysis <i>Kayo Miura and Yusuke Shinno</i>	200
Comparison of “What numbers make sense?” problems solving in the Czech Republic and Lithuania <i>Karolína Mottlová and Ieva Kilienė</i>	208
Arguments in mathematics and physics: An interdisciplinary, systemic communicational approach to teacher education about scientific inference and evidence <i>Andreas Moutsios-Rentzos</i>	216
Constructing a deductive argument when solving word problems <i>Ieva Kilienė and Rimas Norvaiša</i>	224
Reasoning and proving tasks on geometry in two distinct textbooks: One topic, two lenses <i>Jarmila Novotná and Mária Slavičková</i>	232
Exploring students’ conceptions of proof in high-school and university: A proposal for collaborations in Europe <i>Cécile Ouvrier-Bufferet</i>	240
Using analogies to give meaning to the principle of mathematical induction <i>Valentina Postelnicu, Mario A. Gonzalez and Florin Postelnicu</i>	248
Argumentation and interaction: Discussing a new theoretical perspective <i>Pier Luigi Ferrari and Marta Saccoletto</i>	256
Understanding cultural aspects of argumentation and proof: A case study of Pythagorean theorem in a Japanese classroom <i>Yusuke Shinno and Takeshi Miyakawa Hiroshima</i>	264
Argumentation in the mathematical modelling cycle in the classroom <i>Horacio Solar, Andrés Ortiz, Victoria Arriagada and Marco Catalán</i>	272
Changing the significance of proof-related reasoning in upper- secondary school textbooks – a comparison between different German federal states and Hungary related to the law of sines <i>Kinga Szűcs</i>	280
Relating the dialogic and dialectic in mathematical argumentation <i>Sheena Tan</i>	288
Revisiting the transition from pragmatic to conceptual proofs <i>Thibaut Trouvé</i>	296
Analysis of the pre-service mathematics teachers’ errors in proving tasks <i>Peter Vankúš and Michaela Vargová</i>	304

Teachers' orientations of noticing and its underlying mechanisms in the context of Lakatos-style proving activity <i>Mei Yang, Andreas J. Stylianides and Mateja Jamnik</i>	312
Understanding students' argumentation in solving mathematical tasks <i>Da Zhou</i>	320
Proof understandings of preservice mathematics teachers <i>Emine Gaye Çontay</i>	328
Common characteristics of the definitions of mathematical proof at the tertiary level <i>Anairis de la Cruz Benito</i>	330
The functions of argumentation in mathematics education <i>Manuel Goizueta and Jorge Olivares</i>	332
Arguing from different perspectives on quantities and change <i>Luisa Gunia and Christine Knipping</i>	334
Teaching characteristics in the introduction of proof for supporting de-ritualization <i>Dimitri Lipper and Thomais Karavi</i>	336
Argumentation and participation in the context of (neo-)Socratic conversations with mathematical content in primary schools <i>Nicole Megel</i>	338
TWG2: Arithmetic and number systems	340
An introduction to TWG2: Arithmetic and number systems <i>Pernille Bødtker Sunde, Renata Carvalho, Ioannis Papadopoulos and Elisabeth Rathgeb-Schnierer</i>	341
Design of a semi-structured interview to capture flexibility in mental calculation <i>Timo Flückiger and Elisabeth Rathgeb-Schnierer</i>	345
Decimal number system. Professional noticing of mathematics teachers <i>Mónica Arnal-Palacián and Nuria Begué</i>	353
Exploring the components of the competence to construct a mental representation of mathematical problems in a fourth-grade student during mathematical logic problem solving <i>Nathalie Bisailon and Michel Lyons</i>	361
Development and evaluation of a numerosity estimation test for elementary school students <i>Leonie Brumm and Elisabeth Rathgeb-Schnierer</i>	369
Partitive division and quotative division in children's thinking: New individual case studies and considerations for teaching <i>Myriam Burtscher and Michael Gaidoschik</i>	377
Prospective teacher's representations of fractions <i>Renata Carvalho</i>	385
Identifying components of number sense in the teaching practices of first year of primary education: a case study <i>Elvira Fernández-Ahumada, Natividad Adamuz-Povedano, Jesús Montejo-Gámez and Enrique Martínez-Jiménez</i>	393
Multiplicative problem-solving strategies used by students with autism spectrum disorder <i>Raúl Fernández-Cobos, Irene Polo-Blanco and Juncal Goñi-Cervera</i>	401
Understanding fraction operations: The case of fraction division <i>Sofia Graça, João Pedro da Ponte and António Guerreiro</i>	409
Digital self-assessment in learning multiplication <i>Laura Graewert, Daniel Thurm and Bärbel Barzel</i>	417
Different types of understanding of associativity and their usefulness for generalised arithmetic <i>Robert Gunnarsson and Cathrine Englund Eriksson</i>	425

Age- and gender-related differences in Danish students' use of number-based strategies when solving multidigit addition tasks <i>Lóa Björk Jóelsdóttir, Pernille Bødtker Sunde and Paul Andrews</i>	433
Using conceptual metaphors as lenses to understand fifth-grade students' understanding of fraction constructs <i>Leonie Johann and Marja van den Heuvel-Panhuizen</i>	441
Exploring students' understanding of multiplication with eye tracking: A study on the use of strategies in array representations <i>Jingyi Lai, Lukas Baumanns, Anna Lisa Simon, Achim J. Lilienthal and Maike Schindler</i>	449
Problem-posing training and its impact on the quality of the posed problems <i>Ioannis Papadopoulos and Nafsika Patsiala</i>	457
Primary school students' use of unnecessary brackets while evaluating arithmetic expressions <i>Ioannis Papadopoulos</i>	465
The illustrations effect in Word Problems-solving involving fractions <i>Sofia Peregrina Fuentes, Carlos Valenzuela García, Maria T. Sanz and Emilia López-Iñesta</i>	473
Solutions in open-ended tasks by elementary school students with different achievement levels in mathematics <i>Elisabeth Rathgeb-Schnierer and Silke Friedrich</i>	475
Semiotic function: music notation vs fraction knowledge <i>Maria T. Sanz, Carlos Valenzuela, Emilia López-Iñesta and Guillermo M. Luengo</i>	483
Using a rich task to foster number sense <i>Jørgen Sjaastad</i>	491
A bridge between tools and conventions: a transition to ratio and proportion concepts after ratio table <i>Sinem Sozen-Ozdogan, Didem Akyuz and Michelle Stephan</i>	499
TWG3: Algebraic thinking	507
An introduction to TWG3: Algebraic thinking <i>Maria Chimoni, Cecilia Kilhamn, Luis Radford and Jorunn Reinhardtsen</i>	508
Eighth grade students' proportional reasoning: Strategies used in missing value and comparison problems <i>Kübra Çelikdemir and Merve Koştur</i>	512
The association of algebraic thinking with working memory, control of processing, and speed of processing <i>Maria Chimoni, Demetra Pitta-Pantazi and Constantinos Christou</i>	520
Algebraic reasoning: Connecting research to practice <i>Lorraine Day</i>	528
Algebraic wicked problems: Bridging the gap between early school algebra and socially relevant issues? <i>Jenny Fred, Paola Valero and Hendrik Van Steenbrugge</i>	536
Strategies used by a student with autism when solving a pictorial pattern task under an instructional approach <i>Juncal Goñi-Cervera, Irene Polo-Blanco, Hélia Oliveira and Ana Cláudia Henriques</i>	544
Algebraic generalisation of a student with MLD: Skills despite difficulties and implications for diagnostic tests <i>Francesca Gregorio</i>	552
The work done when solving linear equations: Altering experiences due to change of numbers <i>Anna Holmlund</i>	560
Designing mathematical tasks to promote preservice teachers' algebraic reasoning and argumentation <i>Konstantina Kaloutsi, Jorunn Reinhardtsen and Martin Carlsen</i>	568
“ <i>p</i> stands for packs” – diagnosing the <i>letter as object</i> misconception with SMART online tests <i>Katrin Klingbeil, Fabian Rösken and Bärbel Barzel</i>	576

Supporting students' comparisons of different conceptions of variables by telling stories – Insights from a design research study <i>Stefan Korntreff</i>	584
Rethinking the bridging function of graphical model method in the arithmetic-algebra transition: The issue of strategy choice <i>Orsolya Dóra Lócska, Zoltán Kondé and Zoltán Kovács</i>	592
Relational thinking in Grade 1: A study framed in the new Portuguese mathematics curriculum <i>Célia Mestre, Cristina Martins, Cândida Tourais and Isabel Guerra</i>	600
Identification of pre-algebra problems of generalization that discriminate mathematical giftedness <i>Mónica Mora, Angel Gutiérrez and Adela Jaime</i>	608
Grade five students' different experiences of patterns <i>Martin Nyman, Annelie Adolfsson, Anneli Blomqvist and Tove Wållberg</i>	616
The didactical transformation of the concept of variables <i>Reinhard Oldenburg</i>	624
Deepening preservice teachers' algebraic thinking by noticing children's thinking <i>Joana Cabral, Hélia Oliveira and Fátima Mendes</i>	632
Mathematical generalization in school algebra – an overview <i>Kristina Palm Kaplan, Helena Eriksson and Birgit Gustafsson</i>	640
Does solving arithmetic linear equations offer transferable algebraic competence? <i>Jöran Petersson, Niclas Larson, Kristina Palm Kaplan and Paul Andrews</i>	642
Symbolic equational algebraic thinking in young students <i>Luis Radford</i>	650
The role of scaling in the development of students' functional thinking <i>Daniela Šabaková</i>	658
Do students' ways of solving linear equations influence their later mathematical learning? Evidence from a study of Swedish upper secondary students <i>Tyge Sjöstrand, Jörgen Niklasson, Christos Kyprianou, Attila Szabo and Paul Andrews</i>	660
Working with early number algebraically: The Mental Starters Assessment Project <i>Hamsa Venkat, Mike Askew and Mellony Graven</i>	668
How do college students conceptualize algebraic properties? <i>Claire Wladis, Benjamin Sencindiver and Kathleen Offenholley</i>	676
TWG4: Geometry teaching and learning	684
An introduction to TWG4: Geometry teaching and learning <i>Alik Palatnik, Lina Brunheira, Taro Fujita, Chrysi Papadaki and Petra Surynková</i>	685
Design of a questionnaire item to assess the acquisition of Van Hiele level 5 with regard to the definition process <i>Alberto Arnal-Bailera and Víctor Manero</i>	689
Reasoning for the relationships on particular quadrilaterals in a dynamic geometry environment: A comparison of field-dependent and field-independent cognitive style students <i>Yurdagül Aydınyer and Behiye Ubuz</i>	697
Developing students' visuospatial abilities in geometry using various tangible and virtual 3D geometric models <i>Daniela Bímová, Jiří Břehovský, Petra Pirklová, Klaus-Peter Eichler, Moritz Seibold and Asif Mushtaq</i> ...	705
Some misconceptions of 9th grade students about triangles <i>Veronika Bočková and Gabriela Pavlovičová</i>	713

Exploratory analysis of a survey item designed to capture teachers' mathematical digital competency in geometry <i>Nicola Bretscher and Eirini Geraniou</i>	715
The role of representations in developing a conceptual understanding of trigonometry <i>Maximilian Büttner and Kirstin Erath</i>	723
Doubling the side, doubling the area: Managing representation with a Geoboard tool <i>Alessandro Gambini and Giada Viola</i>	731
Are geometric concepts of vectors bookable? An analysis of the didactic transpositions in mathematics textbooks <i>Sebastian Bauer, Stefan Halverscheid and Kolja Pustelnik</i>	739
Using the theories of Fischbein and Shulman to study teachers' algorithmic knowledge of the concept of the altitude of a triangle <i>Nader Hilf</i>	747
Preservice teachers' perceptions on the role of the geometric working spaces model in learning and teaching geometry <i>Yeşim İmamoğlu, Zeynep Çiğdem Özcan, Melek Pesen, Simge Akbal and Emine Erkin</i>	749
Diversity and coherence of geometry topics in the designated mathematics curriculum: The case of Germany <i>Ana Kuzle</i>	757
Interaction of spatial and theoretical aspects in spatial geometry problem solving through construction: The case of 3D sketching <i>Alik Palatnik</i>	765
Visualisation and spatial manipulation in argumentation: Creating a hypothesis <i>Chrysi Papadaki</i>	773
Collaborative geometry problem solving in secondary education: The issue of control strategies <i>Ioannis Papadopoulos and Georgios Ventistas</i>	781
On students' ways of dealing with area and perimeter's relation <i>Jérôme Proulx</i>	789
Geometrical reasoning and the influence of fluid intelligence over geometrical competence <i>Alicia Rubio-Sánchez, Inés M. Gómez-Chacón and Isabel Gómez-Veiga</i>	797
Plane representations in a 3-dimensional dynamic geometry environment: An analysis of soft constructions <i>Camilo Sua, Angel Gutiérrez and Adela Jaime</i>	805
Concrete 3D printed manipulatives in geometry learning and problem solving: usefulness of instructional 3D printed tools in the topic of polygons and tilings <i>Petra Surynková</i>	813
Contexts and connections in prospective elementary school teachers training <i>Juan-Pablo Vargas, Yuly Vanegas and Joaquín Giménez</i>	821
Lower secondary school students reflecting their two-week experience with quadrilaterals and GeoGebra <i>Lukáš Vizek and Libuše Samková</i>	829
TWG5: Probability and statistics education	837
An introduction to TWG5: Probability and statistics education <i>Caterina Primi, Aisling Leavy, Sibel Kazak, Daniel Frischmeier, Orlando Rafael Gonzalez and Martin Andre</i>	838
Preservice mathematics teachers' reasoning about data analysis: Balloon activity <i>Nazlı Akar and Mine İşıksal Bostan</i>	842
The Impact of an Undergraduate Course on Teaching Statistics and Probability on Preservice Mathematics Teachers' Statistical Knowledge <i>Kemal Akoğlu and Selen Çaylı</i>	850

A case study about the understanding of addition and product of probabilities of a technological high school student <i>José Luis Ávila Betancourt and Ana María Ojeda Salazar</i>	856
Processing graphs as an illustration of how engineering students build a machine learning model <i>Katharina Bata, Angela Schmitz and Andreas Eichler</i>	858
How odd are odds? Students' difficulties in converting relative frequencies into odds <i>Karin Binder, Nicole Steib, Theresa Büchter, Katharina Böcherer-Linder, Andreas Eichler, Stefan Krauss, Markus Vogel and Patrick Wiesner</i>	866
Interpretation of statistical information and the role of contextual knowledge – a pilot study among fourth-grade students <i>Lisa Birk</i>	874
Comparative analysis of teaching correlation and regression in three European countries <i>Balázs Boller</i>	882
To develop risk literacy – a design-based research approach <i>Marie-Theres Brehm and Angelika Bikner-Ahsbals</i>	884
Analysing students' notes when calculating in Bayesian situations <i>Theresa Büchter, Nicole Steib, Karin Binder, Katharina Böcherer-Linder, Andreas Eichler, Stefan Krauss and Markus Vogel</i>	892
The role of context knowledge for middle school students' development of critical statistical literacy <i>Christian Büscher</i>	900
The impact of registers of semiotic representation on problem solving for students entering tertiary education in France: case of the mean <i>Roxane Cattán-Jallet, Charlotte Derouet, Camille Doukhan, Lynn Farah and Corinne Hahn</i>	908
Modelling cycle to analyse probabilistic modelling work in high school <i>Charlotte Derouet</i>	910
Teacher's training in inferential statistics with a parallel application of classical and Bayesian approach <i>Péter Fejes-Tóth and Ödön Vancsó</i>	912
Statistics inquiry and the students' engagement in the change of paradigm <i>Maria-Josep Freixanet, Montserrat Alsina and Marianna Bosch</i>	914
Establishing a top-down approach to recognizing professional teacher knowledge regarding probability: A didactically oriented reconstruction of the laws of large numbers <i>Judith Huget</i>	922
The Connection between Numerical Competencies and Young Children's Probabilistic Thinking – A Poster Proposal <i>Lena S. Jaeger and Miriam M. Lüken</i>	930
A view on in-service teachers' understanding of Simpson's paradox <i>Raz Jaff and Hans Wilhelm Mørch</i>	932
Making sense of distribution: A case of <i>dépayement</i> <i>Stine Gerster Johansen and Uffe Thomas Jankvist</i>	940
Pre-service mathematics teachers' statistical problem-solving process for telling a story through the use of photographs as data sources <i>Sibel Kazak</i>	948
From school mathematics to artificial neural networks: Developing a mathematical model to predict life expectancy <i>Stephan Kindler, Sarah Schönbrodt and Martin Frank</i>	956
Combinatorial reasoning in problem solving strategies: a pilot study with pre-service teachers <i>Luca Lamanna, Laura Branchetti and Giorgio Bolondi</i>	964

Peirce's and Toulmin's perspectives to examine the informal inferential reasonings of preschool children <i>Maritza Méndez-Reina and Soledad Estrella</i>	972
The role of technology in school data science <i>Reza Moeti, Abolfazl Rafiepour and Mohammad Reza Fadaee</i>	980
Questioning the world – A dialectic of study and research <i>Camilla Hellsten Østergaard and Dorte Moeskær Larsen</i>	982
The development of pre-service teachers' informal inferential reasoning: A case of a statistical reasoning learning environment <i>Beyza Olgun and Didem Akyüz</i>	990
Probabilistic knowledge in professional resources for teachers: a case study at Japan's lower secondary school level <i>Koji Otaki</i>	998
Spanish secondary school students' building and interpreting cumulative frequency tables <i>Jocelyn D. Pallauta, Pedro Arteaga, María M. Gea and Carmen Batanero</i>	1006
Can attitudes towards statistics mediate the relationship between math anxiety and statistics anxiety? A multiple mediation model with Italian university students <i>Caterina Primi and Maria Anna Donati</i>	1014
The impact of blocky programming on probabilistic thinking of seventh-grade students <i>Abolfazl Rafiepour and Mohammad Radmehr</i>	1022
Two situations to deal with the subjective meaning of probability in the elementary school: students' productions and teachers' knowledge <i>Luis J. Rodríguez-Muñiz, Carmela Suárez and Laura Muñiz-Rodríguez</i>	1030
Mathematical practices when solving problems involving joint and conditional probabilities without prior instruction <i>José M. Rubio-Chueca, José M. Muñoz-Escolano and Pablo Beltrán-Pellicer</i>	1038
Exploring (In)formal Statistical Inference in the Classroom: An Inferentialist Perspective <i>Abdel Seidowry</i>	1046
Supporting English language learners in multilingual statistics classrooms: A collaborative study <i>Sashi Sharma</i>	1054
The teaching of statistical literacy in France: A curricular study <i>Charlotte Tarabant and Floriane Wozniak</i>	1062
Teaching probability at secondary school level with technological tools: issues at stake for mathematics teacher education <i>Mathieu Thibault, Jean-François Maheux and Caroline Lajoie</i>	1070
A study and research path for teacher education in statistics: dealing with the transparency of data treatment <i>Janielly Verbisck, Berta Barquero, Marilena Bittar and Marianna Bosch</i>	1078
Combinatorial strategies based on isomorphism <i>David Zenkl and Nad'a Vondrová</i>	1086
TWG6: Applications and modelling	1094
An introduction to TWG6: Applications and modelling <i>Jonas Bergman Ärlebäck, Susana Carreira, Britta Eyrich Jessen, Gilbert Greefrath, Yana Lacek, Katrin Vorhölter</i>	1095
An activity theory analysis of group interactions in mathematical modelling with the aid of digital technologies <i>Obed Opoku Afram</i>	1103
A reflection process to overcome blockages when introducing modelling <i>Noé Aguilar and Claudia Acuña</i>	1111

Characterising pre-service primary teachers' posed Fermi problems <i>Jonas Bergman Ärlebäck and Lluís Albarracín</i>	1119
A framework for exploring the use of Fermi problems to elicit students' and pre-service teachers' intercultural awareness in mathematics classrooms <i>Jonas Bergman Ärlebäck, Serife Sevinc, Lluís Albarracín, Daniel Orey, Irene Ferrando Palomares and Milton Rosa</i>	1127
Shared drawings in a mathematical modelling activity: An exploratory study <i>Caterina Bassi and Domenico Brunetto</i>	1135
Initiating meta-cognitive communication in students' discussions about process steps when solving real-world problems <i>Bianca Beer and Susanne Prediger</i>	1143
Elementary students demonstrate proportional and protoquantitative reasoning in models for sharing costs fairly <i>Amy Been Bennett, Julia Aguirre, Erin Turner, Elzena McVicar and Erin Carll</i>	1151
Positioning theory framework for analysing teaching of mathematical modelling, focusing on students' opportunities <i>Ingeborg Katrin Lid Berget</i>	1159
Designing escape game as a way to train primary school teachers to develop knowledge and competencies about modelling in mathematics <i>Charlotte Bertin</i>	1161
Cognitive and affective analysis based on two samples of Spanish and Turkish prospective teachers' solving a modelling task <i>Irene Ferrando Palomares and Serife Sevinc</i>	1169
Investigating critical mathematical thinking when applying mathematics to real-world problems <i>Vince Geiger, Kim Beswick, Jill Fielding, Thorsten Scheiner, Gabriele Kaiser, Merrilyn Goos and Katherine Fernandez</i>	1177
Future teachers' experiences with modelling and for teaching modelling: A Realistic Mathematics Education perspective <i>Carolina Guerrero-Ortiz and Chris Rasmussen</i>	1185
Mathematics teachers' analysis of modelling situations in the classroom: A vignette-based study <i>Stefanie Hartmann, Jens Krummenauer and Sebastian Kuntze</i>	1193
Norwegian primary teachers' mathematical modelling knowledge <i>Pamela Helder Naylor and Arne Jakobsen</i>	1201
Mathematicians' and teachers' views on the connection between mathematics and other fields <i>Anna Hoffmann and Ruhama Even</i>	1203
Make it real: Students' mathematical modelling realized with 3D printing <i>Simone Jablonski, Tim Läufer and Matthias Ludwig</i>	1211
Mathematical modelling and problem solving connected through technology: A case study with an in-service teacher <i>Hélia Jacinto, Susana Carreira and Gilbert Greefrath</i>	1219
Constraints and conditions for teaching data science as part of mathematical modelling in upper secondary mathematics <i>Britta Eyrich Jessen</i>	1227
Students' use of assumptions to solve a modelling task with much or little personal interest in the real-world context of the task <i>Melanie Kämmerer</i>	1235

A triplet of data/context, mathematical model, and statistical model: Conceptualising data-driven modelling in school mathematics <i>Takashi Kawakami</i>	1243
Teaching mathematical modelling along with didactic resources <i>Mitsuru Kawazoe and Koji Otaki</i>	1251
The complexity and the difficulty of organising an appropriate experimental milieu as means of validation during an individual mathematical exploration <i>Yana Lacek</i>	1259
An educational experience on modelling for in-service mathematics teachers from the Panamanian school system <i>Carlos Ledezma, Rita Borromeo Ferri, Luisa Morales-Maure and Vicenç Font</i>	1267
Math, modelling and art: One step to STEAM through physical and digital resources combined in a maker culture approach <i>Diego Lieban, Rafael W. Bueno, and Zsolt Lavicza</i>	1275
How working with a professional model affects students' views on the validity of simulation results <i>Christoph Lieben</i>	1277
Analysis of models used by student primary teachers when addressing a geometric estimation task <i>Esperanza López Centella and Jesús Montejo Gámez</i>	1279
High school students' interpretation when solving a MEA in the context of water scarcity <i>Dinorah Méndez-Huerta, Verónica Vargas-Alejo and Luis E. Montero-Moguel</i>	1287
Students' measurement estimation skills and their relation to the process of solving Fermi problems <i>Marlena Meyer and Gilbert Greefrath</i>	1295
Redesigning a mathematical task to an interdisciplinary modelling task <i>Kosuke Mineno, Takashi Kawakami and Akihiko Saeki</i>	1297
Risk literacy in mathematical modelling: Approach to handle uncertainties in the context of microplastic <i>Svenja Müller</i>	1299
Middle school students' preferences for modelling tasks contexts <i>Merlin Pastak</i>	1307
An analysis of mathematical modelling in university textbooks to address Klein's second discontinuity in teacher education <i>Luis Miguel Paz-Corrales, Avenilde Romo-Vázquez and Maleni Pérez-Sarmiento</i>	1309
Analysis of a mathematical modelling activity performed in the industry: The design of controllers used in durability tests and its didactical potential <i>Maleni Pérez-Sarmiento, Avenilde Romo-Vázquez and Luis Miguel Paz-Corrales</i>	1317
Theory-related pedagogical content knowledge for teaching simulations and mathematical modelling with digital tools - empirical analysis of the promotion of pre-service teachers <i>Jascha Quarder, Sebastian Gerber, Hans-Stefan Siller and Gilbert Greefrath</i>	1325
One way to link together school mathematics with the learner's reality <i>Stheven Rodriguez-Amador and Francisco Cordero</i>	1333
Relationship between strategies and appropriateness criteria for modelling problems involving estimates. An exploratory study with expert mathematics teachers <i>Carlos Segura and Irene Ferrando</i>	1341
Varied word problems as a tool to develop pupils' problem solving <i>Pavel Sovič</i>	1349
Tasks in outdoor mobile learning: A study proposal for defining quality in task design <i>Rebecca S. Stäter and Matthias Ludwig</i>	1351

The influence of retelling the task situation on making assumptions when working on mathematical modelling tasks with missing data <i>Anna Surel and Gilbert Greefrath</i>	1353
A new take on mathematical modelling through the analysis of two tasks, a case study in Patagonia, Argentina <i>Mathias Tejera, Pablo Carranza, Zsolt Lavicza and Kristof Fenyvesi</i>	1361
Modelling COVID-19 data with simulations: A recursive process <i>Susana Vasquez, Berta Barquero and Marianna Bosch</i>	1363
Modelling as an approach to implementing Education for Sustainable Development in mathematics teaching <i>Katrin Vorhölter and Hans-Stefan Siller</i>	1371
When horizontal mathematization enters the in-service training of primary school teachers: a case study in France <i>Sonia Yvain-Prébiski</i>	1379
TWG8: Affect and the teaching and learning of mathematics	1387
An introduction to TWG8: Affect and the teaching and learning of mathematics <i>Çiğdem Haser, Chiara Andrá, Inés M. Gómez-Chacón, Janina Krawitz and Hanna Viitala</i>	1388
Trait- and state-like motivations, emotions and perceived competence: a study on freshmen STEM students <i>Chiara Andrá and Cristina Scalvini</i>	1392
Students recalling favourite math experience: How does problem- based approach promote mathematical engagement? <i>Emőke Báró, Zoltán Kovács and Eszter Kónya</i>	1400
Exploring emotions in primary school students through mathematical problem solving <i>Nuria Begué and Mónica Arnal-Palacián</i>	1408
Rethinking the emotional characteristics of students’ attitudes towards mathematics <i>Liping Ding and Farzad Radmehr</i>	1416
Dyscalculia and affects towards mathematics. A systematic review <i>Estefanía Espina, José M. Marbán and Ana I. Maroto</i>	1424
Doctoral Supervisors’ conceptions of mathematics education research and mathematics education researcher <i>Çiğdem Haser</i>	1426
Relationship between difficulty level descriptor, self-efficacy, and choice of mathematics task <i>Maria Herset</i>	1434
What do students’ think about mathematics tasks? Do they like or dislike them? <i>Ylva Høgset and Marja van den Heuvel-Panhuizen</i>	1442
“You have to know that! Shame on you!” Pre-service primary teachers’ shame experiences in mathematics during teacher education <i>Lars Jenßen and Bettina Roesken-Winter</i>	1450
Experiences of competence and autonomy during a teaching intervention on mathematical modelling <i>Janina Krawitz, Stanislaw Schukajlow, Katharina Wiehe and Katrin Rakoczy</i>	1458
Relationships among mathematics beliefs, fluid intelligence, cognitive reflection and mathematics achievement in secondary school students <i>Alicia Rubio-Sánchez, Inés M. Gómez-Chacón and Isabel Gómez-Veiga</i>	1466
Teachers' perception and beliefs regarding mathematics anxiety among Year 3 and Year 4 students in Mexico <i>Beatriz Ruiz</i>	1474
Mathematical problem solving beliefs of upper secondary school students: Perspective from curriculum reform in Zambia <i>Edgar J. Sintema</i>	1482

How teachers' mathematics anxiety affects their students' mathematics anxiety and mathematics confidence <i>Jamie Smith and Nikolaos Fotou</i>	1490
How students view the difficulty of mathematical tasks: factors that influence their perceptions <i>Camilla Spagnolo and Marta Saccoletto</i>	1498
Collaborative problem solving in Finnish lower secondary mathematics classrooms and its effect on mathematics-related affect <i>Hanna Viitala</i>	1506
Mathematics teachers' mathematics educational values in teaching: A case study in Hong Kong <i>Qiaoping Zhang and Chung Yin Lam</i>	1514
TWG9: Mathematics and language	1522
An introduction to TWG9: Mathematics and language <i>Jenni Ingram, Kirstin Erath, Alexander Schüler-Meyer, Ingólfur Gíslason and Máire Ní Ríordáin</i>	1523
Exploring the role of everyday language in comprehending mathematical concepts: A case study of sets and infinity <i>Matthaios Antonopoulos, Ioannis Vlachos and Theodossios Zachariades</i>	1531
Learning to speak mathematically at the Japanese supplementary school in Sweden <i>Mayu Aoki, Yukiko Asami-Johannson, Carl Winsløw</i>	1539
Using the US standard system for measurement in Germany – Bilingual education in primary school <i>Eileen Baschek</i>	1547
Patterns of interaction in classroom dialogues in mathematics class in elementary school <i>Ann-Christin Beforth, Frank Lipowsky and Elisabeth Rathgeb-Schnierer</i>	1549
Talking mathematically – Negotiating mathematical concepts in bilingual settings <i>Malte Bürgstein, Marei Fetzer and Elke Söbbeke</i>	1551
Three complementary frameworks to capture student peer discussion in problem solving <i>Haydeé Ceballos, Theo van den Bogaart, Jeroen Spandaw, Stan van Ginkel and Paul Drijvers</i>	1559
Exploring the patterns of participation framework for studying multilingual mathematics teachers' identities <i>Danaí Dafnopolou</i>	1567
The creation of a bilingual glossary of mathematics terms <i>Marie Therese Farrugia</i>	1575
The role of language awareness in an algebraic-thinking task <i>Eugenia Ferrari and Silke Lekaas</i>	1577
The complexity of task design for utilising the epistemic potential of multiple languages in developing pattern understandings <i>Eugenia Ferrari, Silke Lekaas and Tamsin Meaney</i>	1585
What can mathematics teacher-students learn from dialogue with AI? <i>Ingólfur Gíslason</i>	1593
Sociomathematical norms in peer-cultures in free play situations in kindergarten <i>Esther Henschen, Anna-Marietha Vogler and Martina Teschner</i>	1595
Goals, decisions and responsiveness: Prioritising complexities in interaction <i>Jenni Ingram</i>	1603
Teacher and student perspectives on teacher actions in one-to-one tutoring <i>Inga-Maja Ludviksen Jernsletten</i>	1611
Identifying static- and emergent-shape thinking in students' language about graphs of functions <i>Elizabeth Kimber and Cathy Smith</i>	1613

Investigating pre-service teachers' activated didactic categories and individual language resources on functional relationships <i>Patrizia Kis-Fedi and Carina Büscher</i>	1621
Students with special educational needs verbalise their calculation strategies <i>Tabea Knobbe</i>	1629
Silent video tasks in pre-service mathematics teacher training <i>Bjarnheiður (Bea) Kristinsdóttir</i>	1631
A meta-analytic review on early mathematical interventions for multilingual children – Preliminary results <i>Katri Luomanieni, Andreas Gegenfurtner, Sanni Kankaanpää, Jake McMullen and Minna Hannula-Sormunen</i>	1633
Explorative study on language means used to explain the meaning of multiplying fractions <i>Jessica Mähnert and Kirstin Erath</i>	1635
Malawian secondary mathematics teachers' take-up of language responsive teaching through lesson study <i>Lisnet Mwadzaangati and Jill Adler</i>	1643
Ordering multiple adjectives for naming special prisms and pyramids in the Turkish mathematics language: The case of Ada <i>Samet Okumus and Dilek Girit Yildiz</i>	1651
Practices of noticing important aspects of mathematics teaching talk (about linear equations) with secondary-school mathematics teachers <i>Núria Planas and José M. Alfonso</i>	1659
Designing a workshop on noticing aspects of mathematical discourse for teaching angles <i>Juan Gabriel Rave-Agudelo and Núria Planas</i>	1667
How contrasting multiple languages can support conceptual understanding: The cases of newly arrived and resident multilingual students <i>Shintia Revina and Alexander Schüler-Meyer</i>	1675
Teaching the concept of angles to multilingual learners <i>Frode Rønning</i>	1683
Patterns of interaction in support situations in mathematics classrooms <i>Emine Shaka, Natalie Fischer and Elisabeth Rathgeb-Schnierer</i>	1685
Quotitive division in the bilingual classroom: Exploring structure to support the development of conceptual understanding with primary- aged multilingual learners <i>Jo Skelton</i>	1687
Students' strategies for presenting the problem <i>Anna Teledahl</i>	1695
A sketch of analysis of the teacher's moves towards the implementation of "scientific debates" in the mathematics classroom <i>Daniel Zimmer, Laure Ninove and Vanessa Hanin</i>	1703
TWG10: Diversity and mathematics education - social, cultural and political challenges	1711
An introduction to TWG10: Diversity and mathematics education - social, cultural and political challenges <i>Laura Black, Anette Bagger, Timo Dixel, Juuso Nieminen and Sabrina Salazar</i>	1712
Intercultural perspective on needs and approaches for teacher educations programs <i>Giuseppe Bianco, Benedetto Di Paola and Giovanni Giuseppe Nicosia</i>	1716
Diversity dimensions in Norwegian mathematics textbooks <i>Oda Heidi Bolstad and Bjørn Smestad</i>	1724
Mathem-Ethics for teenagers: An experiment about "loci" <i>Silvia Cerruto and Daniela Ferrarello</i>	1732

‘New generation’ barrier for low SES students: Non-routine problems in mathematics <i>Oğuzhan Doğan, Hülya Kılıç and Ayşegül Kılıç</i>	1740
The ethnomathematics of the Bedouin society: Integrating socio cultural elements into mathematics education <i>Fouze Abu Qouder and Miriam Amit</i>	1748
Including the Sustainable Development Goals in the mathematical training of preservice teachers <i>Fuertes-Prieto, M. A., Delgado-Martín, L. and Alonso-Ruano, B. M.</i>	1756
Social and political issues in mathematics education: An overview of an ongoing participatory research with in-service secondary school teachers <i>David Guillemette, Yasmine Abtahi, Richard Barwell, Achraf Hajby, Mireille Saboya and Marie-Frédéric St-Cyr</i>	1758
‘Doing Difference’ in mathematics education: How do mathematics teachers construct differences in learners? <i>Anna Hummel and Simone Reinhold</i>	1766
Working towards equity for diverse learners: Drawing on values and ways of being as strengths in mathematics classrooms <i>Jodie Hunter and Roberta Hunter</i>	1768
Designing a method for identifying gendered views of mathematics <i>Yuriko Kimura</i>	1776
Teachers talk about implementing problem-solving based instruction at a high diverse elementary school <i>Richard Kitchen</i>	1778
Creating opportunities to learn mathematics through the cultural framing of communication and participation patterns <i>Generosa Leach and Roberta Hunter</i>	1787
At the crossroads of ethnomathematics and Brazilian carnival: A study with samba schools <i>Jessica Lins de Souza Fernandes, Joana Celia dos Passos and Rita de Cassia Pacheco Gonçalves</i>	1795
The imponderable character of doing mathematics: A student’s conflicting discourses before a timed high-stakes mathematics test <i>Nikolaos (Nikos) Makrakis</i>	1803
Socio-cultural expectations of the mathematics classroom topology <i>Mariam Makramalla</i>	1811
Theorising power in classroom assessment of mathematics <i>Juuso Henrik Nieminen and Daniel L. Reinholz</i>	1819
How arguments, in students interaction, might open up for reflecting on mathematics in action at primary school level <i>Anna-Karin Nordin</i>	1827
(Re-)Creating inequality through Teaching for Mastery <i>Julie Alderton and Nick Pratt</i>	1835
Context matters: Results of a homogeneous group of students dealing with sociopolitical issues in the mathematics classroom <i>Daniela Steflitsch</i>	1843
Teachers’ view of twice-exceptional students – outline of the challenges in recognizing mathematical giftedness and supporting needs of hearing impairment <i>Dirk Weber, Sarah Beumann and Ralf Benölken</i>	1851
Emil’s Friday for future – perspective of Paul Ernest’s interest groups on teaching about carbon footprint <i>Ysette Weiss and Rainer Kaenders</i>	1859
The development of socio-mathematical agency <i>Pete Wright, Caroline Hilton and Joel Kelly</i>	1867

Reverse impact: Co-constructing mathematics education with communities facing socio-environmental crises <i>Ulises Xolocotzin, Daniela Tierra-Damián and Armando Solares-Rojas</i>	1875
Collaborative learning as a tool for the formation of moral values in the process of teaching mathematics <i>Anahit Yenokyan</i>	1882
TWG12: History in mathematics education	1884
An introduction to TWG 12: History in mathematics education <i>Antonio M. Oller-Marcén, Olivera Đokić, Tanja Hamann, Jenneke Krüger and Bjørn Smestad</i>	1885
A proposal by Ángel Llorca for teaching and learning geometry <i>Dolores Carrillo Gallego and José Ginés Espín Buendía</i>	1889
The use of GeoGebra for exploring some constructions of Euclid, Archimedes and Apollonius <i>Maria Chiara Cibien, Agnese Del Zozzo and Enrico Rogora</i>	1897
Integrating history of mathematics on the creative process <i>Elmha Moura and Alexandra Rodrigues</i>	1905
On the role of psychologists in the international New Math movement <i>Dirk De Bock and Wendy Goemans</i>	1913
The mathematical film: Dynamic geometry in the post-War era <i>Wendy Goemans and Dirk De Bock</i>	1921
<i>Elementa Euclidis</i> : A Jesuit mathematics schoolbook <i>Tanja Hamann and Barbara Schmidt-Thieme</i>	1929
Useful knowledge: Euclid or Descartes? <i>Jenneke Krüger</i>	1937
Researching the use of old professional journals as a tool for infant education prospective teachers training <i>José M. Muñoz-Escolano and Antonio M. Oller-Marcén</i>	1945
The conditions for the formation of numeracy and related terms <i>Amy O'Brien</i>	1953
The conceptual change approach in mathematics education: From topological primacy to projective geometry <i>Olivera Đokić and Marija Vorkapić</i>	1961
Euclid must go! Norwegian compulsory school exams 1962-1979 <i>Hilde Opsal and Bjørn Smestad</i>	1969
The notion of confrontation as a resource in mathematics education to study the development of mathematical knowledge in complex analysis <i>José Gerardo Piña-Aguirre and Rosa María Farfán Márquez</i>	1977
Simple fractions – not so simple! Insights from a study of Hebrew mathematical texts of the 12-16 centuries <i>Stela Segev</i>	1985
Leonard Nelson’s conception of critical thinking mathematics classroom <i>Shafie Shokrani</i>	1993
Geometrical reasoning in the intersection between historical original sources and the dynamic geometry environment GeoGebra <i>Marianne Thomsen and Uffe Thomas Jankvist</i>	2001
An outcome of debating infinitesimals via primary sources: access to the practice of developing mathematics <i>Mark Watford and Kathleen Michelle Clark</i>	2009
TWG13: Early years mathematics	2017
An introduction to TWG13: Early years mathematics <i>Bozena Maj-Tatsis, Camilla Björklund, Dorota Lembrér, Esther Levenson, Andrea Maffia, and Marianna Tzekaki</i>	2018

How do we make an E? Participation theory and theoretical consideration of supportive activities on block play in the family <i>Ergi Acar Bayraktar and Birgit Brandt</i>	2022
Adults' evaluations of a child's triangle knowledge <i>Ruthi Barkai, Esther S. Levenson, Pessia Tsamir and Dina Tirosh</i>	2030
Aesthetics in embodied task design in mathematics <i>Morten Bjørnebye, Anita Helseth and Jorryt van Bommel</i>	2038
Teachers' strategies in helping children to discern cardinality when playing a designed lottery game <i>Camilla Björklund and Hanna Palmér</i>	2046
A reconceptualisation of mathematical reasoning competency from a Kindergarten Class perspective <i>Mette Amalie Bundgaard</i>	2054
Examining the pre-school aged children's counting skills: A teaching experiment based on subitizing games <i>Deniz Eroglu</i>	2062
Emergent ideas of division: One child's response to a task-based mathematical interview <i>Jill Cheeseman and Ann Downton</i>	2070
Developing preservice early childhood educators' capacity to engage young children with mathematics using a model based on Bronfenbrenner's ecological theory <i>Audrey Cooke</i>	2078
Young children's spontaneous representations of multiplicative semantic structures before formal introduction <i>Ann Downton and Andrea Maffia</i>	2086
Reasoning discursive actions implemented by young children during a classification task <i>Sarah Dufour and Nelly Julien</i>	2094
Fair sharing and division – mathematical reasoning regarding integers and fractions in preschool and preschool class <i>Helena Eriksson, Maria Hedefalk, Peter Markkanen, and Lovisa Sumpter</i>	2096
Play-responsive mathematics teaching in a Swedish preschool <i>Lena Karlsson</i>	2104
Prediction of mathematic performance in primary school by cross- domain and domain-specific precursor skills <i>Jakob Kelz</i>	2112
Reasoning as a language-based action in early years mathematics <i>Astrid Hågensen Kleven</i>	2120
Young primary students' difficulties related to data-based argumentation – a qualitative analysis <i>Jens Krummenauer, Franziska Gutensohn, Johanna Aichele, Maria Emhart and Sebastian Kuntze</i>	2128
"How do you know?" Playfully revealing young children's perception of a given mathematical task <i>Irit Lavie and Michal Dvir</i>	2136
"There is mathematics in there too": Parents' and teachers' discussions about mathematics for young children <i>Dorota Lembrér</i>	2144
Instructional drawings for teaching about triangles: The case of prospective preschool teachers in India <i>Esther S. Levenson and Anna Neena George</i>	2152
Preschool children's mathematical performance in Japan <i>Nanae Matsuo, Nagisa Nakawa and Koji Watanabe</i>	2160
Children educated in heterogeneous and homogeneous kindergarten classes: Comparing their results in early years mathematics <i>Eva Nováková</i>	2168
What can an algebra bridge between number sense and equal sign in early childhood look like? <i>Camilla Rodal and Solfrid Storeli</i>	2176

The intersection of programming and mathematics in an educational context relevant to young students <i>Anna Sjö Dahl</i>	2184
Word problems as a tool to diagnose pupils' difficulties in mathematics <i>Jana Slezáková and Darina Jirotková</i>	2192
Young children's reasoning in a combinatorics task <i>Konstantinos Tatsis and Božena Maj-Tatsis</i>	2200
Could pre-schoolers become "good" problem solvers? <i>Marianna Tzekaki and Souzana Papadopoulou</i>	2208
Five minutes: Young students' understanding of time <i>Jorryt van Bommel, Hanna Palmér and Andreas Ebbelind</i>	2216
Kindergarten preservice teachers' descriptions of the learning potential of mathematical digital games <i>Mona Karbaschi Vee</i>	2224
Analysing children's awareness of the cultural rules of numerals in the world around them <i>Chronoula Voutsina and Debbie Stott</i>	2232
Exploring the potential of using talk moves with young students when striving towards an equitable mathematics education <i>Maria Walla</i>	2234
TWG14: University mathematics education	2242
An introduction to TWG14: University mathematics education <i>Irene Biza, Olov Viirman, Matija Bašić, Ignasi Florensa, Ghislaine Gueudet, Mathilde Hitier, Igor' Kontorovich, Athina Thoma and Megan Wawro</i>	2243
Towards relational thinking by Matlab LiveScript in linear algebra <i>Giovannina Albano, Angela Donatiello and Agnese Iliaria Telloni</i>	2251
Implicit Function Theorem: Students' autonomy in learning <i>Matija Bašić and Željka Milin Šipuš</i>	2259
Praxeologies-in-action during struggle with problem-solving <i>Elin Berggren, Constanta Olteanu, Miguel Perez and Håkan Sollervall</i>	2267
Students' arguments about the growth of a two-variable function <i>Andreas Bergwall</i>	2275
Online search routines in undergraduate students' agentic participation in mathematical discourse <i>Irene Biza and Elena Nardi</i>	2283
University students' understanding of exponential and logarithmic concepts: in case of real-world situations <i>Vahid Borji, Petra Surynková, Emily Kuper and Jarmila Robová</i>	2291
Teaching Mathematics with Digital Interactive Mathematical Maps (DIMM) for Geometry, Algebra and Calculus <i>Matthias Brandl, Urs Hackstein, Mirela Vinerean and Yvonne Liljekvist</i>	2299
Investigating students' worldviews of complex multiplication and derivatives <i>Rebecca Dibbs and Mehmet Celik</i>	2301
Coordinating the Activity Theory and the Anthropological Theory of the Didactic to analyze the secondary-tertiary transition <i>Camille Doukhan</i>	2309
Can one integrate by adding up lines? <i>Tommy Dreyfus, Dafna Elias, Anatoli Kouropatov and Lia Noah-Sella</i>	2311
Posing and solving problems about the functions of two variables <i>Özkan Ergene and Büşra Çaylan Ergene</i>	2319

Usage of prerecorded mathematics video-lectures – Comparison of the intentions of an instructor with students’ usage <i>Frank Feudel, Anja Panse and Thorsten Rohwedder</i>	2327
The best of both worlds – synergizing video learning and face-to-face instruction in undergraduate axiomatic geometry <i>Yael Fleischmann</i>	2335
Evolution of mathematics courses in engineering: new syllabus, same activity? <i>Ignasi Florensa, Iria Fraga, Kristina Markulin, Víctor Martínez-Junza and Noemí Ruiz-Munzón</i>	2337
Semiotic control in calculus: the case of limits of functions at the beginning of the university <i>Macarena Flores González and Fabrice Vandebrouck</i>	2345
Introducing mathematics undergraduate students to the theorem prover lean <i>Irene Garnelo and Michael Liebendörfer</i>	2353
The relevance of advanced mathematics courses for secondary school mathematics teachers: Voices of Tanzanian teachers <i>Alejandro S. González-Martín, Florence Kyaruzi, Gasper Mwangi and Honorata Kihaga</i>	2355
Thoughts on different types of mathematical enculturation at the secondary-tertiary transition <i>Lukas Günther and Reinhard Hochmuth</i>	2363
From real to abstract analysis: the development of structuralist praxeologies in the analysis path <i>Thomas Hausberger and Reinhard Hochmuth</i>	2371
“Like finding the acceleration”: A praxeological analysis of a calculus/mechanics task with and without its physics context <i>Mathilde Hitier and Alejandro González-Martín</i>	2379
Computability as a key aspect of future teachers’ knowledge of real numbers and functions <i>Rongrong Huo</i>	2387
Designing bridging tasks to work on Klein's first discontinuity <i>Leslie Jiménez and Macarena Flores González</i>	2389
Exploring a lecturer’s identification of students: a case study of teaching in university analysis lectures <i>Thomais Karavi and Angeliki Mali</i>	2391
Insights about functions from example-generation tasks: combining e-assessment and written responses <i>George Kinnear, Paola Iannone and Ben Davies</i>	2399
Student learning experiences in mathematics-oriented challenge-based courses <i>Zeger-Jan Kock, Ulises Salinas-Hernández and Birgit Pepin</i>	2407
Research-infused teacher-led innovations in university mathematics education <i>Igor’ Kontorovich, Hongjia Chen, Ian Jones, Nicolette Rattenbury and Padraic Bartlett</i>	2415
Students’ use of symbolical mathematical language and their development in their first year of study <i>Julian Körtling and Andreas Eichler</i>	2423
Navigating the transition: Empowering engineering students to conquer university mathematics with an online course <i>Fulya Kula and Tugce Akkaya</i>	2431
Calculus at the school to university transition: early stages of a structuralist perspective in real analysis <i>Ludwig Laukert, Thomas Hausberger and Reinhard Hochmuth</i>	2433
The evolution of spatial ability using GeoGebra for a multidimensional calculus course <i>Maria Antonietta Lepellere</i>	2441
How mathematical identity informed collaborative learning opportunities for a first-year international student from China <i>Kim Locke, Igor’ Kontorovich and Lisa Darragh</i>	2443

A case study of collaboration between mathematicians and mathematics educators under the perspective of assemblage theory <i>Georgios Mavrommatis</i>	2451
Trends in expectancy for success and value beliefs at the secondary-tertiary transition into STEM fields <i>Martin Mayerhofer, Michael Eichmair and Marko Lüftenegger</i>	2453
Extending example spaces in topology to aid undergraduate students' transition to generalization and abstraction <i>Annamaria Miranda</i>	2461
The evolving design and implementation of MathsFit: supporting first-year non-specialist mathematics students <i>Claire Mullen and Anthony Cronin</i>	2463
The impact of giving different goals (to teach and being tested) on engineering students' approaches to learning <i>Mahboubeh Nedaei</i>	2471
Discursive shifts in substantiation of narratives concerning sequence limit in prospective mathematics teachers <i>Kristýna Nižňanská</i>	2479
First-year students' commognitive conflicts concerning sequences and functions <i>Nikoletta Palamioti, Theodossios Zachariades, Irene Biza and Despina Potari</i>	2481
Engineering mathematics: Reflections on the dialectics of <i>in</i> and <i>for</i> <i>Jana Peters and Reinhard Hochmuth</i>	2489
Towards a designated undergraduate students' mathematics identity through a thinking group approach <i>Annamaria Miranda, Luca Picariello and Cristina Coppola</i>	2491
Introducing eigenspaces: semiotic analysis and didactic engineering <i>Margherita Piroi</i>	2499
Ostensives and the development of mathematical praxeologies – the case of polynomials <i>Jelena Pleština and Željka Milin Šipuš</i>	2507
The importance of moving between embodied, symbolic and formal worlds of mathematical thinking: The case of the tree concept <i>Farzad Radmehr and Maryam Taghizadeh Bilondi</i>	2509
Potential conflict factors in learning exact differential equations: Analysis of students' work <i>Svitlana Rogovchenko and Yuriy Rogovchenko</i>	2517
Implementing flipped learning in mathematics higher education: an action research project <i>Lucia Sagredo-Sanchez</i>	2525
The inextricable intertwining of communication and higher order mathematical thinking <i>Moritz Seibold, Klaus-Peter Eichler, Daniela Bímová and Sandra Gleißberg</i>	2527
Gender equality and students' group work in mathematics at tertiary level <i>Rozenn Texier-Picard and Ghislaine Gueudet</i>	2535
Natural Number Game: Students' activity using an interactive theorem prover <i>Athina Thoma and Paola Iannone</i>	2543
Adapting standard mathematics exercises to promote de-ritualization <i>Olov Viirman and Magnus Jacobsson</i>	2551
Zooming into instructional interactions online: A case of an interactive tutorial in the pandemic <i>Miriam N. Wallach and Igor' Kontorovich</i>	2559
Student reasoning about determinants with GeoGebra <i>Megan Wawro, Matthew Mauntel and David Plaxco</i>	2567

TWG15: Teaching mathematics with technology and other resources	2569
An introduction to TWG 15: Teaching mathematics with technology and other resources <i>Ornella Robutti, Bärbel Barzel, Melih Turgut, Gülay Bozkurt and Daniel Thurm</i>	2570
Exploring the TPACK of pre-service mathematics teachers in real classrooms: A techno-pedagogical integration matrix perspective <i>Hatice Akkoç and Tuğba Hangül</i>	2574
Chaining of instrumental orchestrations <i>Seyfettin Alan and Hatice Akkoç</i>	2582
Mathematics teachers as designers of E-textbook: Sources for resources and TPACK development stages <i>Ismael Almahdi</i>	2590
Sociocultural influences while implementing <i>TouchTimes</i> <i>Sandy Bakos</i>	2598
A novice teacher's work with resources to enhance mathematical competencies <i>Burcu Nur Baştürk-Şahin and Menekşe Seden Tapan-Broutin</i>	2606
How the use of different technologies mobilises different domains of professional knowledge <i>Maria do Carmo Botelho, Tânia Coelho and Helena Rocha</i>	2614
A prospective mathematics teacher's instrumental orchestration: Instrumentation of 2D and 3D screens to teach cylinder <i>Gülay Bozkurt and Melike Yiğit Koyunkaya</i>	2616
Computational thinking and mathematics teaching: An experience on initial teacher education <i>Neusa Branco and Susana Colaço</i>	2624
A decision theatre as a digital environment for teaching mathematics <i>Alexander Brödner, Joshua Wiebe, Sinah Gürtler and Sarah Wolf</i>	2632
Mathematics teachers experiencing a hackathon to design a digital game <i>Roberto Capone, Eleonora Faggiano and Osama Swidan</i>	2634
A teachers' debate on the design and use of a digital medium for mathematics as a window to their practices <i>Dimitris Diamantidis and Chronis Kynigos</i>	2642
Pre-service teachers' competence in using dynamic worksheets to promote functional thinking <i>Alex Engelhardt, Henrik Ossadnik and Jürgen Roth</i>	2650
Norwegian mathematics teachers' beliefs about collaborative learning via computer programming <i>Aleksandra Khara Fadum and Helga Kufaas Tellefsen</i>	2652
Didactical suitability of a proposal for 3D printing and augmented reality with preservice teachers <i>Teresa F. Blanco and Antía Fernández-López</i>	2660
Design of asynchronous mathematical discussions on Padlet: Analysis of students' social modes and teacher's roles <i>Sara Gagliani Caputo, Annalisa Cusi and Laura Branchetti</i>	2662
Factors contributing to preservice teachers' implementation of technology in problem-solving in mathematics <i>Ramesh Gautam</i>	2670
Upper secondary school teachers' views on CAS from an epistemic and pragmatic point of view <i>Ragna í Geil and Ingi Heinesen Højsted</i>	2678
Survey design considerations for capturing teachers' mathematical digital competency: A vignette approach <i>Eirini Geraniou and Nicola Bretscher</i>	2686
Teachers' challenges in integrating technology in mathematics teaching through the lens of the instrumental distance concept <i>Mariam Haspekian and Carolyn Kieran</i>	2694

Teachers' attitudes towards technology during emergency remote learning <i>Anat Klemer, Ruti Segal, Shirley Miedijensky, Ronit Herscu-Kluska and Anatoli Kouropatov</i>	2702
Make it real: Student teachers' professional development through fabrication of their own manipulatives <i>Tim Läufer and Matthias Ludwig</i>	2710
Teachers' perspectives on programming through emerging technologies in mathematics education <i>Trygve K. Løken, Asif Mushtaq, Klaus-Peter Eichler and Daniela Bimova</i>	2712
Effect of GeoGebra explorations combined with art-based practices on in-service teachers' understanding of geometric transformations <i>Irina Lyublinskaya and Marta Cabral</i>	2720
A framework for secondary mathematics technology-enhanced formative assessment (TEFA) literacy <i>Alyssa L. MacMahon and Irina Lyublinskaya</i>	2728
BlueBots and irrational numbers: Contingency moments in teaching mathematics and computational thinking in teacher education <i>Steinar Mathisen, Trude Sundtjønn and Henrik Forssell</i>	2736
The effectiveness of an interactive, autonomous teaching application on the topic of boundedness among pre-service mathematics teachers <i>Borbála Neogrády-Kiss, Csaba Csapodi and Ádám Besenyei</i>	2744
Teacher-researchers can develop techno-mathematical literacy by posing and teaching problems with digital tools <i>Tikva Ovadiya</i>	2752
Sustainability of professional development programmes to teach mathematics using technology <i>Pedro Pimenta, António Domingos and Maria Cristina Costa</i>	2760
From translation to transposition: Italian mathematics teachers as interlingual and intercultural mediators <i>Ornella Robutti and Giulia Bini</i>	2768
Models on teachers' knowledge to teach with digital technology: A systematic review <i>Helena Rocha</i>	2770
Gathering inputs for the development of a gamified experience <i>Cecilia Russo, Fabian Vitabar and Zsolt Lavicza</i>	2778
Science and mathematics teachers' attitudes and emotions regarding technology integration <i>Ruti Segal, Shirley Miedijensky, Anat Klemer, Ira Raveh, Irit Lavie and Iris Wagner-Gershgoren</i>	2780
Pre-service teachers' usage of and beliefs about teaching mathematics with digital mathematical tools: What matters in China? <i>Yihua Shen, Yang Shen and Shuhui Li</i>	2788
Learning, designing and reflecting: Prospective teachers acquire digital competencies by designing digital learning environments <i>Annabelle Speer and Andreas Eichler</i>	2796
Teachers' critical reflections on digital fabrication for making manipulatives to support mathematical teaching <i>Henrik Stigberg, Susanne Koch Stigberg, Marianne Maugesten, Odd Tore Kaufmann</i>	2804
Latent class analysis of distance learning arrangements in secondary mathematics education during COVID-19 <i>Anni S. Sydänmaanlakka, Jokke I. A. Häsä, Marja E. Holm and Markku S. Hannula</i>	2812
Professional development online: The impact of the ASYMPTOTE MOOC on (future) teachers' meta-didactical praxeologies <i>Eugenia Taranto, Despina Koutsomanoli Filippaki, Simon Barlovits, Claudia Lázaro, Léon Anhalt, Stylianos Triantafyllou, Maria Flavia Mammanna and Matthias Ludwig</i>	2820
Formative assessment of computational thinking in mathematics – A case study <i>Aino Ukkonen</i>	2828

Instrumental orchestration during emergency remote mathematics teaching <i>Candaş Uygan and Gülay Bozkurt</i>	2836
Key aspects of using personas in mathematics teacher education <i>Robert Weinhandl, Lena Kleinferchner, Viktoria Riegler, Carina Schobersberger</i>	2844
Spreadsheet-based math-CT activities: Solving statistics problems <i>Wahid Yunianto, Rully Charitas IP, Dyah Susilowati and Zsolt Lavicza</i>	2852
TWG16: Learning mathematics with technology and other resources	2854
An introduction to TWG16: Learning mathematics with technology and other resources <i>Osama Swidan, Rogier Bos, Eleonora Faggiano, Seçil Yemen Karpuzcu, Florian Schacht, Jana Trgalová</i>	2855
Disclosing parametric functions in the transition from pen-and-paper to GeoGebra <i>Sara Bagossi and Eugenia Taranto</i>	2859
Effective or not? The impact of mobile learning on students' interest, self-efficacy, and performance in outdoor mathematics education <i>Simon Barlovits and Matthias Ludwig</i>	2867
Computational thinking in mathematics teaching in secondary school <i>Valérie Batteau and Jana Trgalová</i>	2875
Student-made videos in the spotlight: How to raise interest with a learning video? <i>Brigitta Békési, Eva Ulbrich, Tony Houghton and Zsolt Lavicza</i>	2883
Identifying computational thinking activities in a geometric problem <i>Carina Büscher</i>	2885
Upper-secondary school students' mistakes in using loops while coding cubic solids in a block-based coding environment <i>Martin Cápaj, Eva Schmidthaler and Janka Medová</i>	2893
A fuzzy cognitive analysis on motivation, engagement, and participation: New trends and further ideas <i>Roberto Capone and Mario Lepore</i>	2895
Mathematical discussion in problem solving activities supported by technology: An achievable goal in primary school <i>Sofia Cavalletti, Veronica Manzoni, Giorgio Bolondi, Paolo Cazzaniga and Chiara Giberti</i>	2903
Learning spatial orientation with use of a virtual city <i>Sylvia Coutat, Jean-Luc Dorier and Sabrina Matri</i>	2911
Geogebraization: A process for dynamizing classical mathematical texts <i>Agnese Del Zozzo</i>	2919
The effects of covariational reasoning on conceptual learning of functions with digital-enhanced experiments <i>Susanne Digel, Lena Bolz and Jürgen Roth</i>	2921
Computational thinking learning concepts for elementary school students in face-to-face and distance education <i>Jeanne Dobgenski and Maria Elisabette Brisola Brito Prado</i>	2929
Software-supported development of the ability to rotate mentally <i>Klaus-Peter Eichler, Helena Nussbaumer, Moritz Seibold, Daniela Bímová, Jiří Břehovský and Trygve Loken</i>	2931
Follow the actors – Mathematical learning in digital settings <i>Marei Fetzler and Julia Bräuer</i>	2939
The occurrence and distribution of programming and computational thinking in mathematics education research <i>Martyna K. Fojcik</i>	2947

Programming language as an object-to-think-with: An enactivist perspective <i>Wendy Ann Forbes and Joyce Mgombelo</i>	2949
The design of a digital game to reduce math anxiety in the classroom <i>Léon Imanuel, Sonia Palha and Anders Bouwer</i>	2957
Monitoring automatically gained difficulty rankings with mathematics educational theories and experts <i>Eva-Maria Infanger, Nilay Aral, Edith Lindenbauer and Zsolt Lavicza</i>	2965
Digital games and mathematics: Designing a game for learning probability in secondary school <i>Ljerka Jukić Matić, Darija Marković and Mirela Jukić Bokun</i>	2973
The yoyo-bird: A transformative digital game for meaning making on trigonometric functions <i>Myrto Karavakou and Chronis Kynigos</i>	2975
Bridging the literature gap: An innovative framework for understanding secondary school students' emerging technology use in out-of-class mathematics learning contexts <i>Xinyue Li</i>	2983
How do students evaluate learning mathematics with digital textbooks? An exploratory study in a Chinese secondary school <i>Na Li and Lianghuo Fan</i>	2985
Designing a tool for authoring digital problem-solving tasks in an app – an integrative learning design study <i>Sophie Mense, Karina Höveler, Pauline A. Blohm and Lisa C. Willemsen</i>	2993
Cryptography as a field to foster interactions between mathematics and informatics, and algorithms. Analysis of a didactical situation <i>Simon Modeste, Evmorfia-Iro Bartzia, Michael Lodi, Marco Sbaraglia, Viviane Durand- Guerrier and Simone Martini</i>	3001
Visualization of the conversion of length units with a digital measuring tape <i>Lea Marie Müller and Melanie Platz</i>	3009
Reconstruction of opportunities to understand the function concept from online explainer videos <i>Martin Ohrndorf, Insa Meißner, Florian Schmidt-Borcherding and Maike Vollstedt</i>	3011
Technology as a support in the development of didactic activities for covariational reasoning <i>Helen Perez, Armando Cuevas-Vallejo and José Orozco-Santiago</i>	3019
Incorporation of an AI object recognition app and Cuisenaire rods in a <i>child-rods-app-task</i> body-artifact dynamic functional system <i>Michael Rumbelow</i>	3027
A qualitative analysis of the use of Book Creator functions while processing Fermi questions <i>Christoph Schäfer</i>	3029
'Practicing Place Value': How third graders use and benefit from a training app fostering place value understanding with and without teacher support <i>Johanna Scharlau and Daniel Walter</i>	3037
Programming to animate letter models: A context for mathematical competence? <i>Katia Schiza and Chronis Kynigos</i>	3039
Geometrical argumentation of pre-service teachers at university level. A comparison of two different DGS tasks in interactive books <i>Julia Marie Stechemesser and Florian Schacht</i>	3047
Modifying the didactical tetrahedron to describe aspects of mathematical digital collaborative learning <i>Stephan Tomaszewski</i>	3055
Developing computational thinking: An unplugged problem-solving approach <i>Tran Kiem Minh and Nguyen Ngoc Thanh</i>	3063
How combining self-assessment and automatic assessment might help to support learning <i>Carina Tusche, Daniel Thurm and Shai Olsher</i>	3071

Visualizing flexible learning paths in 9th grade algebra <i>Annika Volt and Mart Laanpere</i>	3073
Interactive videos can foster comprehension: Results of a pilot study <i>Sina Wetzel and Matthias Ludwig</i>	3075
Applying mathematics in a simulation model: An interdisciplinary learning workshop in the context of sustainable mobility <i>Joshua Wiebe, Sinah Gürtler, Anina Mischau and Sarah Wolf</i>	3077
Design of an online professional development platform for mathematics teachers' noticing of students' mathematical thinking <i>Seçil Yemen Karpuzcu, Özge Dışbudak Kuru, Mine Işıksal Bostan, Reyhan Tekin Sitrava</i>	3085
TWG17: Theoretical Perspectives and Approaches in Mathematics Education Research	3093
An introduction to TWG17: Navigating ways forward amidst diversity <i>Heather Lynn Johnson, Felix Lensing, Mariam Haspekian, Abdel Seidouvy, Cecilie Carlsen-Bach</i>	3094
A framework for views of prospective mathematics teachers on mathematics didactics <i>Dana Eilers</i>	3102
Toward a semiotic account of practice-based mathematics teacher education <i>Patricio Herbst and Daniel Chazan</i>	3104
Semiotic Process Cards as a tool to analyse learner's mathematical interactions <i>Melanie Huth and Christof Schreiber</i>	3112
Approaches to address a diversity of theories in mathematics education: importing, adapting, and networking theories <i>Heather Lynn Johnson</i>	3120
On the need for a methodology of didactical content analysis <i>David Kollosche</i>	3128
The demarcation problem: On methodological implications of adopting a reflexive stance in mathematics education research <i>Felix Lensing</i>	3136
Limits and transforming power of didactics <i>Pedro Nicolás and Josep Gascón</i>	3144
Networking the documentational approach and Valsiner's zone theory <i>Amalie Sødal</i>	3152
Advancing Duval's semiotic theory under the holistic psychology of emotion and cognition <i>Tsung-Ju Wu and Fou-Lai Lin</i>	3160
Vygotskyan perezhivanie as a bridging concept between phenomenological methods and a Vygotskyan activity theory lens <i>Zagorianakos Andonis</i>	3168
TWG18: Mathematics Teacher Education and Professional Development	3170
An introduction to TWG18: Building bridges between different perspectives: Emerging themes from international research into mathematics teacher education and professional development <i>Marita Friesen, Tracy Helliwell, Raffaele Casi, Andreas Ebbelind, Janne Fauskanger, Pere Ivars, Macarena Larrain and Libuše Samková</i>	3171
Inquiry to me: Four mathematics teachers' conceptualizations of inquiry preceding a professional development course <i>Marte Bråtalien</i>	3179
Experience a sense of being, becoming and belonging to an educational design project as professional development <i>Andreas Ebbelind, Hanna Palmér and Jorryt van Bommel</i>	3187

Changes in pre-service teachers' conceptions of functional thinking <i>Kerstin Frey and Ute Sproesser</i>	3195
Insights into out-of-field teachers' self-reports: Fostering the understanding of addition and subtraction as a basis for children to overcome difficulties in mathematics <i>Lara Marie Graf, Uta Häsel-Weide, Karina Höveler and Marcus Nührenböcker</i>	3203
How would you explain to someone what mathematics is about? A phenomenological study of preservice teachers' perspectives on the nature, learning and teaching of mathematics <i>Flavio Guiñez, Camila Lizama, Helena Montenegro and Paula Guerra</i>	3211
The role of dissonance in mathematics teacher education <i>Andreas Ebbelind and Tracy Helliwell</i>	3219
Investigating preservice mathematics teachers' reflections on engaging university peers in proving-related activities <i>Karoline Kongshavn</i>	3227
Representational accuracy in pre-service teachers' analyses of a classroom vignette on a students' difficulty with fractions <i>Sebastian Kuntze, Pere Ivars and Jens Krummenauer</i>	3235
Professional development for human flourishing <i>Jennifer Lewis, Christopher Nazelli, S. Asli Özgün-Koca, Kate Rollert French and Lenuel Hernandez</i>	3243
Students' design of inquiry with didactic tools: A Japanese case <i>Tatsuya Mizoguchi, Berta Barquero and Marianna Bosch</i>	3251
The impact of incorrect solutions on the attitude and problem-solving abilities of prospective mathematics teachers towards combinatorics <i>Zoltán Paulovics, Csaba Csapodi and Zoltán Lóránt Nagy</i>	3259
Professional competencies of prospective teachers in the area of patterns and structures - a pilot study <i>Dinah Reuter</i>	3267
What do vocational school teachers in the field of early childhood education plan for their early mathematics education classes? – An analysis of hypothetical lesson plans on early numeracy <i>Alix Richter and Julia Bruns</i>	3275
Empowering mathematics teachers: Three models for professional development in Chile <i>Farzaneh Saadati</i>	3283
Investigating natural number bias of future primary school teachers: The case of fractions and concept cartoons <i>Libuše Samková</i>	3285
Developing elementary mathematics teachers' knowledge of mathematical reasoning processes <i>Lurdes Serrazina, Lina Brunheira and Margarida Rodrigues</i>	3293
Mathematics education philosophies of mathematics teacher educators <i>Fatih Taş and Yüksel Dede</i>	3301
An insight into prospective elementary teachers' mathematical knowledge for teaching: An example of fraction division <i>Anne Tossavainen and Monica Johansson</i>	3303
Developing functional thinking from teacher education to primary school: Students walk along the number line <i>Bartjan Vollmuller, Michiel Veldhuis, Mara Otten, Sonja Stuber, Madhuvanti Anantharajan</i>	3311
Teacher professionalization – developing diagnostic competence by supporting mathematically interested and gifted children <i>Sabine Vietz and Tobias Huhmann</i>	3313

Identifying pre-service mathematics teachers' non-routine problem perceptions through non-routine problem posing <i>Yeliz Yazgan and Tuğçe Kozaklı Ülger</i>	3321
Conceptions of student teachers on the limit concept <i>Christoph Ableitinger, Stefan Götz and Roland Steinbauer</i>	3329
What and how mathematics teachers notice while co-planning instruction <i>Raymond Bjuland and Janne Fauskanger</i>	3337
Interdisciplinary task design for pre-service teacher education: learning potentials at the boundary between mathematics and physics <i>Laura Branchetti, Francesca Morselli and Lorenzo Pollani</i>	3345
Informal mathematics in teacher education: The teachers' voice <i>Raffaele Casi and Cristina Sabena</i>	3353
Using video-based module situated in pedagogies of practice framework to develop pre-service teachers' professional noticing skills in perimeter measurement <i>Büşra Çaylan Ergene and Mine Işıksal Bostan</i>	3361
Expert primary teachers' noticing of exemplary mathematics lessons: A case study in Mainland China <i>Yiru Chen, Qiaoping Zhang and Yang Cao</i>	3369
Supporting pre-service teachers' development: What can we learn from case studies of three experienced school-based mathematics mentors <i>Cosette Crisan</i>	3371
Prospective teachers' mathematical knowledge: Relating representations as design principle for developing knowledge of non- local mathematics for teaching in abstract algebra <i>Anna Dellori and Lena Wessel</i>	3379
Perspective taking as a means to foster pre-service teachers' noticing of students' mathematical thinking <i>Carolyn D. Fellenz and Susanne Schnell</i>	3387
Online professional development for teaching algebra: Towards the design of an asynchronous, adaptive self-learning module <i>Marita Friesen, Felix Kapp, Bärbel Barzel, Anika Dreher, Lars Holzäpfel, Macarena Larrain, Lukas Weith, Tobias Domokos and Antonia Hiemenz</i>	3395
Perceptions of continuity of pre-service teachers in Austria <i>Stefan Götz, Antonia Spannagl and Roland Steinbauer</i>	3403
Teachers' interactions in the joint planning of a teaching experiment <i>Katharina Knaudt, Raja Herold-Blasius and Christoph Selter</i>	3411
Diagnosis of noticing at the beginning of primary mathematics teachers' education programs: A case study <i>María Victoria Martínez, Josefa Perdomo-Díaz and Francisco Rojas</i>	3419
We are digital natives – preservice teachers' professional mathematical identities <i>Marianne Maugesten, Odd Tore Kaufmann and Tamsin Meaney</i>	3427
Prospective mathematics teachers' professional development through meta discussion on a pedagogical model <i>Michele Giuliano Fiorentino, Maria Alessandra Mariotti, Antonella Montone</i>	3435
How can we contemplate “productivity” in students' mathematical interthinking? <i>Margrethe Naalsund, Marte Bråtalen and Joakim Skogholt</i>	3443
Moves for eliciting and promoting reasoning and proving <i>Reidun Persdatter Ødegaard</i>	3451
Pre-service teachers' perception of LGE-tasks through a digital assessment system <i>Siri Ovedal-Hakestad and Niclas Larson</i>	3459

Developing an analytical tool for researching geometry discourse among pre-service teachers from India and Sweden <i>Harita Pankajkumar Raval and Lisa Österling</i>	3461
Strengthening teachers' formative assessment literacy with SMART <i>Fabian Rösken, Katrin Klingbeil and Bärbel Barzel</i>	3469
Noticing the relevance of subject matter in typical teacher tasks – a situated learning intervention for preservice teachers <i>Kata Sebök</i>	3471
The focus on the student perspective in a microteaching task for mathematics teacher trainees <i>Réka Szász</i>	3479
The impact of content knowledge and pedagogical content knowledge on the development of diagnostic thinking <i>Jan Philipp Volkmer, Andreas Eichler and Elisabeth Rathgeb-Schnierer</i>	3487
Professional reflective competence of prospective primary school teachers in mathematics based on video vignettes <i>Susanne Wöller</i>	3495
Noticing skills of prospective and novice mathematics teachers with different teacher perspectives <i>Kübra Yıldırım and Gülseren Karagöz Akar</i>	3503
TWG19: Mathematics teaching and teacher practice(s)	3511
An introduction to TWG19: Mathematics teaching and teacher practice(s) <i>Reidar Mosvold, Mark Hoover, Helena Grundén, Siún Nic Mhuirí and Chrysoula Choutou</i>	3512
Observation is hard: The challenges involved in developing a shared understanding of classrooms <i>Fay Baldry</i>	3520
Pre-service teachers' generalization strategies solving figurative growth patterns <i>Ana Barbosa and Isabel Vale</i>	3528
Socio-mathematical norms related to explanations: A case of a gifted and talented mathematics classroom <i>Aslı Çakır and Hatice Akkoç</i>	3536
A teacher's noticing of students' mathematical thinking in action <i>Emine Çatman Aksoy and Mine Işıksal Bostan</i>	3544
Connections and tensions between mathematics and visual art classrooms <i>Chrysoula Choutou</i>	3552
How mini–whiteboards can help teachers raise their level of communication in whole class plenary talks <i>Thomas F. Eidissen</i>	3560
Imaginary dialogues in secondary class mathematics lessons – understanding students' mathematical thinking processes <i>Petra Gössinger and Stefan Götz</i>	3568
Experts' views on teaching financial literacy <i>Jiří Helus</i>	3576
<i>Listening being</i> to recognize and relieve harm in mathematics teaching <i>Mark Hoover and Charles Phillips</i>	3578
Teachers' in-the-moment response to student thinking: The case of pattern generalization <i>Mine Işıksal Bostan, Reyhan Tekin Sitrava, Zeynep Özel and Seçil Yemen Karpuzcu</i>	3586
Creating space for socio-mathematical agency in the primary classroom <i>Joel Kelly, Caroline Hilton and Pete Wright</i>	3594
Analysis of metacognitive activities in pre-service teachers' lessons – case study <i>Márton Kiss and Eszter Kónya</i>	3602

The four quadrants of mathematical preparedness: Insights from a Delphi study <i>Derek Maher and Aibhín Bray</i>	3604
Analysing improvisation in the mathematics classroom <i>Nick McIvor, Jeremy Hodgen and Sai Loo</i>	3612
The impact of the SIMPLE strategy on word problem solving – a case study <i>Qëndresa Morina</i>	3620
Variation in mathematics teacher questioning across instructional settings <i>Reidar Mosvold</i>	3622
Mathematical modelling and multiple solutions: Students’ pathways through a long division unit <i>Siún Nic Mhuirí and Patrick Neary</i>	3630
Orienting to the phenomenon of teachable moments through teachers’ lived experiences <i>Amanjot Toorand and Joyce Mgombelo</i>	3638
Spontaneous noticing and responding to students’ non-conventional fractional thinking <i>Zelha Tunç Pekkan and Özlem Kayıtmaz</i>	3646
Challenging problems through engineering design with pre-service teachers <i>Isabel Vale and Ana Barbosa</i>	3654
Promoting children’s procedural knowledge in problem solving via metacognition <i>Aikaterini Vissariou and Despina Desli</i>	3656
Addressing different socio-mathematical norms as teaching knowledge <i>Christina Svensson and Rickard Wester</i>	3664
A comparative case study of the mathematics pedagogy in two Chinese schools: How “student-centered” is a proclaimed reformed pedagogy? <i>Ying Zhang and Andreas J. Stylianides</i>	3666
TWG20: Mathematics Teacher Knowledge, Beliefs and Identity	3674
An introduction to TWG20: Knowledge, beliefs and identity <i>Francesca Martignone, Miguel Montes, Federica Ferretti, Veronika Hubeňáková, Jimmy Karlsson, Nadia Kennedy and Miguel Ribeiro</i>	3675
Unpacking the reasoning task solution approaches of pre-service teachers in the Teacher Education and Development Study in Mathematics <i>Helen Alfaró</i>	3679
The influence of an internship in South Africa on Norwegian preservice mathematics teachers’ development of a professional identity <i>Mette Andresen and Bettina Dahl</i>	3687
Knowledge mobilised in a teaching experiment about conjecturing and proving by elementary preservice teachers <i>Matías Arce, Laura Conejo and Eric Flores-Medrano</i>	3695
The role of practicum teachers in mathematics teacher identity development <i>Okan Arslan</i>	3703
An investigation of two mathematics teachers’ covariational reasoning <i>Mervenur Belin, Işıl İşler-Baykal, Gülseren Karagöz Akar</i>	3711
Designing tasks for prospective teachers: specialised knowledge on the inclusive teaching of problem-solving <i>Ángeles Chico, Nuria Climent and Inmaculada Gómez-Hurtado</i>	3719
How do prospective primary teachers interpret ‘to promote understanding’? <i>Chico, J., Martín-Díaz, J.P., Montes, M. and Badillo, E.</i>	3727
Analysis of the implementation of a professional task designed around the MTSK model <i>Nuria Climent, M^a Isabel Pascual, Juan P. Martín, Myriam Codes and Luis C. Contreras</i>	3735

In-service Italian primary mathematics teachers' knowledge and beliefs about possible students' mistakes in mathematics large-scale tests <i>Federica Ferretti, Francesca Martignone and Giada Viola</i>	3743
Pre-service teachers' difficulties with the semiotic representation of the decimal number line <i>Macarena Flores González and Elann Lesnes</i>	3751
Beliefs of teachers about complex and authentic problems <i>André Greubel, Hans-Stefan Siller, Martin Hennecke</i>	3759
Design of a “framework-document” for training schoolteachers to teach mathematics <i>Claire Guille-Biel Winder, Édith Petitfour and Frédérick Tempier</i>	3767
The concept of <i>practices</i> in mathematics teachers' competence: A theoretical exploration <i>Christine Luise Hirsch and Nils Buchholtz</i>	3775
Pre-service Mathematics Teachers Thoughts about Reasoning and Proof in Mathematics Lessons <i>Katarína Jánošková, Katarína Hrušková, Dominika Valášková and Lenka Vráblová</i>	3783
Knowledge of different forms of complex numbers through quantitative reasoning: The case of teachers <i>Gülseren Karagöz Akar, Mervenur Belin, Nil Arabacı, Yeşim İmamoğlu and Kemal Akoğlu</i>	3785
Decision-making in planning for mathematics teaching <i>Jimmy Karlsson</i>	3793
Professional Identity Development of Prospective Mathematics Teachers <i>Nadia Stoyanova Kennedy</i>	3795
Two pre-service mathematics teachers' beliefs about mathematics and its teaching and their lesson plans for linear function topic <i>Monika Křišáková and Veronika Hubeňáková</i>	3803
Pre-service teachers' subjective theories on students with exceptional difficulties in learning mathematics <i>Philipp Larmann and Matthias Ludwig</i>	3811
Relation of a test for assessing pre-service primary teachers' pedagogical content knowledge to other educational variables <i>Malte Lehmann, Bettina Roesken-Winter and Lars Jenßen</i>	3819
Relationships between the specialized knowledge of a mathematics teacher and his noticing: a starting point <i>Ledher M. López and Diana Zakaryan</i>	3827
Ethical dimension in the use of interpretative tasks in mathematics teacher education: fraction division <i>Maria Mellone, Arne Jakobsen, Miguel Ribeiro and Alessio Parlati</i>	3835
A blueprint for measuring mathematical knowledge for teaching college algebra at community colleges <i>Vilma Mesa, Inah Ko, Claire Boeck, Irene Duranczyk, Bismark Akoto, Dexter Lim, Patrick Kimani, Laura Watkins, April Ström, and Mary Beisiegel</i>	3843
Investigating the relation between primary school teachers' beliefs, self-efficacy, and knowledge: The case of fraction multiplication and division <i>Eleni Odysseos and Marios Pittalis</i>	3851
Mathematical knowledge for teaching problem solving: pre-service mathematics teachers' beliefs about problem-solving <i>Emma M. Owens and Brien C. Nolan</i>	3859
Design considerations and decisions of mathematics teachers in creating dialogical situations for their students <i>Reut Parasha and Boris Koichu</i>	3867
Theorizing teacher's knowledge on connections from the MTSK perspective <i>Milena Policastro and Miguel Ribeiro</i>	3875
First-class teachers' beliefs about manipulatives and their use <i>Clara Ries, Stephanie Schuler and Gerald Wittmann</i>	3883

Preservice teachers' feedback to students' solutions on linking linear function equation and graph: a Polish and Slovak case <i>Ingrid Semanišínová and Mirosława Sajka</i>	3891
Pre-service teachers' knowledge of students' misconceptions and difficulties about functions <i>Matej Slabý</i>	3899
'Nature of mathematics weiterDenken' - an empirical study among pre-service teachers and mathematics students at the University of Vienna and its implications for academic teaching and learning <i>Felix Woltron</i>	3901
TWG21: Assessment in Mathematics Education	3903
An introduction to TWG21: Assessment in mathematics education <i>Francesca Morselli, Christina Drüke-Noe, Chiara Giberti, Gözde Kaplan-Can, Johanna Rämö</i>	3904
Assessment of students' procedural knowledge and relations to student characteristics <i>Christoph Ableitinger, Christian Dorner and Georg Krammer</i>	3908
Teachers' beliefs about assessment: A study in Italy and Portugal <i>Nélia Amado and Francesca Morselli</i>	3916
Higher order thinking skills through automatic formative assessment: Generating examples in ordinary differential equations to actively engage students in developing conceptual understanding <i>Alice Barana, Marina Marchisio, Fabio Roman and Matteo Sacchet</i>	3924
Interpreting the gap between foreign and native speaker students in national standardised assessment <i>Marta Barbero, Clelia Cascella, Chiara Giberti and Matteo Viale</i>	3932
Design principles and task characteristics of mathematics class tests <i>Christina Drüke-Noe</i>	3940
Formal intentions in a new mathematics curriculum versus example tasks published before the standardized examinations – do they align? <i>Stig Eriksen and Pauline Vos</i>	3948
Realising formative assessment strategies with SMART – a study of teachers' use of an understanding-orientated digital diagnosis tool in the topic of linear functions <i>Anica Eumann and Bärbel Barzel</i>	3956
Competency goals and oral exams in mathematics <i>Tomas Højgaard</i>	3958
A case of a preservice middle school mathematics teacher's planned and enacted formative assessment strategies <i>Gözde Kaplan-Can and Çiğdem Haser</i>	3966
The inconsistency between curricular expectations and assessment results <i>Hülya Kılıç, Oğuzhan Doğan and Ayşegül Kılıç</i>	3974
One task for assessing students' ability to order a list of outcomes <i>Miriama Kmeciková, Nikoleta Ratvajová and Veronika Hubeňáková</i>	3982
Norwegian mathematics teachers' dilemmas related to "Going Gradeless" assessment practice <i>Iveta Kohanová and Hilde Rotabakk</i>	3984
Diagnosing preservice mathematics teachers' conditional probability and Bayesian reasoning <i>Oguz Koklu and Muhammet Arican</i>	3986
Mathematical competence and classroom assessment: A framework for task classification <i>Iina Kröger and Andreas Büchter</i>	3994
Mathematical written tests as formative assessment practice <i>Alice Lemmo</i>	4002
Teachers' self-reported teaching practices in formative assessment <i>Jeanne-Celine Linker and Christoph Selter</i>	4010

Blind versus visible checkbox grading: Does not seeing the grades when assessing mathematics enhance inter-rater reliability? <i>Filip Moons and Ellen Vandervieren</i>	4012
“My mind is getting used to always find a better solution process”: Formative assessment and self-regulation in secondary school algebra <i>Francesca Morselli and Simone Quartara</i>	4020
Students’ self-assessment predictors and practices in an undergraduate mathematics course <i>Johanna Rämö, Jokke Häsä and Zi Yan</i>	4028
The impact of linguistic and mathematical difficulty - determining characteristics of students' performance in test items in mathematics <i>Antonia Rewer, Gilbert Greefrath</i>	4036
Assessing procedural and conceptual calculus knowledge <i>Franziska Sommerlade and Andreas Eichler</i>	4044
Mathematical assessment for first-year STEM students: Development, experience, and outlook at the University of Innsbruck <i>Pia Tscholl, Florian Stampfer and Tobias Hell</i>	4052
Formative assessment in mathematics teacher education – Perspectives of pre-service teachers in a group discussion <i>Martina Geisen and Joerg Zender</i>	4060
TWG22: Curricular Resources and Task Design in Mathematics Education	4068
An introduction to TWG22: Curricular resources and task design in mathematics education <i>Shai Olsher, Annalisa Cusi, Dubravka Glasnović Gracin and Hendrik Van Steenbrugge</i>	4069
Mathematics teacher knowledge activated when choosing a textbook <i>Luciana Vieira Andrade, Hélia Jacinto and Rúbia Barcelos Amaral</i>	4077
The potential of educational resources for teaching practice in Telesecundaria in Mexico: An exploratory study <i>Alexandra Angel, Olimpia Figueras and Carlos Valenzuela García</i>	4085
Curriculum-aligned digital formative assessment for elementary school geometry <i>Hassan Ayoob and Shai Olsher</i>	4093
Topological approach to historical analytic narrative in the ‘Game Theory lab’ activities <i>Giovanna Bimonte, Francesco Saverio Tortoriello and Ilaria Veronesi</i>	4101
Mathematics teachers' uses of resources in the context of teaching reasoning-and-proving: Insights from a cross-national study <i>Erdiç Çakıroğlu, Iveta Kohanová, Işıl İşler-Baykal, Mária Slavičková, Benedetto Di Paola, Jakub Michal and Siri-Malén Høynes</i>	4109
On the bumpy road to decisions: Reconciling different agendas and perspectives in cross-community collaborative task design <i>Adi Eraky, Ronnie Karsenty and Alon Pinto</i>	4117
Teaching situations designed by experienced teachers to foster students’ mathematical flexibility <i>Menucha Farber and Boris Koichu</i>	4119
Undergraduate students’ use of interactive mathematics textbooks: A study of user’s action paths <i>Saba Gerami, Shi Qi Lim and Vilma Mesa</i>	4127
Time as a resource in mathematics education: Comparing teacher and student perspectives <i>Dubravka Glasnović Gracin and Goran Trupčević</i>	4135
Exploring the structure of multiplication tasks in Norwegian textbooks within a developmental approach <i>Nils-Jakob Herleiksplass, Viktor Freiman and Raymond Bjuland</i>	4143

An analytical framework for programming tasks in mathematics textbooks <i>Mari Solberg Jensen, Antoine Julien, Abolfazl Rafiepour and Alexander Schmeding</i>	4151
Teachers' problem posing for facilitating mathematical transition from primary to secondary school <i>Sotirios Katsomitros and Konstantinos Tatsis</i>	4159
Design principles for tasks to support student learning in mathematics across primary-secondary transition: An iterative approach <i>Tandeep Kaur, Eilish McLoughlin and Paul Grimes</i>	4167
Stories of congruent triangles in two geometry textbooks <i>Jane-Jane Lo, Lili Zhou and Jinqing Liu</i>	4169
Conceptions of spanning sets and linear independence emerging from examples and student responses to reading questions in an interactive linear algebra textbook <i>Vilma Mesa, Eric Khiu, Saba Gerami and Tom Judson</i>	4177
WILMA as a way of connection between maths educators in universities and teachers in practice <i>Monika Dillingerová and Emilia Mitková</i>	4185
Granularity of CT tasks embedded in secondary mathematics lessons <i>Kristin Parve and Mart Laanpere</i>	4187
Reflecting from the start – curriculum design to foster continuous reflective practice in mathematics pre- service teacher education <i>Ingolf Schäfer and Erik Hanke</i>	4189
Tasks stimulating reflections on mathematics <i>Edith Schneider</i>	4197
Proposing educative features of the curriculum materials that can enhance teachers' noticing <i>Sumeyra Tutuncu, Jeremy Hodgen and Jennie Golding</i>	4205
The inscription of desired images of children through contextualized tasks in mathematics textbooks from Flanders and Turkey <i>Hendrik Van Steenbrugge and Ayşe Yolcu</i>	4213
Mathematical praxeologies in the Chinese and Singaporean secondary school textbooks: The case of probability <i>Sikai Wang, Carl Winsløw and Binyan Xu</i>	4221
Using different versions of a mathematical problem for visualizing students' conceptions of perimeter <i>Jonas Jäder and Lotta Wedman</i>	4229
Textbooks in interaction in primary school mathematics lessons <i>Rebekka Will</i>	4237
TWG23: Implementation of Research Findings in Mathematics Education	4239
An introduction to TWG23: Implementation of research findings in mathematics education <i>Mario Sánchez Aguilar, Linda Marie Ahl, Boris Koichu and Morten Misfeldt</i>	4240
A Delphi-inspired study on the concepts of large and small-scale projects in mathematics education research <i>Mario Sánchez Aguilar, Linda Marie Ahl, Uffe Thomas Jankvist and Ola Helenius</i>	4244
Factors of Influence for Implementing Curriculum Reform: An analysis based on a systematic literature review <i>Linda Marie Ahl, Mario Sánchez Aguilar, Uffe Thomas Jankvist, Morten Misfeldt & Johan Prytz</i>	4252
The dynamic of implementation of adapted Lesson Studies in France <i>Michèle Artigue and Blandine Masselin</i>	4260
From design principles to implementation heuristics: A networking of theories perspective on implementation research <i>Cecilie Carlsen Bach, Rikke Maagaard Gregersen, Mathilde Kjær Pedersen, Ingi Heinesen Højsted and Uffe Thomas Jankvist</i>	4268

Implementation of active, bodily experience mathematics learning activities: Differences among primary and secondary school teachers <i>Alessandra Boscolo</i>	4276
Researching implementation in mathematics education innovations: a practice-oriented theory of change model <i>Mark Boylan</i>	4284
What can we learn from teachers' reflection on implementation of problem solving in middle school? <i>Jason Cooper and Esther Gruenhut</i>	4292
What do heterarchical social network approaches to policy research have to offer IRME? <i>Jennie Golding</i>	4300
On the implicitness of the 'theory of change' in implementations in mathematics education research <i>Ola Helenius, Uffe Thomas Jankvist, Linda Marie Ahl, & Mario Sánchez Aguilar</i>	4308
What can researchers and teachers co-learn from data on students' problem-solving experiences? <i>Boris Koichu, Aamer Badarneh and Menucha Farber</i>	4316
Evaluating the enactment of a scripted teaching model <i>Tuula Koljonen, Ola Helenius and Linda Marie Ahl</i>	4324
Implementability of computational thinking in Danish compulsory school mathematics – a survey conducted in a pre-implementation context <i>Andreas L. Tamborg and Liv Nøhr</i>	4332
Implementation through adaptation: Relations between mathematics, programming, and computational thinking <i>Morten Misfeldt, Thomas Brahe, Uffe Thomas Jankvist, Raimundo Elicer, Eirini Geraniou, Kajsa Bråting and Andreas Lindenskov Tamborg</i>	4340
Piloting an innovation for developing students' written mathematical communication competency <i>David Nordqvist, Linda Marie Ahl & Ola Helenius</i>	4348
Innovation and the problem of measuring effects in large scale professional development projects <i>John Prytz, Linda Marie Ahl and Uffe Thomas Jankvist</i>	4356
A comparison of the implementation of two large-scale innovations in mathematics education: TRIUMPHS and 'Boost for Mathematics' <i>Iresha Ratnayake, Linda Marie Ahl, Johan Prytz and Uffe Thomas Jankvist</i>	4364
Implementing a teaching model for enhancing students' formal written mathematical communication <i>Julia Tsygan, Ola Helenius and Linda Marie Ahl</i>	4372
TWG24: Representations in Mathematics Teaching and Learning	4380
An introduction to TWG24: Representations in mathematics teaching and learning <i>Carla Finesilver, Anna Baccaglioni-Frank, Elisa Miragliotta and Kate C. O'Brien</i>	4381
Representation of numbers and variables in Austrian Sign Language <i>Flavio Angeloni, Annika M. Wille and Christian Hausch</i>	4385
Unknown and variable: The semiotic potential of a digital balance- model <i>Chiara Bonadiman</i>	4393
"I have never encountered an exercise as confusing as this one": Mathematical representations and affect in an urban escape booklet <i>Amalie Thorup Eich-Høy</i>	4401
Multiple representations in a digital game on arithmetic word problems <i>Evrin Erbilgin and Gregory Michael Adam Macur</i>	4403
Cubeling: Building connections between spatial reasoning and computational thinking <i>Heiko Etzold and Kevin Larkin</i>	4411

The roles of representations in inclusive mathematics education: A review of CERME research <i>Carla Finesilver</i>	4419
The role of automated assessment in representing students' strategies for comparing fractions <i>Amal Kadan-Tabaja and Michal Yerushalmy</i>	4427
Investigating the effect of learning part-part instead of part-whole concepts using the Fingu App from an ACAT perspective <i>Ulrich Kortenkamp, Kevin Larkin, Silke Ladel and Dorothee Dahl</i>	4435
Pre-service teachers' intuitive conception about fractions <i>Chloé Lemrich, Marie-Line Gardes and Emmanuel Sander Lausanne</i>	4443
Sense-making in algebraic mathematizing discourse: The profiles of Bea and Nico <i>Elena Macchioni, Giulia Lisarelli, Elisa Miragliotta and Anna Baccaglioni-Frank</i>	4451
Analysis of inequalities graphs with a semiotic artefact <i>Ytzeen F. Méndez Franco and Claudia M. Acuña Soto</i>	4459
Figural component in geometrical reasoning: The case of a blind solver <i>Elisa Miragliotta, Carola Manolino and Andrea Maffia</i>	4467
Becoming diagram: Experimenting with diagrams to explore geometrical and topological concepts in tapestry weaving <i>Kate C. O'Brien</i>	4475
From <i>weight</i> to <i>value</i> : Focusing on verbal signs during a digital artifact-mediated algebraic activities <i>Elisabetta Robotti, Elisa Miragliotta and Chiara Bonadiman</i>	4483
Matching a graph with an image representing the situational context: Students' approaches identified by using eye tracking <i>Aylin Thomaneck, Maïke Vollstedt and Maïke Schindler</i>	4491
Activate students? Let them fold! Mathematical paper folding in secondary education in France and Germany <i>Jacoliene van Wijk, Rogier Bos, Anna Shvarts and Michiel Doorman</i>	4499
Representation of fractions in Danish fourth-grade textbooks <i>Julie Vangsøe Færch and Pernille Ladegaard Pedersen</i>	4507
Patterns of individual learning support with focus on sign activity and communicating about it: A comparison <i>Annika M. Wille and Barbara Ott</i>	4515
TWG25: Inclusive Mathematics Education – Challenges for Students with Special Needs	4523
An introduction to TWG25: Inclusive mathematics education – challenges for students with special needs <i>Petra Scherer, Michael Gaidoschik, Hana Moraová, Helena Roos, Andreas Ulovec</i>	4524
Which factors do pre-service teachers consider most important for successful inclusive mathematics classrooms? Results of an interview study <i>Jennifer Bertram and Petra Scherer</i>	4532
Cooperation processes in inclusive learning settings with a special focus on mathematical potential <i>Anna-Maria Billigen, Elke Söbbeke and Lara Sprenger</i>	4540
Inclusive teaching for part-whole understanding: A case study and related reflections on desirable frameworks <i>Michael Gaidoschik</i>	4548
Students' participation in mathematics in inclusive settings <i>Malin Gardesten</i>	4556
How can the double number line promote students with mathematical learning difficulties to conduct and explain proportional reasoning? <i>Alexander Goldschmidt and Susanne Prediger</i>	4558
How teaching mathematics for social justice can support inclusive practices in the elementary mathematics classroom <i>Caroline Hilton, Joel Kelly and Pete Wright</i>	4566

Participating in inclusive classrooms by solving tasks in practical contexts and with objects of representation <i>Yola Koch</i>	4574
Collective mathematical argumentation in the in-between of inclusion and scientific convention <i>Sebastian Kollhoff and Kerstin Gerlach</i>	4582
Developing differentiated algebra worksheets for inclusive classroom <i>Liina Malva and Triin Kivirähk</i>	4590
Mathematics teachers' professional noticing of gifted students' mathematical thinking within the context of pattern generalization <i>Zeynep Özel, Mine İşıksal Bostan and Reyhan Tekin Sitrava</i>	4598
Expressing mathematical creativity: The case of mediating open- ended tasks for students with learning disorders <i>Maya Ron Ezra and Esther S. Levenson</i>	4606
Moments of inclusion and equity in mathematics education <i>Helena Roos and Anette Bagger</i>	4614
Silent videos in heterogeneous classrooms as impulse for the development of mathematical notions exemplified on "What is a tangent?" <i>Marc Sauerwein</i>	4616
Tensions in teachers' attempts to apply inclusive mathematics education for students with diverse learning needs <i>Maria Vasilopoulou and Chrissavgi Triantafillou</i>	4624
TWG26: Mathematics in the Context of STEM Education	4632
An introduction to TWG26: Mathematics in the context of STEM education <i>Behiye Ubuz, Nelleke Braber, Clelia Cascella and Michelle Stephan</i>	4633
Computational thinking in higher education in the context of STEM education <i>Maria Cristina Costa, Sandra Gaspar Martins and António Domingos</i>	4639
Thinking outside the box: in-service teachers' search for the value of mathematics outside the mathematics classroom <i>Nelleke den Braber, Jenneke Krüger and Marco Mazereeuw</i>	4641
Communicating spatial reasoning – The case of unplugged programming activities <i>Andreas Eckert</i>	4643
Promoting interdisciplinarity in elementary education in the context an Erasmus + STEAM project <i>Nádia Ferreira, Maria Cristina Costa, Francisco Peixoto, Vera Monteiro, José Castro Silva</i>	4651
Teachers' perceptions of utilizing virtual field trips for teaching STEAM <i>Gizem Güzeller and Didem Akyüz</i>	4653
Supporting the interpretation of formulas in physics education through mathematics lessons <i>Peter Kop</i>	4661
Two inquiry approaches to STEM: The role of mathematics <i>Dorte Moeskær Larsen and Camilla Hellsten Østergaard</i>	4669
Integrating mathematics and science in secondary education classrooms: numbers and algebra <i>Zaira Ortiz-Laso, José Manuel Diego-Mantecon, Sandra García-Fernández and María Sanz-Ruiz</i>	4677
The role of teachers' experiences and beliefs in the conceptualization of mathematics within the design of STEAM activities <i>Gabriella Pocalana, Giulia Bini and Ornella Robutti</i>	4679
STEM integration from a systems thinking perspective: Engineering and mathematical practices <i>Premkumar Pugalenti, Michelle Stephan and David Pugalee</i>	4687

Promoting university STEAM competence: analysis of a learning situation in the Organic Learning Garden <i>María Santágueda-Villanueva, Gil Lorenzo-Valentín, Mireia Adelantado-Renau and Lidón Monferrer-Sales</i>	4695
AI education as a starting point for interdisciplinary STEM projects <i>Sarah Schönbrodt, Katja Hoeffler and Martin Frank</i>	4703
Authentic-STEM: Opening long-term domains of experience for fostering students' and mentors' self-efficacy through mathematics <i>Gero Stoffels</i>	4711
Investigating performativity in evaluation approaches in STEM education with a focus on mathematics: A methodological design <i>Maiken Westen Holm Svendsen</i>	4719
The relationship between students' variational and covariational reasoning levels and their quantitative reasoning in a STEM environment <i>Pelin Turan Kurudirek and Selahattin Arslan</i>	4721
Engineering design process on designing a neighborhood in project- based learning environment: a comparison of field dependent and field independent cognitive style students <i>Behiye Ubuz and Yurdagül Aydınyer</i>	4729
Authentic problems of knowledge society: mathematics teachers' integration of cognitive and social dimensions into stem lesson plans <i>Defne Yabaş, Gaye D. Ceyhan, Zerrin Doganca-Kucuk and M. Sencer Corlu</i>	4737
TWG27: The Professional Practices, Preparation and Support of Mathematics Teacher Educators	4745
An introduction to TWG27: The professional practices, preparation and support of mathematics teacher educators <i>Ronnie Karsenty, Hilda Borko, Alf Coles, Birte Friedrich-Pöhler, Bettina Rösken-Winter and Stefan Zehetmeier</i>	4746
Learning to integrate reflections on the classroom level and teacher professional development level: A step in novice facilitators' growth <i>Thomas Bardy, Silvan Gorrengourt, Lars Holzäpfel, Bärbel Barzel and Susanne Prediger</i>	4750
Responsibilities and practices of mathematics teacher educators in relation to the new climatic regime: "It's a choice I make." <i>Alf Coles</i>	4758
Role and actions of a facilitator leading collective discussions in mathematics teacher education <i>Miriam Criez Nobrega Ferreira, Alessandro Jacques Ribeiro and João Pedro da Ponte</i>	4766
Pedagogical reasoning with pre-service teachers and their supervisors <i>Heleen Vellekoop, Els Franken and Anneke Smits</i>	4774
Supporting teachers' joint construction of teaching and preparation for future work: Examining workgroup facilitation <i>Lynsey Gibbons and Hannah Nieman</i>	4776
Meta-coaching: A novel approach to supporting the practice of mathematics coaches <i>Michael Jarry-Shore, Meghan Smith Durkin, Hilda Borko, Marsha Ing and Thomas Smith</i>	4784
Professionalizing primary school mathematics teacher educators <i>Ronald Keijzer and Michiel Veldhuis</i>	4792
Contributions of <i>Math for All</i> to building local facilitators' capacity for conducting professional learning to support accessible mathematics instruction <i>Babette Moeller, Karen Rothschild and Teresa Duncan</i>	4800
Mathematics teacher educators' conceptions of professional learning <i>Francisco Rojas, Daniela Rojas, Roberto Araneda, Helena Montenegro and Salomé Martínez</i>	4808

Profiles and beliefs of mathematics teacher educators on preservice teacher education: Development of an instrument <i>Francisco Rojas and M^a Isabel Pascual</i>	4816
Facilitators' planning practices for PD activities and their alignment to chosen PD goals: A case study on relevance of unpacking PD goals <i>Dilan Şahin-Gür and Susanne Prediger</i>	4818
How and why facilitators adapt the core ideas of professional development activities on dividing natural numbers <i>Victoria Shure, Bettina Roesken-Winter, Birte Pöhler and Esther Wensing</i>	4826
Facilitators' feedback in a mathematics education course for practicing teachers <i>Trude Sundtjønn and Grethe Kjensli</i>	4834
A teaching session on environmental socio-scientific issues for prospective mathematics teachers: Teacher educators' actions and challenges <i>Katerina Bogiatzi and Chrissavgi Triantafyllou</i>	4842
How and why facilitators adapt the core ideas of professional development activities on multiplying natural numbers <i>Esther Wensing, Birte Pöhler, Bettina Roesken-Winter and Victoria Shure</i>	4850
TWG28: Collaborative Settings in Mathematics Teacher Education	4858
An introduction to TWG 28: Collaborative settings in mathematics teacher education <i>Geoff Wake, Stéphane Clivaz, Paula Gomes, Pernilla Mårtensson and Aoibhinn Ní Shúilleabháin</i>	4859
Teachers' awareness on combinatorics during Lesson Study discussion <i>Valeria Andriano, Roberto Capone, Carola Manolino, Riccardo Minisola and Ornella Robutti Liceo</i>	4864
Collaborative working group between schoolteachers and researchers on the teaching of attributes and measurement in elementary school <i>Camille Antoine</i>	4872
Board work as a resource for lesson study in mathematics <i>Yukiko Asami-Johansson, Koji Otaki and Junki Akamoto</i>	4874
The development of mathematical knowledge of prospective primary teachers in a lesson study <i>Linda Cardoso, João Pedro da Ponte and Marisa Quaresma</i>	4882
Study of a teacher professional development process in the context of a collaborative work to design a resource for teaching problem-solving <i>Maud Chanudet and Stéphane Favier</i>	4890
Mathematical knowledge for teaching problem-solving: Dialogic construction during a lesson study <i>Stéphane Clivaz, Sara Presutti, Valérie Batteau, Luc-Olivier Bünzli, Audrey Daina and Jean-Philippe Pellet</i>	4898
Improving middle school algebra through bi-institutional lesson study <i>Derya Diana Cosan</i>	4906
Implementing Learning Study to foster Mathematical Knowledge for Teaching: A case in preservice teacher education in the Netherlands <i>Dédé de Haan, Siebrich de Vries, Paul Drijvers and Gerrit Roorda</i>	4908
Preparing and orchestrating mathematical whole-class discussions in lesson study <i>Filipa Faria, João Pedro da Ponte and Margarida Rodrigues</i>	4910
Mathematics and chemistry: A co-disciplinary educational path to face early school leaving in vocational school <i>Michele Giuliano Fiorentino, Antonella Montone, Giuditta Ricciardiello</i>	4912
A contributing framework for improving the quality of mathematics teaching and learning in Indonesia <i>Linda Devi Fitriana</i>	4920

Collaboration in inclusive mathematics settings from the point of view of German primary school and special education teachers <i>Martina Geisen</i>	4922
Planning a lesson in lesson study: The role of the facilitator <i>Paula Gomes, Filipa Faria and Micaela Martins</i>	4930
Mathematical trails supporting the collaboration among teachers and didacticians <i>Silvia Haringová and Janka Medová</i>	4938
The process of creating a useful mediating artefact in a professional learning community <i>Frida Harvey</i>	4940
Greater than the sum of our parts: Collaborative facilitation of online lesson study <i>Mairéad Holden and Miriam Ryan</i>	4948
Task-Design Lesson Study – A methodological tool for exploring mathematics teachers’ task-design principles and pedagogic practices <i>Laurie Jacques</i>	4956
How to choose problems? A co-construction between researchers and pedagogical counsellors <i>Caroline Lajoie, Lily Bacon, Nadine Bednarz, Jean-François Maheux, Mireille Saboya and Vanessa Hanin</i>	4964
Understanding the influence of lesson study design on teacher learning in the context of primary school mathematics: A theoretical view <i>Sarah Leakey</i>	4972
Professional theorizing: A key to understanding how prospective teachers improve their knowledge for teaching mathematics <i>Pernilla Mårtensson</i>	4980
Lesson study in initial teacher education: Key aspects that promote the development of prospective teachers’ knowledge <i>Micaela Martins, Nicole Duarte and João Pedro da Ponte</i>	4988
How the situation seeds used in adapted lesson studies support the collaborative work of facilitators and future facilitators <i>Blandine Masselin and Michèle Artigue</i>	4996
Computational thinking in primary teachers’ teaching practice: The contribution of lesson study <i>Ana Lúcia Bento Miguens, João Pedro da Ponte and Marisa Alexandra Ferreira Quaresma</i>	5004
Insights into the collaborative work of expert teachers within a lesson study project <i>Galit Nagari-Haddif, Ronnie Karsenty and Abraham Arcavi</i>	5006
Exploring the prevalence of structured problem solving in research lessons: A post-intervention study from Ireland <i>Aoibhinn Ni Shuilleabhain, Roisin Neururer, Diarmaid Hyland and Conor Sievwright</i>	5014
Fostering prospective teachers’ knowledge development within lesson study: What are the facilitator’s moves? <i>Sara Presutti</i>	5022
Challenges for the development of lesson study in Portugal <i>Marisa Quaresma and João Pedro da Ponte</i>	5030
Investigating the tensions that arise in a researcher-teacher collaboration when theory is to be translated into practice <i>Mona Røsselund</i>	5038
Combining the inquiry-based learning and critical thinking frameworks for interpreting the processes of teachers’ inquiry into teaching through lesson study <i>Svein Arne Sikko and Liping Ding</i>	5046

Prospective teachers researching on practice through lesson study <i>Raquel Vieira and João Pedro da Ponte</i>	5054
Using preservice teachers' reflections about a project-based statistics course to investigate knowledge creation in collaborative settings <i>Pauline Vos and Oda Heidi Bolstad</i>	5062
Collaborative lesson research as an effective space for teacher learning and improved student outcomes <i>Geoff Wake and Marie Joubert</i>	5070
TWG29: Embodied and material studies of mathematical behaviour	5078
An introduction to TWG29: Embodied and material studies of mathematical behaviour <i>Anna Shvarts, Elizabeth de Freitas, Giulia Ferrari, Christina Krause and Ricardo Nemirovsky</i>	5079
Actions behind mathematical concepts: A logical-historical analysis <i>Lonneke Boels, Rosa Alberto and Anna Shvarts</i>	5083
Embodiment at the crossover of sensuous cognition and linguistics: a possible bridge between physics and mathematics education <i>Federico Corni, Hans Fuchs and George Santi</i>	5091
Spatial imaginaries and spherology: Exploring the mathematical shape of space <i>Elizabeth de Freitas</i>	5099
The body probably understands <i>Dafna Efron</i>	5107
The materiality of mathematical imagination <i>Francesca Ferrara and Giulia Ferrari</i>	5115
Rethinking reflectional symmetry through Bee-Bots <i>Giulia Ferrari and Carlotta Soldano</i>	5123
Mathematics and body entangled in workshops with art <i>Cláudia Regina Flores, Mônica Maria Kerscher-Franco and Débora Regina Wagner</i>	5131
Construction of spatial orientation meanings during an outdoor activity with use of dynamic screen-based spatial representations <i>Christina Gkreka</i>	5133
Supporting individual learning of mathematical competences with the app TouchTimes <i>Marina Lentín</i>	5141
Looping the Fibonacci sequence <i>Charlotte Mégrouèche</i>	5143
Aesthetical entanglements in mathematics learning <i>Ricardo Nemirovsky, Vinay Kathotia and Charlotte Mégrouèche</i>	5151
Using manipulatives for teaching mathematics: how creative are the teachers? <i>Benjamin Nicolas-Noir and Thierry Dias</i>	5159
Dancing a proof, or a dancing proof? Extended embodiment in a Pre-service geometry class <i>Ottavio G. Rizzo</i>	5161
Experimental epistemology of mathematics revisited, in the light of inclusive materialism and agential realism <i>Jorge Soto-Andrade and Daniela Diaz-Rojas</i>	5163
Inside mathematics: mystery vs. problem <i>David Wagner</i>	5171
A collective approach to embodied learning in early years mathematics (ELEMS Phase 1) <i>Jennifer Way, Katherin Cartwright and Paul Ginns</i>	5173

Preface: CERME13 in Budapest, and beyond

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The 13th Congress of the European Society for Research in Mathematics Education – CERME13, for short – was a memorable event in many ways. It was the first CERME held “on site” since CERME 11 in Utrecht in February 2019, almost 4½ years before. After the outbreak of the COVID pandemic in early 2020, CERME12 was postponed to 2022 and was finally held online from Bozen-Bolzano in February 2022, as explained in the preface to the proceedings of that conference. At that time, it was still not clear whether the pandemic was nearing the end – although, as we now know, it was – and it was decided to defer the dates of CERME13 from February 2023 to July 2023. This way, we avoided the risks of new virus-related lockdowns in early 2023, which in fact did not materialize – and also real inconveniences related to the war in Ukraine, such as the energy crisis in the winter 2022-2023. In addition to these reasons, the new dates of CERME13, almost exactly between CERME12 and CERME14 (to be held in February 2025), allowed for a more realistic timeframe for organising CERME13 and CERME14 (1½ year for each). From February 2025, we will finally be back to the normal rhythm of biannual congresses in February of odd years, after CERME12 and CERME13 were exceptionally held in somewhat modified modalities and dates.

CERME13 took place in Budapest, the beautiful capital of Hungary. The many ways the event was memorable include, besides the reunion of our community after the pandemic, also the first and maybe only CERME ever held in the summer. It cannot be denied that central Europe is much warmer in this season than in the winter, and the hospitality of our Hungarian colleagues was equally warm and efficient. To the local organising committee, led by Csaba Csapodi, Katalin Gosztonyi and Ödön Vancsó, and to the institutions and staff who supported the committee, we extend our warmest thanks on behalf of all participants, for preparing and realising the complex organisation of the congress in a way that fully matched the quality of its former editions. The Lágymányosi campus, with the Faculty of Science buildings of the Eötvös Loránd University, provided the extensive facilities required for both plenary events and the sessions of the thematic working groups. The traditional gala dinner was held in the impressive halls of the Museum of Fine Arts, and added another memorable and very distinguished dimension to the social aspects of the congress.

The scientific programme is evidently the *raison d'être* of any CERME, and the contributions of the participants – papers and posters – form the core material. Unlike many other research conferences, CERME is not focused on presentations, but on in-depth discussions of the contributions. This, indeed, is the secret behind the constantly increasing attraction and success of CERME. Since the very first CERME, the purpose of promoting *communication, cooperation and collaboration* among mathematics education researchers has been implemented by organising the work of the congress in *thematic working groups*. In these groups, the main activity is in-depth discussion of the submitted contributions, with only very brief “reminder presentations” of the texts, which participants in each group are expected to read before attending the congress. At CERME13, no less than 28 such groups

were organised – both new groups, reflecting new specialized areas of research, and groups which have already convened at several previous editions. To organise these groups, teams of coleaders are invited by the international programme committee. Both to every member of these coleader teams, and to the programme committee – led this year by Paul Drijvers and Hanna Palmér – the ERME community is deeply indebted and thankful. We also thank our plenary speakers, László Lovász and Berta Barquero, as well as the plenary panel led by João-Pedro da Ponte, for providing deep and state-of-the-art accounts of research themes that can inspire the community as a whole.

CERME13 received, as almost all previous CERMEs, an unprecedented high number of participants (941), from more nations than ever before (54), see Table 1.

Table 1: The success of CERME13 in numbers: 941 participants from 54 countries

Armenia	1	Germany	165	Romania	1
Australia	7	Greece	20	Serbia	3
Austria	27	Hong Kong	4	Slovakia	14
Belgium	5	Hungary	49	South Africa	2
Brazil	9	Iceland	3	Spain	51
Canada	19	Ireland	13	Sweden	46
Chile	11	Israel	27	Switzerland	14
China	7	Italy	66	Taiwan	2
Costa Rica	1	Japan	9	Thailand	1
Croatia	5	Lithuania	2	Tunisia	3
Cyprus	2	Malta	1	Turkey	46
Czech Republic	17	Mexico	17	Ukraine	1
Denmark	19	Netherlands	32	UAE	1
Egypt	1	New Zealand	5	UK	43
Estonia	4	Norway	66	USA	35
Faroe Islands	1	Philippines	1	Uruguay	1
Finland	6	Poland	1	Vietnam	1
France	24	Portugal	28		

In fact, the number of contributions to CERME congresses has increased almost linearly throughout the history of CERME, to reach a whopping 834 at CERME13. Of these, 757 were finally accepted and presented. Given the capacity of academic venues – roughly 900-1000 participants – and more subtly, of the organisation of a CERME, which depends entirely on voluntary committees and coleader teams – it has become ever clearer that this linear growth cannot continue, even if only one participant for each contribution were to participate. At CERME13, it was only to a small degree possible for co-authors to register, even for experienced scholars who attended all or most of the previous CERME congresses. These dilemmas were discussed at CERME13, and will continue to be worked on in the committees of ERME. Both CERME, and the other venues and supports that ERME offers to scientific exchange and creation by researchers in Europe and beyond, need to be continuously maintained and developed. The enthusiasm and quality of CERME13 promises a bright future for our research field and for the ERME community. See you at CERME14 in Bozen-Bolzano!

Introduction to the proceedings of the thirteenth Congress of the European Society for Research in Mathematics Education (CERME13)

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About CERME13

The thirteenth Congress of European Research in Mathematics Education (CERME13) took place in Budapest, Hungary, from 10 to 14 July 2023. It was organised by the Alfréd Rényi Institute of Mathematics, hosted by the Eötvös Loránd University, and attracted 941 participants. Paul Drijvers and Hanna Palmér were chair and co-chair of the International Programme Committee, which included Federica Ferretti, Eirini Geraniou, Katalin Gosztonyi, Thomas Hausberger (from August 2022), Jeremy Hodgen, Sibel Kazak, Esther Levenson, Bozena Maj-Tatsis, Simon Modeste (until August 2022), Juuso Henrik Nieminen, and Florian Schacht. Csaba Csapodi chaired the Local Organizing Committee, which was co-chaired by Katalin Gosztonyi and Ödön Vancsó.

CERME13 hosted 27 Thematic Working Groups, listed in Table 1. New TWGs were TWG28 on Collaborative Settings in Mathematics Teacher Education and TWG29 on Embodied and Material Studies of Mathematical Behaviour. TWG7 on Mathematics for work, society and personal development: lifelong learning was suppressed this edition, but will reappear at CERME14. TWG11 on algorithms merged with TWG16 on Learning mathematics with technology and other resources. In total, CERME13 had 143 TWG (co-)leaders, who did an extremely valuable job which is key to CERME's success. The conference was preceded by the YERME-day for young researchers in ERME, that attracted 150 participants. Altogether, CERME13 was a success both in academic terms and in social atmosphere, which the European mathematics education research community enjoyed.

Table 1: Overview of the TWG leadership teams

Thematic Working Group	Leader	Co-leaders
TWG1: Argumentation and Proof	Andreas Moutsios-Rentzos	Orly Buchbinder, Nadia Azrou, Fiene Bredow, Dimitrios Deslis, Viviane Durand-Guerrier, David A. Reid, Mei Yang
TWG2: Arithmetic and Number Systems	Pernille Bødtker Sunde	Elisabeth Rathgeb-Schnierer, Renata Carvalho, Ioannis Papadopoulos
TWG3: Algebraic Thinking	Maria Chimoni	Cecilia Kilhamn, Jorunn Reinhardtson, Luis Radford

Proceedings of CERME13

TWG4: Geometry Teaching and Learning	Alik Palatnik	Lina Brunheira, Taro Fujita, Chrysi Papadaki, Petra Surynková
TWG5: Probability and Statistics Education	Caterina Primi	Daniel Frischemeier, Orlando Gonzalez, Sibel Kazak, Aisling Leavy, Martin Andre
TWG6: Applications and Modelling	Jonas Bergman Ärlebäck	Susana Carreira, Britta Eyrih Jessen, Gilbert Greefrath, Yana Lacek, Katrin Vorhölter
TWG8: Affect and the Teaching and Learning of Mathematics	Çiğdem Haser	Inés M Gómez-Chacón, Chiara Andrà, Janina Krawitz, Hanna Viitala
TWG9: Mathematics and Language	Jenni Ingram	Kirstin Erath, Alexander Schüler-Meyer, Máire Ní Ríordáin, Ingólfur Gíslason
TWG10: Diversity and Mathematics Education: Social, Cultural and Political Challenges	Laura Black	Anette Bagger, Sabrina Salazar, Timo Dexel, Juuso Nieminenn
TWG12: History in Mathematics Education	Antonio M. Oller-Marcén	Jenneke Krüger, Tanja Hamann, Bjørn Smestad, Olivera Đokić
TWG13: Early Years Mathematics	Bozena Maj-Tatsis	Camilla Björklund, Dorota Lembrér, Esther Levenson, Andrea Maffia, Marianna Tzekaki
TWG14: University Mathematics Education	Irene Biza	Olov Viirman, Matija Bašić, Ignasi Florensa, Ghislaine Gueudet, Mathilde Hitier, Igor' Kontorovich, Athina Thoma, Megan Wawro
TWG15: Teaching Mathematics with Technology and Other Resources	Ornella Robutti	Bärbel Barzel, Melih Turgut, Gülay Bozkurt, Daniel Thurm
TWG16: Learning Mathematics with Technology and Other Resources	Osama Swidan	Rogier Bos, Eleonora Faggiano, Seçil Yemen Karpuzcu, Simon Modeste, Florian Schacht, Jana Trgalová
TWG17: Theoretical Perspectives and Approaches in Mathematics Education Research	Heather Johnson	Mariam Haspekian, Abdel Seidouvy, Felix Lensing, Cecilie Carlsen Bach
TWG18: Mathematics Teacher Education and Professional Development	Marita Friesen	Tracy Helliwell, Raffaele Casi, Andreas Ebbelind, Janne Fauskanger, Pere Ivars, Macarena Larrain, Libuše Samková
TWG19: Mathematics Teaching and Teacher Practice(s)	Reidar Mosvold	Mark Hoover, Helena Grundén, Siún Nic Mhuirí, Chrysoula Choutou
TWG20: Mathematics Teacher Knowledge, Beliefs and Identity	Francesca Martignone	Miguel Montes, Federica Ferretti, Veronika Hubeňáková, Jimmy Karlsson, Nadia Kennedy, Miguel Ribeiro
TWG21: Assessment in Mathematics Education	Francesca Morselli	Christina Drüke-Noe, Gözde Kaplan-Can, Chiara Giberti, Johanna Rämö
TWG22: Curricular Resources and Task Design in Mathematics Education	Shai Olsher	Annalisa Cusi, Dubravka Glasnović Gracin, Hendrik Van Steenbrugge
TWG23: Implementation of Research Findings in Mathematics Education	Mario Sánchez Aguilar	Boris Koichu, Morten Misfeldt, Linda Marie Ahl

TWG24: Representations in Mathematics Teaching and Learning	Carla Finesilver	Anna Baccaglini-Frank, Elisa Miragliotta, Kate C. O'Brien
TWG25: Inclusive Mathematics Education – Challenges for Students with Special Needs	Petra Scherer	Michael Gaidoschik, Hana Moraová, Helena Roos, Andreas Ulovec
TWG26: Mathematics in the Context of STEM Education	Behiye Ubuz	Michelle Stephan, Clelia Cascella, Nelleke Den Braber
TWG27: The Professional Practices, Preparation and Support of Mathematics Teacher Educators	Ronnie Karsenty	Hilda Borko, Alf Coles, Birte Friedrich-Pöhler, Bettina Rösken-Winter, Stefan Zehetmeier
TWG28: Collaborative Settings in Mathematics Teacher Education	Stéphane Clivaz	Geoff Wake, Paula Gomes, Pernilla Mårtensson, Aoibhinn Ní Shúilleabháin
TWG29: Embodied and material studies of mathematical behaviour	Anna Shvarts	Elizabeth de Freitas, Ricardo Nemirovsky, Christina Krause, Giulia Ferrari

Editorial information

These proceedings are available as a complete volume on the ERME website. Each individual contribution is also available through the HAL open archive, where it can be found through keywords, title, or author name. This has been the practice since CERME9, to increase the visibility of the huge work done in CERME conferences.

After the preface by the president of ERME, Carl Winsløw, and this brief introduction by the proceedings editors, this volume continues with texts on two plenary activities of CERME13: the plenary lecture by Berta Barquero entitled “Mathematical modelling as a research field: Transposition challenges and future directions,” and the report on the Panel on “Bridging the research-practice gap,” chaired by João Pedro da Ponte, with contributions by Mario Sánchez Aguilar, Nad’a Vondrová, Stefan Zehetmeier, Sarah Seleznyov, and Jorryt van Bommel.

The biggest part of these proceedings consists of the sections corresponding to the TWGs. These sections start with an introduction by the TWG leadership team. According to the team’s choice, this is a 4-page introduction presenting the contributions, or an extended 8-page one, which proposes an additional analysis of the current research on the theme of the TWG, and perspectives for the future. Next, the section consists of the paper and poster contributions – in alphabetical order by first author’s name.

These CERME13 proceedings result from a collaboration, involving the CERME13 IPC and LOC, the TWG leaders and co-leaders. Particular thanks are due to Nathalie Kuijpers for her editorial work. We warmly thank all these people for their involvement, and hope that this volume will contribute to the development of mathematics education research in Europe and beyond.

Higher order thinking skills through automatic formative assessment: Generating examples in ordinary differential equations to actively engage students in developing conceptual understanding

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The application of digital technologies empowers students to use their creativity and develop their critical thinking skills. This is why at the University of Turin, within the first-year module “Mathematics and biostatistics with computer science applications” in the study program in Biotechnologies students are engaged in multiple ways with hybrid ways of delivery mathematical contents, making use of digital interactive resources and automatic assessment empowered with formative practices. In particular, in this paper we are going to analyse students' group and individual activities in which they were asked to provide specific examples of Mathematical objects under specific restrictions, and to reflect on the tasks they performed. We analysed the way students think while performing example eliciting tasks, with a focus on Ordinary Differential Equations.

Keywords: Automatic formative assessment, example eliciting tasks, higher education, higher order thinking, ordinary differential equation.

Introduction

Mathematics education is an important component of the background for many sciences. As some authors highlighted, research in STEM education is increasing in importance internationally and the identity of STEM education journals is becoming clearer over time (Li et al., 2020). The use of automatic assessment combined with formative practices and boosted by interactive components that allows students exploration of concepts can have positive effects on the achievement of learning objectives. Moreover, as a general trend in Mathematics education, there is a growing number of universities and academies that use automatic assessment systems to evaluate their students. The combination of automatic assessment with formative practices helps the students in developing mathematical construction and conceptual understanding.

Li and colleagues (2020) found that research collaboration (publications with multiple authors) has been very common in STEM education research. Indeed, this research was developed thanks to an international collaboration between two universities, the University of Turin and Karlstad University, Sweden (Barana, et al., in press). In this joint project, similar tasks were proposed to Italian and Swedish students in various topics in mathematics, even though the ones described in this paper were only proposed to Italian students.

This paper reports the result of a study performed with first-year students in a Mathematics module of the bachelor's degree in biotechnology during the first semester of the academic year 2022/2023. The context of the study is represented by group activities which students could freely decide to join: we had 121 subscribers. Here we analyse the answers provided by 88 students to the group activity dedicated to Ordinary Differential Equations (ODEs), a topic that made students struggle a lot in previous years of the university module. Recently, ODEs have also been included to the topics that,

according to the Italian National Guidelines, students should study at the end of Scientific Lyceum, so we expect that it is not a new topic for many of the students. In the following sections of the paper, we will explore the Theoretical background in performing the kind of Example Eliciting Tasks (EETs) that we require of students, together with formative practices, since those tasks were performed by students with their peers and with the purpose of consolidating mathematical knowledge. Next, we will provide the research methodology and discuss the results and the comments of students quantitatively and qualitatively. Conclusions and discussion will provide the main findings.

Theoretical background

In literature there are numerous examples of collaborative learning environments designed to let students perform explorative tasks. There are examples on the topic of functions (Brunström & Fahlgren, 2015; Jaworski & Matthews, 2011). This may be boosted by the mediation of technologies, to redesign tasks that are very difficult to assess with other means. In fact, today many instructors choose to utilise Automatic Assessment Systems (AAS) in mathematics modules in higher education (Rønning, 2017). Students' performances can be improved by reducing the cognitive load. Students were helped by the possibility of using dynamic mathematics software (DMS) environments (Sangwin, 2013), combining a dynamical exploration and automatic assessment to promote mathematical understanding (Joubert, 2017). An example of this kind of system is given by Möbius Assessment. There are studies that started investigating the integration of these two types of technology (Luz & Yerushalmy, 2019), but anyway further research is needed to better understand students' learning behaviour. This can be further enhanced by implementing also formative assessment (FA) (Black & Wiliam, 2009). Black and Wiliam definition entails the collection of data about students' understanding, and the use of such data to change the learning path. In previous works, we adopted automatic formative assessment (AFA) in a Digital Learning Environment, thus with the automatic elaboration of responses and provision of feedback (Barana et al., 2021). We have developed and experimented a model for the design of activities with AFA (Barana & Marchisio, 2019), based on Möbius Assessment, whose computational engine is empowered by advanced mathematical capabilities. By making use of programming languages or mathematical packages, it is possible to build interactive tasks based on algorithms where answers, feedback and values are calculated over random parameters and can be shown with different mathematical and graphical representations. Thus, new solutions for computer-based items can be conceived, including dynamic explorations, animations, and symbolic manipulations, which offer students experiences of mathematical construction and conceptual understanding (Sangwin, 2015). According to our model, AFA activities should:

- A. be always available in a Digital Learning Environment, without limitations in data, time and number of attempts;
- B. be algorithm-based, so that random values, parameters, formulas and graphs make questions, and their answers change at every attempt;
- C. be open-ended or almost open-ended, the AAS's mathematical engine assures that open mathematical answers are graded independently of the form in which they are provided;
- D. provide students with immediate feedback while they are focused on the task;

- E. provide students with interactive feedback just after giving an incorrect answer. It has the form of a step-by-step guided resolution that interactively shows a possible solving process;
- F. be contextualised in real-life, thus contributing to the creation of meanings through the association of abstract concepts to concrete experience.

Example eliciting tasks

There is a growing literature on Example Eliciting Tasks (EETs). As a teaching and learning tool in mathematics, EETs are a type of problem-solving activity where students have to provide examples of mathematical entities under certain conditions, rather than simply calculating solutions with standard procedures. This is a way to engage students actively in their mathematical understanding, using also different registers and representation of mathematical entities (Watson & Mason, 2002). This idea has been integrated into automatic assessment (Sangwin, 2003). Studies have shown that EETs can have a positive impact on students' understanding of mathematical concepts (Brunström et al., 2022). For example, research has found that students who participated in EETs had a deeper understanding of mathematical principles and a stronger ability to apply these principles in real-life situations. Additionally, EETs have been shown to improve students' problem-solving skills and ability to think critically. To further challenge students' thinking, some authors suggested asking students for several different examples to extend students' example space (Watson & Mason, 2002) and to support problem solving and to mirror the reasoning processes (Yerushalmy et al., 2017). EETs have also been shown to be effective for students of all abilities, including those who struggle with mathematics, since they allow students to work at their own pace and focus on their strengths and weaknesses. In didactical practices, EETs can be used as a teaching tool alone or as part of a larger lesson plan or activities. They can be used in the classroom, as well as in online or remote learning environments. Additionally, EETs can be used with a wide range of mathematical concepts, from basic arithmetic to advanced calculus. This approach is used to help students understand concepts by actively engaging with the material and making connections to real-life situations.

Mathematics and biostatistics with computer science applications

The context of our research lies in a first-year module of a study program in biotechnology called “Matematica e Biostatistica con Applicazioni Informatiche” (Mathematics and Biostatistics with Computer Science Applications). The module aims both at providing competences in basic mathematics that is useful for a future scientist and at improving several soft and transversal skills, such as digital competences and problem-solving skills. Students also participated to lab sessions in a room equipped with their own PCs (Bring Your Own Device mode), to work with an Advanced Computing Environment (ACE) to explore mathematical concepts and learn how to deal with high order mathematics through the use of technologies, and to solve problems contextualised in the applied sciences constituting the core of their curriculum of studies, such as Life Sciences and Chemistry, as well as Biotechnologies itself. We created an online course available from everywhere and at all times with a dedicated section for most of the course topics, in which ACE worksheets with interactive components and AAS tests flank more traditional resources such as static files. According to our model, tests are designed to allow multiple attempts, and random parameters make questions change at each attempt. This allows students to find repeatable exercises and activities inside our

course, removing at least from a strict point of view the need for looking outside in order to train themselves better. Furthermore, we provided questions with an immediate and interactive feedback, by subdividing them in several parts, with the path proposed to the student dependent on his or her performance. Likewise, we also designed exercises that bore a similar construction with respect to those which constitute the exam, in order to let students become confident of the ways they would have been assessed and graded. The tasks that we are going to consider for this research possess most of the features that have been highlighted in the Theoretical background, with two main differences:

- The tasks are not based on real life applications of Mathematics since they aimed at empowering students' theoretical understanding and ability to generate examples.
- The feedback could be viewed by students only after performing the full test (which was composed by four questions) for the first time during the group activity. This choice has been made since students were performing the tasks in groups, and we tried to avoid one student suggesting the others the correct solutions while performing the tasks. Many activities with feedback in the course make the students read it directly after the task. Students could perform the test more than one time to recap the topics.

Research question and methods

This research has been mainly motivated by the need to find an answer to the following research question: *How students generate examples about ODEs when they have some degree of freedom in their construction?* To answer to this question, we will examine student responses to two tasks concerning ODEs. The collected data are student responses to the tasks typed inside Möbius Assessment AAS. The 88 students worked in groups, autonomously and independently from the lessons, and questions contained group tasks and individual tasks. We considered one student from each of the 35 groups, in order to highlight differences between the various teams. We considered the first attempt of those students. We analysed the two following group tasks:

Task 1

This question asks you to provide an example of a function that satisfies a differential equation.

a) *Write two different functions that are solutions of the following differential equation:*

$$\frac{d}{dx} y(x) + y(x)^2 = 1$$

b) *Describe how you found the expressions of the functions.*

For this task, the tricks that students had to pay attention were:

- There was only one possible way to solve this problem through usually-taught resolution methods, separating the variables.
- The application of the usually-taught resolution methods was more difficult than the standard ones, the inversion of the integrated hand sides to find a suitable function for x requires to be skilled with these aspects of Calculus.
- The main aim of the task was finding examples, so students are encouraged to find the shortest way to the solution, in this case the easiest ones were the constant functions -1 and 1.

In light of this, we classified the groups in some categories, according to the kinds of functions they provided as examples, and if they were actually solutions of the ODE or not. This quantitative aspect is then related with the qualitative description of how they proceeded in order to give the answers to the task, by focusing for example on why they followed a certain line of reasoning.

Task 2

This question is individually answered but can be investigated in a group. The question asks you to provide two examples of differential equations that have the function $f(x)$ as a solution.

a) *Write two different differential equations that have the following function as their solution:*

$$f(x) = a_1 \cdot \exp(a_2 \cdot x) + b_1 \cdot \exp(b_2 \cdot x) + c_1 \cdot x + c_2$$

(where the parameters are random integers between -5 and 5, with a_2 , b_2 , c_2 nonzero)

b) *Describe how you came up with the function expressions.*

In this task, the challenge for students was to recognize that the usual ways of obtaining a solution of linear ordinary differential equations (no matter the order of the ODE), and looking for proper coefficients of the homogeneous solution and the particular solution, although resolutive, were not necessarily the best methods to find the examples with the most possible simplicity. Indeed, it would have been quite simpler to differentiate n times the function $f(x)$ and equating it to the n -th derivative of y , for every n (in order to answering the task, $n=1$ and $n=2$ are sufficient). Moreover, this task swaps the role of “solving an equation” into “finding an equation”.

Again, we devised a proper classification for the groups, by considering on the one side who actually answered $y' = f'(x)$ or $y'' = f''(x)$ (or both of them), and on the other side who on the contrary constructed more complicated ODEs (either of first or second order), by still paying attention also to the correctness of the answers. We put here too in relation the quantitative submissions with the qualitative explanations, allowing us to understand what they thought to do.

Results and discussion

Let us start with the analysis of Task 1. Among the 35 groups of students, we were able to collect 29 groups' answers, since six of them did not respond. Our classification reads as follows:

- 28% of groups of students answered with two constant functions: they all submitted -1 and 1, thus correctly answering to the task;
- 14% of groups answered with a constant and a non-constant function: all of them but one by correctly providing both examples, while a group alone made an error on the non-constant;
- 55% of groups answered with two non-constant functions, but only just over half of them provided correct answers, with the other ones answering two erroneous functions;
- one group answered with a single non-constant erroneous functions, by leaving the second field blank.

By relating these findings with the qualitative description, we were able to observe that:

- Students calculated the full solution just to input constant functions: for instance, one of the groups answering with a constant and a non-constant function, declared to having substituted

two values for C in the general integral. They probably substituted the easiest values for C in the general integral, but they forgot to look for the easiest way to write the answer. In fact, it is worth to note that, while teaching how to solve an ODE by separating the variables, the fact that constant functions are to be found (if present) otherwise explicitly emerges; nonetheless, most students appeared to skip that part. Relating this evidence to the research question, we can state that students tend to follow rigid patterns, also when they are not asked to do so, by having the possibility of being more flexible. A possible motivation about the skipping of finding constants can lie in the fact that these students could be used to follow rigid algebraic processes without exploring the problem from an analytic perspective or looking for simpler solutions. This could be due to the way they learned Mathematics at secondary level. Since the steps required to obtain the solutions to an ODE are more algebraic than analytic, students likely tend to see the constant solutions as less related with the differential problem, even if they actually provide other solutions to the ODE.

- two groups said to have looked firstly for solving the ODE by separating the variables, and only secondly for reasoning and finding the constant functions, when they found the standard resolution method too difficult. In a certain sense, answering with the constants has been a makeshift solution for them;
- generally speaking, most groups separated the variables, but this resulted in wrong answers in almost half of the cases, and furthermore they underwent a needlessly more challenging procedure, with respect to what really necessary in order to answer correctly.

By considering now the analysis of Task 2, we collected 27 groups' answers. The main findings were:

- only a group answered in the simpler way: $y' = f'(x)$ and $y'' = f''(x)$;
- 26% of groups gave as one of the answers $y' = f'(x)$, but apart from the just mentioned group, only one other group correctly gave the other answer;
- 74% of groups gave as at least one answer an ODE having order 1 or 2 more articulated than $y' = f'(x)$ and $y'' = f''(x)$, which resulted to be wrong in almost half of the cases: sometimes due to minor errors (like sign), but other times completely, even with some situations where in the description they explained the procedure with the right steps;
- less frequent errors concerned for example integrating $f(x)$ rather than differentiating it, or writing expressions in x without equating them to an expression in y and its derivatives.

Again, the majority of groups followed a procedure which was not the simplest one and often brought errors, essentially confirming the answer to the research question inferred by Task 1: students tend to generate examples by “answering using rigid procedures”, even if they are not required to do so.

It has to be noted also how 19% of groups provided only an example rather than two, a percentage considerably higher than in Task 1 (where only one group did so). The fact that this problem is the inverse of what usually asked, i.e. equations have to be *found* rather than *solved*, could have constituted a difficulty also in framing it.

Future work

Our next steps will consider that, apart from the descriptive answers in the two tasks, we asked students also to keep track of their actions by redacting some logs, in which they were questioned to

report what they learned through lab activities that they were unable to learn during the lectures; to explain how working in group supported or not their comprehension of mathematical concepts relative to the task subjects, by giving also an example; and to state what they perceived as simpler and what was more difficult, explaining also why.

By analysing the answers, we will have a deeper insight of the students' behaviour when facing tasks which require conceptual understanding and high-order thinking. It would be interesting to compare the findings with other countries (such as Sweden) to investigate if these results are the product of a national school system or if they are more generalizable. Moreover, we will be able to detect to what extent these activities help students' learning with respect to a more traditional course not considering anything of comparable, how teamwork plays a role in better understanding what they work on, and to target difficulties. This would suggest how to reinforce with proper teachers' actions where students have to put more effort to achieve their goals, such as in commitments comparable to Tasks 1 and 2, where most of them did not answer optimally. This would allow us to devise subsequent actions aimed at improving the benefits arising from this approach, which can be in turn studied in order to determine how they constitute progress, and if other further actions are still appropriate. In future work we are going to provide further feedback to the students to extend their example space and make them reflect on the example-generation process.

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