SIS | 2022

51st Scientific Meeting
of the Italian Statistical Society

Caserta, 22-24 June

Università degli Studi della Campania *Luigi Vanvitelli*

SIS Società Italiana di Statistica

www.unicampania.it

Book of the Short Papers

**Editors: Antonio Balzanella, Matilde Bini,
Carlo Cavicchia, Rosanna Verde**

Città di Caserta

800 ANNI 1222-2022 Università degli Studi di Padova

Dipartimento di Scienze Statistiche

Stata

sas

Università degli Studi del Sannio Benevento

Pearson

Matilde Bini (Chair of the Program Committee) - *Università Europea di Roma*
Rosanna Verde (Chair of the Local Organizing Committee) - *Università della Campania "Luigi Vanvitelli"*

PROGRAM COMMITTEE

Matilde Bini (Chair), Giovanna Boccuzzo, Antonio Canale, Maurizio Carpita, Carlo Cavicchia, Claudio Conversano, Fabio Crescenzi, Domenico De Stefano, Lara Fontanella, Ornella Giambalvo, Gabriella Grassia - Università degli Studi di Napoli Federico II, Tiziana Laureti, Caterina Liberati, Lucio Masserini, Cira Perna, Pier Francesco Perri, Elena Pirani, Gennaro Punzo, Emanuele Raffinetti, Matteo Ruggiero, Salvatore Strozza, Rosanna Verde, Donatella Vicari.

LOCAL ORGANIZING COMMITTEE

Rosanna Verde (Chair), Antonio Balzanella, Ida Camminatiello, Lelio Campanile, Stefania Capecchi, Andrea Diana, Michele Gallo, Giuseppe Giordano, Ferdinando Grillo, Mauro Iacono, Antonio Irpino, Rosaria Lombardo, Michele Mastroianni, Fabrizio Maturo, Fiammetta Marulli, Paolo Mazzocchi, Marco Menale, Giuseppe Pandolfi, Antonella Rocca, Elvira Romano, Biagio Simonetti.

ORGANIZERS OF SPECIALIZED, SOLICITED, AND GUEST SESSIONS

Arianna Agosto, Raffaele Argiento, Massimo Aria, Rossella Berni, Rosalia Castellano, Marta Catalano, Paola Cerchiello, Francesco Maria Chelli, Enrico Ciavolino, Pier Luigi Conti, Lisa Crosato, Marusca De Castris, Giovanni De Luca, Enrico Di Bella, Daniele Durante, Maria Rosaria Ferrante, Francesca Fortuna, Giuseppe Gabrielli, Stefania Galimberti, Francesca Giambona, Francesca Greselin, Elena Grimaccia, Raffaele Guetto, Rosalba Ignaccolo, Giovanna Jona Lasinio, Eugenio Lippiello, Rosaria Lombardo, Marica Manisera, Daniela Marella, Michelangelo Misuraca, Alessia Naccarato, Alessio Pollice, Giancarlo Ragozini, Giuseppe Luca Romagnoli, Alessandra Righi, Cecilia Tomassini, Arjuna Tuzzi, Simone Vantini, Agnese Vitali, Giorgia Zaccaria.

ADDITIONAL COLLABORATORS TO THE REVIEWING ACTIVITIES

Ilaria Lucrezia Amerise, Ilaria Benedetti, Andrea Bucci, Annalisa Busetta, Francesca Condino, Anthony Cossari, Paolo Carmelo Cozzucoli, Simone Di Zio, Paolo Giudici, Antonio Irpino, Fabrizio Maturo, Elvira Romano, Annalina Sarra, Alessandro Spelta, Manuela Stranges, Pasquale Valentini, Giorgia Zaccaria.

PUBLISHED BY PEARSON

WWW.PEARSON.COM

ISBN 9788891932310

Risk Parity strategy for portfolio construction: a kurtosis-based approach

Risk Parity strategy per la costruzione di un portafoglio: un approccio basato sulla curtosi

Maria Debora Braga, Consuelo Rubina Nava and Maria Grazia Zoia

Abstract Investors interested in the homogeneous distribution of responsibility for the dispersion of portfolio returns can pursue not only the equal constituents' contribution to portfolio volatility as, alternatively, they can focus on portfolio's extreme outcomes (either positive or negative). This leads to the proposal of a kurtosis-based risk parity strategy (KRP) that is a new risk parity strategy based on portfolio kurtosis as reference measure. In this paper, the portfolio weights, obtained with the KRP strategy. are compared with those estimated following the traditional standard deviation-based risk parity strategy. An analysis of KRP strategy performance is carried out through an out-of-sample study which takes advantage of real data from a global equity investment universe.

Abstract *Gli investitori, interessati a una distribuzione omogenea del rischio tra i vari asset di portafoglio, possono concentrarsi non solo sull'equa contribuzione dei costituenti alla volatilità di portafoglio poiché, in alternativa, essi si possono focalizzare sui risultati estremi di portafoglio (sia positivi che negativi) prendendo in considerazione la curtosi di portafoglio. In questo contesto, si sviluppa una strategia di parità di rischio basata sulla curtosi (strategia KRP) di portafoglio quale misura di rischio anziché sulla sua volatilità. I pesi per la costruzione del portafoglio ottenuti con la strategia KRP sono messi a confrontati con quelli ottenuti con la tradizionale strategia di parità di rischio basata sulla volatilità. Le performance della strategia KRP sono studiate con un approccio out-of-sample basato su dati reali di un universo di investimento azionario globale.*

Maria Debora Braga
SDA Bocconi School of Management, Via Roberto Sarfatti 10, Milano, e-mail: mariadebora.braga@sdabocconi.it

Consuelo Rubina Nava
Università degli Studi di Torino, Lungo Dora Siena 100, Torino, e-mail: consuelorubina.nava@unito.it

Maria Grazia Zoia
Università Cattolica del Sacro Cuore di Milano, Largo Gemelli 1, Milano e-mail: maria.zoia@unicatt.it

Key words: Kurtosis, Risk parity, Risk diversification, Asset allocation

1 Introduction

Since 2008, in finance, a new strategy for portfolio construction, known as risk parity (RP), has appeared [4]. Its main feature is the wealth allocation (accordingly to a suitable defined risk-measure) among asset classes in such a way that each asset contributes in the same manner to the selected portfolio risk-measure. Thus, risk contribution forms the basis for the development of the RP strategy [3]. So far, portfolio volatility has been the most used reference risk-measure.

Here we develop the new version of the RP strategy where portfolio volatility is replaced by the portfolio kurtosis as the reference risk-measure. The existing literature has already dealt with other risk measures but never with the kurtosis. This is also due to the absence of a close form expression of the portfolio kurtosis. This is also due to difficulty of deriving closed form expressions for the marginal risk contribution of each asset and, consequently, the set-up of the optimization problem, necessary to identify optimal portfolio weights.

In this paper, a risk parity strategy based on portfolio kurtosis as reference measure (KRP – kurtosis-based risk parity) is investigated, applied, and its performance is compared with the traditional standard deviation-based risk parity (SRP). In such a way a novel strategy able to better accommodate the needs of investors interested in the homogeneous distribution of responsibility for portfolio returns' huge dispersion, as that measured by portfolio kurtosis, is developed. A novel and effective expression for portfolio kurtosis is proposed, which integrates the results proposed in [1]. The proposed methodology is applied on real data and compared with the classical RP allocation strategy based on volatility in order to explore its peculiar features.

The paper is organized as follow. Section 2 proposes an original formula for portfolio kurtosis. Section 3 describes the dataset used for the empirical application and provides main results. Section 4 concludes the paper.

2 Methodology

Let consider a portfolio composed by N assets with returns $\boldsymbol{R} = [R_i]$ and associated weights $\boldsymbol{w} = [w_i]$ with $i = 1, \dots, N$. The weights w_i are the percentage of asset class i in the portfolio and satisfy the following normalizing constraint $\sum_{i=1}^{N} w_i = 1$. Let $\boldsymbol{\mu}$ and $\boldsymbol{\sigma}$ be the $N \times 1$ vectors of the expected returns and standard deviations for the selected asset classes in the investment universe: μ_i is the expected return of asset class i while σ_i is its risk. Furthermore, let $\boldsymbol{C} = [\rho_{ij}]$ and $\boldsymbol{\Sigma} = [\sigma_{ij}]$ be the $N \times N$ correlation matrix and covariance matrix. Accordingly, the expected return of the portfolio P and its variance are:

$$\mu_P = \boldsymbol{w}'\boldsymbol{\mu}, \ \sigma_P^2 = \boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}.$$

Let's now recall for the purpose of this contribution some results concerning the kurtosis of two generic random variables R_i and R_j

$$K_i = K(R_i) = \mathbb{E}\left[\left(\frac{R_i - \mu_i}{\sigma_i}\right)^4\right] = \frac{\mathbb{E}[(R_i - \mu_i)]^4}{[\mathbb{E}[(R_i - \mu_i)]^2]^2} = \frac{\mu_{i,4}}{\sigma_i^4}$$

with $\frac{R_i - \mu_i}{\sigma_i}$ representing the standardized returns. Thus,

$$K(w_i R_i) = \mathbb{E}\left[\left(\frac{w_i R_i - w_i \mu_i}{w_i \sigma_i}\right)^4\right] = \frac{\mu_{i,4}}{\sigma_i^4} = K(R_i)$$

$$K(R_i + R_j) = \frac{\sum_{k_1+k_2=4} \frac{4!}{k_1! k_2!} \sigma_1^{k_1} \sigma_2^{k_2} \, coK(R_\cdot, R_\cdot, R_\cdot, R_\cdot)}{(\sigma_i^2 + \sigma_j^2 + 2cov(R_i, R_j))^2}$$

where the cokurtosis (coK) is defined as follows

$$coK(R_i, R_i, R_i, R_j) = \frac{\mathbb{E}[(R_i - \mu_i)^3 (R_j - \mu_j)]}{\sigma_i^3 \sigma_j};$$
$$coK(R_i, R_i, R_j, R_j) = \frac{\mathbb{E}[(R_i - \mu_i)^2 (R_j - \mu_j)^2]}{\sigma_i^2 \sigma_j^2}.$$

The cokurtosis is invariant over linear transformation, i.e. $coK(w_i R_i, w_i R_i, w_j R_j, w_j R_j) = coK(R_i, R_i, R_j, R_j)$. Moreover, the particular case of $coK(R_i, R_i, R_i, R_i)$ reduces to the kurtosis of R_i, i.e. $coK(R_i, R_i, R_i, R_i) = K(R_i)$, still invariant over linear transformation. However, it is worth noting that $K(w_i R_i + w_j R_j) \neq K(R_i + R_j)$.

The previous results can be extended to obtain a portfolio kurtosis including N asset, which can be expressed as follows

$$K_P = K\left(\sum_{i=1}^N w_i R_i\right) = \frac{\mathbb{E}\left[\sum_{i=1}^N w_i R_i - \sum_{i=1}^N w_i \mu_i\right]^4}{\left(\mathbb{E}\left[\sum_{i=1}^N w_i R_i - \sum_{i=1}^N w_i \mu_i\right]^2\right)^2}$$
$$= \frac{\sum_{\sum_{i=1}^N k_i = 4} \frac{4!}{k_1! \cdots k_N!} coK(R_\cdot, R_\cdot, R_\cdot, R_\cdot) \prod_{i=1}^N w_i^{k_i} \sigma_i^{k_i}}{\left(\sum_{\sum_{i=1}^N k_i = 2} \frac{2!}{k_1! \cdots k_N!} cov(R_\cdot, R_\cdot) \prod_{i=1}^N w_i^{k_i}\right)^2} \quad (1)$$

where $k_i = 0, 1, 2, 3, 4 \ \forall i = 1, \ldots, N$, $cov(R_\cdot, R_\cdot)$ is the covariance between two assets while $coK(R_\cdot, R_\cdot, R_\cdot, R_\cdot)$ is the cokurtosis.

Thus, based on this portfolio kurtosis formula, which can be also written in a closed form for using matrix notation [1], the portfolio kurtosis gradient, needed for the determination of the marginal risk contributions of the asset classes to portfolio kurtosis, can be computed.

In this connection, it is worth noting that the RP approach is based on the idea that portfolio risk must be equally distributed among asset classes, a goal that can be pursued for any risk measure RM, provided it is homogeneous of degree one in the weights. This is also the case of the portfolio kurtosis. All methodological details are reported in [1] while in the next section we show the effect of selecting a KRP strategy compared with a SRP one.

3 Data and Results

The empirical application is based on seven equity indices from January 2001 to December 2020, provided by Morgan Stanley Capital International. In particular, the selected indexes are MSCI EMU, MSCI UK, MSCI USA, MSCI CANADA, MSCI JAPAN, MSCI PACIFIC EX-JAPAN, MSCI EMERGING MARKETS (expressed in euros). Here weekly returns are considered. Further empirical results considering also monthly data can be found in [1].

Main summary statistics, together with the Jarque-Bera test, are displayed in Table 1. Looking at it, we see that the indexes show negative skewness and positive excess kurtosis. The Jarque-Bera test is always significantly rejected.

Table 1 Summary statistics of the standardized returns assets of the investment universe

Asset	Mean	Volatitily	Skewness	Kurtosis	JB test (p-value)
MSCI EMU	0,0010	0,0291	-0,8840	9,4767	0
MSCI UK	0,0006	0,0273	-0,7391	12,7522	0
MSCI USA	0,0015	0,0261	-0,4513	6,7135	0
MSCI CANADA	0,0014	0,0291	-0,7728	10,1454	0
MSCI JAPAN	0,0008	0,0264	-0,1579	5,4569	0
MSCI PACIFIC EX JP F	0,0017	0,0271	-0,8073	10,4945	0
MSCI EM	0,0019	0,0284	-0,1937	8,4607	0

The test bed for our methodological proposal takes advantage of a rolling sample approach [2], considering different estimation window lengths and applying alternative portfolio rebalancing frequencies to determine the portfolio weights with the KRP and SRP strategies. In particular, here we consider an estimation window that covers 5 years (260 weeks) over which we have calculated portfolios with quarterly (i.e. every 12 weeks) and semi-annual (i.e. every 24 weeks) rebalancing windows. Figure 1 presents the results for the selected estimation window, together with the alternative rebalancing frequencies considered.

It emerges that the KRP strategy turns out to lead to a much more unbalanced and erratic portfolio structure compared with the one of the SRP. The Shannon Entropy measure also shows a different behavior for the two risk parity strategies (Figure 2).

According to Figure 2, the Shannon index is close to 2 when the SRP strategy is adopted, while it shows a very unstable and oscillating behavior when the KRP strategy is adopted. It is worth noting that exposures to asset classes within the KRP-based portfolios are inversely linked to their contribution to the portfolio kurtosis.

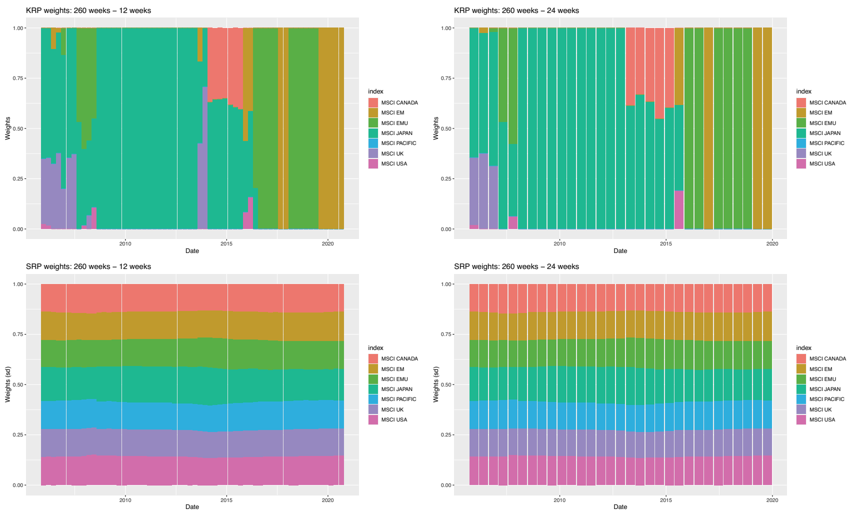

Fig. 1 Bar-charts representing the weights worked out using the KRP (top panels) and the SRP strategy (bottom panels). The estimation windows is 260 weeks long. The left panels refer to a rebalancing frequency of 12 week, while the right ones to 24 weeks.

The relevant characteristics of the out-of-sample returns provided by the two risk parity strategies have been also investigated. The KRP strategy systematically implies a lower kurtosis of out-of-sample returns. Finally, the risk-adjusted performance of the competing risk parity strategies, using the Sharpe ratio, the Sortino ratio and the Omega ratio have been evaluated. From Table 2, it results that KRP strategy provides persistently better reward per unit of risk taken.

Table 2 Sharpe, Omega and Sortino ratios obtained with the KRP and SRP strategies by using different rebalancing frequencies

Estimation window	Rebalancing frequency	Risk Parity strategy	Sharpe ratio	Omega ratio	Sortino ratio
260	12	KRP	0,380	1,159	0,520
		SRP	0,144	1,052	0,199
260	24	KRP	0,245	1,102	0,325
		SRP	0,126	1,046	0,171

Fig. 2 Shannon entropy using KRP and SRP strategies. The estimation windows is 260 weeks long. The top panel refer to a rebalancing frequency of 12 week, while the bottom one to 24 weeks.

4 Conclusions

Our empirical results reveal that a kurtosis-based risk parity strategy, compared to the classic risk parity, produces asset allocation solutions characterized by extremely unbalanced portfolio weights.

in addition we note that the "democratization" of kurtosis also helps its mitigation at least in comparison with the SRP strategy.

The methodological proposal of the paper, even if requires a more complex procedure for its implementation and leads to portfolio weights with a more unstable behavior, allows a better risk-return profile than SRP.

References

1. Braga, M.D., Nava, C.R., Zoia, M.G.: Kurtosis-based Risk Parity: Methodology and Portfolio Effects. ArXiv, (2022)
2. DeMiguel, V., Garlappi, L., Uppal, R.: Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? The review of Financial studies, **22**(5), 1915–1953, (2009)
3. Litterman, R.: Hot spots and hedges. Journal of Portfolio Management, (1996)
4. Maillard, S., Roncalli, T., Teïletche, J.: The properties of equally weighted risk contribution portfolios. The Journal of Portfolio Management. **36**, 4, 60–70, (2010)