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Risk Parity strategy for portfolio construction: a kurtosis-based approach

Risk Parity strategy per la costruzione di un portafoglio: un approccio basato sulla curtosi

Maria Debora Braga, Consuelo Rubina Nava and Maria Grazia Zoia

Abstract Investors interested in the homogeneous distribution of responsibility for the dispersion of portfolio returns can pursue not only the equal constituents’ contribution to portfolio volatility as, alternatively, they can focus on portfolio’s extreme outcomes (either positive or negative). This leads to the proposal of a kurtosis-based risk parity strategy (KRP) that is a new risk parity strategy based on portfolio kurtosis as reference measure. In this paper, the portfolio weights, obtained with the KRP strategy, are compared with those estimated following the traditional standard deviation-based risk parity strategy. An analysis of KRP strategy performance is carried out through an out-of-sample study which takes advantage of real data from a global equity investment universe.

Abstract Gli investitori, interessati a una distribuzione omogenea del rischio tra i vari asset di portafoglio, possono concentrarsi non solo sull’equa contribuzione dei costituenti alla volatilità di portafoglio poiché, in alternativa, essi si possono focalizzare sui risultati estremi di portafoglio (sia positivi che negativi) prendendo in considerazione la curtosi di portafoglio. In questo contesto, si sviluppa una strategia di parità di rischio basata sulla curtosi (strategia KRP) di portafoglio quale misura di rischio anziché sulla sua volatilità. I pesi per la costruzione del portafoglio ottenuti con la strategia KRP sono messi a confrontati con quelli ottenuti con la tradizionale strategia di parità di rischio basata sulla volatilità. Le performance della strategia KRP sono studiate con un approccio out-of-sample basato su dati reali di un universo di investimento azionario globale.

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1 Introduction

Since 2008, in finance, a new strategy for portfolio construction, known as risk parity (RP), has appeared [4]. Its main feature is the wealth allocation (accordingly to a suitable defined risk-measure) among asset classes in such a way that each asset contributes in the same manner to the selected portfolio risk-measure. Thus, risk contribution forms the basis for the development of the RP strategy [3]. So far, portfolio volatility has been the most used reference risk-measure.

Here we develop the new version of the RP strategy where portfolio volatility is replaced by the portfolio kurtosis as the reference risk-measure. The existing literature has already dealt with other risk measures but never with the kurtosis. This is also due to the absence of a close form expression of the portfolio kurtosis. This is also due to difficulty of deriving closed form expressions for the marginal risk contribution of each asset and, consequently, the set-up of the optimization problem, necessary to identify optimal portfolio weights.

In this paper, a risk parity strategy based on portfolio kurtosis as reference measure (KRP – kurtosis-based risk parity) is investigated, applied, and its performance is compared with the traditional standard deviation-based risk parity (SRP). In such a way a novel strategy able to better accommodate the needs of investors interested in the homogeneous distribution of responsibility for portfolio returns’ huge dispersion, as that measured by portfolio kurtosis, is developed. A novel and effective expression for portfolio kurtosis is proposed, which integrates the results proposed in [1]. The proposed methodology is applied on real data and compared with the classical RP allocation strategy based on volatility in order to explore its peculiar features.

The paper is organized as follow. Section 2 proposes an original formula for portfolio kurtosis. Section 3 describes the dataset used for the empirical application and provides main results. Section 4 concludes the paper.

2 Methodology

Let consider a portfolio composed by N assets with returns $\mathbf{R} = [R_i]$ and associated weights $\mathbf{w} = [w_i]$ with $i = 1, \ldots, N$. The weights $w_i$ are the percentage of asset class $i$ in the portfolio and satisfy the following normalizing constraint $\sum_{i=1}^{N} w_i = 1$. Let $\mathbf{\mu}$ and $\mathbf{\sigma}$ be the $N \times 1$ vectors of the expected returns and standard deviations for the selected asset classes in the investment universe: $\mu_i$ is the expected return of asset class $i$ while $\sigma_i$ is its risk. Furthermore, let $\mathbf{C} = [C_{ij}]$ and $\mathbf{\Sigma} = [\sigma_{ij}]$ be the $N \times N$ correlation matrix and covariance matrix. Accordingly, the expected return of the portfolio $P$ and its variance are:
kurtosis of asset, which can be expressed as follows

\[ K_i = K(R_i) = E\left[ \left( \frac{R_i - \mu_i}{\sigma_i} \right)^4 \right] = \frac{E[(R_i - \mu_i)^4]}{E[(R_i - \mu_i)^2]^2} = \frac{\mu_{i4}}{\sigma_i^4} \]

with \( \frac{R_i - \mu_i}{\sigma_i} \) representing the standardized returns. Thus,

\[ K(w_i R_i) = E\left[ \left( \frac{w_i R_i - w_i \mu_i}{w_i \sigma_i} \right)^4 \right] = \frac{\mu_{i4}}{\sigma_i^4} = K(R_i) \]

\[ K(R_i + R_j) = \frac{\sum_{k_1+k_2=4} 4! \frac{k_1! k_2!}{4!} \sigma_i^{k_1} \sigma_j^{k_2} \text{coK}(R_i, R_i, R_i, R_j)}{\left( \sigma_i^2 + \sigma_j^2 + 2\text{cov}(R_i, R_j) \right)^2} \]

where the cokurtosis (\( \text{coK} \)) is defined as follows

\[ \text{coK}(R_i, R_i, R_i, R_j) = \frac{E[(R_i - \mu_i)^3 (R_j - \mu_j)]}{\sigma_i^2 \sigma_j} \]

\[ \text{coK}(R_i, R_i, R_j, R_j) = \frac{E[(R_i - \mu_i)^2 (R_j - \mu_j)^2]}{\sigma_i^2 \sigma_j} \]

The cokurtosis is invariant over linear transformation, i.e. \( \text{coK}(w_i R_i, w_i R_i, w_j R_i, w_j R_j) = \text{coK}(R_i, R_i, R_j, R_j) \). Moreover, the particular case of \( \text{coK}(R_i, R_i, R_i) \) reduces to the kurtosis of \( R_i \), i.e. \( \text{coK}(R_i, R_i, R_i, R_i) = K(R_i) \), still invariant over linear transformation. However, it is worth noting that \( K(w_i R_i + w_j R_j) \neq K(R_i + R_j) \).

The previous results can be extended to obtain a portfolio kurtosis including \( N \) asset, which can be expressed as follows

\[ K_p = K \left( \sum_{i=1}^{N} w_i R_i \right) = \frac{E\left[ \sum_{i=1}^{N} w_i R_i - \sum_{i=1}^{N} w_i \mu_i \right]^4}{E\left[ \sum_{i=1}^{N} w_i R_i - \sum_{i=1}^{N} w_i \mu_i \right]^2} \]

\[ = \sum_{i=1}^{N} w_i \text{coK}(R_i, R_i, R_i, R_i) \prod_{i=1}^{N} w_i^{k_i} \sigma_i^{k_i} \]

\[ \sum_{i=1}^{N} \frac{4! k_i !}{4!} \text{cov}(R_i, R_i) \prod_{i=1}^{N} w_i^{k_i} \]

where \( k_i = 0, 1, 2, 3, 4 \) \( \forall i = 1, \ldots, N \), \( \text{cov}(R_i, R_i) \) is the covariance between two assets while \( \text{coK}(R_i, R_i, R_i, R_i) \) is the cokurtosis.

Thus, based on this portfolio kurtosis formula, which can be also written in a closed form for using matrix notation [1], the portfolio kurtosis gradient, needed for
the determination of the marginal risk contributions of the asset classes to portfolio kurtosis, can be computed.

In this connection, it is worth noting that the RP approach is based on the idea that portfolio risk must be equally distributed among asset classes, a goal that can be pursued for any risk measure RM, provided it is homogeneous of degree one in the weights. This is also the case of the portfolio kurtosis. All methodological details are reported in [1] while in the next section we show the effect of selecting a KRP strategy compared with a SRP one.

3 Data and Results

The empirical application is based on seven equity indices from January 2001 to December 2020, provided by Morgan Stanley Capital International. In particular, the selected indexes are MSCI EMU, MSCI UK, MSCI USA, MSCI CANADA, MSCI JAPAN, MSCI PACIFIC EX-JAPAN, MSCI EMERGING MARKETS (expressed in euros). Here weekly returns are considered. Further empirical results considering also monthly data can be found in [1].

Main summary statistics, together with the Jarque-Bera test, are displayed in Table 1. Looking at it, we see that the indexes show negative skewness and positive excess kurtosis. The Jarque-Bera test is always significantly rejected.

<table>
<thead>
<tr>
<th>Asset</th>
<th>Mean</th>
<th>Volatility</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>JB test (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSCI EMU</td>
<td>0.0010</td>
<td>0.0291</td>
<td>-0.8840</td>
<td>9.4767</td>
<td>0</td>
</tr>
<tr>
<td>MSCI UK</td>
<td>0.0006</td>
<td>0.0273</td>
<td>-0.7391</td>
<td>12.7522</td>
<td>0</td>
</tr>
<tr>
<td>MSCI USA</td>
<td>0.0015</td>
<td>0.0261</td>
<td>-0.4513</td>
<td>6.7135</td>
<td>0</td>
</tr>
<tr>
<td>MSCI CANADA</td>
<td>0.0014</td>
<td>0.0291</td>
<td>-0.7728</td>
<td>10.1454</td>
<td>0</td>
</tr>
<tr>
<td>MSCI JAPAN</td>
<td>0.0008</td>
<td>0.0264</td>
<td>-0.1579</td>
<td>5.4569</td>
<td>0</td>
</tr>
<tr>
<td>MSCI PACIFIC EX JP F</td>
<td>0.0017</td>
<td>0.0271</td>
<td>-0.8073</td>
<td>10.4945</td>
<td>0</td>
</tr>
<tr>
<td>MSCI EM</td>
<td>0.0019</td>
<td>0.0284</td>
<td>-0.1937</td>
<td>8.4607</td>
<td>0</td>
</tr>
</tbody>
</table>

The test bed for our methodological proposal takes advantage of a rolling sample approach [2], considering different estimation window lengths and applying alternative portfolio rebalancing frequencies to determine the portfolio weights with the KRP and SRP strategies. In particular, here we consider an estimation window that covers 5 years (260 weeks) over which we have calculated portfolios with quarterly (i.e. every 12 weeks) and semi-annual (i.e. every 24 weeks) rebalancing windows. Figure 1 presents the results for the selected estimation window, together with the alternative rebalancing frequencies considered.

It emerges that the KRP strategy turns out to lead to a much more unbalanced and erratic portfolio structure compared with the one of the SRP. The Shannon Entropy measure also shows a different behavior for the two risk parity strategies (Figure 2).
According to Figure 2, the Shannon index is close to 2 when the SRP strategy is adopted, while it shows a very unstable and oscillating behavior when the KRP strategy is adopted. It is worth noting that exposures to asset classes within the KRP-based portfolios are inversely linked to their contribution to the portfolio kurtosis.

The relevant characteristics of the out-of-sample returns provided by the two risk parity strategies have been also investigated. The KRP strategy systematically implies a lower kurtosis of out-of-sample returns. Finally, the risk-adjusted performance of the competing risk parity strategies, using the Sharpe ratio, the Sortino ratio and the Omega ratio have been evaluated. From Table 2, it results that KRP strategy provides persistently better reward per unit of risk taken.

**Table 2** Sharpe, Omega and Sortino ratios obtained with the KRP and SRP strategies by using different rebalancing frequencies

<table>
<thead>
<tr>
<th>Estimation window</th>
<th>Rebalancing frequency</th>
<th>Risk Parity strategy</th>
<th>Sharpe ratio</th>
<th>Omega ratio</th>
<th>Sortino ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>260</td>
<td>12</td>
<td>KRP</td>
<td>0.380</td>
<td>1.159</td>
<td>0.520</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SRP</td>
<td>0.144</td>
<td>1.052</td>
<td>0.199</td>
</tr>
<tr>
<td>260</td>
<td>24</td>
<td>KRP</td>
<td>0.245</td>
<td>1.102</td>
<td>0.325</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SRP</td>
<td>0.126</td>
<td>1.046</td>
<td>0.171</td>
</tr>
</tbody>
</table>
Fig. 2 Shannon entropy using KRP and SRP strategies. The estimation window is 260 weeks long. The top panel refers to a rebalancing frequency of 12 weeks, while the bottom one to 24 weeks.

4 Conclusions

Our empirical results reveal that a kurtosis-based risk parity strategy, compared to the classic risk parity, produces asset allocation solutions characterized by extremely unbalanced portfolio weights.

In addition, we note that the “democratization” of kurtosis also helps its mitigation at least in comparison with the SRP strategy.

The methodological proposal of the paper, even if requires a more complex procedure for its implementation and leads to portfolio weights with a more unstable behavior, allows a better risk-return profile than SRP.

References