Prompting embodied instrumented covariation with the digital Tracer

Sara Bagossi[1], Carlotta Soldano[1], Eugenia Taranto[2], and Giada Viola[3]

[1]University of Turin, Italy; sara.bagossi@unito.it; carlotta.soldano@unito.it; [2]Kore University of Enna, Italy; eugenia.taranto@unikore.it; [3]University of Ferrara, Italy; giada.viola@unife.it

This study explores a learning environment that involves tasks designed by exploiting a digital artifact called digital Tracer. Such an artifact enables an embodied design of tasks that may prompt a covariational perspective on (functional) relationships traditionally taught by adopting a static approach. The paper aims to investigate how peers work together with the digital Tracer (what can be called an intercorporeal functional dynamic system) in order to solve some tasks elaborated with an embodied design. Employing the framework of embodied instrumentation, the preliminary analysis presented in this study examines a group of three university students, one of whom acted as an observer, while solving tasks that required them to follow two given traces (a broken line and a circumference) using the digital Tracer.

Keywords: Embodied instrumentation, covariation, intercorporeal functional dynamic system, Tracer.

Introduction

The ability to interpret the graphical representation of a function or a curve is a fundamental goal in mathematics that should be developed from the last years of primary school. However, this is far from easy for students, especially if they should understand such graphs as a representation of "a continuum of states of covarying quantities" (Saldanha & Thompson, 1998, p. 305) as it might happen in the conceptualization of dynamic situations. Indeed, the points on the graph should be understood as a covariation between the co-varying quantities, mathematically representable as an abscissa and an ordinate, which might be in a relation of functional dependence.

In this paper, we investigate the potentialities of a digital-embodied design involving the use of an artifact named *digital Tracer* to approach covariational reasoning, realized within a multi-touch dynamic geometry environment. Such an artifact displays a Cartesian plane with two bars that can be moved using the touch of fingers. By moving these bars, users can move and control their intersection point to draw or trace lines in the given plane, engaging their bodies in the activity.

Literature in Mathematics Education has shown how perceptuomotor integration in solving tasks can benefit mathematical learning and understanding (e.g., Nemirovsky et al., 2013). In order to incorporate embodied learning experiences with the digital Tracer into educational contexts, the five design principles (DPs) of Palatnik et al. (2023, pp. 7-8), which we briefly recall here, can be considered. DP1: *Involve students' bodies in the learning process*, either through perception-based design artifacts or action-based design; DP2: *Offer immediate sensorimotor interactions with artifacts* through which create stable forms of engagement with the environment and promote mathematical understanding through various means of communication, including verbal and nonverbal language; DP3: *Attend to the semiotic sensitivity of the design* by reflecting on the signs included in artifacts that can be produced by students or teachers while solving the task and their relationship with mathematical knowledge, as well as how students interpret and attribute meanings to these signs; DP4: *Include a variety of semiotic registers and artifacts that potentiate mathematical perception* *and discourse*, recognizing that learning and mathematics understanding happens when learners are able to convert between different semiotic registers and to treat within one register; DP5: *Foster multimodal engagement and "languaging"* by integrating various modes of communication, such as speech, gestures, and drawings, into the learning process, acknowledging that learning new concepts often involves discovering new ways of acting in the environment and participating in cultural discourse. The five principles are reminiscent of Papert's constructionism (1980), according to which learning and understanding are more effective through hands-on experience with technologies and through the exploration of individuals. Indeed, the interaction with the environment plays a central role, and scaffolding is used to support learning.

This research aims to investigate the efficacy of an embodied design of tasks, elaborated with the 5 DPs in mind, using the digital Tracer, for supporting a covariational perspective. This exploratory study allows us as researchers to describe the process of resolution of a small group of university students involved in the experiment, by adopting embodied instrumentation as the theoretical framework, and to evaluate the embodied design of the tasks.

Theoretical framework: Embodied instrumentation

The theoretical framework of embodied instrumentation stems from a radical embodied perspective (Shvarts et al., 2021): learning can only happen through the body, and such a body meets the *affordances* of the environment (Gibson, 1986) including various digital tools, culturally-established artifacts, and even other individuals acting as observers. Body potentialities help to perceive the affordances of the environment and then act on them. *Perception* and *action* are connected in loops driven by intentionality and oriented towards the anticipation of the environment's affordances. Fulfilling the intentionality, an individual (brain and body) and an artifact form what is called *body-artifact functional system* (Shvarts et al., 2021). When solving a mathematical task, a body-artifact functional system activates action-perception loops to recognize and act on mathematical affordances of cultural artifacts, leading to the emergence of *stabilized behaviors*. But what about when more than one individual is involved in the resolution of a task? Shvarts and Abrahamson (2020) refer to a student and a teacher as forming an *intercorporeal functional dynamic system* (IFDS), in which one student is a sub-system. When individuals are engaged in a coupled coordination of motor actions, various *attentional anchors* (Abrahamson & Sanchez-Garcia, 2016), i.e. foci of attention, may pop up as new perceptual structures and then contribute to stable performances. Within an IFDS, the two bodies attune to perform a common action and work together toward the resolution and conceptualisation of a mathematical task. The construct of *embodied instrumented covariation* (Bagossi, 2022) has already been introduced to condense the idea that a bodily-sensorial experience by using an interactive multimodal technology may mediate the elaboration of an intuitive idea of covariation enabling the learners to perceive the mathematical processes. How such an instrumented approach that integrates bodily involvement and coordination can aid in understanding covariational relationships, specifically learning about variations that occur simultaneously, is still scarcely explored. In light of the theoretical framework outlined and the insights coming from an explorative study like ours, the research question we are willing to focus on can be formulated as: How does an intercorporeal functional dynamic system engage with the digital Tracer to explore covariational relationships?

The digital Tracer

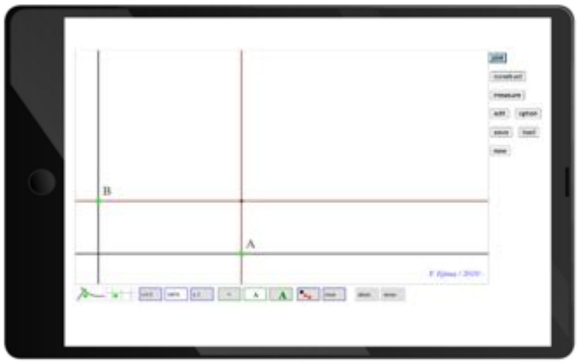

Figure 1: The digital Tracer

The digital Tracer (Figure 1) is the corresponding digital version of the physical Tracer (made of a wooden panel and two moving bars) introduced in Ferretti et al. (2024). Inspired by the mechanism of the game Etch A Sketch, the digital Tracer is an artifact conceived using the dynamic geometry software GC/htlm5, an updated version of Geometric Constructor, one of the free dynamic geometry software used in Japan since 1989 and developed by Prof. Y. Iijima. It is compatible with both iPad and Android tablets and supports multi-touch interaction. The digital Tracer consists of two perpendicular axes (black lines in Figure 1), two points (A and B) belonging to these lines, and the two bars of the Tracer represented by other lines (brown lines in Figure 1) that pass through these points and are perpendicular to the previous lines. By dragging points A and B, it is possible to control the point of intersection of the bars so that it can follow a specific path. By activating the "Trace" command, clicking on the fourth lowest button placed at the bottom of the screen, the intersection point of the bars will leave a colored trace when moved. The digital Tracer can be used by a single user, but in our experiment, as we will explain in more detail, we had the students work in pairs on the screen showing the digital Tracer. To describe a given curve or function, A should be moved simultaneously with B, because it is through the combination of these movements that the trace is left on the path or not. In order to move the intersection point along the given path when there are two users, it is essential that they coordinate their movements.

Method

Embodied design with the Tracer

We designed three tasks involving the use of the physical and digital Tracer:

1. Follow four given traces (in pairs and then individually): steps, a broken line, a circumference, and a parabola (see Figures 2a and 2b).
2. Reproduce the trace (in pairs): two broken lines with different slopes and two parabolas with different widths.
3. Produce two parallel lines (in pairs).

Such mathematical tasks meet those embodied design principles outlined by Palatnik et al. (2023). Indeed, students' bodies are involved in the learning process (DP1) through the movement of the two bars by means of their arms or fingers and the attunement made possible by the eye-hand coordination of the bodies involved. It offers sensorimotor interaction with the Tracer (DP2) through the movement of the two bars. The result of such interaction can be made visible by using a marker with the physical Tracer or by activating the Trace option on the digital Tracer (DP3). The formulation of the tasks mainly relies on the graphical register, but students are also invited to elaborate verbally on their actions (DP4), hence the activity fosters languaging (DP5). Eventually, we believe that the multimodal engagement prompted by such an embodied design is coherent with the mathematical content of the activities. In fact, through the movement of the bars and the resulting intersection, students can feel the co-variation of the two bars, which may be associated with variables x and y.

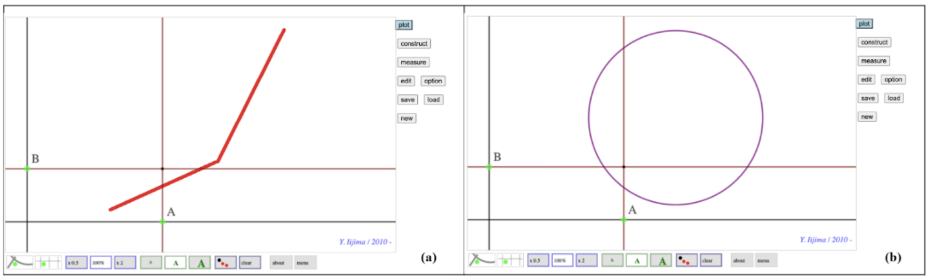

Figure: 2a. Broken line trace; 2b. Circumference trace

Case study: participants

In this paper, we focus on a case study involving three students from a Master degree program in Mathematics, Anna, Bea, and Chiara. During the experiment, the researcher gave the students three tasks, initially executed using the physical Tracer, and then performed with the digital Tracer. In what follows we will focus on the resolution in pairs of task 1 where students were asked to follow the traces of a broken line and a circumference (Figures 2a and 2b): this intentionality determines the assembly of the IFDS that we will analyze. During the experiment, Anna and Bea worked in pairs to solve the tasks using the digital Tracer: these three elements form the IFDS we are willing to investigate (Figure 3). Concerning the affordances of the environment we should consider that the two students had previously worked with the physical Tracer to solve the same tasks. Moreover, Chiara acted as an external observer who sometimes prompted some reflections at the end of the task.

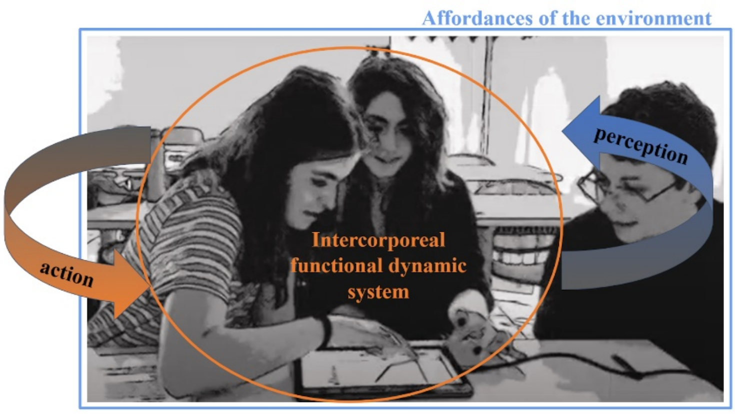

Figure 3: Intercorporeal functional dynamic system and environment's affordances. From left to right, Anna, Bea, and Chiara

Data collection and analysis

The entire experiment (90 minutes more or less) was videorecorded. In this contribution, a preliminary analysis of two episodes is presented to outline some considerations related to our research question. The episodes were analyzed qualitatively by focusing on the key elements of the embodied instrumentation framework: *intentionality* (Which is the aim of the IFDS?); *bodily experience* (Which embodied attunements between people emerge? Which attentional anchors emerge within the IFDS? Which action-perception loops emerge?); *stabilized behaviors* (Which stable techniques are adopted by the IFDS when using the digital Tracer?); *conceptualization* (Which verbal formalizations are elaborated by the IFDS when solving the task?). The qualitative analysis of the two selected episodes was conducted through descriptive coding based on the four elements previously outlined. The authors analyzed the episodes independently and discordant interpretations were discussed and revised until reaching an agreement.

Results

Episode 1: Broken line

The two students were requested to follow the given trace of a broken line, and this is the goal they try to achieve by leaving a nice trace (*intentionality*). The legacy of the two girls' use of the physical Tracer is evident. In fact, they immediately know that one of them will have to consider only one bar and therefore point A and the other will consider the other bar and, therefore, point B (*intentionality*).

Figure: 4a. Chiara attuning with the IFDS; 4b. Different behaviors of Anna and Bea while moving the bars; 4c. Final trace produced

Their bodies attune to perform this action by positioning their finger on the screen and adjusting their posture. Even Chiara who is not involved in performing actions with the Tracer attunes herself, so becoming part of the IFDS (Figure 4a). Having tacitly agreed on the points to be moved by each of them (and this derives from their previous experience with the physical Tracer, in which Anna moved the abscissa bar and Bea the ordinate bar) (*stabilized behavior*), the two girls silently and with concentration perform the movements. Anna and Bea agree on the starting point (bottom left of the broken line), and after positioning the intersection point of the two bars in that extreme, they start moving the two bars, and they direct their attention to the intersection point of the two bars (*bodily experience*). While Anna moves her finger along the x-axis, the finger of Bea tends to follow the curve and does not stay strictly along the axis, as can be observed in Figure 4b (*bodily experience*). The hand-eye coordination is evident: each person observes the trace progressively left by the intersection point, while simultaneously moving points A and B on the axes with their index fingers (another *stabilized behavior* with which they move the points). Only at the end, they hint a smile, a little bitterly, an expression that can be observed even on Chiara's face: they have finished this performance, but the result is not optimal (Figure 4c). Anna says "the first trait ... um ... I can hardly see because I put my finger on it" (meaning that she did not have a full view of the line to trace because her hand and wrist partially covered the line - *bodily experience*).

Episode 2: Circumference

In this episode, Anna and Bea work together to trace the given circumference, and they agree on starting from the lowest point of the circumference and moving in a counterclockwise sense (*intentionality*). Anna perceives a lack of fluidity in tracing the circumference, especially comparing the previous experience she had with Bea with the physical Tracer. When approaching the extreme right point of the circumference, she perceives from the left blue trace (Figure 5a) that she is mistakenly progressing in the positive direction of the x-axis (*bodily experience*), and the two girls' attunement is interrupted (Figure 5b). Hence, a disruption of the IFDS happens, which is made evident by a joint laugh of the girls breaking the tension of the previous coordination. Anna says, "Before, I was used to stopping when we had to change direction, I'm not used to doing everything in one go", and again: "I'm not used to changing direction quickly" (*conceptualization*). These different behaviors emerge even though both students are directing their attention to the intersection point rather than on their fingers (*intentionality* and *bodily experience*). Bea is the first who observes that she and Anna behaved differently: "We do two completely different things: you [Anna] always keep your finger in the same place, I on the other hand, I don't know why, but with my finger, I follow the progression [of the trace] a bit" (*conceptualization* on the *stabilized behavior*). Anna reacts by saying, "I always keep my finger on the same point because first I go in one direction and then I have to change; I move within a very small range." Throughout this dialogue, the eyes of the three girls, even those of Chiara who is silent, all look at the tablet screen and follow the finger movements on it. Chiara has an attentive and focused gaze; sometimes she smiles, and this is a tacit confirmation of agreement with the colleagues' statements (*bodily experience*).

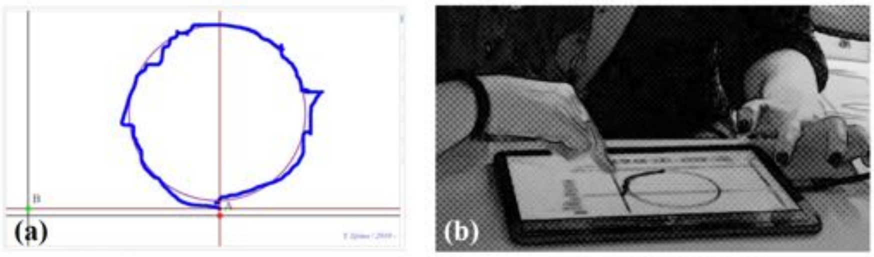

Figure: 5a. Final trace produced by Anna and Bea; 5b. Disruption of the IFDS

Discussion

The intentionality of the IFDS is strongly focused on drawing the traces as precisely as possible. Attention remains concentrated on responding to this intentionality: Anna and Bea do not look at their hands as they follow the trace, nor at the colored trace that gradually emerges with their movements. Rather, they look at the trace assigned to them. While performing the tasks, Anna and Bea attune and engage in loops of action-perception in which they go back and forth, move together, and stop, elaborating on stabilized behaviors. However, little conceptualisation emerges. In previous experiments, in which we had first involved primary school children (Ferretti et al., 2024) and then school teachers in training, free observations and then conceptualisation of the lived experience spontaneously emerged. In this experimentation with undergraduate mathematics students, we realized that conceptualisation does not naturally arise, but rather needs to be stimulated. The embodied design should certainly provoke some initial insights, but then a reflective stage is needed to trigger conceptualization (Alberto et al., 2022). Considering the qualitative nature of the tasks here proposed, such a reflective stage could be achieved through open questions, warmed up by a researcher or a teacher, asking for the what, how and why of the sensory-motor experience.

It is worth drawing attention to another aspect of our environment. Shvarts and Abrahamson (2020), define the IFDS as consisting of a student, working with an artifact, and a tutor. We, on the other hand, have two peers, Anna and Bea, interacting simultaneously with the artifact. Notably, Anna and Bea in enacting their actions with the artifact do not speak. They feel and act. Then, in our environment, there is also Chiara. She has the role of observer but becomes somehow part of the IFDS: she attunes, nods, smiles, and follows with her gaze all the movements of her colleagues. Moreover, at the end of the performance of tracing the circumference, all three burst out laughing: all three knew how tense they were in their effort to trace the circumference as smoothly as possible.

Final remarks

The IFDS proves to be a valid construct for investigating how the digital Tracer can be used to explore covariational relationships. Indeed, the interactivity of an artifact like the Tracer may help the learner give meaning to functions, or relationships, as dynamic entities, as opposed to a static view of functions supported by the other approaches to functional thinking, such as the correspondence one. Moreover, a specific task design such as the one proposed here may support a smooth image of change, in contrast to a chunky one: indeed, the focus is on the sensorial perception of the variations of the quantities at stake, favouring a qualitative approach. However, retrospectively analyzing the videos, we realized that the university students created windows for conceptualization, but the proposed task design did not allow full use of them. Even though in this paper we present two episodes focused on the digital Tracer, the whole sequence allows for a comparison between the physical and digital Tracers and for reasoning as to which of the two artifacts has more potential to achieve the goal of the task. For example, when Anna and Bea observe their failure in making a smooth trace with the digital Tracer, we could have explicitly asked them to compare their experience with the one previously had with the physical Tracer: the concepts of smoothness and approximation might have come out more. Therefore, we reiterate that conceptualisation needs to be triggered and more could be done in the design to prompt a reflection on covariational aspects. We believe that such an embodied approach may help students not only perceive, given the different speeds with which one can move the bars, what variation means, but also conceptualize the given trace as a dynamic object. In this way, it is possible to see the covariation between A and B not only as a result of the coordination of the two users' actions, expressed as a colored trace, but also to feel and perceive the covariation through the bodies and their movements' coordination. Eventually, our embodied task design, elaborated to support the conceptualization of covariational relationships, focuses on a qualitative approach so far. Hence, a quantitative stage (Alberto et al., 2022) could be introduced to reinforce the idea of covariation between quantities, for example by inserting a grid in the digital Tracer. These will be points of future reflection.

Acknowledgment

We would like to thank the MaT&L Research group (especially Federica Ferretti and Ferdinando Arzarello) for making the experiment described here possible and for the precious feedback. Our gratitude goes also to Anna Shvarts for the invaluable exchange of ideas throughout the analysis and writing process. Finally, we thank the INdAM GNSAGA "Istituto Nazionale di Alta Matematica - Gruppo Nazionale per le Strutture Algebriche, Geometriche e le loro Applicazioni".

References

Abrahamson, D., & Sánchez-García, R. (2016). Learning is moving in new ways: The ecological dynamics of mathematics education. *Journal of the Learning Sciences*, *25*(2), 203–239. https://doi.org/10.1080/10508406.2016.1143370

Alberto, R., Shvarts, A., Drijvers, P., & Bakker, A. (2022). Action-based embodied design for mathematics learning: A decade of variations on a theme. *International Journal of Child-Computer Interaction*, 32, 1–23. https://doi.org/10.1016/j.ijcci.2021.100419

Bagossi, S. (2022). *Second-order covariation: An analysis of students' reasonings and teacher's interventions when modelling real phenomena*, Unpublished Ph.D. Thesis, Università degli studi di Modena e Reggio Emilia.

Ferretti, F., Giberti, C., Bagossi, S., Taranto, E., & Arzarello, F. (2024). Embodied instrumented covariation: a case study from primary school with the Tracer. *For the Learning of Mathematics* - in press.

Gibson, J. J. (1986). *The ecological approach to visual perception*. Psychology Press.

Nemirovsky, R., Kelton, M. L., & Rhodehamel, B. (2013). Playing mathematical instruments: Emerging perceptuomotor integration with an interactive mathematics exhibit. *Journal for Research in Mathematics Education*, *44*(2), 372–415. https://doi.org/10.5951/jresematheduc.44.2.0372

Palatnik, A., Abrahamson, D., Baccaglini-Frank, A., Oi-Lam, N., Shvarts, A., & Swidan, O. (2023). Theory and practice of designing embodied mathematics learning. In M. Ayalon, B. Koichu, R. Leikin, L. Rubel & M. Tabach (Eds.). *Proceedings of the 46th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 159–189). PME 46.

Papert, S. (1980). *Mindstorms. Children, Computers and Powerful Ideas.* Basic Books, Inc.

Saldanha, L., & Thompson, P. W. (1998). Re-thinking covariation from a quantitative perspective: Simultaneous continuous variation. In S. B. Berenson, & W. N. Coulombe (Eds.), *Proceedings of the Annual Meeting of the Psychology of Mathematics Education – North America*, Vol 1, 298–304. North Carolina State University.

Shvarts, A., & Abrahamson, D. (2020). Intercorporeal dynamic functional system: A dual eye-tracking study of student-tutor collaboration. *The Body in Mathematics*. Sense/Brill.

Shvarts, A., Alberto, R., Bakker, A., Doorman, M., & Drijvers, P. (2021). Embodied instrumentation in learning mathematics as the genesis of a body-artifact functional system. *Educational Studies in Mathematics*, *107*(3), 447–469. https://doi.org/10.1007/s10649-021-10053-0