A non-euclidean clockwork orange: from reality to mathematics and back

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Abstract. This paper presents a teaching experience in non-Euclidean geometry involving the use of artefacts and physical experiences. The teaching practice was set up for a group of 25 4th-year high-school students. The aim of this activity was to encourage the "process of translation from 'reality' to mathematics and back". Further, the students were stimulated to evaluate the authenticity of the proposed context, which was designed for learning about spherical geometry and its value in helping students understand and organise the abstract key facts of the topic. The overall aim was to shift back to the starting point and apply the acquired theoretical knowledge in order to interpret a real-world problem.

Résumé. Cet article présente une expérience pédagogique dans la géométrie non-euclidienne impliquant l'utilisation d'artefacts et d'expériences réelles. La pratique pédagogique a été mise en place pour un groupe de 25 élèves du quatrième année de l'école secondaire. Le but de cette activité était d'encourager le "processus de traduction de la 'réalité' aux mathématiques et retour". En plus, les étudiants ont été stimulés à évaluer l'authenticité du contexte proposé, conçu pour apprendre la géométrie sphérique et sa valeur pour aider les élèves à comprendre et à organiser les faits clés abstrait du sujet. L'objectif général était de revenir au point de départ et d'appliquer les connaissances théoriques acquises afin d'interpréter un problème réel.

1. Introduction and theoretical framework

According to Jablonka and Gellert's reflection on mathematisation and demathematisation (Jablonka & Gellert, 2007), "if classroom talk concentrates on the public language of description in order to help students construct meaning, then the mathematical knowledge of students tends to remain in the public domain of its origin. If, on the other hand, the classroom talk is mainly esoteric, then the individual construction of meaning appears to be more difficult."

In high-school, non-Euclidean geometry is often treated in a particular way: from an epistemological point of view, it is a highly significant subject. However, there is a risk of losing the cognitive ties to reality if the subject matter, as taught in schools, is kept strictly in the abstract realm. Further, the key role of non-Euclidean geometry, considering the interpretation of geometry in real-world settings, is endangered by the demathematisation of such by, for example, Google Maps and GPS navigation.

The overall aim of this teaching experience was to encourage the "process of translation from 'reality' to mathematics and back" called for by Jablonka and Gellert (Jablonka & Gellert, 2007). In addition, the purpose was to evaluate the effectiveness of the authenticity of the proposed context for the learning of spherical geometry. The evaluation included the teaching experience's value in terms of its support for students' understanding and organisation of the key abstract facts about non-Euclidean geometry. Further, the students were questioned about their newly developed ability to shift back and apply the acquired knowledge to interpret the real-world problem that was given to them.

2. Method and activity

This learning experience has been inspired by Lénárt's work on comparative geometry and by the article "Grapefruit Math" by Evelyn Lamb, published in the Scientific American online issue in May 2015. The added value of this specific project is the use of artefacts and physical experiences that effectively support students' learning by means of connecting abstract concepts of thought and discoveries to real world tangible results.

The activity was carried out with a class of 25 18 y.o. students attending the 4th year of Liceo Scientifico. The results were gathered by the teacher through the observation of classroom discussion and the assessment of students' homework.

After an introductional theoretical approach to non-Euclidean geometry, which primarily entailed the discussion on the role of the fifth postulate and the consequences of its negations with reference to the validity of the "classical" theorems about parallels and transversals and the sum of a triangle internal angles, students were asked to bring an orange, three elastic bands and a protractor for the upcoming lesson.

In said lesson they were confronted with the following tasks:

- Draw three points on the orange that do not lay on the same great circle.

- Put two elastic bands, each passing through two of the three points as in 'figure 1', and derive the expression for the area of each lune.

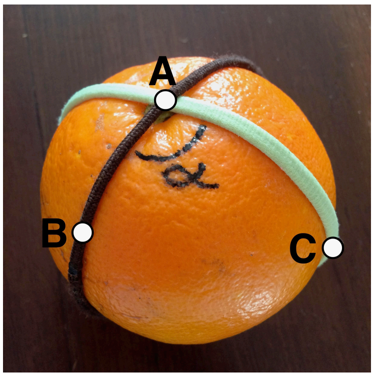

Figure 1. from Elena's work: the construction of the lune

- Add a third elastic band to form a spherical triangle as in 'figure 2'.

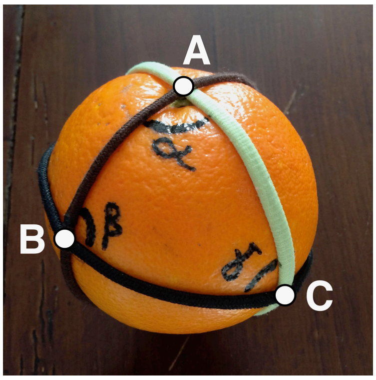

Figure 2. from Elena's work: the construction of the spherical triangle

- Use a protractor to measure the internal angles of the spherical triangle, add them up and contextualise the result.

- Derive the formula for the surface area of the spherical triangle.

The final questions encouraged critical reflection on the subject matter that required from the students to switch from the real-world model (the triangle on the orange) to the mathematical model (the formula for the surface of the triangle). After the students had completed these tasks, they were confronted with those follow-up questions, which aimed to link the mathematical interpretation to their everyday life:

- What happens if the triangle on the sphere gets smaller?

- What does that have to do with your everyday life as a resident of a giant sphere?

The students discussed their results and findings in class. After that, each student summarised her/his conclusions in a written essay that was finally submitted through the school's Moodle platform and assessed by the teacher.

3. Results and conclusion

The key aspect of this learning experience lies within the fact that "the essence of the process is that students are searching for the truth themselves, through experiments with palpable and virtual models, and through discussion or debate with one another or their teacher." (Lénárt, 2009).

The outcomes of this activity proved the efficiency of mathematical modelling as a didactic principle. On the one hand, the students were able to assess the mathematical behaviour of great circles as "straight lines" on the sphere by themselves. Thereby, the theoretical assumption presented in class beforehand gained a contextualised meaning due to personal experience. The overall, shared comment was:

I couldn't believe that the rubber bands stayed put only on great circles!

On the other hand, it clearly showed that by integrating real objects in order to draw in on abstract mathematical concepts, the students were more willing to apply the mathematical concept to real world observations thereafter. Further, this made them get an idea of why – in a small scale - we cannot identify our reality as non-Euclidean, as Chiara neatly pointed out in her paper:

This happens because the surface area of the shrunk spherical triangle is so small compared to the one of the sphere that the curviness of its sides becomes negligible and it is perceived as a Euclidean triangle.

References

Jablonka, E., & Gellert, U. (2007). Mathematisation – Demathematisation. In U. Gellert & E. Jablonka (Eds.), *Mathematisation and demathematisation: Social, philosophical and educational ramifications*, (pp. 1–18). Rotterdam: Sense.

Lamb, E. (2015). 'Grapefruit Math', Scientific American online, May 19, 2015. https://blogs.scientificamerican.com/roots-of-unity/grapefruit-math/. Accessed 7 June 2017.

Lénárt, I. (2009). Paper geometry vs orange geometry – Comparative geometry on the plane and the sphere. https://www.mav.vic.edu.au/files/conferences/2009/21Lenart.pdf. Accessed 7 June 2017.