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The trouble with modus ponens. Some explorations on modus ponens and its failures.

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CONTENTS

INTRODUCTION. The election, the lottery, and the trouble with modus ponens.

1. Defining modus ponens

As the title of this dissertation already suggests, the main topic of this thesis is the logical principle named “modus ponens”. What the thesis’ title also suggests is that there may be “a trouble” with such a principle. That is, the modus ponens rule, this fundamental rule of classical logic, usually regarded as unassailable, could be a source of “trouble”. The title also says that there are “failures” of modus ponens that are “explored” in the thesis. The aim of this introductory chapter is to provide an explanation of the sense in which modus ponens can be taken to fail, and also to give an overview of the problems I discuss in this thesis, and which provide me with the evidence for my surprising claim that modus ponens “fails”.

A first obvious question is how we should define modus ponens, i.e., what exactly I have in mind when I say that this principle “does not hold”. Indeed, philosophers distinguish (and discuss) different versions of the principle. Labels may vary, but here are some fundamental varieties. (Note that “$\rightarrow$” stands for the indicative conditional, “$\supset$” stands for the material conditional, and *Bel* is a rational belief operator.)

Truth-preserving modus ponens. If P is true and $P \rightarrow Q$ is true, then Q is true.

Epistemic modus ponens. If $Bel(P \rightarrow Q)$, and $Bel(P)$, then $Bel(Q)$.

Classical-logic modus ponens. If $P \supset Q$ is true and P is true, then Q is true.

*Epistemic modus ponens**. If $Bel(P \supset Q)$, and $Bel(P)$, then $Bel(Q)$.

In fact, my thesis mostly deals with this last version of the principle, i.e., with epistemic modus ponens*. Notably, in chapter 3 I provide an argument to the effect that epistemic modus ponens* fails. However, I will also give an argument against classical-logical modus ponens, albeit of a more indirect kind, as the argument assumes that we should provide a unified solution to two of the puzzles I discuss. So my claim that "modus ponens fails" concerns, first and foremost, these two versions of the principle (epistemic modus ponens* and classical-logic modus ponens).

However, I will argue that epistemic modus ponens does not hold either. Indeed, throughout this thesis I assume (unless otherwise specified) that indicative conditionals are not given a material interpretation. More specifically, I assume that $Bel(P \to Q)$ entails $Bel(P \supset Q)$, but not vice versa. Now, if this is the case (i.e., if $Bel(P \to Q)$ implies $Bel(P \supset Q)$), then if epistemic modus ponens* fails, epistemic modus ponens also fails. (More details will be given, of course, in the next chapters.)

The evidence against the three principles I just mentioned (epistemic modus ponens*, classical-logic modus ponens and epistemic modus ponens) will come essentially from the discussion of two puzzles, i.e., McGee's so-called "counterexample to modus ponens" (McGee 1985) and Kyburg's Lottery Paradox (Kyburg 1961). In this introduction, I will briefly present these problems, categorize the main attempts to solve them and situate my conclusions with respect to such accounts.

2. McGee's counterexample to modus ponens

Opinion polls taken just before the 1980 election showed the Republican Ronald Reagan decisively ahead of the Democrat Jimmy Carter, with the other Republican in the race, John Anderson, a distant third. Those apprised of the poll results believed, with good reason:

[1] If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

[2] A Republican will win the election.

Yet they did not have reason to believe

[3] If it's not Reagan who wins, it will be Anderson.

Famously, the above argument has been proposed by Vann McGee (1985, p. 462), who regards it as a "counterexample to modus ponens". The author is not explicit about the "kind" of modus ponens involved in the argument, and the ambiguities of the original article have given rise to different interpretations. I will deal with the question how the example should be interpreted in chapter 1, where I suggest (based, among others, on McGee's own clarifications in a later article) that epistemic modus ponens is the principle applied in the example. However, some philosophers have given a semantic interpretation of the puzzle (one in which truth-preserving modus ponens is involved): two influential authors belonging to this category are Kolodny and MacFarlane (2010). Moreover, a recent approach, related to the so-called "informational paradigm" in logic (Yalcin 2007; Bledin 2014), provides an interpretation of the puzzle in informational terms (Bledin 2015; more below).

Be that as it may, what McGee clearly specifies in his paper is that his challenge concerns the indicative conditional, and not the material conditional. Indeed, if we interpret indicative conditionals materially (i.e., if we equate $P \rightarrow Q$ to $\sim P \vee Q$), (3) boils down to the disjunction "either Reagan wins or Anderson wins", which is intuitively very plausible, for the simple reason that Reagan will very likely win.

In what follows I present the main accounts of (1)-(3). They are, for most of them, critical of McGee's conclusion that it is a counterexample to modus ponens (no matter which version of modus ponens we take to be the one involved in the argument). Indeed, as far as its credibility as a counterexample is concerned, the reception of the argument has been rather lukewarm. "McGee (1985) and Lycan (2001) have argued against the validity […] of Modus Ponens […], but it is fair to say that their arguments have found little acclaim (Douven 2016, p. 42; I modified the author's quotation conventions in order to make them coherent with mine). However, in spite of being common, this sceptical response is far from universal. Kolodny and MacFarlane (2010), for instance, propose a semantics for indicative conditionals and deontic modals which does not validate modus ponens, and regard McGee's scenario as evidence in its favour (also see Cantwell 2008).

A first category of accounts is that of treatments that involve dissociating truth conditions, or conditions for rational belief (depending on the way we interpret (1)-(3)) from conditions for assertion. Gricean accounts belong to this category: Sinnott-Armstrong *et al.* (1986), for instance, adopt this kind of strategy; so does Lowe (1987) (more on his approach below). As is well known, the Gricean account of indicative conditionals asks us to dissociate truth conditions (or conditions for rational belief) from assertability conditions. According to Grice, the indicative conditional's truth conditions are those of the material conditional. However, if the only reason why $P \rightarrow Q$ (i.e., $\sim P \vee Q$) is true/rationally acceptable is that either P is false or Q is true, then it is not conversationally appropriate to assert the conditional. Indeed, why would we assert the disjunction $\sim P \vee Q$ if we know that $\sim P$ is true? (Grice 1989). Concerning (1)-(3), a Gricean can argue that if we assume the material conditional, we should believe (3), but should not assert it, because the only reason why we should believe (3) is that we should believe "Reagan will win".

Among the defenders of the Gricean strategy, it is worth mentioning Lowe (1987), who thinks that there are good reasons to give a material interpretation of the conditionals involved in McGee's scenario. More specifically, he contends that in (1) the nested consequent of the conditional has the form of a material conditional, and that, as a consequence, we should believe (3), although, for Gricean reasons, we should not assert it. In Lowe's view, the putative puzzle results from an ambiguity in the intended meaning of the conditional "If it's not Reagan who wins, it will be Anderson": according to Lowe, the consequent of (1) should be interpreted as a material conditional, whereas the intended interpretation of (3) is non-material. In other terms, according to the author, in McGee's example the perceived problem results from equivocation, rather than from a genuine failure of modus ponens.

A second category of criticisms of McGee's argument revolves around premise (1), which is an embedded conditional. For instance, Appiah (1987) argues that (1)-(3) is not, in fact, an instance of modus ponens, because the compound conditional in (1) actually has the form $(P \wedge Q) \rightarrow R$ (where "$\wedge$" is the conjunction symbol). Lowe (1987), whose viewpoint I just discussed, is also among those authors who focus on this feature of the argument.

McGee himself has proposed a treatment of the puzzle which belongs to this category (McGee 1985; 1989), although, of course, a treatment which is not dismissive of the example itself. McGee's approach takes as its starting point Stalnaker's semantics. According to Stalnaker, import-export (i.e., the principle according to which $P \rightarrow (Q \rightarrow R)$ is equivalent to $(P \wedge Q) \rightarrow R$) does not hold (Stalnaker 1968). The defenders of

Stalnaker's approach can thus argue that (1)-(3) does not challenge (truth-preserving) modus ponens because (1) is false. This view presupposes some sort of error theory: if import-export does not hold, (1) can be false while at the same time (1*) "If a Republican wins the election and it's not Reagan who wins, then it will be Anderson" is true. The fact that we intuitively regard (1) as true can be ascribed to our tendency to confuse (1) with (1*).

McGee's proposal consists in amending Stalnaker's logic so as to obtain that modus ponens is invalid for embedded conditionals, whereas (unlike what happens in Stalnaker's original proposal) import-export comes out valid.

A third kind of approach to McGee's scenario goes well beyond McGee's puzzle itself and is part of a broader debate about the right semantics for modal (notably epistemic and deontic) vocabulary. The protagonists of this strand of research (among which are Cantwell (2008), Kolodny and MacFarlane (2010), Yalcin (2012), Moss (2015), and Stojnić (2017)) would like to provide a unified account of the (alleged) counterexamples to modus ponens and modus tollens by focusing on the presence, in such examples, of embedded conditionals, operators like "ought to", "probably", "certainly", or adverbs like "a lot" or "gently". Indeed, embedded conditionals, epistemic operators and the like are present in (almost) all the examples one can find in the literature[1].

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[1] Even though my own approach is very distant from what is proposed in this kind of literature, I do agree that a unified treatment of the different scenarios is what we should aim at. Indeed, in this footnote I will show that from each of the most prominent scenarios other than McGee's (i.e., from Carroll's (1894), Kolodny and MacFarlane's (2010), and Yalcin's (2012) scenarios) it is possible to generate a McGee-like argument. I will also show that a Yalcin-style example (see below) can be provided starting from McGee's story.

The remarks I propose are unsystematic and have a limited scope; however, I believe that they should be taken as hints of the existence of a single structure underlying the different scenarios. That is, they should be taken as suggesting that, in spite of the superficial differences, the four abovementioned scenarios (McGee's, Carroll's, Kolodny and MacFarlane's, and Yalcin's) are all constructed in the same way.

First of all, note that (1)-(3) can be transformed into a counterexample to modus tollens (see Gauker 1994, but also Kolodny and MacFarlane 2010):

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

(4) If it's not Reagan who wins, it's not the case that Anderson will win.

(5) The winner won't be a Republican. (!)

Clearly, (5) does not seem to follow from (1) and (4), as "a Republican will win" seems perfectly acceptable if the winning Republican is Reagan.

Now consider another putative counterexample to modus tollens, that is, Yalcin's example (2012, pp. 1001-1002):

"An urn contains 100 marbles: a mix of blue and red, big and small. The breakdown:

	blue	red
big	10	30
small	50	10

A marble is selected at random and placed under a cup. This is all the information given about the situation. Against this background, the following claims about the marble under the cup are licensed:

(P1) If the marble is big, then it's likely red.
(P2) The marble is not likely red.

However, from these, the following conclusion does not intuitively follow:

(C1) The marble is not big."

However, according to Yalcin (2012, p. 1002), "[…] this conclusion would follow, were Modus Tollens […] valid".

Now, it turns out that starting from Yalcin's scenario we can generate a McGee-style argument (namely a counterexample to modus ponens involving a compound conditional):

(P3) If the marble is not red, then if it's big, it's blue.

(P4) The marble is not red.

(C2) If the marble is big, then it's blue. (!)

In spite of the premises being intuitively acceptable, the conclusion does not seem to follow. A modus tollens, compound-conditional version of Yalcin's argument can also be generated:

(P3) If the marble is not red, then if it's big, it's blue.

(P5) If the marble is big, then it's not blue.

(C3) The marble is red. (!)

Here too, modus tollens seems to fail.

Now let me present Kolodny and MacFarlane's scenario (2010, pp. 115-116).

"Ten miners are trapped either in shaft *A* or in shaft *B*, but we do not know which. Flood waters threaten to flood the shafts. We have enough sandbags to block one shaft, but not both. If we block one shaft, all the water will go into the other shaft, killing any miners inside it. If we block neither shaft, both shafts will fill halfway with water, and just one miner, the lowest in the shaft, will be killed.

Action	if miners in *A*	if miners in *B*
Block shaft *A*	All saved	All drowned
Block shaft *B*	All drowned	All saved
Block neither shaft	One drowned	One drowned

We take it as obvious that the outcome of our deliberation should be

[6] We ought to block neither shaft.

Still, in deliberating about what to do, it seems natural to accept:

[7] If the miners are in shaft *A*, we ought to block shaft *A*.

[8] If the miners are in shaft *B*, we ought to block shaft *B*.

We also accept:

[9] Either the miners are in shaft *A* or they are in shaft *B*.

But [7], [8], and [9] seem to entail

[10] Either we ought to block shaft *A* or we ought to block shaft *B*.

And this is incompatible with [6]. So we have a paradox."

Starting from Kolodny and MacFarlane's story, I can show that, once again, an argument involving an embedded conditional can be generated:

(11) If we ought to block neither shaft, then if the miners are in shaft *A*, we ought not to block shaft *A*.

(6) We ought to block neither shaft.

(12) If the miners are in shaft *A*, we ought not to block shaft *A*. (!)

We intuitively accept both (11) and (6), but we do not accept (12).

I say "almost" because there is at least one author who has presented (putative) counterexamples to modus ponens that do not display any of these features, i.e., Lycan (1993; 2001). His examples consist of complex arguments whose aim is to show that a tension is present between the validity of modus ponens and that of antecedent strengthening (i.e., the rule according to which $P \rightarrow Q$ entails $(P \wedge R) \rightarrow Q$). Note, however, that Willer's examples are also unusual in their structure, as they rely both on Thomason conditionals and on Moore's Paradox (Willer 2010).

A fourth class of (purported) solutions to McGee's puzzle is that of the contextualist accounts (see, most prominently, Gillies 2004). According to the contextualist, our intuition that modus ponens fails in (1)-(3) results from a shift in context that takes place between the premises and the conclusion. For example, Gillies (2004) suggests that the only reason why we think that (3) is false is that when evaluating (3) we "forget" that we have assumed (2).

A modus tollens version can also be provided:

(11) If we ought to block neither shaft, then if the miners are in shaft *A*, we ought not to block shaft *A*.

(7) If the miners are in shaft *A*, we ought to block shaft *A*.

(10*) We ought to block one of the shafts. (!)

Again, it seems rational to believe the two premises and to disbelieve the conclusion.

So I have shown that no matter which scenario we start from (whether McGee's, Yalcin's, or Kolodny and MacFarlane's), we can always generate a compound-conditional, McGee-style counterexample to both modus ponens and modus tollens. This seems to suggest that a same structure hides behind these three scenarios.

To reinforce my point, let me add that in McGee's scenario it is possible to proceed the other way around: we can go from an argument involving an embedded conditional (i.e., McGee's original argument) to a Yalcin-style counterexample (to modus tollens), featuring no embedded conditionals. Recall the election scenario. Both "If Reagan doesn't win, then Carter will probably win" and "Carter won't probably win" are sensible claims. However, at least if we stick to Yalcin's perspective, we should not be ready to accept the unqualified conclusion that Reagan will win.

Concerning Carroll's scenario, I will postpone my remarks to chapter 3, where I will show that it also enables us to generate a McGee-like argument. Indeed, chapter 3 will extensively deal with Carroll's scenario, which will play a fundamental role in my argument against epistemic modus ponens*.

Finally, as hinted at above, among the most recent accounts is the one by Bledin (2015). Bledin's treatment takes its starting point from the radical turn Yalcin (2007) and Bledin (2014) himself have proposed concerning the way we conceive logical inference: the authors' proposal is that logical inference is not about truth, rational belief (or acceptability), and their preservation, but rather about the preservation of information content. "An argument is *logically valid* if and only if any body of information that incorporates all of its premises also incorporates its conclusion by virtue of logical form" (Bledin 2015, p. 65).

Turning to conditionals, the content of an indicative conditional is incorporated in a body of information just in case the incorporation of the antecedent (i.e., the minimal modification of the data at our disposal based on the antecedent's information content) in our present information state makes it the case that the consequent is also incorporated.

If we consider McGee's case, it seems that the election scenario incorporates the assumption that Reagan will win. This blocks all statements featuring the negation of this claim in the antecedent of a conditional. It follows that (1) turns out to be false.

My own account of McGee's puzzle does not belong to any of these categories. In chapter 1, after clarifying that the most sensible interpretation of the example seems to be an epistemic one, I show that the standard reading according to which if we assume the material conditional the puzzle is dissolved is false. A specificity of my argument is that clarifying the relations between McGee's puzzle and the Lottery Paradox is central to it. As a result, before going back to my solution to the puzzle, and situating it with respect to its competitors, I need to introduce the second puzzle this thesis will revolve around, i.e., the Lottery Paradox.

3. The Lottery Paradox

A scenario proposed for the first time by Kyburg (1961), now become a classic, has proved that two very plausible principles about rational belief are incompatible. The very plausible principles in question are Belief Closure and the so-called "Lockean Thesis":

Belief Closure. Rational belief is closed under classical logic.

Lockean Thesis. If and only if, given her evidence, *P* is very probable (where "very probable" means "equal to or higher than a specified threshold value *t*"), then the agent should believe *P*.

The scenario can be formulated along these lines. A fair 1000-ticket lottery with one winner is organized. For each ticket, it is very unlikely that it will win, as the probability that a given ticket wins is 0.001. If $t = 0.999$, by the Lockean Thesis, we should believe, of each ticket, that it loses. By multiple applications of Belief Closure, we should also believe the conjunction "ticket n°1 will lose ∧ ticket n°2 will lose . . . ∧ ticket n°1000 will lose". However, given that the lottery is fair and has exactly one winner, the negation of "ticket n°1 will lose ∧ ticket n°2 will lose . . . ∧ ticket n°1000 will lose" has a probability of 1; therefore, by the Lockean Thesis, we should believe it. So we should believe both "ticket n°1 will lose ∧ ticket n°2 will lose . . . ∧ ticket n°1000 will lose" and its negation. As it is generally accepted that we should not believe two pairwise inconsistent sentences, we conclude that Belief Closure and the Lockean Thesis are incompatible.

It has been claimed that a large part of the classical literature on rational belief and rational degrees of belief actually reduces to a debate between those who argue that we should block the Paradox by rejecting the Lockean Thesis on the one hand, and those who reject Belief Closure on the other hand (Leitgeb 2014). That is, the protagonists of the debate on the Lottery Paradox can be essentially divided into two categories: the deniers of the Lockean Thesis and the deniers of Belief Closure. (Note that in this dissertation I will follow the standard convention of taking the deniers of the Lockean Thesis to include all the Thesis' "modifiers", i.e., all those who propose to amend the Thesis somehow.)

The category of the Lockean Thesis critics is the largest one. It includes authors like Lehrer (1975; 1990), Kaplan (1981a; 1981b; 1996), Stalnaker (1984), Pollock (1995), Ryan (1996), Evnine (1999), Nelkin (2000), Adler (2002), Douven (2002), Smith (2010; 2016), Lin and Kelly (2012a; 2012b), and Kelp (2017). Three particularly salient subcategories can be isolated within this group. A classical (and somehow old-fashioned) option consists in modifying the Lockean Thesis by supplementing it with a defeat condition. Central to this idea is that the so-called "lottery propositions" deserve a special treatment, which can be ensured by keeping the Lockean Thesis unchanged apart from adding to it a clause which applies as selectively as possible to "lottery propositions". The clause prevents the failure of Belief Closure when such propositions come into play. A typical proponent of this strategy is Pollock (1995; but see also Ryan 1996 and Douven 2002), who has introduced the concept of "collective defeat". According to Pollock, classical deductive consistency is a necessary condition for justification: whenever the agent realizes that her beliefs are inconsistent, she loses justification for (some or all of) the beliefs included in the inconsistent set; that is, (some or all of) her judgements incur a "collective defeat".

Another very classical type of strategy restricts the Lockean Thesis to a very weak, somehow trivial version of the Thesis itself, i.e., to a version of it in which the threshold t is equal to 1. Levi (1980), Gärdenfors (1986), Van Fraassen (1995), Arló-Costa (2001), and Arló-Costa and Parikh (2005) defend this strategy. In spite of having been, as Roorda (1995) calls it, "the received view", it is not a really popular option anymore.

A third subcategory within the critics of the Lockean Thesis, one which is much more widespread nowadays, may be dubbed "the statistical evidence approach" (see, most prominently, Nelkin 2000, and Smith 2010; 2016). The common trait behind these proposals is the idea that evidence which is merely statistical or probabilistic does not suffice for rational belief (or rational acceptance, or justification). For instance, Smith's normalcy view (2010; 2016) dismisses probabilistic considerations and instead explains justification in terms of what is normal on one's evidence. In Smith's perspective, an instructive scenario is the one in which we find out that we have won the lottery: as surprising as we may find this outcome from a probabilistic viewpoint (the probability that our ticket would win was very low), this event needs no further explanation; after all, someone had to win, i.e., it is a perfectly *normal* outcome. In other terms, before the draw, a lottery ticket owner is not justified in believing that his ticket will lose, on simple statistical grounds. Indeed, were he to find out that he won, this would not require any explanation in terms of some otherwise reliable process having gone wrong[2].

A last subgroup within the Lockean Thesis deniers includes those philosophers who propose a modification of the Thesis itself which does not simply amount to adding a defeat clause, and at the same time, unlike the authors in the previous subcategory, do not completely discard the Thesis either. For instance, Leitgeb's stability theory of belief (2014; 2015) suggests that we should supplement the Lockean Thesis with the condition that the probability of *P* should remain higher than 0.5 when the agent gains evidence which is consistent with *P*. Arló-Costa and Pedersen (2012) defend a similar proposal, under the label "high probability cores". Lin and Kelly (2012a; 2012b) are also part of this subgroup: following Levi (1996), they argue that the Lottery Paradox can be avoided by modifying the threshold constraint so that it is sensitive to the ratios between the agent's degrees of belief[3].

Let us now leave the Lockean Thesis deniers (or modifiers) and turn to our second big category, i.e., to the enemies of Belief Closure. Important examples of philosophers in this group are Kyburg (1961), Klein (1985), Foley (1992), Hawthorne and Bovens (1999), Kyburg and Teng (2001), Christensen (2004), Hawthorne and Makinson (2007), Kolodny (2007), and Easwaran and Fitelson (2015).

One of the most recent and influential pieces in this area is Easwaran and Fitelson's 2015 paper, where the authors explore a number of formal coherence constraints for full beliefs alternative to Belief Closure (Easwaran and Fitelson 2015). As an inspiration for their project, they cite Foley's view that "[a]t first glance, the requirement that we avoid recognizable inconsistency seems little enough to ask in the name of rationality. It asks only that we avoid certain error. It turns out, however, that this is far too much to ask" (Foley 1992, p. 186). A specificity of their exploration is that they ground it on a compelling analogy between coherence requirements for full beliefs on the one hand and decision-theoretic principles of rational choice on the other hand. I will come back to their proposal in the thesis' conclusion.

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[2]For a sceptical viewpoint on statistical evidence as a basis for justification also see Buchak 2014 and Staffel 2016.

[3]For an overview of this kind of proposals see Staffel (forthcoming).

Although in a spirit which is quite different from Easwaran and Fitelson's, Hawthorne and Makinson (2007) also examine a range of alternatives to Belief Closure, in the form of weaker, non-classical logics of belief.

An interesting subset in the broader category of the Belief Closure opponents is that of those authors who do not simply reject Belief Closure, but also deny the very existence of any sort of coherence requirement of rationality for full beliefs. Christensen (2004), for example, is an advocate of this radical stance. He contends that only partial beliefs are subject to a formal, synchronic requirement of rationality: probabilism. Famously, Kolodny's standpoint (2007) is even more radical, as he denies in general that there may be coherence requirements of rationality, for any attitude whatsoever (full belief, partial belief, or otherwise).

I have said that students of the Lottery Paradox can be essentially divided into two categories: the Lockean Thesis deniers and the Belief Closure deniers. However, a third category is sometimes mentioned (see, for instance, Logins (forthcoming)). According to the authors in this (putative) third category, it does not really follow from the lottery scenario that we should hold two pairwise inconsistent judgements. Indeed, some of them argue that the perceived problem results from one or more terms in the formulation of the Paradox being ambiguous. Others blame, instead, our tendency to neglect the fact that the truth conditions for attributions of rational belief are actually context-dependent. Typically, both kinds of philosophers seem to trace back the ambiguity (or the context-dependence) to the Lockean Thesis (or to some related principle, depending on the specific version of the Paradox they discuss, which may differ in some respects from the one I consider here: see Lewis (1996), Cohen (1998) and Logins (forthcoming); given its strong contextualist component, Leitgeb's account (2014; 2015) is sometimes regarded as part of this category too).

Note, however, that these views actually boil down to the claim that the Lockean Thesis/the related principle should be abandoned, or at least modified, i.e., to our second big category of accounts. Indeed, what the proponents of this approach are saying is that the Lockean Thesis/the related principle does not hold unrestrictedly, but only on some of its readings, or in certain contexts.

Let me conclude this overview of the proposed solutions to the Lottery Paradox by mentioning, besides the Lockean Thesis deniers, the Belief Closure deniers, and the contextualists, a fourth class of responses, one which I will not discuss in this thesis and only briefly mention here. I am talking about the very radical strategy which consists in rejecting the whole framework the Lottery Paradox is based on. More precisely, I am thinking of the strategy which consists in denying that the concept of full belief has any reality or epistemological interest. This "hard core Bayesian" approach is discussed in Foley 1992. For an eliminativist approach outside the Bayesian paradigm see Sorensen's reconstruction of Quine's stance in Sorensen 2018.

4. *This thesis*

Now that I have given an overview of the problems I will tackle, I can go on to introduce my dissertation's content.

This thesis will comprise three chapters, corresponding to three research papers. The papers are self-contained; each of them has its own abstract and its own reference list. They have been conceived independently of each other and can thus be read independently. However, a common moral is drawn in the thesis' conclusion.

In the first chapter ("From McGee's puzzle to the Lottery Paradox") I show that (a slight variant of) the election scenario McGee uses to generate his attack to modus ponens is just a lottery scenario. One main conclusion ensues, according to which we should expect a unified solution to both McGee's puzzle and the Lottery Paradox. This conclusion defies the existing accounts of McGee's problem.

In the second chapter ("Cut-off points for the rational believer") I argue that the Lottery Paradox is in fact a (probabilistic) Sorites. One important consequence of reformulating the Lottery Paradox in soritical terms is that the most popular solution to it, i.e., denying the Lockean Thesis, becomes less attractive. The reason is that keeping Belief Closure entails a counterintuitive view, which I dub "the cut-off point view". However, I also show that rejecting Belief Closure is not enough. More precisely, it is not enough to solve a puzzle which is closely related to Kyburg's and which puts us before the following dilemma: we should either accept the cut-off point view or reject *classical-logic* modus ponens. That is, not merely Belief Closure, but a fundamental principle of classical logic.

In the third chapter ("Against Belief Closure") I give a straightforward argument to the conclusion that we should solve the Lottery Paradox by denying Belief Closure. I build on my previous result that (a slight variant of) McGee's election scenario is a lottery scenario. Indeed, this result implies that the sensible ways to deal with McGee's scenario are the same as the sensible ways to deal with the lottery scenario: we should either reject the Lockean Thesis or Belief Closure. I demonstrate, then, that a McGee-like argument (which is just, in fact, Carroll's barbershop paradox (1894)) can be provided in which the Lockean Thesis plays no role: this proves that denying Belief Closure is the right way to deal with both McGee's scenario and the Lottery Paradox.

In other terms, my dissertation's main conclusion is that we should account for some long-standing problems (McGee's puzzle, but also the Lottery Paradox and Carroll's barbershop paradox) by dropping Belief Closure. However, I also claim that rejecting Belief Closure is not enough. Indeed, I conclude my thesis by showing that the only way to provide a unified solution to both the Lottery Paradox and a new variant of it I propose in chapter 2 is by denying classical-logic modus ponens (and not merely Belief Closure).

Let me conclude with some clarifications on the way my solutions to McGee's puzzle and the Lottery Paradox should be situated with respect to the categories of proposals I have presented in sections 2 and 3.

Concerning the Lottery Paradox, the situation is quite straightforward: my approach belongs to the second broad category above, that of the Belief Closure deniers. Regarding McGee's puzzle, recall what I have said concerning the assumption I will make throughout this thesis that $Bel(P \rightarrow Q)$ entails $Bel(P \supset Q)$. As already specified, a consequence of this reasonable (and popular) assumption is that if epistemic modus ponens* fails, epistemic modus ponens also fails. Now, if my claim that epistemic modus ponens* fails is correct, and if I am right in arguing that the principle in (1)-(3) is epistemic modus ponens, this means that the problem revealed by McGee's scenario is much broader than one concerning embedded conditionals (second type of solutions in section 2), or modal operators and the like (third type of solutions). Nor has the problem something to do in particular with context shifts (fourth category of solutions). Or rather, more precisely, even supposing that in (1)-(3) there is a specific issue with context shift, this would not undermine my general result that epistemic modus ponens does not hold. It could perfectly be the case that in (1)-(3) a context shift takes place, or that a Gricean strategy (first category of solutions) can account for (1)-(3) specifically. However, from my analysis of McGee's scenario we are able to draw the general conclusion that both epistemic modus ponens* and epistemic modus ponens fail.

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CHAPTER 1. From McGee's puzzle to the Lottery Paradox.

Abstract. Vann McGee has presented a putative counterexample to modus ponens. After clarifying that McGee actually targets an epistemic version of such a principle, I show that, contrary to a view commonly held in the literature, assuming the material conditional as an interpretation of the natural language conditional "if … then …" does not dissolve the puzzle. Indeed, I provide a slightly modified version of McGee's famous election scenario in which (1) the relevant features of the scenario are preserved and (2) both (epistemic) modus ponens and modus tollens fail, even if we assume the material conditional. I go on to note that in the modified scenario (which I call "the restaurant scenario") (epistemic) conjunction introduction does not hold. More specifically, I show that the restaurant scenario is actually a version of the lottery scenario Kyburg uses in his Lottery Paradox. One main conclusion ensues, according to which we should expect a unified solution to both McGee's puzzle and the Lottery Paradox. This conclusion defies the existing accounts of McGee's puzzle.

1. The election scenario

In a well-known article, McGee (1985, p. 462) has proposed the following scenario:

Opinion polls taken just before the 1980 election showed the Republican Ronald Reagan decisively ahead of the Democrat Jimmy Carter, with the other Republican in the race, John Anderson, a distant third. Those apprised of the poll results believed, with good reason:

[1] If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.

[2] A Republican will win the election.

Yet they did not have reason to believe

[3] If it's not Reagan who wins, it will be Anderson.

McGee (1985) speaks of a "counterexample to modus ponens". In fact, the question whether, and in which sense, (1)-(3) deserves such a label, remains, as of today, highly controversial. Still, there is at least one claim on which students of McGee's example seem to agree, i.e., the claim that the puzzle is dissolved if we assume a material interpretation of the natural language conditional "if … then …". Indeed, if we assume the material conditional, we should interpret (3) as the disjunction "either Reagan wins or Anderson wins", which is very plausible, for the simple reason that Reagan is hot favourite. That is, as McGee himself specifies, if we interpret (1)-(3) according to the material conditional, we believe both the premises and the conclusion (McGee 1985).

Interestingly, starting from McGee's scenario it is also possible to generate what looks like a counterexample to modus tollens (see Gauker 1994, but also Kolodny and MacFarlane 2010):

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.
(4) If it's not Reagan who wins, it's not the case that Anderson will win.
(5) The winner won't be a Republican.

(5) does not seem to follow from (1) and (4). Indeed, "a Republican will win" is very plausible, as long as the winning Republican is Reagan.

In this case as well, it seems that if we assume the material conditional the puzzle disappears. Indeed, we can only apply modus tollens to (1) and (4) if (4) and the nested consequent of (1) contradict each other. However, if we assume the material conditional (4) and the nested consequent of (1) cannot be seen as contradictory.

In this paper I will challenge the idea that modus ponens does not fail in McGee's scenario if we assume the material conditional. Indeed, in what follows I show that even if we give a material interpretation of the conditionals in (1)-(3) and (1)-(5) McGee's puzzle is not dissolved. Before that, however, I will have to provide some clarifications about the nature of McGee's example.

2. McGee on his "counterexample"

Truth-preserving modus ponens may be defined as the principle according to which if P is true and $P \rightarrow Q$ is also true, then Q is true as well. (In what follows, $P \rightarrow Q$ will denote the indicative conditional, and $P \supset Q$ will denote the material conditional. I will also assume that the indicative conditional is not given a material interpretation.)

For an argument to be a counterexample to this principle, it must be the case that *P* and $P \rightarrow Q$ are true and *Q* is false. The title of McGee's paper ("A counterexample to modus ponens") may at first suggest that McGee regards (1)-(3) as a counterexample in this sense. However, in the body of the article, McGee describes (1)-(3) (and the other, structurally similar, examples he provides) as cases in which "one has good grounds for believing the premises of an application of modus ponens but yet one is not justified in accepting the conclusion" (p. 462). Moreover, McGee's example revolves around what "those apprised of the poll results" *had reason to believe*, and not about truth.

A later paper by the author contains some important elucidations. In McGee 1989, he explicitly admits that his examples concern the preservation of acceptability, versus truth preservation: "Such examples show that *modus ponens* fails in English [...] More precisely, the examples show that *modus ponens* does not preserve warranted acceptability. As I [McGee] pointed out (1985, p. 463) and as Sinnot-Armstrong, Moor, and Fogelin (1986) have emphasized, the examples have no direct bearing on the question whether *modus ponens* is truth-preserving" (McGee 1989, p. 512 and fn. 20).

That is, McGee seems to target a principle that may be formulated along these lines:

Epistemic modus ponens. If $Bel(P \rightarrow Q)$, and $Bel(P)$, then $Bel(Q)$, where *Bel* is a rational belief operator.

If we now turn to (1)-(5), it seems that, by McGee's own criteria, we should regard it as a failure of the following schema:

Epistemic modus tollens. If $Bel(P \rightarrow Q)$, and $Bel(not\ Q)$, then $Bel(not\ P)$.

As for rational belief (or acceptability) itself[1], McGee does not provide many details about the way it should be defined in order for his examples to go through. However, in McGee 1985, he mentions high probability as a reason for believing the premises of his examples, and low probability as a reason for disbelieving their conclusions. That is, he

[1]Even though there may be subtle differences between the concept of (rational) acceptance and that of (rational) belief, these are not relevant for my purposes; as a result, in this paper, I will take the two terms to be synonyms.

speaks of such reasons in terms of likelihood ("[i]t is more likely that […]"; "[…] it is virtually certain that […]"; "[…] it is entirely certain that […]"; McGee 1985, p. 163). Although he does not endorse it explicitly, he seems to adopt a principle called "Lockean Thesis":

Lockean Thesis. If and only if, given one's evidence, *P* is very probable (where "very probable" means "equal to or higher than a specified threshold value *t*"), then one should believe *P*. (Or equivalently: if and only if, given one's evidence, *P* is very probable (where "very probable" means "equal to or higher than a specified threshold value *t*"), then it is rational to believe *P*.)[2]

In what follows I argue that McGee's scenario gives us reasons to believe that, under the assumption that the Lockean Thesis holds, epistemic modus ponens and modus tollens fail, even if natural language conditionals are given a material interpretation (i.e., even if $P \rightarrow Q$ and $P \supset Q$ are taken to be equivalent). More precisely, under the assumption that the Lockean Thesis holds, McGee can be taken to show that the two following principles of the logic of belief are falsified (where ⊃ is, as we already know, the material conditional and ~ is the negation symbol):

*Epistemic modus ponens**. If $Bel(P \supset Q)$, and $Bel(P)$, then $Bel(Q)$.

*Epistemic modus tollens**. If $Bel(P \supset Q)$, and $Bel(\sim Q)$, then $Bel(\sim P)$.

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[2] In what follows I will apply the Lockean Thesis to (among others) indicative conditionals. That is, I will make use of the principle that if and only if an indicative conditional $P \rightarrow Q$ is very probable, then one should believe, or accept, $P \rightarrow Q$. Now, in the wake of Lewis' famous triviality results (Lewis 1976), a number of philosophers have argued that indicative conditionals are not propositions. So I avoid committing myself to the controversial claim that indicative conditionals have propositional content. In other terms, when I will talk about the probability of $P \rightarrow Q$ I will not be talking about the probability of $P \rightarrow Q$ *being true*. Following Adams (1975), I will take such a probability to be the probability of *Q* conditional on *P*. That is, in what follows, the Lockean Thesis (when applied to indicative conditionals) will read: if and only if the probability of *Q* conditional on *P* is high, then one should believe, or accept, $P \rightarrow Q$ (provided that $P \rightarrow Q$ is a simple conditional and that $p(P) \neq 0$, see section 3 below).

Furthermore, it could be objected that it is not so obvious that "one should believe *P*" and "it is rational to believe *P*" are equivalent. And indeed, a whole debate has arisen, in recent years, on the nature of the relations between epistemic obligations and rational belief. However, for this paper's purposes I will ignore this debate and the niceties it involves, for nothing I will say here will depend on such niceties.

Of course, the failure of epistemic modus ponens* (or modus tollens*) entails the failure of a more general principle, often called "Belief Closure":

Belief Closure. Rational belief is closed under classical logic.

In the literature on rational belief and rational degrees of belief it is commonly held that the Lockean Thesis and Belief Closure cannot be jointly satisfied. Indeed, joint acceptance of Belief Closure and the Lockean Thesis gives rise to the so-called Lottery Paradox (Kyburg 1961). In this paper, I will show that the latter is intimately linked to McGee's election scenario.

3. The Argument Schema

In this paper I will make the reasonable assumption that if we are justified in believing (2) above it is because of its high probability (for recent papers that make a similar assumption, see Stern and Hartmann 2018 as well as Neth 2019; as specified above, evidence for this assumption can also be found in McGee's original paper). Essentially, I will assume that McGee endorses the principle I called "Lockean Thesis". However, note that, in spite of my hypothesis being reasonable, it is actually not necessary for my purposes to rely on the claim that McGee indeed made this assumption. What only needs to be the case for this article's purposes is that there is a plausible interpretation of McGee's puzzle in which the Lockean Thesis is assumed. The same holds for the assumption that epistemic modus ponens is the principle involved in (1)-(3): no matter what McGee really had in mind, there is a reasonable and easily accessible interpretation of his puzzle that involves epistemic modus ponens: this is all I need for my aims here.

So let us go on and assume that the reason why we should believe (2) is that it is highly probable. It follows that if (1)-(3) is to be taken as a potential counterexample to (epistemic) modus ponens, the reason why we should believe (1) must be the same (that is, its high probability); and the reason why we should not believe (3) must be that its probability is not high enough.

One popular way of interpreting the conditionals in McGee's example is compatible with the author assuming the Lockean Thesis. According to this interpretation, (1) has a probability of 1 because, supposing that a Republican wins, the conditional probability that Anderson will win given that Reagan doesn't win is 1. In this view, (2) is also likely, because the unconditional probability that a Republican will win is high. However, the conditional probability that Anderson wins, given that Reagan doesn't win, is low, that is, (3) is unlikely.

This interpretation of the premises can be made more precise by adopting what is often called, in the literature on conditionals, "Adams' Thesis"; that is, by assuming that the acceptability of an indicative conditional is equal to the probability of its consequent given its antecedent (see Adams 1975). In the literature on conditionals, many versions of the Thesis can be found, involving subtle differences; however, the one below should be enough for my purposes. Note that it only holds for simple conditionals, $P \rightarrow Q$, such that $p(P) \neq 0$:

Adams' Thesis. The acceptability of $P \rightarrow Q$ is equal to the probability of Q given P (i.e., of Q conditional on P)[3].

Stern and Hartmann (2018) also adopt an account of the conditionals in (1)-(3) based on Adams' Thesis. However, as they observe, the latter does not provide us with an analysis of (1), as Adams' Thesis only applies to simple conditionals and (1) is an embedded conditional. Indeed, consider an indicative conditional of the form $P \rightarrow (Q \rightarrow R)$: "If we were to apply [Adams' Thesis] to this conditional, it would seem that $Acc(P \rightarrow (Q \rightarrow R)) = p((R|Q)|P)$, but there is no such probability expression as $p((R|Q)|P)$" (Stern and Hartmann 2018, p. 608; here and below, I modified the authors' notation to make it coherent with mine). However, there is such a probability expression as $p(R|P \wedge Q)$. Here, I will follow Stern and Hartmann (2018) in assuming that choosing to analyse $Acc(P \rightarrow (Q \rightarrow R))$ as $p(R|P \wedge Q)$ is safe. This step can be motivated by a plausible principle of conditional logic, i.e., import-export, according to which $P \rightarrow (Q$

[3] As specified in footnote 2, this approach eludes the so-called triviality results, for we are only interested here in the acceptability (or believability) conditions for indicative conditionals, and not in the question whether indicative conditionals are propositions (see Stern and Hartmann 2018).

$\rightarrow R$) is equivalent to $(P \wedge Q) \rightarrow R$[4]. An acceptability version of the principle can be formulated as below (see, again, Stern and Hartmann 2018).

Acceptability Import-Export. $Acc(P \rightarrow (Q \rightarrow R)) = Acc((P \wedge Q) \rightarrow R)$

By Acceptability Import-Export and Adams' Thesis, we obtain that $Acc(P \rightarrow (Q \rightarrow R)) = p(R|P \wedge Q)$[5]. That is, our attitudes towards (1), (2), and (3) are represented as indicated in (1'), (2'), and (3') respectively:

(1') $p(R|P \wedge Q)$
(2') $p(P)$
(3') $p(R|Q)$

I will call this way of representing our attitudes towards McGee's argument's premises and conclusion "the Argument Schema". According to it, both (1) and (2) have a high degree of acceptability (as (1') and (2') are both high), whereas (3) is only acceptable to a low degree (because (3') is low). The Argument Schema is clearly compatible with McGee assuming the Lockean Thesis: by the latter, we should (fully) accept both (1) and (2), while we should (fully) reject (3).

Note that I do not mean to claim that (1')-(3') is the only representation of our attitudes towards (1)-(3) compatible with McGee assuming the Lockean Thesis. However, (1')-(3') certainly is one very natural way of representing them, which I will thus take as the main reference here.

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[4]Import-export is usually regarded as a very natural principle. For example, (1) clearly seems equivalent to "If a Republican wins the election and it's not Reagan who wins, then it will be Anderson". However, import-export also has its critics; for instance, it is invalid in Stalnaker's theory of conditionals (see Stalnaker 1968). Recently, Mandelkern (forthcoming) has argued in favour of restricting the validity of import-export, based on a surprising result concerning the relation between import-export and classical conjunction. A survey of the putative counterexamples to import-export for indicative conditionals can be found in Khoo and Mandelkern 2019.

[5]As the authors specify, "this follows only when [Acceptability Import-Export] is restricted to settings where $p(P \wedge Q) > 0$ (since [Adams' Thesis] applies only in these settings)" (Stern and Hartmann 2018, fn. 15).

4. Setting the stage

I mean to challenge the idea that epistemic modus ponens* does not fail in McGee's scenario after all. My key point here will be that even if we give a material interpretation of the conditionals in (1)-(3) and (1)-(5) McGee's puzzle is not dissolved.

Let us postulate a material interpretation of the conditionals in McGee's scenario: McGee's story allows us to reason from the following three premises, which are the material conditional versions of (1), (2) and (4). (Here again, $\supset$ stands for the material conditional, ~ for the negation and $\wedge$ for the conjunction. Moreover, X is "Carter loses the election" (i.e., "a Republican wins"), Y is "Reagan loses" and Z is "Anderson loses".)

(a) $X \supset (Y \supset {\sim}Z)$
(b) X
(c) $Y \supset Z$

Of course, there is a clear difference between premise (a) and premises (b) and (c): (a) has a probability of 1 whereas (b) and (c), in spite of being highly probable, are not certain. (The reason why (a) has a probability of 1 is, of course, that it is equivalent to "either Carter will win or Reagan will win or Anderson will win", i.e., to the assumption that someone will win the election, which does seem to have a probability of 1 for all relevant purposes.) However, by the Lockean Thesis, we should also believe both (b) and (c).

Now, consider the following proof:

(1) $X \supset (Y \supset {\sim}Z)$ [(a)]
(2) X [(b)]
(3) $Y \supset Z$ [(c)]
(4) $Y \supset {\sim}Z$ [epistemic modus ponens* 1,2]
(5) $Y \supset (Z \wedge {\sim}Z)$ [epistemic conjunction of the consequents*[6] 3,4]

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[6]The inference rule that lets us infer $P \rightarrow (Q \wedge R)$ from $P \rightarrow Q$ and $P \rightarrow R$ is often called "CC" in the literature on conditional reasoning. However, as "CC" sometimes stands for a different principle (i.e., it is

(6) ~*Y* [epistemic modus tollens* 5]

What the derivation above shows is that rational belief in (a), (b) and (c) entails rational belief in the conclusion that Reagan will win (i.e., in the conclusion that ~*Y* is the case). This conclusion seems unproblematic: after all, "Reagan will win" is a perfectly plausible conclusion. Or at least, it is a perfectly plausible conclusion given that I am assuming the Lockean Thesis. Indeed, "Reagan will win" is highly probable, i.e., by the Lockean Thesis, we should believe it.

Now, I will show that an argument can be provided that has a structure similar to McGee's, but where it is obviously the case that we should not believe ~*Y*. Indeed, one can provide at least one argument where, unlike what happens in McGee's original scenario, the probability of ~*Y* is less than *t*, where *t* is the threshold assumed for rational belief. In the next section, I propose an example that satisfies this requirement.

Yet, you may say, even assuming that one can come up with such an example, we need not regard epistemic modus ponens* as responsible for the puzzling conclusion, and epistemic modus tollens* is not necessarily guilty either. Actually, there seem to be three possible culprits, which correspond to the three inference rules used in the derivation: epistemic modus ponens*, epistemic modus tollens* and epistemic conjunction of the consequents*. The next section will reveal the identity (identities) of the culprit(s).

5. The restaurant scenario

I am sitting in a restaurant with my Italian friend Pasquale. I know that Pasquale always orders one of the day's specials. Today's specials are pizza, pasta and roast beef. I know that Pasquale loves both pizza and pasta, and that he does not like roast beef very much. I estimate that there is a 0.4 probability that Pasquale will have pizza, a 0.4 probability that he will have pasta and a 0.2 probability that he will have roast beef.

Assume the material conditional and set $t = 0.6$. In this context, I should believe both (6) "If Pasquale doesn't have pizza, then he will have pasta" and (7) "Pasquale won't

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used as an abbreviation for "cautious cut"), I decided to choose a more explicit label. (See Douven (2016) and Unterhuber (2013) for more about standard labels for rules of conditional logic.)

have pizza”. Indeed, they both have a probability of at least 0.6. Now, from (6) and (7), using epistemic modus ponens*, I should infer (8) “Pasquale will have pasta”. But (8) only has a probability of 0.4; so I should not believe (8), that is, epistemic modus ponens* fails.

Let us now turn to epistemic modus tollens*. By the Lockean Thesis, I should believe (9) “Pasquale won’t have pasta”, which has a probability of 0.6. Now, from (9) and (6) I should draw, by epistemic modus tollens*, the conclusion that (10) “Pasquale will have pizza”. But (10) only has a probability of 0.4; therefore, I should not believe (10), i.e., epistemic modus tollens* fails.

So we have both a failure of epistemic modus ponens* and a failure of epistemic modus tollens*. Or, more precisely, given $t = 0.6$, in the above examples rational belief is not closed under modus ponens* and modus tollens*, respectively.

It turns out that the arguments (6)-(8) and (6)-(10) above allow us to identify the culprit(s) among the inference rules used in the derivation I proposed in section 4. Indeed, let us assume that X is “Pasquale doesn’t have pizza”, Y is “Pasquale doesn’t have pasta” and Z is “Pasquale doesn’t have roast beef”. As in McGee’s scenario, in the restaurant scenario we should also believe the relevant instances of (a), (b) and (c). According to the derivation in section 4, however, from (a), (b) and (c) we should draw the conclusion that $\sim Y$ is the case, i.e., that Pasquale will have pasta. But this is absurd, as “Pasquale will have pasta” only has a probability of 0.4.

In McGee’s example $\sim Y$ was highly probable: by the Lockean Thesis, we had to believe it. In the restaurant example, instead, the probability of $\sim Y$ is low, which entails that, by the Lockean Thesis, we should reject it.

The derivation proposed in section 4 involved three different logical principles: epistemic modus ponens*, epistemic modus tollens* and epistemic conjunction of the consequents*. Thanks to the restaurant variant of McGee’s scenario I were able to propose a version of this same derivation in which an unacceptable conclusion is generated. Now, thanks to the restaurant variant we also know who is the culprit (or rather, who are the culprits) for this conclusion. Indeed, in (6)-(8) and (6)-(10) epistemic conjunction of the consequents* does not play any role: (6)-(8) only involves epistemic modus ponens*, and (6)-(10) only involves epistemic modus tollens*. This shows that epistemic conjunction of the consequents* is not needed in order to generate the unwelcome conclusion; instead, using epistemic modus ponens* or epistemic modus tollens* is enough to derive it.

6. From McGee's puzzle to the Lottery Paradox

It can be noted that epistemic modus ponens* and modus tollens* are not the only logical principles that fail in the restaurant scenario. Indeed, epistemic conjunction introduction[7] does not hold either: given t = 0.6, we should believe "Pasquale won't have pizza", "Pasquale won't have pasta" and "Pasquale won't have roast beef", but we should not believe the conjunction of these three propositions (actually, we should believe its negation). That is, the failure of epistemic modus ponens* and modus tollens* is not the only relevant feature of the restaurant scenario. Indeed, it can be shown that the restaurant example has the same structure as the Lottery Paradox (Kyburg 1961).

Famously, Kyburg's puzzle goes as follows: suppose that I participate in a fair 1000-ticket lottery with exactly one winner. In this context, I have very good reasons to believe that my ticket will lose. Indeed, the probability that it will win is 0.001. I believe the same about the ticket of the person next to me, and about all the other tickets. Nonetheless, if I apply this reasoning to every ticket from n°1 to n°1000 I reach the conclusion that all tickets will lose, which is false.

In both Kyburg's scenario and mine, a disjunction must be satisfied: in the lottery scenario, one ticket must win; in the restaurant example, it is assumed that Pasquale will pick one of the day's specials. However, at the same time, in both scenarios we should not believe any of the disjuncts: in the lottery scenario each of the 1000 tickets is unlikely to win; in the restaurant scenario, none of the day's specials is likely to be Pasquale's choice. That is, the restaurant scenario is a lottery scenario, at least if we adopt the standard definition of a lottery scenario as a scenario where, given t higher than 0.5, the Lockean Thesis and epistemic conjunction introduction come into conflict,

[7]Epistemic conjunction introduction can be defined as the principle according to which if $Bel(P)$, and $Bel(Q)$, then $Bel(P \wedge Q)$.

i.e., one should end up believing a contradiction. This definition clearly applies to the restaurant scenario, as in it a probability of 0.6 is assumed as a threshold for rational belief.

So if we assume the above definition of a lottery scenario, then a slight modification of McGee's original example leads to a version of the Lottery Paradox, namely one with three tickets and a probability threshold for rational belief of 0.6. All one has to do in order to obtain such a scenario is to decrease the probability of "Reagan will win"; more specifically, one has to assign to "Reagan will win" a probability lower than the threshold: this is enough to generate a probability distribution where the probability of each of the three disjuncts is below *t*. That is, this is enough to generate a lottery scenario (provided, of course, that some specific proportions are respected between the probabilities of the propositions; the restaurant scenario exemplifies such proportions).

I would like to stress that such a modification (i.e., the one that takes us from McGee's original scenario to the restaurant story) is an innocent one: the fact that in the original scenario "Reagan will win" is very likely can be regarded as a contingent feature of the scenario itself. That is, transforming McGee's original story into the restaurant story by no means betrays the original scenario.

Clearly enough, in the restaurant scenario the relevant properties of McGee's example are preserved: this happens because even if in the restaurant scenario the probability of ~*Y* ("Pasquale will have pasta", corresponding to "Reagan will win" in the election story) is low, the probability of *X* ("Pasquale won't have pizza"/"A Republican will win") is high. That is, even if in the election scenario the probability of "Reagan will win" were to be lower than it is, McGee should still have to regard (1)-(3) and (1)-(5) as failures of modus ponens and modus tollens respectively. Removing the contingent fact that one of the disjuncts in the original scenario has a probability higher than the threshold simply allows us to gain further insight into the puzzle.

The point can be made even clearer as follows. Let, again, *X* be "Carter loses the election" (i.e., "a Republican wins"), *Y* be "Reagan loses" and *Z* be "Anderson loses". McGee's original argument has the following form:

$X \rightarrow (Y \rightarrow {\sim}Z)$

X

$\therefore Y \rightarrow {\sim}Z$

Consider now the restaurant scenario and assume that X = "Pasquale doesn't have pizza", Y = "Pasquale doesn't have pasta" and Z = "Pasquale doesn't have roast beef": the probability of "Pasquale won't have pizza" is high, even though the probability of "Pasquale will have pasta" is low. (Note that the Argument Schema can be applied in the restaurant case as well: the probability that Pasquale will have roast beef, given that he does not have pizza or pasta (i.e., $p(R|P \wedge Q)$) is 1, the probability that he will not have pizza (i.e., $p(P)$) is 0.6, while the probability that he will have roast beef given that he does not have pasta (i.e., $p(R|Q)$) is very low (much lower than 0.6). So assuming t = 0.6, we should believe the argument's premises, and should disbelieve its conclusion.)

Interestingly, the above (namely, the fact that X can be likely, even if ${\sim}Y$ is not) undermines those accounts of McGee's puzzle according to which (1)-(3) is not a modus ponens argument, but rather contains a fallacy of equivocation of certain kinds. Paoli (2005), for instance, argues that (1)-(3) is not an instance of modus ponens because "A Republican wins" should be given a different interpretation in (1) and (2). An essential role in distinguishing the two interpretations is played by the fact that, according to the author, in (2) "A Republican will win" simply stands for "Reagan will win". Fulda (2010) also claims that (1)-(3) is not a modus ponens, but rather an enthymeme, in which "Regan will win" is the suppressed premise. We now see that both attempts to dismiss McGee's argument are misguided, as they focus on a very contingent feature of the argument: the fact that in McGee's original scenario one of the disjuncts ("Reagan will win") seems rationally acceptable to begin with.

7. The need for a unified solution

My main conclusion will be that it is impossible to solve McGee's puzzle without thereby solving the Lottery Paradox, and the other way around. In this section, I will address one potential objection to this conclusion. The objection is based on a difference between the restaurant story and McGee's original story, i.e., on the fact that in the restaurant scenario both "kinds" of modus ponens (epistemic modus ponens and epistemic modus ponens*) can only fail if *t* is relatively low, namely, if it is equal to 0.6. It goes as follows: what makes (1)-(3) interesting is that each of its two premises seems to have a very high probability (higher than 0.6), and still epistemic modus ponens seems to fail, although not for the material conditional, but rather for the indicative conditional (i.e., epistemic modus ponens seems to fail, unlike epistemic modus ponens*).

In other terms, the complaint is this: if we assume that a probability of 0.6 is not sufficient for rational belief (whereas a greater probability does suffice) it is no longer clear whether epistemic modus ponens and modus ponens* would still fail. And if these principles were unscathed for *t* higher than 0.6, then we would still be allowed to regard McGee's original puzzle as a genuine puzzle as far as *indicative* conditionals are concerned, but there would be no puzzle about the restaurant scenario.

A first reply to these remarks is, quite simply, that it is very rarely the case that defenders of the Lockean Thesis commit to a specific value for *t*. Actually, there seem to be only very few authors who have a strong preference for a specific threshold[8].

This notwithstanding, let us go on and assume that the objection can be thoroughly articulated, so that in the restaurant scenario epistemic modus ponens and modus ponens* do not really fail, because a higher probability is needed for rational belief. After all, it does not seem unreasonable to think that a probability of 0.6 is not (at least not always) sufficient for rational belief (think of the defenders of a contextualist version of the Lockean Thesis, who may argue that in the restaurant context there are specific reasons to reject a 0.6 threshold). However, in fact, epistemic modus ponens and modus ponens* do fail for *t* higher than 0.6.

As a far as epistemic modus ponens is concerned, Stern and Hartmann (2018) have proved that it is always possible to find $p(R|P \wedge Q)$ and $p(P)$ such that they are both high, while at the same time $p(R|Q)$ is low (provided that $p(P)$ is not 1; for the very minor probabilistic constraints that $p(R|P \wedge Q)$ and $p(P)$ impose on $p(R|Q)$ see Stern and Hartmann 2018, fn. 18). Concerning epistemic modus ponens*, instead, it is

[8]One of them is Achinstein (2001), who claims that a probability greater than 0.5 is both necessary and sufficient for rational belief. Recently, Shear and Fitelson (2019) have argued that the inverse of the golden ratio ($\phi^{-1} \approx 0.618$) should be regarded as a non-arbitrary bound on the belief threshold.

actually a well-known fact that failure of Belief Closure is not limited to cases in which *t* is equal to 0.6. As long as an appropriate number of tickets is chosen, failure of Belief Closure can always be observed, no matter the specific threshold *t* (the only condition is that *t* must be strictly between 0.5 and 1). Kyburg's original scenario is a case in point: in it *t* is much greater than 0.6, but Belief Closure still fails. As a result, no matter the kind of lottery scenario we are considering (with 3 tickets, 1000 tickets, or with a still different number of tickets), accusing the specific threshold value adopted (0.6, 0.999, etc.) of being responsible for the failure of Belief Closure seems hopeless. As I will spell out below, this point extends to epistemic modus ponens* specifically.

A precision, though: I am not denying that a solution to both Kyburg's original Paradox and its restaurant version could be provided in contextualist terms. However, such a solution would involve a *modification* of the Lockean Thesis, whose original definition would be rejected. The most famous proposal of this kind is found in Leitgeb (2014; 2015), according to which we should supplement the Lockean Thesis with the condition that the probability of *P* should remain higher than 0.5 when the agent learns new information compatible with *P*.

I conclude that the sensible ways to deal with McGee's scenario are the same as the sensible ways to deal with the lottery scenario. In both cases, we seem to have two main options: giving up either the Lockean Thesis or Belief Closure. (Coherently with what is standard in the literature, and as just suggested concerning Leitgeb's proposal, I regard those authors who propose to modify the Lockean Thesis as belonging to the group of the Lockean Thesis deniers.)

Note that, if we decided to reject Belief Closure, we would be forced to deny at least three principles: epistemic modus ponens*, epistemic modus tollens* and epistemic conjunction introduction. Indeed, I showed that in the restaurant scenario the three of them fail. In fact, this also holds for Kyburg's scenario: even though the Lottery Paradox is generally presented as involving (epistemic) conjunction introduction, we can generate lottery-like paradoxes by using other principles (see Douven 2016). It is instructive to see briefly how.

Let us consider Kyburg's original scenario. In it, we should believe "Ticket n°1 wins ∨ ticket n°2 wins… ∨ ticket n°1000 wins" (where ∨ is the disjunction symbol), which is equivalent to “(Ticket n°1 loses ⊃ ticket n°2 loses… ⊃ ticket n°999 loses) ⊃ ticket n°1000 wins”. We should also believe, about each of the tickets between n°1 and n°999, that it will lose. However, we should not believe that ticket n°1000 will win (in fact, we should believe that it will lose). That is, epistemic modus ponens* fails.

In this scenario, epistemic modus tollens* does not hold either. Indeed, we should accept

(Ticket n°1 loses) ⊃ ticket n°2 loses… ⊃ ticket n°999 loses ⊃ ticket n°1000 wins.

Ticket n°2 loses.

…

Ticket n°999 loses.

Ticket n°1000 loses.

Nevertheless, we should reject

Ticket n°1 will win

and accept its negation. That is, in Kyburg’s original scenario, exactly as in the restaurant scenario, there are at least three ways to generate an unacceptable conclusion: using epistemic modus ponens*, epistemic modus tollens*, or (as in the original version of Kyburg’s puzzle) epistemic conjunction introduction.

8. Back to McGee’s original argument

What I just said concerns the way we should deal with the restaurant scenario and the lottery scenario in general. But what can we say concerning specifically the original version of McGee’s argument?

In fact, the dilemma raised by the Lottery Paradox applies in a straightforward manner to (1)-(3). That is, if we reject either the Lockean Thesis or Belief Closure McGee’s original argument is blocked.

Suppose that we deny the Lockean Thesis: we would not be compelled to accept (1) and (2) anymore, which would block the derivation of (3). (This is a straightforward consequence of the fact that the Lockean Thesis is an implicit assumption in McGee’s puzzle, or at least in the version of McGee’s puzzle I am considering here (see my understanding of (1)-(3) in section 3 above); therefore, if we renounce the Lockean Thesis, the puzzle vanishes.)

If, instead, we denied Belief Closure (i.e., as we have seen, at least epistemic modus ponens*, epistemic modus tollens* and epistemic conjunction introduction), this would also solve the puzzle. The reason is the following: suppose that we reject epistemic modus ponens*; it seems that, *a fortiori*, we should reject epistemic modus ponens. This is because it is natural to regard the indicative conditional as stronger than the material conditional; i.e., it is generally assumed that if we should believe an indicative conditional, we should also believe the corresponding material conditional. One main argument to this conclusion goes as follows: suppose that we rationally believe the negation of $P \supset Q$, i.e., $P \wedge {\sim}Q$; it seems natural to infer that we rationally believe the negation of the corresponding indicative conditional. This reasonable assumption entails that if epistemic modus ponens* (modus tollens*) turned out to fail, epistemic modus ponens (modus tollens) would also fail. That is, if we rejected modus ponens*, McGee's original puzzle would also be solved, as (1)-(3) would not be an instance of a valid logical schema anymore, whether we assume the material conditional or a stronger conditional.

Of course, the same holds for (1)-(5): suppose that we reject epistemic modus tollens*: (1)-(5) would not instantiate a valid principle anymore, whether, again, we assume the material conditional or a stronger conditional.

So I showed that the two puzzles (McGee's and the Lottery) have the same structure; i.e., that a slight modification of McGee's election scenario is a lottery scenario. This entails that the two scenarios put us before the same dilemma: should we deny the Lockean Thesis or Belief Closure? I then noted that, no matter which of these two principles we choose to deny, McGee's original argument is blocked. In other terms, exactly as the Lottery Paradox, McGee's 1985 paper can be taken to show that under the assumption that the Lockean Thesis holds, Belief Closure fails. Of course, the condition for this conclusion to follow is that my hypothesis according to which in McGee's original example both epistemic modus ponens and the Lockean Thesis are involved is true. However, as already specified, even though other interpretations of (1)-(3) are perhaps possible, any author tackling McGee's problem should account at least for this very plausible and easily accessible understanding of the argument. A consequence of this fact is that any student of McGee's puzzle should give up either the

Lockean Thesis or Belief Closure. In other terms, any account of McGee's puzzle that does not involve either giving up the Lockean Thesis or Belief Closure is unsatisfactory. The interesting and important point here is that the vast majority of the existing accounts of McGee's problem do not address the rejection of either principle[9].

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[9] Among others, this includes the accounts of the puzzle by Appiah (1987), Lowe (1987), Piller (1996), Katz (1999), Bennett (2003), Gillies (2004), Paoli (2005), Cantwell (2008), Fulda (2010), Kolodny and MacFarlane (2010), Edgington (2014), Moss (2015), Stojnić (2017), Schulz (2018), Stern and Hartmann (2018), and Neth (2019).

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CHAPTER 2. Cut-off points for the rational believer.

Abstract. I show that the Lottery Paradox is just a (probabilistic) Sorites, and argue that this should modify our way of looking at the Paradox itself. In particular, I focus on what I call "the cut-off point problem" and contend that this problem, well known by students of the Sorites, ought to play a key role in the debate on Kyburg's puzzle.

Very briefly, I show that, in the Lottery Paradox, the premises "ticket n°1 will lose", "ticket n°2 will lose"… "ticket n°1000 will lose" are logically equivalent to soritical premises of the form "buying *n* tickets does not allow me to win the lottery ⊃ buying *n* + 1 tickets does not allow me to win the lottery" (where "⊃" is the material conditional). As a result, failing to believe, for some ticket, that it will lose comes down to introducing a cut-off point in a chain of soritical premises. I call the view that, for some ticket, we should not believe that it loses the "the cut-off point view".

One important consequence of this reformulation of the Lottery Paradox is that the most popular solution to the puzzle, i.e., denying the Lockean Thesis, becomes less attractive. The reason is that keeping Belief Closure entails the (rather counterintuitive) cut-off point view. In order to make the counterintuitive character of this view emerge as clearly as possible I consider a heap variant of the original lottery scenario: in this scenario (which is generally used in the context of a different puzzle, viz. the Sorites) the worrying consequences of the cut-off point view become evident.

Finally, I demonstrate that denying Belief Closure is not enough. More precisely, it is not enough to solve a puzzle which is closely related to Kyburg's and which puts us before the following dilemma: we should either accept the cut-off point view or reject classical-logic modus ponens. That is, not merely Belief Closure, but a fundamental principle of classical logic.

1. The lottery scenario

In the literature on rational belief and rational degrees of belief it is usually claimed that the two following principles cannot be jointly satisfied:

Belief Closure. Rational belief is closed under classical logic.

Lockean Thesis. If and only if, given her evidence, *P* is very probable (where "very probable" means "equal to or higher than a specified threshold value *t*"), then the agent should believe *P*.

Indeed, joint acceptance of Belief Closure and the Lockean Thesis gives rise to the well-known Lottery Paradox, first proposed by Kyburg (1961).

Consider a fair 1000-ticket lottery with exactly one winner. The probability, for each ticket, that it will win is very low, i.e., it is 0.001. It follows that if $t = 0.999$, then, by the Lockean Thesis, one should believe, of each ticket, that it will lose. By multiple applications of Belief Closure, one should also believe the conjunction “ticket n°1 will lose ∧ ticket n°2 will lose . . . ∧ ticket n°1000 will lose”. However, given that the lottery is fair and has exactly one winner, the negation of “ticket n°1 will lose ∧ ticket n°2 will lose . . . ∧ ticket n°1000 will lose” has a probability of 1; therefore, by the *Lockean Thesis*, one should believe it. So one should believe both “ticket n°1 will lose ∧ ticket n°2 will lose . . . ∧ ticket n°1000 will lose” and its negation. As it is generally accepted that one should not believe two pairwise inconsistent sentences, we conclude that Belief Closure and the Lockean Thesis are incompatible.

As Leitgeb (2014) puts it, we can classify a huge part of the classical literature on rational belief according to which principle is dropped: for instance, Isaac Levi (1967) accepts Belief Closure but rejects the Lockean Thesis, while Henry Kyburg (1961) accepts the Lockean Thesis and rejects Belief Closure.

The most widespread solution to the puzzle, however, consists in denying the Lockean Thesis: among the authors who adopt this option are Lehrer (1975; 1990), Kaplan (1981a; 1981b; 1996), Stalnaker (1984), Pollock (1995), Ryan (1996), Evnine (1999), Nelkin (2000), Adler (2002), Douven (2002), Smith (2010; 2016; 2018), and Kelp (2017).

Philosophers who believe, instead, that the Lottery Paradox puts pressure on Belief Closure include Klein (1985), Foley (1992), Hawthorne and Bovens (1999), Kyburg and Teng (2001), Christensen (2004), Hawthorne and Makinson (2007), Kolodny (2007), Easwaran and Fitelson (2015).

Leitgeb’s view seems to be an exception to this categorization (deniers of the Lockean Thesis vs. deniers of Belief Closure). Indeed, Leitgeb (2014; 2015) defends a form of contextualism which, he contends, allows us to keep both the Lockean Thesis and Belief Closure. I will come back to his proposal in the last section of the paper.

In this paper, I show that the Lottery Paradox is just a (probabilistic) Sorites, and argue that this should modify our way of looking at the Paradox itself. In particular, I focus on what I will call “the cut-off point problem” and contend that this problem, well known by students of the Sorites, ought to play a central role in the debate on Kyburg’s puzzle.

Very briefly, I will show that, in the Lottery Paradox, the premises "ticket n°1 will lose", "ticket n°2 will lose"…"ticket n°1000 will lose" are logically equivalent to soritical premises of the form "buying *n* tickets does not allow me to win the lottery ⊃ buying $n + 1$ tickets does not allow me to win the lottery" (where "⊃" is the material conditional). As a result, failing to believe, for some ticket, that it will lose comes down to introducing a cut-off point in a chain of soritical premises. I call the view that, for some ticket, we should not believe that it loses the "the cut-off point view".

One important consequence of this reformulation of the Lottery Paradox is that the most popular solution to the puzzle, i.e., denying the Lockean Thesis, becomes less attractive. The reason for this is that keeping Belief Closure entails the (rather counterintuitive) cut-off point view. In order to make the counterintuitive character of this view emerge as clearly as possible I will consider a heap variant of the original lottery scenario: in this scenario (which is generally used in the context of a different puzzle, viz. the Sorites) the worrying consequences of the cut-off point view become evident.

Finally, I demonstrate that denying Belief Closure is not enough. More precisely, it is not enough to solve a puzzle which is closely related to Kyburg's and which puts us before the following dilemma: we should either accept the cut-off point view[1] or reject classical-logic modus ponens. That is, not merely Belief Closure, but a fundamental principle of classical logic.

The next section will be devoted to reformulating Kyburg's original puzzle so that its connection with the Sorites becomes obvious. However, before that, one more preliminary remark is needed. In presenting the Lottery Paradox, I used the expression "Belief Closure". I could have been more specific, though: in Kyburg's puzzle a specific principle is applied, i.e., the closure of rational belief under conjunction introduction. From now on, I will call such a principle "epistemic conjunction

[1]As it will become clear in what follows, in this further puzzle the expression "cut-off point" refers to something a bit different from what it denotes in Kyburg's original puzzle. Specifically, in this additional puzzle we have a cut-off point if and only if one in a series of soritical conditionals of the form "I should believe that buying *n* tickets does not allow me to win the lottery ⊃ I should believe that buying $n + 1$ tickets does not allow me to win the lottery" is false.

introduction", in order to distinguish this epistemic version of the conjunction introduction schema from its non-epistemic, classical-logic counterpart. Concerning the other logical principles, my choice of the labels will be coherent with the one I made in this thesis' introduction; i.e., when referring to the closure of rational belief under classical-logic modus ponens and modus tollens, I will use the labels "epistemic modus ponens*" and "epistemic modus tollens*" respectively.

2. The Wide Scope Paradox

Consider again our fair 1000-ticket lottery, with $t = 0.999$. Also assume that tickets are numbered from 1 to 1000. By the Lockean Thesis, in this scenario one should believe, for instance, "ticket n°1 wins ∨ ticket n°2 wins…∨ ticket n°999 wins", as its probability is 0.999. That is, one should believe that the set which includes tickets from n°1 to n°999 contains the winning ticket. At the same time, one should not believe "ticket n°1 wins ∨ ticket n°2 wins…∨ ticket n°998 wins", as this sentence only has a probability of 0.998. That is, we should suspend our judgement on whether the winning ticket is to be found between ticket n°1 and ticket n°998 (included).

From now on, instead of saying that one should (or should not) believe the sentence "ticket n°1 wins ∨ ticket n°2 wins… ∨ ticket n°*n* wins" I will say that one should (or should not) believe "buying *n* tickets allows me to win the lottery". That is, I will assume that the probability of "buying *n* tickets allows me to win the lottery" equals the probability of "ticket n°1 wins ∨ ticket n°2 wins…∨ ticket n°*n* wins". Of course, as a matter of fact someone who buys 999 tickets could buy the tickets from n°2 to n°1000 and not those from n°1 to n°999. This simplification will not affect my point, though, and it will make my presentation smoother[2].

Let me now introduce another logical equivalence that will be of much help in what follows. "Ticket n°1 will lose" (that is, given the above, "buying 1 ticket does not allow me to win") is just equivalent to a negated conjunction, i.e., to the negation of "buying 0 tickets does not allow me to win ∧ buying 1 ticket allows me to win". That is,

[2]Instead of "buying *n* tickets allows me to win the lottery" I could have used a different formulation, i.e., for instance, "someone who buys *n* tickets wins the lottery". The only reason why in what follows I will not adopt this alternative formulation (or a still different one) is that the use, in the former, of the first person makes things easier to formulate.

“ticket n°1 will lose” (or “buying 1 ticket does not allow me to win”) is equivalent to “buying 0 tickets does not allow me to win ⊃ buying 1 ticket does not allow me to win”. More generally, it can be noted that “ticket n°*n*+1 will lose” = “~(buying *n* tickets does not allow me to win ∧ buying *n* + 1 tickets allows me to win)” = “buying *n* tickets does not allow me to win ⊃ buying *n* + 1 tickets does not allow me to win” = “~(the winning ticket is not among the tickets from 1 to *n* included ∧ it is among the tickets from 1 to *n* + 1 included)”, where ~ is the negation symbol.

What is the point of introducing these equivalences? First, these equivalences can be used to show that starting from a lottery scenario we can generate a (probabilistic) Sorites. I will call this argument “Wide Scope Paradox” (WSP), in order to distinguish it from a related argument that I will label “Narrow Scope Paradox”, and that I present in section 5. I have called it “WSP” because in it the rational belief operator has wide scope over the conditional premises; in the Narrow Scope Paradox instead, as we will see, the belief operator has narrow scope over the antecedent and the consequent of the same conditionals.

Consider the following sentences. Remember that we have set $t = 0.999$.

(1) Buying 1000 tickets allows me to win the lottery.
(2) Buying 0 tickets does not allow me to win the lottery.

Also consider 1000 sentences of the form:

Buying *n* tickets does not allow me to win the lottery ⊃ buying *n* + 1 tickets does not allow me to win the lottery.

For convenience, I will call the above conditionals “P-conditionals”. The paradox consists in the fact that by multiple applications of epistemic modus ponens* we conclude (3), which contradicts (1):

(3) Buying 1000 tickets does not allow me to win the lottery. (!)

Note that we are bound to believe all the premises. Indeed, (1) and (2) both have a probability of 1, and each of the P-conditionals has a probability of 0.999. Why 0.999? Consider, for instance, the conditional "buying 499 tickets does not allow me to win ⊃ buying 500 tickets does not allow me to win": it is equivalent to "buying 499 tickets allows me to win ∨ buying 500 tickets does not allow me to win". The reason why this disjunction has a probability of 0.999 is that its probability is calculated by adding the probability of "buying 499 tickets allows me to win", i.e., 0.499, to the probability of "buying 500 tickets does not allow me to win", i.e., 0.5; the probabilities must be added here because the sentences we are dealing with are mutually inconsistent. Another way of reaching the same result is by focusing on what would make the P-conditionals false: in order to falsify one of them, it is both necessary and sufficient that the winning ticket is exactly ticket n°n+1, which has a probability of 0.001.

So one should believe (1), (2) and each of the conditional premises; nonetheless, one should not believe (3), which has a probability of 0, whence the problematic outcome: by epistemic modus ponens*, we should believe two pairwise inconsistent sentences, i.e., (1) and its negation.

Now, it is not simply the case that we can generate a (probabilistic) Sorites starting from the lottery scenario. Actually, the premises of the original puzzle by Kyburg and those of WSP are equivalent.

Recall the logical equivalences I have introduced. It can be noted that (1) is equivalent to the sentence that, in the original version of the puzzle, says that the 1000-ticket lottery is fair and has one winner (i.e., to the disjunction "ticket n°1 wins ∨ ticket n°2 wins…∨ ticket n°1000 wins"). (2) corresponds, instead, to a premise which is left implicit in the original argument, i.e., to a premise which is trivially true in the scenario and which says that the lottery is not a 0-ticket lottery which has a winner. Finally, "buying n tickets does not allow me to win the lottery ⊃ buying n + 1 tickets does not allow me to win the lottery" is equivalent to "ticket n°n+1 will lose"; that is, the P-conditionals are equivalent to the premises of Kyburg's original argument that say that ticket n°1 will lose, ticket n°2 will lose, etc.

So WSP is just a reformulation of the standard Lottery Paradox[3]. However, one clear difference between Kyburg's original argument and WSP is that in the original argument epistemic conjunction introduction is used, whereas in WSP epistemic modus ponens* is applied. Now, it turns out that it is possible to reformulate WSP so that epistemic conjunction introduction is used.

Consider the conjunction "buying 0 tickets does not allow me to win ∧ buying 1000 tickets allows me to win"; call it (C). It is equivalent to (D):

(D): (buying 0 tickets does not allow me to win ∧ buying 1 ticket allows me to win) ∨ (buying 1 ticket does not allow me to win ∧ buying 2 tickets allows me to win)… ∨ (buying 999 tickets does not allow me to win ∧ buying 1000 tickets allows me to win).

As we have seen, given t = 0.999, one should believe both (1) and (2). So by epistemic conjunction introduction one should believe (C); but then one should believe (D). However, each disjunct of (D) only has a probability of 0.001, so one should believe its negation. Now, by applying epistemic conjunction introduction to ~(buying 0 tickets does not allow me to win ∧ buying 1 ticket allows me to win), ~(buying 1 ticket does not allow me to win ∧ buying 2 tickets allows me to win)… ~(buying 999 tickets does not allow me to win ∧ buying 1000 tickets allows me to win), one should believe the negation of (D). That is, by epistemic conjunction introduction, one should believe both a sentence and its negation, exactly as in Kyburg's original version of the puzzle.

It could be asked what is the point of introducing this reformulation of the Lottery Paradox, i.e., of introducing WSP. Actually, I think that WSP is interesting in itself, as it shows that the Lottery Paradox just is a (probabilistic) Sorites. However, this is not all

[3]This strengthens a point by Dorothy Edgington (1992; 1997). Indeed, Edgington claims that the Lottery Paradox and the Sorites are structurally similar, so that a common strategy should be applied to solve both. However, unlike mine, her view presupposes the acceptance of a degree-theoretic framework (i.e., the idea that there are such things as degrees of truth). Moreover, and most importantly, my claim is stronger than Edgington's: the Lottery Paradox is not merely similar to the Sorites Paradox; the Lottery Paradox just *is* a (probabilistic) Sorites.

there is to WSP. The main reasons why in what follows I will use WSP instead of the original formulation of the Paradox are matters of clarity for my current purposes. Indeed, using WSP makes it more natural to rerun the Lottery Paradox starting from a heap scenario; as a result, the advantages of switching to a heap scenario are more apparent and the main point of the paper will emerge more clearly.

3. Setting the threshold at 1

If we reject the Lockean Thesis the Lottery Paradox is blocked. However, it is traditionally assumed that accepting $t = 1$ allows us to keep both the Lockean Thesis and Belief Closure (among the authors who argue that we should accept $t = 1$ are Gärdenfors (1986), Van Fraassen (1995), Arló-Costa (2001), Arló-Costa and Parikh (2005)). One main consequence of this solution is that we are forced to accept the cut-off point view, i.e., as specified above, we are bound to disbelieve[4], for some ticket, that it will lose (in the original scenario, for *each* ticket, we are bound to disbelieve that it will lose). In order to better evaluate the consequences of this view, let us put aside for a moment the standard lottery scenario and use instead a classical example from the literature on vagueness, i.e., the heap example.

Note that replacing the lottery scenario with a different scenario (and more specifically, a heap scenario) is a perfectly legitimate move. Indeed, the Lottery Paradox is not a puzzle about lotteries. On the contrary, the point of the Paradox is a general one, and consists in showing that the Lockean Thesis (with t short of 1) and Belief Closure are incompatible. Moreover, we are perfectly allowed to assign probabilities to the premises of the Sorites, based on our evidence[5].

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[4]Throughout this paper, by "disbelieving *P*" I mean "not believing *P*", which includes suspending one's judgement on *P* and believing the negation of *P*.

[5]Clearly, this way of looking at the Lottery Paradox is at odds with those accounts of the latter which argue that we should deny the Lockean Thesis because evidence which is "merely probabilistic" is not enough for rational belief (see, for instance, Nelkin 2000, or Smith 2010, 2016 and 2018). These accounts usually focus on the original version of the Paradox or, when they consider variants of it, they focus on cases in which the relevant evidence is statistical.

However, it can be noted that the formulation I have given of the Lockean Thesis leaves it open whether the kind of evidence the agent relies on is "merely probabilistic" or not. That is, in my formulation of the Lockean Thesis, which I take to be standard, evidence need not be "merely probabilistic". A consequence of this fact is that, as I say in the body of the paper, we should regard the original version of the Lottery

For reminder, here is the classical, textbook version of the Sorites Paradox.

Consider (1') and (2').

(1') 1000 grains are a heap[6].

(2') 0 grains are not a heap.

Also consider 1000 sentences of the form:

n grains are not a heap ⊃ n + 1 grains are not a heap

Let us call these sentences "P'-conditionals". (1'), (2') and the P'-conditionals seem true. However, multiple applications of *classical-logic* modus ponens let us infer the following puzzling conclusion:

(3') 1000 grains are not a heap. (!)

I would like to stress that (1')-(3') is about truth, not rational belief: as students of the Sorites classically put it, the argument's premises are intuitively true whereas the conclusion is intuitively false. However, it is easy to transform "the classical Sorites" into a version of WSP: as it will become clear below, one only needs to assign probabilities to the premises of (1')-(3'), set an appropriate threshold for belief, and apply epistemic modus ponens* instead of classical-logic modus ponens.

So how should we assign probabilities to the Sorites' premises? Keep in mind that we are concerned here with the probabilities a rational agent assigns to sentences based on the relevant evidence. Now, given that we know both that 1000 grains are a heap and

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Paradox as simply *illustrating* the conflict between the Lockean Thesis and Belief Closure. This is an important point, as it seems clear that the conflict does not vanish if we consider evidence which is not "merely probabilistic". In other terms, reducing the Lottery Paradox to a problem concerning statistical evidence alone does not do justice to the challenge it illustrates, which is much more general.

[6]Here "1000" could be replaced with any sufficiently high number. This of course holds for the Lottery Paradox too: instead of a 1000-ticket lottery we could consider a 5000-ticket lottery, or a 1 million-ticket lottery, etc.

that 0 grains are not a heap, it seems that we should assign a probability of 1 to (1') and (2') respectively. What about the P'-conditionals? Clearly, we cannot assign to all of them a probability of 1. The reason is simple: consider the conjunction "0 grains are not a heap ∧ 1000 grains are a heap", which I will call (C'). It is equivalent to (D'):

(D') (0 grains are not a heap ∧ 1 grain is a heap) ∨ (1 grain is not a heap ∧ 2 grains are a heap)… ∨ (999 grains are not a heap ∧ 1000 grains are a heap).

Given that she knows that (C'), a rational agent should assign to (C') a probability of 1. But then she should also assign a probability of 1 to (D'); i.e., the sum of the probabilities of the disjuncts of (D') must be 1. Therefore, it cannot be the case that all the P'-conditionals (which are just the negations of the disjuncts of (D')) have a probability of 1. That is, it cannot be the case that all the disjuncts of (D') have a probability of 0. Still, each of them can be assigned a very low probability. If we do so (i.e., if we assign each of them a very low probability), WSP can be formulated starting from a heap scenario. I will call this alternative formulation "Soritical Wide Scope Paradox" (SWSP). It goes as follows: assume that t is very high, but short of 1. By the Lockean Thesis, we should believe (1'), (2'), and each of the P'-conditionals. However, again by the Lockean Thesis, we should not believe (3'), which has a probability of 0; in fact, we should believe its negation, which is just (1'). That is, we end up having to believe both (1') and its negation. (Exactly as for WSP, we can provide a version of SWSP in which epistemic conjunction introduction is used, instead of epistemic modus ponens*. Indeed, we should believe (D') (which has a probability of 1), but if its disjuncts are assigned very low probabilities, we should believe the negation of each of them, so that, by epistemic conjunction introduction, we should believe the negation of (D'). That is, we should believe both (D') and its negation[7].)

One more precision is in order. We are supposing that the disjuncts of (D') are all assigned low probabilities. That is, the probability distribution we are considering is a

[7]For the sake of completeness, note that, both in the case of SWSP and of WSP, epistemic modus tollens* could also be used to generate the unacceptable conclusion. Indeed, as we have seen, by the Lockean Thesis we should believe (1') ((1) in WSP) and all the conditional premises. However, by multiple applications of epistemic modus tollens*, we should believe the negation of (2') (the negation of (2) in WSP), which has a probability of 0.

"uniform" one (one in which all the disjuncts have the same probability), or at least one in which all the disjuncts have a probability greater than 0. However, for the paradox to arise, we are not at all obliged to assign our probabilities this way. On the contrary, we can assume a probability distribution in which some disjuncts have a probability of 0 (in fact, as many as we wish, provided that the probabilities of the disjuncts in D' sum up to 1).

Here we come to a crucial point. Consider again SWSP. Exactly as WSP, we could block it either by dropping the Lockean Thesis or by dropping Belief Closure. However, as announced in the title of this section, we could be willing to keep both principles by setting $t = 1$. However, if we assume that $t = 1$ we are forced to conclude that, for at least one n, it is not the case that one should believe "n grains are not a heap ⊃ $n + 1$ grains are not a heap". Indeed, for at least one n, the probability of "n grains are not a heap ∧ $n + 1$ grains are a heap" must be greater than 0 (otherwise the probability of (C') would be 0, whereas, by hypothesis, it is 1). (Of course, if there is only one n such that the probability of "n grains are not a heap ∧ $n + 1$ grains are a heap" is greater than 0, then, given that the probability of (C') has to be 1, the probability of that disjunct must be 1.)

Clearly, if our evidence is distributed uniformly over the disjuncts of D' we should neither believe the conditional "0 grains are not a heap ⊃ 1 grain is not a heap", nor any of the other P'-conditionals. Conversely, if we have absolutely no evidence for some of the disjuncts (i.e., if we assign a zero probability to, say, the first twenty disjuncts), the cut-off point will come "later" in the distribution (e.g., we should believe "19 grains are not a heap ⊃ 20 grains are not a heap", but we should not believe "20 grains are not a heap ⊃ 21 grains are not a heap"). Anyway, what matters is that in both cases we are forced to disbelieve at least one of P'-conditionals.

So we have seen that the cut-off point view follows from the acceptance of $t = 1$. Now, it seems clear that in order to solve WSP one must also solve SWSP, which is a simple variant of WSP, in which a heap scenario is used instead of a lottery one.

Many authors already reject the cut-off point view for the original lottery scenario; notably, all those who defend the Lockean Thesis with t short of 1. Switching to a heap scenario raises an interesting problem for those who accept the Lockean Thesis with t =

1, as perhaps some of them will find the outcome that we should disbelieve at least one of the P'-conditionals unpalatable. Of course, "some" does not mean "all of them". Still, the fact that if we set $t = 1$ we must accept the cut-off point view with respect to SWSP is something we should keep in mind when evaluating a solution to the Lottery Paradox, and this was the point I wanted to make in this section.

4. Rejecting the Lockean Thesis altogether

As we know, a possible way of solving WSP consists in accepting the Lockean Thesis with t short of 1 and rejecting Belief Closure. Another possible way out of the puzzle is keeping both the Lockean Thesis and Belief Closure, while setting t at 1. However, we have seen that the latter option forces us to adopt the cut-off point view with respect to (S)WSP.

Let us now turn to the third and last option, which consists in rejecting the Lockean Thesis across the board and accepting, instead, a different norm of belief. The alternative norms I will consider are the most popular competitors of the Lockean Thesis, i.e., the truth norm and the knowledge norm of belief. In this section, I show that accepting either of these alternative norms still forces us to endorse the cut-off point view with respect to (S)WSP.

The truth norm may be defined as the norm according to which we should believe *P* if and only if *P* is true. During its history, the truth norm has been precisified in various ways; however, the subtleties of the different definitions are not relevant here[8]. As far as WSP is concerned, this norm provides a clear verdict: the argument has one false premise. Indeed, there is a ticket (the winning one) of which we should not believe that it will lose. That is, one of the P-conditionals is false.

Regarding the knowledge norm, i.e., the norm according to which we should believe *P* if and only if we know *P*[9], it also provides a straightforward solution to WSP: we

[8]Among others, Wedgwood 2002, Boghossian 2003, Shah 2003, Gibbard 2005, Bykvist and Hattiangadi 2007, Engel 2007, and Thomson 2008 contain stimulating remarks on the way the truth norm should be made precise.

[9]The knowledge norm is adopted by a growing number of epistemologists; its most famous defender is Timothy Williamson (see Williamson 2000). Note, though, that in Williamson's work the defence of such

should believe both (1) and (2), as we know that if we buy all the tickets we will win, and that if we do not buy any ticket we will lose. However, we should disbelieve all the P-conditionals. This is because we do not know, of each ticket, that it will lose.

Now consider SWSP. If we accept the truth norm, there are only two ways out of the puzzle: one consists in embracing the cut-off point view, the other in denying Belief Closure[10]. The problem that faces the truth norm's advocate is the following: is there one grain such that when added to a collection of grains which is not a heap turns it into a heap? If the answer is yes, then, by the truth norm, there is one *n* such that we should not believe the conditional "*n* grains are not a heap ⊃ *n* + 1 grains are not a heap". If, instead, she believes that there is not such a *n*, she must reject Belief Closure.

Similar remarks hold for the knowledge norm's defender, even though the problem she faces is slightly different: is there one *n* such that we know that *n* grains are not a heap but we do not know that *n* + 1 grains are not a heap? Depending on her answer, the knowledge norm's advocate will be either endorsing the cut-off point view or denying Belief Closure.

However, we have seen that SWSP is an innocent variant of WSP, and that, as a result, we should give a unified answer to the two puzzles. This means that, given that both the truth norm's and the knowledge norm's advocates endorse the cut-off point view with respect to WSP, they should also endorse it with respect to SWSP. In other words, if we accept either the truth norm or the knowledge norm of belief, we are bound to accept the cut-off point conclusion with respect to SWSP.

Of course, rejecting the Lockean Thesis does not automatically entail that we should endorse either the truth norm or the knowledge norm. Indeed, among the most classical proposals concerning the Lottery Paradox is that of amending the Lockean Thesis by adding a defeat clause (Pollock 1995 is a good example of this kind of approach). Other

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a norm is only implicit and must be derived from the author's defence of the knowledge norm of assertion.

[10]Of course, the truth norm's advocate may also solve SWSP by claiming that (2') is false or that (3') is true. The claim according to which (3') is true has been defended in the literature on the Sorites by Peter Unger (1979). However, I will not deal with these very unpopular options here. And anyway, as it will become clear below, these options are not relevant for my argument's purposes.

(more recent) accounts do not simply add to the Lockean Thesis a defeat condition, but propose an outright modification of the threshold constraint (see Lin and Kelly 2012a and 2012b. Actually, Leitgeb's account (2014; 2015) can also be regarded as part of this category, as Leitgeb proposes to modify the Lockean Thesis to the effect that the probability of *P* should remain higher than 0.5 conditional on any proposition consistent with it; see Staffel (forthcoming)). However, these proposals also entail the cut-off point view. So, if I am right in claiming that the solution to WSP should be extended to SWSP, the advocates of these accounts should also endorse the cut-off point view with respect to SWSP[11]. More generally, given that disbelieving (1) or (2) does not seem to be an option, it appears that if we want to preserve Belief Closure we are forced to disbelieve at least one of the P-conditionals.

Let us take stock: we can keep Belief Closure only if we accept the cut-off point view as far as SWSP is concerned. Indeed, in order to provide a unified solution to WSP and SWSP we only have three options:

(i) accepting the Lockean Thesis with *t* short of 1, which implies rejecting Belief Closure.

(ii) accepting the Lockean Thesis with $t = 1$, which allows us to keep Belief Closure, but forces us to accept the cut-off point view.

(iii) rejecting the Lockean Thesis across the board, which also allows us to keep Belief Closure, but forces us to endorse the cut-off point view.

[11] Another option consists in denying the Lockean Thesis while adopting, at the same time, an eliminativist approach to the notion of full rational belief. That is, it consists in rejecting the whole framework in which the Lottery Paradox is formulated. According to this very radical approach, which I will put aside here, talk about full belief should be entirely replaced by talk about degrees of belief. For a discussion of this option see Foley 1992.

5. The Narrow Scope Paradox

In this last section I will show that, despite the appearances, option (i), i.e., accepting the Lockean Thesis with *t* short of 1, does not allow us to avoid the cut-off point conclusion (at least with respect to the argument I am going to present).

Consider (4) and (5).

(4) I should believe that buying 1000 tickets allows me to win the lottery.
(5) I should believe that buying 0 tickets does not allow me to win the lottery.

Also consider 1000 conditionals of the form "I should believe that buying *n* tickets does not allow me to win the lottery ⊃ I should believe that buying *n* + 1 tickets does not allow me to win the lottery".

Repeated applications of *classical-logic* modus ponens lead to (6):

(6) I should believe that buying 1000 tickets does not allow me to win the lottery. (!)

I will call this puzzle "Narrow Scope Paradox" (NSP). As explained above, the reason for this label is that in NSP the rational belief operator has narrow scope over the antecedent and the consequent of the P-conditionals (whereas in WSP it has wide scope over them).

Now, suppose that we adopt option (i), i.e., that we accept the Lockean Thesis with *t* short of 1: option (i) clearly implies that one of the conditionals in (4)-(6) is false. That is, here too, we have a cut-off point, even though of a different kind than in (S)WSP: what I mean by a cut-off point here is that one of the conditionals of the form "I should believe that buying *n* tickets does not allow me to win the lottery ⊃ I should believe that buying *n* + 1 tickets does not allow me to win the lottery" is false (see fn. 1). For convenience, and in spite of the differences with (S)WSP, I will extend the use of the expression "cut-off point view" to the view that one of the conditionals in NSP is false.

Where the cut-off point falls of course depends on the value of *t*. If we assume, as above, that $t = 0.999$ and that the 1000-ticket lottery is fair, then the false premise will be "I should believe that buying 1 ticket does not allow me to win the lottery ⊃ I should believe that buying 2 tickets does not allow me to win the lottery". Indeed, by the Lockean Thesis, one should believe that buying 1 ticket does not allow her to win. However, one should not believe that buying 2 tickets does not allow her to win ("buying 2 tickets does not allow me to win" has a probability of 0.998). In other words, even if the defender of (i) manages to avoid the cut-off point conclusion in the case of WSP, she cannot avoid it in the case of NSP.

This remark can be extended to a heap variant of NSP. As it was already the case with WSP, we can generate NSP starting from a heap scenario instead of a lottery one, i.e., we can generate a Soritical Narrow Scope Paradox (SNSP). One just has to replace (4) with "I should believe that 1000 grains are a heap", (5) with "I should believe that 0 grains are not a heap" and the conditionals in (4)-(6) with sentences of the form "I should believe that *n* grains are not a heap ⊃ I should believe that $n + 1$ grains are not a heap". Finally, (6) must be replaced with "I should believe that 1000 grains are not a heap".

Here too (i.e., in the case of SNSP too) the advocate of (i) will be obliged to say that there is a cut-off point. Where this cut-off point is will again depend on the value of *t* and on the specific probability distribution associated with her evidence.

As announced in section 1, the above has some interesting consequences concerning Leitgeb's account of the Lottery Paradox. According to Leitgeb (2014; 2015), the context in which we ask ourselves whether a given ticket *n* wins and that in which we focus on the fact that some ticket will win (i.e., that the lottery is fair and has one winner) are different and allow us to set different thresholds for rational belief.

More specifically, in a context in which we focus on the fact that some ticket will win, Leitgeb's theory of belief constrains us to set $t = 1$ (and therefore to suspend our judgement on each of the tickets). Instead, a context in which we concentrate on whether a given ticket *n* will win is one in which we can set $t = 0.999$, and this will not cause the Lottery Paradox to arise, provided that we "partition" (i.e., that we subdivide) the probabilities in our distribution as imposed by the theory. According to Leitgeb, the

Lottery Paradox results from fallaciously mixing premises that come from these different contexts.

However, imagine that instead of talking about tickets we were talking about grains: in the context in which $t = 1$ we would have to accept the cut-off point conclusion with respect to SWSP. In the context in which, instead, we ask ourselves whether some specific ticket will win (whether some specific grain turns something that is not a heap into a heap) and we are assuming $t = 0.999$, we would have to say that one of the conditionals in SNSP is false.

It is noteworthy that in (S)NSP the principle that is applied is not Belief Closure, but classical-logic modus ponens. Indeed, this is an important fact: until I only considered (S)WSP, the dilemma was between accepting the Lockean Thesis with t short of 1 on the one hand and accepting both Belief Closure and the cut-off point view on the other hand. Thanks to (S)NSP we are now aware that rejecting Belief Closure is not enough to avoid the cut-off point view (at least not with respect to this further puzzle): rejecting *classical-logic* modus ponens[12] is necessary. That is, not merely Belief Closure, but a fundamental principle of classical logic. The reason is that dropping Belief Closure would allow us to block (S)WSP, but not (S)NSP. Instead, giving up classical-logic modus ponens would solve both (S)WSP and (S)NSP: if classical-logic modus ponens is invalid, Belief Closure fails; however, the opposite direction of the conditional does

[12] Or rather, classical-logical modus ponens plus at least two other principles, i.e., classical-logic conjunction introduction and classical-logic modus tollens. Indeed, NSP can be generated by using indifferently modus ponens, conjunction introduction and modus tollens. I have explicitly formulated the modus ponens version, but the versions in which conjunction introduction and modus tollens are used are easy to work out. To provide a formulation of NSP in which conjunction introduction is used it suffices to notice that "I should believe that buying 1000 tickets allows me to win the lottery ∧ I should believe that buying 0 tickets does not allow me to win the lottery" is equivalent to "(I should believe that buying 0 tickets does not allow me to win ∧ I should believe that buying 1 ticket allows me to win) ∨ (I should believe that buying 1 ticket does not allow me to win ∧ I should believe that buying 2 tickets allows me to win)… ∨ (I should believe that buying 999 tickets does not allow me to win ∧ I should believe that buying 1000 tickets allows me to win)". If we regard all its disjuncts as false, then, by conjunction introduction, this last disjunction is both true and false (provided, of course, that we assume that "I should believe that buying 1000 tickets allows me to win the lottery ∧ I should believe that buying 0 tickets does not allow me to win the lottery" is true).

Concerning the modus tollens version, the contradiction is generated by assuming both (4) and all the conditionals in (4)-(6), and by applying modus tollens as many times as needed.

not hold. In other words, if we want to solve (S)NSP, we should either endorse the cut-off point view or give up classical-logic modus ponens.

I will not take a stand here on which of these two very radical alternatives is the best. Of course, this new dilemma could be regarded as favouring the cut-off point view, i.e., as a clear indication of the fact that, puzzling as they may be, cut-off points are unavoidable. However, the validity of classical-logic modus ponens has been challenged in the past. Dialetheists, for instance, argue that the derivation of Q from $P \supset Q$ and P can fail, although in very special circumstances, when both P and $\sim P$ are true (see, most notably, Priest 1979 and Beall 2009). For their part, relevant logicians have questioned the validity of disjunctive syllogism (which is just modus ponens for the material conditional modulo double negation principles; see Anderson and Belnap 1975)[13].

Anyway, I will not tackle this issue here. In this paper I wanted to show that keeping Belief Closure becomes a less appealing option when one sees what happens if instead of a lottery scenario a different material is used, notably, a heap scenario. However, it is also worth noting that rejecting Belief Closure is not enough: as we have just seen, if we want to avoid the cut-off point conclusion with respect to (S)NSP we should embrace an even more radical solution, i.e., denying classical-logic modus ponens.

The above also teaches us something important concerning the most popular norms of belief on the market. Indeed, it can be noted that they all entail the cut-off point view (either only with respect to (S)NSP or with respect to both (S)WSP and (S)NSP): whether we assume the Lockean Thesis (with $t = 1$ or with t short of 1), the truth norm or the knowledge norm, we end up with a cut-off point "somewhere". More precisely, if we assume the Lockean Thesis with t short of 1, we end up with a cut-off point (only) in (S)NSP. If, instead, we assume either the Lockean Thesis with $t = 1$, the truth norm or the knowledge norm, we end up with a cut-off point both in (S)WSP and in (S)NSP. Indeed, accepting any of these three norms makes it the case that for some ticket we should not believe that it loses/that for some grain we should *not* believe that adding it to something which is not a heap does *not* turn it into a heap. So some conditional in

[13]I should also mention here the advocates of the so-called "degree-theoretic view of vagueness"; indeed, many "degree-theorists" reject modus ponens when degrees of truth are involved in the inference.

(S)WSP is unacceptable. But it is also the case that (S)NSP has one false conditional: for some *n*, we should believe "buying *n* tickets does not allow me to win" ("*n* grains are not a heap"), while we should not believe "buying *n* + 1 tickets does not allow me to win" ("*n* + 1 grains are not a heap").

Consequently, if we want to avoid cut-off points altogether, it is not enough to give up classical-logic modus ponens: denial of modus ponens would block both (S)WSP and (S)NSP, but were we to keep any of these three norms of belief, the cut-off points would still be there. As a result, if we want to avoid cut-off points, we should reject all three norms; i.e., we should deny classical-logic modus ponens (to block the paradoxes) *and* we should go through a quite radical rethinking of the way we conceive norms of belief.

Of course, this result too could be regarded as favouring the cut-off point view, i.e., as proof of the fact that cut-off points cannot be avoided. On the contrary, I think that the cut-off points' opponents could take up the challenge. Notably, it seems to me that the challenge can be broken into three "smaller" ones: the cut-off points' enemies should (i) propose a suitable non-classical framework in which (S)NSP can be dealt with; (ii) come up with weaker (but still sensible) coherence constraints on rational belief (weaker than Belief Closure)[14]; iii) propose a norm of belief which can be naturally associated with such constraints, and which does not entail cut-off points. These certainly are hard challenges, but hard is not impossible.

More generally, I believe that it is premature for both sides (the cut-off points' advocates and their opponents) to claim success: more work has to be done in order to understand the structure of (S)WSP and (S)NSP, as well as their mutual relations. Hopefully, from such work decisive arguments will result against or in favour of cut-off points. For the time being, I take it to be the main lesson of this paper that the "cut-off point problem" (i.e., the question whether our solution to the Lottery Paradox and its variants should allow for cut-off points) ought to play a key role in the debate on the Lottery Paradox. In the literature on the Sorites, this question has always been central. The present article is a plea for writers on rational belief and rational degrees of belief to focus on this issue, which has been neglected so far.

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[14]An interesting attempt to provide a compelling alternative to Belief Closure as a coherence requirement for rational belief can be found in Easwaran and Fitelson 2015.

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CHAPTER 3. Against Belief Closure.

Abstract. I argue that we should solve the Lottery Paradox by denying that rational belief is closed under classical logic. To reach this conclusion, I build on my previous result that (a slight variant of) McGee's election scenario is a lottery scenario (see chapter 1). Indeed, this result implies that the sensible ways to deal with McGee's scenario are the same as the sensible ways to deal with the lottery scenario: we should either reject the Lockean Thesis or Belief Closure. After recalling my argument to this conclusion, I demonstrate that a McGee-like example (which is just, in fact, Carroll's barbershop paradox) can be provided in which the Lockean Thesis plays no role: this proves that denying Belief Closure is the right way to deal with both McGee's scenario and the Lottery Paradox. A straightforward consequence of my approach is that Carroll's puzzle is solved too.

1. Outline of the paper

Students of rational belief and rational degrees of belief generally agree that the two following principles are incompatible:

Belief Closure. Rational belief is closed under classical logic.

Lockean Thesis. If and only if, given one's evidence, *P* is very probable (where "very probable" means "equal to or higher than a specified threshold value *t*"), then one should believe *P*. (Or equivalently: if and only if, given one's evidence, *P* is very probable (where "very probable" means "equal to or higher than a specified threshold value *t*"), then it is rational to believe *P*.)[1]

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[1]In what follows I will apply the Lockean Thesis to (among others) indicative conditionals. That is, I will make use of the principle that if and only if an indicative conditional $P \rightarrow Q$ is very probable, then one should believe, or accept, $P \rightarrow Q$. Now, as a consequence of Lewis' famous triviality results (Lewis 1976), a number of philosophers have argued that indicative conditionals are not propositions. So I avoid committing myself to the controversial claim that indicative conditionals have propositional content. In other terms, when I will talk about the probability of $P \rightarrow Q$ I will not be talking about the probability of $P \rightarrow Q$ *being true*. Following Adams (1975), I will take such a probability to be the probability of *Q* conditional on *P*. That is, in what follows, the Lockean Thesis (when applied to indicative conditionals) will read: if and only if the probability of *Q* conditional on *P* is high, then one should believe, or accept, $P \rightarrow Q$ (provided that $P \rightarrow Q$ is a simple conditional and that $p(P) \neq 0$, see section 2 below).

Also note that, in spite of the subtle differences that may exist between the notion of (rational) acceptance and that of (rational) belief, I will use these two terms interchangeably; indeed, such differences are of no relevance for this paper's purposes. As it is clear from my definition, "it is rational

The well-known Lottery Paradox (Kyburg 1961) is an illustration of the incompatibility between these two principles. One way of presenting the puzzle is the following. Consider a fair 1000-ticket lottery with exactly one winner. There is a very low probability, for each ticket, that it will win, namely a probability of 0.001. Consequently, if $t = 0.999$, then, by the Lockean Thesis, it is rational to believe, of each ticket, that it will lose. By multiple applications of Belief Closure, it is also rational to believe the conjunction "ticket n°1 will lose ∧ ticket n°2 will lose . . . ∧ ticket n°1000 will lose" (where "∧" is the conjunction symbol). However, given that the lottery is fair and has exactly one winner, the negation of "ticket n°1 will lose ∧ ticket n°2 will lose . . . ∧ ticket n°1000 will lose" has a probability of 1; therefore, by the Lockean Thesis, it is rational to believe it. So it is rational to believe both "ticket n°1 will lose ∧ ticket n°2 will lose . . . ∧ ticket n°1000 will lose" and its negation. We conclude that Belief Closure and the Lockean Thesis are incompatible, for it is generally agreed that it is not rational to believe two pairwise inconsistent sentences.

In the present paper I propose a solution to the Lottery Paradox. The solution I put forth consists in rejecting Belief Closure. Indeed, as we will see, a convincing argument to this conclusion can be provided. My proposal builds on my previous result that (a slight variant of) McGee's election scenario is a lottery scenario (see chapter 1). This result implies that the sensible ways to deal with McGee's scenario are the same as the sensible ways to deal with the lottery scenario: we should either reject the Lockean Thesis or Belief Closure. I show, then, that a McGee-like argument (which is just, in fact, Carroll's 1894 barbershop paradox) can be provided in which the Lockean Thesis plays no role: this proves that denying Belief Closure is the right way to handle both McGee's scenario and the Lottery Paradox.

A corollary of this conclusion is that Carroll's puzzle (1894) is also solved: besides McGee's problem and the Lottery Paradox, my proposal accounts for Carroll's barbershop paradox, which turns out to be a simple variant of McGee's argument.

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to believe *P*" and "one should believe *P*" will be used interchangeably too. Although aware of the discussions surrounding the relation between rational belief and epistemic obligations, I will not deal with them here, as the niceties involved in this discussion are, once again, of no relevance for my purposes.

However, I will only attend to these issues (i.e., I will only propose my solutions to these three puzzles) starting from section 5. Sections from 2 to 4 included do not actually introduce new material: they just summarize some results obtained in chapter 1, which I need in order to introduce my new conclusions. More precisely, this paper will be structured as follows. In section 2 I recall McGee's scenario and propose my interpretation of it. In section 3 I present the "restaurant scenario". The importance of this scenario lies in that, first, it preserves the relevant features of McGee's election story, and that, second, as I will show, it is just a lottery scenario. Section 4 clarifies the consequences the discovery of this variant of McGee's scenario has on the way we should handle McGee's original argument. The new results, namely my solutions to the three puzzles (the Lottery Paradox, McGee's problem and Carroll's barbershop paradox), which constitute this paper's original contribution, are described starting from section 5.

2. *McGee's argument*

Famously, McGee (1985, p. 462) has proposed the following scenario:

> Opinion polls taken just before the 1980 election showed the Republican Ronald Reagan decisively ahead of the Democrat Jimmy Carter, with the other Republican in the race, John Anderson, a distant third. Those apprised of the poll results believed, with good reason:
>
> [1] If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.
>
> [2] A Republican will win the election.
>
> Yet they did not have reason to believe
>
> [3] If it's not Reagan who wins, it will be Anderson.

In chapter 1, I have challenged a standard claim concerning McGee's scenario, i.e., the claim that modus ponens does not fail in it if we assume the material conditional. It is not difficult to see why this claim is widely held: if we assume the material conditional, (3) is equivalent to the disjunction "either Reagan wins or Anderson wins", which is very plausible, for the simple reason that Reagan is "decisively ahead" of his competitors. Now, it can be shown that giving a material interpretation of the natural language (indicative) conditional "if…then…" does not block McGee's puzzle (see chapter 1). More precisely, I have shown that, under the supposition that the Lockean Thesis holds, McGee can be regarded as demonstrating that the following principles of the logic of belief are falsified (where "⊃" is the material conditional, "~" is the negation symbol, and *Bel* is a rational belief operator). (As in chapter 1, I use * to identify those principles in which the material conditional is involved versus those in which the conditional is a non-material indicative, which I denote as $P \rightarrow Q$.)

*Epistemic modus ponens**. If $Bel(P \supset Q)$, and $Bel(P)$, then $Bel(Q)$.

*Epistemic modus tollens**. If $Bel(P \supset Q)$, and $Bel({\sim}Q)$, then $Bel({\sim}P)$.

In other words, in a way similar to the Lottery Paradox, McGee's (1985) article can be taken to show that Belief Closure and the Lockean Thesis are incompatible.

Note that the condition for such a conclusion to follow (i.e., the conclusion that McGee's paper shows that Belief Closure and the Lockean Thesis cannot be jointly satisfied) is that some specific assumptions concerning McGee's scenario are granted. The first (call it "A1") is that the principle applied in (1)-(3) is what I call, in chapter 1, "epistemic modus ponens" (with no star):

Epistemic modus ponens. If $Bel(P \rightarrow Q)$, and $Bel(P)$, then $Bel(Q)$.

This principle can be contrasted with truth-preserving modus ponens, in which the notion of truth is involved, instead of that of rational belief:

Truth-preserving modus ponens. If P is true and $P \rightarrow Q$ is true, then Q is true.

A1 just says that McGee targets the former principle.

The second assumption (which I will call "A2") is that McGee endorses the Lockean Thesis.

Both A1 and A2 are well supported by McGee's remarks (see McGee 1985 and

1989), and have been recently endorsed by students of McGee's problem (for a recent example, see Stern and Hartmann 2018). Actually, McGee explicitly acknowledges that epistemic modus ponens is used in (1)-(3). Indeed, referring to (1)-(3) and to the two other (structurally similar) examples he provides in (1985), he says: "Such examples show that *modus ponens* fails in English […] More precisely, the examples show that *modus ponens* does not preserve warranted acceptability. As I [McGee] pointed out (1985, p. 463) and as Sinnot-Armstrong, Moor, and Fogelin (1986) have emphasized, the examples have no direct bearing on the question whether *modus ponens* is truth-preserving" (McGee 1989, p. 512 and fn. 20). Let me also stress that in presenting (1)-(3) McGee specifies that "those apprised of the poll results" *believed* with good reason (1) and (2), whereas they had no reason to *believe* (3) (McGee 1985, p. 462). Another explicit reference to the fact that epistemic modus ponens, or something along its lines, is applied in (1)-(3) is contained in the author's remark that on some occasions "one has good grounds for believing the premises of an application of modus ponens but yet one is not justified in accepting the conclusion" (McGee 1985, p. 462).

Concerning A2, it seems reasonable to argue that the reason why we should believe (2) is its high probability (besides Stern and Hartmann 2018, Neth 2019, among recent papers, subscribes to this claim). This seems confirmed by the fact that when presenting his counterexamples, McGee speaks of the reasons to believe their premises (or to disbelieve their conclusions) in terms of likelihood ("[i]t is more likely that […]"; "[…] it is virtually certain that […]"; "[…] it is entirely certain that […]"; McGee 1985, p. 163).

However, in spite of my hypothesis that A2 holds being reasonable, it is in fact not necessary for my purposes to rely on the claim that McGee indeed assumed the Lockean Thesis, i.e., that in the version of the puzzle McGee had in mind the Lockean Thesis is involved. What only needs to be the case for this article's purposes is that there is a plausible interpretation of McGee's puzzle in which both epistemic modus ponens and the Lockean Thesis are assumed. Now, as I have just shown, this clearly seems to be the case.

Let us go on. If we take high probability to be what justifies belief in (2), then if (1)-(3) is to be regarded as a potential failure of modus ponens, it must be the case that the reason why we should believe (1) also is its high probability, and that the reason why we should not believe (3) is that its probability is not high enough (see chapter 1).

Interestingly, there is one very popular way of interpreting indicative conditionals which is compatible with McGee accepting the Lockean Thesis. This way of interpreting indicative conditionals (or rather, their acceptability) is mostly known as "Adams' Thesis" (see Adams 1975):

Adams' Thesis. The acceptability of $P \rightarrow Q$ is equal to the probability of Q given P (i.e., of Q conditional on P), provided that $P \rightarrow Q$ is a simple conditional and that $p(P) \neq 0$[2].

If we also assume import-export, i.e., a principle usually regarded as highly plausible for the logic of conditionals[3] (or, more precisely, if we assume the counterpart principle of acceptability: $Acc(P \rightarrow (Q \rightarrow R)) = Acc((P \wedge Q) \rightarrow R)$), we obtain that our attitudes towards (1)-(3) are represented as follows:

(1') $p(R|P \wedge Q)$
(2') $p(P)$
(3') $p(R|Q)$

The reason why our attitude towards (1) is the one indicated in (1') is that, by Acceptability Import-Export and Adams' Thesis, we obtain that $Acc(P \rightarrow (Q \rightarrow R)) = p(R|P \wedge Q)$. (More precisely, as Stern and Hartmann (2018) specify, "this follows only when [Acceptability Import-Export] is restricted to settings where $p(P \wedge Q) > 0$ (since [Adams' Thesis] applies only in these settings)". See Stern and Hartmann 2018, fn. 15; I modified the authors' notation to make it coherent with mine.)

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[2] As I note in chapter 1 and remind in footnote 1 above, the approach I develop here eludes the so-called triviality results, for in the present context we are only interested in the acceptability (or believability) conditions for indicative conditionals, and not in the question whether indicative conditionals are propositions.

[3] Import-export certainly enjoys a huge popularity as a logical schema, and the case we are considering seems to confirm its solidity: (1) is intuitively equivalent to "If a Republican wins the election and it's not Reagan who wins, then it will be Anderson". However, import-export has sometimes been challenged: famously, Stalnaker's logic of conditionals does not validate it (Stalnaker 1968). For a recent criticism of this principle see Mandelkern (forthcoming). A review of the alleged failures of import-export for indicative conditionals is provided in Khoo and Mandelkern (2019).

Note that the above does not rule out that there may be other ways of representing our attitudes towards (1)-(3) compatible with the Lockean Thesis. However, (1')-(3') certainly is one very natural way of representing them, which I will thus take as the main reference here.

In chapter 1 I have shown that, if A1 and A2 are granted, a slightly modified version of McGee's election scenario can be provided, in which (i) both epistemic modus ponens* and epistemic modus tollens* fail, and (ii) the relevant features of the scenario are preserved. Here we will have to consider once again that example, which I dub "the restaurant scenario".

3. From McGee's puzzle to the Lottery Paradox

I am sitting in a restaurant with my Italian friend Pasquale. I know that Pasquale always orders one of the day's specials. Today's specials are pizza, pasta and roast beef. I know that Pasquale likes both pizza and pasta very much, and that he does not especially enjoy roast beef. I estimate that there is a 0.4 probability that Pasquale will have pizza, a 0.4 probability that he will have pasta and a 0.2 probability that he will have roast beef.

Assume the material conditional and set $t = 0.6$. In this context, I should believe both (a) "If Pasquale doesn't have pizza, then he will have pasta" and (b) "Pasquale won't have pizza". Indeed, they both have a probability of at least 0.6. Now, from (a) and (b), by epistemic modus ponens*, I should infer (c) "Pasquale will have pasta". But (c) only has a probability of 0.4; so I should not believe (c), that is, epistemic modus ponens* fails.

Let us now turn to epistemic modus tollens*. By the Lockean Thesis, I should believe (d) "Pasquale won't have pasta", which has a probability of 0.6. Now, from (d) and (a) I should draw, using epistemic modus tollens*, the conclusion that (e) "Pasquale will have pizza". But (e) only has a probability of 0.4; therefore, I should not believe (e), i.e., epistemic modus tollens* fails.

So epistemic modus ponens* and modus tollens* fail in the restaurant scenario. Or, more precisely, assuming $t = 0.6$, in the above arguments rational belief is not closed under modus ponens* and modus tollens* respectively. But this is not all; a principle I will label "epistemic conjunction introduction" fails too:

Epistemic conjunction introduction. If $Bel(P)$, and $Bel(Q)$, then $Bel(P \wedge Q)$.

Clearly, given $t = 0.6$, we should believe "Pasquale won't have pizza", "Pasquale won't have pasta" and "Pasquale won't have roast beef"; however, we should not believe the conjunction of these three sentences.

Here we come to a crucial point: assume the standard definition of a lottery scenario as a scenario where, given t higher than 0.5, the Lockean Thesis and epistemic conjunction introduction are incompatible (typically because one should end up believing two pairwise inconsistent sentences). By this definition, the restaurant scenario is just a version of the lottery scenario Kyburg uses in his Lottery Paradox, albeit one with only three tickets and a probability threshold for rational belief of 0.6. In other terms, insofar as we adopt the above definition of what a lottery scenario is, the restaurant scenario is a lottery scenario.

So, under the assumption that the Lockean Thesis holds, epistemic modus ponens* and epistemic modus tollens* both fail in the restaurant example, i.e., the condition (i) above is satisfied. Now, it is also possible to show that (ii) holds with regard to the restaurant scenario, i.e., that in it the relevant traits of the election example are preserved. Assume that X = "Carter loses the election" (i.e., "a Republican wins"), Y = "Reagan loses", and Z = "Anderson loses". Given these interpretations of X, Y and Z, (1)-(3) has the following form:

$X \rightarrow (Y \rightarrow \sim Z)$

X

$\therefore Y \rightarrow \sim Z$

Consider now the restaurant scenario. Let X be "Pasquale doesn't have pizza", Y be "Pasquale doesn't have pasta" and Z be "Pasquale doesn't have roast beef": clearly, McGee would still have to regard this instance of the above argument form as a failure of modus ponens. This is very important. Indeed, it can be noted that, unlike “Reagan wins” in the election scenario, in the restaurant scenario “Pasquale has pasta” has a low probability. This means that even if in the election scenario the probability of “Reagan will win” were lower than it is, McGee should still view (1)-(3) as a failure of modus ponens. In other terms, for modus ponens to fail in McGee’s sense it is not necessary that one of the disjuncts in the probability distribution has a probability higher than t: even if in the restaurant example the probability of $\sim Y$ (“Pasquale has pasta”, corresponding to “Reagan wins” in the election scenario) is low, the probability of X (“Pasquale doesn’t have pizza”/“A Republican wins”) is still high. This proves that it is a merely contingent fact that one of the disjuncts in the original scenario has a probability higher than the threshold. Removing this contingent element allows us to gain a deeper understanding of the structure underlying McGee’s scenario.

So both (i) and (ii) hold with respect to the restaurant scenario. Now, as we have seen, the restaurant scenario is just a lottery scenario. My conclusion is that we should expect a unified solution to both McGee’s puzzle and the Lottery Paradox; i.e., that the sensible ways to handle McGee’s scenario are the same as the sensible ways to handle the lottery scenario.

This conclusion is unscathed, I submit, by a potential objection also addressed in chapter 1. According to this objection, the restaurant scenario does not really preserve all the relevant features of McGee’s original story, because in the former epistemic modus ponens only fails if we set t no higher than 0.6. Indeed, in the restaurant counterpart of McGee’s argument the probability that Pasquale will have roast beef, given that he does not have pizza or pasta (i.e., $p(R|P \wedge Q)$) is 1, the probability of “Pasquale will not have pizza” (i.e., $p(P)$) is 0.6, while the probability that Pasquale will have roast beef given that he does not have pasta (i.e., $p(R|Q)$) is low (much lower than 0.6). Now, the election scenario seems different: in it, a threshold higher than 0.6 seems to be compatible with the failure of epistemic modus ponens. In other terms, someone who thinks that a probability of 0.6 is not enough for rational belief would still regard (1)-(3) as a counterexample to epistemic modus ponens, but would argue that there is no puzzle about the restaurant scenario.

A first response to this objection is simply that advocates of the Lockean Thesis are usually not committed to a specific value for t[4]. Even so, in chapter 1, I go on and concede that a threshold of 0.6 may not suffice (at least not always) for rational belief. Still, I conclude that this is not enough to show that the specific threshold assumed in the restaurant case is responsible for the fact that epistemic modus ponens fails there. Indeed, were this to be the case, then epistemic modus ponens would not fail for thresholds greater than 0.6. However, examples in which epistemic modus ponens fails for t higher than 0.6 are easy to work out: Stern and Hartmann (2018, pp. 609 and 610) have proved that it is always possible to find $p(R|P \land Q)$ and $p(P)$ such that they are both high, while at the same time $p(R|Q)$ is low. (More precisely, they have proved that there is only one case in which there are lower bounds for $p(R|Q)$, viz. the case in which $p(P)$ is equal to 1; see Stern and Hartman 2018, fn. 18).

This point is related to a further one. Someone who argues that a threshold over 0.6 should be endorsed, in general or in this scenario specifically (think of the advocates of a contextualist version of the Lockean Thesis, who may believe that in the restaurant case there are specific reasons to reject a 0.6 threshold) could claim that I did not show that, if we assume the Lockean Thesis, then Belief Closure fails in that scenario, but only that if we assume the Lockean Thesis *with t = 0.6*, then Belief Closure fails. However, as is well known, failure of Belief Closure in cases in which the Lockean Thesis is assumed is not limited to cases in which t is equal to 0.6. As long as a suitable number of tickets is also chosen, failure of Belief Closure can always be observed, no matter the specific threshold t (the only proviso is that t must be strictly between 0.5 and 1). Kyburg's original scenario is one such case: in it t is greater than 0.6, but Belief Closure still fails. As a result, no matter which version of the Lottery Paradox we are considering (with 3 tickets, 1000 tickets, or with a still different number of tickets), accusing the specific threshold value assumed (0.6, 0.999, etc.) of being responsible for the Paradox itself seems hopeless.

Note that I am not denying that an account of both the restaurant scenario and Kyburg's scenario could be given in contextualist terms. However, such an account would involve a *modification* of the Lockean Thesis, whose original form would be

[4] Achinstein 2001 is one notable exception. Also note that Shear and Fitelson (2019) have claimed that a value they dub "the golden threshold", i.e., a value corresponding to the inverse of the golden ratio ($\phi^{-1} \approx 0.618$), should be regarded as a non-arbitrary Lockean threshold.

rejected. The most famous account of this kind is found in Leitgeb (2014; 2015), according to which we should supplement the Lockean Thesis with the condition that the probability of *P* should remain higher than 0.5 when the agent gains evidence which is consistent with *P*. (For more on contextualist accounts of the Lottery Paradox see section 6 below.)

So I conclude that McGee's original scenario and Kyburg's scenario should be dealt with in the same way. More specifically, it looks like we have two main options: rejecting the Lockean Thesis or rejecting Belief Closure. (Note that, conforming to what is standard in the literature, and as I have just suggested regarding Leitgeb's account, I take rejections of the Lockean Thesis to include modifications of the latter; on this point, also see section 6 below.) It can be observed that denying Belief Closure would in fact entail denying at least three principles: epistemic modus ponens*, epistemic modus tollens*, and epistemic conjunction introduction. Indeed, as we have seen, in the restaurant scenario the three of them fail. And this is also the case in Kyburg's scenario: even though the original version of the Lottery Paradox involves (epistemic) conjunction introduction, we can generate lottery-like paradoxes by using other principles, notably (epistemic) modus ponens* and modus tollens* (see, for instance, Douven 2016[5]).

[5] It may be helpful to see how these lottery-like paradoxes are constructed. In the original scenario, it is rational to believe "Ticket n°1 wins ∨ ticket n°2 wins… ∨ ticket n°1000 wins", which is equivalent to "(Ticket n°1 loses ⊃ ticket n°2 loses… ⊃ ticket n°999 loses) ⊃ ticket n°1000 wins". It is also rational to believe, about each ticket between n°1 and n°999, that it loses. However, it is *not* rational believe that ticket n°1000 wins (in fact, it is rational to believe that it loses). That is, epistemic modus ponens* fails.

In this scenario, epistemic modus tollens* fails too. Indeed, it is rational to accept

(Ticket n°1 loses) ⊃ ticket n°2 loses… ⊃ ticket n°999 loses ⊃ ticket n°1000 wins.

Ticket n°2 loses.

…

Ticket n°999 loses.

Ticket n°1000 loses.

Nevertheless, it is rational to reject

Ticket n°1 wins

and accept its negation.

4. Back to McGee's argument

So far, my proposal concerns the way we should handle McGee's and Kyburg's scenarios *in general*; however, I now would like to dwell on the consequences of what I have shown concerning (1)-(3) specifically. Of course, such consequences are not especially relevant if our only aim is to solve the Lottery Paradox. Still, this question is of the utmost importance if, as I plan to do here, we also wish to provide an account of McGee's original argument. Now, what I have shown so far straightforwardly applies to (1)-(3). No matter whether we decide to drop the Lockean Thesis or Belief Closure, in both cases the derivation of (3) from (1) and (2) is blocked. Suppose that we drop the Lockean Thesis: we are not compelled to believe (1) and (2) anymore. (This clearly follows from the fact that the Lockean Thesis is an implicit assumption in McGee's puzzle, or at least in the version of McGee's puzzle I am considering here (see my interpretation of (1)-(3) in section 2 above); as a result, if we abandon the Lockean Thesis, the puzzle disappears.) Suppose, instead, that we give up Belief Closure, i.e., as we have seen, at least epistemic modus ponens*, epistemic modus tollens* and epistemic conjunction introduction: this also blocks (1)-(3). The reason is that if epistemic modus ponens* turns out to fail, epistemic modus ponens also fails. Or at least, this is the case under the reasonable (and popular) assumption that $Bel(P \rightarrow Q)$ entails $Bel(P \supset Q)$. Here is a convincing argument to this conclusion I also provide in chapter 1: suppose that we rationally believe $P \wedge {\sim}Q$ (i.e., the negation of $P \supset Q$); this seems sufficient for rationally believing the negation of $P \rightarrow Q$.

So I showed that (a slight variant of) McGee's scenario is just a lottery scenario. This implies that the only sensible ways to deal with McGee's scenario are either by denying Belief Closure or by denying the Lockean Thesis. As we have seen, giving up either Belief Closure or the Lockean Thesis also blocks (1)-(3). A consequence of this result is that it undermines any account of McGee's puzzle that does not involve dropping either Belief Closure or the Lockean Thesis. Indeed, as I have argued above, there is a reasonable and easily accessible interpretation of (1)-(3) that assumes both epistemic modus ponens and the Lockean Thesis. As a result, even though other interpretations of (1)-(3) are perhaps possible, any student of McGee's problem should deal at least with this one. Now, from my interpretation of (1)-(3), it follows, as we have seen, that (a slight variant of) McGee's scenario is actually a lottery scenario, so that we should to deal with it either by dropping Belief Closure or by dropping the Lockean Thesis. This leads us to the conclusion that any author tackling McGee's puzzle should renounce either the Lockean Thesis or Belief Closure. What is interesting and important here is that the great majority of the existing discussions of McGee's puzzle do not address the rejection of either principle[6].

Now of course the next question is: who is the "culprit" between Belief Closure and the Lockean Thesis? Answering this question would solve both the Lottery Paradox and McGee's puzzle. Now suppose that an argument can be provided which has the same structure as McGee's, but in which the Lockean Thesis plays no role: clearly, this would show that denying Belief Closure is the right way to react to both McGee's scenario and the Lottery Paradox. In the next section, I will argue that an argument proposed by Carroll in 1894 is such an argument.

5. The barbershop and the election

In 1894, Lewis Carroll proposed his famous "barbershop paradox" (Carroll 1894). The scenario can be summarized as follows: Carr, Allen and Brown are three barbers who never leave their shop at the same time, as one of them has to be there in order to keep the shop open. Moreover, due to the consequences of an illness, Allen never goes out without Brown. On the one hand, we should believe (4):

(4) If Carr is out, then if Allen is out, Brown is in.

Nevertheless, we should also believe (5):

[6] I am thinking, *inter alia*, of the accounts proposed by Appiah (1987), Lowe (1987), Piller (1996), Katz (1999), Bennett (2003), Gillies (2004), Paoli (2005), Cantwell (2008), Fulda (2010), Kolodny and MacFarlane (2010), Edgington (2014), Moss (2015), Stojnić (2017), Schulz (2018), Stern and Hartmann (2018), and Neth (2019).

(5) If Allen is out, then Brown is out.

(5) and the nested consequent of (4) seem to contradict each other and therefore seem to imply, by (epistemic) modus tollens, that we should believe (6) Carr is in. However, it looks like we should not accept this conclusion, for it is perfectly possible that Carr is out, provided that Allen is in. That is, intuitively, epistemic modus tollens (i.e., the principle according to which if $Bel(P \rightarrow Q)$, and $Bel(\sim Q)$, then $Bel(\sim P)$) fails[7].

In what follows, I mean to show that the culprit behind McGee's puzzle is epistemic modus ponens, and not the Lockean Thesis. To do this, I will have to demonstrate that our intuition that epistemic modus ponens fails in (1)-(3) does not depend on the assumption that the Lockean Thesis holds. One way of reaching this conclusion is by showing that there is at least one argument such that (i) it has the same structure as (1)-(3) and that (ii) even if we assume a different norm of belief (different from the Lockean Thesis) we still have the intuition that epistemic modus ponens fails in that argument. This is exactly what I plan to do here. First, I will demonstrate that starting from either of the two scenarios (McGee's or Carroll's) we can generate both what looks like a failure of epistemic modus ponens and what looks like a failure of epistemic modus tollens. Such putative failures clearly have the same structure in the two scenarios. I will then show that in the arguments generated starting from Carroll's scenario the Lockean Thesis does not play any role. This conclusion is a straightforward consequence of the fact that Carroll's story does not involve probabilities.

Consider, first of all, the logical structure of Carroll's argument. Let X, Y and Z be "Carr is out", "Allen is out" and "Brown is out" respectively. (4)-(6) has the following form:

[7] Actually, Carroll formulates his puzzle in terms of truth, not in terms of rational belief. Indeed, he specifies that the *truth* of (6) does not seem follow from the *truth* of (4) and (5) (Carroll 1894). My rendering of his puzzle, however, is in terms of rational belief. The reason is that this makes the analogy with McGee's scenario emerge more clearly. Note that introducing this variant of the puzzle is perfectly legitimate, as in Carroll's scenario it clearly seems that we should believe both (4) and (5), and that we should not believe (6).

$X \rightarrow (Y \rightarrow \sim Z)$

$Y \rightarrow Z$

$\therefore \sim X$

Now, it can be noted that starting from McGee's scenario we can also generate what looks like a counterexample to epistemic modus tollens[8]: one just has to replace "Carr is out" with "Carter loses the election" (i.e., "a Republican wins"), "Allen is out" with "Reagan loses" and "Brown is out" with "Anderson loses". What we obtain is an argument having exactly the same form as (4)-(6):

(1) If a Republican wins the election, then if it's not Reagan who wins it will be Anderson.
(7) If it's not Reagan who wins, it's not the case that Anderson will win.
(8) The winner won't be a Republican. (!)

It seems rational to believe both (1) and (7); however, we should not believe (8). Indeed, "a Republican will win" is highly plausible, as long as the winning Republican is Reagan.

There is an important difference, though, between (1)-(8) and Carroll's argument. This difference has to do, as announced, with the role played by the Lockean Thesis in McGee's scenario. Indeed, in (1)-(8), it seems that the reason why we should believe (7) is that it is very likely. Now, if we are to regard (1)-(8) as a potential failure of epistemic modus tollens the reason why we should believe (1) must be the same (i.e., that it is very likely), while the reason why we should not believe (8) must be that it is not likely enough[9].

The point I will make below is that Carroll's story, instead, does not involve

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[8]For a discussion of the modus tollens version of McGee's argument see Gauker 1994, but also Kolodny and MacFarlane 2010.

[9]By proceeding in a similar way as for McGee's original argument, our attitudes towards (1)-(8) may be represented as follows:

(1') $p(R|P \wedge Q)$
(7') $p(\sim R|Q)$
(8') $p(\sim P)$

probabilities, and this entails, of course, that the Lockean Thesis does not play any role in Carroll's argument. Notably, I will show that we can perfectly assume another norm of belief (other than the Lockean Thesis) and still have the intuition that epistemic modus tollens fails in (4)-(6). The truth norm of belief will serve as a key illustration.

The Lockean Thesis certainly is a very popular norm of belief. However, it does have some (also very popular) competitors: one of them is the so-called "truth norm". According to one possible definition of this norm, we should believe *P* if and only if *P* is true. The opposition between the truth norm and the Lockean Thesis is the contemporary heir to a very classical debate in epistemology, whose origins may be traced back to the opposition between William James' alethic approach to belief (James 1896) and Clifford's evidential conception of it (Clifford 1877). Several different versions of the truth norm have been proposed over the years. Advocates of some kind of truth norm include Wedgwood (2002; 2009), Boghossian (2003), Shah (2003), Gibbard (2005), O'Hagan (2005), Shah and Velleman (2005), Engel (2007), Whiting (2010), Littlejohn (2012; 2014), McHugh (2012; 2014), Raleigh (2013), Turp (2013), and Greenberg (forthcoming). I will not dwell here on these different proposals, as the subtleties they involve are not relevant in the present context. Rather, the definition I have given above will be enough for my purposes.

Now suppose that we accept the truth norm. As applied to indicative conditionals, this norm of course implies that indicative conditionals have truth conditions. Both claims are controversial (both the claim that the truth norm is the right norm of belief and that indicative conditionals have truth conditions). I am not saying that we should endorse these claims though, just that we can assume them for the sake of the argument. Now, it can be noted that were we to accept the truth norm, we would still have the intuition that in (4)-(6) epistemic modus tollens fails. Indeed (as Carroll himself points out, see fn. 8), (4) and (5) both seem true; (6), however, does not seem true[10].

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[10]Let us briefly consider, in passing, another well-known norm of belief (actually one which is becoming increasingly popular among epistemologists), i.e., the so-called "knowledge norm". According to one possible definition of it, one should believe *P* if and only if one knows *P*. It is noteworthy that were we to adopt this norm, my point would still hold: it clearly seems that we know both (4) and (5); however, we do not know (6).

It is also the case that, starting from Carroll's scenario, we can generate a modus ponens version of (4)-(6):

(4) If Carr is out, then if Allen is out, Brown is in.
(9) Carr is out.
(10) If Allen is out, Brown is in. (!)

As in (1)-(3), we regard the premises as rationally acceptable but we reject the conclusion. The only (relevant) difference between (1)-(3) and (4)-(10) concerns, again, the role of the Lockean Thesis. As noted in section 2, it seems that in McGee's story what grounds our acceptance of "A Republican will win" (i.e., of (2)) is its high probability. As a result, if we are to look at (1)-(3) as a potential failure of (epistemic) modus ponens, the reason why we should accept (1) and should reject (3) respectively must be that (1) is likely enough, while (3) is not (see my interpretation of (1)-(3) in section 2 above). In his scenario, instead, Carroll explicitly assumes that (4) and (9) are both true (Carroll 1894, pp. 436 and 437). That is, were we to assume the truth norm, we would still have the intuition that (4)-(10) is a counterexample to epistemic modus ponens.

Once again, let me stress that my claim is not that in Carroll's scenario truth-preserving modus ponens and modus tollens fail[11]. My only claim here is that were we to accept the truth norm of belief we would still have the intuition that (4)-(10) and (4)-(6) are failures of epistemic modus ponens and epistemic modus tollens respectively[12].

In more general terms, it can be observed that McGee's and Carroll's scenarios have the same structure or, more precisely, that McGee's scenario is just a "weaker" version of Carroll's. Why "weaker"? Consider Carroll's story and replace, as above, "Carr is out" with "Carter loses the election" (i.e., "a Republican wins"), "Allen is out" with

[11]I have already defined truth-preserving modus ponens (see section 2). Unsurprisingly, I take truth-preserving modus tollens to be the principle according to which if $P \rightarrow Q$ is true and $\sim Q$ is true, then $\sim P$ is true.

[12]Interestingly, once again, my point would also hold if we were to accept the knowledge norm, instead of the truth norm. Imagine that you just saw Carr sitting at the bar: intuitively, you would then know both (4) and (9); however, you would not know (10).

"Reagan loses", and "Brown is out" with "Anderson loses". If probabilities are "injected" "in the right way" (i.e., if the right proportions are respected in the probability distribution) what we obtain is McGee's election story. It is in this sense that we may regard McGee's scenario as "weaker" than Carroll's: it seems to involve a probabilistic component that is absent in the latter, meaning that it can only generate "counterexamples" in which the Lockean Thesis (or some related principle) is involved. This is not the case for Carroll's scenario: the "counterexamples" we can construct starting from it are stronger because the Lockean Thesis is clearly not responsible for them.

6. *Rejecting Belief Closure*

The above provides the basis for a historical point. Much before McGee, Carroll had already presented a "counterexample to modus ponens" (and modus tollens) very similar to McGee's, and actually even stronger than McGee's. Indeed, as we have just seen, in Carroll's scenario the Lockean Thesis clearly does not play any role.

Let us take stock. I conclude that Belief Closure is responsible for the Lottery Paradox. Indeed, I have shown that (a slight variant of) McGee's scenario is just a lottery scenario. This implies that the only sensible ways to deal with McGee's scenario are either by rejecting Belief Closure or by rejecting the Lockean Thesis. The fact that an argument structurally identical to (1)-(3) can be provided in which the Lockean Thesis plays no role can be regarded as proof of the fact that denying Belief Closure is the right response to both McGee's scenario and the Lottery Paradox.

Lottery Paradox scholars are usually regarded as belonging to two categories: those who reject (or at least propose to modify) the Lockean Thesis[13] and those who reject Belief Closure. Both categories have a long history: the former, which is the largest one, includes authors like Lehrer (1975; 1990), Kaplan (1981a; 1981b; 1996), Stalnaker (1984), Pollock (1995), Ryan (1996), Evnine (1999), Nelkin (2000), Adler (2002),

[13] I take this category to include those authors who restrict the Lockean Thesis to the effect that we should set $t = 1$. Levi (1980), Gärdenfors (1986), Van Fraassen (1995), Arló-Costa (2001), and Arló-Costa and Parikh (2005) are among the proponents of such a restriction.

Douven (2002), Smith (2010; 2016), Lin and Kelly (2012a; 2012b), and Kelp (2017). Among the Belief Closure deniers are, instead, Klein (1985), Foley (1992), Hawthorne and Bovens (1999), Kyburg and Teng (2001), Christensen (2004), Hawthorne and Makinson (2007), Kolodny (2007), Easwaran and Fitelson (2015). Contextualist accounts seem to form a category of its own: according to contextualists, one or more terms featuring in the formulation of the Paradox are ambiguous (or, alternatively, the truth conditions for attributions of rational belief are context-dependent). Typically, these authors seem to focus on the Lockean Thesis (or on some related principle, depending on the specific version of the Paradox they discuss, which may differ in some respects from the one I am considering here). Notably, they focus on the ambiguities (putatively) contained in the definition of the Lockean Thesis/the related principle or on its (alleged) context-dependence (see Lewis 1996, Cohen 1998, Leitgeb 2014, 2015, and Logins forthcoming; on Leitgeb's approach also see section 3 above). However, note that in fact these (purported) solutions to the Lottery Paradox are not really an independent category (see, again, Logins forthcoming). Indeed, they reduce to the claim that the Lockean Thesis/the related principle does not hold unrestrictedly, but only on some of its readings, or in certain contexts, which entails that their proponents should be viewed as Lockean Thesis deniers, or modifiers (first category above).

Anyway, I have shown that the Lockean Thesis does not play any role in the Lottery Paradox: this clearly undermines, among others, a contextualist approach according to which we should restrict the truth of the Thesis to some of its readings, or to certain contexts.

Note that a straightforward account of McGee's puzzle results from what I have shown in this paper. Indeed, as we have seen, it follows from the failure of epistemic modus ponens* that epistemic modus ponens fails too. This accounts for (1)-(3), i.e., McGee's original puzzle is solved. (Of course, the modus tollens version of McGee's puzzle is also solved, for from the failure of epistemic modus tollens* it follows that epistemic modus tollens fails too.)

As a final remark, let me stress that if the assumptions I made concerning McGee's argument (viz. A1 and A2 above) did not hold this would not undermine my account of the Lottery Paradox. Of course, it would then be true that I did not solve McGee's original puzzle, but only a version of McGee's puzzle with respect to which A1 and A2 hold. However, this would not be a problem for my account of the Lottery Paradox, as in order to derive the conclusion that Belief Closure is responsible for the latter it is enough to assume a version of McGee's puzzle with respect to which A1 and A2 do hold, and which does not necessarily coincide with the one McGee had in mind. However, as already specified, mine would still be a very reasonable and easily accessible interpretation of McGee's problem, thus one that students of the latter should account for.

A proviso, though: it suffices that A1 holds for McGee's original puzzle to be solved. The reason is the one above: under the supposition that $Bel(P \rightarrow Q)$ entails $Bel(P \supset Q)$, if epistemic modus ponens* fails, then epistemic modus ponens fails. The same applies, of course, to Carroll's arguments, or at least to the epistemic versions of them I consider in this article: on the assumption that the principles involved in (4)-(6) and (4)-(10) are epistemic modus tollens and modus ponens respectively, if epistemic modus tollens* and modus ponens* fail, both arguments are blocked.

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CONCLUSION. Modus ponens in trouble.

In chapter 1, I have shown that McGee's election scenario essentially boils down to a lottery scenario, and that, as a result, a unified treatment of the two scenarios should be provided. Chapter 2 focuses on the Lottery Paradox. Although in it I do not defend a specific solution to the Paradox, I do show the potential disadvantages of keeping Belief Closure. Indeed, keeping such a principle entails what I have called "the cut-off point view". Chapter 2 also shows that the debate on the validity of classical-logic modus ponens should play an important role in discussions on the Lottery Paradox. Indeed, there are only two possible unified solutions to the (Soritical) Wide Scope Paradox ((S)WSP) and the (Soritical) Narrow Scope Paradox ((S)NSP): the first consists in endorsing the cut-off point view with respect to both arguments; the second involves rejecting classical-logic modus ponens. (Denying classical-logic modus ponens indeed entails that Belief Closure fails, i.e., it suffices to block both (S)WSP and (S)NSP. However, the converse entailment does not hold: if Belief Closure fails, it is not the case that classical-logic modus ponens fails too). That is, in order to provide a unified account of our two paradoxes we should either endorse a cut-off point solution or a cut-off point *free* solution to both puzzles[1].

Finally, in chapter 3 I give a straightforward argument to the conclusion that we should solve the Lottery Paradox by denying Belief Closure. I build on my previous result that (a slight variant of) McGee's election scenario is a lottery scenario. This result entails that the sensible ways to deal with McGee's scenario are the same as the sensible ways to deal with the lottery scenario: we should either deny the Lockean Thesis or Belief Closure. I demonstrate, then, that an argument having the same structure as McGee's (which is just, in fact, Carroll's barbershop paradox) can be generated in which the Lockean Thesis does not play any role: this can be regarded as

[1]Of course, we could also provide a unified account of the two puzzles by denying that we should believe "buying 0 tickets does not allow me to win"/"0 grains are not a heap" or by claiming that we should in fact believe "buying 1000 tickets does not allow me to win" (i.e., "buying the totality of the tickets of a 1000-ticket lottery does not allow me to win")/"1000 grains are not a heap". As mentioned in chapter 2, Peter Unger (1979) has defended the thesis that "1000 grains are not a heap" is true. However, I will put aside this very implausible thesis here. (Note that in the lottery case this thesis seems even more implausible.)

proof that rejecting Belief Closure is the right way to deal with both McGee's scenario and the Lottery Paradox.

So this is what I have done. So far, my dissertation's main conclusion is that we should account for some long-standing problems (the Lottery Paradox, but also McGee's counterexample to modus ponens and Carroll's barbershop paradox) by renouncing Belief Closure. However, a further (important) consequence can be drawn from my results: in light of what I have shown, it seems that we can only provide a unified solution to (S)WSP and (S)NSP by denying classical-logic modus ponens. The reason is that chapter 3 demonstrates that we should solve (S)WSP by rejecting Belief Closure. As a result, the only remaining unified solution consists in dropping classical-logic modus ponens[2]. That is, it seems that we should endorse a cut-off point *free* solution to both (S)WSP and its narrow-scope variant.

I take this to be a fundamental consequence of my results. However, as noted in chapter 2, there is a proviso: actually, in order for our (unified) solution to be cut-off point free denying classical-logic modus ponens is not enough; we also have to reject the three most popular norms of belief on the philosophical market. That is, we should both deny classical-logic modus ponens (to block the paradoxes) and reconsider our most entrenched norms of belief. Indeed, as we have seen, all three norms (the Lockean Thesis with both t short of 1 or t = 1, the truth norm or the knowledge norm) entail the cut-off point view either with respect to (S)WSP alone or with respect to both (S)WSP and (S)NSP.

It follows that the most promising research project is one I described in chapter 2 as a particularly challenging one: providing a cut-off point free account of both (S)WSP and (N)SNP entails (i) treating (S)NSP within a suitable non-classical framework; (ii) in order to deal with (S)WSP, adopting coherence constraints on rational belief which are weaker than Belief Closure, and (iii) introducing and adopting an alternative norm of belief, which does not entail cut-off points.

Fortunately, such a project does not have to be pursued from scratch. A 2015 article by Easwaran and Fitelson provides an interesting starting point (see Easwaran and

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[2]In fact, as specified in chapter 2, we should deny classical-logic modus ponens *and* at least two more principles, namely classical-logic conjunction introduction and classical-logic modus tollens. The reason is that all three principles can be used to obtain a paradoxical conclusion (see chapter 2).

Fitelson 2015, but also Easwaran 2016). Like the author of this thesis, Easwaran and Fitelson argue that we should solve the Lottery Paradox by denying Belief Closure. They note that the main reason why dropping Belief Closure is usually not regarded as particularly appealing is that no convincing alternative has been proposed so far. That is, no (compelling) alternative "coherence requirement for rational belief" has been put forth, where by this expression they mean a constraint on sets of synchronically held full beliefs.

Now, our authors claim that an analogy between coherence constraints on belief and principles of rational choice in decision theory provides us with such a compelling alternative. A fundamental concept in order to spell out their proposal is that of distance from vindication, i.e., that of distance of a given belief set from the ideal belief set (defined as a set which only includes true judgements). "As a heuristic, you can think of [the ideal set] as the set of judgements […] that an omniscient agent (*i.e.*, an agent who is omniscient about the facts at world *w*) would have" (Easwaran and Fitelson 2015, fn. 29). Distance in vindication is measured in number of judgements; the set of judgements is taken to be finite, so counting is no problem.

After dismissing (among others) Belief Closure as too strong, the authors consider two minimal constraints (labelled WADA (Weak Accuracy-Dominance Avoidance) and SADA (Strict Accuracy-Dominance Avoidance) respectively) which essentially correspond to decision-theoretic non-dominance: "[w]e could say that being accuracy dominated reveals that you are in a position to recognize a priori that another option is guaranteed to do better at achieving the "epistemic aim" of getting as close to the truth as possible" (Easwaran and Fitelson 2015, fn. 35).

However, in spite of being initially regarded as promising (especially WADA), both constraints are finally deemed too weak by the authors. Indeed, either of them rules out as irrational belief sets containing pairwise inconsistent judgements. This leads to a proposal called "(R)": according to it, for any rational belief/disbelief set, there must be a probability function which makes the beliefs in the set more probable than the disbeliefs. In other terms, there must be a probability function (not necessarily anyone's actual degrees of belief) for which the Lockean Thesis holds with respect to the belief set with a threshold of 0.5.

Towards the end of the article, Easwaran and Fitelson provide a sketch of a complete analogy between constraints on rational belief and decision-theoretic principles. In particular, they suggest that "[i]f we think of closeness to vindication as a kind of epistemic utility […], then we may think of (R) as an expected epistemic utility maximization principle. On this reading, (R) is tantamount to the requirement that an agent's belief set should maximize expected epistemic utility, relative to some evidential probability function" (Easwaran and Fitelson 2015, p. 84). By analogy with decision theory, this requirement is regarded as more compelling than maximizing utility in the actual world (which the authors identify with acceptance of the truth norm of belief) or maximizing utility in some possible world (which they show to be equivalent to Belief Closure).

More thoroughly, accepting the truth norm corresponds to accepting that it is rational to believe whatever yields the highest "epistemic payoff" in the actual world. As we have seen, Easwaran and Fitelson regard this requirement as too demanding. However, they also reject a requirement which looks much more plausible (and in fact even quite weak), i.e., that any rational belief set should yield the highest "epistemic payoff" in at least one possible world. As evidence for their case they mention the so-called "miner puzzle" (see Kolodny and MacFarlane (2010), who take it from Parfit 1988; also see this dissertation's introduction). In the miner scenario, the rational act is one which does not maximize utility in any possible world.

A reasonable hypothesis I would like to make in this conclusive chapter is that from Easwaran and Fitelson's framework it should be possible to derive a proper logic. A fundamental notion in defining logical consequence is that of satisfiability: according to one usual way of defining logical consequence, a formula is said to follow from a set of formulas if and only if the formula consisting of the conjunction of the premises and of the conclusion's negation is not satisfiable (i.e., it is false in every interpretation). Now, Easwaran and Fitelson provide us with a notion of coherence, albeit different from (weaker than) classical deductive consistency. A reasonable hypothesis (which has been suggested to me by Matteo Plebani) is that, starting from the authors' proposed coherence requirement (R), we should be able to generate a logical consequence relation. Of course, the logic generated would be quite weak, as it would lack (at least) modus ponens, modus tollens and conjunction introduction. However, if the hypothesis is correct, this logic would both provide us with an alternative to Belief Closure and with a framework in which we could treat (S)NSP. That is, both (i) and (ii) of the research project above would be fulfilled[3].

I would like, nevertheless, to point out a limit of our authors' proposal. Indeed, they seem to retain (R) as a requirement for rational belief; however, consider (S)NSP: (R) entails a cut-off point with respect to it. This means that (at least in its present form) Easwaran and Fitelson's approach cannot provide a unified solution to our two paradoxes: their framework should be improved by replacing (R) with a norm of belief which does not entail cut-off points. So one fundamental research question is the following: is it possible to come up with a requirement which is both stronger than WADA and SADA (i.e., which does not imply that we can rationally hold contradictory judgements) and which, at the same time, does not entail cut-off points? Answering this question would mean fulfilling point (iii) in my research project: the norm of belief we finally settle on should do justice to our intuition that the premises of our puzzles are rationally believable/true and at the same time should block the unwanted conclusions.

I would like to close this dissertation by proposing two quotes, which I regard as particularly effective in capturing the spirit in which this thesis has been written. The first one is by Graham Priest (1979, pp. 219-220):

> Of course, we know how to avoid the paradoxes formally. We can avoid the semantic paradoxes, e.g., by a hierarchy of Tarski meta-languages, and the set theorectic ones, e.g., by the class/set distinction of von Neumann. But these are not solutions. A paradox is an argument with premises which appear to be true and steps which appear to be valid, which nevertheless ends in a conclusion which is false. A solution would tell us which premise is false or which step invalid; but moreover it would give us an *independent reason* for believing the premise or the step to be wrong. If we have no reason for rejecting the premise or the step other than it blocks the conclusions, then the 'solution' is *ad hoc* and

[3]Of course, Easwaran and Fitelson's framework is not the only possible starting point. Logics lacking modus ponens, modus tollens and conjunction introduction have already been proposed in the literature on rational belief and rational degrees of belief. The so-called "system O", for instance, lacks these three rules (see Hawthorne 1996 and 2007, but also Hawthorne and Makinson 2007). However, it seems to me that Easwaran and Fitelson's framework, with its (compelling and heuristically rich) analogy to principles of rational decision represents a particularly promising starting point.

unilluminating. Virtually all known 'solutions' to the paradoxes fail this test and this is why I say that no solution has yet be found.

Priest's target in this excerpt are the proposed solutions to the so-called semantic and set-theoretic paradoxes. However, similar considerations apply to the puzzles I focused on in this thesis (i.e., McGee's counterexample and its variants, as well as the Lottery Paradox and its variants). I believe that very often too little attention is devoted to the problem of identifying the "culprit(s)" of a paradox or a puzzle, while many efforts are devoted, instead, to constructing new logics or to devising new sophisticated formal tricks to block a puzzle's conclusion. This is why I decided to concentrate, in this thesis, on the task which consists in trying to provide a "diagnosis" for a (some) paradox(es). As I see it, we have two main tools at our disposal for this task: the first one is the discovery of new variants of a given puzzle. The role that this kind of discoveries has played, historically, in the study of paradoxes is clearly of the utmost importance (think of the so-called "revenge paradoxes"). The second one is the discovery of analogies between existing paradoxes, which is also a characterizing feature of this dissertation.

Some forty years later, a student of Wittgenstein described as follows his teacher's main message: "first, to keep in mind that things are as they are; and secondly, to seek illuminating comparisons to get an understanding of how they are" (Basil Reeve, paraphrased in Monk 1990, p. 451). In a similar spirit, the analogies and variants I have proposed in this dissertation were aimed at establishing that some famous puzzles show exactly what, intuitively, they appear to show. Consider McGee's counterexample, or Carroll's puzzle: in both cases, our intuition is that the argument's premises are rationally acceptable, while the argument's conclusion is not. The analogies and variations I have proposed show that these appearances are truthful.

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