A behavioral model of consumer response to price information

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Abstract

This paper introduces a model of choice to capture heuristics and reference-dependence in consumer's response to price information. The model is consistent with a kinked and upward-sloping demand curve. The price and cross-price elasticities of demand can be positive or negative, asymmetric, and product-dependent. The model offers an explanation for quality-dependent price stickiness, justifies the adoption of complex pricing strategies, and allows for the derivation of closed-form expressions for the optimal price and reference price set by a monopolist. The model is fully characterized by testable restrictions on demand data, which provide a method for identifying the reference price.

Reference price, Multinomial logit, Price-quality heuristic, Pricing strategy, Price rigidity, Lambert W
JEL Classification: D01, D11, D91

1 Introduction

Extensive empirical evidence shows that consumers' response to prices is often "behavioral." Consumers rely on "reference" prices – internal, subjective prices used to evaluate the appropriateness of actual prices (e.g., Monroe, 1973; Kalyanaram and Winer, 1995; Mazumdar et al., 2005). Moreover, consumers may behave contrary to the standard economic intuition that higher prices *reduce* demand (e.g., Ng, 1987; Cosaert, 2018; Dusansky and Koç, 2007; Genesove and Mayer, 2001). For example, consumers use prices as a proxy for quality (the "price-quality" heuristic), leading to an upward-sloping demand curve (e.g., Scitovszky, 1944; Pollak, 1977; Gneezy et al., 2014).

Understanding consumers responses to price variations is essential for marketers who want to anticipate consumer behavior and implement successful pricing strategies. But it is also

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central to policy makers who wants to anticipate the effect of fiscal or monetary policies (e.g., Eichenbaum et al., 2011; Kim, 2019).

Existing "behavioral" models of consumer choice have limitations. Models of the price-quality heuristic (e.g., Gneezy et al., 2014) do not consider reference prices. Models that integrate reference prices (e.g., Winer, 1986; Lattin and Bucklin, 1989; Putler, 1992; Hardie et al., 1993; Kopalle et al., 1996; Bell and Lattin, 2000; den Boer and Keskin, 2022) assume that the demand for a product is always decreasing with price, and that the evaluation of (relative) prices is independent of the product's quality. Moreover, as is common in models of reference-dependent behavior, these models assume an exogenous reference price, making their predictions sensitive to an external parameter.[1]

In this paper, I introduce and characterize axiomatically a model of random consumer choice, called the Reference Price Quality (RPQ) model, which addresses these limitations and provides new predictions about consumers' responses to price information. The key assumption in the RPQ model is that the consumer positively distorts the perception of a product's quality when its price is close to the reference price.

The RPQ model has several advantages. Firstly, it is consistent with a variety of observed consumer responses to price information that other models cannot accommodate. Secondly, the RPQ model maintains tractability, as demonstrated by the closed-form characterization of the optimal price and reference price that a monopolist set in the presence of outside options. Thirdly, the RPQ model features a relatively simple axiomatic characterization. Importantly, these axioms enable me to uniquely determine the reference price(s) used by individuals.

The RPQ model extends the multinomial logit demand model (e.g., Guadagni and Little, 1983). In the latter, the structural value of a product of quality q and price p is given by $v(q) - c(p)$, for some functions v, c. In the RPQ model, the structural value of a product of quality q and price p is $\sigma(p|p^*)v(q) - c(p)$, where p^* is the reference price and σ the *distortion function*. The key assumption is that observing a price above or *below* the reference price results in a lower relevance for quality; this means that the function σ is weakly single-peaked (i.e., single-plateaued) at the reference price.

Like other models of reference-price dependent consumer choice, demand in the RPQ model

[1] This critique is shared by many models of reference-dependent preferences (Kahneman and Tversky, 1979). However, there are empirical approaches for estimating reference points (e.g., Baucells et al., 2011; Allen et al., 2017; Baillon et al., 2020).

can be kinked. However, unlike these models, in the RPQ model, demand can be upward-sloping. This is because increasing a price that is below the reference price can have a positive effect on the perception of quality. If the positive effect is greater than the negative effect of incurring a higher monetary cost, the product's demand will increase. Furthermore, the RPQ model predicts that upward-sloping demand is more likely when the product is of high quality. This property is consistent with the empirical evidence that high-quality products have stickier prices (Kim, 2019) than low-quality ones. Reducing the price of a high-quality product is more likely to reduce demand than for a low-quality one. Thus, the model captures a reduced form of the price-quality heuristic, but refines it by making it reference-price dependent.

In terms of the price and cross-price elasticities of demand, the RPQ model predicts that they can be positive or negative, asymmetric, and dependent on the products' quality (see, for example, the empirical evidence in Dossche et al., 2010; Biondi et al., 2020; Iizuka and Shigeoka, 2021; Yaman and Offiaeli, 2022).

I also study the demand effects of varying the reference price. Although the results are generally ambiguous, if the actual price of a product exceeds the reference price and the prices of alternative products are lower than the reference, increasing the reference price can lead to a larger demand for the product.

I demonstrate the applicability of the RPQ model by studying the optimal price that a monopolist sets in the presence of an outside option when demand follows the RPQ model. Under general conditions, the optimal price can be expressed in closed-form through the Lambert W function (an easy-to-simulate and approximate function, see e.g., Corless et al., 1996; Aravindakshan and Ratchford, 2011). I also study the case in which the monopolist can set both the price and the reference price (e.g., in the long run). In this case, the optimal reference price is equal to the posted price and the optimal posted price takes a "logit-like" closed form. If there is no demand "premium" for observing a price equal to the reference price, the short-run monopolist's profit, under RPQ demand), is always lower than the profit obtained under logit demand (the long-run profit). This leads to a new interpretation for the logit optimal price and profit as the price and profit that emerge in the RPQ model when the monopolist has the ability to choose both the price and the reference price.

Furthermore, a simple two-period version of the RPQ model provides a new explanation for the widespread adoption of complex pricing strategy, such as the markdown (MD), as opposed to simpler alternatives, such as the everyday-low-price (EDLP) (e.g., Adida and Özer, 2019; Özer and Zheng, 2016). When a consumer has a high reference price, facing a low price reduces the relevance of quality. If this effect is stronger than the positive impact of paying a lower price, the MD pricing strategy results in higher demand than the EDLP strategy.

One advantage of the RPQ model is that it can be fully characterized in terms of testable restrictions on commonly available data (such as scanner data). Prior to characterizing the RPQ model, I first characterize a general version of the multinomial price-quality logit model, which includes models of reference-price dependent behavior present in the literature. In addition to some basic axioms, the defining properties of the general multinomial logit demand are the independence between the product's quality and its price, and a downward-sloping demand for all products. In terms of observable restrictions, the first property implies that the relative demand between two qualities is independent of prices. The second property translates into a monotonicity condition. In the characterization of the RPQ model, I relax both independence and monotonicity. Most notably, I establish a testable condition on choice probabilities that allows to uniquely identify the reference price (or an interval of reference prices). Reference prices are those at which quality differences are most relevant for the relative differences in demand. Indeed, at the reference price, the distortion of quality is maximal, so even small quality differences are magnified. This intuition clarifies why the interplay between quality and price is essential for identifying reference prices in the model. Without this interplay, reference prices cannot be determined from choice data.

To illustrate this point further, I provide the axioms that characterize the standard reference-price models, where the cost function c is increasing and piecewise linear, and compare them with those characterizing the RPQ model. I demonstrate that reference prices can only be identified under these specific parametric restrictions on c, meaning that any modifications to the cost function would require testing different conditions.

Lastly, I extend the model in two directions. First, I consider a model that generalizes the RPQ model by allowing for a price-quality interaction that is potentially independent of the reference price. Special cases of this model have appeared in the literature on the price-quality interaction (e.g., Crawford et al., 2015; Li et al., 2020). In the second extension, I allow for "context-dependent" distortion effects, so that the price distorts quality depending on the other available products. This model allows me to provide a reference-price-based explanation for context effects, such as the asymmetric dominance and the compromise effects (Simonson, 1989). Similarly to the baseline model, I provide a characterization of the context-dependent model through properties of choice data. As a by-product, I show that any positive demand probabilities can be expressed in a context-dependent logit-like formulation.

2 The model

I consider a consumer choosing among homogeneous products (i.e., products belonging to the same category, such as cookies). I represent each product k by a pair (q_k, p_k), where q_k denotes the quality and p_k the price of k. I assume that $q_k \in [q_0, q_1]$, with $q_0 < q_1$, and $p_k \in [0, \infty)$ for all products k. The consumer randomly selects a product from a finite and non-empty set of products (a choice set). I denote by $\mathbb{A}$ the family of all choice sets. The demand for a product k in a set A is the probability that k is selected from A, denoted by $P((q_k, p_k)|A)$.[2] The choice probabilities satisfy $P : A \times \mathbb{A} \to [0, 1]$ and $\sum_{(q_k,p_k)\in A} P((q_k, p_k)|A) = 1$ for all $A \in \mathbb{A}$.

Definition 1. *The choice probabilities P_{RPQ} have a Reference Price Quality (RPQ) model representation if there exist $p^* \in [0, \infty)$, a function $\sigma(\cdot|p^*) : [0, \infty) \to [0, \infty)$ and weakly increasing functions $v : [q_0, q_1] \to [0, \infty)$, $c : [0, \infty) \to [0, \infty)$ such that:*

$$P_{RPQ}((q_k, p_k)|A) = \frac{e^{\sigma(p_k|p^*)v(q_k)-c(p_k)}}{\sum\limits_{(q_l,p_l)\in A} e^{\sigma(p_l|p^*)v(q_l)-c(p_l)}}, \tag{RPQ}$$

for all $A \in \mathbb{A}$. Moreover, $\sigma(p|p^*) \geq \sigma(p'|p^*)$ if $p^* \geq p \geq p'$ or $p' \geq p \geq p^*$.

The RPQ model assumes that choice probabilities have a logit-like functional form, where the structural value of a product of quality q_k and price p_k is given by $u(q_k, p_k) = \sigma(p_k|p^*)v(q_k) - c(p_k)$. The function v measures the perceived quality, and the function c the perceived monetary cost of product k. The function σ distorts the product's perceived quality as a function of the

[2] I take the standard interpretation of random demand generated by a single consumer as arising from unobservable changing tastes. These are, for example, due to neural computational constraints (Webb, 2018) or to variations in attention, experimentation or perception (e.g., Guo, 2016; He, 2023). Alternatively, random demand can arise from the demand of a population of individuals with deterministic but unobservable preferences.

actual price relative to a reference price. The distortion function satisfies a weak form of *single-peakedness*—called single-plateauedness (see e.g., Moulin, 1984)—at p^*: the closer a price is to the reference price, the (weakly) more relevant quality becomes.

The following are some examples of the distortion function. The first, called Piecewise Linear, is defined as $\sigma_{PL}(p|p^*) = \max\{0, \hat{\sigma}_{PL}(p|p^*)\}$ where:

$$\hat{\sigma}_{PL}(p|p^*) = \begin{cases} \zeta - \eta(p - p^*) & \text{if } \; p > p^* \\ \zeta - \gamma(p^* - p) & \text{if } \; p \leq p^*, \end{cases} \tag{PL}$$

for some $\zeta \geq 0$, $0 \leq \gamma \leq \eta$. The inequality $\gamma \leq \eta$ represents "loss aversion" (see Kalyanaram and Winer, 1995): consumers are more sensitive to losses (observed prices above the reference price) than gains (observed prices below the reference price). The parameter ζ represents the "premium" of observing a price equal to the reference price. Figure 1 shows a possible specification of the piecewise linear distortion and the corresponding demand. A related example with

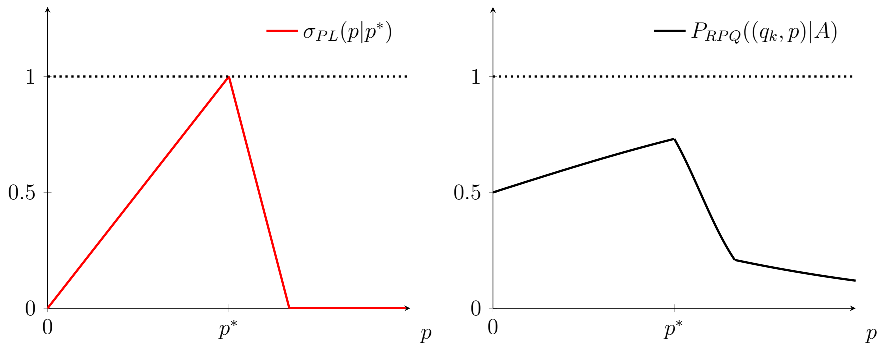

Figure 1: Left panel: a Piecewise linear distortion with $\zeta = 1$ and loss aversion. Right panel: the probability of selecting (q_k, p) in $A = \{(q_k, p), (q_l, p_l)\}$, as a function of p.

similar interpretations is the Quadratic distortion, defined as the positive part of the function:

$$\sigma_Q(p|p^*) = \xi - \frac{\kappa}{2}(p^* - p)^2, \tag{Q}$$

for some $\xi, \kappa \geq 0$. Figure 2 shows a possible specification of the quadratic distortion and the corresponding demand. A third example is the "Acceptable Price Range" distortion (e.g.,

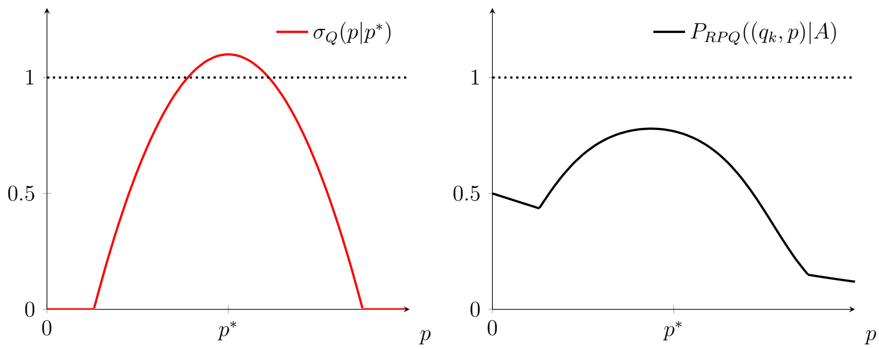

Figure 2: Left panel: a Quadratic distortion with $\xi = 1.1$ and $\kappa = 1$. Right panel: the probability of selecting (q_k, p) in $A = \{(q_k, p), (q_l, p_l)\}$, as a function of p.

Monroe, 1971; Janiszewski and Lichtenstein, 1999):

$$\sigma_{APR}(p|p^*) = \begin{cases} 1 & \text{if } \; p \in [p^* - \delta_1, p^* + \delta_2] \\ 0 & \text{if } \; p \notin [p^* - \delta_1, p^* + \delta_2], \end{cases} \tag{APR}$$

for some $\delta_2 \geq 0$ and $0 \leq \delta_1 \leq p^*$. When the actual price is not too far from the reference price, it is acceptable, and the product's quality matters. If the price is "unacceptable," quality becomes irrelevant, and demand is solely driven by the monetary cost of the product (see Figure 3). A last example of distortion function is the "Salience" distortion:

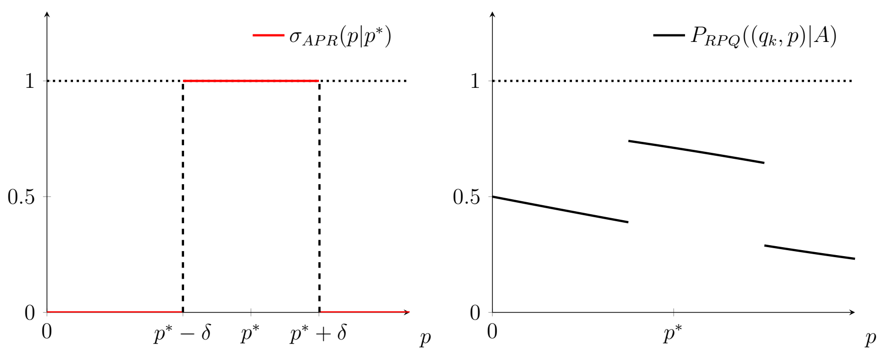

Figure 3: Left panel: a symmetric (i.e., $\delta_1 = \delta_2 = \delta$) Acceptable Price Range distortion. Right panel: the probability of selecting (q_k, p) in $A = \{(q_k, p), (q_l, p_l)\}$ as a function of p.

$$\sigma_S(p|p^*) = \begin{cases} 1 + \theta & \text{if } \; p = p^* \\ 1 & \text{if } \; p \neq p^*, \end{cases} \tag{Salience}$$

for some $\theta \geq 0$. When the actual price is equal to the reference price, it attracts the consumers' attention, and the perceived value of quality is amplified. All other prices are non-distortive.

The RPQ choice probabilities can be derived from an additive random utility model (ARUM), expressed as follows:

$$P_{RPQ}((q_k, p_k)|A) = \mathbb{P}(\sigma(p_k|p^*)v(q_k) - c(p_k) + \varepsilon_k \geq \sigma(p_l|p^*)v(q_l) - c(p_l) + \varepsilon_l,\ \forall (q_l, p_l) \in A), \quad (1)$$

assuming the error terms ε are i.i.d. and distributed according to a Gumbel distribution (see, for example, Train, 2009). The ARUM formulation helps to understand the price-quality "inference" in the RPQ model. A higher $\sigma(p_k|p^*)$ increases the weight of quality in the structural value of k. Since the noise components are i.i.d., they are unaffected by a higher σ. Consequently, a higher distortion reduces choice variability and it increases the probability of selecting k. In this sense, the price "signals" quality. When all products have the same price, the RPQ model can be interpreted as a psychophysical model of stimuli discrimination, where σ measures "randomness" in the choice of quality. The higher σ, the closer the choices are to deterministic utility maximization (see Appendix A).

Special cases. The RPQ model generalizes the multinomial logit demand model proposed by Guadagni and Little (1983), where a product's demand is given by:

$$P_{Logit}((q_k, p_k)|A) = \frac{e^{v(q_k) - \beta p_k}}{\sum\limits_{(q_l, p_l) \in A} e^{v(q_l) - \beta p_l}}. \quad \text{(Logit)}$$

This corresponds to a RPQ model with $\sigma(p|p^*) = 1$ and $c(p) = \beta p$ for some $\beta \geq 0$.

The RPQ model also generalizes models of consumer choice that account for reference prices (Winer, 1986; Lattin and Bucklin, 1989; Hardie et al., 1993; Bell and Lattin, 2000). In these models, the probability of selecting a product is given by:

$$P_{RD}((q_k, p_k)|A) = \frac{e^{u_{RD}(q_k, p_k)}}{\sum\limits_{(q_l, p_l) \in A} e^{u_{RD}(q_l, p_l)}}, \quad \text{(RD)}$$

where

$$u_{RD}(p, q) = \begin{cases} v(q) + \eta^+(p^* - p) - \beta p & \text{if } p \leq p^*, \\ v(q) - \eta^-(p - p^*) - \beta p & \text{if } p > p^*, \end{cases}$$

and $\eta^+, \eta^-, \beta \geq 0$. The RD model is a particular case of the RPQ model with $\sigma(p|p^*) = 1$ for all prices and a cost function $c(p) = (\beta + \eta^+)p - \eta^+ p^*$ if $p \leq p^*$ and $c(p) = (\beta + \eta^-)p - \eta^- p^*$ otherwise.

In both the logit and the reference-dependent models, the demand is downward-sloping, and the value of quality is independent of the price. In the RPQ model instead, the demand can be *upward-sloping* (see Figures 1, 2 and 3), and price and quality are intertwined. To illustrate these properties, consider two products $A = \{(q_k, p_k), (q_l, p_l)\}$ and assume that $\sigma_{APR}(p_l|p^*) = \sigma_{APR}(p_k|p^*) = 0$ and $c(p) = \beta p$. Since both prices p_k and p_l are unacceptable (e.g., they are too low), the products' quality becomes irrelevant, and the demand for k is solely driven by prices:

$$P_{RPQ}((q_k, p_k)|A) = \frac{e^{-\beta p_k}}{e^{-\beta p_k} + e^{-\beta p_l}}.$$

Suppose that the price of k increases by Δ and becomes acceptable, so that $\sigma_{APR}(p_k + \Delta|p^*) = 1$. The demand for $(q_k, p_k + \Delta)$ in A^Δ, where $A^\Delta = \{(q_k, p_k + \Delta), (q_l, p_l)\}$, is given by:

$$P_{RPQ}((q_k, p_k + \Delta)|A) = \frac{e^{v(q_k) - \beta(p_k + \Delta)}}{e^{v(q_k) - \beta(p_k + \Delta)} + e^{-\beta p_l}}.$$

It is immediate to observe that if $v(q_k) \geq \beta\Delta$, the RPQ model predicts a *higher* demand for product k after its price increased, i.e., $P_{RPQ}((q_k, p_k)|A) \geq P_{RPQ}((q_k, p_k + \Delta)|A^\Delta)$. The interpretation is that a higher price reduces the structural value of k by $\beta\Delta$, leading to a negative effect on its demand. However, if the higher price is acceptable, it has a positive "signaling value" that increases the value of k by an amount equal to $v(q_k)$. If the positive effect outweighs the negative effect, the demand for product k will *increase*. Moreover, this conclusion depends on the quality of the product. Specifically, it is possible that increasing the price by Δ may boost the demand for a high-quality product (if $v(q_k) \geq \beta\Delta$) but decrease the demand for a low-quality product (if $v(q_l) \leq \beta\Delta$). Therefore, an upward-sloping demand curve is more likely to occur with high-quality products. This conclusion is generally true in the RPQ model (see Section 3).

3 The shape of demand: elasticities

Changes in demand are typically measured by price and cross-price elasticities. In this section, I investigate these elasticities in the RPQ model and also explore how demand changes in response to variations in the reference price. For simplicity, I assume that both σ and c are differentiable functions (or at least they posses one-sided derivatives).

3.1 Price elasticity

Consider the derivative of the demand for product $k = (q_k, p_k) \in A$ with respect to its own price:[3]

$$\frac{\partial P_{RPQ}((q_k, p_k)|A)}{\partial p_k} = P_{RPQ}((q_k, p_k)|A)\left[1 - P_{RPQ}((q_k, p_k)|A)\right]\left[\sigma_{p_k}(p_k|p^*)v(q_k) - c_{p_k}(p_k)\right],$$

where σ_{p_k} and c_{p_k} are the derivatives of σ and c in p_k, respectively. Since probabilities are strictly positive:

$$\frac{\partial P_{RPQ}((q_k, p_k)|A)}{\partial p_k} \geq 0 \iff \sigma_{p_k}(p_k|p^*)v(q_k) \geq c_{p_k}(p_k). \tag{2}$$

The condition (2) involves all the relevant quantities of the RPQ model: the price, quality and reference price of product k. A higher price increases demand if the marginal cost $c_{p_k}(p_k)$ is smaller than the marginal benefit $\sigma_{p_k}(p_k|p^*)v(q_k)$. Because the distortion function is weakly single-peaked at the reference price, the inequality (2) is never satisfied if the actual price is larger than the reference price. This is because in that case, $\sigma_{p_k} \leq 0$, which implies $\sigma_{p_k}(p_k|p^*)v(q_k) \leq 0 \leq c_{p_k}(p_k)$. However, the inequality can be satisfied if the actual price is lower than the reference price. Furthermore, *ceteris paribus*, the inequality (2) is harder to satisfy when $v(q_k)$ is small. Therefore, for (perceived) low-quality products, a price increase is less likely to increase demand compared to high-quality products.

This property is consistent with the empirical evidence that price stickiness is quality-dependent (Kim, 2019). In particular, high-quality products have stickier prices than low-quality ones. In the RPQ model, higher quality products are more likely to display an upward-sloping demand curve if the price is below the reference price (see Figure 4). Thus, a price reduction for high-quality product is more likely to depress demand than for low-quality ones,

[3]For choice probabilities of the form $P((q_k, p_k)|A) = \frac{e^{u(q_k,p_k)}}{\sum_{(q_l,p_l)\in A} e^{u(q_l,p_l)}}$, the partial derivative with respect to a product's own price is given by $\frac{\partial P((q_k,p_k)|A)}{\partial p_k} = P((q_k, p_k)|A)\left[1 - P((q_k, p_k)|A)\right]\frac{\partial u(q_k,p_k)}{\partial p_k}$. In the RPQ model, $u(q_k, p_k) = \sigma(p_k|p^*)v(q_k) - c(p_k)$, so that the result follows.

inducing downward price stickiness.

From the marginal variation, I can derive the price elasticity of demand, which is defined as $E^{p_k}_{(q_k,p_k),A} = \frac{\partial P((q_k,p_k)|A)}{\partial p_k} \cdot \frac{p_k}{P((q_k,p_k)|A)}$. In the RPQ model, the price elasticity is given by:

$$E^{p_k}_{(q_k,p_k),A} = p_k \left[1 - P_{RPQ}((q_k,p_k)|A)\right] \left[\sigma_{p_k}(p_k|p^*)v(q_k) - c_{p_k}(p_k)\right],$$

which has the same sign as the marginal variation. Thus, it can be positive or negative. Moreover, it can be asymmetric around the reference price. Evidence of asymmetric price elasticity can be found in Dossche et al. (2010); Biondi et al. (2020); Iizuka and Shigeoka (2021); Yaman and Offiaeli (2022), for example. Moreover, the elasticity is quality-dependent.

To further illustrate condition (2), let consider the Piecewise Linear function $\sigma_{PL}(p|p^*)$ of Eq. (PL), and assume $c(p) = \beta p$. In this case, I have:

$$\frac{\partial P_{RPQ}((q_k,p_k)|A)}{\partial p_k} > 0 \iff \begin{cases} -\eta v(q_k) > \beta & \text{if } \ p_k > p^* \\ \gamma v(q_k) > \beta & \text{if } \ p_k < p^*. \end{cases}$$

Suppose that the price of k is lower than the reference price p^*. A positive variation of p_k may result in a higher demand for k if the marginal cost β is smaller than the "marginal value" $\gamma v(q_k)$ of observing a price closer to the reference p^*. If the actual price is higher than the reference price p^*, a larger price p_k will never result in higher demand for k. As mentioned earlier, the variation of the demand is a function of the product's quality. The inequality $\gamma v(q_k) > \beta$ is more likely to hold when $v(q_k)$ is high. Figure 4 shows a situation in which the demand for a low-quality product is kinked but downward-sloping (blue line), while the demand for a high-quality product (black line) is upward-sloping before the reference price and downward-sloping after it.

3.2 Relation with models of reference-price

Before discussing the cross-price elasticity, I compare demand variations in the RPQ model and the reference-price dependent demand of Equation RD. The latter predicts a kinked and monotone decreasing demand function, similar to the blue line in Figure 4. Indeed, the structural

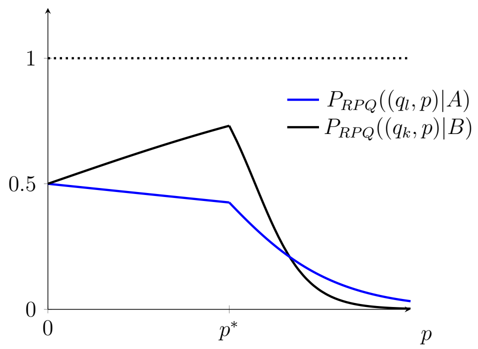

Figure 4: Blue line: the probability of selecting (q_l, p) as a function of p in $A = \{(q_l, p), (q_m, p_m)\}$, with $c(p) = \beta p$, $\sigma_{PL}(p|p^*)$ and $\gamma v(q_l) < \beta$. Black line: the probability of selecting (q_k, p) as a function of p in $B = \{(q_k, p), (q_m, p_m)\}$, with $c(p) = \beta p$, $\sigma_{PL}(p|p^*)$ and $\gamma v(q_k) > \beta$.

value u_{RD} can be rewritten as:

$$u_{RD}(p, q) = \begin{cases} v(q) + \eta^+ p^* - p(\beta + \eta^+) & \text{if } \ p \leq p^* \\ v(q) + \eta^- p^* - p(\beta + \eta^-) & \text{if } \ p > p^*. \end{cases}$$

It is immediate to observe that the demand for a product is always downward-sloping. Indeed, the condition (2) for an upward-sloping demand becomes $c_{p_k}(p_k) \leq 0$ in the case of u_{RD}. But $c_{p_k}(p_k) = \beta + \eta^+$ if $p_k < p^*$ and $c_{p_k}(p_k) = \beta + \eta^-$ if $p > p^*$, and both $\beta + \eta^+$ and $\beta + \eta^-$ are positive.

Consider the structural value in the RPQ model with σ_{PL} (assuming it is strictly greater than zero for simplicity) and $\zeta = 1$:

$$u(p, q) = \begin{cases} v(q) + v(q)\gamma p^* - p(\beta - v(q)\gamma) & \text{if } \ p \leq p^* \\ v(q) + v(q)\eta p^* - p(\beta + v(q)\eta) & \text{if } \ p > p^*. \end{cases}$$

This version of the RPQ model is qualitatively similar to the RD model if $\beta \geq v(q)\gamma$. In this case, a higher price always reduces demand (see the blue line in Figure 4). However, the RPQ model is more general, since the demand can be upward-sloping when the posted price is below the reference price, and quality is intertwined with price. In Section 6, I provide the falsifiable restrictions of the RD model, and show that, unlike the RPQ model, they require strong parametric assumptions.

3.3 Cross-price elasticity

A change in the price of a product $l = (q_l, p_l) \in A$ affects the demand for $k = (q_k, p_k) \in A$ according to:

$$\frac{\partial P_{RPQ}((q_k, p_k)|A)}{\partial p_l} = -P_{RPQ}((q_k, p_k)|A)P_{RPQ}((q_l, p_l)|A)\left[\sigma_{p_l}(p_l|p^*)v(q_l) - c_{p_l}(p_l)\right].$$

The condition $\sigma_{p_l}(p_l|p^*)v(q_l) \geq c_{p_l}(p_l)$, as showed earlier, implies that the demand of l will increase. Due to the logit-like properties of the RPQ model, this results in a decrease in the demand for k. Similar to the own price-elasticity, I can define the cross-price elasticity by normalizing the derivative of the demand

$$E^{p_l}_{(q_k,p_k),A} = -p_l P_{RPQ}((q_l, p_l)|A)\left[\sigma_{p_l}(p_l|p^*)v(q_l) - c_{p_l}(p_l)\right].$$

The cross-price elasticity has the same sign as the cross-price variation, and it can be positive or negative, asymmetric, and dependent on the products' quality. If $\sigma(p|p^*) = 1$ for all prices p, as in the logit or in the RD model, then $E^{p_l}_{(q_k,p_k),A} = p_l P_{RPQ}((q_l, p_l)|A)c_{p_l}(p_l)$, which is positive since $c_{p_l} \geq 0$. Thus, according to these models, products are always substitutes. However, the cross-price elasticity in the RPQ model may be negative if $\sigma_{p_l}(p_l|p^*)v(q_l) \geq c_{p_l}(p_l)$. In such cases, a positive change in the price of an alternative product may *reduce* the demand for k, similar to what happens with complementary products. However, complementarity in demand may be spurious in the RPQ model, as it can occur as a joint consequence of the upward-sloping demand curve for l and the logit-like substitution patterns of the RPQ model (a higher demand for a product reduces the demand for all the alternative products).

3.4 Marginal changes in the reference price

Lastly, I analyze the demand effect of a marginal change in the reference price:

$$\frac{\partial P_{RPQ}((q_k, p_k)|A)}{\partial p^*} = P_{RPQ}((q_k, p_k)|A)\left[\sigma_{p^*}(p_k|p^*)v(q_k) - \sum_{(q_l,p_l)\in A} P_{RPQ}((q_l, p_l)|A)\sigma_{p^*}(p_l|p^*)v(q_l)\right], \tag{3}$$

which implies:

$$\frac{\partial P_{RPQ}((q_k,p_k)|A)}{\partial p^*} \geq 0 \iff \sigma_{p^*}(p_k|p^*)v(q_k) \geq \sum_{(q_l,p_l)\in A} P_{RPQ}((q_l,p_l)|A)\sigma_{p^*}(p_l|p^*)v(q_l). \quad (4)$$

The condition states that, if the marginal value of facing a higher reference price for product k is "above average," then the demand for product k will increase. By the weak single-peakedness property of σ, the derivative $\sigma_{p^*}(p_k|p^*)$ is positive (negative) if the posted price p_k is above (below) the reference price. Therefore, a sufficient condition for inequality (4) to hold is that p_k is above the reference price while all p_l are below it. With two products $A = \{(q_k,p_k),(q_l,p_l)\}$, condition (4) becomes[4] $\sigma_{p^*}(p_k|p^*)v(q_k) \geq \sigma_{p^*}(p_l|p^*)v(q_l)$.

4 Applications

In this section, I first illustrate the applicability of the RPQ model by considering the optimal price and reference price that a monopolist sets when facing RPQ demand in the presence of an outside option. Then, I apply the RPQ model to explain why complex pricing strategies (the Markdown (MD) pricing strategy) are more prevalent than simpler ones (the Everyday-low-price (EDLP) pricing strategy (Adida and Özer, 2019; Özer and Zheng, 2016)).

4.1 Optimal price for a monopolist

A monopolist sells a product (q,p) and consumers can either buy it or choose an outside option (with utility normalized to 0). The demand for (q,p) is given by

$$P_{RPQ}((q,p)|A) = \frac{e^{\sigma(p|p^*)v(q)-\beta p}}{1+e^{\sigma(p|p^*)v(q)-\beta p}},$$

where, for simplicity, I assumed $c(p) = \beta p$. I begin by analyzing the case in which the reference price is fixed (e.g., in the short-run), so that the monopolist can only choose the price $\hat{p}$ to maximize profit Π:

$$\max_p \Pi = \max_p (p-C)P_{RPQ}((q,p)|A),$$

[4]This follows from $\sigma_{p^*}(p_k|p^*)v(q_k) \geq P((q_k,p_k)|A)\sigma_{p^*}(p_k|p^*)v(q_k) + P((q_l,p_l)|A)\sigma_{p^*}(p_l|p^*)v(q_l)$, which becomes $(1-P((q_k,p_k)|A))\sigma_{p^*}(p_k|p^*)v(q_k) \geq P((q_l,p_l)|A)\sigma_{p^*}(p_l|p^*)v(q_l)$, but $(1-P((q_k,p_k)|A)) = P((q_l,p_l)|A)$.

where C is the marginal cost. Explicit solutions are typically hard to find because of the interaction between linear and exponential terms (see, e.g., Aravindakshan and Ratchford, 2011). The following result, however, demonstrates that, under fairly general conditions, the structural value of the product at the optimal price can be expressed independently of $\hat{p}$ through the Lambert W function.[5] Consequently, it can be possible to obtain a closed-form expression for the monopolist's optimal price by isolating it from the structural value.

Proposition 1. *Suppose that $\hat{p}$ is the optimal price and $\sigma(p|p^*)$ is differentiable at $\hat{p}$. If $\sigma(\hat{p}|p^*)v(q) + \sigma_p(\hat{p}|p^*)v(q)(C - \hat{p}) = f(C, v(q), p^*)$ for some function f (i.e., the expression $\sigma(\hat{p}|p^*)v(q) + \sigma_p(\hat{p}|p^*)v(q)(C - \hat{p})$ is independent of $\hat{p}$), then*

$$\sigma(\hat{p}|p^*)v(q) - \beta\hat{p} = -\beta C - 1 + f(C, v(q), p^*) - W\left(e^{f(C,v(q),p^*)-1-\beta C}\right), \tag{5}$$

where W is Lambert W function.

Equation 5 can be rewritten as $\sigma_p(\hat{p}|p^*)v(q)(\hat{p}-C) - \beta\hat{p} = -\beta C - 1 - W\left(e^{f(C,v(q),p^*)-1-\beta C}\right)$ and, if $\sigma_p(\hat{p}|p^*)$ is independent of $\hat{p}$, then the optimal price has the following closed-form expression

$$\hat{p} = C + \frac{1 + W\left(e^{f(C,v(q),p^*)-1-\beta C}\right)}{\beta - \sigma_p v(q)}. \tag{6}$$

The condition $\sigma(\hat{p}|p^*)v(q) + \sigma_p(\hat{p}|p^*)v(q)(C - \hat{p}) = f(C, v(q), p^*)$ is always satisfied if σ is linear around $\hat{p}$ (in this case, $\sigma_p(\hat{p}|p^*)$ will be independent of $\hat{p}$, so that equation (6) holds).[6] In particular, the condition is satisfied almost everywhere when $\sigma = \sigma_{PL}$ (or when $\sigma = \sigma_S$ or $\sigma = \sigma_{APR}$). Therefore, I can find closed-form expressions for the monopolist's optimal price, as shown in the following corollary:

Corollary 1. *Suppose that $\hat{p}$ is the optimal price and $\sigma_{PL}(p|p^*)$ is differentiable at $\hat{p}$, then $\gamma v(q) < \beta$ and: if $\sigma_{PL}(\hat{p}|p^*) = 0$,*

$$\hat{p} = C + \frac{1 + W\left(e^{-1-\beta C}\right)}{\beta},$$

[5]The Lambert W function is implicitly defined as $W(x)e^{W(x)} = x$ (see Corless et al., 1996; Aravindakshan and Ratchford, 2011). It is positive and increasing for $x \geq 0$ and it has the property that $\ln W(x) = \ln x - W(x)$.

[6]Indeed, suppose that $\sigma(p|p^*) = mp + k$ for all p in a interval around $\hat{p}$. Then, $\sigma(\hat{p}|p^*)v(q) + \sigma_p(\hat{p}|p^*)v(q)(C - p) = (m\hat{p} + k)v(q) + mv(q)(C - \hat{p})$, which simplifies to $kv(q) + mv(q)C = f(C, v(q), p^*)$ and is independent of $\hat{p}$.

if $\sigma_{PL}(\hat{p}|p^*) > 0$,

$$\hat{p} = \begin{cases} C + \frac{1+W\left(e^{\zeta v(q)-\eta v(q)(p^*-C)-1-\beta C}\right)}{\beta+\eta v(q)} & \textit{if} \quad \hat{p} \geq p^* \\ C + \frac{1+W\left(e^{\zeta v(q)-\gamma v(q)(p^*-C)-1-\beta C}\right)}{\beta-\gamma v(q)} & \textit{if} \quad \hat{p} < p^*, \end{cases}$$

where W *is Lambert* W *function.*

The logit optimal price (see Aravindakshan and Ratchford, 2011) is obtained if $\zeta = 1$ and $\eta = \gamma = 0$, resulting in $\hat{p} = C + \frac{1+W\left(e^{v(q)-1-\beta C}\right)}{\beta}$. The monopolist's margin in the RPQ model is always positive but may be either strictly larger or strictly smaller than the margin under the logit demand. For example, assuming $\zeta = 1$ and that the marginal cost is smaller then the reference price, if the optimal price is higher than the reference price, the monopolist's margin in the RPQ model is strictly smaller than that in the logit case (the next section shows that this is always true when $\zeta = 1$). This is due to the fact that $W\left(e^{v(q)-\eta v(q)(p^*-C)-1-\beta C}\right) \leq W\left(e^{v(q)-1-\beta C}\right)$ and $\beta + \eta v(q) \geq \beta$. Similarly, the monopolist's profit under the RPQ demand can be smaller or larger than the profit under the logit demand.

4.2 Optimal reference and posted prices

Suppose that the monopolist can set both the posted price and the reference price (e.g., in the long-run). First, for a fixed posted price, the optimal reference price is equal to the posted price. Indeed, σ is (weakly) single-peaked at p^*; therefore, for a fixed price $\bar{p}$, the optimal reference price is equal to $\bar{p}$:

$$\bar{p} \in \arg\max_{p} (\bar{p} - C) \frac{e^{\sigma(\bar{p}|p)v(q)-\beta\bar{p}}}{1 + e^{\sigma(\bar{p}|p)v(q)-\beta\bar{p}}}.$$

Therefore, the optimization problem reduces to finding the optimal posted price that maximizes the following expression:

$$\max_{p} (p - C) \frac{e^{\sigma(p|p)v(q)-\beta p}}{1 + e^{\sigma(p|p)v(q)-\beta p}}.$$

If $\sigma(p|p)$ is constant across reference prices (i.e., $\sigma(p|p) = \sigma(p'|p') = \bar{\sigma}$ for all prices p, p'), the solution to the above problem is given by:

$$\bar{p} = C + \frac{1 + W(e^{\bar{\sigma}v(q)-1-\beta C})}{\beta},$$

which corresponds to the optimal price in a modified logit model in which the structural value of the product is $\bar{\sigma}v(q) - \beta p$ rather than $v(q) - \beta p$. It follows that the monopolist's short-run profit (i.e., when the reference price is fixed) is bounded by the long-run profit, thus:

$$\Pi \leq (\bar{p} - C)\frac{e^{\bar{\sigma}v(q)-\beta\bar{p}}}{1 + e^{\bar{\sigma}v(q)-\beta\bar{p}}} = \frac{W\left(e^{\bar{\sigma}v(q)-1-\beta C}\right)}{\beta},$$

where the equality derives from $\frac{e^{\bar{\sigma}v(q)-\beta\bar{p}}}{1+e^{\bar{\sigma}v(q)-\beta\bar{p}}} = \frac{W\left(e^{\bar{\sigma}v(q)-1-\beta C}\right)}{1+W\left(e^{\bar{\sigma}v(q)-1-\beta C}\right)}$ (see Aravindakshan and Ratchford, 2011). Lastly, if $\bar{\sigma} = 1$, which indicates that there is no demand premium for observing a price equal to the reference price, the monopolist's profit under the logit model (i.e., the long-run profit) will always be greater than the profit under the RPQ demand (i.e., the short-run profit). Thus, I provided a new interpretation for the logit optimal price and profit as the price and profit of a monopolist facing the RPQ demand when he has the ability to choose the price and the reference price.

4.3 Optimality of complex pricing strategies.

Complex pricing strategies, such as frequent price changes, are more common than simpler strategies in which prices are kept constant over time (e.g., Adida and Özer, 2019; Özer and Zheng, 2016).

To explain this puzzling pattern, I consider a two-period ($t = 1, 2$) version of the RPQ model and only one product k. The consumer's choice is either to buy k or nothing n (with the value of n normalized to 0). There are two possible prices for product k, $p_l < p_h$. I denote by p_1, p_2 the price of k in period one and two, respectively. A pricing strategy is called everyday-low-price (EDLP) if $p_1 = p_2 = p_l$, while the Markdown (MD) strategy is such that $p_1 = p_h > p_l = p_2$. Assuming that the reference price is p_h, the overall demand under the EDLP strategy is then given by:

$$P_1^{EDLP}(q, p_l) + P_2^{EDLP}(q, p_l) = \frac{e^{\sigma(p_l|p_h)v(q)-c(p_l)}}{1 + e^{\sigma(p_l|p_h)v(q)-c(p_l)}} + \frac{e^{\sigma(p_l|p_h)v(q)-c(p_l)}}{1 + e^{\sigma(p_l|p_h)v(q)-c(p_l)}}.$$

With the MD strategy, the overall demand is given by:

$$P_1^{MD}(q, p_h) + P_2^{MD}(q, p_l) = \frac{e^{\sigma(p_h|p_h)v(q)-c(p_h)}}{1 + e^{\sigma(p_h|p_h)v(q)-c(p_h)}} + \frac{e^{\sigma(p_l|p_h)v(q)-c(p_l)}}{1 + e^{\sigma(p_l|p_h)v(q)-c(p_l)}}.$$

I can state the following immediate result:

Proposition 2. *MD leads to higher demand than EDLP, i.e.,* $P_1^{MD}(q, p_h) + P_2^{MD}(q, p_l) \geq P_1^{EDLP}(q, p_l) + P_2^{EDLP}(q, p_l)$ *if and only if* $c(p_h) - c(p_l) \leq v(q)(\sigma(p_h|p_h) - \sigma(p_l|p_h))$

The MD pricing strategy involves two contrasting forces. On the one hand, an initial high price may lead to lower demand due to the higher monetary cost of acquiring the product. On the other hand, this initial high price can also boost demand by matching the reference price. If the latter effect outweighs the former, the overall demand under MD pricing is larger than that under EDLP pricing. For example, with $\sigma = \sigma_{PL}$ and $c(p) = \beta p$, the inequality in Proposition 2 becomes $\beta \leq \gamma v(q)$. If instead, $\sigma = \sigma_S$, the condition becomes $\beta(p_h - p_l) \leq \theta v(q)$. Note that both the Logit model and the reference-price dependent model are incompatible with a higher demand for the MD strategy. In both models, σ is equal to 1 and c is increasing. Therefore, the inequality in Proposition 2 becomes $c(p_h) - c(p_l) \leq 0$, which is never satisfied because c is increasing. Lastly, the inequality in Proposition 2 is more likely to hold for high quality products. Suggesting that MD pricing strategies may be more effective for high quality products.

5 Axiomatic characterization

In this section, I introduce the properties of demand data that characterize the RPQ model, making it falsifiable. I start by characterizing a general version of the multinomial logit demand model that encompasses the reference-price dependent model in Equation (RD). Next, I show which conditions need to be relaxed to obtain the RPQ model. I assume that the analyst can observe the consumer's choice among finitely many products. Therefore, there are finitely many observed quality levels, denoted by $\mathcal{Q} \subset [q_0, q_1]$ with $|\mathcal{Q}| \geq 2$. Additionally, I assume that $q_0 \in \mathcal{Q}$. There are finitely many observed prices, denoted by $\mathcal{P} \subset [0, \infty)$ with $|\mathcal{P}| \geq 2$. I denote by $\mathcal{A}$ the family of all choice sets in the dataset. The first three assumptions are standard:

Axiom (Positivity - P)**.** *For all* $A \in \mathcal{A}$ *and* $(q_k, p_k) \in A$, $P((q_k, p_k)|A) > 0$.

This assumption is rather weak. It cannot be rejected in any finite dataset since, empirically, a small but strictly positive probability is indistinguishable from a zero probability. The second property is the standard Independence of Irrelevant Alternatives (IIA):

Axiom (IIA)**.** *For all* $A, B \in \mathcal{A}$ *and* $(q_k, p_k), (q_l, p_l) \in A \cap B$,

$$\frac{P((q_k, p_l)|A)}{P((q_l, p_l)|A)} = \frac{P((q_k, p_k)|B)}{P((q_l, p_l)|B)}.$$

The last assumption is that, for a fixed price, the demand increases with quality.

Axiom (Quality Monotonicity - QM)**.** *For all* $p \in \mathcal{P}$*, all* $q, q' \in \mathcal{Q}$ *with* $q \leq q'$ *and all choice sets* A *such that* $(q, p), (q', p) \in A$*,* $P((q, p)|A) \leq P((q', p)|A)$*.*

I say that choice probabilities satisfy the *Basic Axioms* if they satisfy Positivity, IIA and Quality Monotonicity.

In the multinomial logit demand model (and in the reference-price dependent model), the structural value of a product (q, p) is $v(q) - c(p)$. This implies a separation between quality and price (v and c are independent), and a downward-sloping demand for all products (c is increasing). The following two axioms capture these properties:

Axiom (Odds Independence - OI)**.** *For all* $q_k, q_l \in \mathcal{Q}$*, all* $p_k, p_l \in \mathcal{P}$*, and all* $A \in \mathcal{A}$ *with* $(q_k, p_k), (q_k, p_l), (q_l, p_l), (q_l, p_k) \in A$*:*

$$\frac{P((q_k, p_k)|A)}{P((q_l, p_k)|A)} = \frac{P((q_k, p_l)|A)}{P((q_l, p_l)|A)}.$$

Odds Independence means that *relative* quality preferences are independent of the price. The next condition requires that, *ceteris paribus*, a higher price decreases the likelihood of selecting a product:

Axiom (Monotonicity (M).)**.** *For all* $q_k, q_l \in \mathcal{Q}$ *and all* $p_k, p_l \in \mathcal{P}$ *with* $p_k \geq p_l$*,* $P((q_k, p_k)|(q_l, p_l) \cup (q_k, p_k)) \leq P((q_k, p_l)|(q_k, p_l) \cup (q_l, p_l))$

The previous axioms characterize the general multinomial logit demand:

Theorem 1 (General Logit)**.** *The choice probabilities satisfy the Basic Axioms, Odds-Independence and Monotonicity if and only if there are weakly increasing functions* $v : \mathcal{Q} \to \mathbb{R}_+$ *and*

$c : \mathcal{P} \to \mathbb{R}_+$ such that:

$$P((q_k, p_k)|A) = \frac{e^{v(q_k)-c(p_k)}}{\sum_{(q_l,p_l)\in A} e^{v(q_l)-c(p_l)}},$$

for all $(q_k, p_k) \in A$ and $A \in \mathcal{A}$. The functions v and c are unique up to location (i.e., up to translation by a constant).

In Section 6, I will characterize the Logit case $c(p) = \beta p$ in Proposition 5 and the reference-price model RD in Proposition 4. In the RPQ model, choice probabilities are strictly positive, the IIA holds, and higher quality is more valuable. However, price and quality are intertwined and the demand for some products may increase with price. Therefore, both Odds Independence and Monotonicity need to be relaxed. Let $L_{q_k,q_l}(p, A)$ denote the log-odds of selecting q_k and q_l in a choice set A when they have the same price p:

$$L_{q_k,q_l}(p, A) = \ln \frac{P((q_k, p)|A)}{P((q_l, p)|A)}.$$

Log-odds represent the relative demand between two quality levels, q_k and q_l, at a given price p. The following axiom employs log-odds to establish the type of "independence" that holds in the RPQ model:

Axiom (Log-odds Independence - LOI)**.** *For all $p, p' \in \mathcal{P}$ and all $A, B \in \mathcal{A}$:*

$$\frac{L_{q_k,q_m}(p, A)}{L_{q_k,q_m}(p', B)} = \frac{L_{q_l,q_n}(p, A)}{L_{q_l,q_n}(p', B)}, \tag{7}$$

for all $q_k, q_l, q_m, q_n \in \mathcal{Q}$ such that either ratio is well defined.

Regarding the interpretation, Log-odds Independence implies that *relative changes* in demand between pairs of quality levels are independent of the price. Suppose that all log-odds are non-zero, then Log-odds Independence allows me to express the relative demand between two quality levels at price p as a quality-independent function of the relative demand at price p', $\frac{P((q_k,p)|A)}{P((q_m,p)|A)} = \left(\frac{P((q_k,p')|B)}{P((q_m,p')|B)}\right)^{\psi(p,p')}$, where $\psi(p, p') = L_{q_l,q_n}(p, A)/L_{q_l,q_n}(p', B)$ is independent of q_l and q_n.

The next condition weakens Monotonicity and states that the demand for the product of lowest quality decreases when its price increases:

Axiom (Worst Monotonicity - WM)**.** *If $p_k \geq p_l$, then $P((q_0, p_k)|(q_0, p_k) \cup (q_l, p_l)) \leq P((q_0, p_l)|(q_0, p_l) \cup (q_l, p_l))$.*

The last condition is needed to identify the reference price(s) and to impose weak single-peakedness of the distortion function. Note that the IIA implies that log-odds are independent of the choice set A. Hence, I can omit the reference to the choice set and write $L_{q_k,q_l}(p)$. Consider $q_k, q_l \in \mathcal{Q}$ and $\bar{p} \in \mathcal{P}$ such that $L_{q_k,q_l}(\bar{p}) > 0$ (which exists unless all probabilities are uniform). I define P^* to be the (set of) price(s) given by:

$$P^* = \underset{p \in \mathcal{P}}{\operatorname{argmax}} \frac{L_{q_k,q_l}(p)}{L_{q_k,q_l}(\bar{p})}.$$

The normalization only serves to show that, by Log-Odds Independence, these ratios are independent of q_k, q_l. The following condition requires that log-odds have a peak at (the potentially multiple) $p^* \in P^*$:

Axiom (Weak Single-Peakedness - WSP)**.** *For all $p^* \in P^*$, if $p \geq p' \geq p^*$ or $p^* \geq p' \geq p$ then $L_{q_k,q_l}(p') \geq L_{q_k,q_l}(p)$.*

Intuitively, the prices in P^* are the *revealed* reference prices. The WSP condition ensures that the further a price is away from the reference price, the lower is the relative likelihood of choosing one quality over the other. Reference prices are those at which quality differences are most relevant for the relative difference in demand. It is crucial to highlight, that in models where quality and price are independent, such as the logit or the RD models, all prices are revealed reference prices (i.e., $P^* = \mathcal{P}$), because the log-odds are independent of the price. I can now state the main result of this section:

Theorem 2 (Reference-Price-Quality)**.** *The choice probabilities satisfy the Basic Axioms, Log-odds Independence, Worst Monotonicity, and Weak Single-Peakedness if and only if there is $p^* \in \mathcal{P}$ and there are weakly increasing functions $v : \mathcal{Q} \to \mathbb{R}_+$ with $v(q_0) = 0$, $c : \mathcal{P} \to \mathbb{R}_+$, and a function $\sigma(\cdot|p^*) : \mathcal{P} \to \mathbb{R}_+$ such that:*

$$P((q_k, p_k)|A) = \frac{e^{\sigma(p_k|p^*)v(q_k) - c(p_k)}}{\sum_{(q_l,p_l) \in A} e^{\sigma(p_l|p^*)v(q_l) - c(p_l)}},$$

for all $(q_k, p_k) \in A$ *and all* $A \in \mathcal{A}$*. The function* $\sigma(\cdot|p^*)$ *satisfies* $\sigma(p'|p^*) \geq \sigma(p|p^*)$ *if* $p^* \geq p' \geq p$ *or* $p \geq p' \geq p^*$*.*

If $P^* = p^*$, then p^* is the unique reference price. However, consider the Acceptable Price range σ_{APR} defined in Equation APR. In this case, all prices in $[p^* - \delta_1, p^* + \delta_2]$ are revealed reference prices and are in P^*. For example, suppose that $p^* = 2$ and $\delta_1 = \delta_2 = 2$, then $P^* = \mathcal{P} \cap [0, 4]$. However, $p_1^* = 1$, $\delta_1' = 1$, and $\delta_2 = 3$ determine the same acceptable price range $P^* = \mathcal{P} \cap [0, 4]$. Thus, in some cases, it is not possible to identify a unique reference price from choice data. However, the Weak Single-Peakedness property ensures that P^* is an interval in $\mathcal{P}$. I conclude by stating the uniqueness properties of the RPQ model.

Proposition 3 (Uniqueness)**.** *The set* P^* *is unique. If* $\bar{\sigma}, \bar{v}, \bar{c}$ *also represent the choice probabilities, and the latter are not uniform, then there are* $a, b, d \in \mathbb{R}$ *such that* $\bar{\sigma} = a\sigma$*,* $\bar{v} = \frac{1}{a}v + b$ *and* $\bar{c} = c + ab\sigma + d$*.*

6 Falsifiability of reference price dependent models

In this section, I present the conditions that characterize the widely used model of Equation (RD) and the Logit model. I show that these conditions are much more stringent than those characterizing the RPQ model.

Assuming the piecewise linear and monotone cost function of Equation RD, one could identify the reference price by exploiting the change in the slope of c around p^*. Specifically, for prices below the reference price, the cost function is linear with a slope of $-\bar{\beta} - \eta^+$. For prices above the reference price, it is linear with a slope of $-\bar{\beta} - \eta^-$. The next axiom formalizes this property in terms of choice probabilities:

Axiom (Piecewise Constant Sensitivity - PCS)**.** *There is* $p^* \in [0, \infty)$ *such that, for all* $q_k \in \mathcal{Q}$*, all* $p_k, p_l \in [0, \infty)$ *with* $p_k, p_l \leq p^*$*, and all* $A \in \mathcal{A}$ *with* $(q_k, p_k), (q_k, \frac{1}{2}p_k + \frac{1}{2}p_l), (q_k, p_l) \in A$

$$\frac{P((q_k, p_k)|A)}{P((q_k, \frac{1}{2}p_k + \frac{1}{2}p_l)|A)} = \frac{P((q_k, \frac{1}{2}p_k + \frac{1}{2}p_l)|A)}{P((q_k, p_l|A)}.$$

An identical condition holds for all $p_k, p_l > p^*$*.*

Suppose the $p_l \leq p_k \leq p^*$. Keeping quality fixed, the relative increase in demand due to passing from a price p_k to a lower price $\frac{1}{2}p_k + \frac{1}{2}p_l$ is the same as the relative increase in demand of passing from $\frac{1}{2}p_k + \frac{1}{2}p_l$ to p_l. An identical property holds for prices above the reference. It is important to highlight that I am assuming an exogenous reference price p^* because the separation of quality and price does not allow me to "identify" reference prices as done with the RPQ model. Additionally, I assume that one can observe demand at all prices in $[0, \infty)$ (although this assumption can be partially relaxed to hold for a closed interval of prices).

The PCS assumption alone is not sufficient to characterize the cost function in the RD model. This is because the latter is continuous, while PCS can be satisfied by a discontinuous cost function. For example, $\hat{c}(p) = \gamma^+ p + \theta_1$ if $p \leq p^*$ and $\hat{c}(p) = \gamma^- p + \theta_2$ would satisfy PCS. Therefore, I need to impose continuity of the odds ratios with respect to perturbations of the price:

Axiom (Continuity - C)**.** *For all $q_k, q_l \in \mathcal{Q}$, all $p \in [0, \infty)$, $\epsilon > 0$, and all $A \in \mathcal{A}$ with $(q_l, p) \in A$, $\lim_{\varepsilon \to 0} \frac{P((q_k, p+\varepsilon)|A)}{P((q_l, p)|A)} = \frac{P((q_k, p)|A)}{P((q_l, p)|A)}$.*

I have the following result:

Proposition 4 (Reference price model)**.** *The choice probabilities satisfy the Basic Axioms, Odds-Independence, Monotonicity, Continuity and Piecewise Constant Sensitivity if and only if they have the General Logit representation of Theorem 1 and the cost function is $c(p) = \beta p - \eta^+(p^* - p)$, for some $\eta^+, \beta \geq 0$ when $p \leq p^*$ and $c(p) = \beta p + \eta^-(p - p^*)$, for some $\eta^-, \beta \geq 0$ when $p > p^*$.*

To falsify standard models of reference price, rich choice data and continuity of the cost function are required in addition to an *exogenously* specified reference price. Moreover, the identification of the reference price critically depends on parametric assumptions about the cost function, and specifying a different cost function (such as $c(p) = \eta(p - p^*)^2 + \beta p$) would require finding different conditions on choice data.

I conclude this section with the characterization of the multinomial logit. This is a particular case of the RD model with a zero reference price $p^* = 0$. Therefore, it necessarily satisfies the next axiom, which corresponds to PCS with $p^* = 0$:

Axiom (Constant Sensitivity - CS). *For all $q_k \in \mathcal{Q}$, all $p_k, p_l \in [0, \infty)$, and all $A \in \mathcal{A}$ with $(q_k, p_k), (q_k, \frac{1}{2}p_k + \frac{1}{2}p_l), (q_k, p_l) \in A$,*

$$\frac{P((q_k, p_k)|A)}{P((q_k, \frac{1}{2}p_k + \frac{1}{2}p_l)|A)} = \frac{P((q_k, \frac{1}{2}p_k + \frac{1}{2}p_l)|A)}{P((q_k, p_l|A)}.$$

I have the following result:

Proposition 5 (Logit with linear cost)**.** *The choice probabilities satisfy the Basic Axioms, Odds-Independence, Monotonicity, and Constant Sensitivity if and only if they have the General Logit representation of Theorem 1 and the cost function is $c(p) = \beta p$ for some $\beta \geq 0$.*

It is worth highlighting that I did not assume continuity in the previous result, but I nonetheless obtained a continuous cost function (this is due to the assumption of observing demand at all price levels).

7 Extensions

In this section, I study two extensions of the RPQ model. In the first, there is a price-quality interaction, but there is not necessarily a reference price(e.g., Crawford et al., 2015; Li et al., 2020). In the second, the quality distortion is context-dependent.

7.1 Price-quality interaction without a reference price.

In the first generalization of the RPQ, called the General PQ model, the structural value of a product of quality q and price p is given by $\sigma(p)v(q) - c(p)$, where σ is a positive function. This model is characterized by relaxing the WSP axiom from the axioms used to characterize the RPQ model in Theorem 2:

Proposition 6 (Price-Quality Interaction)**.** *The choice probabilities satisfy the Basic Axioms, Log-odds Independence, and Worst Monotonicity if and only if there is there are weakly increasing functions $v : \mathcal{Q} \to \mathbb{R}_+$ (with $v(q_0) = 0$), $c : \mathcal{P} \to \mathbb{R}_+$, and a function $\sigma : \mathcal{P} \to \mathbb{R}_+$ such that:*

$$P((q_k, p_k)|A) = \frac{e^{\sigma(p_k)v(q_k) - c(p_k)}}{\sum_{(q_l, p_l) \in A} e^{\sigma(p_l)v(q_l) - c(p_l)}}$$

for all $(q_k, p_k) \in A$ *and all* $A \in \mathcal{A}$.

Special cases of the General PQ model have appeared in the literature studying price-quality interaction. For instance, Crawford et al. (2015) and Li et al. (2020) propose that the value of a product is given by $u(q,p) = \alpha_0 + \alpha_1 q - \beta p + \alpha_2 qp$. This corresponds to a General PQ model with a distortion $\sigma(p) = \alpha_1 + \alpha_2 p$ for some $\alpha_1, \alpha_2 \in \mathbb{R}$, a linear cost function $-c(p) = -\alpha_0 - \beta p$ for some $\alpha_0 \in \mathbb{R}$ and $\beta \geq 0$, and $v(q) = q$. In this model, the demand for a product is upward-sloping if $\alpha_2 q \geq \beta$ (see condition (2)).

7.2 Context effects

The second extension of the RPQ model relaxes the logit-like functional form. It is well-known that the logit model is not suitable for modeling context-dependent behaviors, such as the asymmetric dominance effect and the compromise effect (Simonson, 1989). These phenomena violate a property called *regularity*, which states that the demand of a product cannot increase if a new product is added to the choice set. Formally:

Definition 2. *The demand* P *satisfies regularity if* $P((q_k, p_k)|A) \geq P((q_k, p_k)|B)$, *for all products* (q_k, p_k) *and all choice sets* $A \subseteq B$.

To address violations of regularity, I will consider the following context-dependent generalization of the PQ model:

$$P((q_k, p_k)|A) = \frac{e^{\sigma_A(p_k)v(q_k) - c_A(p_k)}}{\sum\limits_{(q_l,p_l)\in A} e^{\sigma_A(p_l)v(q_l) - c_A(p_l)}}, \tag{8}$$

where $\sigma_A : \mathcal{P} \to \mathbb{R}$ is a choice set-dependent distortion and c_A is a cost function for each choice set A.

Suppose that the initial choice set A contains only two products $k = (q_k, p_k)$ and $l = (q_l, p_l)$. A violation of Regularity occurs if adding m to A increases the probability of selecting k and/or l. For simplicity, let $c_A(p) = c_B(p)$ for all choice sets $A, B \in \mathcal{A}$. Simple algebra shows that:

Proposition 7. *The context-dependent PQ model of Equation* (8) *violates Regularity if and only if*

$$e^{v(q_k)(\sigma_{A\cup m}(p_k) - \sigma_A(p_k))} > e^{v(q_l)(\sigma_{A\cup m}(p_l) - \sigma_A(p_l))} + e^{\sigma_{A\cup m}(p_m)v(q_m) - c(p_m) - \sigma_A(p_l)v(q_l) + c(p_l)}.$$

To observe a violation of Regularity, the additional distortion of the quality of $k = (q_k, p_k)$ when m is added to A (the left-hand side of the inequality) must be larger than the additional distortion of the quality of l plus the relative value of m over l (the right-hand side of the inequality). If Regularity is violated, it must be the case that the additional distortion of k when m is present overcomes the additional distortion of l. Indeed, a necessary condition is that

$$v(q_k)\left(\sigma_{A\cup m}(p_k) - \sigma_A(p_k)\right) > v(q_l)\left(\sigma_{A\cup m}(p_l) - \sigma_A(p_l)\right).$$

If the distortion of the quality of l is unaffected by the presence of m (i.e., $\sigma_{A\cup m}(p_l) = \sigma_A(p_l)$), the necessary condition becomes

$$\sigma_{A\cup m}(p_k) > \sigma_A(p_k), \tag{9}$$

which simply states that the distortive effect of p_k is larger when m is present.

Two robust violations of Regularity are the *asymmetric dominance effect* and the *compromise effect* (Simonson, 1989). In the asymmetric dominance effect, a decoy product is added to a choice set to increase the demand for a targeted product. The decoy is a product that is dominated by the targeted product (e.g., it has a higher price and lower quality than the target) but not dominated by the alternative product. The asymmetric dominance effect occurs when the probability of selecting the targeted product increases after adding the decoy. In the compromise effect, the demand for a product increases when it becomes a "compromise" between two extreme alternatives. For example, if k has higher quality and a higher price than l, adding m that has higher quality and a higher price than k increases the demand for k.

The necessary condition (9) implies that the presence of a decoy or an extreme product boosts demand of the target because the price of the latter becomes more "salient."

7.2.1 Context-dependent "salience"

To illustrate an additional application of the context-dependent PQ model, I consider the wine purchase example discussed by Bordalo et al. (2013). In a wine shop, a cheap but low-quality wine (l) sold at \$10 per bottle may be preferred over an expensive and high-quality wine (h) sold at \$20 per bottle. However, this preference may be reversed in a restaurant, even if the price difference remains the same. For instance, the restaurant may sell the high-quality wine at \$60 and the low-quality wine at \$50. Let $A = \{(q_l, 10), (q_h, 20)\}$ and $A' = \{(q_l, 50), (q_h, 60)\}$, and $c_A(p) = c_{A'}(p) = p$. The context-dependent PQ model is consistent with $P((q_h, 60)|A') \geq P((q_h, 20)|A)$. Indeed, I have:

$$P((q_h, 60)|A') \geq P((q_h, 20)|A) \iff v(q_l)(\sigma_{A'}(50) - \sigma_A(10)) \leq v(q_h)(\sigma_{A'}(60) - \sigma_A(20)). \quad (10)$$

Suppose that $v(q_h) \geq v(q_l)$, then a sufficient condition for the inequality (10) is $\sigma_{A'}(50) - \sigma_A(10) \leq \sigma_{A'}(60) - \sigma_{A'}(20)$. Let's consider the special cases in which only the reference price is context-dependent: $\sigma_A(p) = \sigma_{PL}(p|p^*_A)$ and $\sigma_{A'}(p) = \sigma_{PL}(p|p^*_{A'})$. If the reference price at the wine shop is \$10 and it is \$60 at the restaurant, I have

$$\sigma_{PL}(50|60) - \sigma_{PL}(10|10) = -\gamma 10 \leq \eta 10 = \sigma_{PL}(60|60) - \sigma_{PL}(20|10),$$

which is always satisfied.

7.2.2 Characterization

I conclude this section with the characterization of the context-dependent PQ model. In Theorem 6, I show that the axioms characterizing the general PQ model are the basic Axioms (Positivity, Quality Monotonicity and the IIA) and the LOI. The characterization of its context-dependent version essentially relaxes the IIA. I have the following result:

Theorem 3 (Context Effects)**.** *Suppose that, for some products $(q_k, p_k), (q_l, p_l)$, $P((q_k, p_l)|Z) \neq P((q_l, p_l)|Z)$, where $Z = \{(q, p) : q \in \mathcal{Q},\ p \in \mathcal{P}\}$. The choice probabilities satisfy Positivity, Quality Monotonicity, and Log-odds Independence if and only if there exist a weakly monotone function $v : \mathcal{Q} \to \mathbb{R}$ and functions $c_A : \mathcal{P} \to \mathbb{R}_+$, $\sigma_A : \mathcal{P} \to \mathbb{R}$ for each $A \in \mathcal{A}$ such that:*

$$P((q_k, p_k)|A) = \frac{e^{\sigma_A(p_k)v(q_k) - c_A(p_k)}}{\sum\limits_{(q_l, p_l) \in A} e^{\sigma_A(p_l)v(q_l) - c_A(p_l)}},$$

for all $(q_k, p_k) \in A$ and all $A \in \mathcal{A}$.

An important intermediate step in the proof of Theorem 3 is Lemma 1 in Appendix B. This Lemma states that choice probabilities that satisfy Positivity always have a representation given by

$$P((q_k,p_k)|A) = \frac{e^{v_A(q_k,p_k)}}{\sum_{(q_l,p_l)\in A} e^{v_A(q_l,p_l)}},$$

where $v_A : A \to \mathbb{R}$ for each choice set A.

Figure 5 summarizes the relationship among the representation results proved in the text.

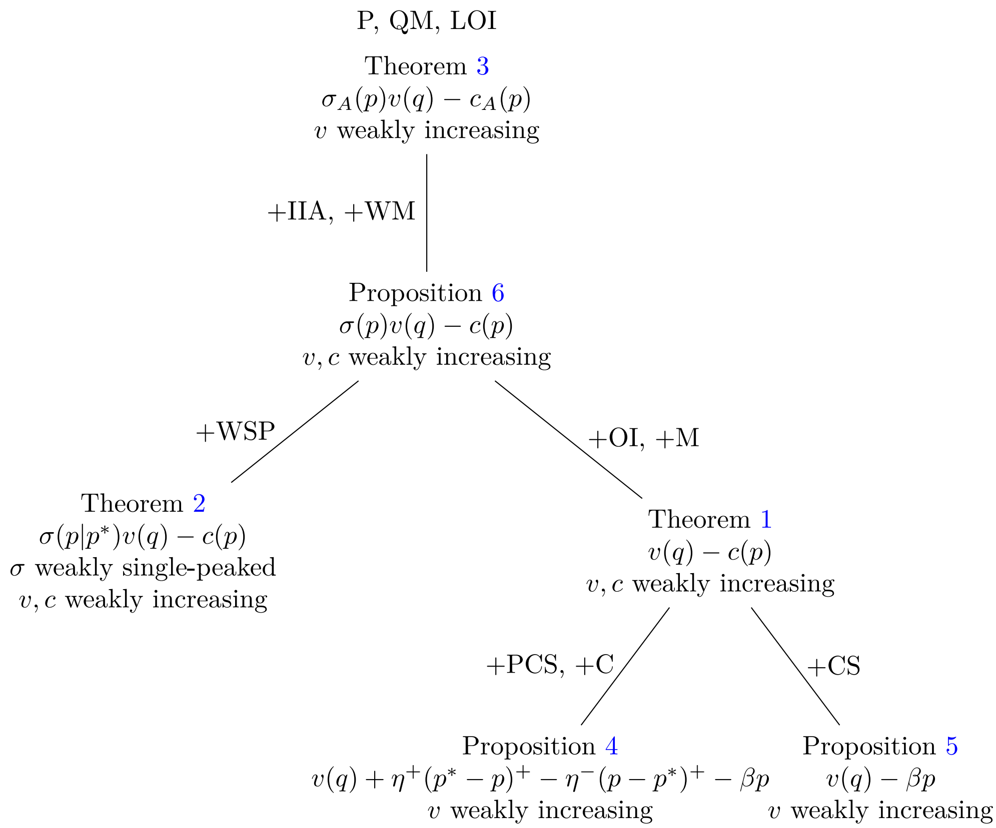

Figure 5: Summary of the axioms and the associated representation results.

8 Related literature

This paper contributes to the literature on random choice and to the literature on behavioral consumer choice.

In the context of the random choice literature, the axiomatic characterization of my model is one of the first to exploit the bi-dimensional nature of products.[7] In general, the results of the present paper can inform models of discrete choice over bi-dimensional objects, such as dated outcomes or two-person allocations. Although with a different scope and primitives,

[7] In this literature, Falmagne and Iverson (1979) studied the Weber's law for random choice over bi-dimensional objects.

Cerreia-Vioglio et al. (2023) axiomatized a dynamic version of the multinomial logit in which noise decreases over time. In their model, the structural value of an alternative a at time t is $v(a)/\lambda(t) + \alpha(a)$. This can be viewed as a model in which time distorts the value of a, as the price distorts quality in the generalized PQ model of Proposition 6.

The literature on "behavioral" consumers' responses to price information is extensive (Monroe, 1971, 1973; Cheng and Monroe, 2013, e.g.,). Within this literature, the paper contributes to the research on reference prices (see Briesch et al., 1997; Mazumdar et al., 2005, for reviews), as well as the research on the interdependence of price and value (e.g., Scitovszky, 1944; Cosaert, 2018; Ng, 1987; Pollak, 1977; Dodds et al., 1991; Gneezy et al., 2014). This paper offers a first axiomatic characterization of two models that are popular in this literature: the multinomial logit model of Guadagni and Little (1983) (see Theorem 1 and Proposition 5), and the model of random choice with reference prices (Theorem 1 and Proposition 4). Moreover, the RPQ model extends and fleshes out these models. Indeed, models of reference price typically disregard the interaction between prices or reference-prices and quality and assume that a higher price reduces demand. Moreover, they often assume an exogenous reference price.[8] This paper the first to allow for the identification of reference price(s) from choice data.

Concerning the literature on the price-quality heuristic, the present model results from an additive random utility in which the consumer is uncertain about the overall value of a product. The function distorting quality can be interpreted as if the price "signals" quality by increasing the weight of the quality over the random component, thereby reducing choice variability. This approach differs from Gneezy et al. (2014), where consumers are uncertain about the product's quality and form expectations based on prices. Typically, approaches to the price-quality heuristic (e.g., Bagwell and Riordan, 1991; Wolinsky, 1983; Pollak, 1977; Gneezy et al., 2014) are independent of reference prices, so I extend these approaches to include the effects of reference prices. Additionally, I provide a first behavioral characterization of a model allowing for the interaction between price and quality (Proposition 6).

Although not the primary focus of the paper, the context-dependent extension introduced in Section 7.2 contributes to the vast literature developed to rationalize context-effects (e.g., Tversky, 1972; Guo, 2016; Steverson et al., 2019; Webb et al., 2021; He, 2023).

[8]One exception is Baucells and Hwang (2017), who proposed a model of intertemporal consumption that features a time-varying reference price to account for various behavioral biases.

A RPQ and the psychophysics of price and quality perception.

Suppose that all products k in A have the same quality $q_k = q$, as in Anderson and De Palma (1992). In this case, the RPQ model becomes a model of *price-stimulus discrimination*. This is consistent with the Behavioral Pricing Theory (e.g., Cheng and Monroe, 2013), which poses that the perception of prices is subject to the same perceptual noise that affects the perception of other physical stimuli (e.g., sound or light). Let denote A_q the set $\{p_k \in [0, \infty) : (q, p_k) \in A\}$. Then,

$$P((q, p_k)|A) = P_q(p_k|A_q) = \frac{e^{\sigma(p_k|p^*)v(q)-c(p_k)}}{\sum\limits_{p_l \in A_q} e^{\sigma(p_l|p^*)v(q)-c(p_l)}}$$

and the demand for (q, p_k) in A can be interpreted as the choice of p_k from A_q, denoted by $P_q(p_k|A_q)$. In the particular case of binary choice sets $A = \{(q, p_k), (q, p_l)\}$, so that $A_q = \{p_k, p_l\}$, the choice probabilities in the RPQ model becomes

$$P((q, p_k)|A) = P_q(p_k|A_q) = \frac{1}{1 + e^{-v(q)(\sigma(p_k|p^*)-\sigma(p_l|p^*))-(c(p_l)-c(p_k))}},$$

which is increasing in $c(p_l) - c(p_k)$ and in $\sigma(p_k|p^*) - \sigma(p_l|p^*)$. Moreover, the dependence on $v(q)$ makes the stimuli discrimination *quality-dependent*. If the price p_k is "more distortive" than p_l, so that $\sigma(p_k|p^*) \geq \sigma(p_l|p^*)$, products with higher (perceived) quality makes price discrimination easier than low quality products. That is, $P((q_k, p_k)|(q_k, p_k), (q_k, p_l)) \geq P((q_l, p_k)|(q_l, p_k), (q_l, p_l))$ if $v(q_k) \geq v(q_l)$.

Suppose now that all products in A have the same price p. In this case, the RPQ model becomes:

$$P((q_k, p)|A) = \frac{\left(e^{v(q_k)}\right)^{\sigma(p|p^*)}}{\sum\limits_{(q_l, p) \in A} \left(e^{v(q_l)}\right)^{\sigma(p|p^*)}}.$$

It is customary to interpret $\sigma(p|p^*) \geq 0$ as a measure of rationality (e.g., Holt and Laury, 2002). Indeed, for $\sigma(p|p^*) = 0$, the demand is uniform $P((q_k, p)|A) = \frac{1}{|A|}$. For a large $\sigma(p|p^*)$ (going to infinity) the demand becomes deterministic and the choice from A coincides with the product that maximizes v. In the RPQ model with homogeneous prices, the distortion function becomes a measure of how "random" is consumers choice.

B Proofs

Proof of Proposition 1. Since $\hat{p}$ is optimal and σ is differentiable at $\hat{p}$, the first-order condition is satisfied at $\hat{p}$. The condition is

$$P_{RPQ}((q,p)|A) + (p - C)P_{RPQ}((q,p)|A)(1 - P_{RPQ}((q,p)|A))(\sigma_p(p|p^*)v(q) - \beta) = 0.$$

The optimal price cannot be smaller than C, therefore $\sigma_p(\hat{p}|p^*)v(q) - \beta < 0$ (otherwise, the left-hand side would be strictly positive). Then, $\hat{p} = C - \frac{e^{\sigma(\hat{p}|p^*)v(q)-\beta\hat{p}}}{\sigma_p(\hat{p}|p^*)v(q)-\beta} - \frac{1}{\sigma_p(\hat{p}|p^*)v(q)-\beta}$. Multiplying both sides by $\sigma_p(\hat{p}|p^*)v(q)-\beta$ and then adding $\sigma(\hat{p}|p^*)v(q)$ to both sides, I obtain $(\sigma_p(\hat{p}|p^*)v(q) - \beta)\hat{p} + \sigma(\hat{p}|p^*)v(q) = C(\sigma_p(\hat{p}|p^*)v(q) - \beta) - e^{\sigma(\hat{p}|p^*)v(q)-\beta\hat{p}} - 1 + \sigma(\hat{p}|p^*)v(q)$. Rearranging I obtain

$$e^{\sigma(\hat{p}|p^*)v(q)-\beta\hat{p}} + \sigma(\hat{p}|p^*)v(q) - \beta\hat{p} = \sigma(\hat{p}|p^*)v(q) + v(q)\sigma_p(\hat{p}|p^*)(C - \hat{p}) - \beta C - 1. \tag{11}$$

By the assumption in the proposition, there is $f(C, v(q), p^*)$ such that $\sigma(\hat{p}|p^*)v(q)+v(q)\sigma_p(\hat{p}|p^*)(C-\hat{p}) = f(C, v(q), p^*)$. Then, taking exponentials on both sides of Equation (11), I obtain $e^{e^{\sigma(\hat{p}|p^*)v(q)-\beta\hat{p}}}e^{\sigma(\hat{p}|p^*)v(q)-\beta\hat{p}} = e^{f(C,v(q),p^*)-\beta C-1}$. By defining $W = e^{\sigma(\hat{p}|p^*)v(q)-\beta\hat{p}}$, the previous equality can be written as $We^W = e^{f(C,v(q),p^*)-\beta C-1}$. This expression is related to the Lambert W function implicitly defined as $W(x)e^{W(x)} = x$, so that $e^{\sigma(\hat{p}|p^*)v(q)-\beta\hat{p}} = W\left(e^{f(C,v(q),p^*)-\beta C-1}\right)$. Taking logs on both sides and using the property that $\ln W(x) = \ln x - W(x)$, I have:

$$\sigma(\hat{p}|p^*)v(q) - \beta\hat{p} = f(C, v(q), p^*) - \beta C - 1 - W\left(e^{f(C,v(q),p^*)-\beta C-1}\right),$$

which gives the result. □

Proof of Corollary 1. If $\sigma = \sigma_{PL}(\hat{p}|p^*)$, then $\sigma(\hat{p}|p^*) = 0$, $\sigma(\hat{p}|p^*) = \zeta - \eta(\hat{p} - p^*)$, or $\sigma(\hat{p}|p^*) = \zeta - \gamma(p^* - \hat{p})$. Thus, the condition in Proposition 1 is satisfied with $f(C, v(q), p^*) = 0$, $f(C, v(q), p^*) = \zeta v(q) - \eta v(q)(p^* - C)$ if $\hat{p} > p^*$ or $f(C, v(q), p^*) = \zeta v(q) + \gamma v(q)(C - p^*)$ if $\hat{p} \leq p^*$. Then, using the result in Proposition 1, the optimal price has a closed form solution given by

$$\hat{p} = C + \frac{1 + W\left(e^{-1-\beta C}\right)}{\beta},$$

$$\hat{p} = C + \frac{1 + W\left(e^{\zeta v(q) - \eta v(q)(p^* - C) - 1 - \beta C}\right)}{\beta + \eta v(q)},$$

or by

$$\hat{p} = C + \frac{1 + W\left(e^{\zeta v(q) + \gamma v(q)(C - p^*) - 1 - \beta C}\right)}{\beta - \gamma v(q)}.$$

As showed above, $\beta > \gamma v(q)$. $\square$

Proof of Theorem 1. Necessity of the Axioms is straightforward. For sufficiency, by the IIA and Positivity, for all $A \in \mathcal{A}$ and all $(q_k, p_k) \in A$, the choice probabilities can be written as $P((q_k, p_k)|A) = \frac{u'(q_k,p_k)}{\sum_{(q_l,p_l)\in A} u'(q_l,p_l)}$ for a random scale $u' : \mathcal{Q} \times \mathcal{P} \to \mathbb{R}_{++}$. Let fix an arbitrary $q^* \in \mathcal{Q}$ and order $u'(q^*, p_0) \leq u'(q^*, p_1) \leq \ldots \leq u'(q^*, p^{\max})$. Let define $u(q,p) = \frac{u'(q,p)}{u'(q^*,p^{\max})}$. With such a u, $P((q_k, p_k)|A) = \frac{u(q_k,p_k)}{\sum_{(q_l,p_l)\in A} u(q_l,p_l)}$ and $u(q^*, p) \leq 1$ for all $p \in \mathcal{P}$. Now, let define $c(p) = -\ln u(q^*, p) \geq 0$, and similarly define $v_p(q) = \ln \frac{u(q,p)}{u(q^*,p)}$. By Odds Independence, $v_p(q)$ is independent of p and I denote it by $v(q)$. Then, $e^{v(q)-c(p)} = \frac{u(q,p)}{u(q^*,p)} u(q^*, p) = u(q,p)$. Lastly, Quality Monotonicity states that $P((q,p)|A) \geq P((q',p)|A)$ when $q' \geq q$ and $(q,p),(q',p) \in A$, that implies $v(q) - c(p) \leq v(q') - c(p)$, so v is weakly increasing. Monotonicity implies that $(1 + e^{v(q_l)-c(p_l)-v(q_k)+c(p_k)})^{-1} = P((q_k, p_k)|(q_k, p_k) \cup (q_l, p_l)) \leq P((q_k, p_l)|(q_k, p_l) \cup (q_l, p_l)) = (1 + e^{v(q_l)-v(q_k)})^{-1}$, when $p_k \geq p_l$. Thus, $v(q_l) - c(p_l) - v(q_k) + c(p_k) \geq v(q_l) - v(q_k)$ or $c(p_k) \geq c(p_l)$. For uniqueness, suppose that $\bar{v}, \bar{c}$ also represent the probabilities. By the uniqueness of the Logit model, there is $k \in \mathbb{R}$ such that $v(q) - c(p) = \bar{v}(q) - \bar{c}(p) + k$. Fix an arbitrary $\bar{p} \in \mathcal{P}$, then $v(q) = \bar{v}(q) - \bar{c}(\bar{p}) + c(\bar{p}) + k$. Thus, $v = \bar{v} + l$, where $l = -\bar{c}(\bar{p}) + c(\bar{p}) + k$ is a constant. Then, $\bar{v}(q) + l - c(p) = \bar{v}(q) - \bar{c}(p) + k$ implies $c(p) = \bar{c}(p) + l - k$. $\square$

Before proving Theorem 2, I prove the more general Proposition 6.

Proof of Proposition 6. Necessity of the Basic Axioms is straightforward. For Log-odds Independence, it clearly holds when $v(q_k) \neq v(q_m)$ or $v(q_m) \neq v(q_l)$. If for all $q_k, q_l \in \mathcal{Q}$, $v(q_k) = v(q_l)$, Log-odds Independence holds vacuously, since no ratio is well-defined. Worst Monotonicity follows directly from $v(q_0) = 0$ and the fact that c is weakly increasing. For sufficiency, by Positivity and the IIA property, for all $A \in \mathcal{A}$ and all $(q_k, p_k) \in A$, $P((q_k, p_k)|A) = \frac{u'(q_k,p_k)}{\sum_{(q_l,p_l)\in A} u'(q_l,p_l)}$ for some $u' : \mathcal{Q} \times \mathcal{P} \to \mathbb{R}_{++}$. By the IIA, Log-odds are independent of the choice set, so I can write $L_{q_k,q_l}(p)$. I say that $q_k, q_l \in \mathcal{Q}$ are p-distinguishable, if $P((q_k, p)|(q_k, p) \cup (q_l, p)) \neq 0.5$ (thus $L_{q_k,q_l}(p) \neq 0$). If there are no p-distinguishable qualities for all $p \in \mathcal{P}$, it must be that $u'(q,p)$ is constant for all $q \in \mathcal{Q}$ and all $p \in \mathcal{P}$. In this case, the proof is complete by setting $v(q) = 0$ for all $q \in \mathcal{Q}$ and $\sigma(p) = 1$ and $c(p) = 0$ for all $p \in \mathcal{P}$. Suppose there are $q, q' \in \mathcal{Q}$ that are $\bar{p}$-distinguishable. By Log-Odds Independence q and q_0 are also $\bar{p}$-distinguishable. Indeed, $L_{q,q'}(p)/L_{q,q'}(\bar{p})$ is a well-defined real number since q and q_0 are $\bar{p}$ distinguishable. By LOI, $L_{q,q'}(p)/L_{q,q'}(\bar{p}) = L_{q,q_0}(p)/L_{q,q_0}(\bar{p})$, so that $L_{q,q_0}(p)/L_{q,q_0}(\bar{p})$ must be a well-defined real number, meaning that q and q_0 are $\bar{p}$-distinguishable. I can order $u'(q_0, p_0) \leq u'(q_0, p_1) \leq \ldots \leq u'(q_0, p^{\max})$ and define $u(q,p) = \frac{u'(q,p)}{u'(q_0,p^{\max})}$. With such a u, $P((q_k,p_k)|A) = \frac{u(q_k,p_k)}{\sum_{(q_l,p_l)\in A} u(q_l,p_l)}$ and $u(q_0,p) \leq 1$ for all $p \in \mathcal{P}$. By definition, either $L_{q,q_0}(\bar{p})$ or $L_{q_0,q}(\bar{p})$ must be greater than 0. Assume w.l.o.g. that $L_{q,q_0}(\bar{p}) > 0$. Let define

$$\sigma_q(p) \equiv \frac{L_{q,q_0}(p)}{L_{q,q_0}(\bar{p})},$$

which is a well-defined real number because q, q_0 are $\bar{p}$-distinguishable. By Log-odds Independence, $\sigma_{q_k}(p) = \frac{L_{q_k,q_0}(p)}{L_{q_k,q_0}(\bar{p})} = \frac{L_{q_l,q_0}(p)}{L_{q_l,q_0}(\bar{p})} = \sigma_{q_l}(p)$, for all $q_k, q_l \in \mathcal{Q}$. Thus, the function σ_q is independent of q and I denote it by $\sigma(p)$. Now, I define $v(q) \equiv L_{q,q_0}(\bar{p})$ and $c(p)$ as $c(p) \equiv -\ln u(q_0,p) \geq 0$. Notice that $v(q_0) = 0$. It follows that $e^{\sigma(p_k)v(q_k)-c(p_k)} = e^{\frac{L_{q_k,q_0}(p_k)}{L_{q_k,q_0}(\bar{p})} L_{q_k,q_0}(\bar{p})} u(q_0,p_k) = e^{L_{q_k,q_0}(p_k)} u(q_0,p_k) = \frac{P((q_k,p_k)|A)}{P((q_0,p_k)|A)} u(q_0,p_k) = u(q_k,p_k)$. Lastly, Quality Monotonicity implies that, for all p, $P((q,p)|A) \leq P((q',p)|A)$ when $q' \geq q$ and $(q,p),(q',p) \in A$, which implies $\sigma(p)v(q) - c(p) \leq \sigma(p)v(q') - c(p)$, or $0 \leq \sigma(p)(v(q') - v(q))$. If $v(q') \geq v(q)$, the inequality is satisfied if $\sigma(p) \geq 0$ for all p. Take any $q''' \geq q''$, then $0 \leq \sigma(p)(v(q''') - v(q''))$ implies $v(q''') \geq v(q'')$, so v is weakly increasing. If $v(q') \leq v(q)$, the inequality is satisfied if $\sigma(p) \leq 0$ for all p. Take any $q''' \geq q''$, then $0 \leq \sigma(p)(v(q''') - v(q''))$ implies $v(q'') \geq v(q''')$, so v is weakly decreasing. Defining $\hat{\sigma} = -\sigma$ and $\hat{v} = -v$, the result follows. For the second part, if c is weakly increasing, the fact that $v(q_0) = 0$ implies that the choice probabilities satisfy Worst Monotonicity. For the opposite direction, Worst Monotonicity implies that $(1 + e^{\sigma(q_l)v(q_l)-c(p_l)-\sigma(p_k)v(q_0)+c(p_k)})^{-1} = P((q_0,p_k)|(q_0,p_k) \cup (q_l,p_l)) \leq P((q_0,p_l)|(q_0,p_l) \cup (q_l,p_l)) = (1 + e^{\sigma(p_l)v(q_l)-c(p_l)-\sigma(p_l)v(q_0)+c(p_l)})^{-1}$, when $p_k \geq p_l$. Thus, $v(q_0)(\sigma(p_l) - \sigma(p_k)) - c(p_l) + c(p_k) \geq 0$, but $v(q_0) = 0$ and then $c(p_k) \geq c(p_l)$. Hence the conclusion. □

Proof of Theorem 2. Necessity is straightforward. For sufficiency, by Proposition 6, the Basic Axioms, LOI and Worst Monotonicity imply the existence of $\sigma : \mathcal{P} \to \mathbb{R}$ and weakly increasing functions $v: \mathcal{Q} \to \mathbb{R}_+$, $c: \mathcal{P} \to \mathbb{R}_+$ such that $P((q_k, p_k)|A) = \frac{e^{\sigma(p_k)v(q_k)-c(p_k)}}{\sum_{(q_l,p_l)\in A} e^{\sigma(p_l)v(q_l)-c(p_l)}}$. Moreover, by construction $v(q_0) = 0$. Take now p^* such that $p^* \in \text{argmax}_{p\in\mathcal{P}}\, \sigma(p) = P^*$. By Weak Single-Peakedness $\sigma(p') = \frac{L_{q,q^*}(p')}{L_{q,q^*}(\bar{p})} \leq \frac{L_{q,q^*}(p)}{L_{q,q^*}(\bar{p})} = \sigma(p)$ if $p \geq p' \geq p^*$ or $p^* \geq p' \geq p$. Thus, $\sigma(p)$ is weakly single-peaked at p^*, and I conclude by relabeling $\sigma(p) = \sigma(p|p^*)$ to highlight this property. □

Proof of Proposition 3. Uniqueness of P^* is immediate, since the positive normalization $L_{q_k,q_l}(\bar{p})$ does not affect the maximum over prices. Suppose that probabilities are not uniform and that $\bar{v}, \bar{\sigma}, \bar{c}$ also represent the probabilities. Then, for some $p^*, p^\circ \in P^*$, $\frac{\sigma(p|p^*)}{\sigma(\bar{p}|p^*)} = \frac{L_{q_k,q_l}(p)}{L_{q_k,q_l}(\bar{p})} = \frac{\bar{\sigma}(p|p^\circ)}{\bar{\sigma}(\bar{p}|p^\circ)}$, so that $\bar{\sigma}(\cdot|p^\circ) = a\sigma(\cdot|p^*)$, where $a = \frac{\bar{\sigma}(\bar{p}|p^\circ)}{\sigma(\bar{p}|p^*)}$ is a constant. By the uniqueness property of the logit model, $\sigma(p|p^*)v(q) - c(p) = \bar{\sigma}(p|p^\circ)\bar{v}(q) - \bar{c}(p) + d$ for some $d \in \mathbb{R}$. Take an arbitrary $\bar{p} \in \mathcal{P}$, then $\bar{v}(q) = \frac{1}{a}v(q) - \frac{c(\bar{p})}{a\sigma(\bar{p}|p^*)} + \frac{ac(\bar{p})}{a\sigma(\bar{p}|p^*)} - \frac{d}{a\sigma(\bar{p}|p^*)}$, so that $\bar{v} = \frac{1}{a}v + b$, where b is a constant. Using the equality $\sigma(p|p^*)v(q) - c(p) = a\sigma(p|p^*)(\frac{1}{a}v(q) + b) - \bar{c}(p) + d$ again implies $\bar{c}(p) = c(p) + ab\sigma(p|p^*) + d$. □

Proof of Proposition 4. Necessity of the axioms is straightforward. For sufficiency, by Theorem 1, the choice probabilities have a general multinomial logistic representation. By PCS, the equality $\frac{P((q_k,p_k)|A)}{P((q_k,\frac{1}{2}p_k+\frac{1}{2}p_l)|A)} = \frac{P((q_k,\frac{1}{2}p_k+\frac{1}{2}p_l)|A)}{P((q_k,p_l|A)}$ for all $p_k, p_l \leq p^*$, implies $-c(p_k) + c(\frac{1}{2}p_k + \frac{1}{2}p_l) = -c(\frac{1}{2}p_k + \frac{1}{2}p_l) + c(p_l)$. Rearranging gives $\frac{1}{2}c(p_k) + \frac{1}{2}c(p_l) = c(\frac{1}{2}p_k + \frac{1}{2}p_l)$. This is a Jensen's functional equation whose solution is $c(p) = \bar{\beta}p + \theta$ for some arbitrary $\bar{\beta} \geq 0$ (due to Monotonicity) and $\theta \in \mathbb{R}$ (see Aczél, 1966, Th.1 p. 46). For $p > p^*$, Continuity and an identical argument implies $c(p) = \hat{\beta}p + \hat{\theta}$ for some arbitrary $\hat{\beta} \geq 0$ (by Monotonicity) and $\hat{\theta} \in \mathbb{R}$. By Continuity, $\bar{\beta}p^* + \theta = \hat{\beta}p^* + \hat{\theta}$, that implies $\hat{\theta} = p^*(\bar{\beta} - \hat{\beta}) + \theta$. Take any $a, b \geq 0$ with $a + b = \hat{\beta}$, then $c(p) = (a+b)p + p^*(\bar{\beta} - a - b) + \theta$. Rearranging gives $c(p) = ap + b(p - p^*) + \theta + p^*(\bar{\beta} - a)$ for all $p > p^*$. Now, take b' such that $a + b' = \bar{\beta}$, then $c(p) = (a + b') + \theta = (a + b')p + \hat{\theta} - p^*(a + b' - \hat{\beta})$. Rearranging gives $c(p) = ap - b'(p^* - p) + \hat{\theta} - p^*(a - \hat{\beta})$ for all $p \leq p^*$. By definition, $\hat{\theta} - p^*(a - \hat{\beta}) = \theta + p^*(\bar{\beta} - a)$, so this constant term cancels out when plugging c in the choice probabilities. □

Proof of Proposition 5. Necessity of the axioms is straightforward. For sufficiency, by Theorem 1, the choice probabilities have a multinomial logistic representation. By Constant Sensitivity, the equality $\frac{P((q_k,p_k)|A)}{P((q_k,\frac{1}{2}p_k+\frac{1}{2}p_l)|A)} = \frac{P((q_k,\frac{1}{2}p_k+\frac{1}{2}p_l)|A)}{P((q_k,p_l|A)}$ for all p_k, p_l, implies $-c(p_k) + c(\frac{1}{2}p_k + \frac{1}{2}p_l) =$

$-c(\frac{1}{2}p_k+\frac{1}{2}p_l)+c(p_l)$. Rearranging gives $\frac{1}{2}c(p_k)+\frac{1}{2}c(p_l)=c(\frac{1}{2}p_k+\frac{1}{2}p_l)$. This is again a Jensen's functional equation whose solution is $c(p)=\beta p+\theta$ for some $\beta\geq 0$ (due to Monotonicity) and $\theta\in\mathbb{R}$. Substituting into the choice probabilities gives

$$P((q_k,p_k)|A)=\frac{e^{v(q_k)-\beta p_k+\theta}}{\sum\limits_{(q_l,p_l)\in A} e^{v(q_l)-\beta p_l+\theta}}=\frac{e^{v(q_k)-\beta p_k}}{\sum\limits_{(q_l,p_l)\in A} e^{v(q_l)-\beta p_l}}.$$

□

Before proving Theorem 3, I show that any positive choice probabilities admit a context-dependent logit-like representation:

Lemma 1. *The choice probabilities satisfy Positivity if and only if for all $A\in\mathcal{A}$ and all $(q_k,p_k)\in A$,*

$$P((q_k,p_k)|A)=\frac{e^{v_A(q_k,p_k)}}{\sum\limits_{(q_l,p_l)\in A} e^{v_A(q_l,p_l)}}, \tag{12}$$

for some functions $v_A:A\to\mathbb{R}_-$.

Proof of Lemma 1. Necessity is immediate. For sufficiency, for each $A\in\mathcal{A}$, I fix $(q_A^*,p_A^*)\in A$ and define $\bar{v}_A:A\to\mathbb{R}$ as $\bar{v}_A(q_k,p_k)=\ln\frac{P((q_k,p_k)|A)}{P((q_A^*,p_A^*)|A)}$, which is defined because of Positivity. Then,

$$\frac{e^{\bar{v}_A(q_k,p_k)}}{\sum_{(q_l,p_l)\in A} e^{\bar{v}_A(q_l,p_l)}}=\frac{\frac{P((q_k,p_k)|A)}{P((q_A^*,p_A^*)|A)}}{\sum_{(q_l,p_l)\in A}\frac{P((q_l,p_l)|A)}{P((q_A^*,p_A^*)|A)}}=P((q_k,p_k)|A).$$

Now, let define $v_A(q_k,p_k)=\bar{v}_A(q_k,p_k)-\max_{(q,p)\in A}\bar{v}_A(q,p))$, so that $v_A\leq 0$ and

$$\frac{e^{v_A(q_k,p_k)}}{\sum_{(q_l,p_l)\in A} e^{v_A(q_l,p_l)}}=\frac{e^{\bar{v}_A(q_k,p_k)-\max_{(q,p)\in A}\bar{v}_A(q,p)}}{\sum_{(q_l,p_l)\in A} e^{\bar{v}_A(q_l,p_l)-\max_{(q,p)\in A}\bar{v}_A(q,p)}}=P((q_k,p_k)|A).$$

□

Proof of Theorem 3. Necessity of the axioms is straightforward. For sufficiency, by Lemma 1, there are choice set-dependent functions $v_A:A\to\mathbb{R}_-$ such that $P((q_k,p_k)|A)=\frac{e^{v_A(q_k,p_k)}}{\sum_{(q_l,p_l)\in A} e^{v_A(q_l,p_l)}}$ for all $A\in\mathcal{A}$. I say that $q_k,q_l\in\mathcal{Q}$ are p,A-distinguishable, if $P((q_k,p)|A)\neq P((q_l,p)|A)$. By assumption, there are $q,q^*\in\mathcal{Q}$ that are $\bar{p},Z$-distinguishable. By Log-odds Independence, any $q',q''\in\mathcal{Q}$ are $\bar{p},Z$-distinguishable. Indeed, if $L_{q,q^*}(\bar{p},Z)\neq 0$, Log-odds Independence implies $L_{q,q^*}(p,A)/L_{q^*,q}(\bar{p},Z) = L_{q',q''}(p,A)/L_{q',q''}(\bar{p},Z)$. Since the left-hand side ratio is well-defined, so is the right-hand side. Let define $\sigma_{A,q}(p) \equiv \frac{L_{q,q^*}(p,A)}{L_{q,q^*}(\bar{p},Z)}$. By Log-odds Independence, $\sigma_{A,q}(p) = \frac{L_{q,q^*}(p,A)}{Lq,q^*(\bar{p},Z)} = \frac{L_{q',q^*}(p,A)}{L_{q',q^*}(\bar{p},Z)} = \sigma_{A,q'}(p,A)$, for all $q' \in \mathcal{Q}$ such that $(q',p) \in A$, hence the function $\sigma_{A,q}(p)$ is independent of q and I denote it by $\sigma_A(p)$. Now, letd define $v(\hat{q}) \equiv L_{\hat{q},q^*}(\bar{p},Z)$, and define $c_A(p)$ as $c_A(p) \equiv -v_A(q^*,p)$, which is positive since v_A can be rescaled to be negative. Then,

$$e^{\sigma_A(p)v(q)-c_A(p)} = e^{\frac{L_{q,q^*}(p,A)}{L_{q,q^*}(\bar{p},Z)} \cdot L_{q,q^*}(\bar{p},Z)} e^{v_A(q^*,p)} = e^{v_A(q,p)} e^{-v_A(q^*,p)} e^{v_A(q^*,p)} = e^{v_A(q,p)}.$$

Lastly, Quality Monotonicity implies that, for all p, $P((q,p)|Z) \leq P((q',p)|Z)$ when $q' \geq q$ and $(q,p),(q',p) \in Z$, which implies $\sigma_Z(p)v(q) - c_Z(p) \leq \sigma_Z(p)v(q') - c_Z(p)$, or $0 \leq \sigma_Z(p)(v(q') - v(q))$. If $v(q') \geq v(q)$, the inequality is satisfied if $\sigma_Z(p) \geq 0$ for all p. Take any $q''' \geq q''$, then $0 \leq \sigma_Z(p)(v(q''') - v(q''))$ implies $v(q''') \geq v(q'')$, so v is weakly increasing. If $v(q') \leq v(q)$, the inequality is satisfied if $\sigma_Z(p) \leq 0$ for all p. Take any $q''' \geq q''$, then $0 \leq \sigma_Z(p)(v(q''') - v(q''))$ implies $v(q'') \geq v(q''')$, so v is weakly decreasing. The result follows by defining $\hat{\sigma}_Z = -\sigma_Z$ and $\hat{v} = -v$. □

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