Measurements of the Mass and Width of the $\eta_c$ Using the Decay $\psi(3686) \rightarrow \gamma \eta_c$


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CLEO-c experiment [9], using both cays, have reported a significantly higher mass and a \( \mu = 2978 \text{ MeV} \) compared to those of other charmonium states [2]. Early measurements of the properties of the lowest lying charmonium state, the \( \eta_c \), continue to have large uncertainties when compared to those of other charmonium states [2].

In recent years, many new charmonium or charmoniumlike states have been discovered. These states have led to a revived interest in improving the quark-model picture of hadrons [1]. Even with these new discoveries, the mass and width of the lowest lying charmonium state, the \( \eta_c \), continue to have large uncertainties when compared to those of other charmonium states [2]. Early measurements of the properties of the \( \eta_c \) using \( J/\psi \) radiative transitions [3,4] found a mass and width near 2978 MeV/\( c^2 \) and 10 MeV, respectively. However, recent experiments, including photon-photon fusion and \( B \) decays, have reported a significantly higher mass and a much larger width [5–8]. The most recent study by the CLEO-c experiment [9], using both \( \psi(3686) \rightarrow \gamma \eta_c \) and \( J/\psi \rightarrow \gamma \eta_c \), pointed out a distortion of the \( \eta_c \) line shape in \( \psi(3686) \) decays. CLEO-c attributed the \( \eta_c \) line-shape distortion to the energy dependence of the \( M1 \) transition matrix element.

In this Letter, we report measurements of the \( \eta_c \) mass and width using the radiative transition \( \psi(3686) \rightarrow \gamma \eta_c \). We successfully describe the measured \( \eta_c \) line shapes using a combination of the energy dependence of the hindered-\( M1 \) transition matrix element and a full interference with nonresonant \( \psi(3686) \) radiative decays. The analysis is based on a \( \psi(3686) \) data sample of \( 1.06 \times 10^9 \) events [10] collected with the BESIII detector operating at the BEPCII \( e^+e^- \) collider. A 42 pb\(^{-1}\) continuum data sample, taken at a center-of-mass energy of 3.65 GeV, is used to measure non-\( \psi(3686) \) backgrounds.

The \( \eta_c \) mass and width are determined from fits to the invariant mass spectra of exclusive \( \eta_c \) decay modes. Six modes are used to reconstruct the \( \eta_c : K_S K^+ \pi^- K^+ K^- \pi^0 \).
The BESS detector is described in detail in Ref. [11].

The detector has a geometrical acceptance of 93% of 4π. A small cell helium-based main drift chamber provides momentum measurements of charged particles; in a 1 T magnetic field the resolution is 0.5% at 1 GeV/c. It also supplies an energy loss (−dE/dx) measurement with a resolution better than 6% for electrons from Bhabha scattering. The electromagnetic calorimeter (EMC) measures photon energies with a resolution of 2.5% (5%) at 1 GeV in the barrel (endcaps). The time-of-flight system (TOF) is composed of plastic scintillators with a time resolution of 80 ps (110 ps) in the barrel (endcap) and is mainly useful for particle identification. The muon system provides a 2 cm position resolution and measures muon tracks with momenta greater than 0.5 GeV/c.

We use inclusive Monte Carlo (MC) simulated events as an aid in our background studies. The $\psi(3686)$ resonance is produced by the event generator KKMC [12], while the decays are generated by EVTGEN [13] with known branching fractions [2], or by LUNDCHARM [14] for unmeasured decays. The signal is generated with an angular distribution of $1 + \cos^2 \theta$ for $\psi(3686) \rightarrow \gamma \eta_c$, and phase space for multibody $\eta_c$ decays, where $\theta$ is the angle between the photon and the positron beam direction in the center-of-mass system. Simulated events are processed using GEANT4 [15], where measured detector resolutions are incorporated.

We require that each charged track (except those from $K_S$ decays) be consistent with originating from within 1 cm in the radial direction and 10 cm along the beam direction of the run-by-run-determined interaction point. The tracks must be within the main drift chamber fiducial volume, $|\cos \theta| < 0.93$. Information from the TOF and $-dE/dx$ is combined to form a likelihood $L_\pi$ (or $L_K$) for a pion (or kaon) hypothesis. To identify a track as a pion (kaon), the likelihood $L_\pi$ ($L_K$) is required to be greater than 0.1% and greater than $L_K$ ($L_\pi$).

Photons are reconstructed from isolated showers in the EMC that are at least 20 degrees away from charged tracks. The energy deposited in the nearby TOF scintillator is included to improve the reconstruction efficiency and the energy resolution. Photon energies are required to be greater than 25 MeV in the fiducial EMC barrel region ($|\cos \theta| < 0.8$) and 50 MeV in the endcap (0.86 < $|\cos \theta| < 0.92$). The showers close to the boundary are poorly reconstructed and excluded from the analysis. Moreover, the EMC timing, with respect to the collision, of the photon candidate must be in coincidence with collision events, i.e., $0 \leq t \leq 700$ ns, to suppress electronic noise and energy deposits unrelated to the event.

The $K_S \rightarrow \pi^+ \pi^-$ candidates are reconstructed from pairs of oppositely charged tracks. The secondary vertex constrained tracks must have an invariant mass $\pm 10$ MeV/$c^2$ of the nominal $K_S$ mass, and a decay length more than twice the vertex resolution. The track information at the secondary vertex is an input to the kinematic fit. Candidate $\pi^0$ and $\eta$ mesons are reconstructed from pairs of photons with an invariant mass in the range $0.118$ GeV/$c^2 < M(\gamma \gamma) < 0.150$ GeV/$c^2$ for $\pi^0$ and $0.50$ GeV/$c^2 < M(\gamma \gamma) < 0.58$ GeV/$c^2$ for $\eta$. The remaining photons are considered as candidates of the transition photon.

Events with either extra charged tracks or nonzero net charge are rejected. The $\eta_c$ candidates are reconstructed from $K_S^+ K^- \pi^0$, $K^+ K^- \pi^0$, $K^+ K^- \pi^+ \pi^-$, $K_S^+ K^- \pi^+ \pi^-$, $K^+ K^- \pi^+ \pi^- \pi^0$, and $3(\pi^+ \pi^-)$, where the $K_S$ is reconstituted in $\pi^+ \pi^-$, and the $\eta$ and $\pi^0$ in $\gamma \gamma$ decays. The inclusion of charge conjugate modes is implied.

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background processes, but do find dozens of decay modes that each make small additional contributions to the background. These decays typically have additional or fewer photons in their final states. The sum of these background events is used to estimate the contribution from other $\psi(3686)$ decays. Backgrounds from the $e^+e^- \rightarrow q\bar{q}$ continuum process are studied using a data sample taken at $\sqrt{s} = 3.65$ GeV. Continuum backgrounds are found to be small and uniformly distributed in $M(X_i)$. There is also an irreducible nonresonant background, $\psi(3686) \rightarrow \gamma X_i$, that has the same final state as signal events. A nonresonant component is included in the fit to the $\eta_c$ invariant mass.

Figure 1 shows the $\eta_c$ invariant mass distributions for selected $\eta_c$ candidates, together with the estimated $\pi^0 X_i$ backgrounds, the continuum backgrounds normalized by luminosity, and other $\psi(3686)$ decay backgrounds estimated from the inclusive MC sample. A clear $\eta_c$ signal is evident in every decay mode. We note that all of the $\eta_c$ signals have an obviously asymmetric shape: there is a long tail on the low-mass side; while on the high-mass side, the signal drops rapidly and the data dips below the expected level of the smooth background. This behavior of the signal suggests possible interference with the nonresonant $\gamma X_i$ amplitude. In this analysis, we assume 100% of the nonresonant amplitude interferes with the $\eta_c$.

The solid curves in Fig. 1 show the results of an unbinned simultaneous maximum likelihood fit in the range from 2.7 to 3.2 GeV/c$^2$ with three components: signal, nonresonant background, and a combined background consisting of $\pi^0 X_i$ decays, continuum, and other $\psi(3686)$ decays. The signal is described by a Breit-Wigner function convolved with a resolution function. The nonresonant amplitude is real, and is described by an expansion to second order in Chebyshev polynomials defined and normalized over the fitting range. The combined background is fixed at its expected intensity, as described earlier. The fitting probability density function as a function of mass ($m$) reads

$$F(m) = \sigma \otimes \left[ \left(e(m)\, e^{i\phi} E_{\gamma}^{7/2} S(m) + \alpha N(m) \right)^2 + B(m)\right],$$

where $S(m)$, $N(m)$, and $B(m)$ are the signal, the nonresonant $\gamma X_i$ component, and the combined background, respectively; $E_{\gamma}$ is the photon energy, $\sigma$ is the experimental resolution, and $\epsilon(m)$ is the mass-dependent efficiency. The $E_{\gamma}^7$ multiplying $|S(m)|^2$ reflects the expected energy dependence of the hindered-M1 transition [16], which partially contributes to the $\eta_c$ low-mass tail as well as the interference effect. The interference phase $\phi$ and the strength of the nonresonant component $\alpha$ are allowed to vary in the fit.

The mass-dependent efficiencies are determined from phase space distributed MC simulations of the $\eta_c$ decays. Efficiencies obtained from MC samples that include intermediate states change the resulting mass and width by negligible amounts. MC studies indicate that the resolution is almost constant over the fitting range. Thus, a mass-independent resolution is used in the fit. The detector resolution is primarily determined by MC simulation for

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**FIG. 1 (color).** The $M(X_i)$ invariant mass distributions for the decays $K_0^{*0} K^-\pi^+$, $K_0^{*0} K^-\pi^0$, $\eta\pi^+\pi^-$, $K_0^{*+}\pi^-\pi^0$, and $\lambda\pi^+\pi^-$, respectively, with the fit results (for the constructive solution) superimposed. Points are data and the various curves are the total fit results. Signals are shown as short-dashed lines, the nonresonant components as long-dashed lines, and the interference between them as dotted lines. Shaded histograms are in red, yellow, green for [continuum, $\pi^0 X_i$, other $\psi(3686)$ decays] backgrounds. The continuum backgrounds for $K_0^{*+}\pi^-$ and $\eta\pi^+\pi^-$ decays are negligible.
fits with and without interference, is of order differences of likelihood and degrees of freedom between in Table I. The solutions for the relative phase of each mode are listed component, the fitting range, and the efficiency.

In the simultaneous fit, the $\eta_c$ mass and width are constrained to be the same for all the decay modes but still free parameters; the two Chebyshev polynomial coefficients and the factor $\alpha$ are also allowed to float. Two solutions for the relative phase are found for each decay mode, one corresponds to constructive and the other destructive interference between the two amplitudes at the $\eta_c$ peak. Regardless of which solution we take, the mass, width of the $\eta_c$, and the overall fit quality are always unchanged [17]. The mass is $M = 2984.3 \pm 0.6$ MeV/$c^2$, and the width is $\Gamma = 32.0 \pm 1.2$ MeV. The goodness of fit is $\chi^2/ndf = 283.4/274$, which indicates a reasonable fit. The solutions for the relative phase of each mode are listed in Table I.

However, without the interference term, the fit would miss some data points, especially where the symmetric shape of a Breit-Wigner function is deformed, and the goodness of fit is $\chi^2/ndf = 426.6/280$. The statistical significance of the interference, calculated based on the differences of likelihood and degrees of freedom between fits with and without interference, is of order $15\sigma$.

The systematic uncertainties of the $\eta_c$ mass and width mainly come from the background estimation, the mass scale and resolution, the shape of the nonresonant component, the fitting range, and the efficiency.

In the fit, the $\pi^0X_i$ background is fixed at its expected intensity, so the statistical uncertainty of the observed $\pi^0X_i$ events introduces a systematic error. To estimate this uncertainty, we vary the number of events in each bin by assuming Gaussian variations from the expected value. We repeat this procedure a thousand times, and take the standard deviation of the resulting mass, width, and phases as systematic errors. We also use different dynamics in generating the $\pi^0X_i$ events (with the same final state, but different intermediate states) for the efficiency correction, and find the differences in resulting mass and width are small. We take 0.24 MeV/$c^2$ in mass and 0.44 MeV in width as the systematic errors for the $\pi^0X_i$ background estimation.

We assign a 0.07 MeV/$c^2$ (0.06 MeV) error in mass (width) for the nonresonant component shape that is obtained by changing the polynomial order. Also we include an additional noninterfering component, which is represented by a 2nd-order polynomial with free strength and shape parameters. The changes in the resulting $\eta_c$ mass and width are 0.10 MeV/$c^2$ and 0.02 MeV, respectively, and the fraction of this component to the total nonresonant rate varies from 0 to 25% depending on the decay mode. These variations are included in systematic errors.

The systematic errors from the uncertainty in the other $\psi(3686)$ decay backgrounds is estimated by floating the magnitude and changing the shape of this component to a 2nd-order polynomial with free parameters. The changes, 0.05 MeV/$c^2$ in mass and 0.06 MeV in width, are taken as systematic errors.

The consistency of the mass scale and resolution between data and MC simulations is checked with the decay $\psi(3686) \rightarrow \gamma \gamma \eta_c$, and possible discrepancies are described by a smearing Gaussian distribution, where a non-zero mean value indicates a mass offset, and a non-zero $\sigma$ represents the difference between the data and MC mass resolutions $(\sigma_{data}^2 - \sigma_{MC}^2)^{1/2}$. A typical mass shift is about $-1.0$ MeV/$c^2$ and the resolution smear is $\sim 3.0$ MeV. Another possible bias is the difference between input and the value after event reconstruction and selection. This is small for both the mass shift ($< 0.3$ MeV) and resolution smear. Both of these are added in the smearing Gaussian distribution. By varying the parameters of the smearing Gaussian distribution from the expected value, we estimate the uncertainties. From a large number of tests, the standard deviation of the resulting mass (width), 0.38 MeV/$c^2$ (0.27 MeV), is taken as a systematic error in mass (width) for the mass scale uncertainty. A 0.35 MeV/$c^2$ (0.60 MeV) systematic error in mass (width) is assigned due to the mass resolution uncertainty.

The systematic error due to the fitting range is estimated by varying the lower end between 2.6 and 2.8 GeV/$c^2$ and the higher end between 3.1 and 3.3 GeV/$c^2$. The changes, 0.05 MeV/$c^2$ in mass and 0.07 MeV in width, are assigned as systematic errors. A mass-dependent efficiency is used in the fit. By removing the efficiency correction from the fitting probability density function, the changes, which are 0.05 MeV/$c^2$ in mass and 0.06 MeV in width, are taken as systematic errors. The stability of the simultaneous fit program is checked by repeating the fit a thousand times with random initialization; the standard deviation of mass and width, 0.14 MeV/$c^2$ and 0.66 MeV, respectively, are taken as systematic errors.

**Table I.** Solutions of the relative phase (in unit of radian) of each decay mode.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Constructive</th>
<th>Destructive</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_SK^+\pi^-$</td>
<td>$2.94 \pm 0.27$</td>
<td>$3.75 \pm 0.26$</td>
</tr>
<tr>
<td>$K^+K^-\pi^0$</td>
<td>$2.63 \pm 0.21$</td>
<td>$3.96 \pm 0.19$</td>
</tr>
<tr>
<td>$\eta\pi^+\pi^-$</td>
<td>$2.41 \pm 0.13$</td>
<td>$4.28 \pm 0.09$</td>
</tr>
<tr>
<td>$K_SK^+\pi^+\pi^-\pi^-$</td>
<td>$2.16 \pm 0.11$</td>
<td>$4.46 \pm 0.07$</td>
</tr>
<tr>
<td>$K^+K^-\pi^+\pi^-\pi^0$</td>
<td>$2.73 \pm 0.19$</td>
<td>$4.00 \pm 0.16$</td>
</tr>
<tr>
<td>$3(\pi^+\pi^-)$</td>
<td>$2.28 \pm 0.10$</td>
<td>$4.43 \pm 0.06$</td>
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</table>
We assume all these sources are independent and take their sum in quadrature as the total systematic error. We obtain the $\eta_c$ mass and width to be

$$M = 2984.3 \pm 0.6 \pm 0.6 \text{ MeV}/c^2,$$

$$\Gamma = 32.0 \pm 1.2 \pm 1.0 \text{ MeV}.$$

Here (and elsewhere) the first errors are statistical and the second are systematic.

The relative phases for constructive interference or destructive interference from each mode are consistent with each other within $3\sigma$, which may suggest a common phase in all the modes under study. A fit with a common phase (i.e., the phases are constrained to be the same) describes the data well, with a $\chi^2/ndf = 303.2/279$. Comparing to the fit with separately varying phases for each mode, we find the statistical significance for the case of five distinct phases to be $3.1\sigma$. This fit yields $M = 2983.9 \pm 0.6 \pm 0.6 \text{ MeV}/c^2$, $\Gamma = 31.3 \pm 1.2 \pm 0.9 \text{ MeV}$, and $\phi = 2.40 \pm 0.07 \pm 0.47 \text{ rad}$ (constructive) or $\phi = 4.19 \pm 0.03 \pm 0.47 \text{ rad}$ (destructive). The physics behind this possible common phase is yet to be understood.

In summary, we measure the $\eta_c$ mass and width via $\psi(3686) \to \gamma \eta_c$ by assuming all radiative nonresonant events interfere with the $\eta_c$. These results are so far the most precise single measurement of the mass and width of $\eta_c$ [2]. For the first time, interference between the $\eta_c$ and the nonresonant amplitudes around the $\eta_c$ mass is considered; given the assumptions of our fit, the significance of the interference is of order $15\sigma$. We note that this interference affects the $\eta_c$ mass and width significantly, and may have impacted all of the previous measurements of the $\eta_c$ mass and width that used radiative transitions. Our results are consistent with those from photon-photon fusion and $B$ decays [5–8]; this may partly clarify the discrepancy puzzle discussed above. The changes of the $\eta_c$ mass and width may also have an impact on the expected $\eta'_{cJ}$ mass and width, and will modify the parameters used in charmonium potential models, where the $\eta_c$ mass is one of the input parameters. From this measurement, we determine the hyperfine mass splitting to be $\Delta M_{hf}(1S)_{\eta_c} = M(\bar{D}/\psi) - M(\eta_c) = 112.6 \pm 0.8 \text{ MeV}/c^2$, which agrees well with recent lattice computations [18–20] as well as quark-model predictions [21], and sheds light on spin-dependent interactions in quarkonium states.

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