Interesting Characteristics of Epidemiological Processes Spreading on Bipartite Networks

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Dynamics on Bipartite Networks
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Bipartite Networks
In bipartite networks nodes are divided in two families and edges are allowed only between individuals belonging to different families.

Vector-Borne Diseases
A disease is called vector-borne when the pathogen is transmitted from an infected individual to another individual by a blood-sucking arthropod, called vector. Many vector borne diseases are a particular kind of zoonosis, infectious diseases that could be transmitted from animals to humans, being of big importance for public health reasons. Vector-borne diseases depend upon three different classes of vectors:
- the pathogens, either viruses, bacteria, protozoa, or helminths (worms);
- the vectors, commonly arthropods such as ticks or mosquitoes;
- the hosts, usually vertebrates, which could be infected by vectors, thus becoming source of infection for vectors successively feeding on them.

An example: Lyme-Borreliosis
Lyme Borreliosis (LB) is the most common vector-borne disease of humans in temperate climates. It can affect at different threat levels several tissues in the human body: skin, heart, nervous system, eyes, kidneys, and liver. LB’s pathogenic agent is spirochetal bacteria belonging to the Borrelia genus. Vectors are hard ticks of the genus Ixodes (in Europe I. ricinus), while hosts are small-medium vertebrate such as reptiles, birds and mammals depending on the host-seeking tick stage.

Epidemic Spreading in Scale-Free Networks
As pathogen transmission seems to have complexity in the infection spreading time, we generate a number of bipartite random networks with different sizes and connection probabilities $p$. Over these we let a disease spreads observing the following rules:

- a sound individual at time $t$ connected to at least one infected nodes will be infectious at time $t+1$
- an infectious node at time $t$ will be susceptible at time $t+1$, no matter it is connect or not with infective individuals

What we observe is that after few iterations is that the system reaches an oscillating equilibrium. In fact we found a 2 periodic solution and even more amazingly we have a (anti-)synchronization phenomena.

Analytical results
Let’s consider random bipartite networks, where $A$ and $B$ are the two families of nodes, and two nodes $x\in A$ and $y\in B$ are connected with probability $p$.
For a node belonging to the $A$ partition the degree distribution is nothing but the binomial:
$$\rho_A(k) = \binom{N}{k} p^k (1-p)^{N-k}$$
and analogous for nodes belonging to $B$ partition. Calling $x(t)$ the prevalence of infected $A$-nodes at time $t$, and respectively $y(t)$ the prevalence of infected $B$-nodes, and using eq. (1) we could detect the probability for a $A$-node to have $k$ neighbors of which $h$ is infected but nothing is:
$$\rho_B(h) = \sum_{k=0}^{N} \binom{N}{k} p^k (1-p)^{N-k} (x(t))^k (y(t))^{N-k}$$
However, a susceptible node get the infection if at least one neighbor is infectious. Therefore, we sum previous for $h=1$...
A. Thus, at time $t+1$ the prevalence in family $A$ is nothing but:
$$x(t+1) = (1-x(t)) - (1-x(t)) (1-y(t))$$
and similarly for the prevalence in family $B$.
Now, in order to check the 2-periodic solutions, which mean oscillations of period 2, we impose that concurrently $x_{t+1} = x_t - y_t$ and $y_{t+1} = y_t$. We numerically find the solutions using the Newton's method. We observe they change as the value of $p$ changes.

Evolutionary Advantage?
We compare the epidemic threshold derived on bipartite networks with those derived on same but unipartite networks. In fact we compare the asymptotic behavior for enlarging population having degree distributed with two power-laws of exponents $\alpha_1$ and $\alpha_2$. Analytically we demonstrate that there exist couples of exponents $(\alpha_1, \alpha_2)$ for which the epidemic threshold tends to zero for bipartite but not for unipartite networks.

Oscillation Behavior
Results presented here are of some interests from a biological point of view. The fact that there exist scale free random bipartite networks on which the transmission probability needed for a pathogen to became endemic tends to zero, in the limit of large population size, while this does not happens on same but unipartite networks could mean that pathogen could take advantage of the peculiar transmission route described by bipartite network. That could result in an evolutionary advantage for these pathogens as their transmission probabilities is less subject to selection pressure.
Moreover the oscillating behavior observed in infection spreading over synthetic bipartite networks is observed in real world: e.g. tick nymphs are active in Spring while larvae are active during the Summer of the same year. Hence, in order for the pathogen to be passed to a next generation of larvae, hosts are required to keep the pathogen from Spring to Summer, having been first infected by nympha and then transmitting to larvae.

Essential bibliography
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