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It Is not just Escalation: the One Dollar Game Revisited

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Abstract

This paper examines the one-dollar auction game ruling out escalation. The aim of the paper is to understand if players’ expectations about competitors’ moves are strong enough to induce at least one player to bid more than the auctioned euro. Any other bid represents an expected loss for the bidder, so he maximizes his own payoff by choosing a bid which produces a null expected payoff. The empirical results and the analysis based on them support theoretical findings. It is possible that the winner pays more than one euro to get one euro because of his expectations about competitors’ bids and because of his indifference over a certain interval. The results are symptoms of some risk aversion. In an English auction escalation leads to this result, but when escalation is ruled out, expectations and indifference of preferences can lead to the same result.

JEL classification codes: C72, C93, D03

Keywords: one-dollar game, sealed auction, rationality, experiment

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1. Introduction

The dollar auction game proposed by Shubik (1971) has been used to study the phenomenon of escalation by both economists and political scientists. This is a modified English auction: both the participants who raise the highest and the second highest bids pay their bid, but only the first gets the auctioned item (in this case a one dollar coin), while the second gets nothing. In this setting, escalation emerges as the consequence of both the competitive mechanism of the English auction, and of the loss imposed on the second highest bidder; in particular the rules of the game push the second highest bidder to increase his bid at each stage, in order to avoid (or minimise) the loss. This mechanism makes some participants (in general two) overbid, i.e. incur in a loss (Shubik, 1971 and Murnighan, 2002). A sort of “bidding fever” (Ku et al., 2005) takes the participants; however this is not necessary something naïf: the two highest bidders start escalating to reduce their potential loss, and, besides this, it is also possible that the reservation price of the bidders includes non-monetary components of utility (say the satisfaction of being the winner), so that the monetary valuation of the item (and of winning the auction) can exceed its market value.

This paper modifies the basic dollar auction with the aim of ruling out the mechanism of escalation, and shows that, nevertheless, some participants bid more than one dollar (one euro in our case). The setting proposed in this paper involves a sealed-envelope auction instead of an English auction, maintaining that both the first and the second highest bidders pay their bids, but only the former takes the coin. This procedure rules out escalation, as each participant can bid only once and all the players bid simultaneously. In addition notice also that each player does not know his/her relative position in the distribution of the bids, and there is no possibility of modifying the bid. The aim of the paper is to understand whether the results of the dollar auction are driven by escalation only, or if people would bid more than the value of the item also in absence of escalation.

1 Suppose that the value of the auctioned item is 1 (the dollar of the baseline treatment). Suppose now that the highest bid is 1$, while the second highest bid is 0.99$. The participant who bids 0.99$ would incur in a net loss equal to the bid. Therefore he has incentive to bid 1.01$ in order to win the auction and to reduce the net loss to 0.01$ < 0.99$. Of course, the other bidder would then bid 1.02$ in order to reduce his net loss from 1$ to 2¢ and so on. The willingness to reduce the net loss induces the escalation process and generally leads at least two players to bid more than the value of the auctioned item.

2 He auctioned a 2,000 HKD and the winner bade more than ten times this value.

3 On this phenomenon see also the so-called “winner’s curse” (Milgorm and Weber, 1982; Bazerman and Samuelson, 1983 and Bajari and Hortaçsu, 2003)

4 Escalation is a dynamic process by definition.
The results show that, although escalation does not occur, some players offer more than one euro. Furthermore, expectations about the first and the second highest bids affect players’ decisions; in particular the value of the expected highest bid correlates positively with the actual bid. The effect of the expected second highest bid can be either negative or positive, depending on whether the expected second highest bid exceeds the value of the auctioned item or not. This means that people play to win, although they are afraid of raising the second highest bid.

2. Brief discussion of the dollar auction.

Applying backward induction, O’Neill (1986) theoretically finds that no escalation occurs if bids are upper bounded: given the presence of the upper bound, every subject knows that, when bidding a positive amount, he/she will probably incur in a loss, whose maximum is predetermined. Hence O’Neill’s theoretical prediction is that escalation is irrational, under the non explicit\(^5\) assumption that individuals get utility from money only. Leininger (1989) implements two models: one works in a discrete space (i.e. the subjects can bid only discrete amounts), the other in a continuous space. The former model confirms the results of O’Neill (1986), while the development of the second model leads to a theorem, which proves that equilibriums with escalating bidding can be rational and eventually “stabilise around a parity level”\(^6\).

More recently Leventoğlu and Slantchev (2007) suggest that escalation is indeed rational and consistent with the extant models of war: the minimization of the loss, if any, is always rational. The actual presence of a credible treat (as in the case of a war of attrition) induces escalation as a rational strategy to counterbalance the enemy’s power and avoid the conflict\(^7\). Krishna and Morgan (1997) theoretically study how the war of attrition can modify the outcome of different types of auction. In particular they prove two theorems. The first assesses that the average revenue of an all-pay auction is at least equal to the expected revenue from a first-price sealed-bid auction. The second shows that the expected value from a war of attrition is at least equal to the expected value from an all-pay auction. And the authors conclude that “the auction forms we have considered may, in certain circumstances, extract all the surplus from the bidders; that is, the bidders' expected payoff in equilibrium may be zero.”\(^8\) If the individual’s surplus includes not only the difference between the monetary value of the auctioned item and the bid,

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\(^5\) In O’Neill’s mentioned paper.


\(^7\) One of the most recent and brightest example of this is the “Cold War”.

but also other non-monetary components, then the bid might even be higher than the simple monetary value of the object.

In the setting used in this research escalation and war of attrition can not occur, as the tit-for-tat mechanism which is the basis of escalation can not work by default. However incentives to bid more than the extrinsic value of the auctioned item can exist: I call these “psychological components of the individual’s utility” (for example the satisfaction of winning the auction). As mentioned before, these may induce the bidder to incur in an (apparent) loss, when the winning bid exceeds the monetary value of the object. As the participants play a sealed-envelope auction, and assuming that they aim at maximising their total (i.e. monetary and psychological) payoff, the bidder behaves also in accordance with his expectations about the two highest bids. The effect of these expectations can be either positive or negative, and in particular two cases are possible: those who expect both the highest and the second highest bids to be lower than or equal to 1€ should bid slightly more than the expected highest bid in order to maximise their payoff (by winning the auction). The others, who expect both the highest and the second highest bids to exceed the euro, should bid any value lesser than their expected second highest bid, decreased by some amount depending on their level of risk aversion\(^9\). However this is equivalent to bid zero (as this means to pay nothing and to receive nothing) and as a consequence we might expect that all the players in the second group bid zero. However this would happen if the players derived no utility from the act of bidding itself and/or of winning the auction, i.e. if they were not willing to pay for the psychological components of their utility; indeed, when this is not the case, the one’s maximum acceptable monetary loss would correspond to the non-monetary utility got from participating to the auction and winning it\(^10\). The participants whose reservation price is higher than 1€ could (rationally) bid more than this, always trying to exceed the expected second highest bid. In this case the winner and the second highest bids could be even larger than one euro.

However, it is possible to show theoretically that, as multiple Nash equilibriums in mixed strategies exist, placing a positive bid could be rational for the individual, even when leaving apart the psychological components of utility. In particular, it can be proved (see the Appendix) that any bid in the range \([0, 0.99]\) is in accordance with the characterisation of the Nash equilibrium of this game.

\(^9\) This is necessary in order to minimise the loss arising from the risk of being the second highest bidder.
\(^10\) This amount is equal to the reservation price of the bidder minus the value of the auctioned item (Bajari and Hortaçsu, 2003 and Maskin and Riley, 2003).
3. Experimental design and procedure

Thirty-six students, played the modified Schubik’s auction. They did not receive any participation fee and paid the bid using their own money\textsuperscript{11}. The auctioned item was a one-euro coin. The rules were explained after the coin was shown by the experimenter to the participants; these rules were essentially the same as Shubik’s\textsuperscript{12}. In order to ensure a neutral environment, the game took place in a classroom of the School of Economics (of which all the participants were students having attended no course in game theory). After explaining the rules and asking control questions, and before starting the game, the experimenter invited all those who did not intend to participate to quit the room. Nobody quitted. In case of ties, both the winner and the “second” highest bidder would have been randomly drawn from this group. In case of a unique highest bid, but two or more second highest bids, then the bidder having to pay without receiving anything would have been randomly drawn from this group.

The students were sitting far from each other in the largest classroom of the School\textsuperscript{13} and invited to write their bid on an isolated table, so that neither other participant nor the experimenter could see what they were writing. The bid was then put into a numbered envelope distributed at the beginning of the experiment. Then the sealed envelopes were returned to the experimenter, while each participant kept a small paper with a number corresponding to the number written on the envelope. At the end of the auction the participants were asked to fill in a questionnaire aimed at collecting information about the usual socio-demographic data such as age, gender, etc., and their expectations about the highest and the second highest bids. In the meantime, the experimenter opened the envelopes. After collecting the questionnaires, the identifying numbers corresponding to the first and the second highest bids were announced, and the students were invited to approach the experimenter: they paid what due and the highest bidder received the coin. The two highest bidders were publicly announced and they paid their bids in front of everybody.

\textsuperscript{11} It might be argued that this introduces a disparity among the participants. Indeed it is likely that not all the participating students had the same amount of money in their pocket. Although this is almost certainly true, it is also true that the participants to a true (i.e. non experimental) auction have different budget constraints, and the same holds for states or firms involved in military escalations, wars of attrition, R&D, etc. Therefore this feature is useful to reproduce conditions that are similar to those of the “real life”. In addition, given the low value of the auctioned item (1€), any participation fee would have introduced a severe bias in the result.

\textsuperscript{12} The winner pays the bid and takes the coin, the second highest bidder pays and receives nothing, the others neither pay nor receive anything back.

\textsuperscript{13} They were all gathered in the same classroom in order to see the number of the competitors. This was given information, but the visual experience is important as well.
4. Results

Participants were 12 male and 24 female students; 70% of the players bade less than 1€, 17.5% bade exactly 1€, and the remaining 12.5% more than that. The average bid was 74.3¢; the average highest expected bid was 3.70€, and the expected second highest bid 3.04 euro on average. The winner bade 3€, and the second highest bidder 2€. The distribution of bids is summarised in figure 1 (density).

Eight people (4 males and 4 females) bade exactly or more than (one case) their expected highest bid and nine people bade more than their expected second highest bid. In five cases the bid was at least equal to 1€, and a player bade more than this amount (namely 1.01€). All these people clearly participated to the auction with the scope of winning it, in spite of (for one of them) a potential, though very small, monetary loss. 29 participants bade less than their expected second highest bid, independently of the fact that this was larger or smaller than 1€. Likely, these people were motivated also by the fear of incurring in a loss if being the second highest bidders. Eventually two bade exactly their expected second highest bid. Perhaps they acted like that just for fun, given the limited amount of the potential loss.

These results suggest that it is not only escalation that induces people to bid high when participating to a dollar game. Apparently also the simple fact of winning and or participating is source of utility. However also the expectations about the highest and the second highest bids can be responsible of amounts that exceed the value of the auctioned item. In this case the rational explanation could be the individual’s overconfidence in his/her fitness of predicting the others’ behaviour (Weyl, 2006). In other words, all those who bade more than 1€, but less than their expected second highest bid, were so confident that somebody would have bidden a very high sum, that they bade more than 1€.

The second step of the analysis involves OLS and tobit regressions (Tables 1 and 2). The tobit leaves out those whose bids were 0 and the “winner”. For the purpose of this step of the investigation, the whole sample is divided in two sub-samples, according to the fact that the

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14 This unbalance does not induce almost any bias: all the analyses do not detect almost any gender effect.
15 And to be announced as the winner in front of the others
The expected second highest bid was higher or lower than 1€. The idea behind this division is that if the expected second highest bid exceeds 1€, then the fear to incur in a loss could (or maybe should) prevail on the motivation of winning the auction.

[Tables 1 and 2]

The results shown in the regressions and in the following figures can be summarised as follows:

a. The effect of the expectation over the highest bid on the actual bid is always positive and significant, whilst the expectation over the second highest bid has two different effects. When this is lesser than or equal to 1€, it tends to lead the actual bid upwards, but when it is higher than 1€, the effect is the opposite. This is consistent with the hypothesis that the participants actually aim at winning the auction, but try to avoid a loss.

b. For those whose expectation over the second highest bid exceeded 1€ (Table 1) the effect of the expectations over the highest and the second highest bids are similar in magnitude, but opposite in sign. In particular they reveal that the actual bid is an inelastic function of the expectations, as both the coefficients (as well as their sum) are lesser than 1. However their absolute values are very close to each other suggesting that the two effects almost balance each other. However the (negative) coefficient of the expected second highest bid is slightly larger (in absolute terms) than the coefficient for the expected highest bid. This suggests that on the one hand people get utility from winning, but on the other hand the reduction of the bid due to the fear of incurring a loss\(^\text{16}\) is more than proportional to the increase led by the desire of winning (-0.462 vs. 0.414 in the OLS estimates and -0.401 vs. 0.352 for the tobit). In other words, the disutility coming from the loss of being the second highest bidder is larger than the psychological gain of winning less the monetary loss in case of victory\(^\text{17}\).

c. When the expected second highest bid is lesser than 1€ (Table 2), then the effects of the two expectations reinforce each other, leading to an actual bid, whose elasticity with respect to the two expectations combined is larger than 1 (1.133 for the OLS and 1.105 for the tobit). This fact supports the presence of the hypothesised psychological component of utility, as allows for bids which are (slightly) larger than 1€.

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\(^{16}\) Either because the probability of raising the second highest bid increases with the expectation over it, or because winning becomes more and more costly, as the highest bid increases with the second highest bid. Notice that if a player believes that the second highest bid exceeds 1€, then he is certain to incur in a loss if bidding an amount higher than or equal to this expectation.

\(^{17}\) Here we are considering the cases where the expected highest bid exceeds 1€, and therefore winning generates a monetary loss.
d. An analysis conducted through quadratic polynomial implementation (Figures 2 and 3) highlights that the expected second highest bid increases the predicted bid as guessed by the theory; this result holds for all the values in the range$^{18}$. 

[Figures 2 and 3]

e. The analysis of the bids in the neighbourhood of 1€ in figure 4 reveals an interesting outcome: when the actual bid is predicted by the expected highest bid, then it is locally decreasing and then increasing again, with a minimum at 1. This is consistent with the hypothesis that players are indifferent between bidding 1 euro or less when the expected highest bid is equal to 1€; this produces the local minimum in the fitted function. 

[Figure 4]

In sum, the most of the participants exhibit a rational behaviour in accordance with the predictions of the Nash equilibrium in mixed strategies. Nevertheless there are some who behave according to the presence of motivations different from the simple maximisation of the monetary payoff. Overbidding can occur also in a sealed-envelope context, i.e. it is not necessarily consequence of escalation only (or, better, it can not be explained through escalation tout court).

5. Conclusions

Also in absence of escalation, the “winner” of the auction (and not the winner only) is willing to pay more than the value of the coin, although, consistently with the theoretical predictions, the most of the players bade an amount of money positive and lower than 1€. Among the sources of utility other than money, the existence of a trade-off between the satisfaction from winning and its cost (in this case a net loss) is a possibility. This is a major result, since it sheds some light also on the behaviour of managers and strategists. R&D and wars are not only a matter of maximisation (or minimisation) of material gains (losses), they also provide the winners with the intangible reward of glory and fame (that can explain also the “bidding fever”), which, in turn, may produce additional economic returns. Moreover notice that not all the strategists pursue personal satisfaction through military escalation or wars: they behave as the experimental subjects who play in accordance with the theoretical predictions of a model involving only people who maximise their monetary payoff.

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$^{18}$ Figure 3 highlights a decreasing path for an expected second highest amount larger than 15 euro. However this is due to the fact that the only player who expected more than 15 euro as second highest bid chose to bid 0.5 euro.
The results presented in this paper can be interpreted in the light of some government programs involving the purchase of a few quantity of a very specific product. The U.S. “Marine One” and the “Air Force One” are typical examples, in which the producers of helicopters and aircrafts compete to supply few very customer-tailored machines. The complexity of the project requires massive investments, and the small volume of the sale could generate a monetary loss for the winner. Nevertheless the reputation and the fame acquired by winning this kind of competition are likely to overcompensate this monetary loss. As a consequence the firms which spend for the project more than the expected revenues act rationally and in accordance with the conceptual model presented in this paper.

References


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**Figure 1. Bids: density**

![Figure 1. Bids: density](image)
Table 1. OLS and Tobit regressions for expected second higher offer higher than one euro.

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>Tobit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highest expected bid</td>
<td>0.414*</td>
<td>0.352**</td>
</tr>
<tr>
<td></td>
<td>(0.187)**</td>
<td>(0.106)***</td>
</tr>
<tr>
<td>Second highest</td>
<td>-0.462*</td>
<td>-0.401**</td>
</tr>
<tr>
<td>expected bid</td>
<td>(0.221)°°</td>
<td>(0.127)***</td>
</tr>
<tr>
<td>Male</td>
<td>0.321</td>
<td>0.567*</td>
</tr>
<tr>
<td></td>
<td>(0.400)</td>
<td>(0.245)°°</td>
</tr>
<tr>
<td>Believe to be</td>
<td>-1.104**</td>
<td>-1.068***</td>
</tr>
<tr>
<td>second</td>
<td>(0.388)°°°</td>
<td>(0.250)***</td>
</tr>
<tr>
<td>Don't want to pay</td>
<td>-0.810</td>
<td>-0.822**</td>
</tr>
<tr>
<td>more</td>
<td>(0.463)*</td>
<td>(0.315)***</td>
</tr>
<tr>
<td>Constant</td>
<td>1.017*</td>
<td>1.04**</td>
</tr>
<tr>
<td></td>
<td>(0.421)</td>
<td>(0.295)</td>
</tr>
</tbody>
</table>

Notes: *** significance level for means (*** 99% level; ** 95% level; * 90% level)
°°° significance level for medians (°°° 99% level; °° 95 % level; ° 90% level)

Table 2. OLS and Tobit regressions for expected second higher offer lesser than one euro.

<table>
<thead>
<tr>
<th></th>
<th>OLS</th>
<th>Tobit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highest expected bid</td>
<td>0.641***</td>
<td>0.628***</td>
</tr>
<tr>
<td></td>
<td>(0.172)°°°</td>
<td>(0.165)***</td>
</tr>
<tr>
<td>Second highest</td>
<td>0.492**</td>
<td>0.477**</td>
</tr>
<tr>
<td>expected bid</td>
<td>(0.217)°°</td>
<td>(0.205)°°</td>
</tr>
<tr>
<td>Male</td>
<td>0.089</td>
<td>0.105</td>
</tr>
<tr>
<td></td>
<td>(0.254)</td>
<td>(0.231)</td>
</tr>
<tr>
<td>Believe to be</td>
<td>-0.230</td>
<td>0.221</td>
</tr>
<tr>
<td>second</td>
<td>(0.178)</td>
<td>(0.162)</td>
</tr>
<tr>
<td>Don't want to pay</td>
<td>0.123</td>
<td>0.142</td>
</tr>
<tr>
<td>more</td>
<td>(0.226)</td>
<td>(0.212)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.492*</td>
<td>-0.487*</td>
</tr>
<tr>
<td></td>
<td>(0.289)</td>
<td>(0.269)</td>
</tr>
</tbody>
</table>

Notes: *** significance level for means (*** 99% level; ** 95% level; * 90% level)
°°° significance level for medians (°°° 99% level; °° 95 % level; ° 90% level)
Figure 2. Quadratic polynomial implementation: bid prediction on expected second highest bid when it is lesser than 1.01 euro.

Figure 3. Quadratic polynomial implementation: bid prediction on expected second highest bid when it is higher than 1 euro.
Appendix: the characterisation of the Nash equilibrium

The monetary payoffs are (let $b$ indicate the bid and $i$ the subject):

$$
\pi_i = \begin{cases} 
1 - b_i & \text{if } b_i = b_{\text{max}} \land b_i > 0 \\
-b_i & \text{if } b_i = b_{\text{2nd}} \\
0 & \text{if } b_i < b_{\text{2nd}} \lor b_i = 0 
\end{cases}
$$

and $\forall b_{\text{max}} \leq 1$. Moreover:
Since each player does not know the others’ bids, he must guess their distribution, thus basing his choice on an expected payoff. We can assume that individual $i$ attaches a positive probability $\alpha_i$ to the outcome $1 - b_i$ and a positive probability $\beta_i$ to the outcome $-b_i$; this implies that the probability attached to a null payoff is equal to $1 - \alpha_i - \beta_i$, with $\alpha_i + \beta_i < 1$, under the hypothesis that the three probabilities sum up to 1. The expected payoff is then equal to $\alpha_i(1 - b_i) - \beta_i b_i$. At this point the player can choose between bidding $0$ or $0 < b_i < 1$. In the second case the player has a positive expected payoff if and only if his bid is larger than a given amount that depends on the subjective probabilities assigned to the three possible outcomes. It follows that a rational individual, who is maximising only his/her monetary payoff, will bid some positive amount only if $\alpha_i(1 - b_i) - \beta_i b_i > 0$. To be verified this inequality requires that

$$b_i < \frac{\alpha_i}{\alpha_i + \beta_i}$$

and it easy to see that, for any value of the subjective probabilities, this condition is equivalent to $b_i < 1$. Therefore we can conclude that the players who bid an amount in the range $[0,1)$ are rational and consistent with the theory. People who bid $1$ are indifferent and apparently risk neutral, or risk lover (as in reality bidding $1$ generates an expected payoff lower than $1$). All those who bid more than $1$ euro are therefore motivated by reasons other than the simple maximization of the monetary payoff.

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19 Notice that if $b_i = 1$, then the expected payoff is always negative, and hence it is never convenient to bid $1$.

20 I.e. to say that a Nash equilibrium in mixed strategies exists.