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Modeling epidemic spreading in star-like networks

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Milano, 5 aprile 2013
Tick-Borne Encephalitis

- endemic in Eurasia from Europe, through Russia to China and Japan
- the virus causes potentially fatal neurological infection
- in last years emergence of the virus in new area and increase of morbidity
- maintained in nature by complex cycle involving Ixodid ticks (*I. ricinus* and *I. persulcatus*) and wild vertebrate hosts
Systemic Transmission

time $t$

$\text{time } t + 1$
Non-Systemic Transmission
Non-Systemic Transmission
Non-Systemic Transmission
Our research question

how do the non-systemic transmission together with the different aggregation patterns influence the pathogen spreading?
Spreading Model

- at time $t$ a fraction, $\pi(t)$, of passengers (ticks) are infectious

$\pi(t) = \beta \cdot \pi(t)$

$\pi(t+1) = f(\pi(t))$
Spreading Model

- at time $t$ a fraction, $\pi(t)$, of passengers (ticks) are infectious
- $\mathbb{P}(k)$ probability that a bus (mouse) transports $k$ passengers (ticks) of them

\[ \pi(t+1) = f(\pi(t)) \]
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- $\beta$ transmission probability for infectious path
- $\mu$ recovery probability
Spreading Model

- at time $t$ a fraction, $\pi(t)$, of passengers (ticks) are infectious
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- $\beta$ transmission probability for infectious path
- $\mu$ recovery probability

\[ \Rightarrow \pi(t + 1) = f(\pi(t)) \]
the probability that a susceptible passenger, having $h$ travel mates, gets the infection is

$$1 - (1 - \beta)^h$$
the probability that a **susceptible passenger**, having \( h \) travel mates, gets the **infection** is

\[
1 - (1 - \beta)^h
\]

Let \( \pi(t) \) be the **prevalence of infection** among passengers at time \( t \), the probability for a susceptible passenger on a bus transporting \( k \) individuals including himself to be **infectious** at time \( t + 1 \) is

\[
1 - (1 - \beta)^{(k-1) \cdot \pi(t)}
\]
Recalling that $\mathbb{P}(k)$ is the probability for a bus to have $k$ passengers, the probability for a passenger to be on a $k$-bus is

$$\frac{\# \text{passengers on a } k\text{-bus}}{\# \text{passengers}} = \frac{k \cdot \# k\text{-bus}}{\# \text{passengers}} =$$

$$= k \cdot \frac{\# k\text{-bus}}{\# \text{bus}} \cdot \frac{\# \text{bus}}{\# \text{passengers}} = k \cdot \mathbb{P}(k) \cdot \frac{1}{\langle k \rangle}$$
thus, the probability for a \textit{susceptible} passenger at time $t$ to be \textit{infectious} at time $t+1$ is

$$\sum_{k=1}^{\infty} \left[ 1 - (1 - \beta)^{(k-1) \cdot \pi(t)} \right] \cdot \frac{k}{\langle k \rangle} \cdot \mathbb{P}(k)$$

and therefore the prevalence among passenger at time $t+1$ is

$$\pi(t+1) = f(\pi(t)) =$$

$$= (1 - \mu) \cdot \pi(t) + [1 - \pi(t)] \cdot \left\{ 1 - \sum_{k=1}^{\infty} (1 - \beta)^{(k-1) \cdot \pi(t)} \cdot \frac{k}{\langle k \rangle} \cdot \mathbb{P}(k) \right\}$$
Equilibria

imposing the stationary condition $\pi(t + 1) = \pi(t) = x$ we can derive the equilibria as solutions of the following equation

$$x = f(x) = (1 - \mu)x + [1 - x] \cdot \left\{ 1 - \sum_{k=1}^{\infty} (1 - \beta)^{(k-1)x} \cdot \frac{k}{\langle k \rangle} \cdot \mathbb{P}(k) \right\}.$$
Equilibria

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$$x = f(x) = (1 - \mu) \cdot x + [1 - x] \cdot \left\{ 1 - \sum_{k=1}^{\infty} (1 - \beta)^{(k-1)\cdot x} \cdot \frac{k}{\langle k \rangle} \cdot \mathbb{P}(k) \right\}.$$

Now:

- $x = 0$ is a solution,
- $f(1) = 1 - \mu \leq 1$,
- $f''(x) < 0$. 

assuming $\langle k \rangle$ and $\langle k^2 \rangle$ finite.
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therefore conditions to have one, and only one, solution \( \hat{x} \in (0, 1) \) is that \( f'(0) > 1 \) or

\[
- \frac{\ln (1 - \beta)}{\mu} > \frac{\langle k \rangle}{\langle k^2 \rangle - \langle k \rangle}.
\]
Stability

recalling that \( f''(x) < 0 \) and that

\[
\begin{align*}
    f'(1) &= -\mu + \sum_k (1 - \beta)^{k-1} \frac{k}{\langle k \rangle} P(k) > -\mu > -1 \\
    f'(\hat{x}) &< 1
\end{align*}
\]

hence \( \hat{x} \) is asymptotically stable when it exists. Therefore:

\[
\begin{align*}
    \text{disease-free equilibrium is asymptotically stable when } \hat{x} \text{ does not exist}. \\
    \text{disease-free is unstable when } \hat{x} \text{ exists. Furthermore, when } \hat{x} \text{ exists it is also asymptotically stable.}
\end{align*}
\]
Conclusion and Discussions

- co-feeding transmission
- spreading on star-like networks
- spreading dynamic on dynamic bipartite networks
- analytical result confirmed by simulations
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