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Workers and Firms Sorting into Temporary Jobs*

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Abstract

The liberalisation of fixed-term contracts in Europe has led to a two-tier regime, with a growing share of jobs covered by temporary contracts. The paper proposes a matching model with direct search in which temporary and permanent jobs coexist in a long-run equilibrium. When temporary contracts are allowed, firms are willing to open permanent jobs in as much as their job filling rate is faster than that of temporary jobs. From the labour demand standpoint, a simple trade-off emerges between an ex-ante job filling rate and ex-post flexible dismissal rate. The model thus features a natural sorting of firms and workers into permanent and temporary jobs. It is also consistent with the observation that workers hired on a permanent contract receive more training. Empirically, we test with Italian longitudinal data whether non-employment spells that lead to a temporary job are shorter on average. We find that, other things being equal, the transition intensity of exit towards temporary jobs is higher than to permanent jobs. The other empirical implications, and notably the effects of training, are coherent with the existing literature.

• Key Words: Matching Models, Temporary Jobs

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1 Introduction

The liberalisation of temporary contracts, or fixed-term contracts as are often defined in the policy debate, has been the main labour market reform in continental Europe during the last decades. The liberalisation applies only to new hires, so that only new jobs and new vacancies can potentially be advertised and filled with temporary contracts. Existing jobs, covered by open-ended contracts, are not directly affected by the reform. As a result, a two-tier regime has emerged in many continental European markets, with a growing share of temporary contracts, which peaked to 14.5 in 2007, at the outset of the great recession [European Commission 2010]. As the stock of open-ended jobs dies out by natural turnover, many observers and policy analysts wonder whether the share of temporary contracts will eventually absorb the entire labour market. This paper shows that the latter implication is not likely, and that permanent and temporary workers are likely to coexist in the long run, even with homogeneous labour from the labour demand standpoint.

In the existing literature, the long-run implications of a labour market with both temporary and permanent contracts are not fully understood. In a pure labour demand setting with risk-neutral homogeneous workers and without market frictions, temporary jobs should indeed take over the entire labour market. Boeri and Garibaldi [2007] study theoretically and empirically the transition from a rigid system with only permanent contracts to a dual system with temporary and permanent contracts. In the aftermath of the liberalisation, no vacancies covered by permanent contracts are posted, and the stock of temporary contracts absorbs the entire workforce. Similar implications are held by various papers [Blanchard and Landier 2002; Cahuc and Postel Vinay 2002] and ad hoc assumptions ensure that temporary and permanent contract coexist in equilibrium$^1$.

This paper studies firms and workers’ sorting into permanent and temporary contracts in an imperfect labour market. Specifically, it studies vacancy posting in permanent and temporary jobs in a world with matching frictions and direct search. From the labour demand standpoint, a filled job with a temporary and flexible contract is more profitable to a firm, since it allows the firm to easily adjust labour in the face of adverse productivity shocks. However, free entry in each submarket implies that in equilibrium jobs advertised with permanent contracts display a larger job filling rate. A simple trade-off thus emerges between an ex-ante slower job filling rate and ex-post more flexible dismissal rate. In other words, firms that post jobs with temporary contracts face longer job filling rate. This mechanism is akin to wage posting and to the competitive search equilibrium initially proposed by Moen [1997].

From the labour supply standpoint, a similar mechanism emerges. For a given wage within the bargaining set, in the spirit of Hall [2005], risk-neutral workers with heterogeneous and unobservable reservation utility, prefer more job security, i.e. a permanent contract. Yet, in as much as job search in the submarket for

$^1$ In Cahuc and Postel Vinay [2002] temporary and permanent contracts coexist in light of a random and exogenous state permission to fill jobs with temporary contracts. In Blanchard and Landier [2002] all jobs start with a temporary contract, and only a fraction is endogenously converted into a permanent job. Garibaldi and Violante [2005] have similar implications.
temporary workers leads to larger job filling rate, also a labour supply trade-off emerges between an ex-ante lower job finding rate and an ex-post larger retention rate\(^2\). As a result the model features a natural sorting of firms and workers into permanent and temporary jobs.

The simple theory has several implications. First, the coexistence of temporary and permanent contracts implies that in equilibrium temporary jobs lead to faster job finding rate for workers. This is true even when workers can graduate to a permanent position via a temporary job. Second, the steady state of the model displays both temporary and permanent jobs, with an equilibrium share of temporary jobs that crucially depends on the average duration of temporary contracts and the structure of productivity shocks. Third, the liberalisation of temporary contracts does not crowd out permanent contracts, and the labour market moves smoothly toward a long-run dual system. Fourth, when firms have the option to undertake costly training in the aftermath of adverse productivity shocks, the theory clearly implies that workers covered with permanent contracts are more likely to be trained.

While the existing empirical literature on temporary/permanent jobs is large, the basic implicit mechanism proposed by the model has not been directly tested. Empirically, we use Italian longitudinal data to test whether unemployment spells that lead to a temporary job are shorter on average. We run a competing risks model on a sample of workers who entered the labour market between 1998 and 2003 and find that, other things being equal, the transition intensity of exit towards temporary jobs is higher than to permanent. We also review the rest of the empirical implications, and notably the effects of training, and we find it fully coherent with the existing literature.

The paper proceeds as follows. Section 2 highlights the structure of the model and the basic equations. Section 3 defines and solves the equilibrium. Section 4 studies analytically the transition toward a dual regime and presents a simple set of simulations. Section 5 introduces the option to train workers in the aftermath of adverse shocks. Section 6 studies the model with on-the-job search. Section 7 presents the empirical analysis. Section 8 discusses other implications of our theory vis-à-vis the existing empirical evidence. Section 9 concludes.

## 2 The matching framework

The labour market consists of a mass one of risk-neutral workers. Workers are fully attached to the labour market and if they are out of work, they actively search for a job. Employed workers are subject to natural turnover and separate from their existing job with a Poisson process with arrival rate equal to \( s \).

Workers differ in their idiosyncratic income from non-employment. The outside flow utility is indicated with \( z \), and we assume that \( z \) is time invariant and not observable to the firms. \( z \) is drawn from a continuous

\(^2\)A similar implication, at least from the labour supply standpoint, emerges in the quantitative general equilibrium model proposed by Alonso-Borrego et al. [2005]. The free entry condition in both markets, a key feature of the mechanism of this paper, is anyway not modelled by Alonso-Borrego et al.
cumulative distribution \( F(z) \) with upper support \( z^u \). Since \( z \) is not observable, workers are identical vis-à-vis the firms.

Firms produce with a constant return to scale technology with labour productivity equal to \( y_h \). Each job has an instantaneous probability \( \lambda \) of experiencing a (permanent) adverse shock. Conditional on an adverse shock, the productivity falls to \( y_l < y_h \). We further assume that the wage paid is strictly larger than \( z^u \) so that the labour market is viable for each worker.

Two types of contracts exist in the economy. Temporary contracts and permanent contracts. Temporary contracts can be broken by the firm at will\(^3\). Firm-initiated separation is not possible with permanent contracts. Firms that hire workers on permanent contracts must rely on workers’ natural turnover for downsizing. Firms create jobs by posting costly vacancies, and firms can freely decide to open either temporary or permanent jobs. Keeping open a vacancy, either temporary or permanent, involves a flow cost equal to \( c \).

For simplicity, we assume that the vacancy cost is identical for both contracts.

Temporary and permanent contracts are offered in different submarkets. In each submarket, the meeting of unemployed workers and vacant firms is described by a well-defined matching function \( m \) with constant returns to scale. Submarkets are indexed by \( i \in [p, t] \) where \( p \) stands for permanent and \( t \) for temporary. Unemployed workers can freely move across submarkets but can not search simultaneously across submarkets. In this respect, search is directed toward a specific submarket (this hypothesis will be relaxed in section 6).

Unemployed workers searching for a permanent job enjoy a fixed exogenous benefit \( b > 0 \). \( b \) is not enjoyed when the worker searches in the temporary submarket. In real-life labour markets, unemployed income often requires a specific on-the-job tenure, and our assumption is fully consistent with this fact.

There are matching frictions in each submarket. We let \( m(u_i, v_i) \) be the flow of new matches, where \( u_i \) denotes the measure of unemployed workers in submarket \( i \) searching for the measure \( v_i \) of vacancies; following standard assumptions, we assume that \( m \) is concave and homogeneous of degree one in \( (u_i, v_i) \) with continuous derivatives. Now define \( h_i = m(u_i, v_i)/u_i = m(1, \theta_i) = h(\theta_i) \) as the transition rate from unemployment to employment for an unemployed worker in submarket \( i \) and \( q_i = m(u_i, v_i)/v_i = q(\theta_i) \) as the arrival rate of workers for a vacancy in submarket \( i \). \( \theta_i = v_i/u_i \) is the submarket-specific labour market tightness. The matching function \( m \) satisfies the following conditions:

\[
\lim_{\theta_i \to 0} h(\theta_i) = \lim_{\theta_i \to \infty} q(\theta_i) = 0 \quad i = p, t
\]

\[
\lim_{\theta_i \to \infty} h(\theta_i) = \lim_{\theta_i \to 0} q(\theta_i) = \infty \quad i = p, t
\]

Upon the meeting of an unemployed worker and a vacant firm, each match signs a long-term contract that fix a wage for the entire employment relationship without ex-post renegotiations. In the spirit of Hall [2005], any wage within the parties’ bargaining set, at the time of job creation, can be supported as an equilibrium. To make the problem interesting, we restrict our attention to wages such that \( y_h > w_p > y_l \)

\(^3\)The interpretation of dismissal at will in the case of temporary workers is twofold: either firms are allowed to fire whenever the shock occurs, or they’re able to set contracts whose duration is exactly \( 1/(s + \lambda) \)}
and \(y_h > w_t > y_l\). This will ensure that, conditional on the realisation of the adverse shock \(\lambda\), permanent contracts involve a loss to the firm. Further, we will focus on a constant wage across submarket, such that 
\[w_p = w_t = w.\]

The equilibrium of the model is characterised by free entry of firms in each submarket, and workers’ sorting condition across submarkets.

### 2.1 Value Functions and Job Creation in the Permanent Market

Let \(U_p(z)\) and \(E_p(z)\) denote, respectively, the expected discounted income for an unemployed worker and for an employed one in the permanent market. The Bellman equations are:

\[
\begin{align*}
  rU_p(z) &= z + b + h(\theta_p)[E_p(z) - U_p(z)] \\
  rE_p(z) &= w + s[U_p(z) - E_p(z)]
\end{align*}
\]

where \(r\) is the pure discount rate, \(z\) is the workers’ specific outside option and \(b\) is the unemployment benefit.

Let \(J^h_p\) and \(J^l_p\) denote, respectively, the present discounted value of a permanent job when productivity is high \((y_h)\) or low \((y_l)\); their formal expression read

\[
\begin{align*}
  rJ^h_p &= y_h - w + \lambda[J^h_p - J^h_p] + s[V_p - J^h_p] \\
  rJ^l_p &= y_l - w + s[V_p - J^l_p]
\end{align*}
\]

When productivity is high, the firm enjoys an operational profit equal to \(y_h - w\). The worker leaves at rate \(s\) and the firm gets the expected value of a vacancy formally indicated with \(V_p\). Conditional on a productivity shock \(\lambda\), the firm has no margin of adjustment and experiences a capital loss equal to the difference between the value of a permanent job in high state and a value in bad state \(J^l_p - J^h_p\). In the low state, the firm runs an operational loss \(y_l - w\) as long as the worker separates at rate \(s\). The asset equation of a vacancy reads

\[rV_p = -c + q(\theta_p)[J^h_p - V_p]\]

Assuming free entry in the permanent market, \(V_p = 0\), we have that

\[c = q(\theta_p)J^h_p\]

The previous condition is one of the key equations of the model. It shows that the flow cost of vacancy posting is equal to expected benefit, where the latter is described as the product of the job filling rate into permanent contract time the value of a filled job.

Finally note that the value of a filled job can be written as

\[
\begin{align*}
  J^h_p &= \frac{y_h - w}{r + s + \lambda} + \frac{\lambda(y_l - w)}{(r + s)(r + s + \lambda)} \\
  J^l_p &= \frac{y_l - w}{r + s} < 0
\end{align*}
\]

The latter expression represents the cost associated to having a permanent contract in case of adverse shock.
2.2 Value Functions and Job Creation in the Temporary Market

Workers employed with a temporary contract are dismissed conditional on the arrival rate \( \lambda \), so that the value of employment reads

\[
re_t(z) = w + (s + \lambda)[U_t(z) - E_t(z)]
\]  
(5)

The value of unemployment depends on the specific outside income and faces a transition probability \( h(\theta_t) \)

\[
rU_t(z) = z + h(\theta_t)[E_t(z) - U_t(z)]
\]  
(6)

Firms in temporary market are free to dismiss workers conditional on the adverse productivity shock; the value of a filled temporary job and of a temporary vacancy read

\[
rJ_{t}^{h} = y_h - w + (s + \lambda)[V_t - J_t^{h}]
\]

\[
rV_t = -c + q(\theta_t)[J_t^{h} - V_t]
\]

Assuming free entry also in the temporary market, \( V_t = 0 \), we have that

\[
c = q(\theta_t)J_t^{h}
\]  
(7)

Similarly to the condition above, equation (7) says that the flow cost of vacancy in the temporary market is equal to expected benefit, where the latter is described as product of the job filling rate into temporary contract time the value of a filled job.

Before turning to the equilibrium definition, we derive the second key condition of our analysis. Using the free entry condition into the temporary market, one can easily show that a filled temporary job has larger value than a permanent job

\[
J_{t,h} = \frac{y_h - w}{r + s + \lambda} > J_{p,h}
\]

We are now in a position to establish a key result of our model. The expected value of vacancy depends on the job filling rate and on the value of a filled job. A labour market with both temporary and permanent jobs is such that

\[
q(\theta_t)J_{t}^{h} = q(\theta_p)J_{p}^{h}
\]

where we have just proved that \( J_{t}^{h} > J_{p}^{h} \). This result tells that the coexistence of temporary and permanent contract implies that

\[
q(\theta_t) < q(\theta_p)
\]

Once the job is filled, the firms prefer a flexible contract. They are thus willing to offer both temporary and permanent contract if the job filling rate for permanent contracts is larger than the job filling rate for temporary contracts. Conversely, this result suggests that the job finding rate of a temporary contract is larger, so that

\[
h(\theta_t) > h(\theta_p)
\]
The previous result is very important for the results of the next section, where we discuss the workers’ sorting condition between the two submarkets.

2.3 Workers’ Sorting

Workers take as given the job finding rate\(^4\) and optimally decide in which submarket to search for a job. Since workers can freely move across submarkets, the optimal allocation will be

\[ U(z) = \text{Max}[U_p(z), U_t(z)] \]

where the expressions for \( U_p(z) \) and \( U_t(z) \) are obtained combining (2) with (1) and (5) with (6)

\[ rU_p(z) = \frac{(z + b)(r + s) + h(\theta_p)w}{r + s + h(\theta_p)} \]  \hspace{1cm} (8)

\[ rU_t(z) = \frac{z(r + s + \lambda) + h(\theta_t)w}{r + s + \lambda + h(\theta_t)} \]  \hspace{1cm} (9)

The values of unemployment, for given job finding rates, are monotonically increasing in \( z \). In what follows, we look for a reservation value of \( R \) such that the marginal worker (the one with idiosyncratic outside option \( z = R \)) is indifferent between searching for a temporary or a permanent job. If such \( R \) exist, workers endogenously sort between the two markets. Note that workers with low \( z \) place a larger willingness to work. Such workers are more willing to take up a job right away, even if such job has shorter duration. The formal value of \( R \) is

\[ R = w - b \left( \frac{(r + s)[r + s + \lambda] + h(\theta_p)}{(r + s)h(\theta_t) - (r + s + \lambda)h(\theta_p)} \right) \]

\(^4\)Once a functional form for the matching function is chosen, \( \theta_i \) is completely determined by the behaviour of the firms.
Figure (1), plots the reservation value. As long as the existence condition holds\(^5\), then \( R < w \) and there exists a proportion of workers \( 1 - F(R) \) searching in the permanent market. It’s easy to see that when \( b = 0 \) the reservation outside option is equal to the wage and all workers look for a temporary job.

### 2.4 labour Market Stocks and Flows

Labour supply is the sum of unemployment and employment in each submarket

\[
\begin{align*}
\quad u_t + n_t & \quad = \quad F(R) \\
\quad u_p + n_p & \quad = \quad 1 - F(R)
\end{align*}
\]

The dynamic evolution of unemployment in the two submarket is given by difference between job creation and job destruction. This implies that

\[
\begin{align*}
\quad \dot{u}_p & \quad = \quad sn_p - h(\theta_p)u_p = s[1 - F(R) - u_p] - h(\theta_p)u_p \\
\quad \dot{u}_t & \quad = \quad (s + \lambda)n_p - h(\theta_t)u_t = (s + \lambda)[F(R) - u_t] - h(\theta_t)u_t
\end{align*}
\]

Unemployment in each submarket is constant when job creation is equal to job destruction; the steady state expressions for the stocks read

\[
\begin{align*}
\quad u_p & \quad = \quad \frac{s[1 - F(R)]}{s + h(\theta_p)} \\
\quad n_p & \quad = \quad \frac{h(\theta_p)[1 - F(R)]}{s + h(\theta_p)} \\
\quad u_t & \quad = \quad \frac{(s + \lambda)F(R)}{s + \lambda + h(\theta_t)} \\
\quad n_t & \quad = \quad \frac{F(R)h(\theta_t)}{s + \lambda + h(\theta_t)}
\end{align*}
\]

### 3 Equilibrium

The equilibrium is obtained by a triple \( \{\theta_t, \theta_p, R\} \), and a distribution of employment across states that satisfy the set of value functions \( \{J^h_i, J^l_i, V_i, E_i(z), U_i(z)\} \) with \( i \in [p,t] \) and:

- Optimal vacancy posting in each submarket. The value of a vacancy is identical across submarkets and driven down to zero by free entry

\[
V_p = V_t = 0
\]

This in turn implies:

- Job creation in the permanent market

\[
q(\theta_p)J^h_p = c \quad \text{(JC, permanent)}
\]

\(^5\)See the appendix.
Job creation in the temporary market

\[ q(\theta_i) J_i^b = c \quad \text{(JC, temporary)} \]

which together say that in equilibrium the expected benefit of a permanent job must be equal to the expected benefit of a temporary job.

- Optimal workers’ sorting. The marginal worker is indifferent between searching in the market for temporary or permanent jobs

\[ U_p(R) = U_t(R) \quad \text{(Sorting)} \]

Once a functional form for \( m(u_i, v_i) \) is chosen, \( \theta_p \) and \( \theta_t \) are determined through job creation conditions; the sorting equation yields \( R \) and the last equations in the previous section determine the stocks. The coexistence of the two submarkets depends on a simple condition, as we show next.

**Proposition.** Temporary and Permanent submarkets coexist in equilibrium as long as the reservation utility \( R \) exists. Further, if \( R \) exists, it is also lower than the wage.

Proof, see appendix.

### 3.1 Comparative Statics

Qualitative aspects of the final equilibrium obviously depend on the values taken by the exogenous parameters. In this section we focus our attention upon the effects of changes from a relevant couple of them, namely the unemployment benefit \( b \) and the shock occurrence rate \( \lambda \), on the unemployment rate, the labour market tightness, the reservation outside option and the value (for the firm) of a filled job.

- An increase in the wage \( w \) leads to a reduction in market tightness in both submarkets and an increase in total unemployment. The effect on the two market tightnesses follows directly from a simple differentiation of equations JC, permanent and JC, temporary, so that \( \frac{\partial \theta_i}{\partial w} < 0 \) while the effect on \( R \) is ambiguous. The latter follows from the fact that both transition rate fall, and it is not clear a priori which of the transition rate falls more.

- From the point of view of the firms, the level of the unemployment benefit does not have any direct effect on the value of a filled job, and using the job creation conditions in the two submarkets, also on the labour market tightness. In symbols

\[ \frac{\partial \theta_i}{\partial b} = \frac{-c[\partial J_i^b/\partial b]}{[J_i^b]^2[\partial q(\theta_i)/\partial \theta_i]} = 0 \text{ since } \partial J_i^b/\partial b = 0 \]

An increase in \( b \) makes the permanent submarket more attractive for the workers. Since market tightness does not change, permanent unemployment increases and temporary unemployment decreases. Formally, using the formal value of \( R \) it is immediate to see that, as long as \( R < w, \partial R/\partial b < 0 \). This
result allows to evaluate the effect on the unemployment rates

\[
\frac{\partial u_p}{\partial \beta} = -\frac{s}{s + h(\theta_p)} \frac{\partial F(z)}{\partial z} \frac{\partial R}{\partial \beta} > 0
\]

and

\[
\frac{\partial u_t}{\partial \beta} = \frac{s + \lambda}{s + \lambda + h(\theta_t)} \frac{\partial F(z)}{\partial z} \frac{\partial R}{\partial \beta} < 0
\]

as expected. The effect on total unemployment is consequently ambiguous\(^6\).

- An increase in the arrival rate \(\lambda\) has various effects, but the overall result is not as clear. If a shock to the productivity of a match becomes more likely, all firms enjoy the operational profit for a shorter period; the value of a filled job, either temporary or permanent, diminishes and firms are less prone to post new vacancies. In formal terms

\[
\frac{\partial J^h}{\partial \lambda} = \frac{y_h - w}{(r + s + \lambda)^2} < 0 \quad \text{and} \quad \frac{\partial J^h}{\partial \lambda} = \frac{-y_h - w}{(r + s + \lambda)^2} < 0
\]

Using the result above with job creation conditions in both markets yields the negative reaction of the labour market tightness

\[
\frac{\partial \theta}{\partial \lambda} = -c \frac{\partial J^h / \partial \lambda}{[J^h]^2 \partial \eta(\theta_t) / \partial \theta_t} < 0
\]

From the point of view of the workers a higher \(\lambda\) makes the duration of a temporary job shorter; a fraction of them would therefore move from the temporary to the permanent tier but, differently from the case of the unemployment benefit, the productivity shock negatively affects the tightness in both submarkets too. In other words a trade off emerges between a higher risk of being fired on the temporary market (which has a negative direct effect upon the reservation outside option) and a possibly too high unemployment duration on the permanent. The net effect of a change in the shock rate upon \(R\) is therefore a priori ambiguous and no prediction can be made upon the unemployment rates.

4 Liberalisation of Temporary Contracts

While the steady state solution clearly implies a long run coexistence of the two type of contracts, the question linked to the liberalisation of temporary contracts has not yet been discussed. In this section we consider the full transition from a rigid regime, a situation where only permanent contracts are allowed, to a dual regime where temporary and permanent contracts coexist in equilibrium.

The rigid regime is formally described as a labour market in which only the permanent submarket exists. We define the introduction of temporary jobs as a permanent unexpected shock to the steady state of the rigid market. The functioning of the liberalisation is as follows. At time \(\tau = 0\) when the shock occurs

\[^6\text{With some algebra it can be shown that an increase in the unemployment benefit increases total unemployment as long as } \lambda < [h(\theta_t) - h(\theta_p)]/h(\theta_p)\].
the stock of unemployed workers of the old regime is immediately split in two: workers with $z \leq R$ start searching in the temporary submarket, while workers with $z$ above the reservation productivity $R$ stay in the permanent one. Firms immediately post vacancies in order to fully absorb any rent. Thereafter, the stock of workers smoothly move toward a new steady state with two submarkets. Note that both the reservation utility $R$ as well as market tightness in the two submarkets are time invariant, and the dynamics of the model can be described analytically.

To keep track of the dynamics of the model after the introduction of temporary contracts, we will consider separately the behaviour of workers whose outside option is below or above the reservation threshold.

- $z \leq R$. At $\tau = 0$ all unemployed workers with an outside utility below the reservation utility start searching for a temporary job at the finding rate $h(\theta_t)$. On the demand side, firms post a number of temporary vacancies such that the tightness jumps to its equilibrium level and fill them at rate $q(\theta_t)$. In addition those workers employed with a permanent contract and idiosyncratic utility below $R$ are gradually dismissed at rate $s$ and become unemployed in the temporary submarket. The steady state is reached when all workers with outside utility below the reservation $R$ move to the temporary submarket. Let’s define with $n_p(z, \tau)$ and $n_t(z, \tau)$ the share of permanent and temporary contract with outside utility less or equal to $z$ at transition time $\tau$. $u_t(z, \tau)$ is similarly defined for the unemployment stock. This implies that at each point in time the distribution of workers with a low outside option reads

$$F(z) = n_p(z, \tau) + n_t(z, \tau) + u_t(z, \tau), \quad z \leq R$$

and the dynamics of the three functions is described by

$$\dot{n}_p(z, \tau) = -sn_p(z, \tau), \quad z \leq R$$

$$\dot{n}_t(z, \tau) = h(\theta_t)u_t(z, \tau) - (s + \lambda)n_t(z, \tau), \quad z \leq R$$

$$\dot{u}_t(z, \tau) = sn_p(z, \tau) + (s + \lambda)n_t(z, \tau) - h(\theta_t)u_t(z, \tau), \quad z \leq R$$

where it is clear that there is no inflow into $n_p(z, \tau)$ for $z \leq R$, but simply an outflow that dies out as all permanent jobs with outside utility below $R$ are slowly destroyed at rate $s$. The flows of temporary contracts is governed by flows that are identical to those of the steady state.

- $z > R$. People with outside utility above the reservation $R$ are either employed with a permanent contract or unemployed and searching for a permanent job. This is true both in the rigid and in the liberalised regime. Accordingly, the distribution of such workers reads

$$F(z) = u_p(z, \tau) + n_p(z, \tau), \quad z > R$$

where $u_p(z, \tau)$ is the stock of unemployed at time $\tau$ and $n_p(z, \tau)$ is the stock of employed workers. The
The dynamics of these two components is given by

\[
\begin{align*}
\dot{u}_p(z, \tau) &= sn_p(z, \tau) - h(\theta_p)u_p(z, \tau), \quad z > R \\
\dot{n}_p(z, \tau) &= h(\theta_p)u_p(z, \tau) - sn_p(z, \tau), \quad z > R
\end{align*}
\]

The system of differential equations can be solved analytically. The details are in the appendix. The readers can find the final results below

\[
\begin{align*}
n_t(z, \tau) &= \left\{ \frac{h(\theta_t)h(\theta_p)F(z)}{[s + h(\theta_p)][h(\theta_t) + \lambda]} - \frac{h(\theta_t)F(z)}{h(\theta_t) + s + \lambda} \right\} e^{-[h(\theta_t) + s + \lambda]|\tau|} + \\
&\quad \frac{h(\theta_t)F(z)}{h(\theta_t) + s + \lambda} e^{-s\tau} \\
u_t(z, \tau) &= F(z) \left\{ \frac{s}{s + h(\theta_p)} + \frac{\lambda h(\theta_p)}{[s + h(\theta_p)][h(\theta_t) + \lambda]} - \frac{s + \lambda}{h(\theta_t) + s + \lambda} \right\} e^{-[h(\theta_t) + s + \lambda]|\tau|} + \\
&\quad \frac{(s + \lambda)F(z)}{h(\theta_t) + s + \lambda} e^{-s\tau} \\
u_p(z, \tau) &= \frac{s [1 - F(z)]}{s + h(\theta_p)} \\
n_p(z, \tau) &= \frac{h(\theta_p)F(z)}{s + h(\theta_p)} e^{-s\tau} + \frac{h(\theta_p)[1 - F(z)]}{h(\theta_p) + s}
\end{align*}
\]

Taking the limit as \(\tau\) goes to infinity and using \(z = R\), one gets easily the expressions for the two tiers steady state (see section 2.4).

### 4.1 Just a ”honeymoon effect”?

Having derived the analytical solution to the transition, we now look into the effects of the liberalisation of temporary contracts, with particular attention to the unemployment rate. Our solution distinguishes between a short run and a long run effect.

In the aftermath of the liberalisation, immediately after the shock, the unemployment rate necessarily falls. The reasoning is as follows. At \(\tau = 0\) the stock of unemployed workers is as large as in the rigid regime, but a fraction \(F(R)\) of workers starts searching into the temporary submarket where the job finding rate \(h(\theta_t)\) is larger. Indeed, market tightness and vacancy posting are a forward looking variable, and immediately jump to exhaust all the rents. While it is true that in the temporary submarket also the separation rate is larger through the destruction rate \(\lambda\), it takes time for such effect to emerge. Further, market tightness is constant during the transition. As a result unemployment, initially, necessarily falls\(^7\).

Figures (2) and (3) plot the dynamics of the unemployment and the employment rates for a given set of parameters values\(^8\). The downward jump represents this ”honeymoon effect”: on impact, the liberalisation of temporary contracts has a positive effect on total employment.
Figure 2: Dynamics of the unemployment rate

Figure 3: Dynamics of the employment rate
The results on the long run effects are more ambiguous. Whether total unemployment is permanently lower than in the rigid regime depends on the relative strength of the job finding and job destruction rates in the two submarkets. The unemployment is permanently reduced if

\[
\frac{u_{p,old}}{s + h(\theta_p)} > \frac{s[1 - F(R)]}{s + h(\theta_p)} + \frac{(s + \lambda)F(R)}{s + \lambda + h(\theta_p)} \Rightarrow
\]

\[
u_p \tau \rightarrow \infty + u_t \tau \rightarrow \infty \Rightarrow
\]

\[
\lambda < \frac{s[h(\theta_t) - h(\theta_p)]}{h(\theta_p)}
\]

i.e. if workers’ turnover is not too high. This statement, however, needs to be further discussed. Using the condition for the existence of \( R \) one gets

\[
\lambda < \frac{(r + s)\{h(\theta_t)w - h(\theta_p)w - b[r + s + h(\theta_p)w]\}}{b(r + s) + h(\theta_p)w}
\]

When \( b = 0 \), from the point of view of (10) this condition is not relevant, becoming monotonically binding for increasing values of the unemployment benefit; this means that for small values of the unemployment benefit the coexistence of permanent and temporary contracts does not prevent the labour market from a higher equilibrium unemployment rate.

5 Training

In this section we consider the possibility that firms, in the aftermath of the adverse productivity shock, may be able to jump back to the high productivity by undergoing costly training. Specifically, we assume that when the negative shock occurs firms can jump back to the high level of productivity \( y_h \) by paying a lump sum cost \( T \) in the form of training. As the wage paid to workers is held fixed, we can abstract from the issue of financing. We will show that there exist two bounds \([T_l, T_u]\) such that if \( T_l < T < T_u \) only firms in the permanent submarket decide to train workers. The asset equations in the permanent market read

\[
rJ^h_p = y_h - w + s[V_p - J^h_p] + \lambda[\max(J^l_p, J^h_p - T) - J^h_p]
\]

\[
rJ^l_p = y_l - w + s[V_p - J^l_p]
\]

\[
rV_p = -c + q(\theta_p)[J^h_p - V_p]
\]

7 Analitically this result is obtained by taking the time derivative of \( u_t \) and evaluating it at \( \tau = 0 \); this yields \( \partial u_t(\tau)/\partial \tau|\tau = 0 < 0 \). Details are in the appendix.

8 We assumed that the matching function is a Cobb-Douglas one with unemployment elasticity \( \alpha \): \( m_i = k_i v_i^{\alpha} \) where \( \alpha = 0.5 \) and \( k_i = 1 \). Time is expressed in years. The pure discount rate \( r = 0.02 \), worker turnover \( s = 0.1 \) and the average waiting time for a productivity shock is about six years (\( \lambda = 0.15 \)). Productivity is either 1 or, conditional on the adverse shock, 0.6. The wage is 0.8 and the exogenous benefit \( b \) for the unemployed on permanent market is 30% of the wage. The cost of keeping open a vacancy is 0.3.

9 The stock of unemployed workers in the old regime is discussed in the appendix.
where the max operator conditional on the \( \lambda \) shocks highlights the training option. On the temporary market the asset equations read

\[
\begin{align*}
 rJ^h_t &= y_h - w + s[V_t - J^h_t] + \lambda[\max(V_t, J^h_t - T)] \\
rV_p^h &= -c + q(\theta_t)[J^h_t - V_t]
\end{align*}
\]

We now formally establish under what conditions workers with a permanent job receive training. Since undergoing training transforms a low productivity job into a high productivity job, a firm with a permanent contract will undergo training if

\[
J^h_p - T > J^l_p
\]

Simultaneously, a firm with a temporary contract will not undergo training if

\[
V > J^h_t - T
\]

The first condition implies

\[
\frac{y_h - w}{r + s + \lambda} + \frac{\lambda(y_l - w)}{(r + s)(r + s + \lambda)} - T > \frac{y_l - w}{r + s} \Rightarrow T < \frac{y_h - y_l}{r + s + \lambda}
\]

while the condition on the temporary workers reads

\[
T > \frac{y_h - w}{r + s + \lambda}
\]

If the cost of training \( T \) is large enough so that the exit strategy turns out to be preferable in the temporary market, but not too large, then only firms in the permanent market are induced to train the workers

\[
\frac{y_h - w}{r + s + \lambda} < T < \frac{y_h - y_l}{r + s + \lambda}
\]

More generally, it is never the case that workers receive training only in the temporary market. Training may be viable on both markets, only in the permanent, or in none of them, depending on the level of \( T \). When \( T \) is bounded as in condition (11) the following interesting results follow:

- The temporary market is not affected by training costs. As a consequence, the value of a filled job is the same as in the model without training.

- The value of filled jobs in the permanent market now reads

\[
J^h_p = \frac{y_h - w - \lambda T}{r + s}
\]

which is larger than in the model without training, but still lower than \( J^h_t \).

- Free entry makes the equilibrium conditions in the temporary submarket independent on \( T \)

\[
\begin{align*}
c &= q(\theta_p)J^h_p \\
c &= q(\theta_t)J^h_t
\end{align*}
\]
This means that in equilibrium the temporary market tightness is the same as without training, while the permanent tightness has now to be higher. As a consequence, on average, in the model with training the job finding rate is higher, the arrival rate of workers for a vacancy is lower, and the steady state overall unemployment is lower.

6 On the Job Search

This section proposes a further extension of the basic model, as it allows workers (either employed or unemployed) in the temporary tier to search for a permanent job. As we keep the wage constant across submarkets, we do not need to explicitly consider wage determination, one of the (many) difficult issues to be faced when one deals with on-the-job search [Nagypal 2006; Shimer 2003]. Nevertheless, the matching function and the definition of market tightness need to be modified and adjusted. In what follows, the number of matches in the permanent submarket reads

\[ m_p(u_p + n_t + u_t, v_p) = m_p(u_p + F(R), v_p) \]

where the pool of workers that search for a job is the sum of workers searching only in the permanent market \( u_p \) and the pool of workers searching in the temporary submarket \( n_t + u_t \). Since the pool of workers in the temporary submarket is the fraction of them with outside utility below \( R \), the second expression immediately follows. As a result, market tightness in the permanent submarket is given by

\[ \theta_p = \frac{v_p}{u_p + n_t + u_t} \]

(12)

The matching function in the temporary submarket is unchanged and is simply given by \( m_t(u_t, v_t) \), with market tightness \( \theta_t = v_t/u_t \).

The value functions in the permanent submarket are defined similarly to those of the baseline model (see section 2.1). The only difference is the expression for \( \theta_p \), that is defined as in (12) as a way to take into account the composition of the pool of workers searching for a permanent job. Free entry in the permanent submarket implies that

\[ q(\theta_p)J_h^p = c \]

where \( J_h^p \) is given by (4).

The value functions for the temporary submarket are different, since workers leave temporary jobs at rate \( s + h(\theta_p) \). When business conditions are good, the value function reads

\[ rJ_t^h = y_h - w + [s + \lambda + h(\theta_p)][V_t - J_t^h] \]

while the value of a vacancy is simply given by

\[ rV_t = -c + q(\theta_t)[J_t^h - V_t] \]
so that free entry implies that

\[ q(\theta_t) J^h_t = c \]

where \( J^h_t \) is now given by

\[ J^h_t = \frac{y_h - w}{r + s + \lambda + h(\theta_p)} \tag{13} \]

The job creation conditions are still the two key equations, but since now \( J^h_t \) depends also on \( \theta_p \) they form a non linear system of two equations in two unknowns that can be solved in cascade\(^{10}\). The last variable to be determined is the reservation utility \( R \). The value of unemployment in the temporary submarket reads

\[ rU_t(z) = z + h(\theta_t) [E_t(z) - U_t(z)] + h(\theta_p) [E_p(z) - U_t(z)] \]

where it is clear that an unemployed worker with low outside utility searches both in the temporary and in the permanent submarket, and can leave the unemployment pool for both types of jobs. Unemployed workers in the permanent submarket behave as in the baseline model, and their asset value equation for the unemployment status is provided by (1). Given the expressions for \( E_t(z) \) and \( E_p(z) \) and after some steps of algebra (see the appendix for details), the reservation utility \( R \) reads

\[ R = w - b r + s + \lambda + h(\theta_t) + h(\theta_p) \]

which implies that \( R < w \). Ensuring also that \( b \) is small enough\(^{11}\), we can easily establish that \( 0 < R < w \).

With respect to the base model, the value of a filled temporary job given in (13) is now lower and not necessarily higher than the value of a permanent one; however, assuming that \( J^h_p < J^h_t \), the structure and functioning of this model is identical to the model without on-the-job search. In particular, the basic mechanism that ensures that temporary and permanent jobs coexist in equilibrium survives to this admittedly more realistic scenario. The fact that the value of a permanent job is unchanged while a temporary one is worth less than before means that firms take into account the possibility that temporary workers leave their job moving toward the permanent tier and are consequently less prone to post temporary vacancies; in equilibrium, this leads to a lower tightness in the temporary submarket where a relatively higher congestion from the point of view of the workers emerges.

### 7 The empirical analysis

This section aims at testing one of the main implications of the model - namely that the arrival rate of temporary job offers is larger than that of permanent offers - and at providing further empirical evidence on

\(^{10}\) Starting from job creation in the permanent submarket one gets \( \theta_p \); using this result with job creation in the temporary submarket also \( \theta_t \) is obtained.

\(^{11}\) Technically the equilibrium of the model must be such that

\[ b < \frac{h(\theta_t)}{r + s + \lambda + h(\theta_t) + h(\theta_p)} \]
a second one, i.e. that temporary workers receive less training with respect to those with open-ended jobs. We do this using Italy as a learning case-study. Its labour market’s recent developments, indeed, closely fit our theoretical model’s dynamics. According to the Oecd’s index of employment protection, during the last twenty years Italy displayed the deepest deregulation of temporary hires among Oecd countries\textsuperscript{12}, while the legislation on open-ended employment relationships was instead kept almost unchanged. This is consistent with the hypothesis that in the aftermath of liberalisation only unemployed workers can be hired under temporary agreements, while in order to quit or transform existing (open-ended) positions, employers have to rely on natural turnover; in other words Italy is a prominent example of reforms “at the margin”, a feature actually shared with many other European countries [European Commission 2010]. On top of that our case-study also reproduces the model’s assumption that permanent workers enjoy a higher unemployment benefit. This is due to two reasons. First, in Italy the access to income-maintenance schemes for the unemployed primary depends on one’s contractual arrangement, with permanent workers enjoying very generous, despite discretionary, dedicated schemes. Second, the access to provisions that are formally available to non-standard workers too depends on the fulfilment of some contribution requirements, so that workers with more continuous careers - those with open-ended jobs - are more likely to receive the benefits. This last aspect concerns all countries where social protection is based on insurance principles: Germany and Japan are two other notable examples [Berton et al. 2012]. Eventually Italy also displays a high degree of contractual persistence - what the model implies unless the outside option $z$ changes in time - and, despite the use of temporary contracts have been fully liberalised, the creation of open-ended jobs is all but negligible [Berton et al. forthcoming].

7.1 Econometric strategy and specification issues

Since our primary empirical goal is to test whether the arrival rate of temporary job offers is larger than that of permanent ones - or, from another perspective, the waiting time for a temporary job is shorter - we are interested in models in which the duration of unemployment can be compared across exits to different labour market states. We thus consider a survival model of unemployment with different exits. More precisely we model unemployment as a single duration process that is terminated by one out of $M$ exhaustive and mutually exclusive possible destinations. We let $T$ be the duration of stay and $\{D_m\}$ a set of $M$ dummies taking the value of one if state $m$ is entered and zero otherwise. Lancaster [1990] defines the transition intensity into state $m$ as

$$\theta_m(t) = \lim_{dt \to 0} \frac{\Pr(t \leq T \leq t + dt, D_m = 1 | T \geq t)}{dt}$$

\textsuperscript{12}See Brandt et al. [2005] for a discussion in the perspective of the Oecd Jobs Study.
Following Allison [1982] we model transition intensities - a negative measure of the duration until each exit - as
\[
\theta_m(t) = \frac{\exp(\beta'_m X)}{1 + \sum_{m=1}^{M} \exp(\beta'_m X)}
\]
where \(X\) is a set of covariates. One can show that the resulting likelihood function takes the same form of the likelihood of a multinomial logit regression applied to a dataset reorganised into a person-period form, i.e. a form in which every individual in every moment in time can be in one out of \(M + 1\) labour market states: unemployment or one of the \(M\) alternative states [Jenkins 1995; 2005].

Matrix \(X\) includes a set of time-dummies \(\{D_t\}\) such that \(D_t = 1[T = t]\). This structure is intended to identify the duration dependence shape without assuming any a-priori parametric functional form and allows us to test the hypothesis under study in a very straightforward way: by taking exits to open-ended jobs as the benchmark of the multinomial logit regression, the hypothesis that the waiting time for a temporary job is shorter than for an open-ended one holds if the coefficients of the time-dummies when \(m = \text{temporary}\) are positive. In symbols:

\[
\beta_{m=\text{temporary}}[D_t] > 0
\]

In our specification \(M = 3\): possible exits from unemployment are to 
- i) full-time open-ended contracts with no social security contribution rebates,
- ii) full-time fixed-term direct hire contracts with no social security contribution rebates, and
- iii) other contracts (including self-employment, independent contractorship, temporary agency work, training, apprenticeship, part-time work and contracts implying rebates on the cost of labour). This choice allows us to compare contracts that only differ with respect to their formal ex-ante duration, without cost differentials that may affect employers’ as well as workers’ choices. It thus closely replicates the theoretical model’s framework.

In order to study whether open-ended jobs imply more investments in training, we follow instead an indirect strategy. We include in matrix \(X\) measures of contract-specific actual experience since the beginning of one’s career and analyse their impact on duration of unemployment until different exits: as long as open-ended contracts actually provide workers with more human capital, persistence in unemployment and the probability of exit to temporary jobs are expected to decrease with the amount of actual experience in open-ended employment.

### 7.2 Data and sample selection

The econometric strategy depicted above requires that the dynamics of individual working careers and the work arrangements are observed in details. For these reasons we decided to use administrative data; in particular, we use the *Work Histories Italian Panel* (Whip). Whip is an employer-employee linked database of individual work histories built using information from the Italian social security administration archives. The series covers the period from 1985 to 2004. Its reference population includes all the individuals for which
a payment to or from the social security administration is due during the observed period: all the employees of the private sector, civil servants with temporary contracts of any type, independent contractors, professionals without a dedicated social security fund, craftsmen, traders and unemployed workers who receive an unemployment benefit. Observables include individual information such as age, gender, place of birth and the place of residence; work-related variables are wage, place of work, sector, firm size, occupation, the relevant collective agreement enforced, the type of contract, beginning and end dates of work relationships.

Sample selection aims at overcoming the main limitations of the data. We select all unemployment spells experienced by workers who entered the labour market between 1998 and 2003 at the age of 19 to 29 at entry. The focus on entrants allows us to observe the whole dynamics of workers’ careers and to keep a detailed track of one’s actual and potential experience; on top of being part of our strategy to assess the theoretical model’s implications in terms of training and human capital, this also helps to minimise possible problems that would otherwise arise if the initial portion of one’s career was not observed, and fully prevents left-truncation issues. The purpose of conditions on age at entry is in turn twofold: on the one hand the lower bound at 19 years old makes our sample more homogeneous, since selected workers are very likely to hold at least a high school degree, what is important since education is not observed; on the other, the upper bound drops from the sample most of those workers wrongly defined as entrants, but having instead had other previous relevant work experiences that, again, are not observed in the data.

The main concerns relate indeed to the identification of unemployment spells. A worker who is not observed at work in Whip may potentially be i) unemployed ii) out of the labour market or iii) employed in an unobserved portion of the labour market. Unemployment is formally identified only when the unemployed workers receive a benefit, while unsupported unemployment, non-participation or unobserved employment cannot be distinguished. In order to minimise the possibility that an individual not observed at work in the data is instead actually working, we drop from the sample all the workers who had even a single temporary work experience in the public sector; unobservable regular employment is indeed almost completely absorbed by open-ended contracts in the public sector and since transitions from private to public sector are extremely unlikely in Italy, by dropping the individuals who had a temporary experience within the public administration we exclude from the sample those who are most likely to get an unobserved contract. Non-participation is then narrowed down by excluding individuals with work experiences as traders or craftsmen (since self-employment has a negligible leakage to dependent work in the private sector: see Berton et al. [forthcoming]), in the agricultural sector (in which, according to anecdotal evidence, temporary layoffs are widely used) and by right-censoring at 18 months all the sampled spells.

The series is then limited since only in the 1998-2003 time-span contractual arrangements can be ob-

\[^{13}\text{For further details see www.laboratoriorrevelli.it/whip.}\]
\[^{14}\text{We define entrants those workers who are never observed in the data before 1998-2003.}\]
\[^{15}\text{Further robustness checks about workers’ attachment to the labor market are discussed in the following. From now on, however, we conservatively speak of non-employment spells.}\]
served in full details. Eventually, we dropped all the spells started after a temporary agency or an independent/freelance contract - for which we have no information on sector and firm size - and those lasting up to two months, since they may hide voluntary job-to-job transitions and thus a relevant share of the search activity.

The resulting sample amounts to 14,248 individuals, for a total of 18,767 unemployment spells and 166,575 monthly person-period observations. Table 1 provides some descriptive statistics.

<table>
<thead>
<tr>
<th>Table 1: Descriptive statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Individuals: 14,248</strong></td>
</tr>
<tr>
<td>Entry year</td>
</tr>
<tr>
<td>1998</td>
</tr>
<tr>
<td>1999</td>
</tr>
<tr>
<td>2000</td>
</tr>
<tr>
<td>2001</td>
</tr>
<tr>
<td>2002</td>
</tr>
<tr>
<td>2003</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Spells: 18,767</th>
</tr>
</thead>
<tbody>
<tr>
<td>Failure event</td>
</tr>
<tr>
<td>Censored</td>
</tr>
<tr>
<td>Full-time open-ended</td>
</tr>
<tr>
<td>Full-time fixed-term</td>
</tr>
<tr>
<td>Other</td>
</tr>
</tbody>
</table>

| Occupation | % | Work area | % |
| Blue collars | 80.1 | North-west | 24.4 |
| White collars | 19.9 | Center | 23.0 |
|              |     | South    | 32.8 |

Source: own elaborations on Whip data.

7.3 Estimation results

Our main results are presented in the top panel (**time spline**) of table 2, under columns two and three. The time-dummy coefficients for exit toward full-time direct-hire temporary jobs with no social security rebates are significantly positive until the first year of search; as implied by our theoretical model, the arrival rate of (the waiting time for) temporary job offers is thus larger (shorter) than that of (for) permanent ones. The following panel (**experience**) displays results concerning the training content of contracts. While actual experience of any type reduces in general the probability to persist in unemployment\textsuperscript{16}, we find that the number of months spent in open-ended employment adjusts this result by reducing the probability to get a fixed-term job in favour of an open-ended one. Also in the light of results proposed by Berton et al. [forthcoming] we interpret this result as evidence in favour of the hypothesis that workers with open-ended jobs accumulate more human capital.

\textsuperscript{16}Estimation results for persistence in non-employment are not reported here and are available upon request to the authors.
The other coefficients suggest that exits to full-time fixed-term jobs are more likely for women - employers might be less prone to hire young women under an open-ended contract for the risk of a poorer attachment to the job, due for instance to maternity leaves and childcare - and for younger individuals, who may participate only occasionally to the labour market; on top of that, age at entry also captures the effect of education, and less educated workers have less bargaining power and may require a longer probation period. Exits to full-time direct-hire temporary contracts turn out to be more frequent also for workers who entered the labour market in 2001 and 2002 - what can be explained by the enforcement, in 2001, of Directive 1999/70/CE, which further deregulated the use of this type of contracts - and for those who already had temporary work experiences (including training and apprenticeship contracts) in the past; this result mirrors the high degree of persistence that characterises temporary employment in Italy [Berton et al. forthcoming]. The negative coefficients of having entered unemployment after a job in the construction sector and a part-time contract
are then due to the joint effect of a high sectoral persistence and of an actually much lower employment protection for open-ended workers for the former, and to a preference for going on searching or for non-participation for the latter. Having had a job in central Italy, in the service sector, in a large firm and as white collar increases instead the probability of getting a full-time direct-hire temporary job.

7.4 Discussion of empirical results

Despite the many advantages that administrative data brings to our empirical analysis - being the possibility to observe the dynamics of individual working careers without measurement errors or recall bias and with full details on one’s working arrangement the most relevant to our purposes - Whip also presents some drawbacks that may affect our results.

We already mentioned about the impossibility to distinguish between unemployment, non-participation and employment in unobserved portions of the labour market. By sampling workers never observed as civil servants we already prevent the major source of unobservable employment, i.e. open-ended contracts in the public sector; however, another relevant area of unobservable employment is represented by the underground economy, which is obviously not recorded in administrative data. To check the robustness of our results to the possibility that a number of workers may exit from unemployment to an irregular job, we re-estimated our model on a sub-sample of workers from central and northern regions only, where the underground economy is less widespread. Results are presented in table 2 under columns six and seven; as the reader can easily see, we can maintain our main conclusions. Analogously, under columns four and five and from eight to eleven, we check the robustness of our results to the presence of non-participation by excluding from the sample the group of workers that are more likely to - maybe temporarily - abandon the labour market: women, seasonal and part-time workers. Again, our conclusions still hold.

Another relevant limitation of the data is given by the small number of covariates. This raises the issue of unobserved heterogeneity. Unfortunately, the usual econometric tools to control for its presence are in our case of little help. On the one hand, fixed-effect strategies are not viable in duration models [see Magnac 2000]; on the other, duration models with random effects rely on the hypothesis of orthogonality between observed and unobserved components, what implicitly rules out the capability to control for a number of relevant (unobserved) covariates like, for instance, individual ability and education. In any case, only 30% of the sample is observed at least twice, and only 10% three times or more: this means that strategies involving repeated observations would imply an extremely strong sample selection. We used however some alternative strategies to circumvent the problem. First, we included in the specification some variables that capture the effect of the main sources of unobserved heterogeneity, namely age at entry for education, and type of contract and wage at entry for individual ability. Second, we estimated robust standard errors by clustering observations by individual. Third, semi-parametric specifications of duration dependence like the one we used are proved to be robust to the presence of unobserved components [Dolton and Van der Klaauw 1999].
Fourth, we argue that the effect of unobserved components - individual ability in particular - would be an underestimate of the parameters of interest. Indeed, the most endowed individuals are more likely to find an open-ended job, and to find it more quickly; as time goes by, therefore, the sample is left with less and less endowed individuals, who are instead more likely to get a temporary contract. Controlling for individual ability would thus reinforce our conclusions.

Our last concern is about the difference between the arrival rate of job offers and the job accepting rate. In our theoretical model job offers are always viable, which implies that there is no difference between the two rates. In real world, however, workers may choose to give up on an offer in order to go on searching for a better one. What we actually measure is thus the duration until acceptance, and not until the arrival of a job offer as the theoretical model would suggest. But since workers are more likely to give up on a temporary job than on a permanent one, our results lay again on the safe side.

8 Additional Evidence and Implications

To the best of our knowledge the issue of unemployment duration until temporary vs. permanent contracts has been seldom studied directly. The implication that the waiting time for a temporary job is shorter holds in the Netherlands [De Graaf-Zijl et al. 2011], in Slovenia [Van Ours and Vodopivec 2008] and in Spain [Bover and Gomez 2004], while an analogous effect does not emerge in France [Blanchard and Landier 2002] or in the US [Hotchkiss 1999]. The empirical evidence in turn largely supports the hypothesis that temporary workers receive less on-the-job training. Arulampalam and Booth [1998] investigate the relationship between employment flexibility and training using UK data, and find that workers on temporary contracts are less likely to receive work-related training. The same result holds also for Spain [Albert et al. 2005] as well as for the majority of European countries [Bassanini et al. 2007; European Commission 2010; Oecd 2002], what recently gave rise to a stream of literature concerning the (negative) impact of temporary employment on labour productivity [Dolado and Stucchi 2008].

The model features several other empirical implications. First, it implies a transitional dynamics on employment in the aftermath of the liberalisation of temporary contracts. These transitional effects have been studied by Boeri and Garibaldi [2007]; they show that most countries that experienced a gradual liberalisation of temporary contracts experienced also employment gains. Such honeymoon effect is clearly present in the mechanism analysed in this paper, as well as the possibility that it may fade out in the long run, as argued by Kahn [2010]. Second, the model implies that workers with a poor non-employment option put a high value on finding a job quickly, thus sorting into the temporary submarket; this is consistent with the idea that higher unemployment benefits allow workers to be more selective in the job search process, thus increasing the job match quality [Belzil 2001; Caliendo et al. 2009; Fitzenberger and Wilke 2010; Van der Klundert 1990]. Last, Jahn and Bentzen [2010] argue that during economic upturns unemployed workers are more confident to find a permanent job quickly, what rations labour demand and tightens the market
for temporary jobs; this is consistent with another key result of our model, namely that a labour demand trade-off between ex-ante slower job filling rate and ex-post more flexible dismissal rate exists.

9 Concluding Remarks

The liberalisation of fixed-term contracts in Europe has led to a two-tier regime, with a growing share of jobs covered by temporary contracts that is particularly pronounced in countries where the employment protection legislation differential with respect to workers with open-ended contracts is largest [Booth et al. 2002]. In this perspective the present paper proposed and solved a matching model with direct search in which temporary and permanent jobs coexist in a long-run equilibrium. The intuition is as follows: when temporary contracts are allowed, firms are willing to open permanent jobs in as much as their job filling rate is faster than that of temporary jobs. From the labour supply standpoint an analogous trade-off between ex-ante lower job finding rate and ex-post larger retention rate emerges. The theory has several further empirical implications. First, the liberalisation of temporary contracts does not crowd out permanent contracts and the system moves smoothly to a dual regime. Second, in the aftermath of liberalisation the economy enjoys an employment gain that may nonetheless completely fade away in the long run. Third, workers covered with open-ended contracts are more likely to receive training. Fourth, the basic functioning of the model survives to the scenario in which employed temporary workers are allowed to search on the job.

The prediction that the job offer arrival rate for temporary workers is higher is supported by our analysis of Italian administrative data. Using a competing-risks duration model of unemployment we find indeed that, other things being equal, the transition rates to temporary jobs is higher than to permanent jobs. The other empirical implications of the model, and in particular that temporary workers receive less training, are consistent with the existing literature.
A Existence

The coexistence of the two submarkets in equilibrium depends on the existence of a positive reservation outside utility strictly lower than the wage. We show this result in two steps.

- Existence of $R$. Since both $U_p(z)$ and $U_t(z)$ are linear and monotonically increasing in $z$, $R$ does exist (and moreover is unique) if and only if $U_t(z = 0) < U_p(z = 0)$ and $\frac{\partial U_p(z)}{\partial z} > \frac{\partial U_t(z)}{\partial z}$. Using equation (8) and (9), the condition on the slopes says that

$$\frac{r + s}{r + s + h(\theta_p)} > \frac{r + s + \lambda}{r + s + \lambda + h(\theta_t)}$$

and the one on the intercepts reads

$$\frac{h(\theta_t)w}{r + s + \lambda + h(\theta_t)} > \frac{h(\theta_p)w + (r + s)b}{h(\theta_t)(w - b)} \quad (14)$$

- Existence of the two submarkets. The existence of a reservation outside option is a necessary but not sufficient condition for the coexistence of temporary and permanent contracts in equilibrium. We already know, in fact, that if $R \geq w$ all workers search for a temporary job. We need then that

$$R < w \Rightarrow w - b \frac{(r + s)[r + s + \lambda + h(\theta_t)]}{(r + s)h(\theta_t) - (r + s + \lambda)h(\theta_p)} < (r + s)h(\theta_t) \Rightarrow \frac{r + s}{r + s + \lambda} > \frac{h(\theta_p)}{h(\theta_t)}$$

and we can conclude that if (14) holds then $R$ exists and is lower than the wage.

B Dynamics

In the rigid market all the workforce is either employed with a permanent contract or unemployed

$$u_p + n_p = 1$$

The differential equations describing the dynamics of these two components therefore does not depend on the outside utility and read

$$\dot{u}_p(\tau) = sn_p(\tau) - h(\theta_p)u_p(\tau)$$

$$\dot{n}_p(\tau) = h(\theta_p)u_p(\tau) - sn_p(\tau)$$

It’s easy to see that when the old regime reaches its steady state the stocks amount to

$$u_p = \frac{s}{s + h(\theta_p)}$$

and

$$n_p = \frac{h(\theta_p)}{s + h(\theta_p)}$$
As we pointed out above, in order to fully describe the dynamic behaviour of employed and unemployed workers in both submarkets we need to separately consider people with outside option below and above the reservation value \( R \). In every moment in time the distribution of the formers reads

\[
F(z) = n_p(z, \tau) + n_t(z, \tau) + u_t(z, \tau), \quad z \leq R
\]  

(15)

When \( \tau = 0 \) the stock of workers who start searching in the new submarket is given by the fraction of unemployed workers of the previous regime whose outside option is lower than \( R \)

\[
u_t(z, \tau = 0) = \frac{sF(z)}{s + h(\theta_p)}, \quad z \leq R
\]  

(16)

Since right after the introduction of the new regime nobody works with a temporary contract \((n_t(z, \tau = 0) = 0)\), the initial condition for permanently employed workers with \( z \leq R \) can be obtained through (15)

\[
n_p(z, \tau = 0) = F(z) - u_t(z, \tau = 0) - n_t(z, \tau = 0) \\
= \frac{h(\theta_p)F(z)}{s + h(\theta_p)}, \quad z \leq R
\]  

(17)

We are now in a position to describe the dynamic behaviour of \( n_p, n_t \) and \( u_t \) provided \( z \leq R \).

- In the rigid market a fraction of workers was employed with a permanent contract even if endowed with a low outside option. From \( \tau = 0 \) onwards, once they are fired (what happens at rate \( s \)), they start searching in the temporary submarket with no possibility to come back to the permanent tier

\[
\dot{n}_p(z, \tau) = -sn_p(z, \tau) \Rightarrow \\
\dot{n}_p(z, \tau) + sn_p(z, \tau) = 0 \Rightarrow \\
\int e^{\tau}[\dot{n}_p(z, \tau) + sn_p(z, \tau)]d\tau = b_1 \Rightarrow \\
n_p(z, \tau)e^{\tau} + b_0 = b_1 \Rightarrow n_p(z, \tau) = Be^{-\tau}, \quad z \leq R
\]

where \( b_0 \) and \( b_1 \) are constants of integration. Using (17) and solving for \( B \)

\[
n_p(z, 0) = B = \frac{h(\theta_p)F(z)}{s + h(\theta_p)}, \quad z \leq R
\]

therefore

\[
n_p(z, \tau) = \frac{h(\theta_p)F(z)}{s + h(\theta_p)} e^{-\tau}, \quad z \leq R
\]  

(18)

i.e. the initial stock of permanent workers with a low outside option decreases at rate \( s \) down to zero; in fact

\[
\lim_{\tau \to \infty} n_p(z \leq R, \tau) = 0
\]
The dynamic behaviour of temporary workers is an empty set, but immediately firms post temporary vacancies and fill them at rate \( h(\theta_t) \) by hiring from the stock of "temporary" unemployed. Temporary matches are then destroyed at rate \( s + \lambda \)

\[
\dot{n}_t(z, \tau) = h(\theta_t)u_t(z, \tau) - (s + \lambda)n_t(z, \tau)
\]

using (15) and (18) one gets

\[
\dot{n}_t(z, \tau) = h(\theta_t)[F(z) - n_p(z, \tau) - n_t(z, \tau)] - (s + \lambda)n_t(z, \tau) \Rightarrow \\
\dot{n}_t(z, \tau) + [h(\theta_t) + s + \lambda]n_t(z, \tau) = h(\theta_t) \left[ F(z) - \frac{h(\theta_p)F(z)}{s + h(\theta_p)} e^{-\tau} \right] \Rightarrow \\
e^{[h(\theta_t) + s + \lambda]z} \left\{ \dot{n}_t(z, \tau) + [h(\theta_t) + s + \lambda]n_t(z, \tau) \right\} = e^{[h(\theta_t) + s + \lambda]z} h(\theta_t) \left[ F(z) - \frac{h(\theta_p)F(z)}{s + h(\theta_p)} e^{-\tau} \right] \Rightarrow \\
n_t(z, \tau)e^{[h(\theta_t) + s + \lambda]z} + b_0 = \int h(\theta_t)F(z)e^{[h(\theta_t) + s + \lambda]z}d\tau - \int \frac{h(\theta_t)h(\theta_p)F(z)}{s + h(\theta_p)} e^{[h(\theta_t) + s + \lambda]z}d\tau \Rightarrow \\
n_t(z, \tau)e^{[h(\theta_t) + s + \lambda]z} = b_0 + \frac{h(\theta_t)F(z)}{h(\theta_t) + s + \lambda} e^{[h(\theta_t) + s + \lambda]z} + b_1 - \frac{h(\theta_t)h(\theta_p)F(z)}{s + h(\theta_p)} [h(\theta_t) + \lambda] e^{[h(\theta_t) + s + \lambda]z} + b_2 \Rightarrow \\
n_t(z, \tau) = B e^{-[h(\theta_t) + s + \lambda]z} + \frac{h(\theta_t)F(z)}{h(\theta_t) + s + \lambda} - \frac{h(\theta_t)h(\theta_p)F(z)}{s + h(\theta_p)} [h(\theta_t) + \lambda] e^{-\tau}, z \leq R
\]

A unique solution for the dynamics of \( n_t(z, \tau) \) is obtained imposing the initial condition \( n_t(z, 0) = 0 \), solving for \( B \) and substituting the expression below into the previous equation

\[
B = \frac{h(\theta_t)h(\theta_p)F(z)}{s + h(\theta_p)} - \frac{h(\theta_t)F(z)}{h(\theta_t) + s + \lambda}, z \leq R
\]

The stock of temporary workers therefore grows from zero to

\[
\lim_{\tau \to -\infty} n_t(z, \tau) = \frac{h(\theta_t)F(z)}{h(\theta_t) + s + \lambda}, z \leq R
\]

The dynamic behaviour of \( u_t \) is not simply the reverse of \( n_t \). The stock of workers looking for a temporary job grows also because people with \( z \leq R \) eventually lose their permanent job at rate \( s \) and move to the temporary tier

\[
\dot{u}_t(z, \tau) = sn_p(z, \tau) + (s + \lambda)n_t(z, \tau) - h(\theta_t)u_t(z, \tau), z \leq R
\]

Again, using (15) with (18)

\[
\dot{u}_t(z, \tau) = sn_p(z, \tau) + (s + \lambda)[F(z) - u_t(z, \tau) - n_p(z, \tau)] - h(\theta_t)u_t(z, \tau) \Rightarrow \\
\dot{u}_t(z, \tau) + [h(\theta_t) + s + \lambda]u_t(z, \tau) = (s + \lambda)F(z) - \frac{\lambda h(\theta_p)F(z)}{s + h(\theta_p)} e^{-\tau} \Rightarrow \\
e^{[h(\theta_t) + s + \lambda]z} \left\{ \dot{u}_t(z, \tau) + [h(\theta_t) + s + \lambda]u_t(z, \tau) \right\} = e^{[h(\theta_t) + s + \lambda]z} \left[ (s + \lambda)F(z) - \frac{\lambda h(\theta_p)F(z)}{s + h(\theta_p)} e^{-\tau} \right] \Rightarrow \\
u_t(z, \tau)e^{[h(\theta_t) + s + \lambda]z} + b_0 = \int (s + \lambda)F(z)e^{[h(\theta_t) + s + \lambda]z}d\tau - \int \frac{\lambda h(\theta_p)F(z)}{s + h(\theta_p)} e^{[h(\theta_t) + s + \lambda]z}d\tau \Rightarrow \\
u_t(z, \tau)e^{[h(\theta_t) + s + \lambda]z} = b_0 + (s + \lambda)F(z) - \frac{\lambda h(\theta_p)F(z)}{s + h(\theta_p)} [h(\theta_t) + \lambda] e^{-\tau}, z \leq R
\]
Imposing the initial condition (16) and solving for \( B \) one gets a unique solution for the dynamics of \( u_t(z, \tau) \)

\[
B = F(z) \left\{ \frac{s}{s + h(\theta_p)} + \frac{\lambda h(\theta_p)}{[s + h(\theta_p)] h(\theta_t) + s + \lambda} \right\}, \quad z \leq R
\]

The stock of unemployed workers on the temporary market therefore goes from the initial level

\[
u_t(z, \tau = 0) = \frac{sF(z)}{s + h(\theta_p)}, \quad z \leq R
\]

to its steady state value

\[
\lim_{\tau \to \infty} u_t(z, \tau) = \frac{(s + \lambda)F(z)}{s + \lambda + h(\theta_t)}, \quad z \leq R
\]

Let us now turn to the stock of workers with \( z > R \). People with large outside utility never move from the permanent tier; in every moment in time they are either employed or unemployed with a permanent contract

\[1 - F(z) = u_p(z, \tau) + n_p(z, \tau), \quad z > R \tag{19}\]

The initial stock of unemployed workers searching for a permanent job is given by the proportion of unemployed workers in the old regime with \( z > R \)

\[
u_p(z, \tau = 0) = \frac{s[1 - F(z)]}{s + h(\theta_p)}, \quad z > R \tag{20}\]

Using (19) one gets the initial condition for \( n_p(z > R, \tau) \)

\[
n_p(z, \tau = 0) = \frac{h(\theta_p)[1 - F(z)]}{s + h(\theta_p)}, \quad z > R
\]

- The stock of unemployed with \( z > R \) increases when permanently employed workers with large outside option leave their jobs and decreases when they find a new one

\[
\dot{u}_p(z, \tau) = su_n_p(z, \tau) - h(\theta_p)u_p(z, \tau), \quad z > R
\]

using (19)

\[
e^{[s + h(\theta_p)]\tau} \{ \dot{u}_p(z, \tau) + [s + h(\theta_p)]u_p(z, \tau) \} = s[1 - F(z)]e^{[s + h(\theta_p)]\tau} \Rightarrow
\]

\[
u_p(z, \tau)e^{[s + h(\theta_p)]\tau} + b_o = s[1 - F(z)] \int e^{[s + h(\theta_p)]\tau} d\tau \Rightarrow
\]

\[
u_p(z, \tau) = Be^{-[s + h(\theta_p)]\tau} + \frac{s[1 - F(z)]}{s + h(\theta_p)}, \quad z > R
\]

As usual, a unique solution is obtained through the imposition of the initial condition in (20); solving by \( B \) one gets

\[
B = \frac{s[1 - F(z)]}{s + h(\theta_p)} - \frac{s[1 - F(z)]}{s + h(\theta_p)} = 0 \Rightarrow
\]

\[
u_p(z, \tau) = \frac{s[1 - F(z)]}{s + h(\theta_p)}, \quad z > R
\]

The stock of unemployed workers in the permanent market does not depend on time: its level is constant during the transition to the new steady state.
The dynamics of \( n_p(z > R, \tau) \) is its exact reverse

\[
\dot{n}_p(z, \tau) = h(\theta_p)u_p(z, \tau) - sn_p(z, \tau), \quad z > R
\]

Using (19)

\[
\dot{n}_p(z, \tau) = h(\theta_p)[1 - F(z) - n_p(z, \tau)] - sn_p(z, \tau) \Rightarrow \\
\dot{n}_p(z, \tau) + [h(\theta_p) + s]n_p(z, \tau) = h(\theta_p)[1 - F(z)] \Rightarrow \\
e^{[h(\theta_p) + s]\tau}\{\dot{n}_p(z, \tau) + [h(\theta_p) + s]n_p(z, \tau)\} = h(\theta_p)[1 - F(z)]e^{[h(\theta_p) + s]\tau} \Rightarrow \\
n_p(z, \tau)e^{[h(\theta_p) + s]\tau} + b_0 = h(\theta_p)[1 - F(z)] \int e^{[h(\theta_p) + s]\tau} d\tau \Rightarrow \\
n_p(z, \tau) = Be^{-[h(\theta_p) + s]\tau} + \frac{h(\theta_p)[1 - F(z)]}{h(\theta_p) + s}, \quad z > R
\]

The imposition of the initial condition for \( \tau = 0 \) yields the unique value of \( B \)

\[
\frac{h(\theta_p)[1 - F(z)]}{s + h(\theta_p)} = B + \frac{h(\theta_p)[1 - F(z)]}{h(\theta_p) + s} \Rightarrow B = 0 \Rightarrow n_p(z > R, \tau) = \frac{h(\theta_p)[1 - F(z)]}{h(\theta_p) + s}, \quad z > R
\]

So also the dynamic equation of \( n_p(z > R, \tau) \) does not depend on time; nonetheless we have to keep in mind that the full dynamics for \( n_p \) depends also on workers with \( z \leq R \).

We are now in a position to describe the whole dynamics of the system. \( n_t(z, \tau) \) and \( u_t(z, \tau) \) are fully determined by workers with \( z \leq R \), while \( u_p(z, \tau) \) by the ones with \( z > R \); \( n_p(z, \tau) \) depends on both

\[
n_t(z, \tau) = n_t(z \leq R, \tau) = \left\{ \frac{h(\theta_i)h(\theta_p)F(z)}{[s + h(\theta_p)][h(\theta_i) + \lambda]} - \frac{h(\theta_i)F(z)}{h(\theta_i) + s + \lambda} \right\} e^{[h(\theta_i) + s + \lambda]\tau} + \\
\left( \frac{h(\theta_i)F(z)}{h(\theta_i) + s + \lambda} - \frac{h(\theta_i)h(\theta_p)F(z)}{s + h(\theta_p)} \right) e^{-s\tau}
\]

\[
u_t(z, \tau) = u_t(z \leq R, \tau) = F(z) \left\{ \frac{s}{s + h(\theta_p)} + \frac{\lambda h(\theta_p)}{[s + h(\theta_p)][h(\theta_i) + \lambda]} - \frac{s + \lambda}{h(\theta_i) + s + \lambda} \right\} e^{[h(\theta_i) + s + \lambda]\tau} + \\
\left( \frac{s + \lambda}{h(\theta_i) + s + \lambda} - \frac{\lambda h(\theta_p)}{s + h(\theta_p)} \right) e^{-s\tau}
\]

\[
u_p(z, \tau) = u_p(z > R, \tau) = \frac{s[1 - F(z)]}{s + h(\theta_p)}
\]

\[
n_p(z, \tau) = n_p(z \leq R, \tau) + n_p(z > R, \tau) = \frac{h(\theta_p)F(z)}{s + h(\theta_p)} e^{-s\tau} + \frac{h(\theta_p)[1 - F(z)]}{h(\theta_p) + s}
\]

Taking \( \lim_{\tau \to \infty} \) and using \( z = R \) one gets the expressions for the two tiers steady state.

### C The "honeymoon effect"

In order to prove the existence of what we called the "honeymoon effect" of the introduction of temporary jobs we take the time derivative of the equation describing the dynamics of total unemployment and evaluate it at \( \tau = 0 \); more
precisely, since permanent unemployment does not display any dynamics (see the subsection above), we will focus on the behaviour of temporary unemployment. If the liberalisation of temporary contracts leads to an immediate reduction of total unemployment, the time derivative of temporary unemployment evaluated at \( \tau = 0 \) must be negative. From section 9.2 we know that

\[
\frac{\partial u_t(\tau)}{\partial \tau} = (s + \lambda + h(\theta_t))F(z) - s[s + \lambda + h(\theta_t)]F(z) + \frac{\lambda h(\theta_p)F(z)[s + \lambda + h(\theta_t)]}{s + h(\theta_p)} + s\lambda h(\theta_p)F(z)
\]

imposing \( \tau = 0 \)

\[
\frac{\partial u_t(\tau)}{\partial \tau}|_{\tau = 0} = (s + \lambda + h(\theta_t))F(z) - s[s + \lambda + h(\theta_t)]F(z) + \frac{\lambda h(\theta_p)F(z)[s + \lambda + h(\theta_t)]}{s + h(\theta_p)} + s\lambda h(\theta_p)F(z)
\]

Omitting the common denominator, which is not relevant for the sign of the expression above, one gets

\[
[s + h(\theta_p)][\lambda + h(\theta_t)](s + \lambda)F(z) - s[\lambda + h(\theta_t)][s + \lambda + h(\theta_t)]F(z) + \lambda h(\theta_p)F(z)[s + \lambda + h(\theta_t)] + s\lambda h(\theta_p)F(z) =
\]

\[
= h(\theta_p)[\lambda + h(\theta_t)](s + \lambda)F(z) - s[\lambda + h(\theta_t)]h(\theta_t)F(z) + \lambda h(\theta_p)F(z)[s + \lambda + h(\theta_t)] + s\lambda h(\theta_p)F(z) =
\]

\[
= [\lambda + h(\theta_t)]F(z)h(\theta_p)(s + \lambda) - sh(\theta_t) - \lambda h(\theta_p) =
\]

\[
= [\lambda + h(\theta_t)]F(z)\{s[h(\theta_p) - h(\theta_t)]\} < 0
\]

**D Search on the job**

The proof of the existence of the equilibrium in the model with on the job search follows the lines of section 9.1: we need to find the conditions for the existence of a positive reservation outside utility that is strictly lower than the wage. Once \( \theta_t \) and \( \theta_p \) are determined by sequentially solving the job creation conditions system (see section 6), both \( U_t \) and \( U_p \) are linear functions of \( z \); a positive \( R \) therefore exists when the intercept of \( U_t \) is larger than the intercept of \( U_p \) and its slope is smaller\(^{17}\). We will then prove that under the same conditions not only \( R \) is positive, but is also strictly lower than \( w \).

\(^{17}\)In principle, the existence of a positive \( R \) would be shown also under the opposite conditions, i.e. a higher intercept and a smaller slope for \( U_p \); however, as a few steps of algebra will make clear, the slope of \( U_p \) is always larger than the one of \( U_t \).
The value functions for the supply side of the permanent submarket look as in section 2.1

\[ rE_p(z) = w + s[U_p(z) - E_p(z)] \]
\[ rU_p(z) = z + b + h(\theta_p) [E_p(z) - U_p(z)] \]

so that the value of unemployment for a permanent worker reads

\[ U_p(z) = \frac{(z + b)(r + s) + h(\theta_p)w}{r[r + s + h(\theta_p)]} \]

In the temporary submarket the asset equations are a bit more complicated, since workers leave their temporary jobs not only because of natural turnover, but also when a permanent vacancy becomes available

\[ rE_t(z) = w + h(\theta_p) [E_p(z) - E_t(z)] + (s + \lambda)[U_t(z) - E_t(z)] \]
\[ rU_t(z) = z + h(\theta_t) [E_t(z) - U_t(z)] + h(\theta_p) [E_p(z) - U_t(z)] \]

Using \( E_t(z), E_p(z) \) and \( U_p(z) \) one gets the expression for \( U_t(z) \)

\[ U_t(z) = \frac{(r + s + \lambda + h(\theta_p))z}{r[r + h(\theta_p)][r + s + \lambda + h(\theta_t) + h(\theta_p)]} + \frac{(r + s)h(\theta_t) + h(\theta_t)h(\theta_p) + h(\theta_p)[r + s + \lambda + h(\theta_p)]}{(r + s)[r + h(\theta_p)][r + s + \lambda + h(\theta_t) + h(\theta_p)]} \]
\[ + \frac{h(\theta_p)[(z + b)(r + s) + h(\theta_p)w]}{r[r + s][r + h(\theta_p)][r + s + h(\theta_p)]} \]

We are now ready to go through the steps of the proof.

- **Condition on the slopes:** \( \partial U_p/\partial z > \partial U_t/\partial z \)

\[ \frac{(r + s)}{r[r + s + h(\theta_p)]} > \frac{r + s + \lambda + h(\theta_p)}{r[r + h(\theta_p)][r + s + \lambda + h(\theta_t) + h(\theta_p)]} + \frac{sh(\theta_p)}{r[r + h(\theta_p)][r + s + h(\theta_p)]} \]

Using and omitting the common denominator (which is not relevant for the sign) one gets

\[ (r + s)[r + h(\theta_p)][r + s + \lambda + h(\theta_t) + h(\theta_p)] - r[r + s + \lambda + h(\theta_p)][r + s + h(\theta_p)] + \]
\[ - sh(\theta_p)[r + s + \lambda + h(\theta_t) + h(\theta_p)] > 0 \Rightarrow \]

\[ r^2h(\theta_t) + rh(\theta_t)h(\theta_p) + rsh(\theta_t) > 0 \text{ always} \]

- **Condition on the intercepts:** \( U_p(0) < U_t(0) \)

\[ \frac{b(r + s) + h(\theta_p)w}{r[r + s + h(\theta_p)]} < \frac{(r + s)h(\theta_t) + h(\theta_t)h(\theta_p) + h(\theta_p)[r + s + \lambda + h(\theta_p)]}{(r + s)[r + h(\theta_p)][r + s + \lambda + h(\theta_t) + h(\theta_p)]} \]
\[ + \frac{h(\theta_p)[(z + b)(r + s) + h(\theta_p)w]}{r[r + s][r + h(\theta_p)][r + s + h(\theta_p)]} \]
Multiplying both sides by the common denominator the expression reads

\[(r + s)[r + h(\theta_p)][r + s + \lambda + h(\theta_p) + h(\theta_t)][b(r + s) + h(\theta_p)w] + \]

\[\quad - r[r + s + h(\theta_p)](r + s + \lambda + h(\theta_p) + h(\theta_t)][b(r + s) + h(\theta_p)w] + \]

\[\quad - [r + s + \lambda + h(\theta_p) + h(\theta_t)][h(\theta_p)s(b(r + s) + h(\theta_p)sh(\theta_p)w) < 0; \]

\[\quad [r^2 + rh(\theta_p) + rs][r + s + \lambda + h(\theta_p) + h(\theta_t)][b(r + s) + \]

\[\quad + (r + s)[r + h(\theta_p)][r + s + \lambda + h(\theta_p) + h(\theta_t)]w + \]

\[\quad - rw[r + s + h(\theta_p)](r + s)h(\theta_t) - rw[r + s + h(\theta_p)]h(\theta_t)h(\theta_p) + \]

\[\quad - rw[r + s + h(\theta_p)h(\theta_t)]r + s + \lambda + h(\theta_p)] < 0; \]

\[\quad [r^2 + rh(\theta_p) + rs][r + s + \lambda + h(\theta_p) + h(\theta_t)][b(r + s) + \]

\[\quad - w[r^3h(\theta_t) + 2r^2sh(\theta_t) + rs^2h(\theta_t) + rsh(\theta_p)h(\theta_t) + r^2h(\theta_p)h(\theta_t)] < 0; \]

\[\quad [r^2 + rh(\theta_p) + rs][r + s + \lambda + h(\theta_p) + h(\theta_t)][b(r + s) - \]

\[\quad - rh(\theta_t)[(r + s)^2 + h(\theta_p)(r + s)] < 0; \]

\[\quad [r + s + \lambda + h(\theta_p) + h(\theta_t)][b] < wh(\theta_t) \Rightarrow \]

\[\quad b < \frac{wh(\theta_t)}{[r + s + \lambda + h(\theta_p) + h(\theta_t)]} \quad \text{(21)} \]

that is the condition for the existence of a positive reservation outside option.

By equating \( U_p(z) \) to \( U_t(z) \) and solving for \( z = R \), we are now in a position to determine its exact value:

\[
\frac{(R + b)(r + s) + h(\theta_p)w}{r[r + s + h(\theta_p)]} = \frac{[r + s + \lambda + h(\theta_p)]R}{[r + h(\theta_p)][r + s + \lambda + h(\theta_t) + h(\theta_p)]} + \]

\[
\frac{(r + s)[r + h(\theta_p)][r + s + \lambda + h(\theta_p) + h(\theta_t)]}{(r + s)[r + h(\theta_p)](r + s + \lambda + h(\theta_t) + h(\theta_p))]w} + \frac{h(\theta_p)s(R + b)(r + s + h(\theta_p)w)}{r(r + s)[r + h(\theta_p)][r + s + h(\theta_p)]};
\]

Collecting terms with \( R \) and multiplying both sides by the common denominator one gets

\[
(r + s)\left\{ (r + s)[r + h(\theta_p)][r + s + \lambda + h(\theta_p) + h(\theta_t)] - r[r + s + h(\theta_p)][r + s + \lambda + h(\theta_p)] + \right\} R = \]

\[
= w[r + s + h(\theta_p)] \left\{ \{r + s)h(\theta_t) + h(\theta_t) + h(\theta_p)[r + s + \lambda + h(\theta_p)] \right\} + \]

\[
+ [r + s + \lambda + h(\theta_p) + h(\theta_t)][h(\theta_p)sb(r + s) + sh(\theta_p)w] + \]

\[
- [b(r + s) + h(\theta_p)w][r + s + h(\theta_p)][r + s + \lambda + h(\theta_p) + h(\theta_t)]; \]
For simplicity we separately consider the two sides of the equation; starting from the rhs

\[ w \left\{ r[r + s + h(\theta_p)](r + s)h(\theta_t) + r[r + s + h(\theta_p)]h(\theta_p)h(\theta_t) + r[r + s + h(\theta_p)]h(\theta_p)[r + s + \lambda + h(\theta_p)] + \
- h(\theta_p)[r + s + \lambda + h(\theta_t) + h(\theta_p)]\right\} + \
- b(r + s)[r + s + \lambda + h(\theta_p) + h(\theta_t)][r^2 + rs + rh(\theta_p)] = \

w[r^2 + rs + rh(\theta_p)](r + s)h(\theta_t) - b(r + s)[r^2 + rs + rh(\theta_p)][r + s + \lambda + h(\theta_p) + h(\theta_t)] = 

= [r^2 + rs + rh(\theta_p)](r + s) \{ h(\theta_t)w - b[r + s + \lambda + h(\theta_p) + h(\theta_t)] \}

The lhs in turn reads

\[(r + s)R \left\{ [r^2 + rs + rh(\theta_p)](r + s + \lambda + h(\theta_p) + h(\theta_t)) - [r^2 + rs + rh(\theta_p)][r + s + \lambda + h(\theta_p)] \right\} = 

= h(\theta_t)(r + s)[r^2 + rs + rh(\theta_p)]

so that

\[ Rh(\theta_t) = h(\theta_t)w - b[r + s + \lambda + h(\theta_p) + h(\theta_t)] \Rightarrow \]

\[ R = w - b\frac{r + s + \lambda + h(\theta_p) + h(\theta_t)}{h(\theta_t)} \]

which implies that \( R < w \); moreover, under condition (21), \( 0 < R < w \).
References


