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A Model of the Italian Cut-off System for Taxing Small Businesses

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Abstract

The Studi di Settore are used by the Italian tax administration to calculate reference revenue levels for small businesses and provide a kind of cut-off level for tax audits. Recently new rules have been introduced in order to render the Studi di Settore more efficient in producing realistic estimates, with the aim of reducing the “legalized evasion” that might arise in case of a systematic downward bias. Voices of the involved categories, however, convinced the Government to partially step back. Building upon the standard firm’s tax evasion model of Cowell (2004) and the approach of Santoro (2006) we show that, under given conditions, a stringency increase might backfire implying a larger overall tax evasion and a smaller tax revenue.

Keywords: tax evasion by firms, cut-off rule.

JEL classification: H25, H26, K42.

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1 Introduction

The Studi di Settore are a quite peculiar reference system used by the Italian Tax Administration in its relationships with small and medium size firms and with independent workers since about ten years. The closest example of a similar approach outside Italy is the Tachshiv in Israel, which, while officially ended in 1975, according to Yitzhaki (2007) continued to be unofficially used afterwards. The more broad category to which these systems might be traced back is that of presumptive taxation, in which indirect and approximated methods are used to quantify the tax base. Equity and efficiency problems raised by presumptive taxation have been studied1 both with reference to developing countries (where rough income categories are often formed in absence of reliable detailed information) and to developed ones (where the resort to presumptions often is not officially declared but arises in practise as a consequence of approximated definitions of the tax base, or of the resort to “reasonable” reference points made by tax auditors).

In Italy, through a software provided by the tax administration, each taxpayer in the groups covered by such a system calculates her estimated gross revenue according to the Studio di Settore pertaining to her field of activity.2 The estimate is based on the data imputed by the taxpayer describing the physical and economic characteristics of her activity, such as the number of employees, the dimensions of the premises, etc. Moreover, the software also calculates indexes that signal possible incoherence or irregularity in the data imputed by the taxpayer. The estimated revenue represents a benchmark: those who report less revenue in filling their income tax form3 have a larger probability of being audited. While also those who comply with their Studio di Settore still have a positive probability of being selected for some type of audit according to the law, the general perception is that their situation is free of risk; hence, for simplicity, in this paper it will thus be assumed that those who report the benchmark revenue are in a sure position.

The Studi di Settore are realized by using standard statistical techniques that single out clusters of taxpayers having similar characteristics, relying on data from past revenue reports and from specific surveys. The Studi di Settore are validated by commissions in which members of the representative organizations of the taxpayers involved participate. Due to their complex process of elaboration and application, the Studi di Settore can exert many roles:

- Providing some reference revenue, agreed upon by the government and the representative organizations of the taxpayers involved, in a context of social negotiations and agreements. The aim is that of relying on the support of the groups involved, i.e., presumably, on the interests of their representative member. As long as the latter is at least a partial complier, she is likely to prefer that her competitors do not benefit from a too huge tax evasion. This approach should thus be apt at avoiding forms of tax evasion epidemics, a phenomenon that in Italy might outburst because of the very large number of small businesses, which can be audited with a reasonable probability only at a large cost. Moreover, if the Italian economy would hopefully evolve toward stronger competition and increasing firm’s dimensions, the system might also progressively support a larger compliance.

- Providing a benchmark to the tax administration for programming tax audits, in order to increase their effectiveness. This could result both from the selection of potentially more productive targets (those who do not conform to the Studi di Settore), and from the possibility of using the estimates in order to reinforce the evidence of evasion in tax lawsuits, thus increasing the probability of sanctioning evasion.

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1 See Balestrino and Galmarini (2005) and Yitzhaki (2007) and the references quoted therein.
2 There were 206 Studi di Settore fully working in June 2007, while further ones are in preparation.
3 The tax report must include also the raw data relevant for the estimation of the benchmark.
Offering to the taxpayers a kind of settlement, based on a detailed estimation of the taxpayer revenue and hence willingness to pay in order to avoid audits. In this context, the Studi di Settore introduce a kind of cut-off rule (see, e.g., Reinganum and Wilde, 1985). The tax administration renounces to audit those who report at least the benchmark revenue in order to save audit costs and to extract from taxpayers who comply some extra revenue that absorbs what would otherwise have been wasted in concealing the taxable income.

How well the Studi di Settore have actually served the aforementioned goals is a very debated question that will not be addressed here. The interest of the Italian public opinion in the Studi di Settore peaked in Summer 2007, when taxpayers had to prepare their tax reports on the basis of some new rules introduced by the 2007 state budget law. The groups involved voiced because the benchmark revenues were increased in many instances and the new rules had not been negotiated with the representative organizations of the taxpayers. The protest led to a partial freezing of the new rules.

This paper aims at clarifying, from a theoretical point of view, what are the economic consequences of manoeuvring the benchmark of the Studi di Settore. The focus is on the individual reaction of the taxpayer to a variation of the benchmark decided upon by the government, a case that portraits what happened in Italy in 2007. The study of the problems of political economy stemming from the negotiations between the government and the representatives of the social groups involved, which would capture some more long term features of the system, is left for future research.

The available literature points out that in general any cut-off rule involves relevant problems, mainly in terms of equity. Horizontal equity is violated as long as taxpayers with the same income but different income indicators receive proposals for “settlements” of different amount. Moreover, inside each group there is a vertical equity problem, since those who have an income larger than the benchmark, but report according to the benchmark, are not audited. Hence above the benchmark taxes are not increasing with income.

These and other critical aspects have been analyzed with reference to the Italian experience by Santoro (2006), who points out that the Studi di Settore have through time become ‘inefficient’: they systematically understate the true firm’s revenue and introduce a form of legalized tax evasion, which has also widened progressively thanks to the tricks put forth by taxpayers in order to exploit the loopholes of the system. Increasing the stringency of the Studi di Settore thus appears to be a way for reducing the legalized evasion and for reinforcing the threat of controls, with beneficial results both in terms of equity and of tax revenue. In the following, however, we show that, on the ground of tax revenue and of the efficiency of the tax system as a whole, this is not necessarily the case.

The paper is organized as follows. Section 2 presents the taxpayer problem in a system where the Studi di Settore are applied by building on standard firm’s tax evasion models and on Santoro (2006). After describing the taxpayer optimal choice in Section 3, some comparative statics follows in Section 4. Section 5 presents our main result on the effects on government revenue of manoeuvring the ‘stringency’ of the Studi di Settore. Finally, Section 6 contains some concluding remarks.

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4For a discussion see Russo (2007). Among the many problems involved, let us recall the fact that, as only gross revenue is considered by the Studi di Settore while taxes depend on the net one, there has been scope for cheating in reporting costs, and thus for unwanted effects on the tax revenue.

5For a Survey, see Marchese (2004).

6For a case in which the actual tax system becomes more progressive than the legal one under a cut-off rule see Scotchmer (1987).
2 The basic problem

Following Cowell (2004) and Santoro (2006) let us consider a representative firm that aims at maximizing the expected profit:

\[ \pi = \{ q [1 - T(\varphi)] - m \} x, \]  

(1)

where \( q \) is the product price, \( x \) is the quantity produced, \( m \) is the constant marginal production cost, \( \varphi \) is the share of revenue concealed, with \( 0 \leq \varphi \leq 1 \), and \( T(\varphi) \) is the total outlay per unit of revenue, which the firm must disburse in order to cope with the tax system. As in Cowell (2004), we do not specify the market type, so that either \( q \) is a parameter (as the market is competitive) or it is given by the inverse demand \( q(x) \) if the firm enjoys some market power.

By considering (1) as the objective of the taxpayer, we assume that the optimal choice for concealment \( \varphi \) does not depend on the price \( q \), on the cost \( m \) and on the quantity produced either. This allows us to tackle directly the ‘minimum outlay’ problem faced by the taxpayer.

The Studio di Settore provides a ‘benchmark’ estimated revenue for each taxpayer, given by:

\[ eqx \leq qx. \]

Parameter \( e \leq 1 \) describes the ‘stringency’ rate adopted by the tax administration in quantifying the taxpayer’s revenue through the Studi di Settore: a larger \( e \) implying a larger willingness to squeeze money out of the taxpayer. In principle, also an excessive quantification of the revenue, i.e., \( e > 1 \) might arise. In this paper, however, we rule out this case since the Italian Studi di Settore typically involve an underestimation.\(^7\) The extension to the case \( e > 1 \) is straightforward and would only make exposition more cumbersome without adding meaningful information to our analysis. Hints about this case will be given later on.

It is assumed, for the sake of simplicity, that the taxpayer has no control over the \( e \) value, which is chosen by the tax Administration. Parameter \( e \) can be set according to the results of a purely technical process aimed at estimating the taxpayer’s actual revenue. The tax administration can exploit the past income reports of all the taxpayers and use statistical techniques and checks about the inconsistencies in reports in order to control for possible manipulations. The tax administration can, however, also set an \( e \) value smaller or larger than the one that results from this analysis, whenever the latter choice is optimal in view of maximizing the government revenue or in relationship with other goals.

Under a proportional tax system, if the taxpayer decides to comply with the Studi di Settore, the unit outlay is \((1 - \varphi) \tau\), with \( 0 \leq \varphi \leq 1 - e \), where \( 0 < \tau < 1 \) is the official tax rate. Let us now consider the possibility of reporting revenue according to the general rules. In this case the firm must consider that reports might be audited.

Following Santoro (2006), the audit probability is assumed to be linear in \( \varphi \); specifically:

\[ p(\varphi) = a [\varphi - (1 - e)], \]  

(2)

where \( 0 < a \leq 1/e \). Parameter \( a \) describes the “baseline” probability. The tax administration could manoeuvre it, e.g., by pouring more or less resources into the auditing activity. The term in square parentheses aims instead at describing the role of the Studi di Settore. The idea is that the tax administration, relying on this methodology and on the income report, receives signals about the tax evasion share \( \varphi \). These signals, however, are more blurred the larger is \((1 - e)\).

\(^7\)This fact contributes to explain why in Italy the average reported income of entrepreneurs is lower than that of dependent workers: 13,800 euros versus 18,000 in 2004 (see the IRPEF data on the Italian Ministry of Economics and Finance website).
To let (2) be meaningful, in the following we shall use it for values \(1 - e < \varphi \leq 1\). Hence, the expected tax rate per unit of revenue is given by:

\[
\mathbb{E}t (\varphi) = [1 - \varphi + (1 + s) p (\varphi)] \tau = \{1 - \varphi + a (1 + s) [\varphi - (1 - e)]\} \tau,
\]

(3)

where \(s > 1\) is the penalty rate. Evading some fraction \(\varphi\) of revenue has a concealment cost,\(^8\) which we shall denote by \(g(\varphi)\), with \(g(0) = 0\), \(g'(\varphi) > 0\) and \(g''(\varphi) \geq 0\).

Taking into consideration this possibility as well, we are able to define a quite general objective for the taxpayer – the total outlay \(T(\varphi)\) – by letting

\[
T(\varphi) = \begin{cases} 
(1 - \varphi) \tau & \text{if } 0 \leq \varphi \leq 1 - e \\
\mathbb{E}t(\varphi) + g(\varphi) & \text{if } 1 - e < \varphi \leq 1.
\end{cases}
\]

(4)

The firm thus minimize \(T(\varphi)\) subject to the constraint \(0 \leq \varphi \leq 1\); formally:

\[
\min_{\varphi \in [0, 1]} T(\varphi)
\]

(5)

By construction, \(T(\varphi)\) is discontinuous in \(\varphi = 1 - e\), thus, even if the constraint is compact, we must be careful in guaranteeing existence of a solution.\(^9\)

**Lemma 1** The function \(T(\varphi)\) defined in (4) is lower semicontinuous. Therefore, being the constraint \([0, 1]\) compact, Weierstrass Theorem applies and a solution of (5) always exists.

A rigorous proof of Lemma 1 is reported in the Appendix. Roughly speaking, even if \(T(\varphi)\) is discontinuous, i.e., its graph is made up of two separate curves (the first describing outlays under compliance with the Studi di Settore, the second under larger evasion), nevertheless a solution exists because the first curve is strictly decreasing \([T''(\varphi) = -\tau < 0\) for \(0 \leq \varphi \leq 1 - e]\) and its right extrema, \(T(1 - e)\), cannot lie above the beginning of the second curve.

**Remark 1** Since \(g(0) = 0\) and \(g\) is strictly increasing, \(\lim_{\varphi \to (1 - e)^+} T(\varphi) = \lim_{\varphi \to (1 - e)^-} [\mathbb{E}t(\varphi) + g(\varphi)] > \tau e = \lim_{\varphi \to (1 - e)^-} T(\varphi)\) always holds with strict inequality whenever \(e < 1\). In other words, as \(\varphi\) crosses the discontinuity point \(1 - e\) the graph of \(T(\varphi)\) jumps upward from the first curve to the second one; such ‘jump’ is strictly positive because the costs of concealment are: \(g(1 - e) > 0\) for all \(e < 1\).

### 3 The optimal outlay

First we need assumptions assuring interiority of any solution \(\varphi^*\) of (5). An optimal solution \(\varphi^*\) must satisfy \(\varphi^* \geq 1 - e > 0\), since it would be meaningless to evade less than the “legalized evasion” under the Studi di Settore, which amounts to \(1 - e\). Indeed, we have seen that \(T(\varphi)\) is strictly decreasing for \(0 \leq \varphi \leq 1 - e\) whenever \(e < 1\).

On the other hand, in order to exclude the possibility of full evasion, let us assume that \(T''(1) = (\mathbb{E}t)'(1) + g'(1) > 0\) holds whenever \(e < 1\); since

\[
(\mathbb{E}t)'(\varphi) + g'(\varphi) = -\{1 - a (1 + s) [2\varphi - (1 - e)]\} \tau + g'(\varphi),
\]

(6)

\(^8\)The concealment cost might be due to the necessity of double accounting, to the loss of control on agents that cooperate in hiding income, etc. If the firm does not bear these costs, its evasion is fully evident and punished with certainty. Note also that the firm not complying with the Studi di Settore must hide \(\varphi\) and not \(\varphi - (1 - e)\), since in case of audit the whole evasion is found out.

\(^9\)Note that, under the standard assumption that no prize is given to audited taxpayers who overreported, if \(e > 1\) the taxpayer problem reduces to the second line of (4) with \(0 \leq \varphi \leq 1\), and no discontinuity occurs.
\((Et)'(1) + g'(1) = -[1-a(1+s)(1+e)]\tau + g'(1),\) and thus the condition we are looking for is \(g'(1) > [1-a(1+s)(1+e)]\tau.\) As we want to consider several values of stringency parameter \(e,\) we shall assume the following slightly stronger condition which is independent of \(e.\)

**A. 1** \(g'(1) > [1-a(1+s)]\tau.\)

Under A.1 any solution \(\varphi^*\) of (5) is such that \(0 < \varphi^* < 1.\)

Being \(T''(\varphi) = (Et)'(\varphi) + g''(\varphi) = 2a(1+s)\tau + g''(\varphi) > 0\) for \(\varphi > 1-e,\) \(T(\varphi)\) is strictly convex over \((1-e, 1],\) and hence there can be at most one (interior) relative minimum \(1-e < \varphi^*_r < 1,\) which must satisfy the F.O.C.\(^{10}\)

\[
g' (\varphi^*_r) = \{1 - a (1+s)[2\varphi^*_s - (1-e)]\} \tau, \tag{7}
\]

where the LHS represents the marginal cost and the RHS the expected marginal benefit or the expected rate of return of tax evasion. On the other hand, \(\varphi^*_r = 1 - e\) is always a relative minimum\(^{11}\). We conclude that, when \(e < 1,\) there can be at most two relative minima: \(\varphi^*_e = 1 - e\) and \(\varphi^*_r > \varphi^*_e\) respectively, where \(\varphi^*_e\) is a stationary point satisfying (7). Clearly, one of them is the solution of (5).

In words, either the best choice of the firm is to comply with the Studio di Settore by evading \(\varphi^*_e = 1 - e,\) or the best choice is opting for a larger evasion \(\varphi^*_r.\) There may be, however, also some circumstances in which the two choices are indifferent; this happens when a value \(\hat{\varphi}\) of the stringency parameter \(e\) exists such that \(E_t(\varphi^*_r) + g(\varphi^*_r) = \tau \hat{\varphi}.\) In such cases the two distinct relative minima, \(\varphi^*_e = 1 - \hat{\varphi}\) and \(\varphi^*_r,\) both become absolute minima and solve (5). Hence, our framework cannot rule out multiple solutions for problem (5). Actually, as we shall see in the following section, such multiplicity of solutions will provide the basis for our main result.

Figure 1 portrays the three possible scenarios for problem (5). In figure 1(a) the solution is \(\varphi^*_r = 1 - e,\) as the second curve of the graph of \(T(\varphi),\) \(E_t(\varphi) + g(\varphi),\) lies above \(T(1-e) = \tau e\) for all \(1-e < \varphi \leq 1;\) this is the case in which the taxpayer complies with the Studio di Settore. Viceversa, in figure 1(b) the solution is the stationary point \(1-e < \varphi^*_r < 1,\) as \(E_t(\varphi^*_r) + g(\varphi^*_r) < \tau e;\) here the taxpayer chooses to evade a larger amount \(\varphi^*_r > 1 - e.\) Finally, figure 1(c) plots the ‘multiple solution’ case where the absolute minimum in (5) is reached on both \(\varphi^*_e = 1 - \hat{\varphi}\) and \(\varphi^*_r,\) and its value is \(\tau \hat{\varphi} = E_t(\varphi^*_r) + g(\varphi^*_r);\) this last scenario represents indifference between compliance with the Studio di Settore and a larger evasion.

Problem (5) is thus equivalent to

\[
V(e) = \min \{\tau e, E_t[\varphi^*_e(e)] + g[\varphi^*_e(e)]\}, \tag{8}
\]

where \(\varphi^*_e(e)\) is the unique solution of (7) when the stringency parameter is \(0 < e < 1\).\(^{12}\) This formulation emphasizes the choice of the taxpayer between compliance with the Studi di Settore, \(\tau e,\) and tax evasion, \(E_t[\varphi^*_e(e)] + g[\varphi^*_e(e)].\) In (8) \(V(e)\) denotes the value as a function of stringency parameter \(e;\) its study will be the subject of next section.

**Remark 2** In this model, under the assumption that \(e \leq 1,\) tax evasion cannot be eradicated by means of parameters controlled by the tax administration. To see this, note that, as \(e \rightarrow 1,\) by construction

\(^{10}\)We denote such relative minimum – provided it exists – by \(\varphi^*_r,\) where the subscript ‘r’ stands for right relative minimum, as opposed to the left relative minimum, \(\varphi^*_l = 1 - e,\) which will be discussed shortly after.

\(^{11}\)To see this, recall that \(T (\varphi)\) is strictly decreasing over \([0, 1-e]\) and \(\lim_{\varphi \rightarrow (1-e)^+} [E_t(\varphi) + g(\varphi)] > \tau e = T(1-e)\) whenever \(e < 1\) (see Remark 1).

\(^{12}\)Note that in general \(\varphi^*_r(e)\) may not exist; such case occurs if \(E_t(\varphi) + g(\varphi)\) is strictly increasing for \(1-e < \varphi \leq 1.\) By Lemma 1, however, Problem (8) has a solution also in this peculiar case; since \(E_t(\varphi) + g(\varphi) > \tau e\) for all \(1-e < \varphi \leq 1,\) \(\varphi^*_r = 1 - e\) is the unique solution. Figure 1(a) shows an example of this type.
in (4) $\mathbb{E}t(\varphi) + g(\varphi)$ becomes relevant for all $0 \leq \varphi \leq 1$. In this case either (7) is satisfied for some $\varphi^*_r(1) > 0$, thus allowing for a positive evasion even under the most stringency, or (7) does not hold for all $0 < \varphi < 1$ [being $\mathbb{E}t(\varphi) + g(\varphi)$ everywhere strictly increasing thanks to assumption A.1] and lower values of $e$ would not induce a positive tax evasion. In the latter case tax evasion would be hampered by its own concealment cost and not by government action. This feature of the model, however, does not represent a problem since we aim at focussing on tax evasion. By assuming $(\mathbb{E}t)'(0) + g'(0) < 0$, or, equivalently, $g'(0) < \tau$, we thus refer to cases in which the government manoeuvre of the tax system parameters $a$, $e$, $s$ and $\tau$ cannot let $\varphi \to 0$, i.e., cannot eradicate tax evasion.

**Figure 1:** three possible scenarios for problem (5).

### 4 Comparative statics: the threshold $\hat{e}$

In this section we aim at investigating how solutions of (8) are affected by different values of the stringency parameter $e$; that is, our goal is to understand how the choices of the tax administration with reference to the stringency of the *Studi di Settore* affect preferences either for compliance with the benchmark or for evasion $\varphi > 1 - e$. The following lemma, whose proof is reported in the Appendix, establishes an important monotonicity property of the optimal solution of (8), paving the way for our main result in next section.

**Lemma 2** Assume that A.1 holds and some value $0 < e < 1$ exists such that $\varphi^*_r(e)$ satisfies (7) and $\mathbb{E}t[\varphi^*_r(e)] + g[\varphi^*_r(e)] \leq \tau e$. Then there is a unique $0 < \hat{e} < 1$ such that $\mathbb{E}t[\varphi^*_r(\hat{e})] + g[\varphi^*_r(\hat{e})] = \tau \hat{e}$;
moreover, \( \mathbb{E} t (\varphi) + g (\varphi) > \tau e \) for all \( \varphi > 1 - e \) and for all \( 0 \leq e < \hat{e} \), while \( \mathbb{E} t [\varphi^*_r (e)] + g [\varphi^*_r (e)] < \tau e \) for all \( \hat{e} < e \leq 1 \).

Lemma 2 says that the firm will be indifferent between complying with the Studio di Settore or evading an amount \( \varphi^*_r (e) \) larger than \( 1 - e \) only in one case: when the tax administration sets the stringency value at the threshold \( \hat{e} \). In this case the firm exactly balances the advantage of avoiding concealment costs with the disadvantage of paying a larger tax under the Studio di Settore, so that its total outlay stays the same. Hence, the unique threshold value \( \hat{e} \) for the administration’s stringency characterizes the situation of indifference between compliance with the Studio di Settore and tax evasion from the taxpayer point of view. It corresponds to the (unique) case in which the objective function \( T (\varphi) \) in (5) reaches its absolute minimum in two distinct points: \( \varphi^*_t = 1 - \hat{e} \) and \( \varphi^*_r (\hat{e}) \), with \( \varphi^*_t < \varphi^*_r (\hat{e}) \).

5 Government revenue

By assuming that the taxpayer chooses the Studio di Settore when the stringency rate is \( \hat{e} \), i.e., when she is indifferent between compliance and evasion, then for all \( 0 < e \leq \hat{e} \) the solution of (8) is \( \varphi^*_t = 1 - e \) and the unit government revenue is \( V (e) = \tau e \). Conversely, if \( e > \hat{e} \), \( V (e) = \mathbb{E} t [\varphi^*_r (e)] + g [\varphi^*_r (e)] < \tau e \) and the taxpayer chooses to evade; in this scenario the unit government revenue is \( \mathbb{E} t [\varphi^*_r (e)] < \mathbb{E} t [\varphi^*_t (e)] + g [\varphi^*_t (e)] < \tau e \). In other words, a slight increase of the stringency parameter above the threshold \( \hat{e} \), by letting the taxpayer switch from compliance to the Studio di Settore to tax evasion causes a drastic fall in terms of government revenues. Specifically, our main result states that there is a nontrivial open interval such that \( \mathbb{E} t [\varphi^*_r (e)] < \tau e \) for all \( e \) belonging to such interval.

Let \( e_s \) be the smallest \( e \), if exists, such that \( \mathbb{E} t [\varphi^*_r (e_s)] = \tau \hat{e} \), and let \( \bar{e} = \min \{ e_s, 1 \} \).

Proposition 1 Under the assumptions of Lemma 2 the unit government revenue is smaller than \( \tau \hat{e} = V (\hat{e}) \) for all \( \hat{e} < e < \bar{e} \). The unit revenue loss tends to \( g [\varphi^*_r (\hat{e})] \) as \( e \to \hat{e}^+ \).

Proof. At the threshold \( \hat{e} \), \( \tau \hat{e} = \mathbb{E} t [\varphi^*_r (\hat{e})] + g [\varphi^*_r (\hat{e})] > \mathbb{E} t [\varphi^*_t (\hat{e})] \). Since \( \mathbb{E} t [\varphi^*_r (e)] \) is a continuous function of \( e \), \( \mathbb{E} t [\varphi^*_t (e)] < \tau \hat{e} \) for all \( \hat{e} < e < \bar{e} \), and the first part of the proposition is established. As \( \lim_{e \to \hat{e}^-} \mathbb{E} t [\varphi^*_r (e)] = \mathbb{E} t [\varphi^*_t (\hat{e})] \), the gap between \( \tau \hat{e} = \mathbb{E} t [\varphi^*_t (\hat{e})] + g [\varphi^*_t (\hat{e})] \) and \( \mathbb{E} t [\varphi^*_r (\hat{e})] \) at the (right-hand side) discontinuity point \( \hat{e} \) is \( g [\varphi^*_r (\hat{e})] \).

Our model (5) built upon a discontinuity point, \( \varphi = 1 - e \), in terms of concealed revenues translates into a discontinuity point, the threshold \( \hat{e} \), in terms of stringency. Such discontinuity emphasizes a possible negative side-effect of enhancing stringency of the Studio di Settore around the threshold \( \hat{e} \), where the shift from compliance with the Studio di Settore to tax evasion implies that the taxpayer invests resources into concealment costs, subtracting them to the government revenue. However, since the stringency parameter also positively affects the expected tax paid by those who do not comply with the Studio di Settore, if \( e \) increases further the unit government revenue may recover and even exceed the level reached at the threshold \( \hat{e} \); this occurs for \( e \geq \bar{e} \), provided that \( \bar{e} < 1 \).

Example 1 Figure 2 plots \( \tau e, \mathbb{E} t [\varphi^*_r (e)] + g [\varphi^*_r (e)] \) as functions of \( e \) for the following parameters values: \( g (\varphi) = 0.1 \varphi^2 \), \( s = 3 \), \( \tau = 0.2 \) and \( a = 0.3 \). For low stringency levels \( e \) the firm prefers to comply with the Studio di Settore since this involves a smaller total outlay per unit of revenue: \( V (e) = \tau e < \mathbb{E} t [\varphi^*_r (e)] + g [\varphi^*_r (e)] \). For \( e > \hat{e} \simeq 0.75 \) the opposite holds and the firm shifts to

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13On the basis of the convention that legal behavior is preferred to illegal one in case of indifference with respect to all other parameters.
the stationary solution $\varphi^*_r(e)$. At the threshold $\hat{e} \simeq 0.75$ the shift from compliance with the Studi di Settore to evasion implies a drop in tax revenue, since the government is no more able to cash in an amount equivalent to the concealment cost, $g[\varphi^*_r(\hat{e})]$. If the stringency $e$ increases further, the expected revenue rate from the stationary solution, $E_t[\varphi^*_r(e)]$, increases. In this example $E_t[\varphi^*_r(e)]$ recovers the level $\tau \hat{e} = e_s \simeq 0.88 < 1$. There might also be cases in which it never reaches the upper threshold: in such circumstances $\tilde{e} = 1$.

![Figure 2](image)

**Figure 2:** The government incurs a revenue loss for efficiency levels $e$ between $\hat{e} \simeq 0.75$ and $\tilde{e} \simeq 0.88$.

In this paper the costs that the tax administration bear in order to organize its activity have been disregarded, but it is clear that a shift from compliance with the Studi di Settore to evasion increases the number of audits that must be conducted and thus the costs for the tax administration. Moreover, the tax revenue becomes to a larger extent dependent on expected sanctions rather than on voluntary payments, with a negative impact as long as the tax administration is risk averse. Also the increase of the stringency $e$ is likely to involve administrative costs for both the tax administration and the taxpayers.

### 6 Conclusions

The policy suggestions that stem from this model are the following. If the parameters of the tax system (i.e., the tax rate $\tau$, the level of sanctions $s$ and the parameters of the probability function other than $e$) are given, while the tax administration aims at maximizing the tax revenue, the stringency of the Studi di Settore should be increased as long as this does not imply surpassing the threshold $\hat{e}$. An increase in $e$ that moves it beyond the threshold $\hat{e}$ would be beneficial only if it implies a (perhaps quite large) jump beyond the upper value $\tilde{e}$, provided that $\tilde{e} < 1$. If $\tilde{e} = 1$ no jump of this type can
be done. Moreover, if the threshold \( \hat{e} \) is surpassed, the *Studi di Settore* just work as a reference point for auditing taxpayers, in order to increase the probability of extracting resources from non compliers through costly audits, and not as a system that helps in enlarging both voluntary compliance and tax revenue.\(^{14}\)

As long as the parameters of the tax system other than \( e \) can be increased at will – specifically, \( a \) and \( s \), controlling detection probability and sanction respectively – tax evasion will tend to vanish. As a matter of fact, \( \mathbb{E} \{ \varphi^*_r (e) \} + g \{ \varphi^*_a (e) \} \) is increasing in \( a \) and \( s \), and thus it would become larger and larger while \( \tau e \) remains constant. In view of problem (8) this implies that the threshold \( \hat{e} \) would be pushed toward its right extrema, \( \hat{e} \to 1 \), thus enlarging at will the scope for beneficial stringency increases.

The model can also be used as a support for analyzing the causes and the consequences of the row that arose in Italy around the *Studi di Settore* in the Summer 2007. That is, one may read the increase in the benchmark revenues introduced by the new 2007 rules as an increase of the stringency parameter \( e \). While any \( e \) increase is likely to be opposed by self interested taxpayers, it seems as though movements near the threshold \( \hat{e} \) are more visible, and thus likely to give rise to stronger reactions by those who are pushed on the verge of shifting from compliance with the *Studi di Settore* to non compliance.\(^{15}\) The very fact that a wide spontaneous protest of the categories involved arose against the stringency increase, running initially also against the representative organizations, accused of not performing their duties, suggests that a significant group of taxpayers felt of being on such a threshold. Moreover, the stringency increase may also have increased the likelihood of \( e > 1 \) at least for some taxpayers, thus triggering a request of overpaying taxes in some cases.

Looking at the problem from the other side, *i.e.* that of the tax administration, it was probably not clearly perceived that increasing a relative price (that of conforming to the *Studi di Settore*) was not a sure recipe for increasing the tax revenue. Faced with widespread protest, the government in Summer 2007 watered down somewhat its initial move aimed at increasing the stringency of the *Studi di sette*. The political justifications of the new rules emphasized that the *Studi di Settore* are mainly a basis for programming profitable audits, like, *e.g.*, the DIF point system used by the American tax administration.\(^{16}\) But a much deeper reform (and probably a costly one) should have been enacted in order to tune the *Studi di Settore* in such a way as to exclude the other many roles that they play. That is, if the *Studi di Settore* must just work as a support for tax auditors, why taxpayers should still be fully informed about the details of the system, asked of cooperating deeply in its implementation and lured into conforming to the benchmark with perspective benefits that clearly belong to the logic of settlements?

\( > \)From a more general point of view, the model presented in this paper offers support for understanding the problems raised by cut-off systems of small business taxation. The literature in this field has pointed out the possible advantages in terms of cost savings for the tax administration (since less audits are needed), the equity problems (as, *e.g.*, in Scotchmer, 1987) and, with reference to cases in which taxpayers are risk averse, the possibility of extracting a risk premium (see, *e.g.*, Glen Ueng and Yang, 2001 and Marchese and Privileggi, 1997 and 2004). In this paper it has been shown that it is possible to extract a premium also from risk neutral taxpayers, as long as they, by accepting the

\(^{14}\)This is also the case when \( e > 1 \) and thus the taxpayer never complies with the *Studio di Settore*. Increasing \( e \) in such circumstances is a way for **increasing** the probability of detection and thus the expected revenue rate for the government.

\(^{15}\)We rely here on the idea of ‘focal points’ that ease social spontaneous coordination (see Sugden and Zamarrón, 2006). On an individual basis such a reaction could also be forcefully justified if the taxpayer’s utility function included psychological or moral costs of tax evasion or exhibited some degree of risk aversion. We thank an anonymous referee for this observation; however, we do not pursue such extensions here.

\(^{16}\)The DIF (Discriminating Function) evaluates the danger of evasion in a report on the basis of the options chosen by the taxpayer, such as the deductions claimed, etc.. The characteristics of the system are kept secret in order to avoid manipulations of information provided by taxpayers.
cut-off, avoid the costs of covering tax evasion. Increasing the cut-off threshold, however, puts at risk the extraction of such a premium, and, even if the audit probability is automatically adjusted upward when the threshold increases, local revenue drops for the Government might arise.

Appendix

**Proof of Lemma 1.** By construction, the only discontinuity point is \( \varphi = 1 - e \); therefore, it is sufficient to show that \( \lim_{\varphi \rightarrow (1-e)^-} T(\varphi) \geq T'(1 - e) \). As \( \mathbb{E} t(\varphi) + g(\varphi) \) is a continuous function on its natural domain, the following holds: \( \lim_{\varphi \rightarrow (1-e)^+} [\mathbb{E} t(\varphi) + g(\varphi)] = \lim_{\varphi \rightarrow (1-e)^-} [\mathbb{E} t(\varphi) + g(\varphi)] = \mathbb{E} t(1-e) + g(1-e) = \tau e + g(1-e) \geq \tau e = T(1-e) \), where the inequality holds because \( g(1-e) \geq 0 \). Since \( T(\varphi) \) is left-continuous on \( \varphi = 1 - e \), i.e. \( \lim_{\varphi \rightarrow (1-e)^-} T(\varphi) = T'(1-e) = \tau e \), \( T(\varphi) \) is lower semicontinuous.

**Proof of Lemma 2.** First note that, by definition (4), \( \lim_{e \rightarrow 0} [\mathbb{E} t(\varphi) + g(\varphi)] = g(1) > 0 = \lim_{e \rightarrow 0} \tau e \), therefore a right-hand neighborhood of 0, \( N_{0^+} \), exists such that \( \mathbb{E} t(\varphi) + g(\varphi) > \tau e \) for all \( \varphi > 1 - e \) and for all \( e \in N_{0^+} \). In other words, for \( e \) sufficiently small the solution of (8) is \( V(e) = \tau e \). But, by assumption, there is a value \( 0 < e < 1 \) such that \( \mathbb{E} t(\varphi^*_e(e)) + g(\varphi^*_e(e)) \leq \tau e \); hence a value \( \hat{e} \) such that \( \mathbb{E} t(\varphi^*_e(\hat{e})) + g(\varphi^*_e(\hat{e})) = \tau \hat{e} \) exists.

To establish uniqueness of \( \hat{e} \) we show that \( (\partial/\partial e) \{ \mathbb{E} t(\varphi^*_e(e)) + g(\varphi^*_e(e)) \} < \tau = (\partial/\partial e) \tau e \). Direct differentiation yields

\[
\frac{\partial}{\partial e} \{ \mathbb{E} t(\varphi^*_e(e)) + g(\varphi^*_e(e)) \} = \left\{- (\varphi^*_e)'(e) + a(1+s) \left[ 2 \varphi^*_e(e) (\varphi^*_e)'(e) \right. \right. \\
\left. \left. + \varphi^*_e(e) - (1-e) (\varphi^*_e)'(e) \right] \tau + g' \left[ \varphi^*_e(e) \right] (\varphi^*_e)'(e) \right\} \\
= \{-1 + a(1+s) [2 \varphi^*_e(e) - (1-e)] \} \tau (\varphi^*_e)'(e) \\
+ a(1+s) \varphi^*_e(e) \tau + g' \left[ \varphi^*_e(e) \right] (\varphi^*_e)'(e) \\
= -g' \left[ \varphi^*_e(e) \right] (\varphi^*_e)'(e) + a(1+s) \varphi^*_e(e) \tau + g' \left[ \varphi^*_e(e) \right] (\varphi^*_e)'(e) \\
= a(1+s) \varphi^*_e(e) \tau,
\]

where in the third equality we have substituted the first addend as in (7). By rearranging terms in (7), it is easily seen that

\[
a(1+s) \varphi^*_e(e) \tau = -g' \left[ \varphi^*_e(e) \right] + \tau - a(1+s) \left[ \varphi^*_e(e) - (1-e) \right] \tau \\
< \tau - a(1+s) \left[ \varphi^*_e(e) - (1-e) \right] \tau \\
< \tau,
\]

where the first inequality holds because \( -g' \left[ \varphi^*_e(e) \right] < 0 \) and the second inequality follows from the fact that \( \varphi^*_e(e) \) is a (minimum) stationary point for \( \mathbb{E} t(\varphi) + g(\varphi) \), and hence, by the discussion in the previous section, \( \varphi^*_e(e) > (1-e) \). Thus, \( (\partial/\partial e) \{ \mathbb{E} t(\varphi^*_e(e)) + g(\varphi^*_e(e)) \} < \tau \). ■

References


