Tax Amnesties and the Self-Selection of Risk-Averse Tax Payers

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Tax amnesties and the self-selection of risk-averse taxpayers

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Abstract

In this paper we model taxpayers participation in an unexpected tax amnesty, which can be entered by paying a fixed amount. Taxpayers are characterized by a Constant Relative Risk Aversion (CRRA) utility function and differ in relative risk aversion coefficient and in income. We show that amnesties may fail as a self-selective device to fully separate big from small evaders and to extract resources from the former. Only taxpayers whose relative risk aversion falls within a given interval participate, while those whose evasion is too small or too large do not enter. The model is used to estimate relative risk aversion and tax evasion of participants in 1991 and 1994 Italian income tax amnesties.

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Key words: tax amnesty, tax evasion, relative risk aversion, self-selection.

1 Introduction

Tax amnesties can sharply vary in their characteristics and provisions from country to country and in different time periods, e.g. they can be offered either on a permanent or on a temporary basis, to all citizen or only to those fulfilling some requirements, at a low or a high relative "price" with reference to the expected payment of the perspective participants under ordinary rules, etc. As amnesties must be appealing in order to command participation, they all involve the risk of diluting the long term incentives to compliance deriving by the threat of punishment for tax evaders. This implication seems particularly dangerous if one considers that usually expected sanctions are already not high enough to justify full compliance according to the standard tax evasion model, based upon the expected utility approach. During an amnesty, however, the main tax enforcement system continues to hold for those who do not participate in the amnesty. Hence, a promising way for discovering whether tax amnesties can play some socially beneficial role is to describe them as devices for inducing selection - or self-selection, depending on the chosen design - of taxpayers, in order to address the treatment specifically to those cases for which it can pass a cost-benefit test from the point of view of social welfare. In the model of Andreoni (1991), for example, tax amnesties select those who, after reporting income, have been hit by a random shock that negatively affects their consumption. As this fact increases their risk aversion, they become willing to modify their tax report and to add a premium to their previous tax payment. A permanent and perfectly anticipated tax amnesty can offer this opportunity. In the Marchese and Cassone (2000) model taxpayers differ in their visibility for tax auditors; taxpayers who

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are immediately visible are more willing to pay taxes. Anticipated temporary tax amnesties introduce a kind of intertemporal price discrimination, as less visible taxpayers evade and wait until a "sales amnesty" becomes available.

In the Chu (1990) model, a kind of immediate and permanent tax amnesty, called FATOTA, allows taxpayers to opt from the outset for a Fixed Amount of Taxes (FAT), instead of reporting income as usual and running the risk of a control. By assuming that all taxpayers have the same preferences and are risk averse[1], while they differ in income, Chu (1990) shows that the fixed tax will be chosen by those who would evade the largest amounts. Welfare improving characteristics of the FATOTA system remind those of plea bargaining in criminal proceedings[2], which, at given conditions, induces the self-selection of the guilty indicted, thus cutting the costs of running trials.

Unanticipated tax amnesties in which the participant has to make a fixed payment can also be modelled as a form of plea bargaining, as they promise the renunciation of (further) controls at a given price, thus offering insurance in order to induce taxpayers self-selection. In this case too savings in audit costs can arise, while the amnesty payment can include a risk premium. On the other hand, even an unexpected amnesty might induce the expectation of further ones, thus reducing perspective compliance. Hence there is a trade-off between the opportunity of immediately (and exceptionally) cashing an extra revenue on the one hand, and introducing adverse incentives for future compliane on the other.

In this paper we will not try to expand the analysis about the general properties of tax amnesties from an efficiency point of view, but we will focus instead on finding out the characteristics of the selection process induced by amnesties in which participants must pay a fixed amount, larger than their tax payment under ordinary rules. We depart from a previous research carried over by Marchese and Privileggi (1997), where a Constant Absolute Risk Aversion (CARA) utility function was used, by assuming that all taxpayers are characterized by a Constant Relative Risk Aversion (CRRA) utility function. Heterogeneity among taxpayers is assumed. Thus taxpayers differ in relative risk aversion coefficient and in income, which are treated as continuous unobservable variables. The model can also be extended in order to describe the reaction to a FAT proposal.

The main result of the paper is that amnesties may fail as a self-selective device to fully separate big from small evaders and to extract resources from the former. If taxpayers display CRRA preferences, very rich taxpayers with very small risk aversion may not enter[3]; participants are only those whose evasion falls within a given interval. This result accords with available empirical research, which provides case studies [see Fisher et al. (1989)] in which only small evaders participate in amnesties. It accords also with conjectures and examples put forth in the literature about plea bargaining [see, e.g., Grossman and Katz (1983)]. However, the modelling of possible self-selection failures of FATOTA or tax amnesties has not been considered in previous literature [see for example Chu (1990), Franzoni (2000), Glen Ueng and Yang (2001) and Marchese and Privileggi (1997)] and does not represent a trivial extension from the plea bargaining literature.

The empirical implications of the model are tested with reference to two amnesties run in Italy in the 90's. Italian governments repeatedly resorted to amnesties in the last fifty years. Hence an assumption needed for our exercise is that taxpayers were either myopic or uninformed. Given the specificities of each amnesty law[4] and the complexity of the political process that lead to enact each of

[1]Chu (1990) presents an example assuming logaritmic utility, which implies DARA (Decreasing Absolute Risk Aversion), and belongs to the family of utility functions we study here.

[2]On this topic see Grossman and Katz (1983).

[3]With CARA preferences this cannot happen and all taxpayers with a percentage of concealed income equal to or greater than a given threshold enter the amnesty [see Marchese and Privileggi (1997)].

[4]The 1994 tax amnesty sharply differs from the 1991 one. With reference e.g. to the price, in the 1994 amnesty it depends on data pertaining to the taxpayer's relative reported income (taxpayers are arranged in classes), while in the 1991 amnesty it depended only on each taxpayer's previous income report.

them, however, accurate anticipation was in fact not easy. On the other hand, assuming that amnesties were not at all anticipated certainly implies a simplification, which can bias our estimates. As taxpayers expecting an amnesty are less frightened by the threat of punishment, anticipation is likely to involve larger tax evasion and also somewhat larger participation in amnesties[5]. Hence our approach would lead to underestimating the interval of risk aversion and of tax evasion that characterizes the participants. To assess the scope of this possible bias, in Section 4 the results of our exercise are compared with data about the distribution of tax evasion and of risk aversion provided by other studies.

The paper is organized as follows. In Section 2 we model the reaction to an unanticipated amnesty by a partial evader who has previously optimally reported his income. The non-trivial analytical difficulties arising with the CRRA utility function specification have been overcome by constructing and solving an analytically approximated version of the model. In Section 3 amnesty participants are characterized with reference to their relative risk aversion and tax evasion. In Section 4 results of a deterministic estimation on data pertaining to the 1991 and 1994 Italian tax amnesties are reported and commented. The conclusions focus on the efficiency and equity implications of tax amnesties.

2 The Taxpayer's Problem

The utility that taxpayers enjoy out of their income w is assumed to be of the standard CRRA form, with constant relative risk-aversion coefficient α:

$$u(w) = \frac{w^{1-\alpha} - 1}{1-\alpha}. \tag{1}$$

In the class (1) we include also the case $\alpha = 1$ by taking $u(w) = \lim_{\alpha \to 1} (w^{1-\alpha} - 1) / (1-\alpha) = \ln w$. To focus upon tax evasion problems it is assumed that income is exogenous and non random, thus ignoring on the one hand the feed-back of taxation upon hours of work, savings, etc., and, on the other, background risks the agent may face.

A progressive tax system is considered, where the income tax is approximated by the following function:

$$t(y) = \gamma y^{\delta}, \tag{2}$$

where $y > 0$ denotes the *reported income*[6] and $0 < \gamma < 1$, $\delta > 1$ are parameters.

As we rule out rewards to honest taxpayers by assumption, a taxpayer will report $y \leq w$, where $w > 0$ denotes the *true income*. For reasons of analytical tractability that will become clear in Section 3, we assume that the sanction to be paid in case of detection is proportional to the amount of concealed income:

$$\widehat{s}(w, y) = \sigma (w - y), \tag{3}$$

where $\sigma > 0$ is a (constant) penalty rate.

Since we are considering a progressive tax system defined as in (2) joined with a sanction function $\widehat{s}(w, y)$ which is linear in concealed income, the selection of a suitable range of values for parameter σ becomes a critical issue that will be extensively discussed in Appendix C. Moreover, some preliminary technical restrictions are needed to let expected utility under the CRRA specification well defined. That is, parameters values must be such that reported income falls into a range comprised between a minimum needed to avoid bankruptcy in case of detected tax evasion and a maximum that insures solvency in case of full compliance and payment of the due tax.

[5]This effects arise, e.g., in the model of Alm and Beck (1990).

[6]Strictly positive income reports is a standard assumption in the literauture on tax evasion in presence of risk aversion. See, for example, Allingham and Sandmo (1972).

To ensure that the taxpayer can always bear the loss in case of detected evasion, it is assumed that

$$\sigma(w - y) \leq w - \gamma y^{\delta}, \tag{4}$$

which implies that, for each given w, there is some[7] m_w such that $0 \leq m_w \leq y$, with $m_w > 0$ whenever $y < w$. On the other hand, to guarantee solvency in case of full compliance, $\gamma y^{\delta} < y$ must hold, *i.e.*, the average tax rate must be less than 100%, which implies

$$y < \gamma^{1/(1-\delta)}.$$

To summarize, let

$$M_w = \min\left(w, \gamma^{1/(1-\delta)} - \varepsilon\right),$$

with $\varepsilon > 0$ arbitrarily small, then the feasible set of values for the reported income y is the closed interval $[m_w, M_w]$.

2.1 Rational Taxpayer's Behavior

We assume that the taxpayer, while filling in her income tax form, ignores the possibility that a tax amnesty may follow, and thus cares only about standard income tax parameters.

A rational taxpayer who earned a true income $w > 0$ will choose to report the income v that maximizes her expected utility

$$\mathbb{E}u(y) = \frac{(1-p)\left(w - \gamma y^{\delta}\right)^{1-\alpha} + p\left[w - \gamma y^{\delta} - \sigma(w-y)\right]^{1-\alpha} - 1}{1-\alpha} \tag{5}$$

with respect to y, where $0 < p < 1$ is the probability of detection. Note that, by considering $u(w) = \ln w$ when $\alpha = 1$, $\mathbb{E}u(y)$ is well defined for all $\alpha > 0$ and for all $m_w < y \leq M_w$. Note also that $\mathbb{E}u(y)$ is strictly concave over $(m_w, M_w]$ for all $\alpha > 0$ and satisfies the Inada condition, hence, there exist a unique $m_w < v \leq M_w$ that maximizes[8] the expected utility.

Empirical evidence about actual tax system parameters shows that expected penalties are not large enough to ensure full compliance[9]; therefore we can further restrict the feasible set by imposing the following condition:

$$\gamma \delta w^{\delta - 1} > p\sigma, \tag{6}$$

which means that the marginal benefit stemming from tax evasion (equal to the marginal tax rate) must be larger than the expected sanction rate. A straightforward computation leads to $\lim_{y \to w^-} [\mathbb{E}u(y)]' < 0$ as long as (6) holds, and therefore the optimal amount of reported income v lies in the interior of the feasible set[10], *i.e.*, $m_w < v < M_w$. Thus, utility maximization is completely described by F.O.C. in (5), which leads to

$$\frac{w - \gamma v^{\delta} - \sigma(w - v)}{w - \gamma v^{\delta}} = \left[\left(\frac{p}{1-p}\right)\left(\frac{\sigma}{\gamma \delta v^{\delta-1}} - 1\right)\right]^{1/\alpha}. \tag{7}$$

[7]Note that m_w cannot be explicitly calculated since (4) cannot be solved with respect to y; moreover $m_w = 0$ if and only if $y = w$.

[8]For clarity in expressions with exponents, we shall use v to indicate the optimal value of variable y, instead of adopting the more widely used notation y^*.

[9]See e.g. Bernasconi (1998).

[10]One could also assume that, besides the partial evaders so far described, in the economy there are also some taxpayers who fully comply for moral reasons and some risk neutral full evaders characterized by a linear utility function. While the former would obviously not participate in any amnesty, the latter would partecipate whenever the requested amount is lower than or equals their expected sanctions. We do not pursue these extensions since we are interested in focusing upon risk averse behavior.

Moreover, we do not consider full evaders since their participation was excluded by the laws granting the amnesties studied in Section 4.

Note that interiority of solution v implies the left hand of (7) to be strictly positive and less than one, which translates into

$$0 < \left(\frac{p}{1-p}\right)\left(\frac{\sigma}{\gamma\delta v^{\delta-1}} - 1\right) < 1. \tag{8}$$

A necessary condition for (8) clearly is

$$\sigma > \gamma\delta v^{\delta-1}, \tag{9}$$

which envisages a sanction rate larger than the marginal tax rate. It will thus be assumed that (9) holds in the following.

The choice of joining an exponential function to determine the amount of taxes due, $t(y) = \gamma y^{\delta}$ in (2), to an affine function to calculate the sanction in case of detection, $s(y) = \sigma(w - y)$ in (3), does not allow for an explicit solution v of the maximization problem. Our goal however is that of estimating the true income w for a given (optimally) reported income v, and for this it is enough to have existence and uniqueness of an interior solution for the maximization problem of rational taxpayers, characterized by (7).

2.2 Participation in an Unexpected Amnesty

Suppose that, after taxpayers reported their optimal income v, but before audits begin, the Tax Administration offers them the possibility of paying some fixed premium x - which accounts both for expected sanctions and risk premium - in order to avoid with certainty any applicable sanction. Ignoring, for the sake of simplicity, inter-temporal discounting, and assuming that no other relevant variables (e.g., true income, penalty rate, etc.) has changed in the meantime, the taxpayer will accept the offer if she is at least indifferent between paying the certain amount x or staying in her position of partial evader. Thus, in order to participating in the amnesty, the following condition must be met:

$$\frac{\left(w - \gamma v^{\delta} - x\right)^{1-\alpha}}{1-\alpha} \geq \frac{(1-p)\left(w - \gamma v^{\delta}\right)^{1-\alpha} + p\left[w - \gamma v^{\delta} - \sigma(w - v)\right]^{1-\alpha}}{1-\alpha}, \tag{10}$$

where the additive constants $-(1-\alpha)^{-1}$ have already been dropped from both sides. Obviously, a necessary condition for (10) to hold is that the extra payment x must be lower than the sanction:

$$0 < x < \sigma(w - v). \tag{11}$$

From the point of view of the Tax Administration, the reported income v is available (observable) information, and the fixed amount x is assumed to be a parameter exogenously provided by some government decision maker[11], while the true income w and the individual constant relative risk-aversion coefficient α are unobservable variables. By considering jointly the optimal behavior of taxpayers as expressed in (7) and the threshold condition (10), we are led to the following system:

$$\begin{cases} \dfrac{w - \gamma v^{\delta} - \sigma(w - v)}{w - \gamma v^{\delta}} = \left[\left(\dfrac{p}{1-p}\right)\left(\dfrac{\sigma}{\gamma\delta v^{\delta-1}} - 1\right)\right]^{1/\alpha} \\ \dfrac{\left(w - \gamma v^{\delta} - x\right)^{1-\alpha}}{1-\alpha} \geq \dfrac{(1-p)\left(w - \gamma v^{\delta}\right)^{1-\alpha} + p\left[w - \gamma v^{\delta} - \sigma(w - v)\right]^{1-\alpha}}{1-\alpha} \end{cases} \tag{12}$$

where the reported income v and the amnesty payment x are given and α, w are the unknowns. All pairs (α, w) solving system (12), characterize the subset of taxpayers who previously (optimally) reported an income v and now participate in the amnesty for the fixed amount x, in terms of their relative risk-aversion α and true earned income w.

[11] We do not address in this paper the symmetric problem of the government, which chooses tax and amnesty parameters in order to maximize some objective function.

2.3 An Alternative Interpretation of the Model: Fixed Amount of Taxes

Model (12) can also be referred to an alternative scenario, in which there are no tax amnesties, but, instead, the government offers a FAT (Fixed Amount of Taxes) as a substitute for ordinary taxation. It is assumed in this case that the government posses some *a priori* belief[12] about a taxpayer's optimally reported income. The first equation of system (12) then links v to the taxpayer's true income w according to (7). As the FAT offer aims at cashing the tax that the agent would have voluntarily paid plus a premium, the fixed amount requested by the government can be written as $\gamma v^\delta + x$, where x includes also the error made by the government in estimating v. All pairs (α, w) solving system (12), characterize the subset of taxpayers whose reported income is v and who accept the FAT, in terms of their relative risk-aversion α and true earned income w, which are unknown variables. The FAT scenario is more general than the amnesty one, as taxpayers facing a FAT offer based on government *a priori* beliefs have nothing to gain by altering their actual tax report, which is requested only if they refuse the FAT. Note that the FAT can be offered immediately as an alternative to ordinary taxation, while the amnesty must be unexpected to work according to system (12).

In practice, implementation of the FAT approach may be somehow difficult for the government, as *a priori* information about taxpayers' (believed) reported income v may be poor. To overcome this difficulty, FAT offers are often designed for income classes rather than for pointwise income values. Tax amnesties, instead, may hinge upon actual tax reports to provide the government with the information needed to determine personalized entrance payments, as the examples quoted in Section 4 show. Each scenario has thus its own advantages and drawbacks[13].

3 Simplifying the Model

Thanks to the choice of the sanction function as in (3) that is linear in the true income w, we can transform the left hand side of the first equation in (12) as follows:

$$\frac{w - \gamma v^\delta - \sigma\left(w - v\right)}{w - \gamma v^\delta} = 1 - \sigma + \frac{\sigma\left(v - \gamma v^\delta\right)}{w - \gamma v^\delta}.$$

With this position, substituting into the first equation yields

$$w - \gamma v^\delta = \frac{\sigma\left(v - \gamma v^\delta\right)}{\sigma - 1 + r^{1/\alpha}}, \tag{13}$$

where

$$r = \left(\frac{p}{1-p}\right)\left(\frac{\sigma}{\gamma\delta v^{\delta-1}} - 1\right). \tag{14}$$

[12] In the standard approach, the taxpayer's true income is the realization w of a random variable $\widehat{w}$, known only by himself and not by the government, while value $v(w)$ of reported income is the outcome of the taxpayer's optimal strategy. The probability distribution of r.v. $\widehat{w}$ conditional to some other information β (like profession, branch of activity, etc.) is common knowledge among taxpayers and the government itself, which, by solving the same optimization problem of the taxpayers, may calculate the induced conditional probability distribution of r.v. $\widehat{v}|\beta = v(\widehat{w})|\beta$ as well. Hence, in this scenario, the value for v used in model (12) could be some proxy of r.v. $\widehat{v}|\beta$, like, for instance, the conditional expected value $\mathbb{E}\left[\widehat{v}|\beta\right]$.

[13] The Italian government until nowadays is, in fact, oscillating between these approaches. Since 1999, a reference reported income (and implicitly a kind of FAT tax) was introduced for small business and self-employment income. The amount of the reference incomes v were determined on the basis of physical and economic indicators, according to the so called *Studi di Settore* (economic branch studies). The method for calculating reference incomes has been agreed upon by the tax administration and the small business and self-employment representative organizations.

On the other hand a new general tax amnesty was launched in 2003, while for the years 2004-2005 an explicit FATOTA-like system (concordato preventivo) was offered to small business and the self-employed.

For a given reported income v, F.O.C. expressed in the first equation of (12), describing the maximizing behavior of taxpayers, provides through (13) a representation for the true income w as a function of the relative risk aversion α:

$$w = \frac{\sigma\left(v - \gamma v^{\delta}\right)}{\sigma - 1 + r^{1/\alpha}} + \gamma v^{\delta}. \tag{15}$$

In order to ensure that w is positive also at low risk aversion levels, that is whenever $\alpha \to 0$, it is assumed that $\sigma > 1$. This means that sanctions are large enough to imply the threat of bankruptcy for full evaders, a perspective that a taxpayer characterized by a CRRA utility function always prefers to avoid. Hence the condition $\sigma > 1$ rationalizes the observed choice, that is the report of the positive income amount v, even at low risk aversion levels.

We shall henceforth focus exclusively on variable α to study solutions of (12).

Dividing the second inequality in (12) by $\left(w - \gamma v^{\delta}\right)^{1-\alpha} > 0$, we have

$$\frac{1}{1-\alpha}\left(1 - \frac{x}{w - \gamma v^{\delta}}\right)^{1-\alpha} \geq \frac{1}{1-\alpha}\left[1 - p + p\left(\frac{w - \gamma v^{\delta} - \sigma\,(w - v)}{w - \gamma v^{\delta}}\right)^{1-\alpha}\right],$$

and by substituting the left side as in (13) and the right side as in the first equation of (12), system (12) boils down to a single inequality where the unknown is the sole variable α:

$$\frac{1}{1-\alpha}\left(A - Br^{1/\alpha}\right)^{1-\alpha} \geq \frac{1}{1-\alpha}\left[1 - p + p\left(r^{1/\alpha}\right)^{1-\alpha}\right]$$

where A and B are constants defined by

$$A = 1 - \frac{(\sigma-1)x}{\sigma\left(v - \gamma v^{\delta}\right)} \tag{16}$$

$$B = \frac{x}{\sigma\left(v - \gamma v^{\delta}\right)}. \tag{17}$$

Since, by (14) and (8), $0 < r < 1$, the right hand side is well defined. To let the left hand side make sense as well, we need $A - Br^{1/\alpha} > 0$ for all $\alpha > 0$, which is equivalent to $A - B > 0$. A direct application of (16) and (17) to this inequality, leads to the following assumption that will hold throughout the whole paper.

A. 1 *Parameters x, γ, δ and the reported income v satisfy*

$$0 < x < v - \gamma v^{\delta}.$$

Assumption A.1 ensures that the taxpayer can afford payment x also when tax evasion is small, *i.e.*, v is very close to w. Note that, by (16), (17) and A.1, also $0 < B < A < 1$ and $0 < A - B < A - Br < 1$ hold.

Under Assumption A.1 we can define a function f on $\mathbb{R}_{++}$ by

$$f(\alpha) = \left(A - Br^{1/\alpha}\right)^{1-\alpha} - p\left(r^{1/\alpha}\right)^{1-\alpha} - (1-p). \tag{18}$$

Note that $f(\alpha)$ is well defined for all $\alpha > 0$, and is C^{∞}. Moreover, $f(1) = 0$. With this notation at hand, system (12) turns out to be equivalent to

$$\begin{cases} f(\alpha) \geq 0 & \text{if } 0 < \alpha \leq 1 \\ f(\alpha) \leq 0 & \text{if } \alpha \geq 1. \end{cases} \tag{19}$$

for all $\alpha > 0$ except[14] at $\alpha = 1$.

Function f defined in (18) does not permit an explicit solution of (19). Hence, we shall characterize solutions of a slightly simplified system. Specifically, we shall use a suitable lower bound $l < f$ for $0 < \alpha < 1$, while for $\alpha \geq 1$ we will be able to characterize solutions only for a subclass of models[15]. We shall see that our technique covers the most meaningful cases.

Let

$$\phi(\alpha) = \left(A - Br^{1/\alpha}\right)^{1-\alpha}$$

$$\varphi(\alpha) = -p\left(r^{1/\alpha}\right)^{1-\alpha}.$$

Clearly $f = \phi + \varphi - (1-p)$. Now define

$$\psi(\alpha) = 1 - \ln(A - Br)(\alpha - 1)$$

and

$$l(\alpha) = \begin{cases} \psi(\alpha) + \varphi(\alpha) - (1-p) & \text{for } 0 < \alpha < 1 \\ f(\alpha) & \text{for } \alpha \geq 1. \end{cases}$$

We shall characterize solutions of the system

$$\begin{cases} l(\alpha) \geq 0 & \text{if } 0 < \alpha \leq 1 \\ l(\alpha) \leq 0 & \text{if } \alpha \geq 1. \end{cases} \tag{20}$$

Lemma 1 *Under A.1, $\psi(\alpha) < \phi(\alpha)$ for all $0 < \alpha < 1$.*

The proof of Lemma 1 is reported in Appendix.

3.1 The main Result

The following result completely describes the solution set of our simplified model, system (20). First we restrict the admissible range of the parameters and the reported income; note that these restrictions are met by all available data considered in Section 4.

A. 2 *Parameters $p, \gamma, \delta, \sigma$ and the reported income v altogether satisfy*

i)

$$r = \left(\frac{p}{1-p}\right)\left(\frac{\sigma}{\gamma\delta v^{\delta-1}} - 1\right) \leq e^{-2};$$

ii)

$$1 < \sigma \leq \frac{1}{2}\left(1 + \sqrt{5}\right).$$

Proposition 1 *Suppose A.1 and A.2 hold. Then the solution set S of system (20) has the following properties.*

[14] However, since we represent taxpayers by means of variable α itself, we are interested in "full measure groups" of taxpayers, that is, non trivial intervals of values of variable α. Therefore, the single value $\alpha = 1$, which corresponds to logarithmic utility, becomes negligible, and we can proceed, with no loss of generality, thorough the study of function f defined in (18) for all $\alpha > 0$.

[15] The technique for approximating solutions of system (19) closely follows the framework developed in Marchese and Privileggi (2002) for studying a cutoff tax system based on a proportional tax rate.

i) *If condition (i) of A.2 holds with equality, then S is a nonempty interval*[16]*: $S = [\underline{\alpha}, \overline{\alpha}]$, with $0 \le \underline{\alpha} < 1 < \overline{\alpha} < +\infty$, if and only if*

$$x < \frac{\sigma\left(v - \gamma v^{\delta}\right)}{\sigma - (1 - r)}\left(1 - r^{p}\right). \tag{21}$$

ii) *If condition (i) of A.2 holds with strict inequality, a sufficient condition for S to be non-empty and of the form $S = [\underline{\alpha}, \overline{\alpha}]$ with $0 \le \underline{\alpha} < 1 < \overline{\alpha} < +\infty$, is the following:*

$$x \le \frac{\sigma\left(v - \gamma v^{\delta}\right)}{\sigma - (1 - e^{-2})}\left\{1 - \left[1 + p\left(\ln r\right)\left(\frac{\ln r}{2} + 1\right)\right]^{2/(2+\ln r)}\right\}. \tag{22}$$

iii) *If*

$$x > \frac{\sigma\left(v - \gamma v^{\delta}\right)}{\sigma - (1 - r)}\left[1 - \exp\left(\frac{4e^{-2}p}{r \ln r}\right)\right], \tag{23}$$

then S is empty.

The proof of Proposition 1 is reported in Appendix A. Proposition 1 is only theoretically meaningful: it provides the intrinsic shape of the solution set of a model that approximates (19). It substantially states that all participants in the amnesty, if any, have a relative risk aversion coefficient falling within some interval which includes 1. From a different perspective, it states that taxpayers with relative risk aversion above some upper bound $\overline{\alpha} > 1$ (that is, who, by (15), concealed only a small income amount $w - v$) do not enter the amnesty. Moreover, whenever $\underline{\alpha} > 0$, also taxpayers with a relative risk aversion coefficient below $\underline{\alpha} < 1$ (that is, who, again by (15), concealed a large income amount) do not enter the amnesty. In order to find values of both extrema $\underline{\alpha}$ and $\overline{\alpha}$, one must rely on numerics, as the following Sections show. A last remark regards the error on the lower bound $\underline{\alpha}$ introduced by considering the approximated model (20) in place of (19). Again numerics show that such an error is small relatively to the size of the whole interval $[\underline{\alpha}, \overline{\alpha}]$, also because it affects only the side on the left of $\alpha = 1$.

The intuition behind Proposition 1 is as follow. The reaction of taxpayers characterized by high risk aversion is due to the low value of the expected sanctions for them, as they concealed only small income amounts. Even though their relatively high risk aversion commands a large relative risk premium, this is not enough to motivate them in order to enter the amnesty. With reference to rich taxpayers with small risk aversion, the explanation is somehow more involved. Remember that in order to enter the amnesty so far modelled, a taxpayer must have reported v and must be ready to pay x. However, a taxpayer who optimally reported v can have at most the following true income:

$$\overline{w} = \frac{\sigma v - \gamma v^{\delta}}{\sigma - 1} < +\infty.$$

This upper bound is obtained by calculating the limit in (15) as $\alpha \to 0^{+}$.

Income $\overline{w}$ is such that, whenever the taxpayer endowed with it reports v, she will receive zero net income in case of detection. If true income was larger than $\overline{w}$, the taxpayer would have risked bankruptcy by reporting v; but a taxpayer characterized by a CRRA utility function always prefers avoiding such a risk. When $\sigma \to 1$, the true income upper bound tends to ∞. However, the higher the sanction rate σ, the lower the $\overline{w}$ value. Thus, if σ is large, the tax evasion of a taxpayer with low risk aversion shall be relatively small, and, as the risk premium she is willing to pay is low, she may prefer

[16] To be precise $S = (0, \overline{\alpha}]$ whenever $\underline{\alpha} = 0$, since $\mathbb{E}u(\cdot)$ is not defined for $\alpha = 0$.

not to enter the amnesty. Through a similar argument, one can infer that a taxpayer with an income larger than $\overline{w}$ (and for any value of α) must report an income larger than v.

As we shall consider in Section 4 a tax amnesty in which the amount x to be paid is chosen by the government on the basis of the reported income, the approach so far developed applies with reference to the relevant v and x values. Clearly, the extent of the interval of participants in the amnesty, described in its general shape by Proposition 1, can vary at different reported income levels, according to the "offer function" chosen by the government in establishing the x amount corresponding to each v value.

4 Estimation on Data Pertaining to the 1991 and 1994 Italian Tax Amnesties

Italian Law no. 413/1991 introduced a general tax amnesty regarding the main Italian taxes[17]. This amnesty was considered a success in terms of participation and revenue. Participation was highly concentrated among taxpayers with self-employment and business income, while it was scanty among wage earners. While this amnesty followed a major one granted ten years before, it was nevertheless not easy for taxpayers to anticipate its timing and characteristics in order to suitable shape their tax reports[18]. Thus treating the amnesty as unanticipated seemed an acceptable starting point, to be checked *ex post* by comparing estimation results with those available from other sources.

With reference to the income tax, for taxpayers not yet audited, par. 38 of the amnesty law provides rules for calculating the extra payment necessary to enter, *i.e.*, for calculating variable x in our model. They are summarized in Table 1. Requested x payment is an increasing function of the income tax already paid by the participant and thus of her reported income v. This schedule seems dictated by the aim of extracting each taxpayer's willingness to pay: i.e., the Italian legislator exploited the information conveyed by income reports about potential "amnesty demand".

Payment x was due for each year for which the amnesty was entered, from 1985 to 1990. To calculate the income tax amount due from each taxpayer, according to her reported income, parameters of the tax function (2) have been estimated with reference to the actual income brackets, for each year for which amnesty could be entered: details are reported in Appendix B. Amount x due was calculated on the basis of the estimated due tax, according to the rules described in Table 1. In view of the discussion in Appendix C, the sanction parameter σ in (3) has been set equal to the tax parameter δ in (2). The audit rate[19] in Italy in the relevant years has been around 1%; this is thus the value used for parameter p.

For every amount x requested, we calculated upper and lower threshold levels of relative risk aversion, $\overline{\alpha}$ and $\underline{\alpha}$, and percentage evasion, $\underline{e}$ and $\overline{e}$, needed for participation in the amnesty. Values of parameters $p, \gamma, \sigma = \delta$ and all reported incomes v, are such that condition (*i*) of Assumption A.2 holds with strict inequality. Moreover, it is important to remark that all values of parameter x turned out to be well below the bounds given by condition (22) in Proposition 1.

Here we report numeric solutions of model (20) for some representative taxpayers. Table 2 reports the results for a taxpayer endowed with average net reported income, and who enjoys average deductibles (case a); Table 3 refers to a taxpayer endowed with average net reported business income[20]

[17] More details are provided in Marchese and Privileggi (1997).

[18] In 2003, in Italy, a general amnesty has been again provided, thus reinforcing expectations for repetition in the future. This information was obviously not available in the period considered in the paper.

[19] For data about controls, see Ministero delle Finanze, Ufficio di Statistica, *Accertamenti effettuati ai fini delle imposte dirette*, Roma, various issues.

[20] The average value is calculated without taking into account taxpayers who report income equal to zero, and refers to

(case b); Table 4 to a taxpayer endowed with average net reported self-employment income[21] (case c). All the taxpayers considered paid a tax which belongs to the first bracket of Table 1.

The estimated relative risk aversion interval goes from a high of around $\alpha = 4.5$ to virtually zero [as some lower threshold values are likely to be strictly positive only because the solution of system (19) is approximated by the solution of system (20)] and is thus well inside the range $1 - 10$ usually considered[22]. The upper risk aversion threshold is not far from that calculated in Marchese and Privileggi (1997), which ranged from 4.6 to 6.3[23].

The lower threshold percentage tax evasion in the examples examined is always comprised between 34 and 35% of the true income [to be compared with 33 and 34% calculated in Marchese and Privileggi (1997)]. The upper threshold percentage tax evasion is $73 - 75\%$, thus suggesting that only quasi-full evaders were left out by the amnesty.

To roughly assess the results, the calculated percentage evasion thresholds can be compared with available evasion estimates from other sources[24]. For the average Italian taxpayer and the relevant time period, the minimum evasion estimate is 19.9%, the maximum is 36%. For self-employment and business income, the corresponding values are 42.9% - 58.1%. For income from wage or pension, evasion is relatively quite low (8.1% - 16.1%). Our results show, for the average taxpayer, a lower-threshold percentage evasion (needed to let the amnesty worth entering) which is close to the top evasion values calculated by other studies. This fact is roughly consistent with the idea that the amnesty was not designed to be appealing for the average taxpayer. For those endowed with the average business or self-employment income instead, the mean value of our estimated thresholds is at most 10% higher than the mean of the evasion estimates available from other studies. The interval we found is thus consistent with an amnesty design aimed to be appealing for mean/high evaders in business or self-employment, a fact that is consistent with available data about participation [see Maré and Salleo (2003)].

Law 656/1994 granted a further amnesty (*concordato di massa*) reserved for entrepreneurs and the self-employed. Rules regulating this amnesty provided an entrance payment based upon relative gross revenue and profitability revealed by the tax reports[25]. Estimation results based on model (20) are shown in Table 5.

They largely confirm results reached for the 1991 tax amnesty. Calculated lower threshold tax evasion and upper threshold risk aversion parallel previous findings in Marchese and Privileggi (1997) with a somehow larger downward correction for risk aversion and upward correction for tax evasion than for the 1991 tax amnesty.

An interesting reference point for assessing risk aversion estimates is represented by results reported in Guiso and Paiella (2001). To elicit risk attitudes they exploited household reactions to a hypothetical lottery offered to the sample of Italian population involved in the periodical Bank of Italy household survey of 1995. According to their estimates relative risk aversion range from 0.2 to 36.3, with a right-skewed distribution, a median value of 4.8 and a mean of 5.38. Our estimates are thus coherent with an amnesty design aimed at being selective and appealing for median-low risk averse agents.

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entrepreneurs in ordinary tax regime (thus excluding cases of forfeit).

[21]The average value is calculated without taking into account the taxpayers who report an income equal to zero, and refers to all types of self-employment.

[22]Epstein and Zin (1990) quote a tradition concerning relative risk aversion, which should not be greater than $\alpha = 10$.

[23]Remember that when CARA is assumed there is no lower risk aversion threshold.

[24]For surveys, see Bernasconi (1995) and Monacelli (1996).

[25]See Marchese and Privileggi (1997) for details. The amnesty was not available for those who had already benefited from the 1991 one according to par. 38. For many taxpayers, the 1994 amnesty was cheaper than the previous one.

It is worth to remark also that the method provided by the law for calculating the entrance payment was new, and hence it was not easy for the taxpayer to anticipate it.

5 Conclusions

Optimal taxation literature has pointed out [see, e.g., Brito et al. (1995)] that differences in risk aversion may signal other relevant taxpayers characteristics, such as income or productive ability. This observation has been mainly exploited to develop models of random taxation. Suitably designed random taxes may in fact help in overcoming problems of asymmetrical information between government and taxpayers. Moreover, differences in risk aversion imply that the least risk averse citizens are those more willing to play lotteries. Random taxation may increase efficiency by offering lotteries which are cheaper to implement[26] than the “tax evasion lottery”, which implies running tax controls [see Pestieau et al. (1998)].

Actual tax systems, however, do not seem explicitly resorting to the introduction of forms of gambling, perhaps because of the traditional uncertain statute of this criterion on moral and also religious ground (remember the prohibition of gambling in many religions). Moreover, if one assumes that taxpayers are risk averse and that a benevolent government is at most risk neutral, randomization seems to be a costly way of inducing taxpayers self selection, whenever it increases the overall amount of risk with reference to the *status quo ante*. Taxpayers self-selection systems that resort to insurance offers, like FATOTA or unexpected tax amnesties, seem thus a more natural and efficient way to pursue the same goals.

Some nice efficiency and welfare improving characteristics of the latter instruments have been clarified in the literature. Both FATOTA and tax amnesties, however, may imply equity problems, as they introduce some kind of discrimination. Specifically, taxpayers with the same true income may pay different total amounts according to their attitudes toward risk. Moreover, if one introduces time discounting and taxpayers anticipation, amnesties may increase tax evasion of prospective participants, with effects upon total tax revenue that could turn from positive into negative. In this paper we have added a further *caveat* for the use of these instruments, by demonstrating that self-selection may fail when a CRRA specification of the taxpayers utility function is considered. Our result is in line with findings in the literature about plea-bargaining: Grossman and Katz (1983) have noted that self-selection of the guilty may be problematic when the indicted differ in risk-aversion. Extension of their observations to either FATOTA or amnesty models, however, is not straightforward, as in their model committing the crime is a discrete choice (not explicitly studied), and each defendant has an exogenously given degree of risk aversion. When taxes are considered instead, the amount of the law breach (tax evasion), is a continuous variable, while risk attitudes (which arguably vary depending on income) contribute to motivating both the amount of the law breach and the willingness to accept a settlement proposal. Self-selection of the guilty of large tax evasion through FATOTA or tax amnesties is thus quite likely to work, as happens in examples appeared in Chu (1990), Franzoni (2000), and Marchese and Privileggi (1997). However, the present paper shows that the case of an imperfect self-selection cannot be ruled out in general. With a CRRA specification, participation in amnesties or FATOTA programs accords at any rate to a predictable pattern. It leaves out small evaders with large risk aversion, and might also leave out large evaders with small risk aversion.

With reference to empirical results, caution is needed in evaluating them because their reliability depends on the (strong) assumption that amnesties were unanticipated. The comparison of results with data available from other sources, however, suggests that the model works reasonably well to describe the behavior of participants in Italian tax amnesties. Results also confirm the findings of a former paper by Marchese and Privileggi (1997), which relied on a different utility function specification, thus passing a kind of sensitivity test. The model studied in Marchese and Privileggi (1997) was constructed by means of an exponential utility function (with constant absolute risk aversion), and

[26] For instances, by offering the possibility of opting for a high tax and giving to those who accept it a ticket for a lottery that provides for a given expected rebate.

was capable of describing only the marginal (lower threshold tax evasion) participants; very little emerged about those (arguably the majority of participants) who received a strictly positive benefit from participating in the amnesty program. The new version presented in this paper provides more information on the characteristics and the evasion extent of taxpayers entering the amnesty. While one may argue that some quasi-full evaders did not participate, the coverage of the Italian amnesties within the target social groups seems large, thus providing a picture different from the case studied in Fisher et al. (1989) where the amnesty considered was appealing mainly to small evaders.

Note, finally, that the model discussed in Section 3 lends itself also to a reversed view of the approach pursued in this paper, where we focussed exclusively on the problem of unravelling the information about taxpayers characteristics conveyed by amnesty participation. Since the solution set of participants in a FAT or amnesty offer is represented by an interval $[\underline{\alpha}, \overline{\alpha}]$ on the real line, a different model can be constructed aimed at finding a value for parameter x that maximizes a given social or government utility function. To solve this problem one must figure out the number of participants in the program when x is offered, which depends on both the length $\overline{\alpha} - \underline{\alpha}$ of the interval and the distribution of taxpayers over the interval itself. Hence, while the problem tackled in this work was finding values for the unknown variable α, such a model would require the distribution of taxpayers risk aversion α to be given. This alternative approach, which could provide further insight into the equity and efficiency implications of tax amnesties, is left for future research.

6 Appendix

A Proofs

Proof of Lemma 1. Since $r < 1$ and A.1 implies $B < A$,

$$(A - Br)^{1-\alpha} < \left(A - Br^{1/\alpha}\right)^{1-\alpha} = \phi(\alpha) \tag{24}$$

for all $0 < \alpha < 1$. Since $(A - Br)^{1-\alpha}$ is strictly convex for all $\alpha > 0$, by the superdifferentiability property the following is true:

$$\psi(\alpha) = 1 - \ln(A - Br)(\alpha - 1) < (A - Br)^{1-\alpha}$$

for all $0 < \alpha < 1$, which, coupled with (24), proves the assert. ■

Lemma 1, explains our construction of function l. Since function ϕ is neither convex nor concave over $(0, 1)$, we replace it with its convex lower bound $(A - Br)^{1-\alpha}$, then we further lower it by taking its first order approximation centered on $\alpha = 1$. Thus, l turns out to be a lower bound for f on the interval $(0, 1)$, with an improved (linearized) shape for component ϕ in f; while, by construction, $l = f$ for all $\alpha \geq 1$. Therefore, solutions to (20) are a subset of solutions to (19); in particular, some points in the "left-side" solution set of (19), a subset of interval $(0, 1)$, are lost through our approximation. Note that, by construction, $l'(1) = f'(1) = -\ln(A - Br) + p \ln r$.

The proof of Proposition 1 will be accomplished through several steps. First we need a preliminary lemma.

Lemma 2 *Under A.1, function* $\phi(\alpha) = \left(A - Br^{1/\alpha}\right)^{1-\alpha}$ *is strictly convex for* $\alpha \geq 1$, *while function* $\varphi(\alpha) = -p\left(r^{1/\alpha}\right)^{1-\alpha}$ *is strictly concave for* $0 < \alpha \leq -(1/2)\ln r$ *and is strictly convex for* $\alpha \geq -(1/2)\ln r$.

Proof. A tedious direct computation of the second derivatives of both ϕ and φ gives the result. ■

The proof of Proposition 1 is based on condition (*i*) in Assumption A.2, which has been chosen because $r \leq e^{-2}$ is met by all available data considered in Section 4. The idea behind the proof of Proposition 1, however, can be replicated through a symmetrical argument to obtain analogous results for the case $r > e^{-2}$, as will become clear in the sequel.

The right hand inequality in condition (*ii*) of Assumption A.2 is a sufficient condition for full liability assumption (4) to be satisfied[27], obtained by means of (15) while the left hand inequality in (*ii*) ensures the positivity of true income also at low risk aversion levels, as observed with reference to (15). Hence, since$(1/2)\left(1+\sqrt{5}\right) \simeq 1.618$, Assumption A.2 narrows the admissible values for the penalty rate σ to a subset of the interval $(1, 1.618]$.

Proof of Proposition 1. *Part (i).* This is a very peculiar (and lucky) circumstance, as, in most cases, data available meet condition (*i*) of A.2 with strict inequality. We consider this case in detail mainly for expository reasons, because it helps in clarifying the main idea behind the whole proof.

Since equality in condition (*i*) of A.2 is equivalent to $-(1/2)\ln r = 1$, by Lemma 2 function $l(\alpha) = \psi(\alpha) + \varphi(\alpha) - (1-p)$ turns out to be strictly concave over $(0, 1)$ and strictly convex over $[1, +\infty)$. This is true since l is the sum of a constant and functions ψ and φ, which are linear and strictly concave respectively over $(0, 1)$, and both strictly convex over $[1, +\infty)$. In other words, l has a unique flex-point at $\alpha = 1$. Moreover, since $l(1) = 0$, S is non-empty and has the form $S = [\underline{\alpha}, \overline{\alpha}]$ if and only of its derivative is strictly negative at $\alpha = 1$, that is,

$$l'(1) = -\ln(A - Br) + p\ln r < 0$$

which, after some algebra, is the same as condition (21), which makes sense thanks to (*ii*) of A.2. Note also that $\overline{\alpha} < +\infty$ since $l(\alpha) \to +\infty$ as $\alpha \to +\infty$.

Part (ii). Strict inequality in condition (*i*) of A.2 is equivalent to

$$-\frac{\ln r}{2} > 1. \tag{25}$$

To simplify notation, let $c = -(1/2)\ln r > 1$. In this case we extend the argument above by constructing a function $h(\alpha)$ that is as much similar as l as possible but is better shaped than l over the interval $(1, c)$ where the required convexity property of l cannot be verified directly. Define

$$\chi(\alpha) = 1 + \frac{\phi(c) - 1}{c - 1}(\alpha - 1)$$

and

$$h(\alpha) = \begin{cases} l(\alpha) & \text{for } 0 < \alpha \leq 1 \text{ and } \alpha \geq c \\ \chi(\alpha) + \varphi(\alpha) - (1-p) & \text{for } 1 < \alpha < c. \end{cases}$$

Like in the construction of function l, where we replaced the badly shaped function ϕ with a linear one, ψ, over $(0, 1)$, function h constitutes an improvement of function l again by linearizing ϕ, which,

[27] It should be remarked, however, that the right inequality in (*ii*) is only a sufficient condition for (4). Actaully it could be relaxed a little, since from both (4) and (15) the necessary and sufficient condition for (4) turns out to be $\sigma \leq 1 + (1/2)\left(\sqrt{4 + r^{2/\alpha}} - r^{1/\alpha}\right)$ which, as $r < 1$, is less restrictive than the right inequality in (*ii*). Nonetheless, since the last condition does depend on α, such an assumption would exogenously impose some lower bound on α itself, thus further complicating the subsequent analysis.

In general, progressive taxation is problematic to model for, on the one hand, the sanction must be high enough to sustain increasing marginal tax rates, while, on the other hand, it should not be prohibitive as full liability is assumed.

by Lemma 2, is convex for all $\alpha \geq 1$. As a result, h turns out to be strictly concave over $(1, c)$, being it the sum of a constant, a linear and a concave function.

Function $h(\alpha)$ turns out to be the same as $l(\alpha)$ outside the interval $(1, c)$, where the same argument of Part (*i*) applies. Specifically, h is strictly concave over $(0, 1)$ and $l(\alpha) \geq 0$ has a non-empty interval $[\underline{\alpha}, 1)$ as solution as long as $l'_{-}(1) = l'(1) < 0$; while h is strictly convex over $(c, +\infty)$. Inside interval $(1, c)$, we have seen $h(\alpha) = \chi(\alpha) + \varphi(\alpha) - (1 - p)$ to be strictly concave. Moreover, since h is obtained by replacing the strictly convex function ϕ with the segment joining two points of its graph, $h(\alpha) > l(\alpha)$ holds true for all $\alpha \in (1, c)$, while $h(1) = l(1)$ and $h(c) = l(c)$. Note that h is not differentiable at points $\alpha = 1$ and $\alpha = c$, where it is only left and right-differentiable, while $l'(1)$ exists.

Hence, $h'_{+}(1) \leq 0 \Longrightarrow h(\alpha) < 0 \Longrightarrow l(\alpha) < 0$ for all $\alpha \in (1, c]$. Furthermore, $l(c) < 0$ plus its convexity over $(c, +\infty)$ implies $l(\alpha) \leq 0$ for all $\alpha \in [c, \overline{\alpha}]$, where $c < \overline{\alpha} < +\infty$. To conclude, $h'_{+}(1) \leq 0 \Longrightarrow l(\alpha) \leq 0$ for all $\alpha \in (1, \overline{\alpha}]$, while, on the other side, $h'_{+}(1) \leq 0 \Longrightarrow l'_{+}(1) = l'(1) < 0$, thus establishing also non-emptiness of interval $[\underline{\alpha}, 1)$. A direct computation shows that condition $h'_{+}(1) \leq 0$ is equivalent to condition (22), and the proof is complete. Note that again, under our construction, we obtain a function h with a unique flex point $\alpha = c$.

Part (iii). By construction, $\psi'(\alpha) = \phi'(1) = -\ln(A - Br)$ over $(0, 1]$. By Lemma 2,

$$\phi'(\alpha) \geq \phi'(1) = -\ln(A - Br) \qquad \forall \alpha \geq 1 \quad \text{and}$$

$$\varphi'(\alpha) \geq \varphi'\left(-\frac{\ln r}{2}\right) = \frac{4e^{-2}p}{r \ln r} \qquad \forall \alpha > 0.$$

Therefore,

$$l'(\alpha) \geq \phi'(1) + \varphi'\left(-\frac{\ln r}{2}\right) = -\ln(A - Br) + \frac{4e^{-2}p}{r \ln r}$$

and $l'(\alpha) > 0$ if $-\ln(A - Br) + (4e^{-2}p)(r \ln r)^{-1} > 0$, which is equivalent to (23); hence, (23) $\Longrightarrow l'(\alpha) > 0$ for all $\alpha > 0$. Since $l(1) = 0$, $l' > 0$ means that l "crosses" level zero increasingly at $\alpha = 1$, and system (20) has empty solution set, as was to be shown. ■

B The Tax Function

The progressive income tax due, as a function of the net taxable income, can be represented by as many linear segments as the number of income brackets; each segment has a higher positive slope the higher is the bracket. OLS estimation technique (with variables in logarithms) was used to interpolate each tax schedule in order to obtain a form like in (2). As the income tax was often modified by the government in the period for which the amnesties described in this paper were available, there were six tax schedules to consider, as reported in Table 6.

C Constructing the Linear Sanction Function

In order to be coherent with the progressivity implied by the tax function (2), the sanction to be applied in case of detection should exhibit some degree of progressivity as well. The natural choice would be:

$$s(w, y) = g\gamma \left(w^{\delta} - y^{\delta}\right), \tag{26}$$

where $g > 1$ is a penalty rate.

On the other hand, we have seen in Section 3 that a crucial step in simplifying system (12) requires a sanction function that is linear in the true income w; therefore we assumed the form $\widehat{s}(w, y) =$

$\sigma(w-y)$ as in (3). In this appendix, we provide some arguments for determining a lower bound[28] for the value of the coefficient σ so that $\widehat{s}(w,y)$ in (3) turns out to be not too different from $s(w,y)$ in (26). To do this, let us discuss more thoroughly some restrictions required by (26).

As we did in Section 2 through condition (4), we assume that cheating taxpayers caught by the authorities are always able to pay the sanction; in terms of (26) this implies that

$$g\gamma\left(w^{\delta}-y^{\delta}\right)\leq w-\gamma y^{\delta}, \tag{27}$$

which, in turns, bounds the reported income y to be not smaller than a certain amount depending on parameters and the true income:

$$y\geq\left[\frac{g\gamma w^{\delta}-w}{(g-1)\gamma}\right]^{(1/\delta)}. \tag{28}$$

Moreover, the right hand side in (28) is defined only if $g\gamma w^{\delta}-w>0$, which yields a lower bound also for the true income w to be considered:

$$w>(g\gamma)^{1/(1-\delta)}. \tag{29}$$

Consider the first order Taylor expansion of (26) on the identity line, that is, on points $(w^{*},y^{*})\in\mathbb{R}^{2}$ such that $w^{*}=y^{*}$, which, in turn, implies $s(w^{*},y^{*})=0$:

$$s(w,y)=g\gamma\delta a^{\delta-1}(w-y)+o\left(\|(w,y)-(a,a)\|^{2}\right),$$

where $a\in\mathbb{R}$ satisfies (29), that is $a>(g\gamma)^{1/(1-\delta)}$. Hence, function

$$L(w,y)=g\gamma\delta a^{\delta-1}(w-y)$$

is the linear approximation of $s(w,y)$ around some point $(a,a)\in\mathbb{R}^{2}$. By denoting

$$\sigma=g\gamma\delta a^{\delta-1}, \tag{30}$$

we get a linear approximation of the progressive sanction $s(w,y)$ in (26) of the same form as in (3). Now, it remains to find suitable values for the critical point a, which translate in suitable values for σ through (30). This will be achieved thanks to condition (29), which will provide a lower bound for the critical point a:

$$\underline{a}=(g\gamma)^{1/(1-\delta)}. \tag{31}$$

By substituting (31) in (30) we get:

$$\underline{\sigma}=\delta \tag{32}$$

The choice of the minimum[29] value $\underline{\sigma}=\delta$ implies the largest participation in the amnesty, as comparative static analysis shows. To see this, consider function f defined in (18) and rewrite it as a function of both α and σ:

$$f(\alpha,\sigma)=\left\{A(\sigma)-B(\sigma)\left[r(\sigma)\right]^{1/\alpha}\right\}^{1-\alpha}-p\left\{\left[r(\sigma)\right]^{1/\alpha}\right\}^{1-\alpha}-(1-p).$$

[28]We calculated also an upper bound $\overline{\sigma}$ (see [16]), but it turned out to depend on reported income, reflecting the progressivity of the tax.

[29]By using such a point as the critical value for our approximation, we actually assume the relative sanction $\widehat{s}$ to be parametrized on the "poorest full complier" who earns the infimum (actually even not feasible) income $w=(g\gamma)^{1/(1-\delta)}$.

Straightforward calculations show that

$$\begin{cases} \dfrac{\partial f}{\partial \sigma} < 0 & \text{if } 0 < \alpha < 1 \\ \dfrac{\partial f}{\partial \sigma} > 0 & \text{if } \alpha > 1, \end{cases} \tag{33}$$

for all feasible values of other parameters.

Now suppose that either condition (21) or (22) of Proposition 1 holds. Since, as we have seen in Section 3, the solution set of system (12) is equivalent to the solution set of system (19), which is,

$$\begin{cases} f(\alpha, \sigma) \geq 0 & \text{if } 0 < \alpha \leq 1 \\ f(\alpha, \sigma) \leq 0 & \text{if } \alpha \geq 1, \end{cases}$$

inequalities (33) mean that, as the sanction σ increases the solution set $[\underline{\alpha}, \overline{\alpha}]$ "shrinks". In other words, this heuristic argument shows that the effects of increasing the sanction is a narrowing of the interval of relative risk aversion which characterizes amnesty participants, which must be the largest for the minimum sanction $\underline{\sigma} = \delta$, all other parameters remaining fixed.

References

[1] Allingham, M. G., Sandmo, A., 1972. Income tax evasion: a theoretical analysis, *Journal of Public Economics* **1**, 323-38.

[2] Alm, J. and Beck, W., 1990. Tax Amnesties and Tax Revenue, *Public Finance Quarterly* 18, 433-53.

[3] Andreoni, J., 1991. The Desirability of a Permanent Tax Amnesty, *Journal of Public Economics* **45**, 143-59.

[4] Bernasconi, M., 1995. Evasione Fiscale in Italia ed Evoluzione Recente degli Strumenti di Contrasto. In Bernardi, L. (ed.), *La finanza pubblica italiana - Rapporto 1995*. Il Mulino, Bologna, pp. 111-35.

[5] Bernasconi, M., 1998, Tax evasion and orders of risk aversion, *Journal of Public Economics* **67**, 123-34.

[6] Brito, D. L., Hamilton J. H., Slutsky S. M., Stiglitz J. E., 1995. Randomization in optimal income tax schedules, *Journal of Public Economics* **56**, 189-223.

[7] Chu, C. Y., 1990. Plea Bargaining with the IRS. *Journal of Public Economics* **41**, 319-33.

[8] Epstein, L. G., Zin, S. E., 1990. First Order Risk Aversion' and the Equity Premium Puzzle. *Journal of Monetary Economics*, **26**, 387-407.

[9] Fisher, R. C., Goodeeries, J. H., Young, J. C., 1989. Participation in Tax Amnesties: The Individual Income Tax. *National Tax Journal* ***42***, 15-27.

[10] Franzoni, L. A., 2000. Amnesties, Settlements and Optimal Tax Enforcement. *Economica* **57**, 153-176.

[11] Glen Ueng, K. L. and C.C. Yang, 2001. Plea Bargaining with the IRS: extensions and further results, *Journal of Public Economics* **81**, 83-98.

[12] Grossman, G. M., Katz, M. L., 1983. Plea Bargaining and Social Welfare. *American Economic Review* **73**, 749-57.

[13] Guiso, L., Paiella, M., 2001. Risk Aversion, Wealth and Background Risk, *CEPR Discussion Paper* **2728**.

[14] Marchese, C., Cassone, A., 2000. Tax Amnesty as Price-discriminating Behavior by a Monopolistic Government. *European Journal of Law and Economics* **9**, 21-32.

[15] Marchese, C., Privileggi, F., 1997. Taxpayers Attitudes Toward Risk and Amnesty Participation: Economic Analysis and Evidence for the Italian Case. *Public Finance/Finances Publiques* **52**, 394-410.

[16] Marchese, C., Privileggi, F., 2001. *Who participates in Tax Amnesties? Self-selection of Risk-Averse Taxpayers*. Working Paper **22**, Department of Public Policy and Public Choice *Polis*, University of Eastern Piedmont, Italy.

[17] Marchese, C., Privileggi, F., 2002. *The Cutoff Policy of Taxation When Taxpayers are Risk Averse* Working Paper **7**, Department of Economics and Finance DISEFIN, University of Genova, Italy.

[18] Maré, M. Salleo, C., 2003. Is One More Tax Amnesty Really That Bad? Some Empirical Evidence from the Italian 1991 VAT Amnesty, paper presented at the 59th IIPF congress, Prague.

[19] Monacelli, P., 1996. Problemi di stima dell'evasione fiscale: una rassegna dei metodi e degli studi effettuati per l'Italia. *Economia Pubblica* **36**, 103-126.

[20] Mookherjee, D., Png, I. P. L., 1989. Optimal Auditing, Insurance, and Redistribution. *Quarterly Journal of Economics* **104**, 399-415.

[21] Pestieau, P., Possen, U. M., Slutsky, S. M., 1998. The value of explicit randomization in the tax code. *Journal of Public Economics* **67**, 87-103.

[22] Yizhaki, S., 1987. On the Excess Burden of Tax Evasion. *Public Finance Quarterly* **15**, 123-37.

D Tables

Brackets of paid tax	**Extra payment due**
$0 - 10$	20% of the paid tax (with a minimum of 0.1)
$10 - 40$	18% of the paid tax
> 40	15% of the paid tax

Table 1: payment x in the 1991 tax amnesty (millions of Italian Lire).

Year	v	$\overline{\alpha}$	$\underline{e}$	$\underline{\alpha}$	$\overline{e}$
1985	13.36	4.45	35%	0.21	75%
1986	14.18	4.44	34%	0.13	74%
1987	15.37	4.44	34%	0.15	74%
1988	16.74	4.44	34%	0.18	73%
1989	18.27	4.48	34%	0.08	75%
1990	19.45	4.48	34%	0.08	75%

Table 2: estimation results, 1991 tax amnesty, case (a) (v = reported income in millions of Italian lire; $\overline{\alpha}$ = upper threshold risk aversion; $\underline{e}$ = lower threshold percentage tax evasion; $\underline{\alpha}$ = lower threshold risk aversion; $\overline{e}$ = upper threshold percentage tax evasion).

Year	v	$\overline{\alpha}$	$\underline{e}$	$\underline{\alpha}$	$\overline{e}$
1985	12.80	4.45	35%	0.20	74%
1986	13.20	4.46	34%	0.11	74%
1987	14.00	4.45	34%	0.13	74%
1988	15.50	4.44	34%	0.16	74%
1989	17.92	4.48	34%	0.07	75%
1990	18.20	4.49	34%	0.06	75%

Table 3: estimation results, 1991 tax amnesty, case (b) (v = reported income in millions of Italian lire; $\overline{\alpha}$ = upper threshold risk aversion; $\underline{e}$ = lower threshold percentage tax evasion; $\underline{\alpha}$ = lower threshold risk aversion; $\overline{e}$ = upper threshold percentage tax evasion).

Year	v	$\overline{\alpha}$	$\underline{e}$	$\underline{\alpha}$	$\overline{e}$
1985	16.10	4.43	35%	0.25	74%
1986	17.60	4.43	34%	0.19	73%
1987	19.60	4.42	35%	0.22	73%
1988	23.10	4.40	35%	0.26	73%
1989	25.84	4.45	35%	0.17	74%
1990	28.05	4.45	35%	0.18	74%

Table 4: estimation results, 1991 tax amnesty, case (c) (v = reported income in millions of Italian lire; $\overline{\alpha}$ = upper threshold risk aversion; $\underline{e}$ = lower threshold percentage tax evasion; $\underline{\alpha}$ = lower threshold risk aversion; $\overline{e}$ = upper threshold percentage tax evasion).

Year	v	$\overline{\alpha}$	$\underline{e}$	$\underline{\alpha}$	$\overline{e}$
1987	19.32	4.26	36%	0.24	73%
1988	20.17	4.18	36%	0.26	73%
1989	22.98	4.04	37%	0.22	74%
1990	24.33	4.07	37%	0.21	74%
1991	25.31	4.08	37%	0.28	74%
1992	30.32	5.64	29%	0.10	73%

Table 5: estimation results for a taxpayer endowed with median business income from manufacturing industry, 1994 tax amnesty (v = reported income in millions of Italian lire; $\overline{\alpha}$ = upper threshold risk aversion; $\underline{e}$ = lower threshold percentage tax evasion; $\underline{\alpha}$ = lower threshold risk aversion; $\overline{e}$ = upper threshold percentage tax evasion).

Year	γ	δ
1985	0.002249	1.274515
1986 − 1988	0.001441	1.292886
1989	0.001537	1.282436
1990	0.001508	1.282429
1991	0.001739	1.279154
1992	0.001615	1.285728

Table 6: parameters of the tax function $t(y) = \gamma y^{\delta}$ for years 1985-1992 (OLS estimation on logarithms of data in Italian Lire).