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Embodied Technological Change and Technological Revolution: Which Sectors Matter?

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\textbf{Abstract}

This paper develops a model that reproduces the essential characteristics of the recent ICT Revolution and its effects on economic growth using the framework of endogenous growth theory. In particular, it considers a multi-sectoral growth model in discrete time with infinite horizon, endogenous growth, embodied technological progress, horizontal differentiation and “lab-equipment” specification of R&D. In this model we show analytically that an increase in the productivity of the different sectors (final good sector, equipment sector, intermediate good sector, R&D sector) has an everlasting effect on growth. The conclusion is that when an ICT-driven growth episode is due to the rise of the productivity of one of these sectors, this episode is likely to have permanent effects in the economy. The numerical simulation of the model, then, allows to get some further insights. Finally, an extension of the model that takes into account the presence of learning and spillover effects is able to reproduce empirically the behavior of US productivity in the recent years.

\textbf{Keywords}: Information Technology, Endogenous Growth, Embodied Technological Progress.

\textbf{Journal of Economic Literature}: C63, O40.
1 Introduction

At the beginning of 2000s the sector of Information and Communication Technologies (ICT) has been considered of fundamental importance in the explanation of the economic performance of several countries.

Table 1 reports, with reference to the two decades from 1980 to 2000 for the US, the behavior of labor productivity (measured by GDP per hour worked) and of multi-factor productivity (that measures the overall efficiency with which combined inputs are used in the economy).

From its analysis it turns out that labor productivity growth continued to improve over the 1990s (in fact the average growth of 2% per year over the decade 1990-2000 has been determined by an average growth of 1.3% per year in 1990-95 and of 2.5% per year in 1995-2000) and that multi-factor productivity has been characterized by a structural improvement from the 1980s to the 1990s. In particular, the strong productivity growth registered in the computer sector (i.e. in the production of hardware) has led some analysts to conclude that the era of a “New Economy” has begun, a sort of “Third Industrial Revolution” in which information and communication technologies can be compared with the great inventions of the past that characterized the traditional Industrial Revolution. On the other hand, more sceptic analysts consider this phenomenon as nothing more than a stock market bubble, whose economic benefits will in the end be of negligible importance (see the interesting survey reported in The Economist, 2000).

For all these reasons a great attention has been devoted, both from an empirical and from a theoretical viewpoint, to the study of what has been called the “ICT Revolution” and of its effects on the economy.

On the empirical side, the main studies (Gordon, 1999, 2000; Jorgenson and Stiroh, 2000; Oliner and Sichel, 2000; Whelan, 2000) emphasize the strong productivity growth in the computer sector (particularly in the years 1995-1999, with an increase of about 42% per year). They also point out problems of measurement of the real contribution of ICT to the growth and productivity of the economy, together with the fact that the productivity growth in the computer sector has not been accompanied by spillovers from this sector to the rest of the economy. Therefore, there are reasonable doubts about the long-term viability of the ICT-driven economic expansion.

On the theoretical side, the most important contributions (Greenwood et al., 1997; Greenwood and Jovanovic, 1998, 1999; Greenwood and Yorukoglu, 1997; Hobijn and Jovanovic, 1999; Jovanovic and Rousseau, 2000) stress the importance of embodiment of technological progress (i.e. the fact that only the new machines incorporate the latest technological advances). But they also emphasize the fact that the ICT Revolution has been accompanied by some “puzzling phenomena”. In particular, on the real side there has been an initial strong decrease in the productivity of the whole economy (the so-called “productivity slowdown”) immediately after the beginning of the ICT Revolution (in the early ’70s), followed only later by a rise (in
the late '90s the rise of productivity in the computer sector has been larger than 40% in the US).

The main explanations proposed are based on the idea that the initial drop in productivity is due to an adoption period of the new technologies. It is characterized by learning costs and slow diffusion (and it is precisely in this phase that the “productivity slowdown” takes place), and it is followed by an age of maturity during which the ICT sector starts driving the whole economy.

A different view is taken by Boucekkine and de la Croix (2003), who propose to explain the essential characteristics of the ICT Revolution in the framework of endogenous growth theory, and to get insights about the determinants and the long term viability of an ICT-driven economic expansion. In particular, they consider the effects of positive supply shocks, and they find that only a positive productivity shock in the R&D sector has long term growth effects (while a similar shock in the capital sector is unable to produce similar effects). As a consequence, only if the ICT-driven growth episode is based on an increase in the productivity of R&D it is possible to conclude that this expansion is likely to have permanent effects in the economy.

The model presented in our paper tries to explain some characteristics of the ICT Revolution that emerge from the data, in particular the behavior of output growth as a consequence of productivity shocks linked to the introduction of new technologies. It is a multi-sectoral endogenous growth model and it reproduces some of the essential characteristics of the ICT-based economy, in particular the embodied nature of technological progress, the preeminent role of the R&D sector, and the link between innovation and market power.

The crucial differences of this model with respect to Boucekkine and de la Croix (2003) concern the composition assumed for the workforce (that is homogeneous, without distinction between skilled and unskilled workers) and the specification adopted for the R&D sector (the so-called “lab-equipment”, first introduced by Rivera-Batiz and Romer, 1991).

Our first finding is that the “lab-equipment” specification allows growth as a consequence of productivity shocks in all sectors (final good sector, equipment sector, intermediate good sector, R&D sector). Interestingly, the intensity of growth determined, in the long run, by these shocks is linked to the size (in terms of GDP) of the sector affected by the shock. More precisely, the effects on growth are stronger when the shocks concern the final good sector (that is very important, since more than 90% of the labor force is employed in this sector) or the R&D sector (that in this model is the true engine of growth). They are weaker when the shocks concern the equipment sector or the intermediate good sector (that are less important, in fact the latter employs less than 10% of the labor force - OECD, 2004). This first result is consistent with the findings of Triplett (1999), according to which we should not expect to see a major impact on growth from investment in computers. The shares of computers in the capital stock and in the input of capital services are small, and an input with a small share cannot give a large contribution to economic
growth. In fact, the software sector (i.e. the intermediate good sector of the model) has not a very strong weight in the economy, and this explains why it has not a strong effect on growth. This result allows to solve (at least partially) the so-called “Solow paradox”, according to which “...we can see computers everywhere except in the productivity statistics” (Solow, 1987). In reality, computers are not everywhere (in terms of GDP), and therefore the fact that they do not give a very strong contribution to growth does not represent a paradox, but it is simply a consequence of their relatively small size in the economy.

Our second finding is that the new technologies initially do not determine a very strong growth in the economy, and only in a second phase they have an important effect on productivity. This result is consistent with the findings of Gordon (2004). This study proposes an explanation based on the notion of “intangible assets”, with a key role of the so-called “weightless economy” (see also Quah, 1999). In particular, it highlights the fact that US productivity grew more rapidly after the mid-2000 peak in ICT investment and in the stock market than in the 1995-2000 period when ICT investment was very strong, and accelerated in 2002-2003 when ICT investment boom collapsed. The idea used by Gordon to explain this pattern is that the productivity revival in the late 1990s was mismeasured, due to the exclusion of massive amounts of “intangible” or “hidden” capital from the investment and capital data in the national accounts. In fact, together with computer hardware there is a large quantity of complementary capital investment represented by intangible capital (including software), new business processes and human capital that are not included in the national accounts as investment and are “hidden” as a business expense. The boom in measured ICT investment in the late 1990s was accompanied also by a boom in unmeasured (i.e. hidden) capital. Therefore, the measured productivity growth was substantially lower than the true productivity growth, while after 2000 the reverse happened. As a consequence, before 2000 there have been substantial investments in intangible assets with no effects on productivity, while after 2000 there have been spillover effects of the intangible assets with an increase in the productivity. This dynamics can explain why US productivity growth has been particularly strong during a period (the years 2000-2003) of relatively low measured ICT investment. This behavior of productivity is replicated quite well in the simulations of our model, even if it does not explicitly include a particular mechanism mimicking the role of intangible assets.

Our third finding is the possibility of reproducing the behavior of US productivity in the recent years. As stressed above, before 2000 there have been strong ICT investments with low effects on productivity, while after 2000 there has been an acceleration in productivity in coincidence with relatively low ICT investments (see Figure 1). The idea in this case is to consider an extension of the model, with learning and spillover effects from the ICT sector to the rest of the economy. Introducing new technologies requires a period of adoption characterized by learning costs and slow diffusion, and only after this period there will be effects in the whole economy, driven by the ICT sector. In particular, a greater use of ICT may also contribute to spillover effects (i.e. one firm’s use of ICT has positive spillovers on the economy as
The presence of a learning specification (i.e. the fact that the productivity of a plant depends positively on its age, due to learning by doing), which then originates a spillover effect, is precisely what allows to reproduce the behavior of US productivity in the last decade, with low productivity growth before 2000 and an increase in productivity growth in 2002-2003.

The paper is organized as follows. Section 2 illustrates the model, with the different sectors in the economy. Section 3 describes the equilibrium conditions, characterizes analytically the balanced growth path and derives the corresponding steady state system, from which it is possible to find some analytical results concerning the effects on growth of different shocks that can interest the economy. Section 4 considers the numerical simulation of a calibrated version of the model, which allows to obtain interesting results concerning the short run response of the system to the shocks and the robustness of the model. Furthermore, the extension of the model that takes into account the presence of learning and spillover effects is introduced. Section 5 concludes, while all the computations are available in a separate Appendix.

2 The model

The model developed is based on Boucekkine and de la Croix (2003) and Romer (1990), and it is a multi-sectoral model in discrete time with infinite horizon, endogenous growth and horizontal differentiation. The economy is characterized by 4 sectors: the final good sector, the equipment sector, the intermediate good sector and the R&D sector. Technological progress is mainly embodied (the idea is that the new softwares can be run only on the most recent hardware) and the innovators have a market power represented by copyrights, in order to stimulate innovation (that corresponds to an expansion in the available varieties of softwares) and growth. All these elements are important to reproduce the essential characteristics of the ICT sector.

As stressed in the Introduction, two crucial elements distinguish this model from that of Boucekkine and de la Croix (2003). The first is the assumption of a homogeneous workforce, since the final good sector uses efficient capital and labor, while in the model of Boucekkine and de la Croix it is assumed that the final good sector uses efficient capital and two types of labor, skilled and unskilled. The second difference is the specification of the R&D sector, since our model assumes the “lab-equipment” specification, according to which the R&D input is expressed in units of the final good (and therefore labor is not an input of the research process), while in the model of Boucekkine and de la Croix it is assumed that the R&D sector uses skilled labor.
2.1 The final good sector

The final good sector produces a composite good (used to consume or to invest in physical capital) using efficient capital (bought from the equipment sector) and labor.

Production is obtained through the following Cobb-Douglas technology (as in Solow, 1960):

\[
Y_t = z_t K_t^\alpha L_t^{1-\alpha} \quad \alpha \in [0,1]
\]  (1)

where \(z_t\) represents total factor productivity, and the stock of capital is defined as:

\[
K_t = \sum_{s=-\infty}^{t} E_s (1-\delta)^{t-s}
\]  (2)

where \(E_s\) represents the efficient capital bought from the equipment sector at time \(s\) and \(\delta\) is the physical depreciation rate (constant).

The discounted profits of investing \(E_t\) in efficient capital are given by:

\[
\pi_t = \sum_{s=t}^{\infty} [Y_s - w_s L_s] R_t^s - d_t E_t
\]

where:

\[
R_t^s = 1 \quad R_t^s = \prod_{\tau=t+1}^{s} \left( \frac{1}{1+r_{\tau}} \right)
\]

represent the discount factors at time \(t\) and \(s\) respectively, \(r_{\tau}\) is the interest rate at time \(\tau\), \(w_s\) is the wage for labor input at time \(s\) and \(d_t\) is the price of efficient capital at time \(t\). The representative firm chooses efficient capital and labor input in order to maximize its discounted profits taking prices as given and subject to its technological constraints:

\[
\max_{E_t,\{L_s\}_{s=t}^{\infty}} \pi_t \quad \text{s.t.} \quad (1), (2)
\]

From the solution of this problem we get:

\[
L_t = \left( \frac{(1-\alpha) z_t}{w_t} \right)^{\frac{1}{\alpha}} K_t
\]  (3)

that is the demand for labor by the final good sector.

2.2 The equipment sector

The equipment sector produces efficient capital (sold to the final good sector) using physical capital (hardware) bought from the final good producers and immaterial
capital (software) bought from the intermediate good producers. Efficient capital is produced with a constant return to scale technology:

\[ E_t = e_t Q_t^\lambda I_t^{1-\lambda} \quad \lambda \in (0, 1) \]  

(4)

where \( e_t \) is a productivity variable, \( I_t \) represents physical capital (hardware) and \( Q_t \) represents immaterial capital (software). The immaterial capital is built from a series of specialized intermediate goods, according to a Dixit-Stiglitz CES function:

\[ Q_t = \left( \int_0^{n_t} x_{i,t}^{\sigma - 1} \frac{\sigma}{\sigma - 1} \right)^\frac{\sigma - 1}{\sigma} \]  

(5)

where \( n_t \) is the number of varieties of intermediate input available in \( t \), \( x_{i,t} \) is the quantity of intermediate input of variety \( i \) used in \( t \) and \( \sigma > 1 \) is the elasticity of substitution between two varieties.

The profits of the equipment sector at time \( t \) are:

\[ \pi'_t = d_t E_t - I_t - \int_0^{n_t} p_{i,t} x_{i,t} di \]

where \( p_{i,t} \) is the price of software of variety \( i \) at time \( t \). The representative firm chooses the investment in physical capital and in immaterial capital in order to maximize profits taking prices as given and subject to its technological constraints:

\[ \max_{I_t, x_{i,t}} \pi'_t \]

s.t. \quad (4), (5)

From the solution of this problem we get:

\[ x_{i,t} = \left( \frac{\phi}{q_t} \right)^\sigma Q_t p_{i,t}^{-\sigma} \]  

(6)

that is the demand for intermediate input \( i \) by the firms of the equipment sector at time \( t \) (here we have defined \( q_t = \frac{Q_t}{I_t} \) and \( \phi = \frac{\lambda}{1-\lambda} \)).

### 2.3 The intermediate good sector

The intermediate good sector produces immaterial capital (software, sold to the equipment sector) and it researches for new varieties, in order to expand the range of software (horizontal differentiation).

#### 2.3.1 The production activity

The variety \( i \) of software is produced according to a linear technology that uses labor as the only input:

\[ x_{i,t} = \tau_t L_{i,t} \]  

(7)
where $L_{i,t}$ is the labor employed in the intermediate good sector and $\tau_t$ represents labor productivity. The producer behaves in a monopolistic way (since market power is given by the presence of copyrights which have an infinite lifetime - i.e. the inventor of a new variety of software obtains these copyrights forever -) and his profit is:

$$\pi''_{i,t} = p_{i,t}x_{i,t} - w_tL_{i,t} = \left(p_{i,t} - \frac{w_t}{\tau_t}\right)x_{i,t}$$

The price of output is chosen in order to maximize this profit subject to the demand formulated by the equipment sector. The problem solved by the firm is therefore:

$$\max_{p_{i,t}} \pi''_{i,t}$$

s.t.

(6)

from which we get:

$$p_{i,t} = \left(\frac{\sigma}{\sigma - 1}\right)\frac{w_t}{\tau_t} \forall i \in [0, n_t]$$

(8)

i.e. the output price is a mark-up over unit labor cost (where the mark-up rate depends on the price elasticity of demand, that is given by $-\sigma$).

### 2.3.2 The research activity

The intermediate good sector, besides producing softwares, researches for new varieties of immaterial capital, in order to expand their range. In this version of the model we assume the so-called “lab-equipment” specification of R&D (introduced by Rivera-Batiz and Romer, 1991), according to which the cost to create a new type of product (i.e. a new variety of software) is fixed at $\eta$ units of $Y$ (i.e. R&D is one of the uses of the flow of current output). The underlying assumption is that the technology that uses research to create new products or ideas has the same factor intensities as the technologies that generate consumables and intermediate goods.

There will be entry of new firms in the economy until the cost to create a new variety of software is equal to the discounted flow of profits linked to one invention. This equilibrium condition can be written as:

$$\eta = \sum_{z=t}^{\infty} R^z_t \pi''_{i,z}$$

and since:

$$\pi''_{i,t} = \left(p_{i,t} - \frac{w_t}{\tau_t}\right)x_{i,t} = \frac{1}{\sigma - 1} \frac{w_t}{\tau_t}x_{i,t}$$

the free-entry condition is:

$$\eta = \sum_{z=t}^{\infty} R^z_t \frac{1}{\sigma - 1} \frac{w_z}{\tau_z}x_{i,z}$$
2.4 Household behavior

After the 4 sectors that characterize the economy we also consider the household present in this economy. The representative household consumes, saves for future consumption and supplies labor. The utility of the representative household at time 0 is:

\[ u_0 = \sum_{t=0}^{\infty} \rho^t \ln C_t \]

i.e. it is the discounted sum of the instantaneous utilities from 0 to \( \infty \), where \( \rho \) is the psychological discount factor and the instantaneous utility function is assumed logarithmic. The corresponding budget constraint is:

\[ A_{t+1} = (1 + r_{t+1}) A_t + w_t L - C_t \]  \hspace{1cm} (9)

where \( A_t \) represents the assets held by the household at time \( t \).

The representative household chooses the assets held in order to maximize the discounted utility subject to the budget constraint:

\[ \max_{\{A_{t+1}\}_{t=0}^{\infty}} u_0 \hspace{1cm} s.t. \hspace{1cm} (9) \]

From the solution of this problem we get:

\[ \frac{C_{t+1}}{C_t} = (1 + r_{t+1}) \rho \]  \hspace{1cm} (10)

3 The equilibrium

We now characterize the equilibrium of the economy in the model, determined by the equilibrium both on the labor market and on the final good market.

First of all, the equilibrium on the labor market implies that the labor force is employed either in the final good sector or in the intermediate good sector:

\[ L = L_t + \int_{0}^{n_t} L_{i,t} di \]  \hspace{1cm} (11)

where the supply of labor can be normalized to 1 (i.e. \( L = 1 \)).

The equilibrium on the final good market, then, implies:

\[ Y_t = C_t + I_t + \eta \Delta n_t \]  \hspace{1cm} (12)

where \( \eta \Delta n_t \) is the cost of research for new varieties (for all the algebraic details concerning this Section, see the separate Appendix).
The first-order optimality conditions and the market equilibrium relationships obtained above can be used to derive the equilibrium conditions in the model. The results obtained are summarized in the following Proposition:

**Proposition 1** Given the initial conditions $K_{-1}$ and $n_{-1}$ an equilibrium is a path:

$$\{w_t, q_t, C_t, I_t, K_t, n_t, r_{t+1}\}_{t \geq 0}$$

that satisfies the following equations:

1. $$(\frac{(1 - \alpha)}{w_t})^{\frac{1}{\alpha}} K_t + n_t \left(\frac{\sigma - 1}{\sigma}\right)^{\sigma} \left(\frac{\phi}{w_t}\right)^{\sigma} q_t^{1-\sigma} I_t^{\sigma-1} = 1$$  \hspace{1cm} (13)

2. $$z_t^{\frac{1}{\alpha}} K_t \left(\frac{1 - \alpha}{w_t}\right)^{\frac{1-\alpha}{\alpha}} = C_t + I_t + \eta \triangle n_t$$  \hspace{1cm} (14)

3. $$\alpha z_t^{\frac{1}{\alpha}} (1 - \lambda) e_t q_t^{\lambda} \left(\frac{1 - \alpha}{w_t}\right)^{\frac{1-\alpha}{\alpha}} = 1 - \left(\frac{1 - \delta}{1 + r_{t+1}}\right) \left(\frac{e_t}{e_{t+1}}\right) \left(\frac{q_t}{q_{t+1}}\right)^{\lambda}$$  \hspace{1cm} (15)

4. $$\frac{C_{t+1}}{C_t} = (1 + r_{t+1}) \rho$$  \hspace{1cm} (16)

5. $$K_t = (1 - \delta) K_{t-1} + e_t q_t^{\lambda} I_t$$  \hspace{1cm} (17)

6. $$\frac{w_t q_t}{\tau_t \phi} = n_t^{\frac{1}{\alpha}} \left(\frac{\sigma - 1}{\sigma}\right)$$  \hspace{1cm} (18)

7. $$\frac{\tau_t^{1 - \sigma} \left(\sigma - 1\right)^{1 - \sigma} \sigma^\sigma \eta r_{t+1}}{\phi^\sigma} \left(\frac{1}{1 + r_{t+1}}\right) = w_t^{1 - \sigma} q_t^{1 - \sigma} I_t$$  \hspace{1cm} (19)

### 3.2 The balanced growth path

After the characterization of the equilibrium, we analyze the balanced growth path of the model. In this case we assume that the exogenous productivity variables $z_t$, $e_t$ and $\tau_t$ and the interest rate $r_t$ are constant in the long term, while each endogenous variable grows at a constant rate along a balanced growth path. In this way it is possible to determine the relations among the different growth factors. The results obtained are expressed in the following Proposition:

**Proposition 2** If $q_t$ grows at a factor $g_q > 1$, then all the other variables grow at strictly positive rates with:

$$g_Y = g_C = g_I = g_n = g_w = (g_q)^{\frac{\lambda}{1-\alpha}}$$

$$g_K = (g_q)^{\frac{\lambda}{1-\alpha}}$$
Hence, along a balanced growth path output, consumption, investment, number of varieties and wages grow at the same rate, while the stock of capital grows faster (since it includes improvements in the embodied productivity). The system is therefore able to display growth of the economy.

In this case, in particular, the growth rate of the number of varieties of softwares $g_n$ is equal to the growth rates of output, consumption, investment and wages. This is due to the “lab-equipment” specification assumed for R&D, and it implies that the technology that uses research to create new products has the same factor intensities as the technologies that generate consumables and intermediate goods. As a consequence, the corresponding growth rates are equal. Another result is that, for given $g_q > 1$, all the growth rates are increasing functions of $\lambda$, the softwares share in efficient capital. This reflects the fact that the engine of growth in the model is the expanding varieties of softwares, so that the bigger the impact of the latter on efficient capital, the higher the resulting long run growth rates.

### 3.3 The stationarized dynamic system and the steady state system

After the balanced growth path, we also study the restrictions on the long run levels, in order to obtain the additional information necessary to determine $g_q$. Computing these restrictions from the dynamic system expressed by equations (13) – (19) we obtain 7 equations with 8 unknowns ($\pi, \bar{q}, \overline{C}, \overline{T}, \overline{K}, \pi, \tau$ and $g_q$ - since all the other growth rates can be expressed in terms of $g_q$ -). The system in terms of levels is therefore undetermined (this is an usual property of endogenous growth models), but we can rewrite it in such a way that the indeterminacy is eliminated. This is done by “stationarizing” the equations by means of some auxiliary variables, that is by rewriting the system in terms of variables that are stationary (i.e. constant) in the steady state. In this way we obtain the stationarized dynamic system corresponding to the original one.

In a similar way we get the steady state system corresponding to the stationarized one. In this case we have a system of 7 equations with 7 unknowns ($\hat{q}, \hat{C}, \hat{T}, \hat{K}, \hat{\pi}, g, r$) that can be solved (at least from a theoretical viewpoint). In reality, given the complexity of the long run steady state, we cannot derive an analytical solution, but we can obtain some interesting intermediate results. In particular, each of the other unknowns can be expressed as a function of the growth factor $g$.

The results obtained are reported in the following Proposition:

**Proposition 3** At any growth factor $g$, there exist explicit functions expressing the long run levels $\hat{q}, \hat{C}, \hat{T}, \hat{K}, \hat{\pi}, r$ exclusively in terms of $g$:

\[
\hat{q} = \Psi_{\hat{q}}(g) \quad \hat{C} = \Psi_{\hat{C}}(g) \quad \hat{T} = \Psi_{\hat{T}}(g)
\]

\[
\hat{K} = \Psi_{\hat{K}}(g) \quad \hat{\pi} = \Psi_{\hat{\pi}}(g) \quad r = \Psi_{r}(g)
\]
The following Corollary then holds:

**Corollary 4** There exists an explicit function $\Psi(g)$ such that the long run equilibrium growth factor value solves the equation $\Psi(g) = 0$.

This means that using the $g$-functional expressions of the long run levels (those of Proposition 3) in any equation of the steady state system we obtain an explicit equation involving only $g$. In this way the system can be reduced to an explicit scalar equation involving the growth factor $g$, and once this equation is solved the remaining long run levels can be determined (at least from a theoretical viewpoint) using the explicit $g$-functions.

From the expressions of Proposition 3 we can obtain some other interesting results. They are summarized in the following Proposition:

**Proposition 5** Assuming that a solution for the steady state system exists, the long run values of $z$ and $e$ affect the stationary values $\hat{q}$, $\hat{n}$, $\hat{K}$ and $\hat{C}$ and the long run value of $\tau$ affects the stationary values $\hat{n}$, $\hat{K}$ and $\hat{C}$. Furthermore $z$, $e$ and $\tau$ have an impact on the long term growth factor $g$.

According to these results, permanent changes in $z_t$ (the productivity in the final good sector), in $e_t$ (the productivity in the efficient capital sector) and in $\tau_t$ (the labor productivity in the intermediate good sector) will affect the long run growth rate of the economy. This is the main difference of our version of the model compared with the version without “lab-equipment”, in which long term growth turns out to be insensitive to changes in $z_t$ and $e_t$. In that version, only if the productivity of R&D is boosted, stimulating the creation of softwares, there is long term growth of the economy. Considering for instance the effects of changes in $z_t$, the difference between the versions of the model without and with “lab-equipment” is based on the fact that in both cases long term growth relies on horizontal differentiation of R&D. However, in the “lab-equipment” version the production function in the R&D sector is implicitly the same of the final good sector, while in the other version the production function in the R&D sector is more labor intensive. The result is that a shock on the total factor productivity of the final good sector has an effect on long term growth in the “lab-equipment” case (because it corresponds to a shock on the productivity of the R&D sector, that is the engine of growth in this kind of model), while it does not have this effect in the other case.

The present model supports therefore the view according to which the recent ICT-based economic expansion (mainly driven by an acceleration of productivity in the production of efficient capital) should have a permanent effect on growth.

These are the results that can be obtained analytically. In order to get further insights it is necessary to resort to the numerical simulation of the model.
4 Simulation of the model

The model described above can be simulated numerically in order to get information concerning, in particular, the behavior of the economy (both in the long run and in the short run) as a consequence of shocks that can hit the system, and the robustness of the model itself.

4.1 Calibration

The calibration of the model is based essentially on the recent data of the US economy, and the different parameters are fixed to values that can be considered reasonable on the basis of the empirical evidence. In addition, these values are chosen in such a way that they match a series of moments of the steady state of the model. In particular, these target moments are those based on the recent study of Atkinson and Andes (2010).

First of all, concerning the parameters related to the technology, the labor share in the final good sector \((1 - \alpha)\) is equal to 0.65 (hence \(\alpha = 0.35\)), close to the value of 0.70 used by Boucekkine and de la Croix (2003) in their model (where, differently from the present study, there is a distinction between skilled and unskilled labor) and to the values chosen in other studies by different authors (see for example Alvarez and Lucas, 2007). With reference to the share of software in the production of efficient capital \(\lambda\), this parameter is used to calibrate the size of the new economy in terms of the labor force employed in the intermediate good and in the R&D sector, and it is set equal to the value 0.85 (significantly different from the value of 0.5 used in another paper of Boucekkine et al., 2002). The consequence of this assumption is that the elasticity of substitution between varieties of softwares \(\sigma\) is equal to 1.31 (again significantly different from the value of 3 of the model of Boucekkine and de la Croix, 2003).

Two other important parameters are the rate of depreciation of physical capital \(\delta\) and the psychological discount factor \(\rho\). The first one is set equal to 10%, consistent with some other studies (see for instance Nadiri and Prucha, 1997, who estimate a rate of depreciation for physical and R&D capital in US of 6% and 12% respectively), while the second one is fixed at 97%, that is exactly the value chosen by Boucekkine and de la Croix (2003) in their work.

Another set of parameters is represented by the values that express the productivities in the different sectors. In particular, the productivity in the final good sector \(z\) and the productivity in the equipment sector \(e\) are fixed, respectively, equal to 3 and 12, because these values are such that the capital/output ratio turns out to be equal to 2. The labor productivity in the intermediate good sector \(\tau\), then, is fixed equal to 0.25 (close to the value of 0.2 used by Boucekkine et al., 2002) because in this way (together with the value chosen for the parameter \(\lambda\) considered above) we have that about 5% of the labor force is employed in the intermediate good and in the R&D sector (these results are compatible with the available data, see for
instance Atkinson and Andes, 2010). The last parameter is the cost of a new variety of software expressed in units of output \( \eta \) (that derives from the “lab-equipment” specification for R&D). It is used to calibrate the size of the R&D sector and is equal to 20, so that the R&D expenditure is approximately equal to 3.5% of GDP (again consistent with available studies, see for instance Atkinson and Andes, 2010). The interest rate, finally, is 4%.

The moments of the steady state that have to be reproduced and the values of the different parameters that allow to reach these targets are reported in Table 2.

With these values the model leads to a growth rate of output equal to 0.82% per year, that can be interpreted as the part of output growth generated by embodied technical progress, and is in line with the available data (see for instance Greenwood et al., 1997).

The benchmark case can now be used first of all to study the effects of different types of shocks that can interest the economy, and then to verify the robustness of the model.

### 4.2 Long run effects of productivity shocks

The first set of results concern the analysis of the long run effects of shocks that can hit the economy. In particular, we consider a shock on \( z \) (the productivity in the final good sector), a shock on \( e \) (the productivity in the equipment-efficient capital-sector) and a shock on \( \tau \) (the productivity in the intermediate good-software-sector). We then study also how the economy reacts to a reduction in \( \eta \) (the cost of a new variety of software in units of output, i.e. the cost of research). All the shocks considered are permanent (from \( t = 0 \)) and have an intensity equal to 1%.

The first simulation concerns a shock on \( z \). From the analytical results (Proposition 5) we know that this has an impact on the long term growth rate (this is a central difference of the version of the model with “lab-equipment” with respect to the one without “lab-equipment”). In fact, the growth rate of production (that, in this version of the model, is also equal to the growth rate of the number of patents, i.e. of softwares) increases in the long run (Figure 2). This is due to the fact that the “lab-equipment” specification implies that the production function in the R&D sector is the same as in the final good sector. An increase in the productivity of the final good sector, therefore, is equivalent to an increase in the productivity of the R&D sector. Since the latter is the engine of growth of the model (through the expansion in the varieties of softwares) this determines an effect on long term growth, with an increase in the growth rate of production of about 4.8% with respect to the initial steady state value. A similar behavior is that of the growth rate of the productivity of labor in the final good sector (Figure 3), that in the long run increases of about 2% with respect to the initial level. Another important quantity is the relative price of capital (given by \( \frac{1}{e \eta} \)), that declines over time. Its growth rate is therefore negative (Figure 4), and in the long run it increases in absolute value of
about 4.5%. Finally, with reference to the allocation of workers between final good sector and intermediate good sector, in the long run there is a small reallocation from the former to the latter (about 0.1% with respect to the initial steady state value, Figure 5).

The second simulation concerns a shock on $e$. Also in this case we know from the analytical results that this has an impact on the long run growth rate (contrary to what happens in the version of the model without “lab-equipment”). As before, this long run effect is due to the fact that the “lab-equipment” specification implies the same production function in the final good sector and in the R&D sector. An increase in the productivity of the equipment sector increases the production of efficient capital and therefore of the final good. This corresponds to an expansion in the R&D sector, and since this sector is the engine of growth in the model the outcome is an effect on long term growth. The final result is an increase in the growth rate of production of about 1.5% with respect to the initial steady state value (Figure 6), and a similar behavior of the growth rate of labor productivity in the final good sector, that increases in an analogous measure (Figure 7). With reference to the relative price of capital, it decreases at an increasing rate (Figure 8). Finally, there is a very small reallocation of the labor force from the final good sector to the intermediate good sector (about 0.05% with respect to the initial steady state level, Figure 9).

The last simulations concern a shock on $\tau$ (with the same kind of results obtained considering a shock on the productivity of the equipment sector) and a decrease in $\eta$ (with results very close to those obtained considering an increase in the productivity of the final good sector).

In conclusion, with reference to the quantitative effects determined by the different types of shocks on the relevant variables in the long run, the increase in $z$ and the decrease in $\eta$ have the strongest effect on growth. In fact, on the one hand, the final good sector affects growth directly, on the other hand the R&D sector represents the engine of growth in this model. On the contrary, the increase in $e$ and the increase in $\tau$ have a less strong effect on growth, since these sectors affect growth indirectly. In addition, the shocks on $z$ and on $\eta$ have a stronger effect also on the growth rate of labor productivity in the final good sector, on the (negative) growth rate of the relative price of capital and on the labor force employed in the intermediate good sector. In fact, the intensity of growth in the long run as a consequence of shocks is linked to the size (in terms of GDP) of the sector that is interested from the shock. The effects on growth are stronger when the shocks concern the final good sector (that is very important, in fact more than 90% of the labor force is employed in this sector) and the R&D sector (that in this model is the true engine of growth). They are weaker when the shocks concern the equipment sector and the intermediate good sector (that are less important, in fact the latter employs less than 10% of the labor force). The quantitative results (expressed as percentage increases from the initial steady state levels - before each shock - to the final steady state levels - after the shock - of the different variables) are reported in Table 3.
4.3 Short run effects of productivity shocks

The second set of results concern the analysis of the short run effects of the shocks considered above.

With reference to a positive shock on $z$, first of all this reduces the cost of production of the final good and initially determines a reallocation of the labor force favorable to the final good sector and at the expenses of the intermediate good sector. As a consequence, in the first period immediately after the shock there is a very strong increase in the growth rate of production (almost 8% with respect to the initial steady state value), due to the contemporaneous increase in the productivity and in the labor force of the final good sector. At the same time, the growth rate of the productivity of labor in the final good sector has a strong increase (more than 3% with respect to the initial steady state level) and the growth rate of the relative price of capital has a very strong increase in absolute value. The labor force employed in the intermediate good sector (i.e. in the production of software) is interested, instead, by an important reduction (about 1.3% with respect to the initial steady state level). As time passes, the labor force employed in the final good sector reduces (because productive capacity has reached its maximum) and there is again a reallocation of this labor force in favor of the intermediate good sector. The growth rates therefore partially reduce, and the long run effect is an increase in the growth rate of production of about 4.8% and in the growth rate of labor productivity in the final good sector of about 2% with respect to their initial steady state values. The growth rate of the relative price of capital partially decreases (in absolute value) and the final result is an increase with respect to the initial steady state level. For the same reason, the initial reduction in the labor force employed in the intermediate good sector is almost completely recovered in the following two periods, then there is a further increase and the long run result is a small reallocation of workers from the final good sector to the intermediate good sector.

With reference to a positive shock on $e$, then, the long run effect is qualitatively very similar to that of the first shock considered, but the short run behavior of the variables is very different. In particular, the productivity increase in the equipment sector increases the profitability of producing efficient capital and the marginal return to both softwares and hardware, stimulating the demand for these inputs. This in turn favors the creation and production of softwares and determines an initial strong increase in the labor fraction employed in the intermediate good sector (about 2% with respect to the initial steady state value), launching the growth of the economy. Since this growth is based on the expansion of the intermediate good sector (and not directly of the final good sector), nevertheless, the increase of the growth rates is less strong than in the first simulation considered, and it requires more time. In fact, initially there is a reduction of both the growth rate of production and the growth rate of labor productivity in the final good sector (due to the reduction of the labor force in the final good sector, that affects growth directly). These growth rates then recover, and in the long run they increase of about 1.5%
with respect to their initial steady state values. At the same time, the growth rate of the relative price of capital has an initial decrease then a strong increase, and the final result is an increase (in absolute value) with respect to the initial steady state level. Furthermore, the initial strong reallocation of workers in favor of the intermediate good sector is not long lasting. In a few periods the labor fraction in the software sector returns to a level that is only slightly higher (about 0.05%) than the initial steady state value (since labor force reallocates in favor of the final good sector, contributing to the increase in the growth rates), and it remains to this level in the long run.

With reference to a positive shock on $\tau$, the results are very close to those obtained in the case of a shock on the productivity of the equipment sector. With reference to a decrease in $\eta$, finally, the results are similar to those obtained considering an increase in the productivity of the final good sector.

The results of the simulations can be used also to compare the different shocks. The first observation is that the various shocks considered affect in a different measure the growth rates and the allocation of the labor force between final good sector and intermediate good sector. Furthermore, it turns out that the shock on the productivity of the final good sector and that on the cost of the R&D sector produce very similar consequences. The same is true for the shock on the productivity of the equipment sector and for that on the productivity of the intermediate good sector. More precisely, the increase in $z$ and the decrease in $\eta$ both determine (initially) a strong increase in the growth rate of production, in the growth rate of labor productivity in the final good sector and in the growth rate of the relative price of capital (that then partially reduce). The increase in $e$ and the increase in $\tau$, instead, have a less strong effect on these growth rates (that initially decrease, then increase). With reference to the allocation of the labor force, then, the increase in $z$ and the decrease in $\eta$ determine (initially) a reallocation favorable to the final good sector and at the expenses of the intermediate good sector (then the situation reverses). The increase in $e$ and the increase in $\tau$, instead, have the opposite effect (initially the labor force employed in the intermediate good sector increases, then decreases).

All these results are summarized in Table 4.

### 4.4 Robustness analysis

The benchmark case can be used also to verify the robustness of the model. To this end we start from the benchmark, we modify significantly some parameters, we simulate again the model and we study the effects of the different shocks.

The first variation with respect to the benchmark is a change in the parameter $\lambda$ (the share of software in the production of efficient capital, that affects the weight of intangible assets). In particular, the value of this parameter changes from 0.85 (as in the benchmark) to 0.95 (hence the change is larger than 10%), while the values of all the other parameters remain unchanged with respect to the benchmark. We
compute the new steady state, then we perform the simulations (exactly as in the initial version of the model) introducing the same types of shocks considered in the benchmark case. First of all, with the new value of $\lambda$ the growth rate in the steady state is equal to 0.96% (instead of 0.82% of the benchmark). The behavior of the economy as a consequence of the shocks, then, is very close to the one obtained in the initial calibration of the model. In particular, both the shocks on $z$ and on $\eta$ determine an initial very strong increase in the growth rates (that then partially reduce) and an initial reduction in the labor fraction of the intermediate sector (that then recovers and in the long run increases with respect to the initial steady state value). On the contrary, the shocks on $e$ and on $\tau$ produce an initial reduction in the growth rates (that then recover and in the long run increase with respect to the initial values) and an increase in the labor force of the intermediate sector (that then decreases and reaches a level, in the long run, only slightly higher than the initial one).

With reference to the magnitude of the effects of the shocks on the different variables in the long run, the values obtained are those reported in Table 5. From their analysis it turns out that they are very close to those obtained in the benchmark case, hence the model proves to be robust with respect to the change introduced in the parameter $\lambda$.

The second variation with respect to the benchmark is that of the parameter $\eta$ (the cost of a new variety of software in units of output, that is a measure of the cost of R&D), that changes from 20 to 17 (hence exactly of 15%), while the other parameters remain at the values of the benchmark. The new growth rate in the steady state in this case is 1.45%. We then simulate the model, introducing the different types of shocks. Also in this situation the qualitative behavior of the economy in response to these shocks is the same of the benchmark case, while the quantitative effects are those reported in Table 6. Again they are close to the values obtained for the original parameterization of the model, that therefore is sufficiently robust also with respect to changes in $\eta$.

The third variation with respect to the benchmark concerns the parameter $e$ (the productivity of the equipment sector, that affects also the capital/output ratio and therefore the importance of the so-called “weightless economy”). It changes from 12 to 14 (i.e. of more than 15%), while again the values of all the other parameters remain unchanged with respect to the benchmark. Also in this case we compute the new steady state of the model and we perform the simulations. The new growth rate in the steady state turns out to be 0.98%, and again the economy reacts qualitatively to the different shocks as in the benchmark case. The quantitative effects of the shocks on the different variables, then, are those reported in Table 7. Again it is possible to observe that they are sufficiently close to the values obtained in the benchmark case, hence also when we introduce a change in $e$ the model proves to be robust.
In conclusion, the model considered is quite robust, since the results obtained for the initial calibration remain valid when we alter significantly some parameters. This is a good property of the model, and represents an important finding of the analysis developed.

4.5 Spillover effects

An extension of the model that we then consider is the presence of spillover effects from the ICT sector to the rest of the economy. It is exactly this element that seems to be lacking from the available data (as outlined in the Introduction). A possible explanation is that the introduction of the new technologies requires a period of adoption characterized by learning costs and slow diffusion, so that only after this period there will be effects in the whole economy, driven by the ICT sector.

With reference to this aspect, we can observe that greater use of ICT may also contribute to spillover effects (i.e. one firm’s use of ICT has positive spillovers on the economy as a whole), improving the overall efficiency of the economy (measured by multi-factor productivity). In fact, the diffusion of ICT may also have impacts that go beyond individual firms as it may help establish ICT networks, which produce greater benefits the more customers or firms are connected to the network. Recent data show that networks are estimated to increase labor productivity by roughly 5%.

Theoretically, an example of this mechanism is the modelization introduced by Greenwood and Jovanovic (1998), that assume the presence of a learning effect. In this case the productivity at time $t$ of a plant as a function of its age $\tau$ is given by:

$$z_{\tau}(t) = z_{\tau} = (1 - z^{*}e^{-\alpha_{1}\tau})^{1-\alpha_{2}}$$

i.e. it does not depend on $t$ but only on $\tau$, and as a plant ages it becomes more productive, due (for example) to learning by doing. We also have:

$$z_{0} = (1 - z^{*})^{1-\alpha_{2}}$$

and the difference $[1 - (1 - z^{*})^{1-\alpha_{2}}]$ represents the “amount to be learned” during the life of the plant (whose productivity is bounded above by 1). In periods of rapid technological progress there are steeper learning curves, therefore $z^{*}$ is likely to be positively related to the rate of investment-specific (or embodied) technological progress $g_{e}$. We assume that:

$$z^{*} = \omega g_{e}^{\nu}$$

since the idea is that the bigger $g_{e}$ is, the less familiar the latest generation of capital goods will look, and the more there will be to learn.

In our model, the presence of a learning effect implies that the productivity of the final good sector is given by:

$$z_{t} = (\overline{z} + \mu \Delta e) \left(1 - z^{*}e^{-\alpha_{1}t}\right)^{1-\alpha_{2}}$$

with $z^{*} = \omega g_{e}^{\nu}$.
where $\mu \geq 0$, and also:

$$z_0 = (\bar{z} + \mu \Delta e)(1 - z^*)^{1-\alpha_2} \quad z_\infty = \bar{z} + \mu \Delta e$$

In this way, considering an increase in $e$ (or in $\tau$) with $\bar{z}$ fixed, there is not only a short run effect on the productivity of labor in the final good sector but also a long run effect, and therefore a “spillover effect”.

Empirically, we can simulate the learning effect considering a permanent shock on $e$ or on $\tau$ and a delayed shock on $z$. Assuming for instance shocks of intensity equal to 1%, the behavior of the productivity of labor in the final good sector is reported in Figure 10.

From this picture it is clear that the effect of the shocks considered on the productivity of labor in the final good sector initially is the same as in the case of a permanent shock on $e$ or on $\tau$. In addition, after some periods there is a further effect that reinforces the previous one (as a consequence of the learning specification introduced, that originates a spillover effect). This is the behavior followed by US productivity in the last decade, that can be explained taking into account the presence of “intangible” or “hidden” capital. In this context, the years before 2000 have been characterized by substantial investments in intangible assets with no effects on productivity (and this corresponds to the first periods in the graph), while the years after 2000 have been characterized by the opposite situation with spillover effects of the intangible assets and an increase in the productivity (and this corresponds to the other periods in the graph). In fact, the data on labor productivity in the US show that at the end of the last decade its annual growth rate dropped from 2.4% in 1999 to 2% in 2001, and then it rose again to 2.9% in 2002 and to 3% in 2003, exactly in line with the results of the simulation.

The extension of the model that considers learning and spillover effects is therefore of particular interest, since it allows to reproduce empirically the dynamics (reported in Figure 1) followed by US productivity in the recent years.

## 5 Conclusion

This paper presents a model that reproduces the essential characteristics of the recent ICT Revolution and its effects on economic growth using the framework of endogenous growth theory. More precisely, it is a multi-sectoral growth model with embodied technological progress, horizontal differentiation and “lab-equipment” specification of R&D. As a consequence of this latter assumption, in particular, we show analytically that an increase in the productivity of the different sectors (final good sector, equipment sector, intermediate good sector) has an everlasting effect on growth.

The numerical simulation of the model, then, allows to get some further insights. In order to do this, we consider a calibrated version of the model, where the values chosen for the different parameters reproduce the empirical evidence that is available
concerning in particular the US economy. In this simulation, then, we study different types of productivity shocks, together with the shock represented by a decrease in the cost of R&D in terms of output (strictly linked to the peculiarity of the “lab-equipment” specification adopted in this sector).

The main finding obtained is that the presence of the “lab-equipment” assumption changes the implications of the model, as a consequence of shocks, with respect to the original version, without such assumption. In the latter, in fact, only a shock on the productivity of the R&D sector influences the growth of the economy in the long run. In addition, in our model the shocks on the productivity of the final good sector and on the cost of R&D on the one hand, and the shocks on the productivity of the equipment sector and of the intermediate good sector on the other hand, affect differently, in the short run, the economy, and influence the growth with different intensity. In particular, it turns out that a shock on the R&D sector or on the final good sector has a stronger effect than a shock on the equipment sector or on the intermediate good sector. The result is that if the ICT Revolution can be interpreted as a permanent shock on R&D or as a spillover (on the final good sector), it will have long run effects on the economy. On the contrary, if it is interpreted as a shock on the equipment sector or on the intermediate good sector (i.e. as the possibility of producing easily new softwares), it will not have strong long run effects. The model is also sufficiently robust, since when we modify significantly some parameters with respect to the benchmark case, both the qualitative and the quantitative implications remain valid. Finally, considering an extension of the model that takes into account the presence of learning and spillover effects we are able to reproduce empirically the behavior of US productivity in the recent years.

References


Figure 1: Growth in US labor productivity

(yearly observations and 5-year moving average)
Figure 2: Growth rate of production - increase in z
Figure 3: Growth rate of labor productivity - increase in $z$
Figure 4: Growth rate of relative price of capital - increase in z
Figure 5: Labor fraction in intermediate sector - increase in $z$
Figure 6: Growth rate of production - increase in $e$
Figure 7: Growth rate of labor productivity - increase in e
Figure 8: Growth rate of relative price of capital - increase in e
Figure 9: Labor fraction in intermediate sector - increase in e
Figure 10: Growth rate of labor productivity - increase in e and z
Figure captions

Figure 1: Growth in US labor productivity

Annual growth rate in output per hour in the nonfarm business sector for the US over the period from 1975 to 2005. Together with yearly observations, also the 5-year moving average is reported.

Figure 2: Growth rate of production – increase in z

Growth rate of output, in the different periods considered in the simulation exercise, as a consequence of an initial and permanent increase in the productivity of the final good sector equal to 1%.

Figure 3: Growth rate of labor productivity – increase in z

Growth rate of the productivity of labor in the final good sector, in the different periods considered in the simulation exercise, as a consequence of an initial and permanent increase in the productivity of the final good sector equal to 1%.

Figure 4: Growth rate of relative price of capital – increase in z

Growth rate of the relative price of capital, in the different periods considered in the simulation exercise, as a consequence of an initial and permanent increase in the productivity of the final good sector equal to 1%.

Figure 5: Labor fraction in intermediate sector – increase in z

Fraction of labor in the intermediate good sector, in the different periods considered in the simulation exercise, as a consequence of an initial and permanent increase in the productivity of the final good sector equal to 1%.

Figure 6: Growth rate of production – increase in e

Growth rate of output, in the different periods considered in the simulation exercise, as a consequence of an initial and permanent increase in the productivity of the equipment sector equal to 1%.
Figure 7: Growth rate of labor productivity – increase in \( e \)

Growth rate of the productivity of labor in the final good sector, in the different periods considered in the simulation exercise, as a consequence of an initial and permanent increase in the productivity of the equipment sector equal to 1%.

Figure 8: Growth rate of relative price of capital – increase in \( e \)

Growth rate of the relative price of capital, in the different periods considered in the simulation exercise, as a consequence of an initial and permanent increase in the productivity of the equipment sector equal to 1%.

Figure 9: Labor fraction in intermediate sector – increase in \( e \)

Fraction of labor in the intermediate good sector, in the different periods considered in the simulation exercise, as a consequence of an initial and permanent increase in the productivity of the equipment sector equal to 1%.

Figure 10: Growth rate of labor productivity – increase in \( e \) and \( z \)

Growth rate of the productivity of labor in the final good sector, in the different periods considered in the simulation exercise, as a consequence of an initial and permanent increase in the productivity of the equipment sector equal to 1% and of a delayed and permanent increase in the productivity of the final good sector equal to 1%.
Table 1: Labor productivity and multi-factor productivity in the US, 1980-2000 (average growth per year)

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>GDP per hour worked</td>
<td>+1.3%</td>
<td>+2%</td>
</tr>
<tr>
<td>Multi-factor productivity</td>
<td>+1%</td>
<td>+1.2%</td>
</tr>
</tbody>
</table>

Source: OECD (2000)

Table 2: Relevant moments of the steady state and values of the parameters, benchmark case

<table>
<thead>
<tr>
<th>Moments of the steady state</th>
<th>Parameter values</th>
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<tbody>
<tr>
<td>Interest rate</td>
<td>4%</td>
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<tr>
<td>Share of labor in intermediate and R&amp;D</td>
<td>5%</td>
</tr>
<tr>
<td>Capital/output ratio</td>
<td>2</td>
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<tr>
<td>R&amp;D expenditure in terms of GDP</td>
<td>3.5%</td>
</tr>
<tr>
<td>Growth rate of output</td>
<td>0.82%</td>
</tr>
<tr>
<td>α</td>
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</tr>
<tr>
<td>λ</td>
<td>0.85</td>
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<tr>
<td>σ</td>
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<td>δ</td>
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<tr>
<td>τ</td>
<td>0.25</td>
</tr>
<tr>
<td>z</td>
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</tr>
<tr>
<td>e</td>
<td>12</td>
</tr>
<tr>
<td>ρ</td>
<td>0.97</td>
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Table 3: Long run quantitative effects (with respect to the initial steady state levels) of different types of shocks, benchmark case

<table>
<thead>
<tr>
<th>shock</th>
<th>on z</th>
<th>on e</th>
<th>on τ</th>
<th>on η</th>
</tr>
</thead>
<tbody>
<tr>
<td>growth rate of production</td>
<td>+4.8%</td>
<td>+1.5%</td>
<td>+1.5%</td>
<td>+4.8%</td>
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<tr>
<td>growth rate of labor productivity</td>
<td>+2.2%</td>
<td>+1.2%</td>
<td>+1.2%</td>
<td>+4.8%</td>
</tr>
<tr>
<td>growth rate of price of capital (neg.)</td>
<td>+4.5%</td>
<td>+1.2%</td>
<td>+1.3%</td>
<td>+4.5%</td>
</tr>
<tr>
<td>labor force in intermediate sector</td>
<td>+0.1%</td>
<td>+0.05%</td>
<td>+0.05%</td>
<td>+0.1%</td>
</tr>
</tbody>
</table>
Table 4: Qualitative effects of different types of shocks, benchmark case

<table>
<thead>
<tr>
<th></th>
<th>increase in $z$</th>
<th>increase in $e$</th>
<th>decrease in $\eta$</th>
<th>increase in $\tau$</th>
</tr>
</thead>
<tbody>
<tr>
<td>growth rate of production</td>
<td>first $\uparrow$</td>
<td>first $\downarrow$</td>
<td>then $\downarrow$ (partially)</td>
<td>then $\uparrow$</td>
</tr>
<tr>
<td>growth rate of labor productivity</td>
<td>first $\uparrow$</td>
<td>first $\downarrow$</td>
<td>then $\downarrow$ (partially)</td>
<td>then $\uparrow$</td>
</tr>
<tr>
<td>growth rate of price of capital (negative)</td>
<td>first $\uparrow$</td>
<td>first $\downarrow$</td>
<td>then $\downarrow$ (partially)</td>
<td>then $\uparrow$</td>
</tr>
<tr>
<td>labor force in intermediate sector</td>
<td>first $\downarrow$</td>
<td>first $\uparrow$</td>
<td>then $\uparrow$</td>
<td>then $\downarrow$</td>
</tr>
</tbody>
</table>

Table 5: Long run quantitative effects (with respect to the initial steady state levels) of different types of shocks, case 1 (higher $\lambda$ with respect to the benchmark)

<table>
<thead>
<tr>
<th></th>
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<th>shock on $\tau$</th>
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<tbody>
<tr>
<td>growth rate of production</td>
<td>+4%</td>
<td>+1.4%</td>
<td>+1.3%</td>
<td>+4%</td>
</tr>
<tr>
<td>growth rate of labor productivity</td>
<td>+2%</td>
<td>+1.2%</td>
<td>+1.2%</td>
<td>+4%</td>
</tr>
<tr>
<td>growth rate of price of capital (neg.)</td>
<td>+4%</td>
<td>+1%</td>
<td>+1.2%</td>
<td>+4%</td>
</tr>
<tr>
<td>labor force in intermediate sector</td>
<td>+0.2%</td>
<td>+0.1%</td>
<td>+0.1%</td>
<td>+0.2%</td>
</tr>
</tbody>
</table>

Table 6: Long run quantitative effects (with respect to the initial steady state levels) of different types of shocks, case 2 (lower $\eta$ with respect to the benchmark)

<table>
<thead>
<tr>
<th></th>
<th>shock on $z$</th>
<th>shock on $e$</th>
<th>shock on $\tau$</th>
<th>shock on $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>growth rate of production</td>
<td>+3%</td>
<td>+1%</td>
<td>+1%</td>
<td>+3%</td>
</tr>
<tr>
<td>growth rate of labor productivity</td>
<td>+2%</td>
<td>+1%</td>
<td>+1%</td>
<td>+3%</td>
</tr>
<tr>
<td>growth rate of price of capital (neg.)</td>
<td>+3%</td>
<td>+1%</td>
<td>+1%</td>
<td>+3%</td>
</tr>
<tr>
<td>labor force in intermediate sector</td>
<td>+0.1%</td>
<td>+0.05%</td>
<td>+0.05%</td>
<td>+0.1%</td>
</tr>
</tbody>
</table>
Table 7: Long run quantitative effects (with respect to the initial steady state levels) of different types of shocks, case 3 (higher $e$ with respect to the benchmark)

<table>
<thead>
<tr>
<th>shock</th>
<th>on $z$</th>
<th>on $e$</th>
<th>on $\tau$</th>
<th>on $\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>growth rate of production</td>
<td>+4%</td>
<td>+1.4%</td>
<td>+1.2%</td>
<td>+4%</td>
</tr>
<tr>
<td>growth rate of labor productivity</td>
<td>+2%</td>
<td>+1.2%</td>
<td>+1.2%</td>
<td>+4%</td>
</tr>
<tr>
<td>growth rate of price of capital (neg.)</td>
<td>+4%</td>
<td>+1%</td>
<td>+1.2%</td>
<td>+4%</td>
</tr>
<tr>
<td>labor force in intermediate sector</td>
<td>+0.1%</td>
<td>+0.05%</td>
<td>+0.05%</td>
<td>+0.1%</td>
</tr>
</tbody>
</table>