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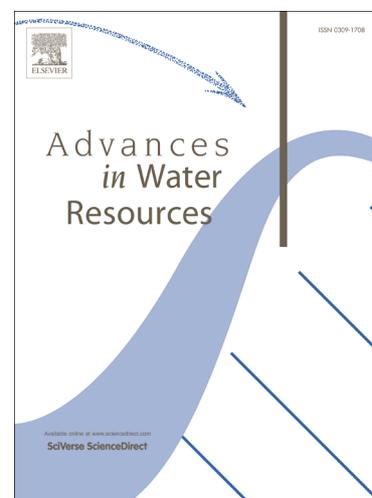
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1 **COUPLED INVERSE MODELING OF A CONTROLLED IRRIGATION EXPERIMENT**
2 **USING MULTIPLE HYDRO-GEOPHYSICAL DATA**

3
4 Matteo Rossi¹, Gabriele Manoli², Damiano Pasetto³, Rita Deiana⁴, Stefano Ferraris⁵,
5 Claudio Strobbia⁶, Mario Putti³ and Giorgio Cassiani¹

6
7 ¹ Department of Geosciences, University of Padova, Via Gradenigo 6, 35131 Padova, Italy

8 ² Nicholas School of the Environment, Division of Earth and Ocean Sciences, Duke University,

9 Durham, North Carolina 27708, USA³ Department of Mathematics, University of Padova, Via

10 Trieste 63, 35121 Padova, Italy

11 ⁴ Department of Cultural Heritage, University of Padova, Piazza Capitaniato 7, 35139 Padova, Italy.

12 ⁵ Department of Agricultural, Forestry and Environmental Economics and Engineering, University

13 of Torino, Via Leonardo da Vinci 44, 10095 Grugliasco, Torino, Italy

14 ⁶ Total SA, Pau, France

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24 **KEYPOINT:** Comparison of hydro-geophysical inversion approaches applied to a

25 surface irrigation experiment.

26 **ABSTRACT**

27 Geophysical surveys can provide useful, albeit indirect, information on vadose
28 zone processes. However, the ability to provide a quantitative description of the
29 subsurface hydrological phenomena requires to fully integrate geophysical data
30 into hydrological modeling. Here, we describe a controlled infiltration experiment
31 that was monitored using both electrical resistivity tomography (ERT) and
32 ground-penetrating radar (GPR). The experimental site has a simple, well-
33 characterized subsoil structure: the vadose zone is composed of aeolic sand with
34 largely homogeneous and isotropic properties. In order to estimate the unknown
35 soil hydraulic conductivity, we apply a data assimilation technique based on a
36 sequential importance resampling (SIR) approach. The SIR approach allows a
37 simple assimilation of either or both geophysical datasets taking into account the
38 associated measurement uncertainties. We demonstrate that, compared to a
39 simpler, uncoupled hydro-geophysical approach, the coupled data assimilation
40 process provides a more reliable parameter estimation and better reproduces the
41 evolution of the infiltrating water plume. The coupled procedure is indeed much
42 superior to the uncoupled approach that suffers from the artifacts of the
43 geophysical inversion step and produces severe mass balance errors. The
44 combined assimilation of GPR and ERT data is then investigated, highlighting
45 strengths and weaknesses of the two datasets. In the case at hand GPR energy
46 propagates in form of a guided wave that, over time, shows different energy
47 distribution between propagation modes as a consequence of the evolving
48 thickness of the wet layer. We found that the GPR inversion procedure may

49 produce estimates on the depth of the infiltrating front that are not as informative
50 as the ERT dataset.

51

52 **KEYWORDS:** hydro-geophysical inversion, electrical resistivity tomography,
53 ground-penetrating radar, infiltration, vadose zone.

54

55 **1. INTRODUCTION**

56 Hydrological research increasingly requires detailed information to feed data-
57 hungry numerical models. For this reason, geophysical data are increasingly called
58 into play to fill the lack of spatial and sometimes temporal resolution of traditional
59 hydrological data. This is particularly true for the vadose zone, where the
60 difficulties for obtaining direct measurements, the general lack of knowledge and
61 the uncertainty on the soil parameters and their spatial heterogeneity often lead to
62 develop numerical models that cannot reproduce the behavior of the real systems,
63 unless they are strongly constrained by multiple, extensive and complementary
64 data.

65 The vadose/unsaturated zone is home to a number of complex key processes
66 that control the mass and energy exchanges in the subsurface (soil water
67 migration) and between the subsurface and the atmosphere (rain infiltration, soil
68 evaporation and plant transpiration). The understanding of vadose zone fluid-
69 dynamics is key to the comprehension of a large number of hydrologically-
70 controlled environmental problems, with strong implications in water resources
71 management and subsurface contaminant hydrology. Unsaturated processes are
72 also key factors in a number of important issues, such as the availability of water

73 for agriculture, slope stability, and floods. The dependence of the hydro-
74 geophysical response on changes in soil moisture content is the key mechanism
75 that allows the monitoring of the vadose zone in time-lapse mode via non-invasive
76 techniques. The use of these techniques can provide high-resolution images of
77 hydro-geological structures in the shallow and deep vadose zones and, in some
78 cases, a detailed assessment of dynamical processes in the subsurface.

79 The estimation of the time and space variations of water content using non-
80 invasive methodologies has been the focus of intensive research over the past
81 three decades. Among the numerous techniques developed in literature for such a
82 goal, such as electromagnetic induction, off-ground ground-penetrating radar,
83 surface nuclear magnetic resonance, in this work we consider electrical resistivity
84 tomography (ERT) and ground-penetrating radar (GPR). These techniques
85 measure the electrical resistivity ρ (Ωm) and the relative dielectric permittivity ϵ_r ,
86 (-) of the porous media, respectively. For both methods the determination of soil
87 water content is based upon existing relationships that link water content to the
88 geophysical quantities measured (e.g., Archie, 1942; Topp et al., 1980; Roth et al.,
89 1990; Brovelli and Cassiani, 2008, 2011).

90 When used to study hydrological dynamics, GPR surveys are often performed to
91 detect changes in soil moisture content via the variation of dielectric permittivity,
92 generally measured from GPR travel times in a variety of configurations (e.g.,
93 Huisman et al., 2003; Cassiani et al., 2006; Cassiani et al., 2008), such as borehole-
94 to-borehole (e.g., Rucker and Ferré, 2004a, 2004b; Rossi et al., 2012) or borehole-
95 to-surface (e.g., Vignoli et al., 2012). However, the most common setup uses GPR
96 antennas from the the ground surface, even though only few studies with this

97 configuration have been focused on the understanding of the dynamics of the
98 water front during irrigation (e.g., Galagedara et al., 2005; Moysey, 2010; Mangel et
99 al., 2012; Lai et al., 2012) or using natural rainfall (Busch et al., 2014). When
100 working solely from the ground surface, three approaches are possible to
101 determine soil moisture content: (a) use the velocity of the direct ground wave, (b)
102 estimating velocity from the reflected events, (c) estimating impedance and thus
103 velocity from the reflected GPR signal. Approaches (a) and (b) share in fact the
104 same operational characteristics, needing the two antennas to be separated from
105 each other. Approach (c) does not require antenna separation and exploits the
106 physics of the reflection mechanism, with its own advantages and disadvantages
107 (e.g., Lambot et al., 2004; Schmelzbach et al., 2012), and with more limited
108 applications so far. When the two antennas are separated from each other, the
109 survey can be conducted in wide angle reflection and refraction (WARR) mode
110 (e.g., van Overmeeren et al., 1997), where one antenna is kept fixed while the other
111 is moved, or common mid point (CMP) (Fisher et al., 1992; Greaves et al., 1996;
112 Steelman et al., 2012), where both antennas are moved simultaneously to keep the
113 same mid-point. Both sounding techniques allow for a good identification of direct
114 waves through the air and the ground. These methods are also employed for the
115 estimation of velocity from the reflected events, even though for this use the
116 normal move-out approach, typical of seismic processing, may not be ideal (see
117 Becht et al., 2006 for a discussion). The estimation of velocity from the direct wave
118 through the ground is the most widely adopted approach for vadose zone
119 applications (e.g. van Overmeeren et al., 1997; Huisman et al., 2001; Hubbard et al.,
120 2002). However, in some cases direct arrivals are not so straightforward to

121 identify and can be confused with other events. This can happen in the presence of
122 critically refracted radar waves (Bohidar and Hermance, 2002) or guided waves
123 (Arcone et al., 2003; van der Kruk et al., 2006; Strobbia and Cassiani, 2007). A
124 water front that infiltrates from the surface can give rise to such ambiguous
125 situations, as the wet and consequently low velocity layer, lying on top of a faster
126 (drier) media, can give rise to critically refracted waves (Bohidar and Hermance,
127 2002) as well as act as a waveguide confined between two faster layers: the air
128 above and the drier media below (Strobbia and Cassiani, 2007), the two situations
129 being defined by the ratio between the wavelength and the layer thickness.
130 Therefore, to study infiltrating fronts, maximum care must be given in
131 understanding the nature of the observed, multi-offset GPR signal, possibly
132 exploiting the entire information content of the data (e.g. Busch et al., 2012).

133 ERT measurements (Binley and Kemna, 2005) have been widely employed to
134 monitor water dynamics, as variations of moisture content (Daily et al., 1992;
135 Binley et al., 1996) and salinity of pore water (Perri et al., 2012) leads to changes in
136 the electrical properties of the media (La Brecque et al., 2004; Cassiani et al.,
137 2009a). However, it is well known that resolution limitations (Day-Lewis et al.,
138 2005) can produce severe mass balance errors (Singha and Gorelick, 2005) even in
139 the most favorable cross-hole configurations. The problem is even more serious
140 when only surface ERT are used to monitor natural or artificial irrigation from the
141 ground surface (Michot et al., 2003; Clément et al., 2009; Caputo et al., 2012;
142 Cassiani et al., 2012; Travelletti et al., 2012) where resolution dramatically drops
143 with depth and a direct conversion of inverted resistivity values into estimates of
144 soil moisture content may prove elusive.

145 Geophysical measurements can be informative of the hydrological response of the
146 soil and subsoil if applied in time-lapse monitoring mode: some geophysical
147 quantities (in this case, ρ and ϵ_r) are useful indicators of changes in the
148 hydrological state variables, such as moisture content or pore water salinity.
149 However, in order to extract this hydrological information, the assimilation of
150 measurements in a hydrological model is needed. Two different approaches may
151 be applied, named respectively “uncoupled” and “coupled” hydro-geophysical
152 inversions (Ferré et al. 2009; Hinnell et al., 2010). The procedure for an *uncoupled*
153 *inversion* can be summarized by the following steps:

- 154 1. the spatial distribution of the geophysical quantity of interest (e.g. electrical
155 resistivity for ERT) is derived from the inversion of geophysical field data;
- 156 2. the application of a petro-physical relationship leads to obtaining, from the
157 geophysical quantity, an estimation of moisture content distribution;
- 158 3. the estimated hydrologic state variable, in its spatio-temporal distribution,
159 is used to calibrate and constrain a hydrological model, thus identifying the
160 corresponding governing parameters.

161 The inversion of geophysical measurements is usually an ill-posed inversion
162 problem that can be tackled introducing prior information. If no solid independent
163 information is available, the most common approach is the introduction of a
164 regularizing functional, commonly a smoothness constraint (Menke, 1984). As a
165 consequence of ill-posedness and regularization, the inversion procedure can lead
166 to artifacts, misinterpretations and unphysical results, especially in the subsurface
167 regions where the sensitivity of the measurements is low (consider e.g. Day Lewis

168 et al., 2005). To overcome these problems, a coupled hydro-geophysical modeling
169 can be applied:

- 170 1. a hydrological model is used to predict the evolution of hydrological state
171 variables – e.g. moisture content – on the basis of a set of hydrological
172 governing parameters, the identification of which is the final aim of the
173 inversion;
- 174 2. a suitable petrophysical relationship (same as for point (2) above)
175 translates hydrological state variables into geophysical quantities, such as
176 resistivity or dielectric permittivity;
- 177 3. the simulated geophysical quantities are used to predict the geophysical
178 field measurements;
- 179 4. a comparison between predicted and measured geophysical field
180 measurements allows a calibration of the complex of hydrological and
181 geophysical models (thus the name “coupled inversion”), leading to the
182 identification of the hydrological parameters, that is the key objective of the
183 study.

184 In this work we follow a coupled approach within the framework of data
185 assimilation (DA). DA schemes are mathematical tools of common use in
186 hydrological applications. The main idea behind DA is using the field
187 measurements to correct numerical simulations obtained with a hydrological
188 model, thus modifying their governing parameters. This is possible by the
189 recursion of forecast steps, which simulate the time-evolution of the probability
190 density function (pdf) of the hydrological process, and analysis (or update) steps,
191 which compute a posterior pdfs of the model parameters and state variables by

192 assimilating the measurements (e.g., McLaughlin, 2002; Moradkhani et al., 2005). A
193 few examples of coupled hydro-geophysical inversion exist in the literature (e.g.,
194 Busch et al., 2014) but the use of DA techniques is less widespread (Rings et al.,
195 2010; Tran et al., 2014).

196 The present work focuses on a field experiment where artificial irrigation is
197 monitored in time-lapse mode from the surface via both ERT and GPR. The
198 homogeneous nature of the site, made of aeolic sand deposits, provides a
199 simplified case study suitable to evaluate the performance of coupled hydro-
200 geophysical inversion and test the information content of different geophysical
201 data. Both GPR and ERT geophysical measurements are assimilated into the
202 hydrological model CATHY (Camporese et al., 2010), that is employed for the
203 numerical simulation of the experiment. We elected to use the iterative sequential
204 importance resampling (SIR) proposed by Manoli et al. (2015) as a DA technique to
205 estimate the model saturated hydraulic conductivity. This technique is particularly
206 designed to assimilate geophysical measurements in a coupled hydro-geophysical
207 model: the geophysical measurements are blended in the simulation to update the
208 state of the system, estimate the model parameters and quantify the model
209 uncertainties.

210 The specific goals of this work are:

- 211 1. to analyze in detail the nature of the WARR GPR data collected during the
212 irrigation experiment, verifying whether or not complex refraction and
213 waveguide phenomena occur during the progression of the wetting front,
214 and how and to what extent this type of data can be processed and inverted;

- 215 2. to assess the effectiveness of incorporating ERT and GPR data in a coupled
216 hydro-geophysical inversion procedure that, using the unsaturated flow
217 equations, point directly at the estimation of the saturated hydraulic
218 conductivity, and to compare this approach with the results of a classical
219 uncoupled inversion approach;
- 220 3. to evaluate to what extent the information that can be obtained from GPR
221 and ERT data corroborate each other, how the independent assimilation of
222 each data type performs, if the assimilation of both geophysical techniques
223 adds information with respect to separate procedures, and finally what is
224 the value of using both techniques to monitor the infiltration process.

225 The paper is organized as follows: Section 2 is dedicated to the description of the
226 hydrological model and the DA procedure used for the coupled inversion of the
227 geophysical data. After presenting the hydrological experiment taken into
228 consideration (Section 3), in Sections 4 and 5 we analyze the GPR and ERT data,
229 respectively. In Section 6 we describe the setup for the DA procedure in this
230 experiment. The benefits of the coupled inversion are presented in Section 7. The
231 major conclusions of this work are summarized in Section 8.

232

233 **2. DATA ASSIMILATION**

234 Data Assimilation methods are typically made of three components: 1) a
235 forward model describing the dynamics of the physical process under study, 2) an
236 observation model that links the simulated system variables to the observed data,
237 and 3) the update procedure, that changes the simulated variables on the basis of
238 the observations. This section describes these three components for our particular

239 application, i.e., the assimilation of ERT and GPR data to calibrate an unsaturated
240 hydrological model with the iterative SIR method.

241

242 **2.1 Hydrological model**

243 The infiltration process in a variably-saturated isotropic porous medium is
244 described by the Richards' equation:

$$S_s S_w(\psi) \frac{\partial \psi}{\partial t} + \phi \frac{\partial S_w(\psi)}{\partial t} = \vec{\nabla} \cdot [\mathbf{K}_s K_r(\psi) (\vec{\nabla} \psi + \eta_z)] + q \quad (1)$$

245 where S_s is the elastic storage term [m^{-1}], S_w is water saturation [-], ψ is water
246 pressure head/suction [m], t is time [s], ϕ is porosity [-], \mathbf{K}_s is the saturated
247 hydraulic conductivity [m s^{-1}] tensor, K_r is the relative hydraulic conductivity [-],
248 $\eta_z = (0, 0, 1)^T$ with z the vertical coordinate directed upward, and q is a
249 source/sink term [s^{-1}]. Eq. (1) is highly nonlinear due to the dependencies of soil
250 saturation and relative hydraulic conductivity on pressure head. These terms are
251 modeled using the water retention curves proposed by van Genuchten and Nielsen
252 (1985).

253

254 **2.2 Geoelectrical and GPR models for data assimilation**

255 The electrical potential field induced in the soil by current injection during the
256 ERT survey, Φ [V], can be modeled as:

$$-\vec{\nabla} \cdot [\rho^{-1} \vec{\nabla} \Phi] = I [\delta(\vec{r} - \vec{r}_{s+}) - \delta(\vec{r} - \vec{r}_{s-})] \quad (2)$$

257 where ρ is the electrical resistivity of the soil [Ωm], I is the applied current [A], δ is
258 the Dirac function, $\vec{r} = (x, y, z)$, and \vec{r}_{s+} and \vec{r}_{s-} are the source and sink electrode

259 positions, respectively. Here, the geophysical model is linked to the hydrologic
 260 model by the petrophysical relationship proposed by Archie (1942):

$$\rho(t_i) = \rho(t_0) \left(\frac{S_w(t_0)}{S_w(t_i)} \right)^n \quad (3)$$

261 where $S_w(t_0)$ is the background water saturation degree and $\rho(t_0)$ is the
 262 corresponding bulk electrical resistivity of the soil. In Eq. (2) the bulk electrical
 263 resistivity at i -th measurement time, $\rho(t_i)$, can be predicted by the knowledge of
 264 the saturation degree at the same time step, $S_w(t_i)$, and vice-versa. Thanks to Eqs.
 265 (2) and (3), we can write the ERT measurements, here indicated with $y_{ERT}(t_i)$, as a
 266 nonlinear function H_{ERT} of the water saturation:

$$y_{ERT}(t_i) = H_{ERT}(S_w(t_i)) + v_{ERT}(t_i) \quad (4)$$

267 where $v_{ERT}(t_i)$ represents a Gaussian measurement error with variance $R_{ERT}(t_i)$,
 268 $v_{ERT}(t_i) \sim N(0, R_{ERT}(t_i))$.

269 For linking the GPR data to the hydrological model we adopt a simplified
 270 approach. The observation model that links the numerical simulations to the GPR
 271 measurements consists in the estimation of the infiltration front depth from the
 272 simulated vertical profiles of water saturation. When the considered porous media
 273 can be considered spatially uniform and the irrigation rate is nearly constant in
 274 time, at any assimilation time (t_1 , t_2 or t_3) the water saturation can be considered
 275 uniform from the surface down to a certain depth d_1 , while from d_1 to a depth d_2 it
 276 decreases to the initial saturation value according to the soil water retention curve,
 277 and finally the water content remains practically constant from d_2 to the bottom of
 278 the domain (considering that the water table is much deeper than the vertical
 279 extent of the infiltration domain). The average value of the two depths d_1 and d_2 is

280 an approximation of the depth of the simulated infiltration front. Indicating the
 281 estimated infiltration front with $y_{GPR}(t_i)$, from the described procedure we have
 282 that:

$$y_{GPR}(t_i) = H_{GPR}(S_w(t_i)) + v_{GPR}(t_i) \quad (5)$$

283 where H_{GPR} is nonlinear operator and $v_{GPR}(t_i)$ is a Gaussian measurement error
 284 with with variance $R_{GPR}(t_i)$, $v_{GPR}(t_i) \sim N(0, R_{GPR}(t_i))$. In the DA process $y_{GPR}(t_i)$ is
 285 compared with the average thickness estimated from GPR measurements.

286 More accurate (and more complex) GPR modeling could be conducted to
 287 construct a forward model e.g. based upon a full-waveform approach (see e.g.
 288 Klotzsche et al., 2012, 2013). However we do not deem this is necessary for this
 289 case study, where the key information that is derived from GPR resides in the
 290 depth of the infiltration front and the electromagnetic (EM) wave propagation is
 291 dominated by guided waves (see Section 4).

292

293 **2.3 Iterative SIR algorithm for Data Assimilation**

294 In Manoli et al. (2015) the hydrological and geophysical models are coupled in a
 295 DA framework to simulate ERT surveys and update the physical state variable (soil
 296 saturation) and the model parameters whenever a geophysical measurement is
 297 available. DA methods allow the incorporation of real system observations onto
 298 the dynamical model to automatically correct the model forecast (i.e., the solution
 299 of Eq. 1) and the model parameters (e.g., the saturated hydraulic conductivity \mathbf{K}_s)
 300 thus reducing the uncertainties related to the model prediction. In the following
 301 we indicate with λ the set of time-independent model parameters in Eq. (1) and
 302 with $p_0(\lambda)$ its prior pdf.

303 The SIR algorithm uses a weighted Monte Carlo (MC) approach to perform the
 304 state and parameter update (e.g., Moradkhani et al., 2005). The MC realizations,
 305 which are also called particles, are initialized by sampling the parameter values
 306 from the prior distribution, $\{\lambda_0^j\}_{j=1}^N$, where N is the total number of MC realizations
 307 and j is the realization index. SIR associates a weight to each realization, w_0^j , which
 308 is initialized to $1/N$. The forecast step is given by the numerical solution of
 309 Richards's equation (1) for each set of parameters, thus describing the space and
 310 time evolution of the infiltration process. Note that weights and parameters are
 311 invariant during the forecast step. At a general time t , each realization is described
 312 by its particular set of parameters, state variables and weight $\{\lambda_t^j, S_w^j(t), w_t^j\}_{j=1}^N$.

313 In an assimilation step t_i , with the idea that the weight represent the 'closeness'
 314 of a realization to the real process, the SIR algorithm changes the weights
 315 according to the Bayes' formula: new weights are assigned to each particle on the
 316 basis of the likelihood function of the measured data with respect to the simulated
 317 data, e.g., $p(y_{ERT}(t_i)|S_w^j(t_i))$ for ERT data. The likelihood functions for the ERT and
 318 GPR data can be obtained from the measurement error pdfs described Eqs. 4 and 5,
 319 respectively. Then, the weights are changed with the following formula (here
 320 written for a general observation y):

$$\tilde{w}_{t_i}^j = w_{t_{i-1}}^j p(y(t_i)|S_w^j(t_i)) \quad (6)$$

$$w_{t_i}^j = \frac{\tilde{w}_{t_i}^j}{\sum_{j=1}^N \tilde{w}_{t_i}^j} \quad (7)$$

321 where (7) is a normalization of the weights. Since some of the updated weights
 322 may be negligible, meaning that the corresponding particles are not representative

323 of the physical process, the SIR introduces a resampling step after the update. In
324 the resampling step, the particles with negligible weights are discarded, while
325 those with large weights are duplicated, in order to retain only the particles that
326 are more representative of the filtering probability. Manoli et al. (2015), similarly
327 to Moradkhani et al. (2005), adapted this step to update also the model
328 parameters: the weighted empirical distribution of the parameters is adopted to
329 sample new parameter values the duplicated particles. The SIR method continues
330 with a repetition of forecast and update steps, and terminates in correspondence
331 of the last geophysical measurement. Since bias may be present in the initial model
332 parameters, and since the hydraulic conductivity distribution may not converge
333 during the sequential assimilation, the posterior distribution computed with the
334 SIR method may not be optimal for the whole simulation. For this reason it is
335 fundamental to iterate the described procedure until the parameter distribution is
336 unchanged during the simulation. At each iteration the procedure initializes the
337 parameters with an averaged posterior distribution, computed on the ensemble of
338 the hydraulic conductivities computed after all the previous updates.

339

340 **3. FIELD SITE AND IRRIGATION EXPERIMENT**

341 The experimental site is located in the campus of the Agricultural Faculty of the
342 University of Turin, Italy, in Grugliasco (45° 03' 52" N, 7° 35' 34" E, 290 m a.s.l.)
343 (Fig.1). The depth of interest is the top 1 m from the ground surface, where the
344 lithology is homogeneous. The stratigraphy is composed of a regular sequence of
345 sandy soil (mesic Arenic Eutrudepts) and the sediments in this area are largely
346 aeolic sands with extremely low organic content. The aeolic sand grains are

347 relatively homogeneous in size with a mass median diameter (d_{50}) of about 200 μm
348 and porosity ranging between 0.35 and 0.4 (Cassiani et al., 2009c). According to
349 the Comprehensive Soil Classification System, the horizon down to about 1-1.5
350 meter depth is an A-horizon made of mineral matter (80% sand, 14% silt and 6%
351 clay).

352 The water table is located around 20 m below the ground surface and therefore
353 the shallow vadose zone, where our experiment took place, is not practically
354 influenced by the underlying saturated zone. At the moment of the survey the
355 vegetation was composed only of natural grass, no cultivation is present (Fig. 1b).

356 An infiltration experiment was performed at the site on August 28, 2009. The
357 irrigation was provided by a 17 m line of sprayers. The soil surface covered by
358 irrigation was approximately a rectangle of 18 m by 2.6 m (Fig. 1). The irrigation
359 lasted for 5 hours and 45 minutes and was performed in 3 steps (Table 1),
360 separated by intervals when a break of the irrigation allowed ERT and GPR
361 acquisitions to be performed (see Fig. 1c for the geometry of the geophysical
362 surveys). At the center of the ERT profile, along the sprinkler line, two Time
363 Domain Reflectometry (TDR) probes were vertically placed in the soil with a
364 length of 0.15 and 0.30 cm.

365 The irrigation intensity was always lower than the infiltration capacity of the
366 soil, so no ponding was observed at the soil surface. The ERT and GPR
367 measurements were performed with the schedule summarized in Table 2, where
368 the time is referred to the starting of the irrigation.

369

370

371 **4. GPR DATA ANALYSIS**

372 The infiltration test was monitored by GPR using a PulseEkko Pro radar system
373 (Sensors and Software Inc., Canada) with 100 MHz antennas. The surveys were
374 repeated in time (Table 2) using a WARR scheme. The WARR profiles were
375 acquired along the sprinkler line (Fig. 1c); the time sampling interval was 0.2 ns
376 and the offset increment between transmitting and receiving antennas was equal
377 to 0.1 m over a 10.5 m line, starting from an initial offset (minimal distance
378 between transmitter and receiver) of 1 m.

379 The background WARR radargram before the irrigation is shown in Fig. 2,
380 where we can clearly recognize the direct ground wave with a velocity of about
381 0.14 m/ns. The evolution of WARR surveys over time (Fig. 3) shows that the
382 infiltration front modifies substantially the appearance of the GPR signal. The
383 radargrams in Fig. 3 are distinctly different from each other: the direct radar wave
384 in air is obviously unaltered over time, while the signal from the soil is
385 progressively delayed. This phenomenon is due to the presence of a wet low-
386 velocity layer, between the surface and the dry sandy soil, which becomes
387 increasingly thicker over the irrigation period. At a first glance, the interpretation
388 of the data may be conducted by identifying the first soil arrival as a critically
389 refracted GPR wave that comes from the -wet - dry interface and arrives at larger
390 intercept times as infiltration progresses, consistently with a deeper wetting front.
391 Although, this event must be present in the data, it is likely to be masked by guided
392 modes of GPR wave propagation as described by Strobbia and Cassiani (2007). The
393 establishment of guided EM waves is the consequence of the geometry of the
394 dielectric properties of the materials involved in the wave propagation. The energy

395 radiated from the transmitting antenna is spread out into the low-velocity layer
396 and reaches the underlying faster layer (dry sand) with an angle greater than the
397 corresponding Snell critical angle, in such a way that the energy is totally reflected.
398 The same phenomenon happens when the reflected energy reaches the boundary
399 between the air and the wet sandy media. The total internal reflections guide the
400 GPR waves horizontally inside the low-velocity layer, while outside of the wet layer
401 there are only evanescent waves with no radiation in the dry material and in the
402 air. A simpler interpretation of the radargrams (Fig. 3A) as a simple consequence
403 of refracted events - albeit possible (see Cassiani et al., 2009b) - would lead to
404 unclear event identification.

405 We analyzed the frequency-wavenumber ($f-k$) spectra of the radargrams with
406 the aim of recognizing guided modes of wave propagation (Fig. 4A). The $f-k$ spectra
407 are obtained by a preprocessing involving several filtering procedures. The first
408 preprocessing step consists in the application of a *de-wow* filter, following the
409 procedure of Gerlitz et al. (1993). The *wow* effect is due to the air-ground pulse
410 interference. In fact, electrostatic and inductive fields near the transmitter lead to
411 the saturation of the receiver electronics and generate a low frequency
412 contribution that decays with distance. The consequence of the *wow* is to move
413 trace amplitude towards positive (or negative) values, resulting in a non-zero-
414 mean trace. Removing the *wow* frequencies should reconstruct a zero-mean trace,
415 where small amplitudes are easier to identify. This filter is based on the
416 subtraction of the median amplitude, calculated inside a mobile window in the
417 time domain. The window size is determined from the maximum *wow* frequency,

418 achieved from the frequency spectra of all unfiltered traces (f - x spectra – i.e. one
419 frequency spectrum for each offset x).

420 The second processing step is a *muting* of the portions of the radargram that are
421 not relevant in the guided mode propagation, so as to highlight the signal of the
422 supposed guided waves. The *muting* process has the aim of cleaning those portions
423 of the radargrams that are not useful in the present study: events that may be
424 considered as noise in a guided wave analysis. So *muting* is applied to remove the
425 air direct wave as well as the reflected events at later times. We applied a Tukey
426 window in time, to prevent ringing in the f - k domain that may be due to an abrupt
427 signal step in the time domain. The Tukey window is set to obtain half of the entire
428 window length as a flat plateau, while the two marginal sectors consist of segments
429 of a phase-shifted cosine.

430 The final filtering process is the application of a finite impulse response (FIR)
431 filter to remove signal noise at low and high frequencies. The FIR filter has a
432 structure that can maintain the true intensity of the signal between 20 and 250
433 MHz. This is a broad window for a signal centered around 100 MHz, since the
434 guided propagation shows apparent frequencies that can be higher than the
435 acquisition capabilities of the receiving antenna. This fact is the consequence of the
436 limitation of our array, that records the GPR echoes only at the ground surface.

437 The filtered radargrams are shown in Fig. 3B. The corresponding f - k spectra (Fig.
438 4A) show the signal evolution over time. The color scale of the power spectral
439 density is the same for the different time-steps, in order to show the differences of
440 energy distribution over time. The energy peaks at times t_1 and t_3 have much
441 higher amplitudes than at time t_2 , when energy peaks are relatively weak as energy

442 is spread over several modes of propagation, while at times t_1 and t_3 a dominant
443 mode is clearly recognizable. This may be the consequence of our spatial sampling
444 that is not able to record with enough intensity the prevailing mode of resonance
445 induced by that particular subsoil geometry, but can also be a symptom of the
446 energy shifting between fundamental (at time t_1) and first higher mode (at time t_3).
447 The positions of the absolute maxima, detected for each frequency, are plotted as
448 magenta dots (Fig. 4A), while the white dots represent the local maxima.

449 Maxima picking in spectral amplitudes leads to obtaining the dispersion curves
450 of Fig. 4B, showing the dependence of phase velocity on frequency. Here red dots
451 correspond to the absolute maxima, while blue dots show local maxima. The
452 dispersion curves at times t_1 show a clearly identifiable fundamental mode, while
453 at time t_3 the first higher mode is much more energetic than the fundamental mode.
454 The switch of the highest energy to higher modes of propagation may lead to the
455 transient step which involves time t_2 , where the power spectral density is spread
456 upon different modes (Fig. 4A).

457 In order to give a hydrological meaning to these results, we need to translate the
458 spectral analysis of guided waves into an estimate of the evolution of the hydraulic
459 process. In particular we are interested in the location of the wetting front at depth,
460 as this information is suitable for the calibration of hydrological models. The depth
461 of this front corresponds to the thickness of the guiding high dielectric permittivity
462 layer. The identification of the layer thickness and dielectric properties requires
463 inversion of the dispersion curves (van der Kruk et al., 2006; Strobbia and Cassiani,
464 2007). We adopted as a forward model the description of the asymmetric slab
465 waveguide given by Strobbia and Cassiani (2007). The approximation of 1-

466 dimensional waveguide is valid as long as we assume that irrigation is practically
467 uniform along the sprinklers' line, and the soil is largely homogeneous. The
468 inversion of dispersion curves was performed using a MC approach. We sampled
469 the controlling parameters, i.e.: velocity of the shallower wet layer, velocity of the
470 deeper dry layer and thickness of the wet layer. The velocity of air can be
471 considered a constant equal to 0.3 m/ns. To reduce the number of ensembles of
472 parameters combinations, we fixed the value of the velocity of the deeper and
473 faster layer to about 0.14 m/ns, i.e. we set it equal to the velocity of the soil before
474 irrigation (Fig. 2). This choice is also in accordance with the TDR measurements
475 (0.3 m prongs) performed before the irrigation, showing a dielectric permittivity of
476 4.55, which corresponds to a EM wave velocity of 0.141 m/ns. The forward model
477 of EM wave propagation assumes the presence of only two ground layers, so we
478 are not able to simulate a smoothed wetting front, that is approximated as a sharp
479 discontinuity of dielectric permittivity. The thickness range is fixed, for all times,
480 between 0.3 m and 1 m, with an increment of 0.05 m. The velocity of wet layer is
481 sampled in the interval from 0.065 m/ns to 0.1 m/ns, at steps of 1.05×10^{-4} m/ns.
482 Both fundamental and first modes are simulated, setting all possible combinations
483 of the parameter space for a total of about 47000 simulations.

484 Fig. 5 shows the results of the inversion procedure, where the goodness of fit
485 between experimental and simulated dispersion curves is calculated using the
486 Nash-Sutcliffe index (NSI) (Nash and Sutcliffe, 1970). Fig. 5A reproduces the
487 experimental curve (black dotted line) plotted together with the best-fitting
488 synthetic curves: the light gray lines have NSI values between 0.85 and 0.95, while
489 the dark gray lines show $NSI > 0.95$. At time t_1 1035 curves of the fundamental

490 mode have a $NSI > 0.95$. At time t_2 the fitting of the measured dispersion curve for
491 the fundamental mode is poor, as NSI does not exceed, for any curve, the value of
492 0.87. For this reason we consider in Fig. 5 only the 1124 simulations with $NSI > 0.85$.
493 The 1232 synthetic curves of the first higher mode are used to represent the
494 experimental first mode at time t_3 , where the NSI is greater than 0.95. We inverted
495 the first higher mode for time t_3 , as at this time the higher mode is much more
496 energetic than the fundamental mode, as shown by Fig. 4. Fig. 5B-C show the
497 distribution of the parameters linked to the best simulations: wet layer thickness
498 and wet layer velocity, respectively.

499 We averaged the parameters of the best simulations to achieve an estimated
500 value for both the velocity and the thickness of the wet layer, at all times. Statistics
501 and ranges of the considered best simulations are summarized in Table 3. The
502 velocity of the wet layer changes slightly over time, with values confined in a
503 narrow range, in all cases very far from the value of the dry sand (0.14 m/ns).

504 We are less confident in the inversion of time t_2 for two reasons: (1) the fitting
505 between measured and calculated data is poor respect to the other time-steps that
506 show high values of NSI ; (2) the experimental dispersion curve is derived from the
507 $f-k$ domain, that shows that energy is smeared between fundamental and first
508 higher mode. Therefore, the dispersion curve at time t_2 may be heavily affected by
509 the unfavorable signal to noise ratio for both the fundamental and the first higher
510 mode.

511 It should also be noted that our MC inversion provides a view of the degree of
512 correlation of the two governing parameters (thickness and velocity of the wet
513 layer). Fig. 6 shows the levels of $NSI > 0.85$ plotted in the parameter space,

514 highlighting some degree of positive correlation. However, at times t_1 and t_3 the
 515 best fitting simulations (NSI>0.984 for t_1 , NSI>0.987 for t_3), marked as a green area,
 516 are centered around small parameter ranges. At time t_2 the green area highlights
 517 the simulations with NSI>0.886. Table 3 reports the standard deviations of the
 518 parameters associated to the best-fitting simulations that are quite small with
 519 respect to the average values.

520

521 **5. ERT DATA ANALYSIS**

522 The ERT data were collected at the surface using a Syscal-Pro resistivimeter
 523 (IRIS Instruments, France). Twenty-four electrodes spaced 20 cm were placed on a
 524 transect perpendicular to the sprinklers' line, for a total length of 4.6 m (Fig. 1).
 525 The acquisition scheme was a dipole-dipole skip zero (dipoles with minimal
 526 distance equal to one electrode spacing). Reciprocal measurements were acquired
 527 and processed to estimate data errors. All the reciprocal measures with the
 528 statistical operator RSD (Relative Standard Deviation) exceeding the 5% were
 529 removed from the dataset. This reciprocal error analysis leads to a different
 530 dataset for each time step. For this reason and to have comparable results, we
 531 performed the inversions considering only the quadripoles that are present in all
 532 datasets. The common datasets preserve 200 measurements over a total of 231
 533 quadripoles, thus data quality is particularly good. We inverted the data as the
 534 ratio of electrical resistances at a specific time with respect to the resistance values
 535 at the background measurement (in our case the time-step before the irrigation):

$$R = \frac{R_i}{R_0} \cdot R_{hom} \quad (8)$$

536 Where R_i is the electrical resistance at the i -th time-step, R_0 is the electrical
537 resistance at the background measure and R_{hom} is the electrical resistance for a
538 homogenous space of 100 Ωm . All the electrical resistances are referred to the
539 same quadripole and R is calculated for each measurement in the dataset. As data
540 errors are difficult to estimate in terms of resistance ratios, some degree of
541 arbitrary choice is present in ratio inversion. Fig. 7A shows the inversion of the
542 resistivity ratios with respect to background (Eq. 8) applying a smoothness
543 constrain of 3%.

544 This time-lapse ratio inversion clearly shows the variation of the electrical
545 resistivity during the experiment (Fig. 7A). The results of the inversion are sections
546 of the percentage variation of resistivity respect to the background values: values
547 equal to 100 Ωm correspond to unchanged resistivity, while values less or more
548 than 100 Ωm show a decreasing or an increasing resistivity, respectively. The
549 inversions were performed using the 2D code developed by Andrew Binley
550 (<http://www.es.lancs.ac.uk/people/amb/Freeware/Freeware.htm>; Slater et al.
551 2000; Cassiani and Binley, 2005; Linde et al., 2006).

552 Fig. 7B shows the results of the ratio of the inverted absolute profiles with
553 respect to the inversion of the background survey. In this case the profiles are
554 inverted with a data error set at 5%, consistent with the reciprocal error removal
555 procedure, and then a pixel by pixel ratio is computed. From the comparison
556 between Fig. 7A and 7B it is apparent that the two approaches are, in this case,
557 essentially equivalent at showing the evolution of the infiltration process. This
558 similarity corroborates the hypothesis of the 2D symmetry of the infiltration

559 process along the sprinkler line, since the ERT monitoring is performed on 2D
560 profiles, assuming a homogeneous resistivity distribution on the third direction.

561 In Fig. 7 the infiltration process is clearly visible. The plume of injected fresh
562 water increases moisture content and consequently reduces resistivity. The shape
563 of the plume is the consequence of a non-uniform distribution of irrigation in the
564 direction perpendicular to the sprinklers' line. The distribution of the artificial
565 precipitation is more likely Gaussian in shape, with considerably more water
566 dropping close to the sprinklers. Time-steps t_5 and t_7 are not shown, as only
567 modest variations are present at these late times after the end of the irrigation.

568

569 **6. SETUP OF THE COUPLED INVERSION**

570 In this work the modeling based on the coupled-inversion described in Section 2 is
571 aimed specifically at the estimation of soil saturated hydraulic conductivity. The
572 physically-based hydrological model CATHY (Camporese et al., 2010) is employed
573 for the numerical solution of Eq. (1) and the simulation of the infiltration
574 experiment. The van Genuchten's parameters necessary for the setup of the
575 numerical model were derived from laboratory experiments: residual saturation is
576 fixed at 0.003 and α (the inverse of the air entry suction) is equal to 5.4 m^{-1} . These
577 values are derived from laboratory experiments and are not considered of
578 paramount importance in the context of the given infiltration experiment. Of
579 course a more complete parameter identification scheme could also include them,
580 as described by Manoli et al. (2015) in the context of using ERT data alone.

581 A careful analysis of Fig. 7 reveal that irrigation was not uniformly distributed in
582 the direction orthogonal to the sprinkler line, probably due to the presence of

583 wind. This was taken into account in order to properly simulate the top boundary
 584 conditions: the irrigation is modeled with a Gaussian distribution centered at 2.5
 585 m, with variance equal to 0.6 m, both values calculated such that the total flux
 586 equals the real irrigation rate. The parameters of the Gaussian distribution are
 587 fixed after a trial procedure where we matched the shape of the measured and
 588 modeled plume (Fig. 7 and Fig. 10).

589 The parameters of Archie's law (Eq. 3), which are necessary to define the ERT
 590 observation operator, are spatially uniform for considered field study. The
 591 exponent n is set to 1.27 as reported in Cassiani et al. (2009c), where the value is
 592 obtained from laboratory calibration on the site's sediments. The initial soil
 593 electrical resistivity $\rho(t_0)$ is set equal to 1300 Ωm , based on the averaged value
 594 obtained by the inversion of background ERT measures. In order to apply Eq. (3),
 595 we need also an estimation of the initial volumetric water content, $\theta(t_0)$. For our
 596 field experiment this is estimated from background GPR and TDR measurements. A
 597 value $\theta(t_0) = 0.07$ is obtained by applying the petrophysical relationship of Topp et
 598 al. (1980):

$$599 \quad \theta = (-530 + 292\varepsilon_r - 5.5\varepsilon_r^2 + 0.043\varepsilon_r^3) \cdot 10^{-4} \quad (5)$$

600 where ε_r is the bulk soil dielectric permittivity. A moisture content value of 7%
 601 corresponds to $S_w(t_0)$ of 0.212 assuming a porosity of 0.33 as estimated by
 602 Cassiani et al. (2009b) for the considered field sediments.

603 In this particular case, we are interested in the value of the saturated hydraulic
 604 conductivity \mathbf{K}_s , that is difficult to identify in unsaturated conditions by direct
 605 measurements. The methodology presented in Manoli et al. (2015) describes
 606 \mathbf{K}_s with a lognormal probability distribution which mean and variance are updated

607 at each assimilation time. Here, the prior values of the hydraulic conductivity mean
608 and variance are summarized in Table 5.

609 The iterative procedure is particularly advantageous when geophysical
610 measurements of different nature (e.g. ERT and GPR) are available for the
611 assimilation, as in the case we consider here. In fact, the independent assimilation
612 of different measurements is to prefer to the joint assimilation of the
613 measurements, since the latter requires the introduction of an artificial
614 normalization to weight the measurements.

615 In this paper the procedure is used to provide the “best possible” estimate of
616 K_s for the site using both ERT and GPR data. We adopt a strategy that is
617 particularly clear in assessing the information content of each dataset and of the
618 two datasets together. In particular, we produce the following four assimilation
619 schemes:

- 620 A. a scheme assimilating only ERT data (similar to the one proposed by Manoli
621 et al., 2015);
- 622 B. a scheme assimilating only GPR data, based on the depth of the infiltration
623 front estimated from the guided wave analysis (see section 3);
- 624 C. a scheme that assimilates alternatively ERT and GPR leading to a final
625 estimate that accounts for both;
- 626 D. a scheme analogous to C, but using GPR and ERT in the reverse order – to
627 check convergence stability (the first iteration starts assimilating GPR data,
628 instead of ERT data).

629 The advantage of assimilating both ERT and GPR measurements is the
630 integration of different information. In this kind of experiment (irrigation

631 monitored on the ground surface), the low sensibility of the ERT array at large
632 depths may be a disadvantage; so the infiltration front may be spread over a broad
633 area, since the most part of the energy is focalized along current paths that cross
634 the wet zone. GPR WARR surveys may be a useful addition to the information
635 obtained from ERT, as GPR can constrain the location of the water front at depth.

636

637 **7. MODELLING RESULTS**

638 The particle filter algorithm assimilates the geophysical data with four different
639 schemes (Fig. 8). Each assimilation scheme leads to a probability distribution of
640 the simulated parameters: in this case K_s is the objective of the coupled inversion.
641 The evolution of the K_s distribution during the assimilation procedures is
642 summarized in Fig. 8. For each assimilation scheme, 3 different prior K_s -
643 distributions are tested to verify the stability of the inversion procedure. It evinces
644 that the convergence towards the estimated K_s value, at the end of the iteration
645 process, is not depending on the initial parameter's range.

646 The estimated values of K_s are only slightly different from scheme to scheme:
647 for case A: $1.0 \cdot 10^{-5} \text{ m s}^{-1}$, for case B: $2.6 \cdot 10^{-5} \text{ m s}^{-1}$, for case C: $1.1 \cdot 10^{-5} \text{ m s}^{-1}$, for case
648 D: $1.1 \cdot 10^{-5} \text{ m s}^{-1}$. Note that the differences in the estimated K_s are almost negligible
649 for practical applications. Assimilating both ERT and GPR we obtain the same
650 K_s value, irrespective of the order of assimilation. The assimilation of only ERT
651 data (Fig. 8A) provides a K_s estimate that is very similar to the ERT-GPR
652 assimilations. The assimilation of the GPR waterfront depths provides a value of
653 K_s about two times larger than the other estimates (Fig. 8B). We attribute this

654 results to the large uncertainty associated to the GPR measurement and analysis,
655 in particular at the time t_2 .

656 Forward hydrological models are then run with the estimated parameters and
657 the results are compared to the geophysical measurements (Tables 4 and 6).
658 Schemes *C* and *D* provide essentially the same hydrological model. The mean and
659 standard deviations of the posterior distributions for the four cases are listed in
660 Table 5 (together with the prior parameters).

661 In Table 4 the waterfront position inverted from the GPR signal is compared to
662 the simulated location of the saturation front. Note that the water front locations
663 estimated from the coupled inversions with the GPR assimilation leads to slightly
664 deeper water front estimations, while ERT and ERT-GPR assimilations conduct to
665 very similar results. The GPR contribution in the combined inversion with ERT
666 drives the estimated waterfront slightly deeper than estimated by ERT only. The
667 waterfront depths from GPR data alone are definitely more problematic to
668 interpret (see also Fig. 9), with uneven penetration speed between time intervals
669 1-2 and 2-3. Note that, as discussed in Section 3, time 2 is a problematic acquisition
670 for GPR, with energy spread over two modes and a more difficult estimation of
671 infiltration front depth.

672 The forward hydrological models are also compared against the ERT field
673 (resistance) dataset (Tab. 5). In this case the simulated hydrological states are
674 converted into geophysical quantities via Eq. 4, and a geophysical forward model
675 (Eq. 3) is run to obtain simulated ERT resistance data. Not surprisingly, the
676 forward model that best matches the field measurements is derived from the
677 assimilation of the sole ERT data. Anyway the assimilation of both ERT and GPR

678 shows a very good fit to the measured ERT, while the assimilation scheme of only
679 GPR-derived waterfronts is distant from the information achieved from ERT
680 survey.

681 Fig. 9 shows the distribution of moisture content predicted by the flow models
682 with the parameters obtained from data assimilation. These saturation profiles are
683 compared against:

- 684 1. the moisture content profiles one could obtain by translating directly the
685 resistivity inverted images (Fig. 7) using the known Archie's law
686 parameters (Eq. 4).
- 687 2. the locations of the infiltration front as estimated from GPR inversion
688 (Section 3).
- 689 3. the estimation of the degree of saturation measured by TDR probes placed
690 at the mid-point of the ERT profile; relative dielectric permittivity is
691 translated into water content using Eq. (5).

692 There is no doubt that the data assimilated simulations are superior at providing
693 estimates of moisture content profiles that, while slightly different from each other,
694 are both consistent with data and model assumptions (most of all, mass balance
695 and hydraulic conductivity homogeneity).

696 The TDR data are used as validation of the modeled water saturation curves (Fig.
697 9). The values are consistent with the hydrological models, that show a rapidly-
698 moving saturation front at the first time steps. Unfortunately the TDR probes reach
699 the maximum depth of only 0.3 m, so no information is available for the deeper
700 portions.

701 For the sake of completeness, we also inverted the synthetic ERT data (Fig. 7C)
702 to provide a direct comparison with the ρ distributions achieved by field
703 measurements (Fig. 7A). In addition, Fig. 10 shows the “true” resistivity structure
704 as simulated by the hydrological model in the combined ERT-GPR data
705 assimilation case. Comparing Figs. 7 and 10, note how inverted and “true”
706 resistivity images tend to diverge at late times, when the front reaches the deeper
707 zones where ERT has the lowest sensitivity, and inversion regularization takes
708 over and smears the images at depth. Consistently, the mass balance derived from
709 ERT data as calculated for the coupled and the uncoupled hydro-geophysical
710 inversions (Table 7) shows the weaknesses of the uncoupled approach for the
711 problem at hand. The uncoupled approach leads to a cumulative volume of injected
712 water over time that strongly overestimates the effective amount of irrigated
713 water.

714 Note that in the literature underestimation of mass balance is more commonly
715 observed (e.g., Singha and Gorelick, 2005), but this fact is dependent primarily on
716 the acquisition scheme and electrode geometrical configuration (e.g. cross-hole
717 versus surface measurements, as in this work).

718

719 **8. CONCLUSIONS**

720 Hydro-geophysical techniques are extremely useful in monitoring the
721 hydrological processes acting in the vadose zone and the data can be effectively
722 translated into hydrological quantities, particularly state variables such as
723 moisture content. The presented field case study analyzes a controlled irrigation

724 test in an unsaturated subsoil with a plain terrain and nearly homogeneous sandy
725 soil.

726 The adopted hydro-geophysical methodology may strongly affect the results of
727 the hydro-geophysical inversion and consequently the hydrological parameter
728 estimations. An approach, that fully couples hydrological modeling and
729 geophysical measurements in a data assimilation procedure, leads to more
730 accurate results. Avoiding the geophysical inversion of the data, we reduce the
731 uncertainty in the hydrological quantities estimation, since no artifacts are
732 inserted in the method by solving an inverse problem. The errors that may be
733 present are due only to data acquisition and model choosing, as in any hydro-
734 geophysical issue. Of course an analysis of the inverted data is generally necessary,
735 not only to ascertain the data quality, but also to direct a correct choice of the
736 hydrological model needed to explain the data (see, e.g., discussion in Camporese
737 et al., 2011). One of the advantages of the coupled approach, that includes a
738 stochastic process, is the proper conservation of mass. This aspect is often a key
739 issue of the uncoupled approach, where the calibration of hydrological models via
740 geophysical inverted data may lead to inconsistent results that may jeopardize the
741 user's confidence in the method.

742 In the present field case both ERT and the infiltration front estimated with the
743 GPR data are considered in the data assimilation process, using a Sequential
744 Importance Resampling (SIR) that allows a flexible assimilation of either or both
745 datasets in a natural, non-subjective manner (i.e. without arbitrary weighting of
746 one dataset with respect to the other). From this procedure the information
747 content of each dataset in the assimilation procedure emerges naturally.

748 In this particular case study, it is apparent that ERT data provide most of the
749 information needed to a robust hydraulic conductivity estimation. GPR, albeit
750 being apparently of easy interpretation in its time-lapse evolution (see Figure 3), at
751 a more in-depth quantitative analysis shows its intricacies linked to the inversion
752 of multi-modal dispersion guided waves. As the energy distribution over different
753 modes changes over time due to the changing geometry of the wet layer, the
754 inversion of GPR data requires particular attention and ultimately delivers weak
755 information on the infiltration process.

756 The comparison between coupled and uncoupled hydro-geophysical inversions
757 shows that, in this particular case, the latter is superior. This happens primarily
758 because the monitoring of type of experiment that we consider (irrigation and
759 infiltration from the ground surface) depends strongly on our ability to image the
760 processes honoring mass balance. In this respect, the uncoupled approach is not
761 capable of reproducing the real state of the system and consequently the mass
762 balance. The uncoupled approach may therefore lead to erroneous parameter
763 estimate. It should be noted how other problems may be less prone to suffering
764 from an uncoupled approach (see e.g. Camporese et al., 2011).

765

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775

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978 **TABLES**

979

Irrigation steps	Irrigation start [min]	Irrigation end [min]	Cumulative water volume [m ³]
1	0	115	2.509
2	146	233	4.127
3	264	327	5.652

980

981 **Table 1.** *Time schedule and irrigated volumes for the infiltration experiment.*

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Geophysical techniques	Starting time of the survey [min]								
	Background								
	t_0	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8
GPRWARR	-10	115	233	327	-	-	-	-	-
ERT	-5	120	240	335	1030	1150	1420	1480	1540

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Table 2. Time schedule of the geophysical acquisitions; time is referred to the irrigation start.

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Time step	Averaged thickness [m]	Standard deviation of thickness [m]	Averaged velocity [m/ns]	Standard deviation of velocity [m/ns]	Number of averaged simulations	NSI range of averaged simulations
t_1	0.46	0.031	0.091	0.0013	197	0.984-0.987
t_2	0.49	0.019	0.074	0.0006	106	0.886-0.889
t_3	0.74	0.016	0.081	0.0007	83	0.987-0.990

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Table 3. *Statistics of the GPR slab waveguide simulations that best fit the*

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measured dispersion curves.

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Time-step	t_1 (m)	t_2 (m)	t_3 (m)	Mean Error (m)
GPR inversion	-0.46	-0.49	-0.74	
Posterior ERT	-0.32	-0.52	-0.66	0.083
Posterior GPR	-0.38	-0.61	-0.79	0.083
Posterior ERT-GPR	-0.34	-0.54	-0.70	0.070

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992 **Table 4.** *Infiltration front depth for the first three time-steps, obtained from GPR-*993 *EM-waveguide inversion and from posterior hydrological forward models. The last*994 *column is the average absolute error between the waterfront positions measured*995 *with the GPR and those estimated with the posterior hydrological forward models.*

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Prior distribution		Posterior distribution							
		ERT		GPR		ERT+GPR		GPR+ERT	
Mean	St. dev.	Mean	St. dev.	Mean	St. dev.	Mean	St. dev.	Mean	St. dev.
m/s	m/s	10^{-5} m/s	10^{-5} m/s	10^{-5} m/s	10^{-5} m/s	10^{-5} m/s	10^{-5} m/s	10^{-5} m/s	10^{-5} m/s
1×10^{-7}	1×10^{-7}	0.99	0.014	2.50	0.148	1.15	0.014	1.11	0.015
1×10^{-5}	1×10^{-5}	1.02	0.008	2.63	0.083	1.14	0.076	1.08	0.018
1×10^{-3}	1×10^{-3}	0.90	0.018	2.86	0.053	1.17	0.032	1.06	0.012

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Table 5: Prior and posterior distributions of the hydraulic conductivity K_s for the different data assimilation schemes

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Time-step	t_1	t_2	t_3	t_4	t_5	t_6	t_7	t_8	Mean
Posterior ERT	3.5	4.3	3.7	3.1	3.1	3.4	3.5	3.6	3.525
Posterior GPR	3.6	4.4	3.7	4.4	4.5	5.1	5.3	5.4	4.550
Posterior ERT-GPR	3.5	4.2	3.6	3.2	3.2	3.6	3.7	3.8	3.600

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Table 6. Root mean square relative error between the field measured electric resistance value and those simulated with the posterior hydrological forward models (results in %). The last column is the mean in time of these errors. The relative error is adopted because the electric resistances vary over several orders of magnitude.

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Irrigation steps	Irrigation time [min]	Cumulative water volume				
		Effective injected volume[m ³]	Coupled model		Uncoupled model	
			Volume [m ³]	% error	Volume [m ³]	% error
1	115	2.509	2.354	6.2	4.181	66.6
2	233	4.127	3.997	3.1	7.713	86.9
3	327	5.652	5.564	1.6	9.559	69.1

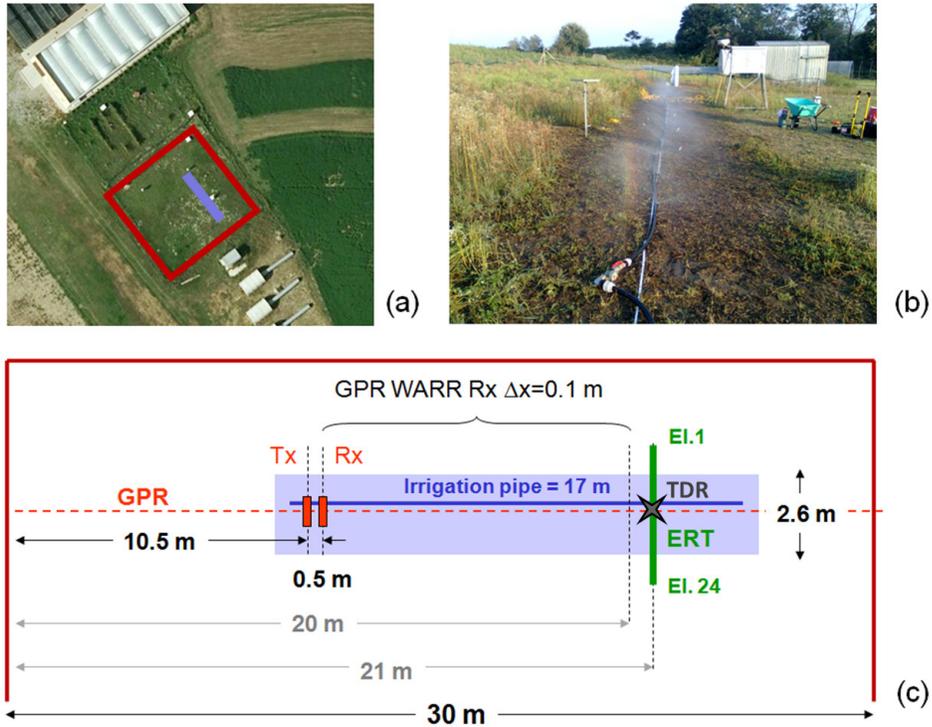
1015

1016 **Table 7.** *Mass balance achieved with coupled and uncoupled hydro-geophysical*

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inversions.

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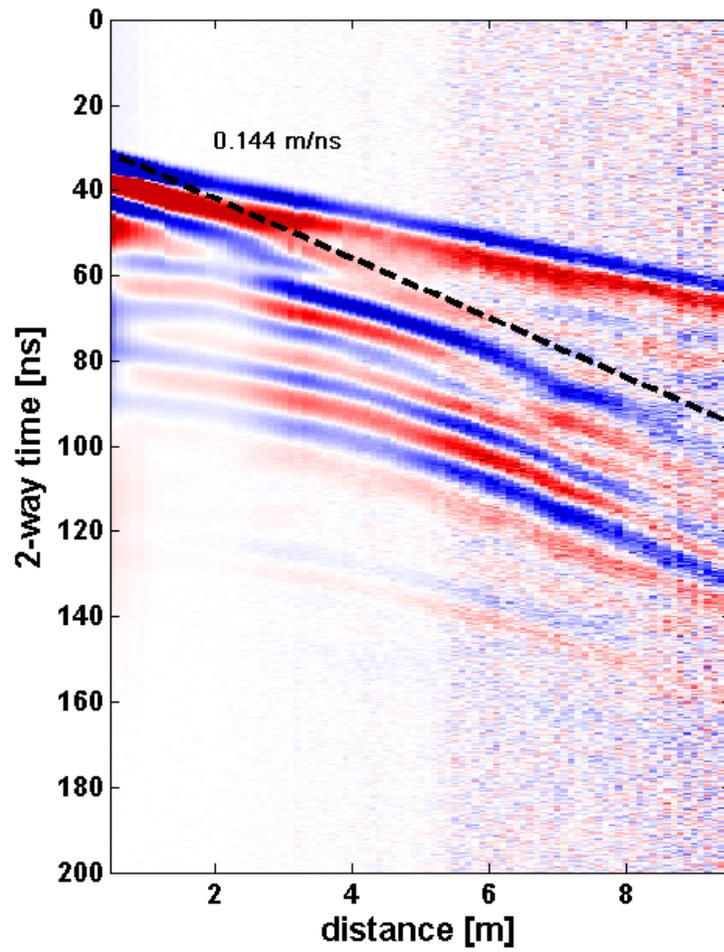
1019 **FIGURES**

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1022 **Figure 1.** Scheme and location of the experiment: (a) aerial view of the field with the
 1023 irrigated zone highlighted in blue; (b) the sprinkler line during the irrigation; (c)
 1024 scheme of the geophysical surveys and position of the irrigated soil.

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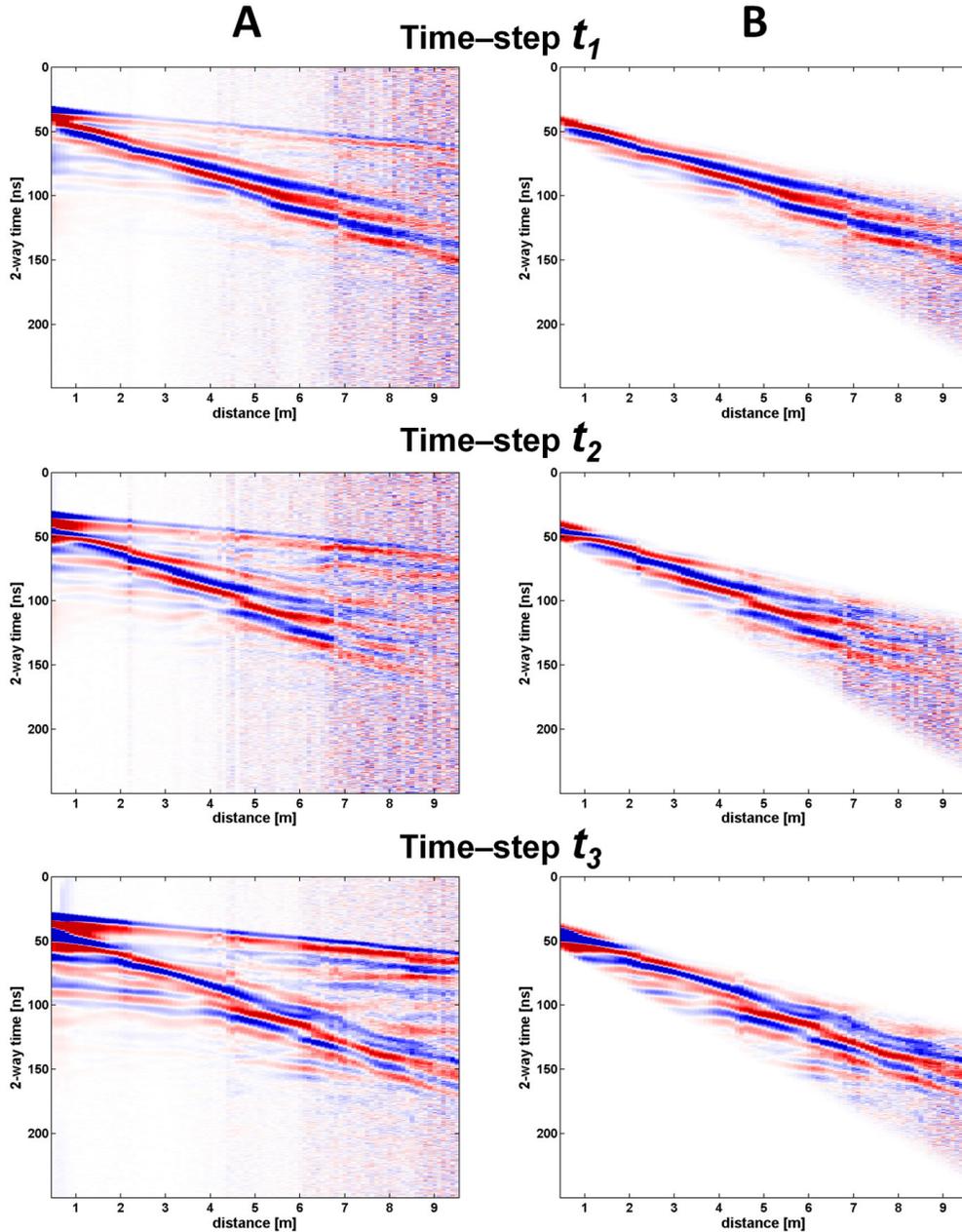


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1028 **Figure 2.** Background WARR survey with the identification of the direct wave
1029 through the ground.

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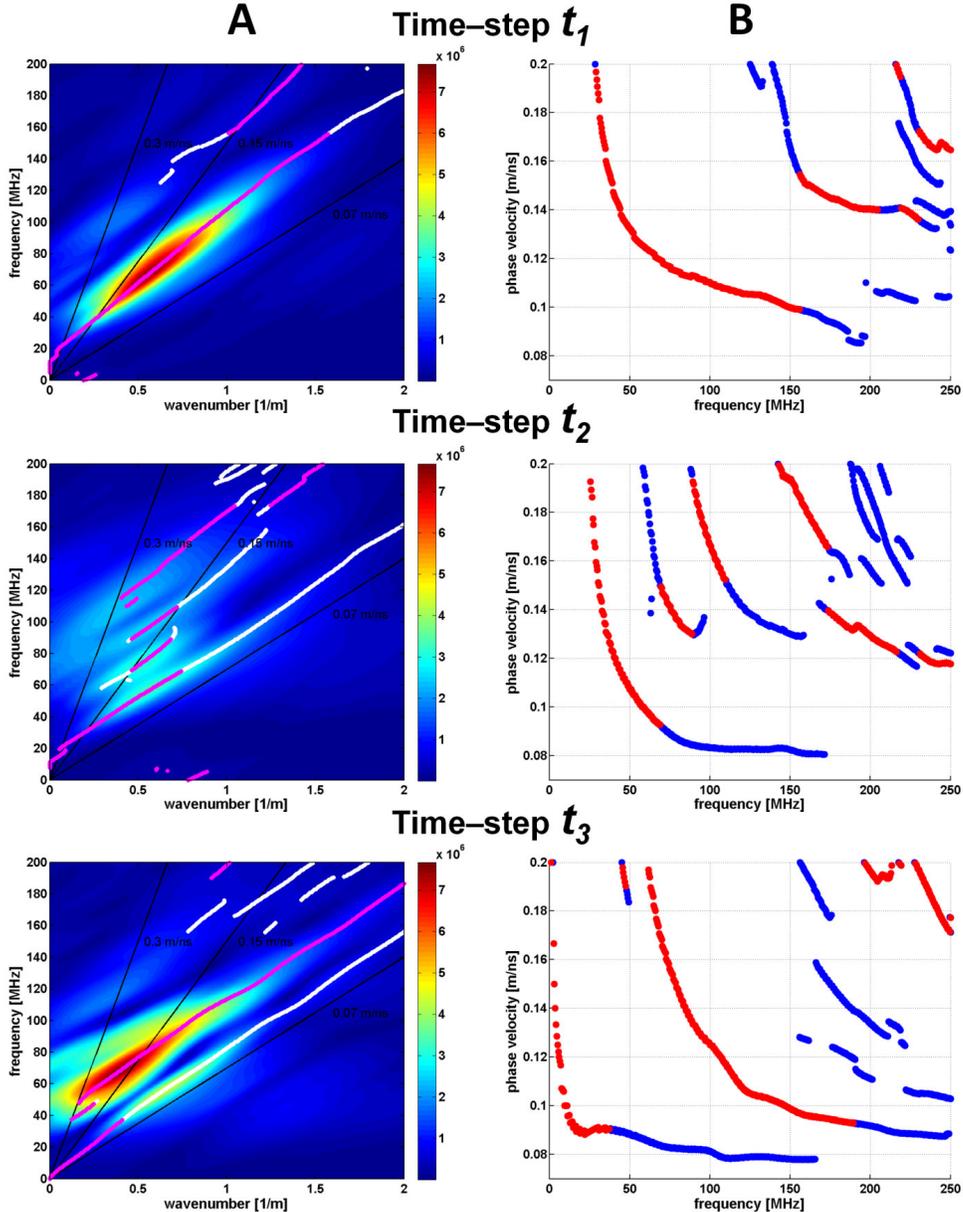
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1033 **Figure 3.** Field measured WARR radargrams at the times t_1 , t_2 and t_3 . A) On the left,

1034 the radargrams are filtered only by the “dewow” procedure (traces are normalized).

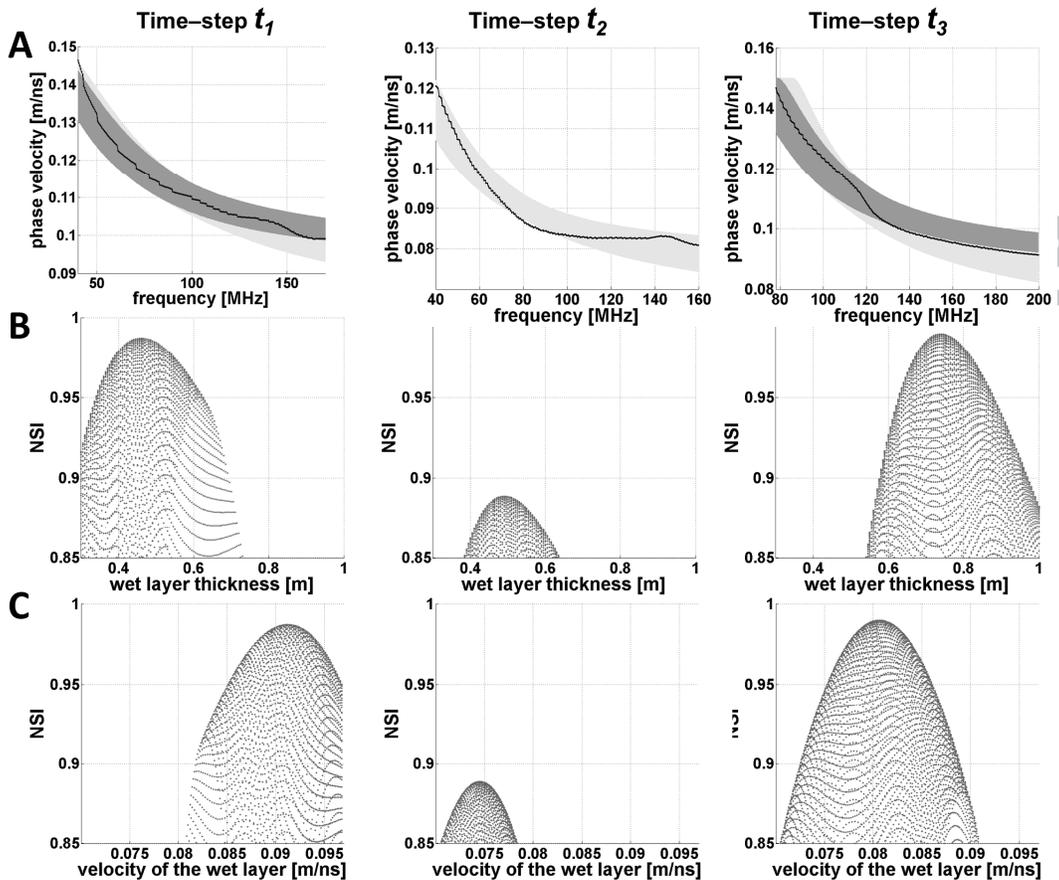
1035 B) On the right, the same radargrams are displayed after the preprocessing (muting

1036 and FIR filter).



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1038 **Figure 4.** Analysis of the GPR soundings in the frequency domain. (a) On the left, the
 1039 f - k domain are displayed with the superimposition of the maxima of the spectral
 1040 density (magenta dots for main maxima, white dots for local maxima). Power
 1041 spectrum density scale in V^2/Hz . (b) On the right, the dispersion curves inferred from
 1042 f - k maxima: red and blue dots correspond to absolute and local maxima, respectively.
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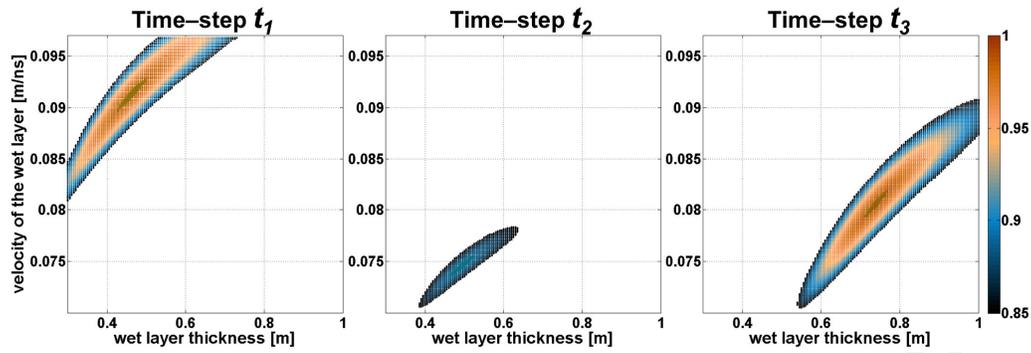
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Figure 5. Parameterizations of the simulations of slab waveguides that best fit the measured dispersion curves. (a) Superposition of the field-derived dispersion curves (black dotted lines) and of the best simulated dispersion curves: light gray lines with $0.85 < NSI < 0.95$ and dark gray lines with $NSI > 0.95$. (b) Wet layer thickness from the best simulations plotted against $NSI > 0.85$. (c) Wet layer velocity from the best simulations plotted against $NSI > 0.85$.



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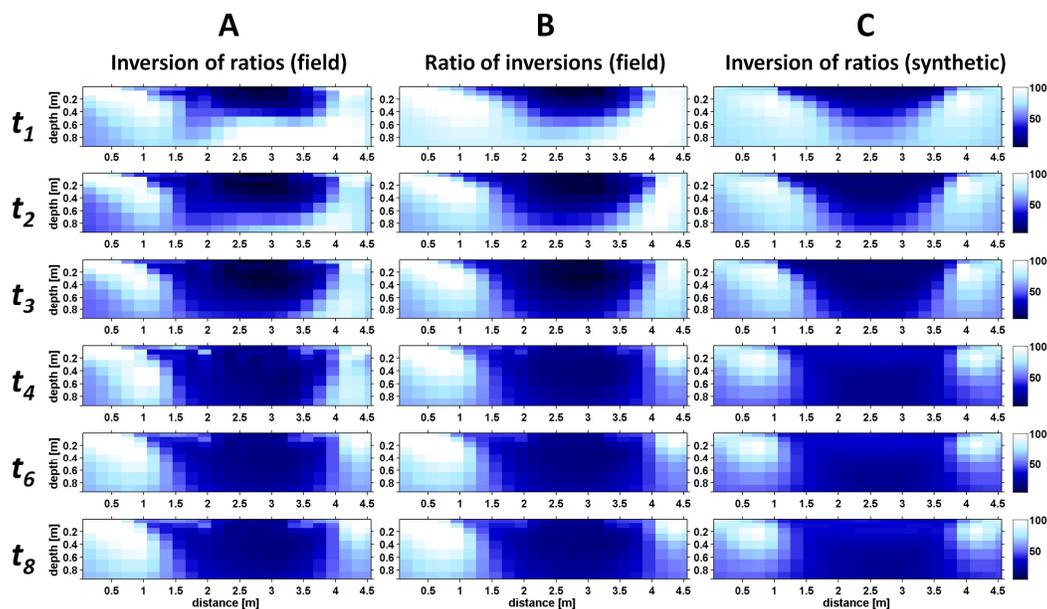
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1056 **Figure 6.** Correlation between the simulated parameters: velocity and thickness of

1057 the layer that guides EM waves; color bar is NSI value. Green polygon highlights the

1058 simulations with highest NSI values for each time-steps: $NSI > 0.984$ for t_1 , $NSI > 0.886$ 1059 for t_2 , $NSI > 0.987$ for t_3 .



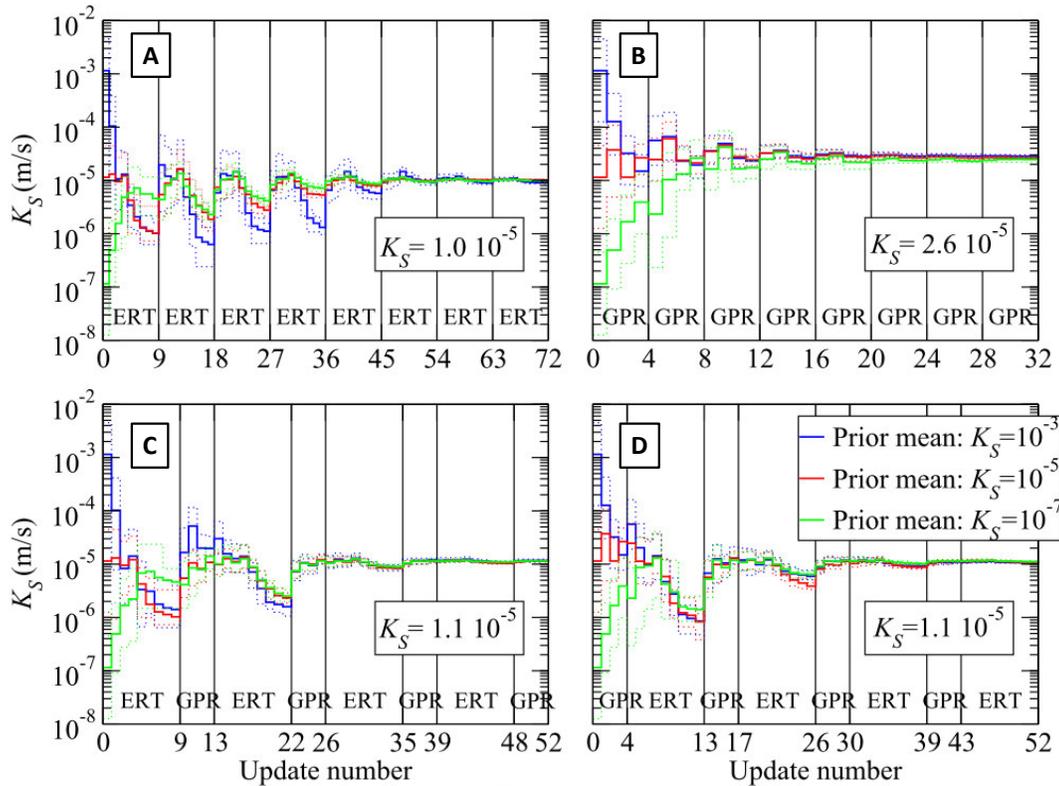
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1061 **Figure 7.** Time-lapse of inverted electrical resistivity profiles displayed as percentage
 1062 of variation respect to background. A) Inversion of the ratio of apparent resistivities,
 1063 measured at the field, respect to background survey. B) Ratio of the inverted profiles
 1064 related to background inversion. C) Inversion of the ratio of synthetic apparent
 1065 resistivities, simulated through the hydrological model, respect to the assumed
 1066 homogeneous background state.

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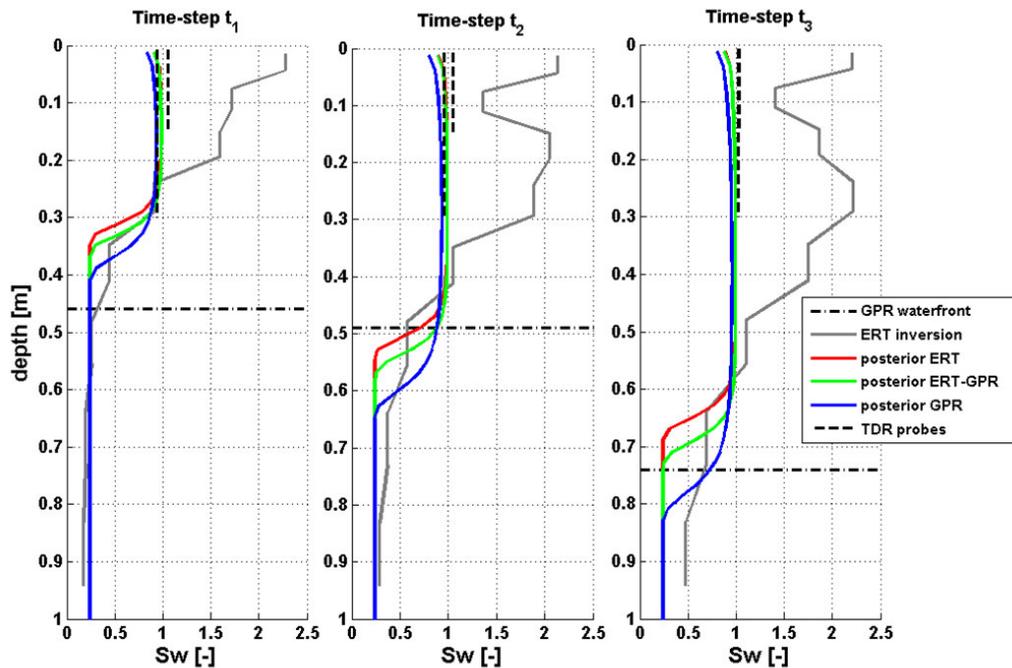
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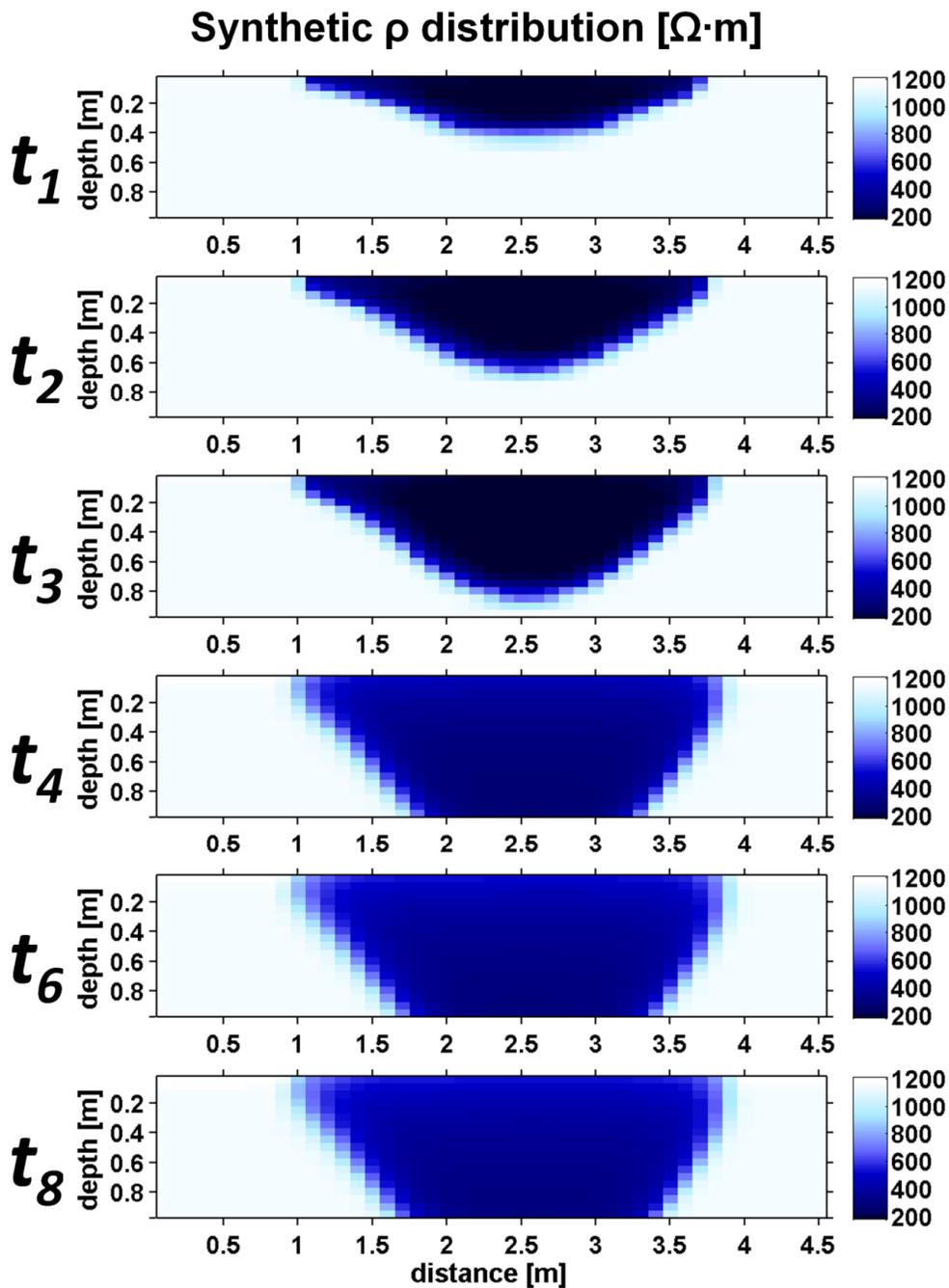
1072 **Figure 8.** K_S -distribution during the iteration of the data assimilation framework.
 1073 The lines of different colors (blue, red and green) point out different initial
 1074 distribution of the parameter: solid line is the mean of the distribution, dashed lines
 1075 are the maximum and minimum values in the range. (a) sequential assimilation of the
 1076 ERT data. (b) sequential assimilation of the waterfront position from GPR data. (c)
 1077 sequential assimilation of ERT and GPR information. (d) sequential assimilation of
 1078 GPR and ERT information. The vertical lines, including the graph extremes, indicate
 1079 the 9 measurement instants (t_0 to t_8).

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Figure 9. Vertical profiles of water saturation, extrapolated on the position of the sprinklers line. Solid lines of red, blue and green colors are the results of forward hydrological models obtained with the K_s estimation assimilating only ERT, only GPR and both techniques, respectively. Gray solid line is the result of the uncoupled ERT inversion. The horizontal black dot-dashed line is the estimation of waterfront location from GPR-EM-waveguide inversion. The vertical black dashed lines are the estimated water saturation achieved by TDR probes (15 and 30 cm length).



1092

1093 **Figure 10.** *Electrical resistivity sections at different time steps, derived by the*
1094 *hydrological model inferred from the assimilation of both ERT and GPR datasets.*