STRATEGIC ANNOUNCEMENTS OF REFERENCE POINTS IN DISPUTES AND LITIGATIONS

ANDREA GALLICE

Working paper No. 3 - March 2012
Strategic announcements of reference points in disputes and litigations

Andrea Gallice*

Department of Economics and Statistics, University of Torino,
Corso Unione Sovietica 218bis, 10134, Torino, Italy.

and

Collegio Carlo Alberto, Via Real Collegio 30, 10024, Moncalieri, Italy.

Abstract

This note shows how the frequent occurrence of seeing exceedingly high claims in disputes and litigations can be rationalized by a model in which claimants display reference dependent preferences, expect the judge to use a generalized social welfare function, and strategically announce their reference points.

Keywords: reference points, claims, litigations.

JEL classification: D03, D63, K41.

1 Introduction

In disputes and litigations it is common to observe litigants asking for exceedingly high claims that are often mutually inconsistent. The existence of subjective

*Contact: gallice@econ.unito.it
behavioral biases that influence agents’ perception of fairness and lead claimants to overestimate how much they deserve certainly contributes to generating such a phenomenon. For instance, it is well known that self-serving bias can create costly impasses in bargaining and negotiations (see Babcock et al., 1995, and Babcock and Lowenstein, 1997). But while behavioral biases unconsciously affect individuals’ claims, the announcement of a very high claim can also be the result of a conscious and strategic decision by the parties. Litigants can in fact purposively exaggerate their claims with the goal of influencing the final allocation that the judge will implement.

In this note, we explore this second option and investigate the strategic aspects related to agents’ announcement of their claims. We show that, in a framework of reference dependent preferences à la Koszegi and Rabin (2006), claimants who expect the judge to make the final decision according to a very general form of social welfare function have an interest in purposively inflating their claims.

Reference dependent preferences (RDPs) capture the famous loss aversion conjecture introduced in the classic article by Kahneman and Tversky (1979). RDPs explicitly acknowledge the fact that agents’ perception of a given outcome is influenced by the comparison between the outcome itself and a certain ex-ante reference point. More precisely, people define gains and losses with respect to the reference point and losses loom larger than gains. RDPs thus seem particularly appropriate to depict the preferences of individuals involved in disputes and litigations. These are in fact typical situations in which agents build their own expectations about the allocation that the authority will implement and inevitably ex-post compare the actual outcome with the expected one.

To sum up, the analysis presented in this paper applies to all those cases
in which reference dependent preferences constitute an appropriate framework, conflicting interests of the agents must be settled by an external authority, and litigants have the possibility to ex-ante declare what they expect to get. Examples include trials, divorces, reimbursements for damages, and political negotiations.

2 The model

We model the situation of two claimants who cannot agree on how to divide a homogeneous and perfectly divisible good (whose amount we normalize to $S = 1$). The litigants thus delegate the choice and the implementation of the final allocation of the good to a judge/planner. Let $x = (x_1, x_2)$ indicate a possible allocation such that $x_i$ is the amount of the good that the planner assigns to claimant $i \in \{1, 2\}$. Feasible allocations are the ones for which $x_i \in [0, 1]$ for any $i$ and $\sum_i x_i \leq 1$.

The litigants expect (perhaps incorrectly) that the judge, in choosing which final allocation to implement, will use a generalized utilitarian social welfare function of the form $W(u) = \sum_i g(u_i)$ where $g(\cdot)$ is an increasing and strictly concave function and $u_i$ is the utility of claimant $i \in \{1, 2\}$. The concavity of $g(\cdot)$ implies that the planner attaches progressively lower weight to additional units of utility. In particular, the more concave is $g(\cdot)$, the more egalitarian will be the final allocation (see Atkinson, 1970). As such $W(u)$ includes all those cases that fall between two well-known extremes. On one hand, as $g(\cdot)$ approaches a linear function, $W(u)$ tends to the purely utilitarian SWF (Bentham, 1789): $W_{ut}(u) = \sum_i u_i$. On the other hand, as $g(\cdot)$ becomes “infinitely” concave, $W(u)$ approaches the maxmin or Rawlsian SWF (Rawls, 1971): $W_{mm}(u) = \min \{u_1, \ldots, u_n\}$. 

3
For what concerns claimants’ utility function, we assume that individual preferences are such that:

\[ u(x_i, r_i) = x_i + \mu(x_i - r_i) \]  

(1)

where the function \( \mu(\cdot) \) is a “universal gain-loss function”. Given the individual reference point \( r_i \in [0, 1] \), \( \mu(x_i - r_i) \) reflects the additional effects that perceived gains or losses have on \( u(\cdot) \) on top of the utility the agent gets from the direct possession/consumption of \( x_i \). In other words, we assume that claimants display reference dependent preferences a la Koszegi and Rabin (2006).\(^1\) The function \( \mu(\cdot) \) satisfies the following properties:

P1: \( \mu(x_i - r_i) \) is strictly increasing in \( x_i \), decreasing in \( r_i \), and such that \( \mu(0) = 0 \).

P2: \( \mu(x_i - r_i) \) is continuous for any \( x_i \) and differentiable for any \( x_i \neq r_i \).

P3: \( \mu'(x_i - r_i) = \gamma \) for \( x_i < r_i \) and \( \mu'(x_i - r_i) = \lambda \) for \( x_i > r_i \) with \( \gamma > \lambda > 0 \).

In line with the original prospect theory formulation of Kahneman and Tversky (1979), the function \( \mu(\cdot) \) is thus characterized by a kink at \( x_i = r_i \) and is steeper in the domain of losses than in the domain of gains. Notice that P3 implies that the function \( \mu(\cdot) \) is linear, as its first derivative with respect to \( x_i \) is a constant. As such, we do not capture the diminishing sensitivity of perceived gains or losses.\(^2\)

---

\(^1\)Koszegi and Rabin (2006) actually introduce a more general family of utility functions given by \( u(x_i, r_i) = m(x_i) + \mu(m(x_i) - m(r_i)) \) where \( m(\cdot) \) is an increasing function that captures the direct effect of \( x_i \) on total utility \( u(\cdot) \). In this note, we thus set \( m(x_i) = x_i \).

\(^2\)In addition to the properties here described in P1 and P2, Koszegi and Rabin (2006) also define an additional property (labeled A3 in their paper): \( \mu''(x_i - r_i) > 0 \) for \( x_i < r_i \) and \( \mu''(x_i - r_i) < 0 \) for \( x_i > r_i \), i.e., the function \( \mu(\cdot) \) is convex for perceived losses and concave for perceived gains. In examples and applications, they then substitute A3 with A3': \( \mu''(x_i - r_i) = 0 \), i.e., a linear functional form like the one we assume here in P3.
We do not investigate the issue of how agents introspectively select their reference points \( r_i \).\(^3\) We focus instead on the issue of how claimants should strategically announce their reference points to the judge with the goal of influencing, obviously in their own interest, the final allocation of the good. We thus introduce \( r_i^a \in [0,1] \), the key variable of the model, which indicates the reference point that agent \( i \) announces to the judge. Notice that \( r_i^a \) may differ from \( r_i \), i.e., what an agent declares to expect \( (r_i^a) \) may differ from what he actually expects \( (r_i) \).

### 2.1 The planner’s problem

In this section we solve the planner’s problem from the claimants’ point of view. Claimants announce to the planner what they expect to get (i.e., the planner knows the vector \( r^a = (r_1^a, r_2^a) \)). They then think, somewhat naively, that the planner sets \( r = r^a \). As such, litigants expect the planner to face and solve the following problem:

\[
\max_{x_1,x_2} W(u) = \left[ g(x_1 + \mu(x_1 - r_1^a)) + g(x_2 + \mu(x_2 - r_2^a)) \right] \quad \text{s.t.} \quad x_1 + x_2 = 1 \quad (2)
\]

The problem has a solution given that \( W(u) \) is a continuous function defined on the closed and bounded space \([0,1] \times [0,1]\) and thus the Weierstrass theorem applies. Moreover \( W(u) \) is concave (it is the sum of two concave functions) such that first order conditions are sufficient. The optimal allocation \( \hat{x} = (\hat{x}_1, \hat{x}_2) \),

---

\(^3\)Agents can set their own \( r_i \) in line with what they have or are used to (as in the traditional status quo formulation of Kahneman and Tversky, 1979), with what they expect (as proposed in Koszegi and Rabin, 2006), or with what they think they deserve, just to name a few possibilities. The choice of \( r_i \) can also be plagued by unconscious behavioral biases. For instance, Gallice (2011) studies the implications of self-serving biased reference points.
where \( \hat{x} = \arg \max W(u) \) and \( \hat{x}_2 = 1 - \hat{x}_1 \), will thus equalize the marginal utilities of the two claimants:

\[
\begin{align*}
[g'(\hat{x}_1 + \mu(\hat{x}_1 - r^a_1))] (1 + \mu'(\hat{x}_1 - r^a_1)) &= \\
= [g'(1 - \hat{x}_1 + \mu(1 - \hat{x}_1 - r^a_2))] (1 + \mu'(1 - \hat{x}_1 - r^a_2))
\end{align*}
\]

(3)

Now notice that, given that the function \( \mu(\cdot) \) is linear, its derivative \( \mu'(\cdot) \) is a constant (property P3). In particular, whenever a) \( \hat{x}_i < r^a_i \) for both agents or b) \( \hat{x}_i > r^a_i \) for both agents, then \( \mu'(\hat{x}_1 - r^a_1) = \mu'(1 - \hat{x}_1 - r^a_2) \) for any \( \hat{x}_1 \). From now on we focus on these two cases (i.e., we ignore the situations such that \( \hat{x}_i < r^a_i \) for \( i \in \{1, 2\} \) and \( \hat{x}_j > r^a_j \) for \( j \neq i \)). We will later check that one of the two cases (more precisely case a) is indeed the one that emerges in equilibrium.

The fact that \( \mu'(\hat{x}_1 - r^a_1) = \mu'(1 - \hat{x}_1 - r^a_2) \) for any \( \hat{x}_1 \) implies that (3) is satisfied if and only if (4) holds:

\[
\begin{align*}
[g'(\hat{x}_1 + \mu(\hat{x}_1 - r^a_1))] &= [g'(1 - \hat{x}_1 + \mu(1 - \hat{x}_1 - r^a_2))]
\end{align*}
\]

(4)

The function \( g(\cdot) \) is strictly concave and monotonically increasing, which implies that its derivative \( g'(\cdot) \) is monotonically decreasing. It follows that (4) holds if and only if the two arguments are the same:

\[
(x_1 + \mu(\hat{x}_1 - r^a_1)) = (1 - \hat{x}_1 + \mu(1 - \hat{x}_1 - r^a_2))
\]

(5)

We are now in the position to study the effects that the announced reference point \( r^a_i \) has on \( \hat{x}_i \). Focusing without loss of generality on claimant \( i = 1 \), we can
express (5) as:

\[ F(\hat{x}_1, r^a_1) = 2\hat{x}_1 + \mu(\hat{x}_1 - r^a_1) - \mu(1 - \hat{x}_1 - r^a_2) - 1 = 0 \quad (6) \]

This is an implicit function that satisfies the assumptions of the implicit-function theorem. In fact, property P2 of the gain-loss function \( \mu(\cdot) \) ensures that partial derivatives \( \frac{\partial F(\hat{x}_1, r^a_1)}{\partial \hat{x}_1} \) and \( \frac{\partial F(\hat{x}_1, r^a_1)}{\partial r^a_1} \) are continuous and different from zero for any \( x_1 < r^a_1 \). Total differentiation of \( F(\hat{x}_1, r^a_1) \) leads to:

\[ \frac{\partial \mu(\hat{x}_1 - r^a_1)}{\partial r^a_1} + \left( 2 + \frac{\partial \mu(\hat{x}_1 - r^a_1)}{\partial \hat{x}_1} - \frac{\partial \mu(1 - \hat{x}_1 - r^a_2)}{\partial \hat{x}_1} \right) \frac{\partial \hat{x}_1}{\partial r^a_1} = 0 \quad (7) \]

such that \( \frac{\partial \hat{x}_1}{\partial r^a_1} \) can be expressed as:

\[ \frac{\partial \hat{x}_1}{\partial r^a_1} = -\frac{\frac{\partial \mu(\hat{x}_1 - r^a_1)}{\partial r^a_1}}{2 + \frac{\partial \mu(\hat{x}_1 - r^a_1)}{\partial \hat{x}_1} - \frac{\partial \mu(1 - \hat{x}_1 - r^a_2)}{\partial \hat{x}_1}} \quad (8) \]

The numerator of the ratio is negative (by property P1) and the denominator is positive. In particular, the second term is positive (again by P1) while the third one is negative given that \( \hat{x}_2 \) decreases as \( \hat{x}_1 \) increases. It follows that \( \frac{\partial \hat{x}_1}{\partial r^a_1} > 0 \). Because of symmetry, the same inequality obviously also holds for claimant \( i = 2 \) such that we can state the main result of this note:

\[ \frac{\partial \hat{x}_i}{\partial r^a_i} > 0 \text{ for any } i \in \{1, 2\} \quad (9) \]

Given that the utility of claimant \( i \) is strictly increasing in \( x_i \), this result indicates that agent \( i \), even though he anticipates that he will possibly get \( \hat{x}_i < r^a_i \),

\[ ^4 \text{In particular, } \frac{\partial u(\hat{x}_i, x_i)}{\partial x_i} = 1 + \mu'(x_i - r_i) > 0 \text{ given that } \mu'(x_i - r_i) > 0 \text{ (see property P3 of the } \mu \text{ function).} \]
should purposely inflate his initial claim. In fact, the final allocation agent \(i\) gets (\(\hat{x}_i\)) is positively anchored to the reference point that the agent announces (\(r^a_i\)). It follows that in general \(r^a_i > r_i\), i.e., the announced reference point will be larger than the “true” reference point (\(r^a_i = r_i\) will happen only when \(r_i = 1\)).

In the Nash equilibrium of this announcement game, both agents will thus announce \(\hat{r}^a_i = 1\). In line with the social welfare function \(W(u)\), the judge will thus implement the Solomonic solution \(\hat{x} = \left\{ \frac{1}{2}, \frac{1}{2} \right\}\). As such, condition \(\hat{x}_i < \hat{r}^a_i\) holds for both claimants and thus confirms the validity of the passages that we implemented in moving from (3) to (4).

The following example uses very simple functional forms for \(g(\cdot)\) and \(\mu(\cdot)\) to illustrate the result established in (9) as well as the equilibrium of the game.

**Example 1** Let \(g(\cdot) = \sqrt{\cdot}\), \(\mu(x_i-r_i) = \begin{cases} x_i - r_i & \text{if } x_i \geq r_i \\ 2(x_i - r_i) & \text{if } x_i < r_i \end{cases}\) for \(i \in \{1, 2\}\) and \(r^a = \{r^a_1, r^a_2\}\). Then \(u_i = \begin{cases} 2x_i - r^a_i & \text{if } x_i \geq r^a_i \\ 3x_i - 2r^a_i & \text{if } x_i < r^a_i \end{cases}\). Claimants thus expect the planner to maximize the function \(W(u) = \sqrt{3x_1 - 2r^a_1} + \sqrt{3x_2 - 2r^a_2}\). First order condition is given by \(3 \left(\sqrt{3x_1 - 2r^a_1}\right)^{-1} = 3 \left(\sqrt{3 - 3x_1 - r^a_2}\right)^{-1}\) and the optimal allocation is then given by \(\hat{x} = (\hat{x}_1, \hat{x}_2)\) with \(\hat{x}_1 = \frac{1}{2} + \frac{1}{3}(r^a_1 - r^a_2)\) and \(\hat{x}_2 = 1 - \hat{x}_1\).

Marginal effects are strictly positive \(\frac{\partial \hat{x}_1}{\partial r^a_1} = \frac{\partial \hat{x}_2}{\partial r^a_2} = \frac{1}{3}\) and obviously they can also be retrieved using equation (8). In equilibrium, we will have \(\hat{r}^a = \{1, 1\}\) and \(\hat{x} = \left\{ \frac{1}{2}, \frac{1}{2} \right\}\).

---

5. The judge’s situation indeed resembles King Solomon’s problem of having to establish the “property” of a baby between two women who both claimed to be his natural mother. As is well known, King Solomon’s suggested solution was to cut the baby in half.
3 Conclusions

This note explored the strategic aspects that may underlie litigants’ decisions to ask for exceedingly high claims. More precisely, the note showed that if claimants are characterized by reference dependent preferences (an assumption that seems particularly appropriate in the context of disputes and litigations), and if they expect the judge to reach his decision in line with the maximization of a very general form of social welfare function, then there is indeed an incentive for agents to announce high reference points. Claimants in fact anticipate that in the final allocation what they will get is positively anchored to their initial claims. As such, they purposively inflate these claims.

References


