Equally Weighted vs. Long-Run Optimal Portfolios

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Equally Weighted vs. Long-Run Optimal Portfolios[1]

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Recent research casts doubt on the ability of portfolio optimizing strategies to outperform simple, equally weighted portfolios (i.e., portfolios where each asset has the same weight) in out-of-sample experiments. However, most results concern the case in which an investor has a 1-month time horizon. In this paper, we examine whether this finding holds for longer investment horizons over which the optimizing strategy exploits linear predictability in returns. Our experiments indicate that power utility investors with longer horizons on average would have benefited, ex post, from an optimizing strategy over the period 1995–2009. This result holds for intermediate-to-high investor risk aversion, and is insensitive to the number of predictors included in the forecasting model and to deduction of transaction costs from measured portfolio performance.

JEL Classification Codes: G11, L85.

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1. Introduction

Finance scholars and practitioners have long recognized that the out-of-sample, ex post performance of ex ante optimal portfolios may be worse than that of simpler strategies, which are sub-optimal from an in-sample, ex ante perspective (see, e.g., Bawa et al., 1979; Friend and Blume, 1970; Jorion, 1985).[2] In particular, Brown (1976), Duchin and Levy (2009), Garlappi et al. (2009a), and Jacobs et al. (2009), among others, report that equally weighting available assets (the so-called 1/N strategy) consistently outperforms almost every optimizing model they scrutinize. This is very troublesome evidence for the entire portfolio management industry, indicating that simple "rules-of-thumb" may lead to higher realized, ex post performance than do optimizing asset management strategies. However, this evidence refers overwhelmingly to short (typically, 1 month) investment horizons, while several institutions and many households invest for the long-term and care about portfolio properties associated with longer time horizons.

Our paper shows that this startling, out-of-sample performance by the naïve, equally weighted strategy fails to generally extend to longer-term portfolio strategies, when asset return predictability is taken into account. This finding, which to our best knowledge is new in the literature, implies that portfolio management methods become increasingly valuable to investors as their planned time horizon grows. This insight seems of relevance to mutual and pension fund managers, who should reflect in their portfolio optimizing decisions the length of the horizons of their shareholders.

Our emphasis on investors' horizons, which we set from 1 to 60 months in this paper, guides several aspects of our research design. First, we allow for predictable risk premia and parameter uncertainty, since these features need to be taken into account by managers of long-term portfolios; at the same time these very features ensure that time horizon affects optimal asset allocation (see, e.g., Barberis, 2000; Campbell and Viceira, 2002). Second, we model, by assuming power utility, an investor who has preferences with regard to skewness and kurtosis of wealth. This setting simplifies to mean variance preferences, as assumed in most of the literature, for short horizons only. Third, our long-horizon emphasis affects the choice of both the type of problem and

[2] In-sample, ex ante analyses assume that investors know precisely the necessary input parameters for portfolio optimization, which is of course not realistic, as parameter uncertainty may play a major role (see Brown, 1976; Klein and Bawa, 1976; Zellner and Chetty, 1965). In contrast, out-of-sample, ex post analyses test the performance of optimization methods under realistic conditions in which only information (including parameter estimates, when needed) at time *t* is used to solve the portfolio problem at time *t*, and the subsequently realized performance is recorded and investigated.

the asset menu. While many papers study stock selection, we focus on strategic asset allocation, because asset allocation has been shown to be the main determinant of portfolio performance (see, e.g., Cumming, Haß, and Schweizer, 2012; Ibbotson and Kaplan, 2000). As a consequence, we include real estate in the asset menu alongside traditional assets, given the presence of this asset class in long-term investors' portfolios. Finally, we model T-bills as a risky asset, because long-term investors clearly face re-investment risk (see, e.g., Campbell and Viceira, 2001).

Our baseline experiment uses U.S. monthly returns on stocks, publicly traded equity real estate vehicles (REITs), long-term government bonds, and T-bills, for the sample period 1972–2007. However, in Section 5 we analyze both shorter and longer sample periods, to include data from the recent financial crisis and to test the robustness of our results. Our data consist of well-known indices that are investable by private investors at low cost via exchange-traded funds. Our key finding is that an investor with a long horizon obtains higher ex post realized utility from optimizing strategies when that investor takes into account the predictability of real returns. Such predictability is captured by simple vector autoregressive (VAR) models that link asset returns to past values of variables (such as dividend yield and the term spread) that are commonly used to capture the state of the economy. The superior performance holds when compared to both naïve strategies and portfolios of intermediate complexity, which derive from an optimization but ignore predictability. It is thus prediction of returns that leads to substantial improvement in the investor's ex post welfare. These conclusions, which hold uniformly for intermediate-to-high investor risk aversion, are robust to changes in the asset menu (with and without REITs), to whether parameter uncertainty is taken into account during the optimization (i.e., whether we compute Classical or Bayesian portfolios), and to the presence of transaction costs: a long-horizon investor always prefers an optimized portfolio to a naïve, equally weighted strategy when such investor uses the best predictive model. These results emphasize the potential irrationality of the observed behavior of individual investors, who nevertheless allocate their wealth by equally weighting assets (see, e.g., Benartzi and Thaler, 2001; Brown, Liang and Weisbenner, 2007).

For short-horizon problems, our results align with the earlier mean-variance literature, despite the different experimental design: equal weighting provides 1-month-horizon investors with higher ex post realized utility than optimal portfolios under constant risk premia. Given the findings of De Miguel et al. (2009a) that naïve diversification leads to lower performance than a mean variance strategy if the sample size exceeds a critical value (which increases in the number of assets), our key finding obtains because the number of assets is relatively small but the sample size is large.

This paper contributes to the literature by providing a systematic decomposition of the economic and statistical drivers behind our results. The key driver is indeed the predictability of returns using VAR models at horizons in excess of 24 months. Although such predictability is weak at shorter horizons (consistent with a growing literature, e.g., Welch and Goyal, 2008), especially in the case of stocks and to some extent REITs, at a 5-year horizon it becomes sufficiently strong to outperform models that forecast returns using recursive means. Interestingly, in relative terms, such predictive power is always rather weak, yet it may give a long-run portfolio optimizer an economically valuable hedge. Given the complex, non-linear relationship between risk premium forecasts and portfolio weights when the investor has power utility, such results are far from obvious. Although adopting an asset menu that includes real estate is realistic, we show that this choice is not critical to our results. In fact, in some experiments, the economic value an investor may derive from exploiting predictability is actually larger for a traditional asset menu that includes only stocks, bonds, and T-bills. Importantly, our results are not significantly affected when we impute ex post transaction costs related to realized portfolio turnover. This evidence indicates that our key findings do not derive from predictability-based strategies that lead investors to trade at an absurdly intense frequency. The degree of risk aversion plays a limited role in the sense that under low risk aversion, for which short sale constraints are often binding, results are more difficult to interpret and long-horizon outperformance of optimizing strategies loses some significance. Finally, our baseline result does not depend on any "wise" choice of the sample period investigated. Because results could be sample-specific, we extend our data to include the 2008–2009 financial crisis. Our results remain consistent although optimal portfolios under predictability perform better than equal weighting only for very long horizons.

We also investigate whether our findings are due to favorable changes induced by a predictability-based strategy in the properties of higher-order moments of the realized wealth distribution. It is well known that, especially in the presence of alternative asset classes, relying on a simple mean–variance framework and ignoring higher moments (such as skewness and kurtosis) may not adequately capture an asset's risk-return profile; see, for example, the discussion in Cumming et al. (2012). Yet, we report that both Sharpe ratios (SRs) and higher moments improve, relative to the equally weighted strategy, as the investment horizon lengthens.

One experiment in particular further contributes to our understanding of why, over long horizons, optimizing models hold up against naïve benchmarks. This concerns specification of the forecasting model for real asset returns—a typical VAR à la Campbell, Chen, and Viceira (2003). In our baseline model, lagged real returns and four predictors—inflation rate, dividend–price ratio, riskless term premium, and default spread—forecast real asset returns. But allowing for several predictors increases the potential for estimation error, together with the number of parameters to be estimated. Thus, we also estimate parsimonious models with one predictor at a time. Interestingly, the richest model usually delivers higher investor welfare than do more parsimonious models, at least for long planned investment horizons. However, a long-horizon investor prefers an optimized portfolio to the equally weighted benchmark, even if such investor uses the worst predictive model.

Our paper does not point out simply that the 1/N benchmark may be beaten. Our most significant contribution consists in showing that, under predictability, longer investment horizons play a key role in beating 1/N. Kirby and Ostdiek (2012) find that market timing skills allow mean–variance efficient portfolios to often perform better than naïve diversification. These authors also emphasize return predictability, but stress its ability to increase the performance of optimal portfolio strategies in the short term only. Other recent work concerning the realized performance of 1/N instead addresses Bayes-type refinements, overlooking both predictability and the role played by the horizon. For instance, DeMiguel et al. (2009b) propose a constrained norm approach, interpreted as the result of using a suitable prior on mean–variance weights, and show that it may outperform 1/N. Tu and Zhou (2010) show that allowing Bayesian informative priors that reflect economic objectives (as opposed to standard statistical methods of prior elicitation) may lead to enough improvement in realized performance to allow mean–variance portfolios to outperform 1/N. Their intuition is that supposedly innocuous diffuse priors on some model parameters may imply strong prior convictions about economic dimensions of a problem, so that following these convictions may lead to optimizing weights that are ex post dominated by equal weight strategies. We compute optimal Bayesian portfolios under predictable returns, but we refrain from using informative priors and focus instead on the effects of investment horizon. Finally, Tu and Zhou (2011) propose rules based on optimal combinations of the 1/N rule and other well-known mean–variance style portfolios; they find that these new rules outperform the 1/N strategy.[3] Although extending our tests to combinations would be interesting, we opt for a simpler research design to highlight the role of the horizon.

Our paper contributes to the literature on optimal long-run portfolios. It is well known that time horizon affects portfolio weights when a changing opportunity set and/or parameter uncertainty are accounted for (Barberis, 2000). Indeed, annualized conditional means of asset returns are

[3] The recent literature also discusses the relative merits of equal- vs. value-weighted strategies; see, for instance, Jun and Malkiel (2008), Ranaldo and Haberle (2008), and Tabner (2012). We explicitly deal with equally weighted strategies only.

increasing or decreasing with the horizon (T) depending on the return generating process, while they are constant in the absence of predictability. Similarly, the conditional variances and covariances of portfolio returns depend on T when the intertemporal correlation of asset returns is non-zero. Forecasts of these conditional moments of returns are also subject to increasing uncertainty as the investment horizon grows. Accordingly, a power utility investor changes her portfolio as her horizon increases when she becomes aware of the growing uncertainty of her forecasts. Most of the literature studies the ex-ante, in-sample performance of optimal portfolios for the long-run. We instead provide an out-of-sample analysis of realized performance of optimal portfolios that consider return predictability, when the investor alternatively overlooks and accounts for parameter uncertainty using Bayesian methods (see Avramov and Zhou, 2011, for a review). Fugazza, Guidolin, and Nicodano (2009) also focus on out-of-sample portfolio performance, but compare gains from static diversification across assets to those from prediction-based, intertemporal-diversification. Diris, Palm, and Schotman (2011) stress that investors should use a shrinkage rather than a uniform prior to time the market, given a 5-year horizon. In our paper instead, even the buy-and-hold strategies described in Barberis (2000) may lead to ex post realized gains with respect to the equal weight benchmark, so long as the portfolio manager exploits predictability over a sufficiently long investment horizon.

The rest of the paper is organized as follows. Section 2 presents the research design. Section 3 describes the data set and presents estimates of a full-scale VAR(1) that includes all predictors and the implied forecasts of risk premia, volatilities and covariances. Section 4 reports realized performances of competing portfolio strategies. Section 5 extends the sample. Section 6 concludes.

2. Research Design

This Section presents the building blocks of our research design. Sections 2.1 and 2.2 introduce the portfolio strategies under examination, i.e., optimizing and equally weighted. Section 2.3 reviews methods for comparing ex post portfolio performance.

2.1 The Buy-and-Hold Portfolio Problem

Let N be the number of asset classes available to the investor. For each strategy, we consider the asset menu typically analyzed in the empirical finance literature (e.g., Campbell et al., 2003), that is, stocks, long-term government bonds, and T-bills ($N = 3$), as well as a more realistic asset menu that includes publicly traded real estate, in the form of equity REITs ($N = 4$). We discuss below the effects and importance of expanding the asset menu. Focusing on the latter case as an example, the investor's terminal real wealth, assuming a unit initial wealth, $W_t = 1$, is given by

where $\omega^j_{t,T}$ is the proportion of wealth invested in the *j*th asset when the horizon is T ≥ 1, and $R^j_{t,T}$ denotes the continuously compounded *real* cumulative returns on asset *j* = 1, …, *N* between *t* and *T*:

$$R^j_{t,T} \equiv \sum_{k=1}^{T} r^j_{t+k}, \quad j = s, b, r \;\; R^f_{t,T} \equiv \sum_{k=1}^{T} r^f_{t+k}. \tag{1}$$

Here, r^s_t, r^b_t, r^r_t denote real, continuously compounded returns on stocks, bonds, and publicly traded real estate, respectively, and r^f_t is real return on T-bills. By construction, $\sum_{j=1}^{N} \omega^j_t = 1$, which explains the residual structure of the weighting assigned to T-bills in (1). The focus on real portfolio performance is, of course, relevant under a long-horizon perspective, as in Hoevenaars, Molenaar, Schotman, and Steenkamp (2009). In fact, we allow for investment horizons ranging from $T = 1$ to $T = 60$ months. Given the observed investor inertia (see, e.g., Choi et al., 2004), we focus on buy-and-hold strategies, under which the investor determines asset allocation at the beginning of the investment horizon and never rebalances afterwards. Expected utility is therefore

$$\max_{\omega_{t,T}} E_t \left[\frac{W^{1-\gamma}_{t+T}}{1-\gamma} \right] \;\; \gamma > 1, \tag{2}$$

where $\gamma = 2,\ 5,\ 10$ are alternative coefficients of risk aversion that span the range of parameters commonly employed in the literature (see, e.g., Barberis, 2000). The maximization in (2) is solved subject to (1) and any other relevant constraints, such as no short sales ($\omega^j_{t,T} \in [0,1]$ for *j* = *s*, *b*, *r*).

We compute optimal asset allocation by maximizing the expectation taken with respect to a joint predictive density of future, *T*-horizon asset returns for a given asset menu. Such joint predictive density of future asset returns may be computed under a number of alternative models. In particular, we assume that future return forecasts may depend both on current returns and on other predictor variables in a linear way. In particular, a simple Gaussian VAR(1) model allows for time-varying risk premia, as in Barberis (2000) or Campbell, Chan and Viceira (2003):

$$\mathbf{z}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\Phi}\mathbf{z}_t + \boldsymbol{\varepsilon}_{t+1}, \tag{3}$$

where $\mathbf{z}_t \equiv [r^s_t \;\; r^b_t \;\; r^r_t \;\; r^f_t \;\; \mathbf{x}'_t]'$, $\boldsymbol{\varepsilon}_t \sim NIID(\mathbf{0}, \boldsymbol{\Sigma})$ (i.e., shocks are identically, independently, and normally distributed), and $\mathbf{x}_t$ represents a vector of *M* predictors. Because a *T*-month horizon investor rolling over 1-month T-bills $T - 1$ times faces both short-term interest rate and inflation risks, we treat r_t^f as risky. In (3), $\boldsymbol{\mu}$ is the vector of intercepts and $\mathbf{\Phi}$ is the matrix of autoregressive coefficients that describe the effects of lagged values of either returns or predictors on the subsequent value of returns. The model in (3) implies an assumption of constant variances and covariances of the shocks to the system, as captured by the covariance matrix $\mathbf{\Sigma}$. The predictors are: consumer price-based inflation rate (CPI), aggregate stock dividend yield, term spread between long- and short-term yields, and default spread between investment- and speculative-grade corporate bonds. Moreover, we estimate and apply, for asset allocation purposes, five different predictability models; the first uses all predictors at the same time ($M = 4$), and the other four use one predictor at a time (i.e., $M = 1$).

An additional no-predictability benchmark obtains when $\mathbf{\Phi}$ is constrained to be a matrix of zeros ($\mathbf{\Phi} = \mathbf{O}$) such that (VAR) simplifies to a Gaussian IID model,

$$\mathbf{z}_{t+1} = \boldsymbol{\mu} + \boldsymbol{\varepsilon}_{t+1} \quad NIID(\mathbf{0}, \mathbf{\Sigma}), \tag{4}$$

in which risk premia are constant over time. The portfolio implied by a Gaussian IID benchmark is (approximately) insensitive to investment horizon and, if portfolio returns were lognormally distributed, would coincide with the sample-based mean variance portfolio analyzed by a number of earlier papers on the performance of equally weighted portfolios.

Because equations (1)–(2) are solved by maximizing expected utility under the joint predictive density of future *T*-horizon returns, it remains to be specified how one goes from any particular parametric estimate of the models in (3) and (4) to such density. We obtain the joint predictive densities using both Classical and Bayesian approaches to estimate the relationship among asset returns and predictors. Under the Classical method, we estimate the parameters that characterize the VAR(1) model. Then, we apply a plug-in approach, i.e., we compute the conditional predictive moments and joint density of future returns by replacing the unknown parameter values with their least-squares estimates. Clearly, under such an approach, the vector $\mathbf{z}_{t+\tau}$ has a normal distribution, with mean vector and covariance matrix that depend on both the estimated least-squares parameters $\hat{\boldsymbol{\mu}}$, $\hat{\mathbf{\Phi}}$, and $\hat{\mathbf{\Sigma}}$, as well as on horizon $\tau = 1, \ldots T$. This approach ignores the fact that the parameter estimators are themselves random variables, thus leaving out an important source of uncertainty (known as *estimation risk* or *parameter uncertainty*).

In the Bayesian case, we follow Barberis (2000) and specify uninformative prior beliefs as to the parameters characterizing the linear relationships among asset returns and predictors. A posterior distribution of such parameters is obtained by application of Bayes' rule. The resulting joint posterior distribution is then used to generate a conditional, predictive density of returns and, therefore, a predictive distribution of future utility levels, from which the expectation in (2) can be computed as a functional of portfolio weights, $\boldsymbol{\omega}_{t,T}$. The Appendix provides further details on the solution to this problem.

2.2 Optimal Asset Allocation Strategies

The optimizing strategies that we compare to the naïve, equally weighted strategy originate from combinations of five dimensions within the general research design of Section 2.1. These are:

1) dynamic statistical relationship linking real asset returns to lagged asset returns and lagged values of the selected predictors, i.e., VAR predictability vs. absence of predictability;
2) treatment of parameter uncertainty, i.e., Classical vs. Bayesian methods;
3) planned investment horizon (T);
4) coefficient of relative risk aversion (γ);
5) asset menu, i.e., including or excluding publicly traded REITs .

In particular, we entertain six alternative econometric models:

1) VAR(1), in which all predictors forecast subsequent real asset returns (strategies called *VAR-ALL* below);
2) VAR(1) in which only lagged inflation forecasts asset returns (*CPI*);
3) VAR(1) in which only the lagged dividend yield forecasts asset returns (*DY*);
4) VAR(1) in which only the lagged riskless term spread forecasts asset returns (*TERM*);
5) VAR(1) in which only the lagged default spread forecasts asset returns (*DEF*);
6) a Gaussian IID model in which there is no risk premia predictability (note that when parameter uncertainty is accounted for, optimal weights are sensitive to investment horizon, even in the absence of predictability, see Barberis, 2000).

Because we consider 2 asset menus (i.e., with or without REITs), 2 alternative ways to compute weights (Classical and Bayesian), 6 investment horizons (1, 3, 6, 12, 24, and 60 months), and 3 risk-aversion coefficients (2, 5, and 10), this 6×2×2×6×3 combination yields a total of 432 alternative optimizing strategies, which we describe in Sections 4 and 5. Importantly, no short sale constraints are imposed in our model, similar to Cumming et al. (2012) and Jondeau and Rockinger (2006).

2.3 The Equally Weighted Strategy

The equally-weighted portfolio rule gives a $1/N$ share to each of the N available assets. Investors enjoy the benefits of naïve diversification by following this strategy, spreading risk over a set of assets with different risk–return trade-offs. As discussed in the literature (see, e.g., DeMiguel et al., 2009a), such cross-sectional diversification delivers large benefits when N goes from 1 or 2 to intermediate values and when added asset classes differ widely in terms of their risk–return trade-off; as N increases to infinity, these benefits decline rapidly. However, *in-sample*, any optimizing portfolio strategy outperforms the $1/N$ strategy by construction: the equally-weighted portfolio can be seen as imposing constraints on the problem (2), and this can only reduce the optimal value of the problem. However, portfolio strategies that are ex ante optimal may disappoint ex post and produce realized portfolio outcomes that are inferior to those of simple benchmarks. This is possible because characterizing an optimal portfolio requires an econometric framework that captures the dynamics of returns as well as their connections to predictors. Further, any econometric model—even if sensible (not obviously mis-specified) ex ante—may turn out to be either mis-specified ex post or plagued by parameter estimation errors (see Jacobs et al., 2009).[4]

We know from a voluminous body of literature that the equal-weighted strategy shows a higher SR in several out-of-sample experiments involving stock portfolios and a one-period horizon (see the references discussed in the Introduction). This obtains because the gains deriving from optimal diversification are often smaller than the loss due to the use, in (often, mean–variance) portfolio optimization, of inputs estimated with large errors. However, it is nevertheless important to examine whether the surprising performance of naïve diversification carries over to types of asset allocation programs and to longer investment horizons relevant to both institutional and individual investors. This extension offers an opportunity to advance our understanding of the properties of equal-weighted portfolios and of how and why they may continue to return realized performance superior to optimizing strategies.

2.4 Measuring Ex Post Performance

To this end, we focus on two indicators of performance, realized SR and certainty-equivalent return (CER). The SR of a strategy has a standard definition

$$SR_i(\gamma,T) \equiv \frac{\frac{1}{K-T}\sum_{\tau=1}^{K-T}[R_{i\tau}(\gamma,T)-r_\tau^f]}{\sqrt{\frac{1}{K-T}\sum_{\tau=1}^{K-T}\{[R_{i\tau}(\gamma,T)-r_\tau^f]-\frac{1}{K-T}\sum_{\tau=1}^{K-T}[R_{i\tau}(\gamma,T)-r_\tau^f]\}^2}}, \tag{5}$$

[4] Misspecification occurs when the portfolio manager uses a functional form that differs from the one that generates the data of interest, providing an inaccurate approximation of the true data-generating process. Parameter uncertainty arises when some parameters of a (correctly specified) model cannot be estimated with accuracy because of some model features (e.g., infrequent breaks that are hard to detect) or insufficient information in the data.

where $R_{i\tau}(\gamma, T)$ is the realized real return over the interval $[\tau-1, \tau]$ from model/strategy *i* when portfolio weights are computed for horizon T and risk aversion γ, and K is the number of realized out-of-sample returns on optimal portfolios. Although quite popular, the SR is an appropriate criterion only for a mean–variance investor, which is not the preference specification employed in this paper. In particular, an increase in $SR_i(\gamma, T)$ is not necessarily associated with higher realized welfare under a power utility function, if it is achieved at the cost of worse higher-order moment properties of portfolio returns. Indeed, investors are averse to both negative skewness and excess kurtosis (see, e.g., Jondeau and Rockinger, 2006), and these preferences are fully captured only by utility functions more general than a simple mean–variance objective.

Comparing investor realized utility across different strategies helps to explain the effect of the presence of non-normal returns, but interpretation across different horizons and risk aversion are problematic. Such comparisons are instead possible by computing and ranking strategies on the basis of annualized CER. This is the solution of the implicit equation $u(W_t(1+(T/12)\times CER(\hat{\boldsymbol{\omega}}_{i,t}(\gamma,T))) = E[u(W_{t+T}(\hat{\boldsymbol{\omega}}_{i,t}(\gamma,T)))]$, where $W_{t+T}(\hat{\boldsymbol{\omega}}_{i,t}(\gamma,T))$ is optimal terminal wealth associated with a given optimal strategy and $u(\cdot)$ is the utility function of the investor. Under the power utility in equation (2), out-of-sample CER is defined as:

$$CER_i(\gamma,T) \equiv \frac{12}{T}\left\{\frac{1}{W_t}\left[\frac{1}{K-T}\sum_{\tau=1}^{K-T}[W_{\tau+T}(\hat{\boldsymbol{\omega}}_{i,\tau}(\gamma,T))]^{1-\gamma}\right]^{\frac{1}{1-\gamma}}-1\right\}. \qquad (6)$$

To compute the series of realized portfolio returns and realized utility in equations (5) and (6), we use a recursive scheme of model estimation and portfolio optimization. We initialize our experiment using data from January 1972 through December 1994 (that is, 276 monthly observations) to estimate the parameters of our 6 alternative econometric models and to produce forecasts of *T*-month ahead means, variances, and covariances of returns on all asset classes. Additionally, we compute predictions of the *T*-month ahead joint density of real returns and use this density to determine optimal portfolio weights in the Classical and Bayesian frameworks. After recording predicted moments, densities, and the corresponding optimal portfolio weights under alternative specifications of γ and horizons, we proceed to expand the recursive estimation window by adding one additional month, which transforms the original sample into a 1972:01–1995:01 period. We use an expanding window, retaining data for the earlier period when adding new data. This guarantees acceptable saturation ratios in the estimation, such as 20 observations per parameter. At this point, predictions and ex *ante* optimal portfolio weights are re-computed and saved. Iterating this recursive scheme until the end of our sample yields a sequence of K = 156 optimal portfolio shares—one for each of the 432 optimal strategies listed in Section 2.2—as well as realized portfolio returns, from which we calculate the ex post performance measures. Longer series of realized portfolio returns are obtained in Section 5.2, where a longer sample, including the 2008 financial crisis, is analyzed, and the effects of the crisis are separately discussed, similar to Cumming et al. (2012).

3. Data and Empirical Portfolio Results

After presenting summary statistics in Section 3.1, Section 3.2 reports estimates of the VAR(1) used in Section 3.3 to produce forecasts of asset returns. Commenting on parameter estimates helps our understanding of portfolio composition in Section 3.4 and its sensitivity to predictability.

3.1 Data and Summary Statistics

Our initial sample spans the period January 1972–December 2007, for a total of 432 monthly observations. The initial date is determined by the availability of prices and realized total returns on publicly traded real estate. In Section 5, we expand the data up to December 2009 to include the recent financial crisis. Stock returns are computed as continuously compounded returns on the value-weighted CRSP index covering all listings on the NYSE, NASDAQ, and AMEX. The 10-year constant-maturity portfolio returns on US government bonds, as well as the 1-month T-bill returns, are from CRSP and the Federal Reserve Bank of St. Louis database (FREDII). The NAREIT website provides monthly returns on US equity REITs. Real returns are calculated by deducting from total returns the realized monthly rate of change in the Consumer Price Index for urban consumers provided by FREDII .

We follow the literature and use the dividend yield computed on the CRSP index along with the term and default spreads as predictors of asset returns (see, e.g., Campbell et al., 2003). As is customary, we compute dividend yield as the ratio of the moving average of the 12 most recent monthly cash dividends paid out by companies in the CRSP universe, and the $t-12$ value-weighted CRSP price index. It is commonly thought (see Fama and French, 1989) that both term and default spreads are leading of business cycle indicators. The term spread is the difference between the yield on a portfolio of long-term US government bonds (10-year maturity) and the yield on 1-month T-Bills. The default spread is measured as the yield difference between Baa corporate bonds and the 10-year constant-maturity Treasury bond. Since much of the literature allows for a relationship between real estate returns and inflation (see, e.g., Karolyi and Sanders, 1998; Ling, Naranjo and Ryngaert, 2000; MacKinnon and Al-Zaman, 2009), we add to our set of predictor variables the inflation rate, measured as the continuously compounded rate of change of the CPI index, and the real short-term rate (difference between 1-month T-bill returns and inflation rate).

Descriptive statistics for monthly returns and predictor variables are reported in Table 1. Mean real stock (bond) returns are close to 0.35% (0.12%) per month, implying annualized returns of 4.2% (1.4%). Estimates of volatility imply annualized values of approximately 15.6% for real stock returns and 7.6% for real bond returns, yielding unconditional SR of 0.04 and -0.02, respectively. The latter value reflects the long period of rising short-term real rates in the 1970s and early 1980s, which caused negative realized bond returns. Equity REITs have an annualized real mean return of 5.9% and annualized volatility of 14.1%. The resulting unconditional SR for equity REITs is relatively high, 0.08, or twice that of stocks.

3.2 Estimation Results

The upper panel of Table 2 displays maximum likelihood estimates of conditional mean coefficients along with robust t-statistics with reference to the VAR(1), which includes all predictors. These estimates refer to the full sample: 1972:01–2007:12.[5] The table shows that future stock returns are positively predicted by dividend yield, as shown in Fama and French (1989). Future stock returns are also negatively predicted by the term spread, the real short rate, and the inflation rate. These are the only statistically significant links between real stock returns and lagged predictor values. Fewer predictors help to forecast subsequent real bond returns, namely, inflation and especially default spread. Further, lagged real stock returns forecast subsequent real bond returns, but the corresponding coefficient is small. The positive relation between equity REIT returns and the dividend yield is similar to the relation uncovered for stocks and may capture a link between commercial real estate and the business cycle. Both lagged real returns on long-term bonds and the real short-term rate predict, with opposite signs, equity REIT returns. A negative association between REITs and lagged inflation has also long been observed in the real estate finance literature (e.g., Liu, Hartzell and Hoesli, 1997), suggesting that publicly traded real estate is not a good inflation hedge in the short-run. Finally, the equation for the real short-term rate illustrates the typical autoregressive dynamics followed by the short rate, with a high AR(1) coefficient estimated at 1.02 (however, the VAR(1) is globally stationary). Further, lagged term spreads, default spreads, and inflation exercise a rather large and statistically significant effect on subsequent real T-bill

[5] Although results are similar across the 144 estimations performed by expanding the sample by one month at a time, the R-square and the statistical significance of the predictors are slightly decreasing over time. This is in line with several studies that document a reduction in the predictability of stock returns after the 1990s.

returns; all forecast higher future real short-term rates. Overall, the full VAR(1) model in Table 2 explains a relatively large share of total equity REIT variance ($R^2 \simeq 8\%$), higher than the proportion of variance explained in the case of stocks (approximately 6%), long-term bonds (7%), but also smaller than that of T-bills (34%), although the latter high R^2 is strongly affected by the autoregressive nature of the real short rate.

The lower panel of Table 2 shows the correlation matrix of the shocks in (3), the portion of real asset returns *not* explained by the predictors. A few of these correlation coefficients are high in absolute value and may hence have large effects on long-run optimal portfolio weights, as explained in Barberis (2000) and Campbell et al. (2003). In particular, shocks to stock returns tend to positively correlate with shocks to REIT returns. Moreover, shocks to both stock and REIT returns tend to be accompanied by shocks that are large and of opposite magnitude in the dividend yield—in this sense, equity REITs tend to share several dynamic properties with equities. Finally, real 1-month T-bill rates display negative and large correlations with shocks to the term spread and the inflation rate.

We also obtained Bayesian estimates, which are used in the portfolio calculations that follow. Given our choice of uninformative priors, the point estimates (means of posterior densities) of all the conditional mean coefficients are identical to those in Table 2 and therefore are not reported.

3.3 Term Structure of Risk and Mean Returns

Predictability of future real asset returns implies that conditional moments vary with investment horizon in non-linear ways (see Campbell and Viceira, 2005), while in the Gaussian IID case, multi-period expected real returns and covariances grow linearly with investment horizon. Most evidence on this issue in the literature refers to ex ante, in-sample analysis. In Table 3, we instead report predicted means, standard deviations, and SRs (upper panel), and correlations (lower panel), as implied by our recursive Classical VAR(1) estimates in Table 2. The table focuses on the $T=1$ and $T=24$ cases only, as representative of short and longer-term horizons comparable to those that have appeared in the literature (for $T \geq 24$, the plots of predicted moments as a function of T are basically flat). All predictions are computed as of the end of our sample period, to derive insight on the risk–return trade-off in the data. The predictions of mean returns are slightly higher for a 2-year than for a 1-month horizon for all asset classes: predicted mean real stock returns increase from 3.7% to 3.8% per year, while in the case of REITs they increase from 4.7% to 5.0% per year. Real bond returns range from 1.1% to 1.3%, and are thus dominated by T-bills, whose predicted mean real returns are 2.0% at $T=1$ month and 2.1% at 24 months. The predicted volatility (annual standard deviation) is approximately constant at 8.4% per year in the case of stocks, somewhat increasing (from 8.4% to 9.6% per year) in the case of publicly traded real estate, and strongly increasing in the case of long-term bond real returns, from 2.4% at a 1-month horizon to 3.6% at a 12-month horizon.

These results characterize longer-term real bond returns and equity REIT returns as mean-averting, in the sense that the predictability affecting these two asset classes makes them increasingly risky as the horizon grows. In particular, the mean aversion in real bond returns is driven by the combination of predictability and residual correlation coefficients involving bonds, the default spread, and the inflation rate: The residual correlation of 0.35 with the default spread and the residual correlation of -0.09 with inflation shocks, along with the sizeable VAR loadings of real bond returns on the lagged default spread and inflation, lead bond returns to drift away from their unconditional mean at a rate that increases with the horizon. On the contrary, stocks are moderately mean-reverting and this is due mainly to the fact that shocks to real stock returns have negative correlation with shocks to dividend yields as well as inflation, although lagged dividend yield (inflation) forecasts future higher (lower) real stock returns. Real REIT returns display an intermediate pattern and turn out—when all predictors are used—to be mildly mean-averting, so that their risk only moderately increases with the horizon.

On balance, bonds have negative predicted SRs for longer-horizon investments, ranging from -0.38 (in annualized terms) to -0.22. This increase is due mainly to the fact that—given their negative risk premium—increasing volatility over expanding investment horizons is good news to their return-to-risk trade-off. The SRs for both stocks and publicly traded real estate are positive and insensitive to investment horizon: an invariant 0.20 per year in the case of stocks, and 0.30-0.32 in the case of real estate. Optimal portfolio weights also depend on correlation patterns. In general real long-term bonds have deteriorating hedge properties as the horizon grows: the predictive correlation between bonds and stocks grows from 0.20 to 0.42, while the correlation between bonds and REITs increases from 0.14 to 0.41. This confirms the results in Campbell and Viceira (2005) on the worsening of the long-term properties of bond returns as the horizon grows. The exception to this is that the correlation between bonds and T-bills declines from 0.25 to 0.05. Correlations involving stocks and real estate and stocks and 1-month T-bills barely change as a function of the horizon. The same applies to the correlation between real T-bill returns and REITs. This leads us to conjure asset allocation results in which REITs and stocks progressively come to dominate long-term portfolios.

3.4 Comparing Portfolio Weights across Strategies

In Figure 1, we visualize means of optimal portfolio weights over the recursive sample period 1994:12–2007:12 as a function of the investment horizon, T . To save space, the figure plots only the means for three sets of strategies: the $1/N$ benchmark; the Gaussian IID model with no predictability; and the Classical optimal VAR-ALL. The composition charts to the left of Figure 1 concern the asset menu that includes REITs; the columns of charts to the right address the case of an asset menu composed of stocks, bonds, and cash only. It is clear that for all combinations of horizons, risk aversions, and asset menus, the departures of optimal portfolio weights from the equally weighted strategy are major.

Let us now focus on the asset menu with real estate and on intermediate risk aversion ($\gamma = 5$) to ease interpretation of the results. In this case, differences between strategies with and without predictability remain modest when the horizon is short. They grow larger for long-horizon strategies, when the interaction between predictability and correlations among shocks to returns and predictors have maximum potential to affect weightings. Predictability tends to favor stock investments at long horizons in comparison to the IID case (e.g., for $T = 60$ and $\gamma = 5$, the mean investment in stocks is 25% under the VAR model vs. 10% under no predictability) and government bond investments at shorter horizons (e.g., for $T = 1$ and $\gamma = 5$, the mean investment in bonds is 27% under the VAR model vs. 17% without predictability). At long horizons, the demand for bonds declines under predictability—as expected on the basis of their mean aversion, uncovered in Section 3.3. Finally, predictability exerts a modest impact on the optimal portfolio share for REITs.

Figure 2 reports means of recursive portfolio weights for the Bayesian case, again as a function of investment horizon (the structure of Figure 2 is the same as Figure 1). Comparing Classical and Bayesian average portfolios under VAR-ALL predictability, the differences are generally minor for stocks and bonds, although the demands for the riskier assets (equity REITs and stocks) fall in favor of a slightly higher demand for T-bills, as one would expect because of parameter uncertainty.

4. Ex Post Realized Performance

In this section, we report summary statistics concerning performance of all 478 (432 plus 36 versions of the equal-weighted strategy) portfolio strategies. Section 4.1 describes our key findings, focusing on the baseline case of intermediate risk aversion, real estate in the asset menu, and return forecasts based on all predictors. It also asks whether such findings may depend on nonnormality in the return distribution. The following sections aim to single out the drivers of the key results reported in Section 4.1 through a number of additional exercises and tests. In particular, Section 4.2 connects patterns of statistical out-of-sample predictability to realized portfolio performance. Section 4.3 examines the results obtained when a single predictor at the time is used to forecast real asset returns. Section 4.4 isolates patterns related to risk aversion. Section 4.5 considers the case of a traditional asset menu that excludes real estate and focuses on the role of REITs. Section 4.6 examines performance results when we account for transaction costs. Given their considerable breadth, complete tabulated results are not reported here but are available from the authors upon request for all tests covered in Sections 4.2–4.6.

4.1 Baseline Results

Table 4 provides details on the realized, ex post performance of competing portfolio strategies for a Classical investor. In each table, there are six panels—one for each horizon—and seven sets of columns, the first column is devoted to the performance of $1/N$, the second to the optimizing strategy under IID returns, columns three through six to each of the individual predictor VAR models, and column 7 to VAR-ALL, when all predictors are included. In each panel, performances are reported for the cases of $\gamma = 2$, 5, and 10. The left-hand (right-hand) side of each column contains information on the realized performance obtained from the asset menu with (without) real estate. Performance figures are boldfaced when they are the best in our out-of-sample recursive experiment, which means *the highest* realized mean portfolio return or SR (CER)—importantly, these are both *annualized*—and *the lowest* realized portfolio volatility. Below, we focus on the baseline case, i.e., we comment only on a subset of the results reported in Table 4 referring to $\gamma = 5$, real estate in the asset menu, and VAR-ALL. Thus, we refer to the left-hand side of columns 1, 2, and 7. Note that even though they imply identical weights, $1/N$ portfolios give rise to different realized performances when they are associated with different asset menus, horizons, and/or preferences.

Table 4 shows that for all horizons up to 1 year, CER is systematically higher under the equally weighted case than in the simple IID model with no predictability. For instance, for $T = 1$, the CER is 5.18% per year under $1/N$ vs. 3.34% under constant risk premia. This regularity, which holds for the SR metric as well, confirms the result in earlier papers that the equally weighted benchmark is hard to beat for naïve mean–variance portfolio optimizers, in spite of our different data set and our focus on asset allocation instead of equity diversification. Importantly, this result also applies up to a 6-month horizon parameterization. For instance, for $T = 1$ month and $\gamma = 5$, the CER yielded by $1/N$ is 5.18% vs. 3.95% under VAR-ALL. This is consistent with De Miguel et al. (2009a), who examine the performance of short horizon portfolios with weights computed using the methods in Campbell et al. (2003).

However, under longer horizons, the distance between equal weighting and optimizing strategies narrows and eventually reverses. For instance, at a 5-year horizon, the highest annualized CER is achieved by VAR-ALL (5.51%); these high CERs are associated with both high SRs (0.34) and high realized mean portfolio returns (9.01%). The equal-weighted benchmark remains the strategy that yields the lowest annualized volatility, but this is insufficient to maximize an investor's welfare (the resulting CER is 5.01%). Note that the difference of 0.50% per year appears to be far from negligible in economic terms, amounting to $[\exp(1.0551\times 5)/\exp(1.0501\times 5)]-1 = 2.53\%$ over 5 years. We later show that once we adjust for a small-sample bias in estimating the difference in the annualized realized CER between the two strategies, this difference becomes larger.

Table 5 instead concerns the Bayesian portfolio strategies. The general qualitative remarks are identical to those reported above. For instance, for $\gamma = 5$ and a 1-month horizon, the naïve $1/N$ strategy delivers the highest CERs (5.18%) and SRs (0.33), outperforming all competing models, including an approximate mean–variance IID one (CER and SR are 4.16% and 0.18, respectively). The ranking is reversed when we consider long horizons. Visually, Table 5 makes it evident that the location of the boldfaced cells moves from left to right as the investment horizon grows. For instance, taking again as a reference the case of $\gamma = 5$ and $T = 60$ months, while $1/N$ yields an annualized CER of 5.01% and a SR of 0.27, these statistics are 5.32% and 0.43 for the best-performing VAR, also in this case VAR-ALL. The difference of 0.31% per year cumulates to $[\exp(1.0551\times 5)/\exp(1.0501\times 5)]-1 = 1.56\%$ over 5 years, while even a spread of 0.16 per volatility point per year in SRs appears economically substantial. Also in this case, the estimations performed below indicate that such differences may be even larger when small-sample biases are taken into account.

Our investor has power utility as opposed to the mean variance preferences in De Miguel et al. (2009a). Better higher-order moments of optimal portfolios over long investment horizons may therefore drive our realized utility results. There is, in fact, evidence of non-normal portfolio returns in our sample, whose properties improve as T increases. For instance, unreported results show that the skewness of realized portfolio returns from VAR-ALL is -0.88 in the Classical framework (-1.18 in the Bayesian), with a 1-month investment horizon and predictability. The corresponding estimates for the *excess* kurtosis are 4.82 and 5.64. As we move to a longer horizon of 24 months, skewness and excess kurtosis respectively decline to 0.29 (0.09 for the Bayesian model) and 0.09 (1.68). These compare to values of -0.78 and 3.83 for $T=1$, and -0.15 and 1.82 for $T=24$ in realized returns from the naïve portfolio. The reader may therefore suspect that the higher CERs associated with optimizing models arise from non-normalities only; it might be that SRs are still higher for long-horizon, equal-weighted portfolios, yet a power utility investor prefers optimizing strategies because of the relative improvement in higher-order moments of realized wealth. However, inspecting realized out-of-sample SRs in Tables 4 and 5 reveals that this conjecture is incorrect. For instance, for a 1-month investor, the SR of a $1/N$ portfolio strategy (0.33) exceeds that of an optimizing Bayesian VAR-ALL (0.32) investor; at longer horizons (e.g., $T=60$), a Bayesian investor derives a SR of 0.27 from the equal-weighted strategy vs. 0.43 from VAR-ALL. The same pattern emerges for a Classical investor. Thus, it appears that changing performance criteria does not affect the conclusions reported above; all properties of the realized portfolio return distribution for the equally weighted strategy worsen as the investment horizon lengthens and are therefore outperformed by optimizing portfolios computed under a VAR model.

How economically meaningful can total improvements between 1.5% and 2.5% over a 5-year horizon really be? On the one hand, these appear far from negligible in light of estimates commonly reported in the literature for comparisons involving different portfolio strategies. For instance, De Miguel et al. (2009a), in their comparisons of CERs from $1/N$ vs. a no short-selling mean–variance benchmark, report (Table 4, p. 1934) an annualized percentage difference (across their six alternative data sets) of 1.18%, and a 1.08% difference in a three-factor (Fama and French) application similar to ours. This is comparable to the welfare gains reported above and fits well the CER differences of 1.28% and 1.02% per year reported in Tables 4 and 5 (for $\gamma=5$ and $T=1$), respectively. Moreover, in De Miguel et al. (2009a), the annualized SR improvement of $1/N$ over a constrained mean–variance strategy is on average (see Table 4, p. 1934) 0.15, comparable to the 0.16 found above. On the other hand, one wonders whether the simple comparisons of average CERs and SRs in Tables 4 and 5 are consistent with the complex statistical properties of realized returns and utilities in our sample.

To tackle precisely these issues, we compute bootstrapped confidence intervals for all key realized performance measures discussed above, making adjustments as in Politis and Romano (1994) to account for the autocorrelation and heteroskedasticity arising from overlapping realized portfolio returns. In fact, it is well known that such unknown forms of dependence that are made very likely both by the overlapping structure of portfolio performances and by the very nature of financial returns at the monthly frequency, are likely to cause small-sample bias in the very estimation of mean performances and to make invalid standard inferences concerning any tests on the differences of performance. By applying a block bootstrapped to realized performance, such biases may be avoided and inferences may be considered valid.

Specifically, B draws of size equal to the size of the out-of-sample period from given strategies i and j are obtained via resampling with replacement monthly realized returns, and by blockwise resampling with replacement overlapping realized returns when $T \geq 2$ months. We choose for both cases along with a block size equal to the number of overlaps in a series, i.e., $T-1$. If $\hat{F}$ denotes the empirical distribution function of the B bootstrap realizations of the difference between strategies and for any relevant, realized performance measure based on the B boostrap sample of realized returns, then a $(1-p)\%$ confidence interval will simply leave $p/2\%$ of the bootstrap below the lower limit of the confidence interval and $p/2\%$ of the bootstrap above the upper limit. Figures 3–6 report two types of results. On the one hand, they quantify and plot measures of differential annualized CERs and SRs identified with the sample mean of the bootstrapped differences. Because of the block nature of the bootstrap scheme, these may depart from the sample differences of out-of-sample averages implied by Tables 4 and 5.

The upper panel of Figure 3 highlights the performance differentials between VAR-ALL and equally weighted strategies that are statistically different from zero using a bootstrapped distribution for the difference and a confidence level of 95%. We focus on VAR-ALL among all the optimizing portfolios because Tables 4 and 5 stress that it tends to outperform (especially at the longest horizons) all the remaining predictability-based strategies. For reasons of space, we focus only on the case of $\gamma = 5$ and 10. In the bar charts, rectangles indicate that the null hypothesis of equal realized performance is rejected for a Classical (solid) or a Bayesian investor (dashed), or for both (dashed bold). In the upper panels, which focus on CER comparisons, boxing of the horizon scales frequently appears with reference to longer horizons (right columns). For instance, still focusing on the case in which REITs belong to the asset menu, for $\gamma = 5$, the increase in CER is statistically significant for both Classical and Bayesian VAR strategies at both 24 and 60 months, and at a 12-month horizon in the Bayesian case. More interestingly, the vertical bars report *annualized* differences that are often slightly larger than those commented on above. For instance, at a 5-year horizon, the annualized difference is estimated to be 1.29% in the Classical case and 1.96% in the Bayesian case. These amount to large economic differences, for instance a cumulative

$[\exp(1.0623 \times 5) / \exp(1.0427 \times 5)] - 1 = 10.3\%$ over 5 years, and they are entirely in line with estimates of economic differences in annualized CERs typical of the literature.

These results and differences are even stronger in Figure 4, which compares not $1/N$ with VAR-ALL, but the best performing VAR among all the models listed in Section 2. For all levels of γ, the differences are of the same magnitude as in Figure 3, but the differences are more precisely estimated to be positive and in favor of models that account for predictability. Thus, we conclude that an optimizing portfolio strategy exploiting predictability delivers significantly higher CERs—at least at longer horizons—than does a naïve, equal-weighted strategy, whether or not the investor accounts for estimation risk. Interestingly, no evidence of statistically significant outperformance of the predictability-driven strategies over the equal-weighted strategy appears when we consider SRs in the lower panels of Figures 3 and 4. In this respect, higher-order moments strengthen the horizon effects implicit in exploiting predictability in portfolio choice. Finally, the results are approximately homogeneous when the cases of $\gamma = 5$ and 10 are compared.

Because it may not be easy to immediately visualize differences in annualized CERs and SRs in Figures 3 and 4, Table 6 presents bootstrapped differences, including any small-sample corrections, and it boldfaces any differences that are significant (p-values of 0.05 or lower). To save space, we focus only on the case of $\gamma = 5$. Clearly, when small-sample biases due to dependence on realized utilities and returns are taken into account, a number of differences appear to be both substantial (for instance, CER differences are never below 0.96% per year beginning at $T = 24$) and statistically significant for intermediate and long horizons.

4.2 Decomposing Results: Single-Predictor Optimizing Strategies

Optimizing strategies based on the VAR-ALL model use four predictors in addition to lags of real asset returns on stocks, bonds, equity REITs, and T-bills. Typically, augmenting the number of predictors increases in-sample accuracy (e.g., the R-square) of a VAR model—and this is so for our VAR estimates as well. For instance, dividend yield helps to predict future returns on stocks, bonds, and the real short rate in the VAR-ALL specification, whereas it cannot predict bond or real T-bill returns in a simpler VAR-DY model. Similarly, the inflation rate helps to predict all future returns in the general specification, but only the future real short-term rate in the VAR-CPI model. The term spread and the default spread predict stock and bond returns as well as the short-term rate when considered together, but only the latter when considered in isolation. Therefore the VAR-ALL specification teases out many partial effects that each predictor may exercise only after conditioning on the remaining predictors. At the same time, these in-sample advantages do not necessarily translate into additional predictive accuracy. As is clear from the forecasting literature, increasing the number of predictors inflates the number of parameters to be estimated to such a degree that the associated estimation error may impair forecasting performance. Therefore, single-predictor optimizing strategies may even outperform VAR-ALL out-of-sample.

It turns out this is not the case in most of our experiments, as is evident from Tables 4 and 5, where columns 3 to 6 refer each to VAR models that include only one predictor, while VAR-ALL is in the right-most column. It is clear that the location of the boldfaced cells tends to concentrate either in columns 1–2 (in correspondence to the equal-weighted strategy and the IID model, respectively) or in column 7, with very few models containing only one predictor emerging as best performers.

To see how this affects the rankings of $1/N$ vs. alternative portfolio rules, Figure 4 reports the difference in CERs (upper panel) and SRs (bottom panel) between the best predictive model and the equal-weighted case using a format comparable to Figure 3. For shorter investment horizons, $T \leq 12$, the best performing model is either VAR-DY or VAR-CPI in both the CER and SR metrics and in whether the investor accounts for parameter uncertainty. Nevertheless, for $T \leq 12$ months, it is still the case that $1/N$ has better out-of-sample portfolio performance. In contrast, for investment horizons in excess of 1 year, the best-performing model is almost always VAR-ALL, even though VAR-CPI shows good performance properties. Thus, the ex post performance for horizons longer than 1 year is worse for the equally weighted strategy than for the optimizing portfolio based on the best forecasting model, while the opposite ranking holds for short investment horizons, which is consistent with the results discussed in Section 4.1.

4.3 Statistical Out-of-Sample Predictability and Portfolio Performance

Even though Sections 4.1–4.2 attempt to establish that predictability strategies significantly outperform, both economically and statistically, a naïve equal-weighted benchmark at long investment horizons, it is nevertheless important to understand the drivers of such differences. In fact, only if a plausible set of drivers explaining the differences can be found, can we legitimately conclude that long-run optimal buy-and-hold portfolios outperform naïve strategies. In our paper, the key candidate is obviously the increasing amount of predictability in the data as the horizon grows. This is shown in Table 7, where we compute ratios of recursive, realized root mean squared forecast errors (RMSFEs) obtained from the five types of VAR(1) models, five alternative forecast horizons, and two alternative sets of test assets over the out-of-sample period 1995–2007. These RMSFEs are obtained from the same, recursive scheme described in Section 2.4. The ratio is computed between the RMSFE of a given model, at a given horizon, when predictions come from a VAR(1) that includes the asset returns specified in the headers as well as the predictors specified in the flank of the table, and the RMSFE from a Gaussian IID model in which predicted returns simply correspond to the recursively estimated sample mean. This echoes the results in Welch and Goyal (2008), which, however, focus only on stocks over a short, 1-month horizon. For simplicity, these results are tabulated for the Classical case only, but are virtually identical for Bayesian estimation methods, given the similarity of the forecasts.

Table 7 shows that for both asset menus (i.e., including real estate or not) and all VAR models, it is very difficult to exploit any predictability in mean stock returns, apart from the longest, 5-year horizon. For instance, when predictions are computed from the VAR-ALL framework, all ratios (slightly) exceed 1—indicating that the realized out-of-sample RMSFE is no better than the Gaussian IID one—apart from the one for the case $T = 60$, when it is 0.978, indicating an RMSFE that is slightly more than 2% better under a VAR model. For some specific VAR models, appreciable stock return predictability can, however, be detected already at $T = 24$, but this is never the case for $T = 12$ months or less. There is instead much more predictability, often at all horizons considered, in the case of other asset classes, especially government bonds and T-bills, where the RMSFE ratios appear to be systematically below 1, even though the gains from using VARs are often below a modest 1%. As one would expect, equity REITs show properties that are intermediate between bonds and stocks. However, even when the ratios are uniformly below 1, with reference to model- and horizon-specific averages across different asset menus and asset classes, the very last column of Table 7 shows the existence of non-negligible differences across 60- vs. 1-month ratios, which always reveal stronger predictability at longer horizons. The differential is approximately 1.5% for all types of VAR, except for the VAR-DY case, where it is just 1%.

Yet, the fact that a predictability score such as the ratio between a predictability model-based RMSFE and a sample mean, no-predictability RMSFE declines—so that forecasting power strengthens—as T grows, hardly proves that this relates to the improving realized portfolio performance discussed in Sections 4.1 and 4.2. For the six investment horizons analyzed in this paper, we therefore compute the correlations between the difference in annualized CER (SRs) between VAR-based portfolio strategies, the naïve $1/N$ benchmark in Tables 4 and 5, and the RMSFE ratios in Table 7 (for simplicity, these are averaged across different assets, but alternative weighting schemes have given similar results). If there were a precise association between improving predictability with horizon T and stronger realized performances, such correlations ought to be negative (this comes from the fact that declining ratios, below 1, indicate improving predictability). For all VAR models, such correlations are negative, rather large in absolute value, and statistically significant. For instance, in the case of the VAR-ALL strategy, the correlation between its annualized CER (SR) spread over the equal-weighted strategy and the RMSFE ratios in Table 7 is -0.63 (-0.55). The weakest among such correlations—a value of -0.58 with respect to CER differences and -0.63 with respect to SR deltas—is found for default spread-based VAR strategies, and these remain large in absolute value and statistically significant.[6]

4.4 Risk Aversion

Most findings for intermediate risk aversion carry over to a situation of high risk aversion ($\gamma = 10$). The SR of the equal-weighted portfolio exceeds by far that of the optimizing strategies for 1-month Classical portfolios (see Tables 4 and 5). This indicates that some of the early literature's focus on the tangency portfolio is not the source of $1/N$ outperformance, even if contrasting it to the tangency portfolio clearly magnifies the effects of estimation risk on optimizing portfolios (see Kirby and Ostdiek, 2012). However, under investment horizons of 2 or more years, optimizing strategies dominate naïve ones according to both the SR and the welfare metric, and welfare differences are statistically different from zero (see the bottom panel of Figure 3).

It is more difficult to find similar patterns for low risk aversion ($\gamma = 2$), as in this case where an investor tends to prefer rather extreme portfolio positions, so that the no short sale constraints are binding most of the time in our sample. This implies the inability to completely exploit predictability patterns or to reduce the weight of asset classes with large exposure to estimation risk. It is still the case that the longer the investment horizon, the better the performance of optimizing strategies in general and in particular of strategies that account for predictability. However, the long-horizon outperformance of VAR-ALL is less clear-cut and the (unreported) statistical significance weaker. On the other hand, Tables 4 and 5 show that even for 1-month horizons optimizing strategies such as VAR-ALL may deliver higher realized CER than the equal-weighted benchmark. In fact, the realized CER of $1/N$ ranks last among portfolio strategies for $T = 1$. Results uncovered by De Miguel et al. (2009a, among others) still hold, though only because the SR of the naïve strategy (0.33) remains higher than that of the optimizing portfolio without

[6] Detailed results are available upon request. Note that these correlations are computed with reference to spreads of annualized CERs and Sharpe ratios vs. $1/N$, not the Gaussian IID benchmark, which would instead be coherent with the structure of Table 6 but uninformative for our purposes.

predictability.[7] Higher-order moments contribute to such divergent CER-SR patterns for short-horizon investments and low risk aversion. Indeed, the naïve strategy has slightly worse portfolio return skewness (-0.78) and excess kurtosis (3.83) than do the Classical (-0.76 and 3.60) or Bayesian no-predictability strategies (-0.58 and 3.83). In conclusion, it seems that the better ex post performance of optimizing portfolios relative to the naïve portfolio strategy over longer horizons holds in our sample irrespective of the degree of risk aversion, with the low risk aversion case somewhat weaker.

4.5 The Role of Real Estate

We now test whether the *out-of-sample* results documented in Sections 4.1 and 4.2 are sensitive to exclusion of real estate in the asset menu. Equivalently, we ask whether and how our earlier findings depend on the fact that, thus far, our asset menu has included REITs, in addition to stocks, bonds, and bills. This question is worth investigating from a real-world perspective, as there are still pension plans that adhere to the traditional choices of cash, bonds, and stocks—even though several other pension plans have offered a capability to invest in real estate since the early 1990s (see, e.g., Benartzi and Thaler, 2001). Finally, excluding real estate from our research design allows us to better link the results in this paper with the existing literature on strategic asset allocation between stocks, bonds, and cash, as in Brennan et al. (1997).

Based on the analysis in De Miguel et al. (2009a), reducing the number of assets should worsen the performance of naïve diversification relative to that of optimal portfolios. However, these authors' experiment reveals that this effect may not obtain if the excluded asset has a high (above in-sample mean–variance efficient portfolio, before exclusion) SR, as equity REITs actually do over our sample period. A check of the appropriate columns in Tables 4–5 reveals that in our work, horizon effects are not affected by the presence of real estate.[8] In the absence of equity REITs, it is still the case that CER is systematically higher under $1/N$ (4.4% per year) for $T=1$ month than under optimizing portfolios with unpredictable returns (2.2%). The equal-weighted strategy also

[7] This occurs even if the optimizing weights are heavily tilted toward stocks and real estate. For instance, the optimal shares invested in stocks and equity REITs are equal to 18% and 82% in the Classical IID case and to 22% and 75% in the Bayesian IID case. Such highly risky portfolio compositions yield higher mean returns but lower Sharpe ratios (of 0.27 and 0.32, respectively) than $1/N$, because their returns are also twice as volatile.

[8] Adding real estate increases both realized CERs and Sharpe ratios, thanks to a larger increase in means than in volatilities. Portfolio volatilities fall with the investment horizon only with real estate in the asset menu. For instance, the volatility of the VAR-ALL portfolio for a Bayesian investor drops from 8.0% per year in the case of $T=1$ to 6.6% for $T=60$ when equity REITs are included, while it increases from 5.3% to 6.6% without REITs.

outperforms the optimizing strategies that capture predictability in terms of both SRs and CERs for a 1-month horizon. Such ranking reverses for a sufficiently long horizon. At a 5-year horizon, for instance, the CER of the optimal portfolio based on all predictors is equal to 4.2% annualized. These high levels result from high SRs (0.15 annualized) and high realized mean portfolio returns (5.7% annualized). $1/N$ remains the strategy that yields the lowest annualized volatility, but it does not maximize an investor's realized ex post utility, as the resulting CER is equal to 3.2%. Also. in this case, an annualized increase in CER of over 1% is far from negligible in economic terms. Figures 3 and 4 concerning bootstrapped differences in CERs for the case of a three-asset menu confirm these conclusions, with many CER differences being highly statistically significant and in favor of VAR strategies for $T = 24$ and 60. In conclusion, although we prefer to report and comment on empirical results that refer to the asset menu that (realistically) includes real estate assets, it is reassuring to find that our key empirical findings are in no way driven by real estate data.

4.6 Turnover Effects and Transaction Costs

Finally, we discuss changes in ex post performance when we account for transaction costs. Even if optimizing portfolios tend to outperform the equal-weighted benchmark, especially at long horizons, this difference in CERs may turn out to be insufficient to cover the differential transaction costs that the optimizing strategies, by linking weightings to state variables, impose on investors. Let us begin by computing turnover as average percentage of wealth traded in each period:

$$Turnover_i(\gamma, T) \equiv \frac{1}{K-T} \sum_{\tau=1}^{K-T} \sum_{i=1}^{N} | \omega_{\tau+1}^i - \omega_{\tau}^i |, \qquad (7)$$

where $(K-T)$ is the number of rebalancing periods over the 1995–2007 sample period (in this case, $K = 156$), N is the number of assets included in the portfolio, ω_{τ}^i is the portfolio weight for asset i before rebalancing, and $\omega_{\tau+1}^i$ is the desired portfolio weight restored by trading the amount $\omega_{\tau+1}^i - \omega_{\tau}^i$. For each strategy, we compute the turnover measure after adjusting for returns occurring between rebalancing points. To save space, we summarize our main findings focusing on an intermediate risk aversion level of 5 and on a relative measure of turnover—i.e., the turnover implied by optimizing investment strategies, divided by that generated by $1/N$. Averaging over Classical and Bayesian investors, the mean recursive monthly turnover ratio is 3.5, with a standard deviation of 2.5 and a wide range of 0.9–24.0. Thus, the lowest turnover recoverable from the full set of optimizing strategies turns out to be 10% below the turnover implied by the naïve portfolio; however, the highest turnover is 2,400% higher than the modest turnover that $1/N$ induces. The turnover ratio is decreasing with the investment horizon, from an average of about 5.3 (for a 1-month horizon) to 2.4 (for $T = 60$). The longer horizons imply less frequent rebalancing; moreover, the long-term optimizing positions tend to be less extreme than the shorter-term ones, resulting in lower rebalancing needs. For example, the optimal weighting assigned to equity REITs in a Bayesian VAR-ALL portfolio falls from 46% in the case of $T = 1$ to 35% when $T = 60$. Predictability does not have a uniform effect on the turnover ratio. For instance, turnover is much higher for IID models (5.1, with a range of 1.8–10.4) than for models with predictability (e.g., 3.1, with a range of 1.1–9.9 in the case of VAR-ALL), when real estate is part of the asset menu.

Consistent with the literature (see, e.g., Jacobs et al., 2009), we assume that each trade gives rise to a proportional cost of 50 basis points, and compute afresh all realized performance measures net of these variable transaction costs. In general, we observe that this structure for transaction costs mainly affects mean ex post portfolio returns, leaving their volatilities almost unchanged. Importantly, it is still the case that $1/N$ outperforms, at least in terms of realized SRs, all optimizing models at short horizons, for all risk aversions and irrespective of estimation method. At longer horizons, $1/N$ continues to produce performance superior to Gaussian IID models; in particular, Bayesian IID strategies have lower SRs than those implied by the equal-weighted strategy, even when they previously ranked higher than the equal-weighted benchmark in terms of gross SR with zero transaction costs. This ranking reversal is associated with high turnover for no-predictability models relative to $1/N$, which lowers their mean ex post return differential.

However, even in the presence of rather high round-trip transaction costs, at long investment horizons, optimizing strategies with predictable returns outperform the equal-weighted benchmark based on CERs when equity REITs are included in the asset menu. The positive transaction cost differential, which impacts negatively on the optimizing strategies, is unable to counter the worsening properties of the $1/N$ portfolio as the investment horizon widens. For instance, a moderately risk-averse investor ($\gamma = 5$) with a long horizon ($T = 60$) scores a CER of 5.2% per year (5.9%) when following the portfolio prescriptions from the VAR-ALL Classical (Bayesian) model, exceeding what this investor obtains by equally weighting the portfolio (4.2% per year, in both cases). However, when real estate is omitted from the asset menu, this result holds for medium to high risk aversion only (see Figures 5 and 6), and the higher turnover rates implied by portfolios based on VAR-ALL may enable the investor to beat $1/N$ only at the longest allowed horizon ($T = 60$). Overall, these results confirm our earlier findings that optimizing strategies that account for the presence of predictability may outperform naïve, equal-weighted strategies for medium-to-high risk-aversion levels.

5. Performance Stability across Samples

5.1 A Shorter Sample

It is well known that the extent of out-of-sample predictability of the equity premium in linear forecasting models is not stable across samples (Welch and Goyal, 2008). Because our long-run portfolio strategies are built to exploit the predictability of asset returns, we are therefore exposed to the dangers of unstable performance. As a check on the extent of this problem, we repeat the entire exercise with reference to a shorter sample period—January 1995 through December 2004—analyzed in Fugazza et al. (2009). Performance results (unreported) confirm the presence of instability. While the equally weighted strategy still outperforms the optimizing strategies for $T = 1$, this is now also the case for most longer horizons as well. Moreover, ex post realized CER falls as a function of investment horizon, whereas it increases, albeit non-monotonically, in the 1995–2007 sample analyzed in Section 4. This implies that optimizing buy-and-hold investors with longer horizons face higher performance volatility than naïve $1/N$ investors.

Interestingly, it is the absence of any major divergences in the (average) structure of the optimal weights that provides an easy explanation for the heterogeneous performances across the two samples. At horizon $T = 24$, for instance, the optimal average portfolio shares in the shorter subsample are equal to 33% (53%) for T-bills, 19% (12%) for stocks, 2% (2%) for bonds, and 54% (37%) for equity REITs, for a Classical (Bayesian) investor with $\gamma = 5$. This is not dissimilar from portfolio composition in the 1995–2007 sample, as seen in Section 3.4. As a result, it is, then, the different timing of portfolio weight changes as well as more modest differences in weights that sum up to yield realized performances that are sufficiently different, across the two samples. Moreover, we find some evidence that the 2005–2007 performance of the asset classes used in our exercise underwent a break that favors optimizing, long-run strategies over $1/N$ during the last (rather well characterized in terms of historical performance) portion of our sample.

5.2 A Longer Sample Period

The pattern described in the previous section opens the possibility that the 2008 crash of stock and real estate markets may also disrupt our baseline results. Thus, we follow the strategy of recent papers, such as Cumming et al. (2012), and extend our sample to include data up to December

2009, which amounts to an extension of our baseline data set by an additional 24 observations. The results are not tabulated here, to save space, but they are available upon request. If one focuses on the simple descriptive statistics, mean asset returns fall and volatilities increase; however, there is no change in the asset ranking according to the unconditional SR. Variation in portfolio shares as the horizon grows is similar to that uncovered for the baseline sample in Section 4.1; for intermediate risk aversion, the Classical portfolio share invested in bonds falls considerably in T, but is compensated by an increase in stocks and REITs. For instance, the portfolio composition at $T = 24$ is close to that found above for the other samples, with 14.3% in stocks, 7.1% in bonds, 57.1% in equity REITs, and 21.5% in T-bills. With respect to performance, we find confirmation of the patterns presented in Section 4. First, a highly risk-averse investor derives higher CER from optimizing strategies that exploit predictability than from the equal-weighted strategy, at all investment horizons. For instance, for $\gamma = 10$ and $T = 1$, we obtain that the CER of Bayesian VAR-ALL is 2.6% (3.1%) per year against 1.9% (0%) for $1/N$ when REITs (do not) belong to the asset menu; these figures are instead 4.4% (3.0%) for VAR-ALL and 3.6% (2.6%) for $1/N$ for a long investment horizon. Including REITs during an extreme real estate bust obviously reduces overall performance; however, this seems to affect the equally weighted performances more, with their constant 25% allocation to REITs, than it does in the case of optimizing models. This means that the latter class of strategies must have detected the financial crisis at some point and moved out of REITs, thus improving their realized performance. The SR statistics are 0.04 for VAR-ALL against 0.07 for $1/N$ when $T = 1$; these figures become 0.89 for VAR-ALL and 0.59 for $1/N$ for a long investment horizon. However, the positive CER difference in favor of optimizing models turns out to be statistically different from zero only for horizons equal to or longer than 12 (6) months when trading is (not) subject to transaction costs.

Results are qualitatively similar for the case of $\gamma = 5$, but this case requires a long horizon of five years for the best of the optimizing models (also in this case, VAR-ALL) to statistically outperform the equal-weighted benchmark in a CER dimension. For instance, when real estate is included in the asset menu, a Bayesian VAR-ALL yields a 1995–2009 realized out-of-sample yearly CER of 2.5% at $T = 1$ 2.7% at $T = 6$, and 4.8% at $T = 60$; these results contrast with annual CERs of 3.9%, 0.1%, and 0.6%, respectively, for the equal-weighted portfolio. Although these differences are as large and economically significant as those reported above for the $\gamma = 10$ case, it is not entirely surprising that the CER differentials may be more precisely estimated when $\gamma = 10$, as the strong 2008–2009 financial crises led to volatile and unstable performances that help differentiate VAR-ALL from a naïve strategy mostly in the case of highly risk-averse investors. Overall, this additional exercise emphasizes that the possibility of predictability-driven strategic asset allocation models outperforming $1/N$ does not critically depend on either including the recent real estate crisis or real estate itself as an asset class. However, some degree of instability appears in smaller samples.

6. Conclusions

Investors with horizons of 1 year and longer would on average have benefited, ex post, from an optimizing portfolio strategy over the period 1995–2007, provided such strategy had been designed to exploit return predictability. This result holds even when portfolio performances are evaluated against a difficult-to-beat, equally-weighted benchmark. While robust to several variations of our baseline research design, this result is stronger when real estate belongs to the asset menu and when the number of predictors for real returns is high, spanning all the most common predictors discussed in the literature. Our findings become clearer when performance is measured in welfare terms, indicating that higher-order moments of the return distribution strengthen horizon effects. Importantly, we also find that the equally weighted strategy usually outperforms at short horizons when the optimal portfolio does not account for return predictability. Thus, our findings are in line with results reported in the literature (see, e.g., De Miguel et al., 2009a, Jacobs et al., 2009; Tu and Zhou, 2011) for short investment horizons, in spite of some differences regarding asset allocation exercise, longer sample periods, and estimation windows. However, when the horizon exceeds 24 months, the evidence turns clearly in favor of predictability-driven strategies, which implies strong horizon effects. Bootstrapping methods, which correct for small-sample biases caused by dependence in overlapping realized performances at horizons exceeding one month and that allow valid inferences, show that for intermediate and high levels of risk aversion, differences in long-horizon CERs are often statistically significant and economically non-negligible.

These results support the usefulness of applied portfolio management techniques in spite of the fact that some features of our research design do not aim to maximize the performance of active portfolio strategies. For instance, portfolios are of a buy-and-hold type; it would be interesting to investigate how fully dynamic extensions that take into account hedging demands would fare in recursive, out-of-sample experiments (see, e.g., Diris, Palm, and Schotman (2011)). Additionally, predictability is captured in a simple manner, through vector autoregressive models that are robust but possibly sub-optimal tools with respect to density forecasting applications, such as those underlying our optimal portfolio calculations, because of their inability to capture spikes in the risk premium and predictability in higher-order moments (see Guidolin and Hyde, 2012; Guidolin and Timmermann, 2008; Jondeau and Rockinger, 2006; Tu, 2010). The asset menu includes only one alternative asset class (REITs), but recent results (Cumming et al. (2012)) cause us to believe that including further assets—with all the computational costs that this would entail—may simply make our results stronger. Finally, even though we also document results derived in a Bayesian portfolio framework, our priors have remain entirely uninformative. Recent work, Tu and Zhou (2010), shows that backing out priors from natural economic objectives (such as CER maximization) may lead to important performance gains, also under predictable returns. Because our Bayesian results in this respect may provide only a lower bound to the extent to which 1/N may be outperformed, it would be interesting to extend our work to long planning horizons under predictability of the use of "intelligent" priors.

References

Avramov, D., and G., Zhou. Bayesian Portfolio Analysis, *Annual Review of Financial Economics*, 2, 25-47.

Barberis, N., 2000, Investing for the Long Run When Returns Are Predictable, *Journal of Finance*, 55, 225-64.

Bawa, V., S., Brown S., and R., Klein, 1979. *Estimation Risk and Optimal Portfolio Choice*. Amsterdam: North Holland.

Benartzi, S., and R., Thaler, 2001, Naive Diversification Strategies in Defined Contribution Saving Plan, *American Economic Review*, 91, 79-98.

Brandt, M., 1999, Estimating Portfolio and Consumption Choice: A Conditional Euler Equations Approach, *Journal of Finance*, 54, 1609-1645.

Brennan, M., E., Schwartz, and R., Lagnado, 1997, Strategic Asset Allocation, *Journal of Economic Dynamics and Control*, 21, 1377-1403.

Brown, S. J., 1976, *Optimal Portfolio Choice under Uncertainty*. Ph.D. Dissertation, University of Chicago.

Brown, J., N., Liang, and S., Weisbenner, 2007, Individual Account Investment Options and Portfolio Choice: Behavioral Lessons from 401(k) Plans, *Journal of Public Economics,* 91, 1992-2013.

Campbell, J., Chan, and L., Viceira, 2003, A Multivariate Model of Strategic Asset Allocation, *Journal of Financial Economics*, 67, 41-80.

Campbell, J., and L., Viceira, 2001, Who Should Buy Long-Term Bonds?, *American Economic Review*, 91, 99-127.

Campbell, J., and L., Viceira, 2002. *Strategic Asset Allocation: Portfolio Choice for Long-Term Investors*. Oxford University Press, Oxford.

Campbell, J., and L., Viceira, 2005, The Term Structure of the Risk-Return Tradeoff, *Financial Analysts Journal,* 61, 34-44.

Cumming, D., L., H., Haß, and D., Schweizer, 2012, Strategic Asset Allocation and the Role of Alternative Investments, forthcoming in *European Financial Management*.

Choi J., D., Laibson, and B., Madrian, 2004, Plan Design and 401(K) Savings Outcomes, *National Tax Journal*, 57, 275-298.

DeMiguel, V., L. Garlappi, and R., Uppal, 2009a, Optimal Versus Naive Diversification: How Inefficient is the 1/N Portfolio Strategy?," *Review of Financial Studies*, 22, 1915-1953.

DeMiguel, V., L., Garlappi, F., Nogales, and R., Uppal, 2009b, A Generalized Approach to Portfolio

Optimization: Improving Performance by Constraining Norms, *Management Science,* 55, 798-812.

Detemple, J., R., Garcia, and M., Rindisbacher, 2003, A Monte Carlo Method for Optimal Portfolios, *Journal of Finance*, 58, 401-446.

Diris, B., Palma, F., and P., Schotman, 2011, Long-Term Strategic Asset Allocation: an Out-of-Sample Evaluation, working paper, Maastricht University.

Duchin, R., and H., Levy, 2009, Markowitz versus the Talmudic Portfolio Diversification Strategies, *Journal of Portfolio Management*, 35, 71-74.

Fama, E., and K., French, 1989, Business Conditions and Expected Returns on Stocks and Bonds, *Journal of Financial Economics*, 25, 23-49.

Friend, I., and M., Blume, 1970, Measurement of Portfolio Performance under Uncertainty, *American Economic Review,* 60, 571-575.

Fugazza, C., M., Guidolin, and G., Nicodano, 2009, Time and Risk Diversification in Real Estate Investments: Assessing the Ex Post Economic Value, *Real Estate Economics*, 37, 341-381.

Welch, I., and A., Goyal, 2008, A Comprehensive Look at The Empirical Performance of Equity Premium Prediction, *Review of Financial Studies*, 21, 1455-1508.

Guidolin, M., and S., Hyde, 2012, Can VAR Models Capture Regime Shifts in Asset Returns? A Long-Horizon Strategic Asset Allocation Perspective *Journal of Banking and Finance*, 36, 695-716.

Guidolin, M., and A., Timmermann, 2008, Size and Value Anomalies under Regime Shifts, *Journal of Financial Econometrics*, 6, 1-48.

Hoevenaars, R., R., Molenaar, P., Schotman, and T., Steenkamp, 2008, Strategic Asset Allocation with Liabilities: Beyond Stocks and Bonds, *Journal of Economic Dynamics and Control*, 32, 2939-2970.

Ibbotson, R., and P., Kaplan, 2000, Does Asset Allocation Policy Explain 40, 90 or 100 Percent of Performance?, *Financial Analysts Journal*, 56, 26-33.

Jacobs, H., S., Müller, and M., Weber, 2009, How Should Private Investors Diversify? An Empirical Evaluation of Alternative Asset Allocation Policies to Construct a World Market Portfolio, University of Mannheim, working paper.

Jondeau E., and M., Rockinger, 2006, Optimal Portfolio Allocation under Higher Moments, *European Financial Management*, 12, 29-55.

Jorion, P., 1985, International Portfolio Diversification with Estimation Risk, *Journal of Business*, 58, 259-277.

Jun, D., and B., G., Malkiel, 2004, New Paradigms in Stock Market Indexing, *European Financial Management*, 14, 118-126.

Kirby, C., and B., Ostdiek, 2012, It's All in the Timing: Simple Active Portfolio Strategies that Outperform Naïve Diversification, *Journal of Financial and Quantitative Analysis*, 47, 437-467.

Kandel, S., and R., Stambaugh, 1996, On the Predictability of Stock Returns: An Asset-Allocation Perspective, *Journal of Finance*, 51, 385-424.

Karolyi, G., and A., Sanders, 1998, The Variation of Economic Risk Premiums in Real Estate Returns, *Journal of Real Estate Financial Economics*, 17, 245-262.

Liu, C., D., Hartzell, and M. Hoesli, 1997, International Evidence on Real Estate Securities as an Inflation Hedge, *Real Estate Economics*, 25, 193-221.

Ling, D. C., A., Naranjo and M., Ryngaert, 2000, The Predictability of Equity REIT Returns: Time Variation and Economic Significance, *Journal of Real Estate Finance and Economics*, 20, 117-136.

MacKinnon, G., and A., Al-Zaman, 2009, Real Estate for the Long Term: The Effect of Return Predictability on Long-Horizon Allocations, *Real Estate Economics*, 37, 117-153.

Ranaldo, A., and R., Haberle, 2008, Wolfe in Sheep's Clothing: the Active Investment Strategies Behind Index Performance, *European Financial Management*, 14, 2008, 55-81.

Politis, D., and J., Romano, 1994, The Stationary Bootstrap, *Journal of the American Statistical*

Association, 89, 1303-1313.

Tabner, I., 2012, In Defence of Capitalisation Weights: Evidence from the FTSE 100 and S&P 500 Indices, *European Financial Management*, 18, 142-161.

Tu, J., and G., Zhou, 2010, Incorporating Economic Objectives into Bayesian Priors: Portfolio Choice under Parameter Uncertainty, *Journal of Financial and Quantitative Analysis*, 45, 959-986.

Tu, J., and G., Zhou, 2011, Markowitz Meets Talmud: A Combination of Sophisticated and Naïve Diversification Strategies, *Journal of Financial Economics*, 99, 204-215.

Zellner, A., and V. K., Chetty, 1965, Prediction and Decision Problems in Regression Models from the Bayesian Point of View, *Journal of the American Statistical Association*, 60, 608-616.

Appendix: Computing Optimal Portfolios under Predictable Returns

Classical Portfolios

In the presence of short-sale constraints, the program that consists of maximizing (3) is solved using numerical Monte Carlo methods:

subject to $\omega_{t,T}^{j} \in [0,1]$ for $j = s, b, r, f$. Here S is a large number of draws from the T-month ahead joint predictive density of real asset returns.

Under the model in equation (4), the (conditional) distribution of cumulative future real returns (i.e. the first four elements in $z_{t,T} \equiv \sum_{k=1}^{T} z_{t+k}$) is multivariate normal with mean and covariance matrix given by the appropriately selected elements of:

$$\begin{aligned} E_{t-1}[\mathbf{z}_{t,T}] &= T\boldsymbol{\mu} + (T-1)\boldsymbol{\Phi}\boldsymbol{\mu} + (T-2)\boldsymbol{\Phi}^2\boldsymbol{\mu} + \ldots + \boldsymbol{\Phi}^{T-1}\boldsymbol{\mu} + (\boldsymbol{\Phi} + \boldsymbol{\Phi}^2 + \ldots + \boldsymbol{\Phi}^T)\mathbf{z}_{t-1} \\ Var_{t-1}[\mathbf{z}_{t,T}] &= \boldsymbol{\Sigma} + (\mathbf{I}+\boldsymbol{\Phi})\boldsymbol{\Sigma}(\mathbf{I}+\boldsymbol{\Phi})' + \ldots + (\mathbf{I}+\boldsymbol{\Phi}+\ldots+\boldsymbol{\Phi}^{T-1})\boldsymbol{\Sigma}(\mathbf{I}+\boldsymbol{\Phi}+\ldots+\boldsymbol{\Phi}^{T-1})', \end{aligned} \tag{A.1}$$

where $\mathbf{I}$ is the N + M-identity matrix and $\boldsymbol{\Phi}^k \equiv \prod_{i=1}^{k}\boldsymbol{\Phi}$. Since the parametric form of the predictive distribution of $\mathbf{z}_{t,T}$ is known, it is possible to approach directly the problem in (3), or equivalently

where $\phi(E_t[\mathbf{z}_{t,T}], Var_t[\mathbf{z}_{t,T}])$ is a multivariate normal with mean $E_t[\mathbf{z}_{t,T}]$ and covariance matrix), by simulation methods. Indeed, it is possible to solve this problem by employing simulation methods similar to Barberis (2000):

$$\max_{\boldsymbol{\omega}_t} \frac{1}{S}\sum_{i=1}^{S}\left[\frac{\{\omega_t^s \exp(R_{t,T}^{s,i}) + \omega_t^b \exp(R_{t,T}^{b,i}) + \omega_t^r \exp(R_{t,T}^{r,i}) + (1-\omega_t^s-\omega_t^b-\omega_t^r)\exp(R_{t,T}^{f,i})\}^{1-\gamma}}{1-\gamma}\right],$$

where $\{R_{t,T}^{s,i}, R_{t,T}^{b,i}, R_{t,T}^{r,i}, R_{t,T}^{f,i}\}_{i=1}^{N}$ are obtained simulating from the process in (1) S times. To obtain sufficiently precise results, we have employed $N = 100{,}000$ Monte Carlo trials in order to minimize any residual random errors in optimal weights induced by simulations.

Bayesian Portfolios

Call θ the vector collecting all the parameters entering the generic VAR(1) model in (4), i.e., $\boldsymbol{\theta} \equiv [\boldsymbol{\mu}'$ $vec(\boldsymbol{\Phi})'$ $vech(\boldsymbol{\Sigma})']'$. The joint predictive distribution for $\mathbf{z}_t$ obtains then by integrating the joint distribution of $\boldsymbol{\theta}$ and returns, $p(\mathbf{z}_{t,T}, \boldsymbol{\theta} \mid \ddot{\mathbf{Z}}_t)$ with respect to the posterior distribution of $\boldsymbol{\theta}$, $p(\boldsymbol{\theta} \mid \ddot{\mathbf{Z}}_t)$:

$$p(\mathbf{z}_{t,T}) = \int p(\mathbf{z}_{t,T}, \boldsymbol{\theta} \mid \ddot{\mathbf{Z}}_t) d\boldsymbol{\theta} = \int p(\mathbf{z}_{t,T} \mid \ddot{\mathbf{Z}}_t, \boldsymbol{\theta}) p(\boldsymbol{\theta} \mid \ddot{\mathbf{Z}}_t) d\boldsymbol{\theta},$$

where $\ddot{\mathbf{Z}}_t$ collects the time series of asset returns and predictors up to time *t*, $\ddot{\mathbf{Z}}_t \equiv \{\mathbf{z}_i\}_{i=1}^{t}$. In turn, the posterior obtains from a standard uninformative prior. The portfolio optimization problem becomes:

$$\max_{\boldsymbol{\omega}_t} \int \frac{W_{t+T}^{1-\gamma}}{1-\gamma} p(\mathbf{z}_{t,T}) \cdot d\mathbf{z}_{t,T},$$

subject to constraints. Also in this case, while the portfolio strategy *ALL* obtains from the full (M = 4) VAR(1), *CPI*, *DY*, *TERM*, and *DEF* are simpler M = 1 cases that can be seen as obtained from imposing restrictions on $\boldsymbol{\Phi}$. When $\boldsymbol{\Phi} = \mathbf{O}$, the no predictability benchmark emerges; if portfolio returns were lognormally distributed, our Bayesian IID case would coincide with the Bayesian diffuse-prior mean variance portfolio analyzed by DeMiguel et al. (2009a) with constant risk premia. In this case, Monte Carlo methods require drawing a large number of times from $p(\mathbf{z}_{t,T})$ and then 'extracting' cumulative returns from the resulting vector. Given the problem

this task is simplified by the fact that predictive draws can drawn from the posterior distribution of the parameters and then, for each set of parameters drawn, by sampling one point from the distribution of returns conditional on past data and the parameters. At this point, equation (4) can be re-stated as:

$$\begin{bmatrix} \mathbf{z}_2' \\ \mathbf{z}_3' \\ \vdots \\ \mathbf{z}_t' \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{z}_1' \\ 1 & \mathbf{z}_2' \\ \vdots & \vdots \\ 1 & \mathbf{z}_{t-1}' \end{bmatrix} \begin{bmatrix} \boldsymbol{\mu}' \\ \boldsymbol{\Phi}' \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon}_2' \\ \boldsymbol{\varepsilon}_3' \\ \vdots \\ \boldsymbol{\varepsilon}_t' \end{bmatrix},$$

or simply $\mathbf{Z} = \mathbf{XC} + \mathbf{E}$, where **Z** is a ($t-1, N+M$) matrix with the observed vectors as rows, $\mathbf{X}$ is a ($t-1, N+M+1$) matrix of regressors, and $\mathbf{E}$ a ($t-1, N+M$) matrix of error terms, respectively. All the coefficients are instead collected in the ($N+M+1, N+M$) matrix $\mathbf{C}$. If we consider the following standard uninformative diffuse prior:

$$p(\mathbf{C}, \boldsymbol{\Sigma}) \propto |\boldsymbol{\Sigma}|^{-\frac{N+2}{2}},$$

then the posterior distribution for the coefficients in θ, $p(\mathbf{C}, \boldsymbol{\Sigma}^{-1} \mid \ddot{\mathbf{Z}}_t)$ can be characterized as:

$$\boldsymbol{\Sigma}^{-1} \mid \ddot{\mathbf{Z}}_t \sim \text{Wishart}(t-N-2, \hat{\mathbf{S}}^{-1})$$
$$vec(\mathbf{C}) \mid \boldsymbol{\Sigma}^{-1}, \ddot{\mathbf{Z}}_t \sim N\left(vec(\hat{\mathbf{C}}), \boldsymbol{\Sigma} \otimes (\mathbf{X}'\mathbf{X})^{-1}\right)$$

where $\hat{\mathbf{S}} = (\mathbf{Z} - \mathbf{X}\hat{\mathbf{C}})'(\mathbf{Z} - \mathbf{X}\hat{\mathbf{C}})$ and $\hat{\mathbf{C}} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Z}$, i.e., the classical OLS estimators for the coefficients and covariance matrix of the residuals.

We adopt the following simulation method. First, we draw *S* independent variates from $p(\mathbf{C}, \boldsymbol{\Sigma}^{-1} \mid \ddot{\mathbf{Z}}_t)$. This is done by first sampling from a marginal Wishart for $\boldsymbol{\Sigma}^{-1}$ and then (after calculating $\boldsymbol{\Sigma}$) from the conditional $N(vec(\hat{\mathbf{C}}), \boldsymbol{\Sigma} \otimes (\mathbf{X}'\mathbf{X})^{-1})$, where $\hat{\mathbf{C}}$ is easily calculated. Second, for each set $(\mathbf{C}, \boldsymbol{\Sigma})$ obtained, the algorithm samples cumulated returns from a multivariate normal with mean vector and covariance matrix given by (A1). In particular, since applying Monte Carlo methods implies a double simulation scheme (i.e., one pass to characterize the predictive density of returns, and a second pass to solve the portfolio choice problem), *S* is set to a relatively large value of 300,000 independent trials that are intended to approximate the joint predictive density of real returns and predictors.

Table 1

Descriptive Statistics

The table reports descriptive statistics for monthly real returns on stocks, bonds, equity REITs and returns on cash investments as well as for predictor variables (dividend yield, inflation, default spread and term spread). The sample period considered is Jan. 1972 – Dec. 2007. Statistics are reported in percentage terms.

	Mean	**Median**	**Std. Dev.**	**Sharpe**	**Min.**	**Max.**	**Skewness**	**Kurtosis**
Real Stock Returns	0.349	0.745	4.493	0.043	-26.39	14.96	-0.843	6.483
Long-term Govt. Bonds, Real Returns	0.117	0.147	2.205	-0.018	-7.631	8.911	0.011	4.028
Real Equity REIT Returns	0.489	0.749	4.057	0.082	-17.39	12.31	-0.633	5.163
Real 3-month T-bill Returns	0.534	0.486	0.287	—	0.072	2.142	1.529	7.303
CPI Inflation rate	0.378	0.320	0.354	—	-0.806	1.790	0.532	3.969
Dividend Yield (annual MA)	2.851	2.640	1.010	—	1.250	6.060	0.707	3.045
Default Spread (Baa-Treasury, annualized)	2.018	1.907	0.561	—	0.923	3.780	0.684	3.130
Riskless Term Spread (annualized)	1.523	1.689	2.194	—	-15.16	5.88	-1.818	11.828

Table 2

VAR(1) MLE Estimates: All predictors, Asset Menu Includes Equity REITs

	$Stocks_t$	$Bonds_t$	$E\text{-}REITs_t$	Dividend $Yield_t$	Term $Spread_t$	Default $Spread_t$	Real Short $Rate_t$	$Inflation_t$
				μ’				
	0.006	-0.008	0.001	0.000	**0.001**	0.000	**-0.004**	**0.003**
	(-0.492)	(-1.314)	(-0.130)	(-0.660)	(-3.283)	(-0.196)	(-4.503)	(-3.505)
				Φ				
$Stocks_{t-1}$	-0.022	**-0.059**	0.074	0.001	0.001	0.000	-0.005	0.004
	(-0.382)	(-2.073)	(-1.432)	(-0.664)	(-0.560)	(-1.151)	(-1.181)	(-1.168)
$Bonds_{t-1}$	0.028	0.027	**0.264**	0.000	**0.007**	**-0.001**	**0.019**	**-0.026**
	(-0.261)	(-0.512)	(-2.731)	(-0.118)	(-1.865)	(-1.950)	(-2.637)	(-3.947)
Equity $REITs_{t-1}$	0.102	-0.015	0.000	-0.003	**0.008**	**-0.001**	**-0.009**	0.002
	(-1.596)	(-0.478)	(-0.007)	(-1.419)	(-3.588)	(-2.461)	(-2.131)	(-0.599)
Dividend $Yield_{t-1}$	**0.801**	0.213	**0.699**	**0.977**	**-0.017**	**-0.002**	**-0.052**	**0.070**
	(-2.842)	(-1.550)	(-2.780)	(-99.93)	(-1.834)	(-2.025)	(-2.705)	(-4.065)
Term $Spread_{t-1}$	**-5.366**	-1.214	**-3.064**	**0.233**	**0.563**	**0.018**	**0.750**	**-0.339**
	(-2.845)	(-1.316)	(-1.819)	(-3.559)	(-9.112)	(-2.988)	(-5.876)	(-2.937)
Def $Spread_{t-1}$	3.239	**6.872**	4.784	**-0.358**	-0.174	**0.976**	**1.061**	**-0.954**
	(-0.664)	(-2.880)	(-1.097)	(-2.115)	(-1.091)	(-61.23)	(-3.215)	(-3.191)
Real Short $Rate_{t-1}$	**-3.127**	-0.687	**-2.940**	0.073	0.037	**0.013**	**1.022**	-0.085
	(-2.066)	(-0.928)	(-2.175)	(-1.391)	(-0.750)	(-2.616)	(-9.984)	(-0.914)
$Inflation_{t-1}$	**-5.437**	**-1.621**	**-4.619**	**0.156**	-0.011	**0.018**	**0.649**	**0.341**
	(-3.196)	(-1.949)	(-3.041)	(-2.648)	(-0.199)	(-3.163)	(-5.642)	(-3.278)
				Implied Correlation matrix for VAR(1) Shocks				
$Stocks_t$	1	0.13	0.547	-0.887	0.099	-0.218	0.109	-0.176
$Bonds_t$		1	0.113	-0.195	-0.361	0.35	0.208	-0.09
$E\text{-}REITs_t$			1	-0.542	0.09	-0.206	0.059	-0.112
Dividend $Yield_t$				1	-0.113	0.186	-0.117	0.191
Term $Spread_t$					1	-0.288	-0.445	-0.006
Def $Spread_t$						1	0.073	0.005
Real Short $Rate_t$							1	-0.886
$Inflation_t$								1

Bold coefficients imply a p-value of 0.1 or lower.

Table 3

Conditional (predicted) moments of returns

The table reports the annualized means and standard deviations (upper panel), in percentage terms, as well as correlations (bottom panel) of conditional returns at one month and 24 months. These derive from Classical VAR(1) estimates.

	Stock	**Bond**	**e-REITs**	**1-m T-bills**
		Horizon: 1 month		
Mean	3.7	1.1	4.7	2.0
Standard Error	(0.001)	(0.000)	(0.001)	(0.000)
Standard Deviation	8.4	2.4	8.4	1.2
Sharpe Ratio	0.20	-0.38	0.32	
		Horizon: 24 month		
Mean	3.8	1.3	5.0	2.1
Standard Error	(0.001)	(0.000)	(0.001)	(0.000)
Standard Deviation	8.4	3.6	9.6	1.2
Sharpe Ratio	0.20	-0.22	0.30	

	Stock Bond	**Stock e-REITs**	**Stock Real Tbill**	**Bond e-REITs**	**Bond Real Tbill**	**e-REITs Real Tbill**
			Horizon: 1 month			
Correlation	0.196	0.590	0.174	0.135	0.253	0.090
Standard Error	(0.004)	(0.003)	(0.002)	(0.001)	(0.002)	(0.002)
			Horizon: 24 month			
Correlation	0.415	0.572	0.290	0.409	0.045	0.067
Standard Error	(0.010)	(0.003)	(0.006)	(0.003)	(0.004)	(0.004)

Table 4

Ex-post Performance of Classical Portfolios

The table reports statistics on the realized performance of classical optimal portfolios (VAR(1) and IID) and 1/N. Performance measures are computed recursively over the period 1995-2007 for an investor with constant relative risk aversion respectively equal to 2, 5, 10 and different investment horizons, from 1 to 60 months. Best realized performances are boldfaced.

☐		(1) 1/N		(2) Gaussian IID		(3) VAR(1) – DY		(4) VAR(1) -- TERM Spread		(5) VAR(1) -- DEF Spread		(6) VAR(1) -- CPI Inflation		(7) VAR(1) – ALL	
		RE	**No RE**	**RE**	**No RE**	**RE**	**No RE**	**RE**	**No RE**	**RE**	**No RE**	**RE**	**No RE**	**RE**	**No RE**
☐								Investment Horizon: 1 month							
$\gamma=2$	**Annual Mean Returns**	6.07	4.97	7.93	**7.60**	8.49	6.76	8.49	6.47	8.44	6.54	**8.50**	6.43	8.40	4.81
	Annualized Volatility	**5.92**	**5.01**	13.97	14.48	13.92	13.07	13.98	12.88	13.96	13.11	13.91	12.79	13.96	11.05
	Annualized Sharpe Ratio	**0.33**	0.17	0.27	**0.24**	0.31	0.20	0.31	0.19	0.31	0.18	0.31	0.18	0.31	0.06
	Annualized CER	5.72	4.72	5.94	**5.45**	6.51	5.00	6.49	4.76	6.45	4.77	**6.53**	4.75	6.41	3.55
$\gamma=5$	**Annual Mean Returns**	6.07	4.97	**6.45**	**7.32**	6.20	4.99	5.97	4.74	6.14	6.49	6.05	4.76	6.18	4.27
	Annualized Volatility	**5.92**	**5.01**	10.87	14.15	8.94	6.12	8.50	5.89	9.20	6.49	8.41	5.75	9.23	6.33
	Annualized Sharpe Ratio	**0.33**	0.17	0.21	**0.22**	0.23	0.14	0.21	0.10	0.22	0.09	0.23	0.11	0.22	0.02
	Annualized CER	**5.18**	**4.35**	3.34	1.95	4.13	4.03	4.09	3.86	3.93	3.67	4.21	3.92	3.95	3.26
$\gamma=10$	**Annual Mean Returns**	**6.07**	4.97	5.42	**7.43**	5.19	4.61	5.06	4.46	5.18	4.52	5.09	4.51	5.16	4.22
	Annualized Volatility	5.92	5.01	8.79	14.09	4.51	3.05	**4.23**	2.89	4.68	3.24	4.24	**2.83**	4.67	3.10
	Annualized Sharpe Ratio	**0.33**	0.17	0.14	**0.23**	0.23	0.15	0.22	0.11	0.22	0.11	0.22	0.18	0.22	0.02
	Annualized CER	**4.26**	3.70	1.12	-4.17	4.14	**4.14**	4.13	4.04	4.04	3.98	4.17	4.10	4.02	3.73
								Investment Horizon: 3 months							
$\gamma=2$	**Annual Mean Returns**	6.16	4.96	8.71	**7.76**	9.28	6.94	9.28	6.61	9.20	6.68	**9.29**	6.53	9.16	7.04
	Annualized Volatility	**6.01**	**4.93**	13.97	14.48	14.03	13.44	14.08	13.14	14.08	13.49	14.03	13.03	14.17	11.31
	Annualized Sharpe Ratio	0.35	0.16	0.32	0.24	0.37	0.21	0.36	0.19	0.36	0.19	**0.37**	0.18	0.35	**0.26**
	Annualized CER	5.91	4.72	6.72	5.50	7.31	5.12	7.29	4.87	7.22	4.85	**7.32**	4.82	7.15	**5.73**
$\gamma=5$	**Annual Mean Returns**	6.26	4.96	**7.11**	**7.38**	6.73	5.04	6.39	4.77	6.62	4.85	6.47	4.83	6.64	4.57
	Annualized Volatility	**6.01**	**4.93**	10.83	14.72	8.86	6.17	8.34	5.84	9.21	6.41	8.34	5.68	9.38	6.13
	Annualized Sharpe Ratio	**0.35**	0.16	0.27	**0.22**	0.29	0.15	0.27	0.11	0.27	0.11	0.29	0.12	0.27	0.07
	Annualized CER	**5.37**	**4.36**	4.09	1.65	4.73	4.10	4.62	3.92	4.47	3.81	4.70	4.03	4.41	3.64
$\gamma=10$	**Annual Mean Returns**	**6.26**	4.96	6.05	**7.45**	5.52	4.68	5.34	4.47	5.47	4.52	5.38	4.54	5.39	4.37
	Annualized Volatility	6.01	4.93	8.87	14.65	4.38	3.14	**4.06**	2.93	4.57	3.26	4.12	**2.89**	4.59	3.01
	Annualized Sharpe Ratio	**0.35**	0.16	0.21	**0.23**	0.31	0.17	0.29	0.11	0.29	0.12	0.30	0.14	0.27	0.07
	Annualized CER	4.46	3.78	1.55	-4.87	**4.54**	**4.19**	4.50	4.04	4.41	3.99	4.51	4.13	4.32	3.92
								Investment Horizon: 6 months							
$\gamma=2$	**Annual Mean Returns**	6.24	4.81	9.06	**7.56**	9.67	6.81	9.65	6.40	9.58	6.48	**9.67**	6.31	9.49	5.39
	Annualized Volatility	**5.70**	**4.73**	14.33	14.61	14.22	13.15	14.27	12.74	14.29	13.15	14.24	12.64	14.39	12.84
	Annualized Sharpe Ratio	0.37	0.14	0.34	**0.23**	**0.39**	0.20	0.39	0.18	0.38	0.18	0.39	0.17	0.37	0.10
	Annualized CER	5.93	4.59	7.06	**5.35**	**7.70**	5.05	7.66	4.74	7.59	4.71	7.69	4.68	7.46	3.66
$\gamma=5$	**Annual Mean Returns**	6.24	4.81	**7.34**	**7.16**	6.93	5.05	6.57	4.68	6.80	4.76	6.70	4.73	6.93	4.82
	Annualized Volatility	**5.70**	**4.73**	10.63	14.56	8.78	6.01	8.09	5.60	9.15	6.17	8.07	5.45	9.34	6.25
	Annualized Sharpe Ratio	**0.37**	0.14	0.30	**0.21**	0.32	0.15	0.30	0.10	0.29	0.10	0.32	0.11	0.30	0.11
	Annualized CER	**5.46**	**4.27**	4.47	1.27	4.99	4.13	4.92	3.89	4.69	3.78	5.06	3.98	4.75	3.83
$\gamma=10$	**Annual Mean Returns**	6.24	4.81	**6.25**	**7.26**	5.67	4.68	5.40	4.45	5.53	4.47	5.51	4.52	5.57	4.35
	Annualized Volatility	5.70	4.73	8.57	14.54	4.32	3.19	**3.90**	2.94	4.47	3.22	3.94	**2.90**	4.41	3.03
	Annualized Sharpe Ratio	**0.37**	0.14	0.25	**0.21**	0.35	0.17	0.32	0.10	0.31	0.10	0.35	0.13	0.32	0.07
	Annualized CER	4.68	3.72	1.85	-6.00	4.72	**4.17**	4.63	4.02	4.52	3.95	**4.72**	4.11	4.59	3.90

Table 4 (continued)
Ex-post Performance of Classical Portfolios

		(1)		(2)		(3)		(4)		(5)		(6)		(7)	
		1/N		**Gaussian IID**		**VAR(1) -- DY**		**VAR(1) -- TERM Spread**		**VAR(1) -- DEF Spread**		**VAR(1) -- CPI Inflation**		**VAR(1) -- ALL**	
		RE	**No RE**	**RE**	**No RE**	**RE**	**No RE**	**RE**	**No RE**	**RE**	**No RE**	**RE**	**No RE**	**RE**	**No RE**
		Investment Horizon: 12 month													
γ=2	**Annual Mean Returns**	6.38	4.66	10.09	**7.57**	10.75	6.90	10.73	6.39	10.66	6.54	**10.79**	6.30	10.67	7.01
	Annualized Volatility	**5.98**	**3.99**	15.18	16.39	15.10	14.97	15.21	14.28	15.25	14.84	15.14	14.14	15.23	14.48
	Annualized Sharpe Ratio	0.37	0.10	0.32	**0.21**	0.44	0.18	0.43	0.16	0.43	0.16	**0.44**	0.15	0.43	0.20
	Annualized CER	6.05	4.40	7.99	4.77	8.68	4.54	8.62	4.24	8.53	4.21	**8.70**	4.20	8.55	**4.79**
γ=5	**Annual Mean Returns**	6.38	4.66	**8.21**	**7.12**	7.76	5.06	7.18	4.62	7.52	4.75	7.35	4.67	7.98	5.18
	Annualized Volatility	**5.98**	**3.99**	10.76	16.40	9.26	7.13	8.11	6.40	9.38	6.98	8.09	6.21	9.88	6.00
	Annualized Sharpe Ratio	0.37	0.10	0.38	**0.18**	0.39	0.13	0.37	0.07	0.36	0.09	**0.40**	0.08	0.39	0.17
	Annualized CER	5.57	3.99	5.55	-0.50	5.74	3.76	5.63	3.56	5.45	3.44	**5.80**	3.67	5.71	**4.28**
γ=10	**Annual Mean Returns**	6.38	4.66	**7.03**	**7.21**	6.03	4.71	5.69	4.42	5.87	4.42	5.80	4.47	6.03	4.41
	Annualized Volatility	5.98	3.99	8.10	16.38	4.40	3.90	**3.74**	3.46	4.43	3.67	3.85	**3.45**	4.22	3.79
	Annualized Sharpe Ratio	0.37	0.10	0.36	**0.19**	0.43	0.14	0.41	0.08	0.39	0.07	**0.43**	0.09	0.42	0.07
	Annualized CER	4.80	3.30	4.00	-8.52	**5.10**	**3.96**	5.02	3.83	4.92	3.72	5.09	3.87	5.06	3.69
		Investment Horizon: 24 months													
γ=2	**Annual Mean Returns**	6.46	4.56	10.86	**7.46**	11.59	6.78	11.55	6.30	11.47	6.46	**11.63**	6.21	11.46	6.22
	Annualized Volatility	**6.67**	**6.13**	17.91	18.74	17.91	17.79	18.03	16.80	18.14	17.54	17.95	16.59	17.97	17.55
	Annualized Sharpe Ratio	0.35	0.07	0.38	**0.18**	0.42	0.15	0.41	0.13	0.40	0.13	**0.42**	0.13	0.41	0.12
	Annualized CER	6.07	**4.20**	8.29	3.71	9.05	3.46	8.97	3.32	8.85	3.18	**9.07**	3.30	8.90	3.04
γ=5	**Annual Mean Returns**	6.46	4.55	8.74	**7.09**	8.12	4.93	7.51	4.50	8.02	4.64	5.95	4.39	**8.89**	4.70
	Annualized Volatility	**6.67**	6.13	12.42	7.09	10.44	8.59	8.97	7.70	10.71	8.41	9.03	7.48	11.92	**5.81**
	Annualized Sharpe Ratio	0.35	0.07	0.37	**0.16**	0.38	0.09	0.38	0.05	0.36	0.06	0.39	0.05	**0.40**	0.10
	Annualized CER	5.48	3.64	5.76	-3.07	5.84	3.07	5.84	2.99	5.66	2.76	5.99	3.14	**6.06**	**3.96**
γ=10	**Annual Mean Returns**	6.46	4.55	**7.33**	**7.21**	6.15	4.61	5.80	4.33	6.06	4.34	5.95	4.38	6.40	4.56
	Annualized Volatility	6.67	6.13	8.54	18.89	4.68	4.92	**3.87**	4.41	4.75	4.63	4.01	**4.39**	5.17	4.85
	Annualized Sharpe Ratio	0.35	0.07	0.37	**0.16**	0.43	0.09	0.43	0.04	0.40	0.04	**0.45**	0.05	0.44	0.09
	Annualized CER	4.55	2.67	**5.31**	-10.54	5.22	**3.49**	5.17	3.43	5.08	3.29	5.25	3.48	5.28	3.44
		Investment Horizon: 60 months													
γ=2	**Annual Mean Returns**	5.78	3.76	9.61	**4.25**	10.72	3.74	10.60	3.45	10.50	3.62	**10.82**	3.40	10.80	3.26
	Annualized Volatility	**6.12**	**5.86**	22.14	20.30	22.06	20.34	22.63	18.39	23.11	20.27	22.69	17.98	22.01	19.48
	Annualized Sharpe Ratio	0.27	-0.07	0.25	**0.01**	0.30	-0.02	0.29	-0.04	0.28	-0.03	0.29	-0.04	**0.30**	-0.05
	Annualized CER	5.48	**3.49**	6.64	1.59	7.84	1.09	7.56	1.22	7.30	1.00	7.77	1.25	**7.94**	0.96
γ=5	**Annual Mean Returns**	5.78	3.76	8.01	4.01	8.08	4.30	6.88	3.44	7.64	3.53	7.14	3.58	**9.01**	**5.66**
	Annualized Volatility	**6.12**	**5.86**	15.79	19.96	12.42	11.51	9.69	8.13	11.93	7.89	9.86	7.92	14.41	10.04
	Annualized Sharpe Ratio	0.27	-0.07	0.25	-0.01	0.32	0.01	0.28	-0.09	0.29	-0.08	0.30	-0.07	**0.34**	**0.15**
	Annualized CER	5.01	3.15	4.68	-0.76	5.26	2.47	5.25	2.36	5.37	2.47	5.48	2.56	**5.51**	**4.22**
γ=10	**Annual Mean Returns**	5.78	3.76	**6.58**	4.09	6.28	4.33	5.49	3.89	5.82	3.87	5.69	3.93	6.65	**4.64**
	Annualized Volatility	6.19	5.86	8.50	20.00	4.92	7.01	**3.44**	5.29	4.21	**4.57**	3.55	5.22	5.79	6.87
	Annualized Sharpe Ratio	0.27	-0.07	0.29	0.00	**0.43**	0.04	0.39	-0.05	0.40	-0.06	0.43	-0.04	0.43	**0.07**
	Annualized CER	4.22	2.71	4.89	-2.46	5.23	3.16	5.01	3.04	5.15	3.18	5.18	3.10	**5.26**	**3.41**

Table 5
Ex-post Performance of Bayesian Portfolios

The table reports statistics on the realized performance of bayesian optimal portfolios (VAR(1) and IID) and 1/N. Performance measures are computed recursively over the period 1995-2007 for an investor with constant relative risk aversion respectively equal to 2, 5, 10 and different investment horizons, from 1 to 60 months. Best realized performances are boldfaced.

		(1) 1/N		(2) Gaussian IID		(3) VAR(1) -- DY		(4) VAR(1) -- TERM Spread		(5) VAR(1) -- DEF Spread		(6) VAR(1) -- CPI Inflation		(7) VAR(1) -- ALL	
		RE	No RE	RE	No RE	RE	No RE	RE	No RE	RE	No RE	RE	No RE	RE	No RE
		Investment Horizon: 1 month													
γ=2	Annual Mean Returns	6.07	4.97	8.19	5.45	8.27	5.91	8.29	5.94	8.27	5.88	8.33	5.94	**8.45**	**6.01**
	Annualized Volatility	**5.92**	**5.01**	12.85	10.31	13.45	11.68	13.44	11.71	13.44	11.67	13.40	11.71	13.55	9.61
	Annualized Sharpe Ratio	**0.33**	0.17	0.32	0.13	0.31	0.15	0.31	0.15	0.31	0.15	0.31	0.15	0.32	**0.19**
	Annualized CER	5.72	4.72	6.51	4.37	6.42	4.50	6.44	4.53	6.42	4.48	6.49	4.53	**6.57**	**5.08**
γ=5	Annual Mean Returns	**6.07**	**4.97**	5.38	4.54	6.04	4.47	5.98	4.47	6.02	4.43	6.02	4.47	6.01	4.21
	Annualized Volatility	**5.92**	5.01	6.86	**4.56**	7.66	5.22	7.68	5.25	7.77	5.31	7.67	5.25	7.96	5.32
	Annualized Sharpe Ratio	**0.33**	**0.17**	0.18	0.09	0.25	0.06	0.24	0.06	0.24	0.05	0.24	0.06	0.23	0.01
	Annualized CER	**5.18**	**4.35**	4.16	4.02	4.53	3.78	4.45	3.77	4.45	3.72	4.49	3.77	4.37	3.49
γ=10	Annual Mean Returns	**6.07**	**4.97**	4.80	4.36	5.01	4.32	5.07	4.29	5.10	4.30	5.10	4.34	5.08	4.19
	Annualized Volatility	5.92	5.01	**3.37**	**2.24**	3.81	2.62	3.81	2.63	3.87	2.67	3.80	2.63	3.95	2.62
	Annualized Sharpe Ratio	**0.33**	**0.17**	0.19	0.10	0.25	0.06	0.24	0.06	0.25	0.11	0.25	0.07	0.24	0.01
	Annualized CER	4.26	3.70	4.21	**4.11**	**4.35**	3.97	4.32	3.94	4.33	3.94	4.35	3.99	4.28	3.84
		Investment Horizon: 3 months													
γ=2	Annual Mean Returns	4.96	6.26	8.86	5.29	8.93	5.84	8.95	5.79	8.96	5.85	8.95	5.80	**9.09**	**7.08**
	Annualized Volatility	**4.93**	**6.01**	13.23	10.13	13.66	11.22	13.62	11.23	13.65	11.34	13.61	11.23	13.72	9.99
	Annualized Sharpe Ratio	0.35	0.16	0.36	0.11	0.35	0.15	0.35	0.15	0.35	0.15	0.35	0.15	**0.36**	**0.29**
	Annualized CER	5.91	4.72	7.10	4.27	7.06	4.58	7.09	4.53	7.09	4.55	7.09	4.53	**7.20**	**6.09**
γ=5	Annual Mean Returns	4.96	**6.26**	5.91	4.35	6.04	4.57	5.99	4.53	6.01	4.50	6.02	4.53	**6.10**	4.42
	Annualized Volatility	**4.93**	6.01	6.82	**4.26**	6.61	4.79	6.54	4.72	6.84	4.99	6.57	4.72	7.10	4.94
	Annualized Sharpe Ratio	**0.35**	**0.16**	0.26	0.05	0.29	0.09	0.28	0.08	0.27	0.07	0.29	0.08	0.27	0.06
	Annualized CER	**5.37**	**4.36**	4.75	3.91	4.94	4.00	4.92	3.97	4.83	3.88	4.93	3.98	4.82	3.82
γ=10	Annual Mean Returns	4.96	**6.26**	**5.60**	4.30	5.16	4.42	5.16	4.39	5.14	4.36	5.18	4.40	5.14	4.30
	Annualized Volatility	4.93	6.01	3.74	**2.16**	3.29	2.45	4.62	4.10	3.40	2.58	**3.27**	2.43	3.42	2.41
	Annualized Sharpe Ratio	0.35	**0.16**	**0.39**	0.07	0.31	0.11	0.31	0.10	0.29	0.08	0.32	0.10	0.29	0.07
	Annualized CER	4.46	3.78	**4.90**	4.07	4.61	**4.13**	4.62	4.10	4.56	4.03	4.64	4.10	4.55	4.02
		Investment Horizon: 6 months													
γ=2	Annual Mean Returns	6.24	4.81	9.34	5.28	9.26	5.65	9.17	5.52	9.26	5.62	9.18	4.70	**9.44**	**6.37**
	Annualized Volatility	**5.70**	**4.73**	13.08	9.78	13.71	10.85	13.63	10.79	13.71	11.08	13.63	10.31	13.90	10.96
	Annualized Sharpe Ratio	0.37	0.14	**0.40**	0.12	0.37	0.14	0.37	0.13	0.37	0.13	0.37	0.05	0.38	**0.20**
	Annualized CER	5.93	4.59	**7.66**	4.33	7.42	4.46	7.35	4.33	7.41	4.37	7.36	3.60	7.55	**5.18**
γ=5	Annual Mean Returns	6.24	**4.81**	5.91	4.33	6.17	4.59	6.05	4.56	6.10	4.40	6.10	4.57	**6.28**	4.45
	Annualized Volatility	**5.70**	4.73	6.48	**4.05**	6.33	4.54	6.14	4.46	6.63	4.75	6.14	4.45	6.88	4.81
	Annualized Sharpe Ratio	**0.37**	**0.14**	0.27	0.05	0.32	0.10	0.31	0.09	0.29	0.05	0.32	0.10	0.31	0.06
	Annualized CER	**5.46**	**4.27**	4.88	3.93	5.17	4.08	5.11	4.07	5.01	3.83	5.17	4.08	5.10	3.87
γ=10	Annual Mean Returns	**6.24**	**4.81**	5.06	4.27	5.25	4.47	5.19	4.43	5.22	4.39	5.24	4.44	5.23	4.31
	Annualized Volatility	5.70	4.73	3.11	**2.20**	3.14	2.49	3.03	2.43	3.31	2.62	**3.02**	2.43	3.27	2.44
	Annualized Sharpe Ratio	**0.37**	**0.14**	0.29	0.06	0.35	0.13	0.35	0.12	0.33	0.09	0.36	0.12	0.33	0.07
	Annualized CER	4.68	3.72	4.59	4.04	4.75	**4.16**	4.73	4.14	4.68	4.05	**4.78**	4.14	4.69	4.01

Table 5 (continued)
Ex-post Performance of Bayesian Portfolios

		(1) 1/N		(2) Gaussian IID		(3) VAR(1) -- DY		(4) VAR(1) -- TERM Spread		(5) VAR(1) -- DEF Spread		(6) VAR(1) -- CPI Inflation		(7) VAR(1) -- ALL	
		RE	**No RE**	**RE**	**No RE**	**RE**	**No RE**	**RE**	**No RE**	**RE**	**No RE**	**RE**	**No RE**	**RE**	**No RE**
		Investment Horizon: 12 month													
γ=2	**Annual Mean Returns**	6.38	4.66	10.23	5.10	10.33	**5.68**	10.11	5.40	10.30	5.52	10.06	5.38	**10.40**	3.06
	Annualized Volatility	**5.98**	**3.99**	13.85	10.86	14.43	12.20	14.25	11.84	14.54	12.48	14.29	11.84	14.68	10.36
	Annualized Sharpe Ratio	0.37	0.10	**0.44**	0.09	0.43	**0.13**	0.42	0.11	0.42	0.11	0.41	0.10	0.43	-0.11
	Annualized CER	6.05	**4.40**	**8.49**	3.90	8.43	4.14	8.23	3.94	8.38	3.88	8.20	3.92	8.45	1.94
γ=5	**Annual Mean Returns**	6.38	**4.66**	6.47	4.20	6.71	4.64	6.48	4.54	6.67	4.49	6.54	4.55	**6.97**	4.53
	Annualized Volatility	5.98	**3.99**	6.44	4.56	6.34	5.25	5.85	5.03	6.69	5.50	**5.82**	5.03	7.06	5.58
	Annualized Sharpe Ratio	0.37	**0.10**	0.36	0.01	0.40	0.09	0.40	0.08	0.38	0.06	**0.41**	0.08	0.40	0.07
	Annualized CER	5.57	**3.99**	5.50	3.69	5.76	3.95	5.67	3.90	5.62	3.71	5.74	3.91	**5.79**	3.73
γ=10	**Annual Mean Returns**	**6.38**	**4.66**	5.32	4.22	5.52	4.51	5.37	4.40	6.25	4.40	5.38	4.38	5.52	4.34
	Annualized Volatility	5.98	3.99	3.07	**2.57**	3.07	3.05	**2.74**	2.87	3.50	3.18	2.83	2.87	3.19	2.90
	Annualized Sharpe Ratio	0.37	0.10	0.38	0.03	0.45	**0.12**	0.44	0.09	**0.60**	0.08	0.44	0.08	0.43	0.07
	Annualized CER	4.80	3.30	4.87	3.91	5.07	**4.05**	5.01	3.99	**5.67**	3.90	4.99	3.98	5.03	3.93
		Investment Horizon: 24 months													
γ=2	**Annual Mean Returns**	6.46	4.56	10.76	5.00	11.06	**5.80**	10.66	5.23	10.87	5.25	10.50	5.21	**11.07**	5.72
	Annualized Volatility	**6.67**	**6.13**	15.93	12.00	16.94	14.49	16.69	13.49	17.20	14.32	16.53	13.49	17.07	11.72
	Annualized Sharpe Ratio	0.35	0.07	**0.42**	0.11	0.41	0.11	0.39	0.08	0.39	0.08	0.38	0.08	0.41	**0.13**
	Annualized CER	6.07	4.20	8.72	3.26	**8.78**	3.64	8.43	3.32	8.51	3.05	8.30	3.30	8.75	**4.40**
γ=5	**Annual Mean Returns**	6.46	4.55	6.62	4.12	7.07	4.68	6.57	4.45	6.95	4.55	6.57	4.46	**7.31**	**4.70**
	Annualized Volatility	6.67	6.13	7.06	**5.52**	7.06	6.61	**6.00**	5.98	7.06	6.48	6.01	5.98	7.49	6.57
	Annualized Sharpe Ratio	0.35	0.07	0.35	0.00	0.41	0.08	0.40	0.05	0.40	0.06	0.40	0.05	**0.42**	**0.08**
	Annualized CER	5.48	**3.64**	5.52	3.39	6.00	3.62	5.80	3.58	5.89	3.49	5.79	3.59	**6.12**	3.63
γ=10	**Annual Mean Returns**	**6.46**	**4.55**	5.40	4.15	5.64	4.49	5.36	4.30	5.51	4.30	5.43	4.31	5.63	4.36
	Annualized Volatility	6.67	6.13	3.10	**3.36**	3.25	4.01	**2.68**	3.63	3.21	3.74	2.75	3.62	3.28	3.53
	Annualized Sharpe Ratio	0.35	**0.07**	0.40	0.00	0.46	0.09	0.45	0.04	0.43	0.04	**0.47**	0.05	0.45	0.06
	Annualized CER	4.55	2.67	4.97	3.64	**5.18**	3.76	5.05	3.70	5.06	3.64	5.10	3.71	5.15	**3.79**
		Investment Horizon: 60 months													
γ=2	**Annual Mean Returns**	5.78	3.76	9.32	2.68	9.79	3.64	8.86	2.86	9.30	3.26	8.90	2.29	**9.79**	**6.14**
	Annualized Volatility	**6.12**	**5.86**	18.59	13.00	18.94	15.38	18.97	12.91	19.68	13.01	18.32	12.89	18.31	12.63
	Annualized Sharpe Ratio	0.27	-0.07	0.28	0.00	0.30	-0.03	0.25	-0.10	0.26	-0.07	0.26	-0.10	**0.31**	**0.16**
	Annualized CER	5.48	3.49	7.11	1.36	7.52	2.03	6.59	1.64	6.86	2.03	6.76	1.64	**7.63**	**5.12**
γ=5	**Annual Mean Returns**	5.78	3.76	6.09	3.44	6.75	4.24	5.92	3.79	6.26	3.81	6.01	3.79	**7.01**	**4.41**
	Annualized Volatility	6.12	5.86	6.86	5.98	6.53	7.54	5.30	6.22	6.07	**5.59**	**5.13**	6.22	6.62	6.61
	Annualized Sharpe Ratio	0.27	-0.07	0.28	0.00	0.40	0.01	0.33	-0.06	0.35	-0.06	0.36	-0.06	**0.43**	**0.04**
	Annualized CER	5.01	3.15	5.21	2.83	5.91	3.34	5.38	3.13	5.59	3.25	5.51	3.13	**6.17**	**3.71**
γ=10	**Annual Mean Returns**	**5.78**	3.76	5.15	3.86	5.51	**4.32**	5.06	4.01	5.24	4.06	5.18	4.00	5.53	4.28
	Annualized Volatility	6.12	5.86	2.62	4.19	2.53	5.07	1.94	4.31	**1.66**	3.49	1.92	4.32	2.25	**3.68**
	Annualized Sharpe Ratio	0.27	-0.07	0.38	0.00	0.54	0.03	0.47	-0.03	**0.66**	-0.02	0.54	-0.03	0.61	**0.03**
	Annualized CER	4.22	2.71	4.86	3.29	5.24	3.54	4.91	3.39	5.12	3.63	5.02	3.39	**5.32**	**3.81**

Figure 1
Classical Optimal Portfolio Weights

This graph reports mean classical optimal portfolio weights for stocks, bonds and cash, for three alternative models of asset returns: Equally Weighted, Gaussian IID, VAR with all predictors (VAR-ALL), under the two alternative cases in which real estate is or is not in the asset menu. The investment horizon varies from 1 to 60 months. Constant relative risk aversion is set to respectively 2, 5 and 10. For each horizon, means of portfolio weights are computed over 120 monthly portfolio allocations on the recursive sample 1995-2007.

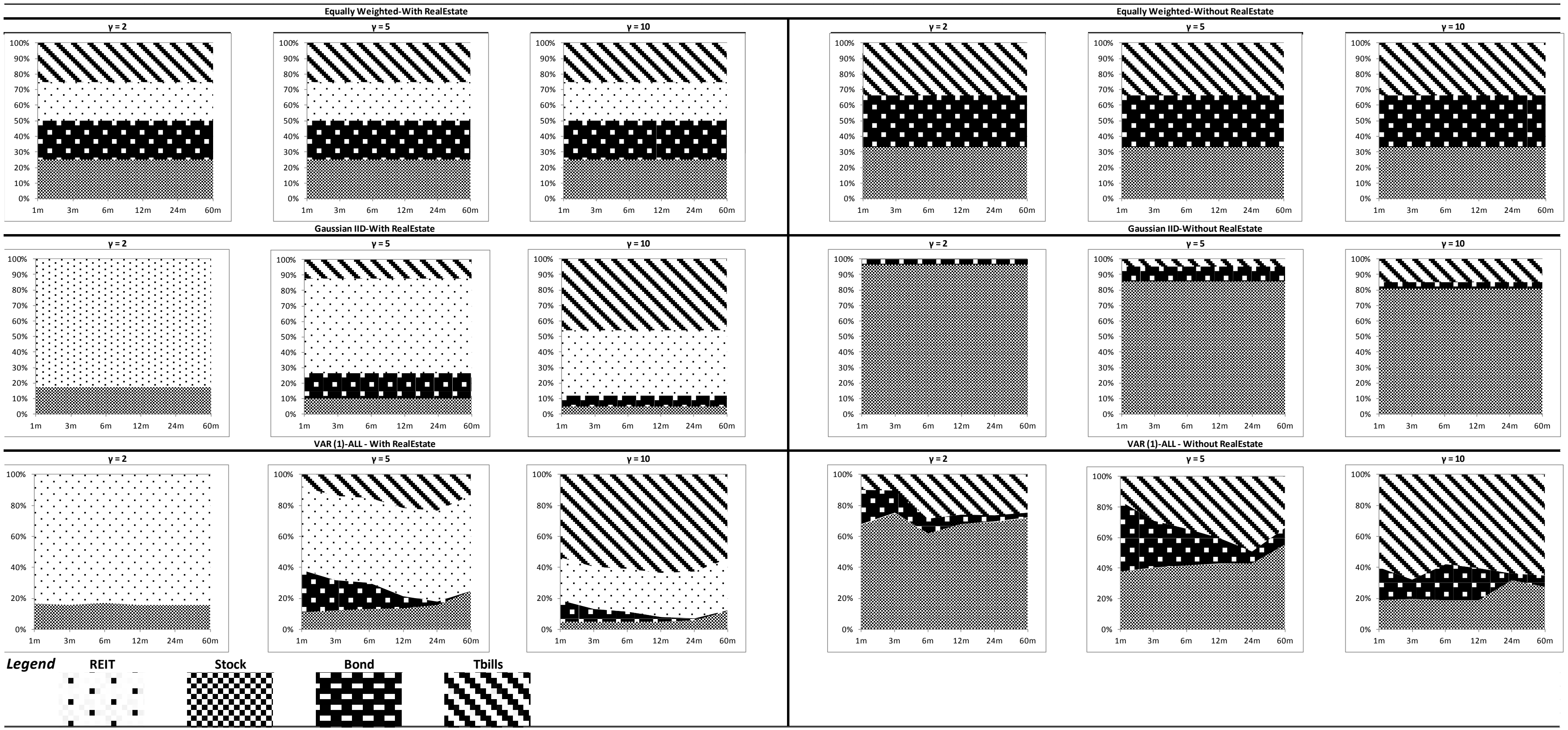


Figure 2

Bayesian Optimal Portfolio Weights

This graph reports mean Bayesian optimal portfolio weights for stocks, bonds and cash, for three alternative models of asset returns: Equally weighted, Gaussian IID, VAR with all predictors (VAR-ALL), under the two alternative cases in which real estate is or is not in the asset menu. The investment horizon varies from 1 (first two rows) to 60 months (last two rows). Constant relative risk aversion is set to respectively 2, 5 and 10. For each horizon, means of portfolio weights are computed over 120 monthly portfolio allocations on the recursive sample 1995-2007.

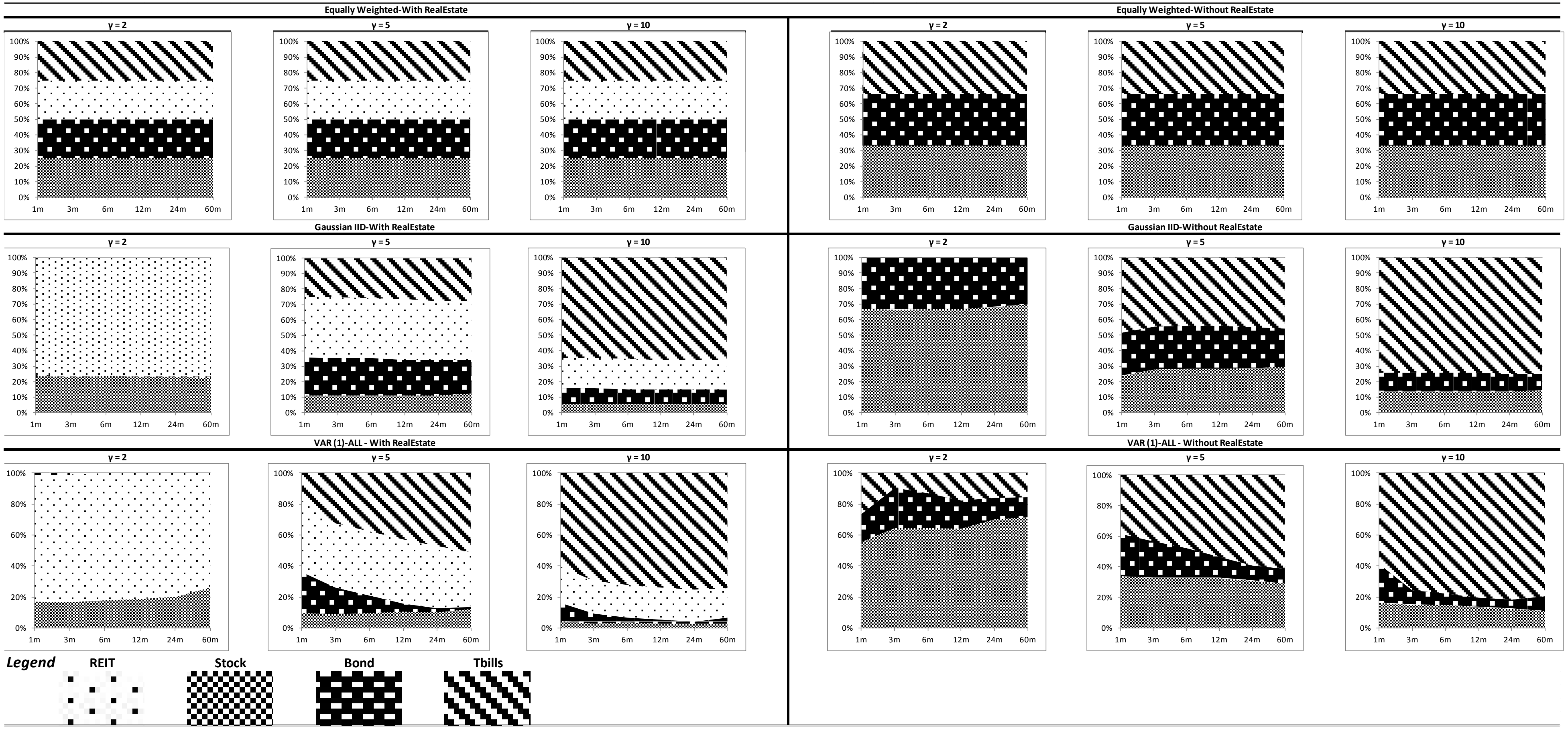
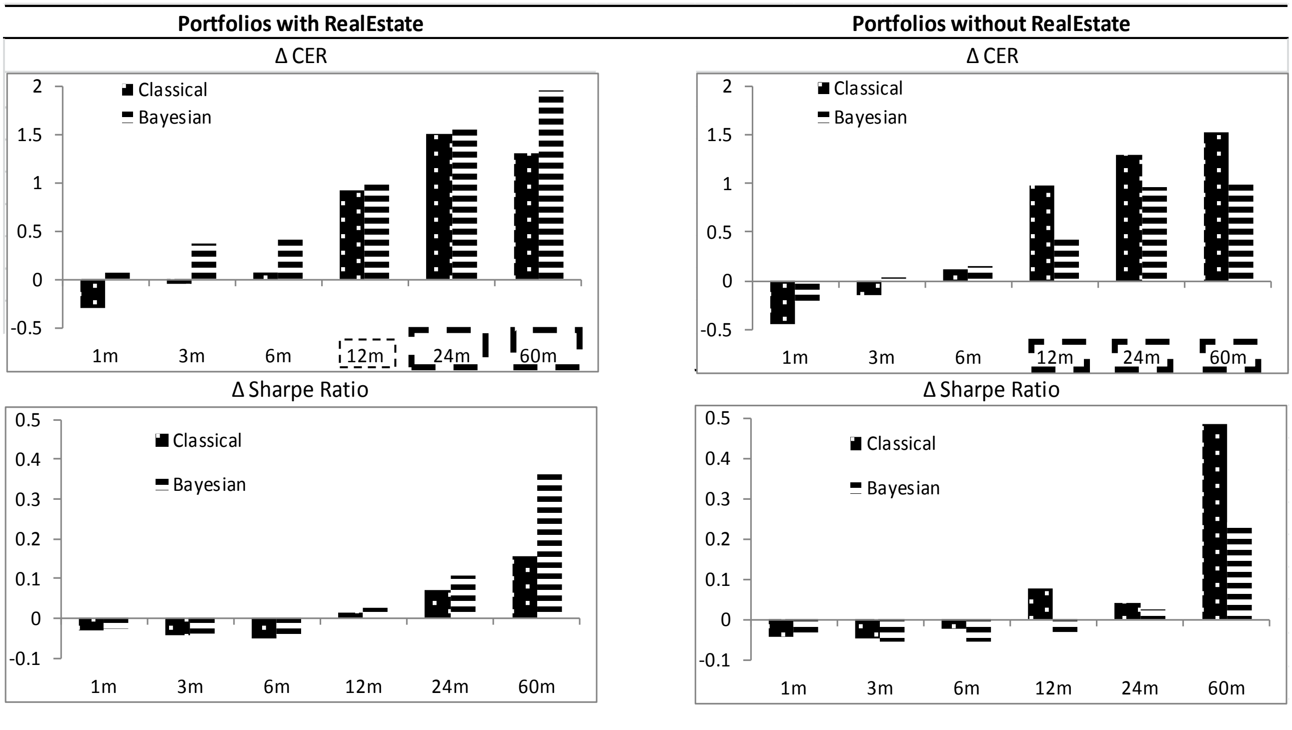


Figure 3

Comparing CERs and Sharpe Ratios of the Equally Weighted Portfolio and VAR-ALL

These graphs report the differences in annualized realized certainty equivalent returns (ΔCER) and Sharpe ratios (ΔSR) between the VAR-ALL and the 1/*N* strategy, for different risk aversion (γ = 5,10) and investment horizons. *Negative* (*Positive*) values imply that *1/N* (*VAR-ALL*) outperforms. Rectangles below the bar charts indicate that the null hypothesis of equal realized performance is rejected for a Classical (solid) or a Bayesian investor (dashed), or for both (dashed bold).

$\gamma = 5$



$\gamma = 10$

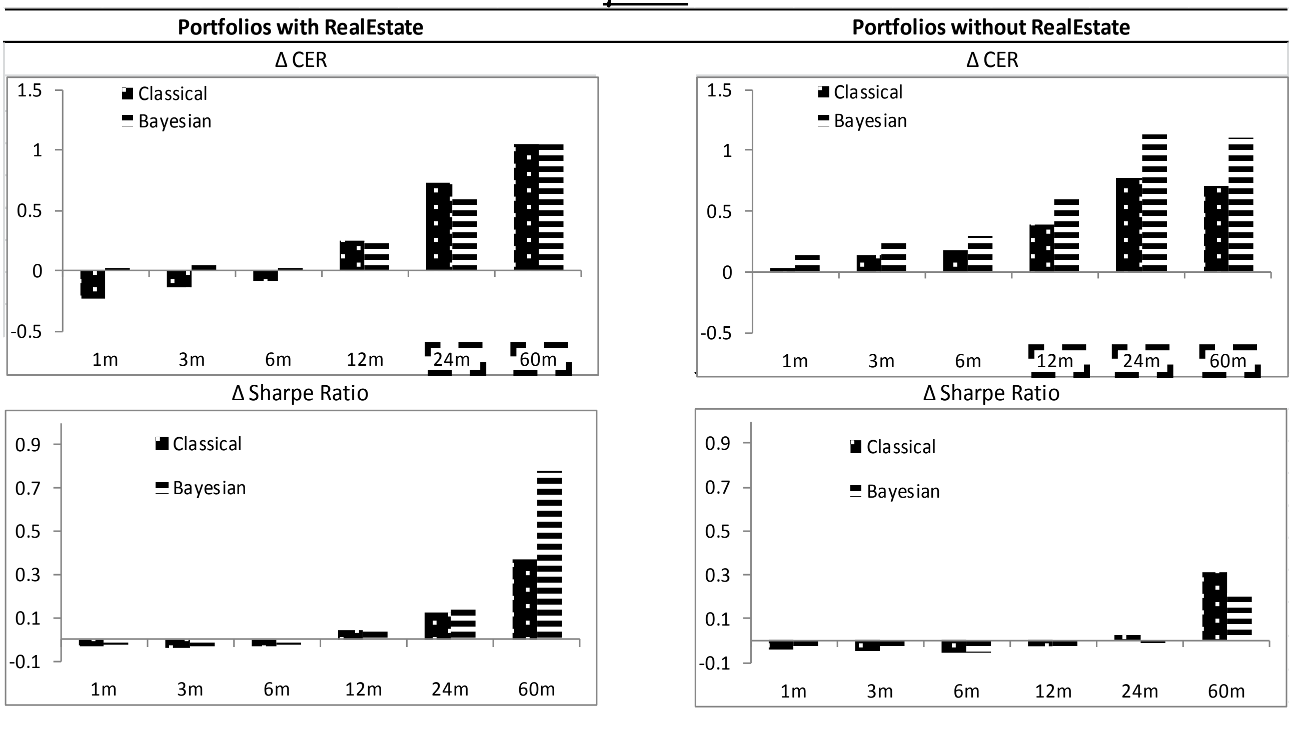
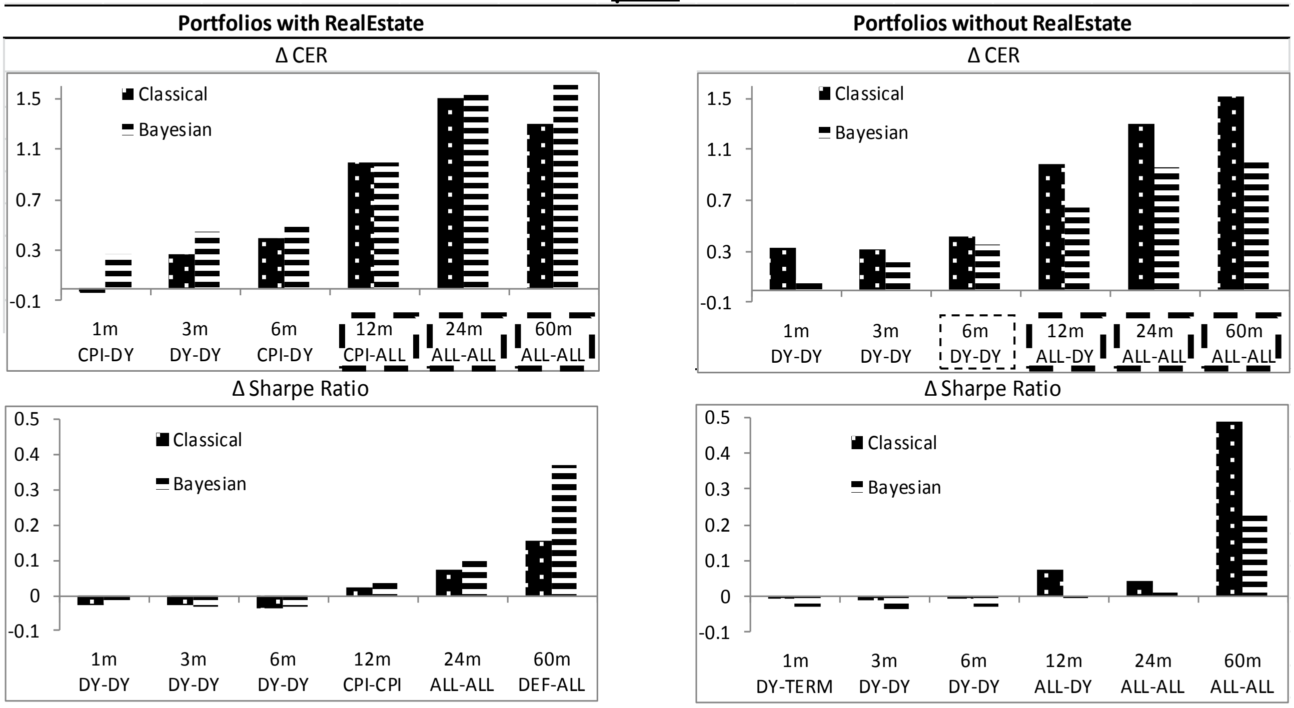


Figure 4

Comparing Performance of the Equally Weighted Portfolio and the Best Optimizing VAR Strategy

These graphs report the differences in annualized realized certainty equivalent returns (ΔCER) and Sharpe ratios (ΔSR) between the best optimizing VAR strategy and the *1/N* portfolio, for different risk aversion (γ = 5,10) and investment horizons. *Negative* (*Positive*) values imply that *1/N* (best optimizing *VAR*) outperforms. Rectangles below the bar charts indicate that the null hypothesis of equal realized performance is rejected for a Classical (solid) or a Bayesian investor (dashed), or for both (dashed bold).

$\gamma = 5$



$\gamma = 10$

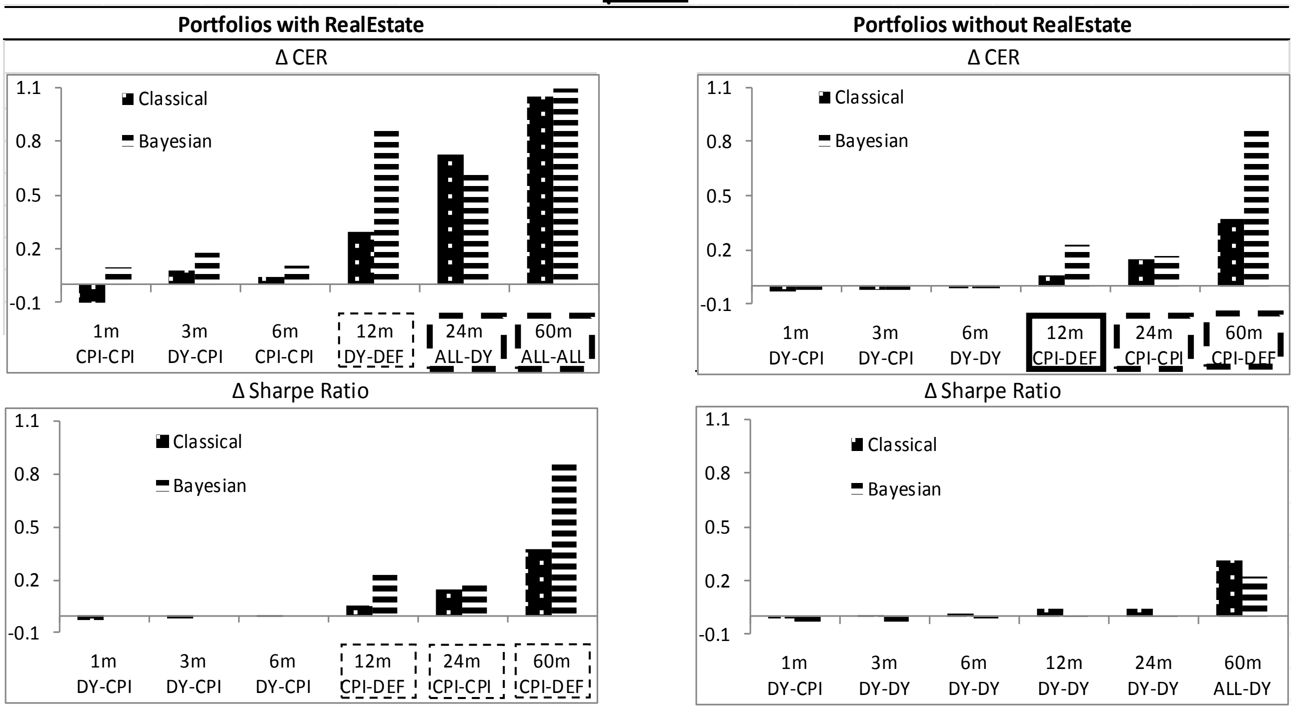


Figure 5
Comparing Performance of the Naive Equally Weighted Portfolio and VAR-ALL Strategy with Transaction Costs

These graphs report the differences in annualized realized certainty equivalent returns (ΔCER) and Sharpe ratios (ΔSR) between the VAR-ALL and the *1/N* strategy, for different risk aversion (γ = 5,10) and investment horizons. *Negative* (*Positive*) values imply that *1/N* (*VAR-ALL*) outperforms. Rectangles below the bar charts indicate that the null hypothesis of equal realized performance is rejected for a Classical (solid) or a Bayesian investor (dashed), or for both (dashed bold). Each trade gives rise to proportional cost of 50 basis points.

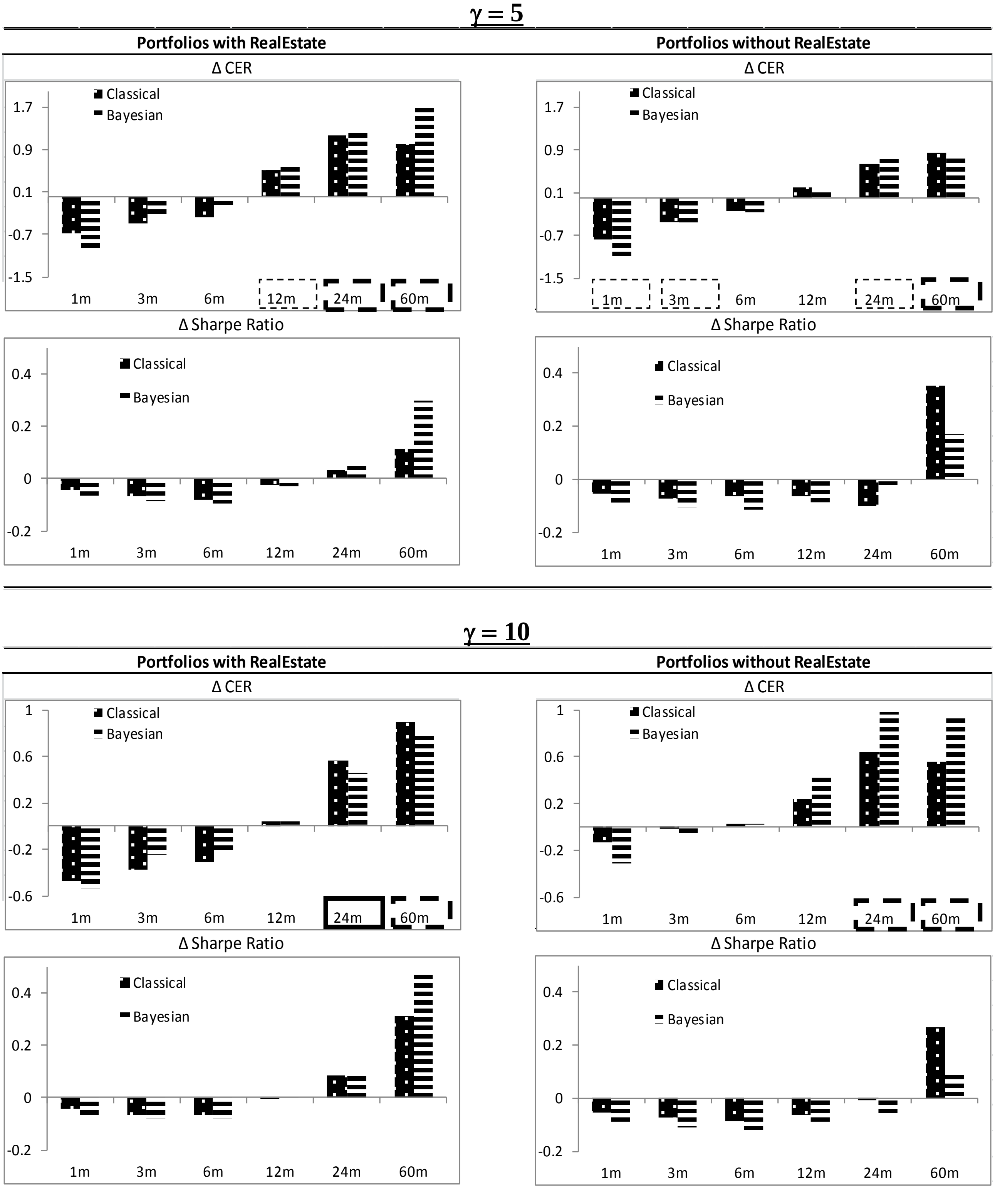
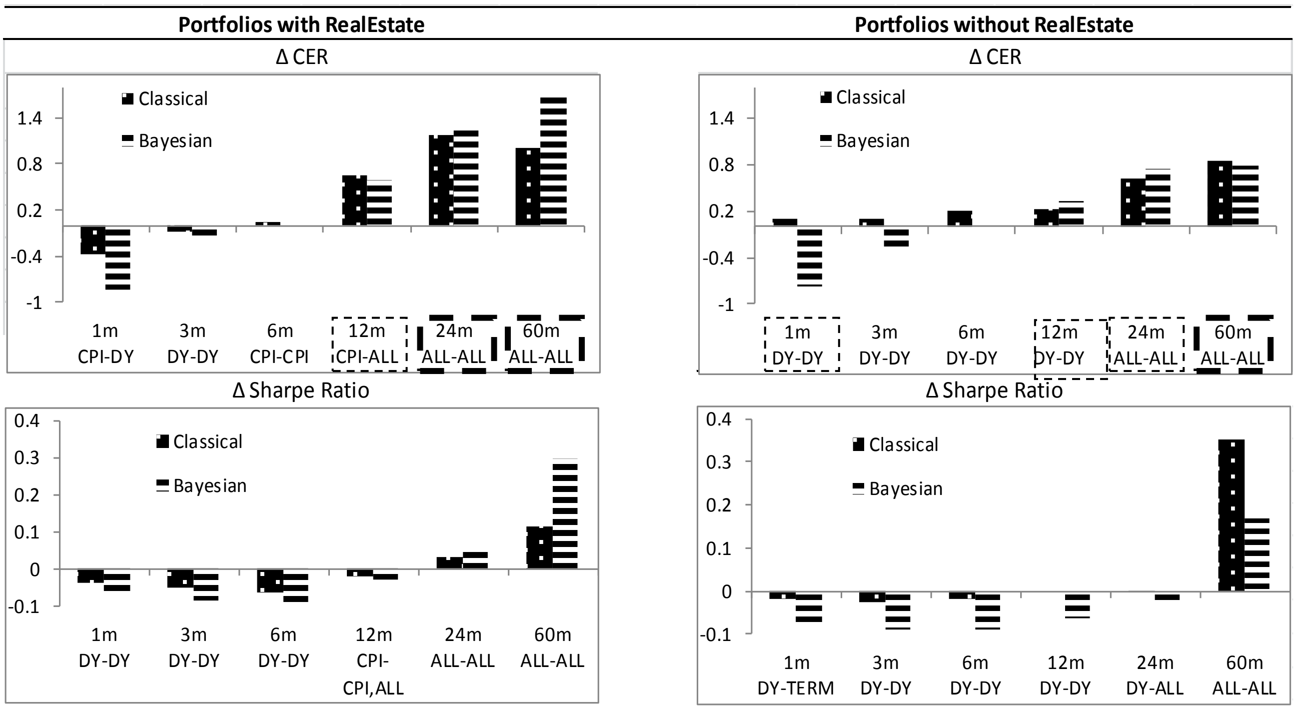


Figure 6

Comparing Performance of the Equally Weighted Portfolio and the Best Optimizing VAR Strategy with Transaction Cost

These graphs report the differences in annualized realized certainty equivalent returns (ΔCER) and Sharpe ratios (ΔSR) between the best optimizing VAR and the *1/N* strategy, for different risk aversion ($\gamma = 5,10$) and investment horizons. *Negative* (*Positive*) values imply that *1/N* (the best optimizing *VAR*) outperforms. Rectangles below the bar charts indicate that the null hypothesis of equal realized performance is rejected for a Classical (solid) or a Bayesian investor (dashed), or for both (dashed bold). Each trade gives rise to proportional cost of 50 basis points.

$\gamma = 5$



$\gamma = 10$