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# NLO Higgs effective field theory and $\kappa$ -framework<sup>1</sup>

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**ABSTRACT:** A consistent framework for studying Standard Model deviations is developed. It assumes that New Physics becomes relevant at some scale beyond the present experimental reach and uses the Effective Field Theory approach by adding higher-dimensional operators to the Standard Model Lagrangian and by computing relevant processes at the next-to-leading order, extending the original  $\kappa$ -framework. The generalized  $\kappa$ -framework provides a useful technical tool to decompose amplitudes at NLO accuracy into a sum of well defined gauge-invariant sub components.

**KEYWORDS:** Higgs Physics, Beyond Standard Model, Effective field theories

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## 1 Introduction

During Run-1 LHC has discovered a resonance which is a candidate for the Higgs boson of the Standard Model (SM) [1, 2]. The spin-0 nature of the resonance is well established [3] but there is no direct evidence for New Physics; furthermore, the available studies on the couplings of the resonance show compatibility with the Higgs boson of the SM. One possible scenario, in preparation for the results of Run-2, requires a consistent theory of SM deviations. Ongoing and near future experiments can achieve an estimated per mille sensitivity on precision Higgs and electroweak

(EW) observables. This level of precision provides a window to indirectly explore the theory space of Beyond-the-SM (BSM) physics and place constraints on specific UV models. For this purpose, a consistent procedure of constructing SM deviations is clearly desirable.

The first attempt to build a framework for SM-deviations is represented by the so-called  $\kappa$ -framework, introduced in refs. [4, 5]. There is no need to repeat here the main argument, splitting and shifting different loop contributions in the amplitudes for Higgs-mediated processes. The  $\kappa$ -framework is an intuitive language which misses internal consistency when one moves beyond leading order (LO). As originally formulated, it violates gauge-invariance and unitarity. In a Quantum Field Theory (QFT) approach to a spontaneously broken theory, fermion masses and Yukawa couplings are deeply related and one cannot shift couplings while keeping masses fixed.

To be more specific the original framework has the following limitations: kinematics is not affected by  $\kappa$ -parameters, therefore the framework works at the level of total cross-sections, not for differential distributions; it is LO, partially accommodating factorizable QCD but not EW corrections; it is not QFT-compatible (ad-hoc variation of the SM parameters, violates gauge symmetry and unitarity).

However, the original  $\kappa$ -framework has one main virtue, to represent the first attempt towards a fully consistent QFT of SM deviations. The question is: can we make it fully consistent? The answer is evidently yes, although the construction of a consistent theory of SM deviations (beyond LO) is far from trivial, especially from the technical point of view.

Recent years have witnessed an increasing interest in Higgs/SM EFT, see in particular refs. [6–8], refs. [9–15], refs. [16, 17], ref. [18], ref. [19], ref. [20], refs. [21, 22], ref. [23], refs. [24–26] and refs. [27–32].

In this work we will reestablish that Effective Field Theory (EFT) can provide an adequate answer beyond LO. Furthermore, EFT represents the optimal approach towards Model Independence. Of course, there is no formulation that is completely model independent and EFT, as any other approach, is based on a given set of (well defined) assumptions. Working within this set we will show how to use EFT for building a framework for SM deviations, generalizing the work of ref. [33]. A short version of our results, containing simple examples, was given in ref. [34] and presented in [35, 36].

In full generality we can distinguish a top-down approach (model dependent) and a bottom-up approach. The top-down approach is based on several steps. First one has to classify BSM models, possibly respecting custodial symmetry and decoupling, then the corresponding EFT can be constructed, e.g. via a covariant derivative expansion [37]. Once the EFT is derived one can construct (model by model) the corresponding SM deviations.

The bottom-up approach starts with the inclusion of a basis of  $\dim = 6$  operators and proceeds directly to the classification of SM deviations, possibly respecting the analytic structure of the SM amplitudes.

The Higgs EFT described and constructed in this work is based on several assumptions. We consider one Higgs doublet with a linear representation; this is flexible. We assume that there are no new “light” d.o.f. and decoupling of heavy d.o.f.; these are rigid assumptions. Absence of mass mixing of new heavy scalars with the SM Higgs doublet is also required.

We only work with  $\dim = 6$  operators. Therefore the scale  $\Lambda$  that characterizes the EFT cannot be too small, otherwise neglecting  $\dim = 8$  operators is not allowed. Furthermore,  $\Lambda$  cannot be too

large, otherwise  $\text{dim} = 4$  higher-order loops are more important than  $\text{dim} = 6$  interference effects. It is worth noting that these statements do not imply an inconsistency of EFT. It only means that higher dimensional operators and/or higher order EW effects (e.g. ref. [38]) must be included as well.

To summarize the strategy that will be described in this work we identify the following steps: start with EFT at a given order (here  $\text{dim} = 6$  and NLO) and write any amplitude as a sum of  $\kappa$ -deformed SM sub-amplitudes (e.g.  $t, b$  and bosonic loops in  $H \rightarrow \gamma\gamma$ ). Another sum of  $\kappa$ -deformed non-SM amplitudes is needed to complete the answer; at this point we can show that the  $\kappa$ -parameters are linear combinations of Wilson coefficients.

The rationale for this course of action is better understood in terms of a comparison between LEP and LHC. Physics is symmetry plus dynamics and symmetry is quintessential (gauge invariance etc.); however, symmetry without dynamics does not bring us this far. At LEP dynamics was the SM, unknowns were  $M_H(\alpha_s(M_Z), \dots)$ ; at LHC (post the discovery) unknowns are SM-deviations, dynamics? Specific BSM models are a choice but one would like to try also a model-independent approach. Instead of inventing unknown form factors we propose a decomposition where dynamics is controlled by  $\text{dim} = 4$  amplitudes (with known analytical properties) and deviations (with a direct link to UV completion) are (constant) combinations of Wilson coefficients.

Our extended  $\kappa$ -framework is a novel technical tool for studying SM deviations; what should be extracted from the data is another story. There are many alternatives, starting from direct extraction of Wilson coefficients or combinations of Wilson coefficients. We do not claim any particular advantage in selecting generalized  $\kappa$ -parameters as LHC observables. Only the comparison with experimental data will allow us to judge the goodness of a proposal. Our belief is based on the fact that SM deviations need a SM basis.

On-shell studies at LHC will tell us a lot, off-shell ones will tell us (hopefully) much more [39–43]. If we run away from the H peak with a SM-deformed theory, up to some reasonable value  $s \ll \Lambda^2$ , we need to reproduce (deformed) SM low-energy effects, e.g. VV and  $tt$  thresholds. The BSM loops will remain unresolved (as SM loops are unresolved in the Fermi theory). That is why we need to expand the SM-deformations into a SM basis with the correct (low energy) behavior. If we stay in the neighbourhood of the peak any function will work, if we run away we have to know more of the analytical properties.

The outline of the paper is as follows: in section 2 we introduce the EFT Lagrangian. In section 3 we describe the various aspects of the calculation; in section 4 we present details of the renormalization procedure, decays of the Higgs boson are described in section 5, EW precision data in section 6. Technical details, as well as the complete list of counterterms and amplitudes are given in several appendices.

## 2 The Lagrangian

In this section we collect all definitions that are needed to write the Lagrangian defined by

$$\mathcal{L}_{\text{EFT}} = \mathcal{L}_4 + \sum_{n>4} \sum_{i=1}^{N_n} \frac{a_i^n}{\Lambda^{n-4}} \mathcal{O}_i^{(d=n)}, \tag{2.1}$$

where  $\mathcal{L}_4$  is the SM Lagrangian [44] and  $a_i^n$  are arbitrary Wilson coefficients. Our EFT is defined by eq. (2.1) and it is based on a number of assumptions: there is only one Higgs doublet (flexible), a linear realization is used (flexible), there are no new “light” d.o.f. and decoupling is assumed (rigid), the UV completion is weakly-coupled and renormalizable (flexible). Furthermore, neglecting  $\dim = 8$  operators and NNLO EW corrections implies the following range of applicability:  $3 \text{ TeV} < \Lambda < 5 \text{ TeV}$ .

We can anticipate the strategy by saying that we are at the border of two HEP phases. A “predictive” phase: in any (strictly) renormalizable theory with  $n$  parameters one needs to match  $n$  data points, the  $(n+1)$ th calculation is a prediction, e.g. as doable in the SM. A “fitting” (approximate predictive) phase: there are  $(N_6+N_8+\dots=\infty)$  renormalized Wilson coefficients that have to be fitted, e.g. measuring SM deformations due to a single  $\mathcal{O}^{(6)}$  insertion ( $N_6$  is enough for per mille accuracy).

## 2.1 Conventions

We begin by considering the field-content of the Lagrangian. The scalar field  $\Phi$  (with hypercharge 1/2) is defined by

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} H + 2\frac{M}{g} + i\phi^0 \\ \sqrt{2}i\phi^- \end{pmatrix} \quad (2.2)$$

$H$  is the custodial singlet in  $(2_L \otimes 2_R) = 1 \oplus 3$ . Charge conjugation gives  $\Phi_i^c = \varepsilon_{ij}\Phi_j^*$ , or

$$\Phi^c = -\frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}i\phi^+ \\ H + 2\frac{M}{g} - i\phi^0 \end{pmatrix} \quad (2.3)$$

The covariant derivative  $D_\mu$  is

$$D_\mu \Phi = \left( \partial_\mu - \frac{i}{2} g_0 B_\mu^a \tau_a - \frac{i}{2} g g_1 B_\mu^0 \right) \Phi, \quad (2.4)$$

with  $g_1 = -s_\theta/c_\theta$  and where  $\tau^a$  are Pauli matrices while  $s_\theta(c_\theta)$  is the sine(cosine) of the weak-mixing angle. Furthermore

$$\mathbf{W}_\mu^\pm = \frac{1}{\sqrt{2}} (B_\mu^1 \mp iB_\mu^2), \quad \mathbf{Z}_\mu = c_\theta B_\mu^3 - s_\theta B_\mu^0, \quad \mathbf{A}_\mu = s_\theta B_\mu^3 + c_\theta B_\mu^0, \quad (2.5)$$

$$F_{\mu\nu}^a = \partial_\mu B_\nu^a - \partial_\nu B_\mu^a + g_0 \varepsilon^{abc} B_\mu^b B_\nu^c, \quad F_{\mu\nu}^0 = \partial_\mu B_\nu^0 - \partial_\nu B_\mu^0. \quad (2.6)$$

Here  $a, b, \dots = 1, \dots, 3$ . Furthermore, for the QCD part we introduce

$$G_{\mu\nu}^a = \partial_\mu g_\nu^a - \partial_\nu g_\mu^a + g_S f^{abc} g_\mu^b g_\nu^c. \quad (2.7)$$

Here  $a, b, \dots = 1, \dots, 8$  and the  $f$  are the  $SU(3)$  structure constants. Finally, we introduce fermions,

$$\Psi_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad f_{L,R} = \frac{1}{2} (1 \pm \gamma^5) f \quad (2.8)$$

and their covariant derivatives

$$D_\mu \Psi_L = (\partial_\mu + g B_\mu^i T_i) \Psi_L, \quad i = 0, \dots, 3$$

$$T^a = -\frac{i}{2} \tau^a, \quad T^0 = -\frac{i}{2} g_2 I, \quad (2.9)$$

$$D_\mu \Psi_R = (\partial_\mu + g B_\mu^i t_i) \Psi_R, \quad t^a = 0 \quad (a \neq 0), \quad (2.10)$$

$$t^0 = -\frac{i}{2} \begin{pmatrix} g_3 & 0 \\ 0 & g_4 \end{pmatrix} \quad (2.11)$$

with  $g_i = -s_\theta/c_\theta \lambda_i$  and

$$\lambda_2 = 1 - 2Q_u, \quad \lambda_3 = -2Q_u, \quad \lambda_4 = -2Q_d. \quad (2.12)$$

The Standard Model Lagrangian is the sum of several terms:

$$\mathcal{L}_{\text{SM}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_\Phi + \mathcal{L}_{\text{gf}} + \mathcal{L}_{\text{FP}} + \mathcal{L}_f \quad (2.13)$$

i.e., Yang-Mills, scalar, gauge-fixing, Faddeev-Popov ghosts and fermions. Furthermore, for a proper treatment of the neutral sector of the SM, we express  $g_0$  in terms of the coupling constant  $g$ ,

$$g_0 = g (1 + g^2 \Gamma), \quad (2.14)$$

where  $\Gamma$  is fixed by the request that the  $Z - A$  transition is zero at  $p^2 = 0$ , see ref. [45]. The scalar Lagrangian is given by

$$\mathcal{L}_\Phi = - (D_\mu \Phi)^\dagger D_\mu \Phi - \mu^2 \Phi^\dagger \Phi - \frac{1}{2} \lambda (\Phi^\dagger \Phi)^2. \quad (2.15)$$

We will work in the  $\beta_h$ -scheme of ref. [45], where parameters are transformed according to the following equations:

$$\mu^2 = \beta_h - 2 \frac{\lambda}{g^2} M^2, \quad \lambda = \frac{1}{4} g^2 \frac{M_H^2}{M^2}. \quad (2.16)$$

Furthermore, we introduce the Higgs VEV,  $v = \sqrt{2}M/g$ , and fix  $\beta_h$  order-by-order in perturbation theory by requiring  $\langle 0 | H | 0 \rangle = 0$ . Here we follow the approach described in refs. [45, 46].

## 2.2 dim = 6 operators

Our list of  $d = 6$  operators is based on the work of refs. [47–50] and of refs. [51–54] (see also refs. [55, 56], ref. [57], refs. [58–62], refs. [63–65] and ref. [66]) and is given in table 1. We are not reporting the full set of dim = 6 operators introduced in ref. [48] but only those that are relevant for our calculations, e.g. CP-odd operators have not been considered in this work. It is worth noting that we do not assume flavor universality.

We need matching of UV models onto EFT, order-by-order in a loop expansion. If  $L = \{\mathcal{O}_1^{(d)}, \dots, \mathcal{O}_n^{(d)}\}$  is a list of operators in  $V^{(d)}$  (the space of  $d$ -dimensional, gauge invariant operators), then these operators form a basis for  $V^{(d)}$  iff every  $\mathcal{O}^{(d)} \in V^{(d)}$  can be uniquely written as a linear combination of the elements in  $L$ .

While overcomplete sets (e.g. those derived without using equations of motion) are useful for cross-checking, a set that is not a basis (discarding a priori subsets of operators) is questionable,

$\mathcal{O}_1 = g^3 \mathcal{O}_\Phi = g^3 (\Phi^\dagger \Phi)^3$	$\mathcal{O}_2 = g^2 \mathcal{O}_{\Phi\Box} = g^2 (\Phi^\dagger \Phi) \Box (\Phi^\dagger \Phi)$
$\mathcal{O}_3 = g^2 \mathcal{O}_{\Phi D} = g^2 (\Phi^\dagger D_\mu \Phi) \left[ (D_\mu \Phi)^\dagger \Phi \right]$	$\mathcal{O}_4 = g^2 \mathcal{O}_{1\Phi} = g^2 (\Phi^\dagger \Phi) \bar{L}_L \Phi^c l_R$
$\mathcal{O}_5 = g^2 \mathcal{O}_{u\Phi} = g^2 (\Phi^\dagger \Phi) \bar{q}_L \Phi u_R$	$\mathcal{O}_6 = g^2 \mathcal{O}_{d\Phi} = g^2 (\Phi^\dagger \Phi) \bar{q}_L \Phi^c d_R$
$\mathcal{O}_7 = g^2 \mathcal{O}_{\Phi 1}^{(1)} = g^2 \Phi^\dagger D_\mu^{(\leftrightarrow)} \Phi \bar{L}_L \gamma^\mu L_L$	$\mathcal{O}_8 = g^2 \mathcal{O}_{\Phi q}^{(1)} = g^2 \Phi^\dagger D_\mu^{(\leftrightarrow)} \Phi \bar{q}_L \gamma^\mu q_L$
$\mathcal{O}_9 = g^2 \mathcal{O}_{\Phi l} = g^2 \Phi^\dagger D_\mu^{(\leftrightarrow)} \Phi \bar{l}_R \gamma^\mu l_R$	$\mathcal{O}_{10} = g^2 \mathcal{O}_{\Phi u} = g^2 \Phi^\dagger D_\mu^{(\leftrightarrow)} \Phi \bar{u}_R \gamma^\mu u_R$
$\mathcal{O}_{11} = g^2 \mathcal{O}_{\Phi d} = g^2 \Phi^\dagger D_\mu^{(\leftrightarrow)} \Phi \bar{d}_R \gamma^\mu d_R$	$\mathcal{O}_{12} = g^2 \mathcal{O}_{\Phi ud} = i g^2 (\Phi^\dagger D_\mu \Phi) \bar{u}_R \gamma^\mu d_R$
$\mathcal{O}_{13} = g^2 \mathcal{O}_{\Phi 1}^{(3)} = g^2 \Phi^\dagger \tau^a D_\mu^{(\leftrightarrow)} \Phi \bar{L}_L \tau_a \gamma^\mu L_L$	$\mathcal{O}_{14} = g^2 \mathcal{O}_{\Phi q}^{(3)} = g^2 \Phi^\dagger \tau^a D_\mu^{(\leftrightarrow)} \Phi \bar{q}_L \tau_a \gamma^\mu q_L$
$\mathcal{O}_{15} = g \mathcal{O}_{\Phi G} = g (\Phi^\dagger \Phi) G^{a\mu\nu} G_{\mu\nu}^a$	$\mathcal{O}_{16} = g \mathcal{O}_{\Phi W} = g (\Phi^\dagger \Phi) F^{a\mu\nu} F_{\mu\nu}^a$
$\mathcal{O}_{17} = g \mathcal{O}_{\Phi B} = g (\Phi^\dagger \Phi) F^{0\mu\nu} F_{\mu\nu}^0$	$\mathcal{O}_{18} = g \mathcal{O}_{\Phi WB} = g \Phi^\dagger \tau^a \Phi F_{\mu\nu}^{a\nu} F_{\mu\nu}^0$
$\mathcal{O}_{19} = g \mathcal{O}_{1W} = g \bar{L}_L \sigma^{\mu\nu} l_R \tau_a \Phi^c F_{\mu\nu}^a$	$\mathcal{O}_{20} = g \mathcal{O}_{uW} = g \bar{q}_L \sigma^{\mu\nu} u_R \tau_a \Phi F_{\mu\nu}^a$
$\mathcal{O}_{21} = g \mathcal{O}_{dW} = g \bar{q}_L \sigma^{\mu\nu} d_R \tau_a \Phi^c F_{\mu\nu}^a$	$\mathcal{O}_{22} = g \mathcal{O}_{1B} = g \bar{L}_L \sigma^{\mu\nu} l_R \Phi^c F_{\mu\nu}^0$
$\mathcal{O}_{23} = g \mathcal{O}_{uB} = g \bar{q}_L \sigma^{\mu\nu} u_R \Phi F_{\mu\nu}^0$	$\mathcal{O}_{24} = g \mathcal{O}_{dB} = g \bar{q}_L \sigma^{\mu\nu} d_R \Phi^c F_{\mu\nu}^0$
$\mathcal{O}_{25} = g \mathcal{O}_{uG} = g \bar{q}_L \sigma^{\mu\nu} u_R \lambda_a \Phi G_{\mu\nu}^a$	$\mathcal{O}_{26} = g \mathcal{O}_{dG} = g \bar{q}_L \sigma^{\mu\nu} d_R \lambda_a \Phi^c G_{\mu\nu}^a$

**Table 1.** List of dim = 6 operators, see ref. [48], entering the renormalization procedure and the phenomenological applications described in this paper.

e.g. it is not closed under complete renormalization and may lead to violation of Ward-Slavnov-Taylor (WST) identities [67–69]. Finally, a basis is optimal insofar as it allows to write Feynman rules in arbitrary gauges. Our choice is given by

$$\mathcal{L}_{\text{EFT}_6} = \mathcal{L}_{\text{SM}} + \sum_i \frac{a_i}{\Lambda^2} \mathcal{O}_i^{(6)}. \quad (2.17)$$

In table 1 we drop the superscript (6) and write the explicit correspondence with the operators of the so-called Warsaw basis, see ref. [48]. We also introduce

$$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L, \quad L_L = \begin{pmatrix} \nu_l \\ l \end{pmatrix}_L \quad (2.18)$$

where  $u$  stands for a generic up-quark ( $\{u, c, \dots\}$ ),  $d$  stands for a generic down-quark ( $\{d, s, \dots\}$ ) and  $l$  for  $\{e, \mu, \dots\}$ . As usual,  $f_{L,R} = \frac{1}{2} (1 \pm \gamma^5) f$ . Furthermore,

$$\Phi^\dagger D_\mu^{(\leftrightarrow)} \Phi = \Phi^\dagger D_\mu \Phi - (D_\mu \Phi)^\dagger \Phi. \quad (2.19)$$

We also transform Wilson coefficients according to table 2. As was pointed out in table C.1 of ref. [70] the operators can be classified as potentially-tree-generated (PTG) and loop-generated (LG). If we assume that the high-energy theory is weakly-coupled and renormalizable it follows that the PTG/LG classification of ref. [70] (used here) is correct. If we do not assume the above but work always in some EFT context (i.e. also the next high-energy theory is EFT, possibly involving some strongly interacting theory) then classification changes, see eqs. (A1-A2) of ref. [15].



$g a_1 = a_\phi$	$g^2 a_2 = -a_{\phi\Box}$	$g^2 a_3 = -a_{\phi D}$	$g\sqrt{2} a_4 = -\frac{M_1}{M} a_{L\phi}$
$g\sqrt{2} a_5 = -\frac{M_u}{M} a_{u\phi}$	$g\sqrt{2} a_6 = -\frac{M_d}{M} a_{d\phi}$	$g^2 a_7 = -a_{\phi 1}^{(1)}$	$g^2 a_8 = -a_{\phi q}^{(1)}$
$g^2 a_9 = -a_{\phi 1}$	$g^2 a_{10} = -a_{\phi u}$	$g^2 a_{11} = -a_{\phi d}$	$g^2 a_{12} = -a_{\phi ud}$
$g^2 a_{13} = -a_{\phi 1}^{(3)}$	$g^2 a_{14} = -a_{\phi q}^{(3)}$	$g^2 a_{15} = g_S a_{\phi G}$	$g a_{16} = a_{\phi W}$
$g a_{17} = a_{\phi B}$	$g a_{18} = a_{\phi WB}$	$g\sqrt{2} a_{19} = \frac{M_1}{M} a_{1W}$	$g\sqrt{2} a_{20} = \frac{M_u}{M} a_{uW}$
$g\sqrt{2} a_{21} = \frac{M_d}{M} a_{dW}$	$g\sqrt{2} a_{22} = \frac{M_1}{M} a_{1B}$	$g\sqrt{2} a_{23} = \frac{M_u}{M} a_{uB}$	$g\sqrt{2} a_{24} = \frac{M_d}{M} a_{dB}$
$g^2 a_{25} = g_S a_{uG}$	$g^2 a_{26} = g_S a_{dG}$		

**Table 2.** Redefinition of Wilson coefficients.

### 2.3 Four-fermion operators

For processes that involve external fermions and for the fermion self-energies we also need  $\text{dim} = 6$  four-fermion operators (see table 3 of ref. [48]). We show here one explicit example

$$\begin{aligned}
 V_{uudd} = & \frac{1}{4} \frac{g^2 g_6}{M^2} a_{qq}^{(1)} \gamma^\mu \gamma_+ \otimes \gamma_\mu \gamma_+ + \frac{1}{8} \frac{g^2 g_6}{M^2} a_{qd}^{(1)} \gamma^\mu \gamma_+ \otimes \gamma_\mu \gamma_- \\
 & + \frac{1}{8} \frac{g^2 g_6}{M^2} a_{qu}^{(1)} \gamma^\mu \gamma_- \otimes \gamma_\mu \gamma_+ + \frac{1}{8} \frac{g^2 g_6}{M^2} a_{ud}^{(1)} \gamma^\mu \gamma_- \otimes \gamma_\mu \gamma_- \\
 & + \frac{1}{16} \frac{g^2 g_6}{M^2} a_{quqd}^{(1)} \gamma_+ \otimes \gamma_+ + \frac{1}{16} \frac{g^2 g_6}{M^2} a_{quqd}^{(1)} \gamma_- \otimes \gamma_-, \quad (2.20)
 \end{aligned}$$

giving the uudd four-fermion vertex. Here  $\gamma_\pm = 1/2(1 \pm \gamma^5)$  and  $g_6$  is defined in eq. (2.28).

### 2.4 From the Lagrangian to the $S$ -matrix

There are several technical points that deserve a careful treatment when constructing  $S$ -matrix elements from the Lagrangian of eq. (2.17). We perform field and parameter redefinitions so that all kinetic and mass terms in the Lagrangian of eq. (2.17) have the canonical normalization. First we define

$$\beta_h = 12 \frac{M^4 a_\phi}{g^2 \Lambda^2} + \beta'_h, \quad \beta'_h = \left( 1 + dR_{\beta_h} \frac{M^2}{\Lambda^2} \right) \bar{\beta}_h \quad (2.21)$$

and  $\bar{\beta}_h$  is fixed, order-by-order, to have zero vacuum expectation value for the (properly normalized) Higgs field.

Particular care should be devoted in selecting the starting gauge-fixing Lagrangian. In order to reproduce the free SM Lagrangian (after redefinitions) we fix an arbitrary gauge, described by four  $\xi$  parameters,

$$\mathcal{L}_{\text{gf}} = -\mathcal{C}^+ \mathcal{C}^- - \frac{1}{2} \mathcal{C}_Z^2 - \frac{1}{2} \mathcal{C}_A^2, \quad (2.22)$$

$$\mathcal{C}^\pm = -\xi_W \partial_\mu W_\mu^\pm + \xi_\pm M \phi^\pm, \quad \mathcal{C}_Z = -\xi_Z \partial_\mu Z_\mu + \xi_0 \frac{M}{c_\theta} \phi^0, \quad \mathcal{C}_A = \xi_A \partial_\mu A_\mu. \quad (2.23)$$

$\Delta R_W = a_{\phi W}$	$\Delta R_{ZZ} = a_{ZZ}$	$\Delta R_{AA} = a_{AA}$
$\Delta R_H = -\frac{1}{4} a_{\phi D} + a_{\phi \square}$	$\Delta R_{\phi^\pm} = 0$	$\Delta R_{\phi^0} = -\frac{1}{4} a_{\phi D}$
$\Delta R_{X^\pm} = \frac{1}{2} a_{\phi W}$	$\Delta R_{Y_Z} = \frac{1}{2} a_{ZZ}$	$\Delta R_{Y_A} = \frac{1}{2} a_{AA}$
$\Delta R_u = -\frac{1}{2} a_{u\phi}$	$\Delta R_d = \frac{1}{2} a_{d\phi}$	$\Delta R_{\beta_h} = a_{\phi W} + \frac{1}{4} a_{\phi D} - a_{\phi \square}$
$\Delta R_{\xi_W} = -a_{\phi W}$	$\Delta R_{\xi_Z} = -a_{ZZ}$	$\Delta R_{\xi_A} = -a_{AA}$
$\Delta R_{\xi_\pm} = a_{\phi W}$	$\Delta R_{\xi_0} = \frac{1}{2} a_{\phi D} + a_{ZZ}$	$\Delta R_{M_H} = \frac{1}{2} a_{\phi D} - 2a_{\phi \square} + 12 \frac{\bar{M}_H^2}{\bar{M}_H^2} a_\phi$
$\Delta R_M = -2a_{\phi W}$	$\Delta R_{c_\theta} = -\frac{1}{4} a_{\phi D} + \bar{s}_\theta^2 (a_{AA} - a_{ZZ}) + \bar{s}_\theta \bar{c}_\theta a_{AZ}$	

**Table 3.** Normalization conditions.

The full list of redefinitions is given in the following equations, where we have introduced  $R_\Lambda = M^2/\Lambda^2$ . First the Lagrangian parameters,

$$M_H^2 = \bar{M}_H^2 \left(1 + dR_{M_H} R_\Lambda\right), \quad M^2 = \bar{M}^2 \left(1 + dR_M R_\Lambda\right), \quad M_f = \bar{M}^f \left(1 + dR_{M_f} R_\Lambda\right), \quad (2.24)$$

$$c_\theta = \left(1 + dR_{c_\theta} R_\Lambda\right) \bar{c}_\theta, \quad s_\theta = \left(1 + dR_{s_\theta} R_\Lambda\right) \bar{s}_\theta, \quad (2.25)$$

secondly, the fields:

$$\begin{aligned} H &= \left(1 + dR_H R_\Lambda\right) \bar{H} & \phi^0 &= \left(1 + dR_{\phi^0} R_\Lambda\right) \bar{\phi}^0 & \phi^\pm &= \left(1 + dR_{\phi^\pm} R_\Lambda\right) \bar{\phi}^\pm \\ Z_\mu &= \left(1 + dR_{ZZ} R_\Lambda\right) \bar{Z}_\mu & A_\mu &= \left(1 + dR_{AA} R_\Lambda\right) \bar{A}_\mu & W_\mu^\pm &= \left(1 + dR_{W^\pm} R_\Lambda\right) \bar{W}_\mu^\pm \\ X^\pm &= \left(1 + dR_{X^\pm} R_\Lambda\right) \bar{X}^\pm & Y_Z &= \left(1 + dR_{Y_Z} R_\Lambda\right) \bar{Y}_Z & Y_A &= \left(1 + dR_{Y_A} R_\Lambda\right) \bar{Y}_A \end{aligned} \quad (2.26)$$

where  $X^\pm, Y_Z$  and  $Y_A$  are FP ghosts. Finally, the gauge parameters, normalized to one:

$$\xi_i = 1 + dR_{\xi_i} R_\Lambda \quad i = A, Z, W, \pm, 0. \quad (2.27)$$

We introduce a new coupling constant

$$g_6 = \frac{1}{\sqrt{2} G_F \Lambda^2} = 0.0606 \left(\frac{\text{TeV}}{\Lambda}\right)^2, \quad (2.28)$$

where  $G_F$  is the Fermi coupling constant and derive the following solutions:

$$dR_i R_\Lambda = g_6 \Delta R_i, \quad (2.29)$$

where the  $\Delta R_i$  are given in table 3. One could also write a more general relation

$$Z_\mu = R_{ZZ} \bar{Z}^\mu + R_{ZA} \bar{A}^\mu, \quad A_\mu = R_{AZ} \bar{Z}^\mu + R_{AA} \bar{A}^\mu, \quad (2.30)$$

where non diagonal terms start at  $\mathcal{O}(g^2)$ . In this way we could also require cancellation of the Z–A transition at  $\mathcal{O}(g_6)$  but, in our experience, there is little to gain with this option. We have

introduced the following combinations of Wilson coefficients:

$$\begin{aligned}
 a_{ZZ} &= \bar{s}_\theta^2 a_{\phi_B} + \bar{c}_\theta^2 a_{\phi_W} - \bar{s}_\theta \bar{c}_\theta a_{\phi_{WB}}, \\
 a_{AA} &= \bar{c}_\theta^2 a_{\phi_B} + \bar{s}_\theta^2 a_{\phi_W} + \bar{s}_\theta \bar{c}_\theta a_{\phi_{WB}}, \\
 a_{AZ} &= 2\bar{c}_\theta \bar{s}_\theta (a_{\phi_W} - a_{\phi_B}) + (2\bar{c}_\theta^2 - 1) a_{\phi_{WB}}.
 \end{aligned}
 \tag{2.31}$$

With our choice of reparametrization the final result can be written as follows:

$$\mathcal{L}(\{\Phi\}, \{p\}) = \mathcal{L}_4(\{\bar{\Phi}\}, \{\bar{p}\}) + g_6 a_{AZ} \left( \partial_\mu \bar{Z}_\nu \partial^\mu \bar{A}^\nu - \partial_\mu \bar{Z}_\nu \partial^\nu \bar{A}^\mu \right) + \mathcal{L}_6^{\text{int}}(\{\bar{\Phi}\}, \{\bar{p}\}),
 \tag{2.32}$$

where  $\{\Phi\}$  denotes the collection of fields and  $\{p\}$  the collection of parameters. In the following we will abandon the  $\bar{\Phi}$ ,  $\bar{p}$  notation since no confusion can arise.

### 3 Overview of the calculation

NLO EFT (dim = 6) is constructed according to the following scheme: each amplitude, e.g.  $H \rightarrow |f\rangle$ , contains one-loop SM diagrams up to the relevant order in  $g$ , (tree) contact terms with one dim = 6 operator and one-loop diagrams with one dim = 6 operator insertion. Note that the latter contain also diagrams that do not have a counterpart in the SM (e.g. bubbles with 3 external lines). In full generality each amplitude is written as follows:

$$\mathcal{A} = \sum_{n=N}^{\infty} g^n \mathcal{A}_n^{(4)} + \sum_{n=N_6}^{\infty} \sum_{l=0}^n \sum_{k=1}^{\infty} g^n g_{4+2k}^l \mathcal{A}_{nlk}^{(4+2k)},
 \tag{3.1}$$

where  $g$  is the SU(2) coupling constant and  $g_{4+2k} = 1/(\sqrt{2} G_F \Lambda^2)^k$ . For each process the dim = 4 LO defines the value of  $N$  (e.g.  $N = 1$  for  $H \rightarrow VV$ ,  $N = 3$  for  $H \rightarrow \gamma\gamma$  etc.). Furthermore,  $N_6 = N$  for tree initiated processes and  $N - 2$  for loop initiated ones. The full amplitude is obtained by inserting wave-function factors and finite renormalization counterterms. Renormalization makes UV finite all relevant, on-shell, S-matrix elements. It is made in two steps: first we introduce counterterms

$$\Phi = Z_\Phi \Phi_{\text{ren}}, \quad p = Z_p p_{\text{ren}},
 \tag{3.2}$$

for fields and parameters. Counterterms are defined by

$$Z_i = 1 + \frac{g^2}{16\pi^2} \left( dZ_i^{(4)} + g_6 dZ_i^{(6)} \right).
 \tag{3.3}$$

We construct self-energies, Dyson resum them and require that all propagators are UV finite. In a second step we construct 3-point (or higher) functions, check their  $\mathcal{O}^{(4)}$ -finiteness and remove the remaining  $\mathcal{O}^{(6)}$  UV divergences by mixing the Wilson coefficients  $W_i$ :

$$W_i = \sum_j Z_{ij}^W W_j^{\text{ren}}.
 \tag{3.4}$$

Renormalized Wilson coefficients are scale dependent and the logarithm of the scale can be resummed in terms of the LO coefficients of the anomalous dimension matrix [11].

Our aim is to discuss Higgs couplings and their SM deviations which requires precise definitions [71–73]:

**Definition** *The Higgs couplings can be extracted from Green’s functions in well-defined kinematic limits, e.g. residue of the poles after extracting the parts which are 1P reducible. These are well-defined QFT objects, that we can probe both in production and in decays; from this perspective, VH production or vector-boson-fusion are on equal footing with gg fusion and Higgs decays. Therefore, the first step requires computing these residues which is the main result of this paper.*

Every approach designed for studying SM deviations at LHC and beyond has to face a critical question: generally speaking, at LHC the EW core is embedded into a QCD environment, subject to large perturbative corrections and we expect considerable progress in the “evolution” of these corrections; the same considerations apply to PDFs. Therefore, does it make sense to ‘fit’ the EW core? Note that this is a general question which is not confined to our NLO approach.

In practice, our procedure is to write the answer in terms of SM deviations, i.e. the dynamical parts are  $\dim = 4$  and certain combinations of the deviation parameters will define the pseudo-observables (PO) to be fitted. Optimally, part of the factorizing QCD corrections could enter the PO definition. The suggested procedure requires the parametrization to be as general as possible, i.e. no a priori dropping of terms in the basis of operators. This will allow us to “reweight” the results when new (differential) K-factors become available; new input will touch only the  $\dim = 4$  components. PDFs changing is the most serious problem: at LEP the  $e^+e^-$  structure functions were known to very high accuracy (the effect was tested by using different QED radiators, differing by higher orders treatment); a change of PDFs at LHC will change the convolution and make the reweighting less simple, but still possible. For recent progress on the impact of QCD corrections within the EFT approach we quote ref. [23].

## 4 Renormalization

There are several steps in the renormalization procedure. The orthodox approach to renormalization uses the language of “counterterms”. It is worth noting that this is not a mandatory step, since one could write directly renormalization equations that connect the bare parameters of the Lagrangian to a set of data, skipping the introduction of intermediate renormalized quantities and avoiding any unnecessary reference to a given renormalization scheme.

In this approach, carried on at one loop in [74], no special attention is paid to individual Green functions, and one is mainly concerned with UV finiteness of S-matrix elements after the proper treatment of external legs in amputated Green functions, which greatly reduces the complexity of the calculation.

However, renormalization equations are usually organized through different building blocks, where gauge-boson self-energies embed process-independent (universal) higher-order corrections and play a privileged role. Therefore, their structure has to be carefully analyzed, and the language of counterterms allows to disentangle UV overlapping divergences which show up at two loops.

In a renormalizable gauge theory, in fact, the UV poles of any Green function can be removed order-by-order in perturbation theory. In addition, the imaginary part of a Green function at a given order is fixed, through unitarity constraints, by the previous orders. Therefore, UV-subtraction terms have to be at most polynomials in the external momenta (in the following, “local” subtraction terms). Therefore, we will express our results using the language of counterterms: we promote bare

quantities (parameters and fields) to renormalized ones and fix the counterterms at one loop in order to remove the UV poles.

Obviously, the absorption of UV divergences into local counterterms does not exhaust the renormalization procedure, because we have still to connect renormalized quantities to experimental data points, thus making the theory predictive. In the remainder of this section we discuss renormalization constants for all parameters and fields. We introduce the following quantities

$$\Delta_{\text{UV}} = \frac{2}{\varepsilon} - \gamma_E - \ln \pi - \ln \frac{\mu_R^2}{\mu^2}, \quad \Delta_{\text{UV}}(x) = \frac{2}{\varepsilon} - \gamma_E - \ln \pi - \ln \frac{x}{\mu^2}, \quad (4.1)$$

where  $\varepsilon = 4 - d$ ,  $d$  is the space-time dimension,  $\gamma_E = 0.5772$  is the Euler - Mascheroni constant and  $\mu_R$  is the renormalization scale. In eq. (4.1) we have introduced an auxiliary mass  $\mu$  which cancels in any UV-renormalized quantity;  $\mu_R$  cancels only after finite renormalization. Furthermore,  $x$  is positive definite. Only few functions are needed for renormalization purposes,

$$A_0(m) = \frac{\mu^\varepsilon}{i\pi^2} \int d^d q \frac{1}{q^2 + m^2} = -m^2 \left[ \Delta_{\text{UV}}(M_W^2) + a_0^{\text{fin}}(m) \right], \quad a_0^{\text{fin}}(m) = 1 - \ln \frac{m^2}{M_W^2}, \quad (4.2)$$

$$B_0(-s; m_1, m_2) = \frac{\mu^\varepsilon}{i\pi^2} \int d^d q \frac{1}{(q^2 + m_1^2)((q+p)^2 + m_2^2)} = \Delta_{\text{UV}}(M_W^2) + B_0^{\text{fin}}(-s; m_1, m_2), \quad (4.3)$$

where the finite part is

$$B_0^{\text{fin}}(-s; m_1, m_2) = 2 - \ln \frac{m_1 m_2}{M_W^2} - R - \frac{1}{2} \frac{m_1^2 - m_2^2}{s} \ln \frac{m_1^2}{m_2^2}, \quad R = \frac{\Lambda}{s} \ln \frac{m_1^2 + m_2^2 - s - \Lambda - i0}{2m_2 m_2}, \quad (4.4)$$

where  $p^2 = -s$  and  $\Lambda^2 = \lambda(s, m_1^2, m_2^2)$  is the Källen lambda function. Furthermore we introduce

$$L_R = \ln \frac{\mu_R^2}{M_W^2}, \quad (4.5)$$

with the choice of the EW scale,  $x = M_W^2$ , in eq. (4.1).

Technically speaking the renormalization program is complete only when UV poles are removed from all, off-shell, Green functions, something that is beyond the scope of this paper. Furthermore, we introduce UV decompositions also for Green functions: given a one-loop Green function with  $N$  external lines carrying Lorentz indices  $\mu_j$ ,  $j = 1, \dots, N$ , we introduce form factors,

$$S_{\mu_1 \dots \mu_N} = \sum_{a=1}^A S_a K_{\mu_1 \dots \mu_N}^a. \quad (4.6)$$

Here the set  $K^a$ , with  $a = 1, \dots, A$ , contains independent tensor structures made up of external momenta, Kronecker-delta functions, elements of the Clifford algebra and Levi-Civita tensors. A large fraction of the form factors drops from the final answer when we make approximations, e.g. vector bosons couple only to conserved currents etc. Requiring that all (off-shell) form factors (including external unphysical lines) are made UV finite by means of local counterterms implies working in the  $R_{\xi\xi}$ -gauge, as shown (up to two loops in the SM) in ref. [75].

A full generality is beyond the scope of this paper, we will limit ourselves to the usual 't Hooft-Feynman gauge and to those Green functions that are relevant for the phenomenological applications considered in this paper.

### 4.1 Tadpoles and transitions

We begin by considering the treatment of tadpoles: we fix  $\bar{\beta}_h$ , eq. (2.21), such that  $\langle 0|\bar{H}|0\rangle = 0$  [45]. The solution is

$$\bar{\beta}_h = i g^2 M_W^2 \left( \bar{\beta}_h^{(4)} + g_6 \bar{\beta}_h^{(6)} \right), \quad (4.7)$$

where we split according to the following equation (see eq. (4.1))

$$\bar{\beta}_h^{(n)} = \beta_{-1}^{(n)} \Delta_{UV} \left( M_W^2 \right) + \beta_0^{(n)} + \beta_{\text{fin}}^{(n)}. \quad (4.8)$$

The full result for the coefficients  $\beta^{(n)}$  is given in appendix A. The parameter  $\Gamma$ , defined in eq. (2.14), is fixed by the request that the Z – A transition is zero at  $p^2 = 0$ ; the corresponding expression is also reported in appendix A.

### 4.2 H self-energy

The one-loop H self-energy is given by

$$S_{\text{HH}} = \frac{g^2}{16\pi^2} \Sigma_{\text{HH}} = \frac{g^2}{16\pi^2} \left( \Sigma_{\text{HH}}^{(4)} + g_6 \Sigma_{\text{HH}}^{(6)} \right). \quad (4.9)$$

The bare H self-energy is decomposed as follows:

$$\Sigma_{\text{HH}}^{(n)} = \Sigma_{\text{HH};UV}^{(n)} \Delta_{UV} \left( M_W^2 \right) + \Sigma_{\text{HH};\text{fin}}^{(n)}. \quad (4.10)$$

Furthermore we introduce

$$\Sigma_{\text{HH};\text{fin}}^{(n)}(s) = \Delta_{\text{HH};\text{fin}}^{(n)}(s) M_W^2 + \Pi_{\text{HH};\text{fin}}^{(n)}(s) s. \quad (4.11)$$

The full result for the H self-energy is given in appendix B.

### 4.3 A self-energy

The one-loop A self-energy is given by

$$S_{\text{AA}}^{\mu\nu} = \frac{g^2}{16\pi^2} \Sigma_{\text{AA}}^{\mu\nu}, \quad \Sigma_{\text{AA}}^{\mu\nu} = \Pi_{\text{AA}} T^{\mu\nu}, \quad (4.12)$$

where the Lorentz structure is specified by the tensor

$$T^{\mu\nu} = -s \delta^{\mu\nu} - p^\mu p^\nu, \quad (4.13)$$

and  $p^2 = -s$ . Furthermore the bare  $\Pi_{\text{AA}}$  is decomposed as follows:

$$\Pi_{\text{AA}} = \Pi_{\text{AA}}^{(4)} + g_6 \Pi_{\text{AA}}^{(6)}, \quad \Pi_{\text{AA}}^{(n)} = \Pi_{\text{AA};UV}^{(n)} \Delta_{UV} \left( M_W^2 \right) + \Pi_{\text{AA};\text{fin}}^{(n)}. \quad (4.14)$$

It is worth noting that the A – A transition satisfies a doubly-contracted Ward identity

$$p_\mu S_{\text{AA}}^{\mu\nu} p_\nu = 0. \quad (4.15)$$

The full result for the A self-energy is given in appendix B.

#### 4.4 W,Z self-energies

The one-loop W,Z self-energies are given by

$$S_{VV}^{\mu\nu} = \frac{g^2}{16\pi^2} \Sigma_{VV}^{\mu\nu}, \quad \Sigma_{VV}^{\mu\nu} = D_{VV} \delta^{\mu\nu} + P_{VV} p^\mu p^\nu, \quad (4.16)$$

where the form factors are decomposed according to

$$D_{VV} = D_{VV}^{(4)} + g_6 D_{VV}^{(6)}, \quad P_{VV} = P_{VV}^{(4)} + g_6 P_{VV}^{(6)}. \quad (4.17)$$

We also introduce the residue of the UV pole and the finite part:

$$D_{VV}^{(n)} = D_{VV;UV}^{(n)} \Delta_{UV} \left( M_W^2 \right) + D_{VV;fin}^{(n)}, \quad (4.18)$$

etc. The full result for these self-energies is given in appendix B. We also introduce

$$D_{VV;fin}^{(n)}(s) = \Delta_{VV;fin}^{(n)}(s) M_W^2 + \Pi_{VV;fin}^{(n)}(s) s = \Delta_{VV;fin}^{(n)}(0) M_W^2 + \left[ \Omega_{VV;fin}^{(n)}(0) + L_s^{VV;fin} \right] s + \mathcal{O}(s^2). \\ L_s^{ZZ;fin} = -\frac{1}{6} \ln \left( -\frac{s}{M^2} \right) \frac{1}{c_\theta^2} N_{gen}, \quad L_s^{WW} = 0. \quad (4.19)$$

#### 4.5 Z–A transition

The Z–A transition (up to one loop) is given by

$$S_{ZA}^{\mu\nu} = \frac{g^2}{16\pi^2} \Sigma_{ZA}^{\mu\nu} + g_6 T^{\mu\nu} a_{AZ}, \quad \Sigma_{ZA}^{\mu\nu} = \Pi_{ZA} T^{\mu\nu} + P_{ZA} p^\mu p^\nu, \quad (4.20)$$

$$\Pi_{ZA} = \Pi_{ZA}^{(4)} + g_6 \Pi_{ZA}^{(6)}, \quad P_{ZA} = P_{ZA}^{(4)} + g_6 P_{ZA}^{(6)}, \quad (4.21)$$

where we have included the term in the bare Lagrangian starting at  $\mathcal{O}(g_6)$ . The full result for the Z–A transition is given in appendix B.

#### 4.6 The fermion self-energy

The fermion self-energy is given by

$$S_f = \frac{g^2}{16\pi^2} \left[ \Delta_f + (V_f - A_f \gamma^5) i \not{p} \right], \quad (4.22)$$

with a decomposition

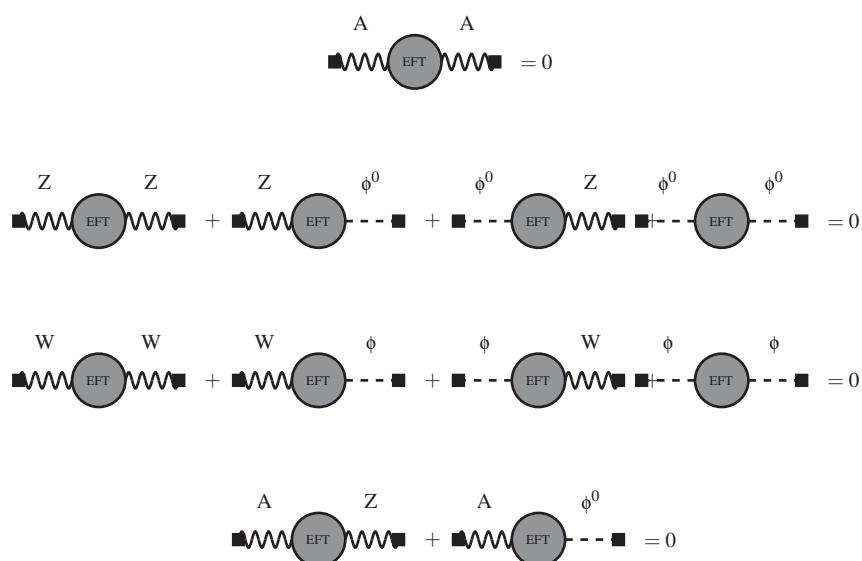
$$\Delta_f = \Delta_f^{(4)} + g_6 \Delta_f^{(6)}, \quad (4.23)$$

etc. The full result for the fermion self-energies ( $f = \nu, l, u, d$ ) is given in appendix B.

#### 4.7 Ward-Slavnon-Taylor identities

Let us consider doubly-contracted two-point WST identity [67–69], obtained by connecting two sources through vertices and propagators. Here we get, at every order in perturbation theory, the identities of figure 1. WST identities [67–69] require additional self-energies and transitions, i.e. scalar-scalar and vector-scalar components

$$S_{SS} = \frac{g^2}{16\pi^2} \left[ \Sigma_{SS}^{(4)} + g_6 \Sigma_{SS}^{(6)} \right], \quad S_{VS}^\mu = i \frac{g^2}{16\pi^2} \left[ \Sigma_{VS}^{(4)} + g_6 \Sigma_{VS}^{(6)} \right] p^\mu. \quad (4.24)$$



**Figure 1.** Doubly-contracted WST identities with two external gauge bosons. Gray circles denote the sum of the needed Feynman diagrams at any given order in EFT.

#### 4.8 Dyson resummed propagators

We will now present the Dyson resummed propagators for the electroweak gauge bosons. The function  $\Pi_{ij}^I$  represents the sum of all 1PI diagrams with two external boson fields,  $i$  and  $j$ , to all orders in perturbation theory (as usual, the external Born propagators are not to be included in the expression for  $\Pi_{ij}^I$ ). We write explicitly its Lorentz structure,

$$\Pi_{\mu\nu, VV}^I = D_{VV}^I \delta_{\mu\nu} + P_{VV}^I p_\mu p_\nu, \quad (4.25)$$

where  $V$  indicates SM vector fields, and  $p_\mu$  is the incoming momentum of the vector boson. The full propagator for a field  $i$  which mixes with a field  $j$  via the function  $\Pi_{ij}^I$  is given by the perturbative series

$$\begin{aligned} \bar{\Delta}_{ii} &= \Delta_{ii} + \Delta_{ii} \sum_{n=0}^{\infty} \prod_{l=1}^{n+1} \sum_{k_l} \Pi_{k_{l-1}k_l}^I \Delta_{k_l k_l}, \\ &= \Delta_{ii} + \Delta_{ii} \Pi_{ii}^I \Delta_{ii} + \Delta_{ii} \sum_{k_1=i,j} \Pi_{ik_1}^I \Delta_{k_1 k_1} \Pi_{k_1 i}^I \Delta_{ii} + \dots \end{aligned} \quad (4.26)$$

where  $k_0 = k_{n+1} = i$ , while for  $l \neq n+1$ ,  $k_l$  can be  $i$  or  $j$ .  $\Delta_{ii}$  is the Born propagator of the field  $i$ . We write

$$\bar{\Delta}_{ii} = \Delta_{ii} [1 - (\Pi\Delta)_{ii}]^{-1}, \quad (4.27)$$

and refer to  $\bar{\Delta}_{ii}$  as the resummed propagator. The quantity  $(\Pi\Delta)_{ii}$  is the sum of all the possible products of Born propagators and self-energies, starting with a 1PI self-energy  $\Pi_{ii}^I$ , or transition  $\Pi_{ij}^I$ , and ending with a propagator  $\Delta_{ii}$ , such that each element of the sum cannot be obtained as a product of other elements in the sum.



In practice it is useful to define, as an auxiliary quantity, the “partially resummed” propagator for the field  $i$ ,  $\hat{\Delta}_{ii}$ , in which we resum only the proper 1PI self-energy insertions  $\Pi_{ii}^I$ , namely,

$$\hat{\Delta}_{ii} = \Delta_{ii} [1 - \Pi_{ii}^I \Delta_{ii}]^{-1}. \quad (4.28)$$

If the particle  $i$  were not mixing with  $j$  through loops or two-leg vertex insertions,  $\hat{\Delta}_{ii}$  would coincide with the resummed propagator  $\bar{\Delta}_{ii}$ . Partially resummed propagators allow for a compact expression for  $(\Pi\Delta)_{ii}$ ,

$$(\Pi\Delta)_{ii} = \Pi_{ii}^I \Delta_{ii} + \Pi_{ij}^I \hat{\Delta}_{jj} \Pi_{ji}^I \Delta_{ii}, \quad (4.29)$$

so that the resummed propagator of the field  $i$  can be cast in the form

$$\bar{\Delta}_{ii} = \Delta_{ii} [1 - (\Pi_{ii}^I + \Pi_{ij}^I \hat{\Delta}_{jj} \Pi_{ji}^I) \Delta_{ii}]^{-1} \quad (4.30)$$

We can also define a resummed propagator for the  $i$ - $j$  transition. In this case there is no corresponding Born propagator, and the resummed one is given by the sum of all possible products of 1PI  $i$  and  $j$  self-energies, transitions, and Born propagators starting with  $\Delta_{ii}$  and ending with  $\Delta_{jj}$ . This sum can be simply expressed in the following compact form,

$$\bar{\Delta}_{ij} = \bar{\Delta}_{ii} \Pi_{ij}^I \hat{\Delta}_{jj}. \quad (4.31)$$

#### 4.9 Renormalization of two-point functions

Dyson resummed propagators are crucial for discussing several issues, from renormalization to Ward-Slavnov-Taylor (WST) identities [67–69]. Consider the W or Z self-energy; in general we have

$$\Sigma_{\mu\nu}^{\text{VV}}(s) = \frac{g^2}{16\pi^2} [D^{\text{VV}}(s) \delta_{\mu\nu} + P^{\text{VV}}(s) p_\mu p_\nu]. \quad (4.32)$$

The corresponding partially resummed propagator is

$$\hat{\Delta}_{\mu\nu}^{\text{VV}} = -\frac{\delta_{\mu\nu}}{s - M_V^2 + \frac{g^2}{16\pi^2} D^{\text{VV}}} + \frac{g^2}{16\pi^2} \frac{P^{\text{VV}} p_\mu p_\nu}{\left(s - M_V^2 + \frac{g^2}{16\pi^2} D^{\text{VV}}\right) \left(s - M_V^2 + \frac{g^2}{16\pi^2} D^{\text{VV}} - \frac{g^2}{16\pi^2} P^{\text{VV}} s\right)}. \quad (4.33)$$

We only consider the case where V couples to a conserved current; furthermore, we start by including one-particle irreducible (1PI) self-energies. Therefore the inverse propagators are defined as follows:

- H partially resummed propagator is given by

$$g^{-2} \hat{\Delta}_{\text{HH}}^{-1}(s) = -g^{-2} Z_{\text{H}} \left(s - M_{\text{H}}^2\right) - \frac{1}{16\pi^2} \Sigma_{\text{HH}}. \quad (4.34)$$

- A partially resummed propagator is given by

$$g^{-2} \hat{\Delta}_{\text{AA}}^{-1}(s) = -g^{-2} s \left(Z_{\text{A}} - \frac{1}{16\pi^2} \Pi_{\text{AA}}\right). \quad (4.35)$$

- W partially resummed propagator is given by

$$g^{-2} \hat{\Delta}_{\text{WW}}^{-1}(s) = -g^{-2} Z_{\text{W}} \left(s - M^2\right) - \frac{1}{16\pi^2} D_{\text{WW}}. \quad (4.36)$$

- Z partially resummed propagator is given by

$$g^{-2} \hat{\Delta}_{ZZ}^{-1}(s) = -g^{-2} Z_Z (s - M_0^2) - \frac{1}{16\pi^2} D_{ZZ}. \quad (4.37)$$

- Z–A transition is given by

$$S_{\mu\nu}^{ZA} + S_{\mu\nu}^{ZA\text{ct}} \quad S_{\mu\nu}^{ZA\text{ct}} = \frac{g^2}{16\pi^2} \Sigma_{\mu\nu}^{ZA\text{ct}} \Delta_{UV}, \quad (4.38)$$

where  $S_{\mu\nu}^{ZA}$  is given in eq. (4.20) and

$$\Sigma_{\mu\nu}^{ZA\text{ct}} = s \, dZ_{AZ}^{(4)} \delta_{\mu\nu} + g_6 \left[ s \, dZ_{AZ}^{(6)} \delta_{\mu\nu} - a_{AZ} \left( dZ_Z^{(4)} + dZ_A^{(4)} \right) p_\mu p_\nu \right]. \quad (4.39)$$

- f resummed propagator is given by

$$G_f^{-1}(p) = \bar{Z}_f (i \not{p} + m_f) Z_f - S_f, \quad (4.40)$$

where the counterterms are

$$Z_f = Z_{Rf} \gamma^- + Z_{Lf} \gamma^+, \quad \bar{Z}_f = Z_{Lf} \gamma^+ + Z_{Rf} \gamma^- \quad \gamma^\pm = \frac{1}{2} (1 \pm \gamma^5), \quad (4.41)$$

$$Z_{If} = 1 - \frac{1}{2} \frac{g^2}{16\pi^2} \left[ dZ_{If}^{(4)} + g_6 dZ_{If}^{(6)} \Delta_{UV} \right], \quad m_f = M_f \left( 1 + \frac{g^2}{16\pi^2} dZ_{m_f} \Delta_{UV} \right), \quad (4.42)$$

where  $M_f$  denotes the renormalized fermion mass and I = L, R. We have introduced counterterms for fields

$$\Phi = Z_\Phi \Phi_{\text{ren}}, \quad Z_\Phi = 1 + \frac{g^2}{16\pi^2} \left( dZ_\Phi^{(4)} + g_6 dZ_\Phi^{(6)} \right) \Delta_{UV}. \quad (4.43)$$

The bare photon field represents an exception, and here we use

$$A_\mu = Z_A A_\mu^{\text{ren}} + Z_{AZ} Z_\mu^{\text{ren}}, \quad Z_{AZ} = \frac{g^2}{16\pi^2} \left( dZ_{AZ}^{(4)} + g_6 dZ_{AZ}^{(6)} \right) \Delta_{UV}. \quad (4.44)$$

In addition, bare fermion fields  $\psi$  are written by means of bare left-handed and right-handed chiral fields,  $\psi_L$  and  $\psi_R$ . The latter are traded for renormalized fields.

For masses we introduce

$$M^2 = Z_M M_{\text{ren}}^2 \quad Z_M = 1 + \frac{g^2}{16\pi^2} \left( dZ_M^{(4)} + g_6 dZ_M^{(6)} \right) \Delta_{UV} \quad (4.45)$$

and for parameters

$$p = Z_p p_{\text{ren}} \quad Z_p = 1 + \frac{g^2}{16\pi^2} \left( dZ_p^{(4)} + g_6 dZ_p^{(6)} \right) \Delta_{UV}. \quad (4.46)$$

The full list of counterterms is given in appendix C. It is worth noting that the insertion of dim = 6 operators in the fermion self-energy introduces UV divergences in  $\Delta_f^{(6)}$ , eq. (4.23), that are proportional to  $s$  and cannot be absorbed by counterterms. They enter wave-function renormalization factors and will be cancelled at the level of mixing among Wilson coefficients.

#### 4.10 One-particle reducible transitions

Our procedure is such that there is a Z–A vertex of  $\mathcal{O}(g_6)$

$$V_{\mu\nu}^{ZA} = g_6 T_{\mu\nu} a_{AZ}, \quad T_{\mu\nu} = -s \delta_{\mu\nu} - p_\mu p_\nu, \quad (4.47)$$

inducing one-particle reducible (1PR) contributions to the self-energies. Since  $p^\mu T_{\mu\nu} = 0$  we obtain

$$\begin{aligned} \Pi^{AA} \Big|_{1PR} &= \frac{g^2 g_6}{16 \pi^2} \frac{s_\theta}{c_\theta} \frac{s}{s - M_0^2} a_{AZ} \Pi_{ZA}^{(4)}, \\ \Pi^{ZZ} \Big|_{1PR} &= \frac{g^2 g_6}{16 \pi^2} \frac{s_\theta}{c_\theta} a_{AZ} \Pi_{ZA}^{(4)}, \\ \Pi^{ZA} \Big|_{1PR} &= \frac{g^2 g_6}{16 \pi^2} s_\theta^2 a_{AZ} \Pi_{AA}^{(4)}. \end{aligned} \quad (4.48)$$

#### 4.11 A – A, Z – A, Z – Z and W – W transitions at $s = 0$

The value  $s = 0$  is particularly important since S, T and U parameters [76] require self-energies and transitions at  $s = 0$ . We introduce the following functions:

$$B_0^{\text{fin}}(-s; m_1, m_2) = B_0^{\text{fin}}(0; m_1, m_2) - s B_{0p}^{\text{fin}}(0; m_1, m_2) + \frac{1}{2} s^2 B_{0s}^{\text{fin}}(0; m_1, m_2) + \mathcal{O}(s^3), \quad (4.49)$$

where, with two different masses, we obtain

$$\begin{aligned} B_0^{\text{fin}}(0; m_1, m_2) &= \frac{m_2^2 a_0^{\text{fin}}(m_2) - m_1^2 a_0^{\text{fin}}(m_1)}{m_1^2 - m_2^2}, \\ B_{0p}^{\text{fin}}(0; m_1, m_2) &= -\frac{1}{(m_1^2 - m_2^2)^3} \left[ \frac{1}{2} (m_1^4 - m_2^4) + m_2^4 a_0^{\text{fin}}(m_2) - m_1^4 a_0^{\text{fin}}(m_1) \right] \\ &\quad - \frac{1}{(m_1^2 - m_2^2)^2} \left[ m_1^2 a_0^{\text{fin}}(m_1) + m_2^2 a_0^{\text{fin}}(m_2) \right], \\ B_{0s}^{\text{fin}}(0; m_1, m_2) &= \frac{1}{(m_1^2 - m_2^2)^5} \left[ \frac{10}{3} (m_1^6 - m_2^6) + 4 m_2^6 a_0^{\text{fin}}(m_2) - 4 m_1^6 a_0^{\text{fin}}(m_1) \right] \\ &\quad + \frac{3}{(m_1^2 - m_2^2)^4} \left[ 2 m_1^4 a_0^{\text{fin}}(m_1) + 2 m_2^4 a_0^{\text{fin}}(m_2) - m_1^4 - m_2^4 \right] \\ &\quad + \frac{2}{(m_1^2 - m_2^2)^3} \left[ m_2^2 a_0^{\text{fin}}(m_2) - m_1^2 a_0^{\text{fin}}(m_1) \right]. \end{aligned} \quad (4.50)$$

For equal masses we derive

$$B_0^{\text{fin}}(0; m, m) = 1 - a_0^{\text{fin}}(m), \quad B_{0p}^{\text{fin}}(0; m, m) = -\frac{1}{6m^2}, \quad B_{0s}^{\text{fin}}(0; m, m) = \frac{1}{30m^4}. \quad (4.51)$$

After renormalization we obtain the results of appendix D, with  $\Pi$  defined in eq. (4.12) and  $\Delta, \Omega$  defined in eq. (4.19). Furthermore  $N_{\text{gen}}$  is the number of fermion generations and  $L_R$  is defined in eq. (4.5). All functions defined in eq. (4.51) are successively scaled with  $M_W$ . In appendix D we have used  $s = s_\theta$ ,  $c = c_\theta$  and  $x_i$  are ratios of renormalized masses, i.e.  $x_H = M_H/M$ , etc.

The expressions corresponding to  $\dim = 6$  are rather long and we found convenient to introduce linear combinations of Wilson coefficients, given in eq. (4.52).

$$\begin{aligned}
a_{\phi W} &= s_\theta^2 a_{AA} + c_\theta s_\theta a_{AZ} + c_\theta^2 a_{ZZ} & a_{\phi B} &= c_\theta^2 a_{AA} - c_\theta s_\theta a_{AZ} + s_\theta^2 a_{ZZ} \\
a_{\phi WB} &= 2c_\theta s_\theta (a_{AA} - a_{ZZ}) + (1 - 2s_\theta^2) a_{AZ} & a_{\phi l} &= \frac{1}{2} (a_{\phi lA} - a_{\phi lV}) \\
a_{\phi u} &= \frac{1}{2} (a_{\phi uV} - a_{\phi uA}) & a_{\phi d} &= \frac{1}{2} (a_{\phi dA} - a_{\phi dV}) \\
a_{\phi l}^{(1)} &= a_{\phi l}^{(3)} - \frac{1}{2} (a_{\phi lV} + a_{\phi lA}) & a_{\phi l}^{(3)} &= \frac{1}{4} (a_{\phi lV} + a_{\phi lA} + a_{\phi lV}) \\
a_{\phi q}^{(1)} &= \frac{1}{4} (a_{\phi uV} + a_{\phi uA} - a_{\phi dV} - a_{\phi dA}) & a_{\phi q}^{(3)} &= \frac{1}{4} (a_{\phi dV} + a_{\phi dA} + a_{\phi uV} + a_{\phi uA}) \\
a_{lW} &= s_\theta a_{lWB} + c_\theta a_{lBW} & a_{lB} &= s_\theta a_{lWB} - c_\theta a_{lBW} \\
a_{dW} &= s_\theta a_{dWB} + c_\theta a_{dBW} & a_{dB} &= s_\theta a_{dWB} - c_\theta a_{dBW} \\
a_{uW} &= s_\theta a_{uWB} + c_\theta a_{uBW} & a_{uB} &= -s_\theta a_{uWB} + c_\theta a_{uBW} \\
a_{\phi WB} &= c_\theta a_{\phi WA} - s_\theta a_{\phi WZ} & a_{\phi W} &= s_\theta a_{\phi WA} + c_\theta a_{\phi WZ} \\
a_{\phi D} &= a_{\phi DB} - 8s_\theta^2 a_{\phi B} & a_{\phi WD}^{(+)} &= 4a_{\phi W} + a_{\phi D} \\
a_{\phi WD}^{(-)} &= 4a_{\phi W} - a_{\phi D} & a_{\phi lW}^{(3)} &= 4a_{\phi l}^{(3)} + 2a_{\phi W} \\
a_{\phi qW}^{(3)} &= 4a_{\phi q}^{(3)} + 2a_{\phi W} & a_{\phi WDB}^{(-)} &= a_{\phi WD}^{(+)} - 4a_{\phi \square} \\
a_{\phi WDB}^{(+)} &= a_{\phi WD}^{(+)} + 4a_{\phi \square} & a_{\phi WB}^{(a)} &= a_{\phi B} - a_{\phi W}
\end{aligned} \tag{4.52}$$

The results for  $\dim = 6$  simplify considerably if we neglect loop generated operators, for instance one obtains full factorization for  $\Pi_{AA}(0)$ ,

$$\Pi_{AA}^{(6)}(0) = -8 \frac{c_\theta^2}{s_\theta^2} a_{\phi D} \Pi_{AA}^{(4)}(0) \tag{4.53}$$

and partial factorization for the rest, e.g.

$$\begin{aligned}
\Pi_{ZA}^{(6)}(0) &= -4 \frac{c_\theta^2}{s_\theta^2} a_{\phi D} \Pi_{ZA}^{(4)}(0) + \frac{s_\theta}{c_\theta} a_{\phi D} \Pi_{ZA}^{(6)\text{nfc}}(0) \\
&\quad - \frac{2}{3} (1 - L_R) \frac{s_\theta}{c_\theta} \sum_{\text{gen}} (a_{\phi lV} + 2a_{\phi uV} + a_{\phi dV}) \\
&\quad - \frac{2}{3} \frac{s_\theta}{c_\theta} \sum_{\text{gen}} \left[ 2a_0^{\text{fn}}(M_u) a_{\phi uV} + a_0^{\text{fn}}(M_d) a_{\phi dV} + a_0^{\text{fn}}(M_l) a_{\phi lV} \right]
\end{aligned} \tag{4.54}$$

$$\begin{aligned}
\Pi_{ZA}^{(6)\text{nfc}}(0) &= -\frac{1}{24} (1 - 14c_\theta^2) - \frac{1}{24} (1 + 18c_\theta^2) L_R - \frac{1}{9} (5 + 8c_\theta^2) (1 - L_R) N_{\text{gen}} \\
&\quad - \frac{1}{12} (3 + 4c_\theta^2) \sum_{\text{gen}} a_0^{\text{fn}}(M_l) - \frac{1}{18} (5 + 8c_\theta^2) \sum_{\text{gen}} a_0^{\text{fn}}(M_u) - \frac{1}{36} (1 + 4c_\theta^2) \sum_{\text{gen}} a_0^{\text{fn}}(M_d) \\
&\quad + \frac{1}{24} (1 + 18c_\theta^2) a_0^{\text{fn}}(M)
\end{aligned} \tag{4.55}$$

The rather long expressions with PTG and LG operator insertions are reported in appendix D. Results in this section and in appendix D refer to the expansion of the IPI self-energies; inclusion of 1PR components amounts to the following replacements

$$\Sigma_{AA}^{(\text{IPI}+\text{1PR})}(s) = -\Pi_{AA}^{(\text{IPI})}(0) s - \frac{[\Pi_{ZA}^{(\text{1PR})}(0) s]^2}{s - M_0^2} + \mathcal{O}(s^2) = -\Pi_{AA}^{(\text{1PI})}(0) s + \mathcal{O}(s^2),$$

$$D_{ZZ}^{(1\text{PI}+1\text{PR})}(s) = \Delta_{ZZ}^{(1\text{PI})}(0) + \left\{ \Omega_{ZZ}^{(1\text{PI})}(0) - \left[ \Pi_{ZA}^{(1\text{PR})}(0) \right]^2 \right\} s + \mathcal{O}(s^2). \quad (4.56)$$

### 4.12 Finite renormalization

The last step in one-loop renormalization is the connection between renormalized quantities and POs. Since all quantities at this stage are UV-free, we term it “finite renormalization”. Note that the absorption of UV divergences into local counterterms is, to some extent, a trivial step; finite renormalization, instead, requires more attention. For example, beyond one loop one cannot use on-shell masses but only complex poles for all unstable particles [71, 77]. Let us show some examples where the concept of an on-shell mass can be employed. Suppose that we renormalize a physical (pseudo-)observable  $F$ ,

$$F = F_B + \frac{g^2}{16\pi^2} \left[ F_{1L}^{(4)}(m^2) + g_6 F_{1L}^{(6)}(m^2) \right] + \mathcal{O}(g^4), \quad (4.57)$$

where  $m$  is some renormalized mass. Consider two cases: a) two-loop corrections are not included and b)  $m$  appears at one and two loops in  $F_{1L}$  and  $F_{2L}$  but does not show up in the Born term  $F_B$ . In these cases we can use the concept of an on-shell mass performing a finite mass renormalization at one loop. If  $m_0$  is the bare mass for the field  $V$  we write

$$m_0^2 = M_{\text{OS}}^2 \left\{ 1 + \frac{g^2}{16\pi^2} \text{Re} \Sigma_{VV;\text{fin}} \Big|_{s=M_{\text{OS}}^2} \right\} = M_{\text{OS}}^2 + g^2 \Delta M^2, \quad (4.58)$$

where  $M_{\text{OS}}$  is the on-shell mass and  $\Sigma$  is extracted from the required one-particle irreducible Green function; eq. (4.58) is still meaningful (no dependence on gauge parameters) and will be used inside the result.

In the Complex Pole scheme we replace the conventional on-shell mass renormalization equation with the associated expression for the complex pole

$$m_0^2 = M_{\text{OS}}^2 \left[ 1 + \frac{g^2}{16\pi^2} \text{Re} \Sigma_{VV;\text{fin}}(M_{\text{OS}}^2) \right] \implies m_0^2 = s_V \left[ 1 + \frac{g^2}{16\pi^2} \Sigma_{VV;\text{fin}}(M_{\text{OS}}^2) \right], \quad (4.59)$$

where  $s_V$  is the complex pole associated to  $V$ . In this section we will discuss on-shell finite renormalization; after removal of UV poles we have replaced  $m_0 \rightarrow m_{\text{ren}}$  etc. and we introduce

$$M_{V\text{ren}} = M_{V;\text{OS}} + \frac{g_{\text{ren}}^2}{16\pi^2} \left( d\mathcal{Z}_{M_V}^{(4)} + g_6 d\mathcal{Z}_{M_V}^{(6)} \right) \quad (4.60)$$

and require that  $s = M_{V;\text{OS}}$  is a zero of the real part of the inverse  $V$  propagator, up to  $\mathcal{O}(g^2 g_6)$ . Therefore we introduce

$$\begin{aligned} M_{\text{ren}}^2 &= M_{W;\text{OS}}^2 \left[ 1 + \frac{g_{\text{ren}}^2}{16\pi^2} \left( d\mathcal{Z}_{M_W}^{(4)} + g_6 d\mathcal{Z}_{M_W}^{(6)} \right) \right], \\ M_{\text{Hren}}^2 &= M_{H;\text{OS}}^2 \left[ 1 + \frac{g_{\text{ren}}^2}{16\pi^2} \left( d\mathcal{Z}_{M_H}^{(4)} + g_6 d\mathcal{Z}_{M_H}^{(6)} \right) \right], \\ c_{\theta}^{\text{ren}} &= c_w \left[ 1 + \frac{g_{\text{ren}}^2}{16\pi^2} \left( d\mathcal{Z}_{c_{\theta}}^{(4)} + g_6 d\mathcal{Z}_{c_{\theta}}^{(6)} \right) \right], \end{aligned} \quad (4.61)$$

where  $c_w^2 = M_{W;\text{OS}}^2/M_{Z;\text{OS}}^2$  and  $s = M_{Z;\text{OS}}^2$  will be a zero of the real part of the inverse  $Z$  propagator.

Finite renormalization in the fermion sector requires the following steps: if  $M_{f;OS}$  denotes the on-shell fermion mass, using eq. (4.41), we write

$$M_f \left[ d\mathcal{L}_{M_f}^{(4)} + g_6 d\mathcal{L}_{M_f}^{(6)} \right] = \Delta_f \left( M_{f;OS}^2 \right) + M_{f;OS} V_f \left( M_{f;OS}^2 \right) \quad (4.62)$$

and determine the finite counterterms which are given in appendix E.

#### 4.12.1 $G_F$ renormalization scheme

In the  $G_F$ -scheme we write the following equation for the  $g$  finite renormalization

$$g_{\text{ren}} = g_{\text{exp}} + \frac{g_{\text{exp}}^2}{16\pi^2} \left( d\mathcal{L}_g^{(4)} + g_6 d\mathcal{L}_g^{(6)} \right), \quad (4.63)$$

where  $g_{\text{exp}}$  will be expressed in terms of the Fermi coupling constant  $G_F$ . The  $\mu$ -lifetime can be written in the form

$$\frac{1}{\tau_\mu} = \frac{M_\mu^5}{192\pi^3} \frac{g^4}{32M^4} (1 + \delta_\mu). \quad (4.64)$$

The radiative corrections are  $\delta_\mu = \delta_\mu^W + \delta_G$  where  $\delta_G$  is the sum of vertices, boxes etc and  $\delta_\mu^W$  is due to the W self-energy. The renormalization equation becomes

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M^2} \left\{ 1 + \frac{g^2}{16\pi^2} \left[ \delta_G + \frac{1}{M^2} \Sigma_{WW}(0) \right] \right\}, \quad (4.65)$$

where we expand the solution for  $g$

$$g_{\text{ren}}^2 = 4\sqrt{2} G_F M_{W;OS}^2 \left\{ 1 + \frac{G_F M_{W;OS}^2}{2\sqrt{2}\pi^2} \left[ \delta_G + \frac{1}{M^2} \Sigma_{WW;fin}(0) \right] \right\} \quad (4.66)$$

Note that the non universal part of the corrections is given by

$$\delta_G = \delta_G^{(4)} + g_6 \delta_G^{(6)}, \quad \delta_G^{(4)} = 6 + \frac{7 - 4s_\theta^2}{2s_\theta^2} \text{In}c_\theta^2, \quad (4.67)$$

but the contribution of dim = 6 operators to muon decay is not available yet and will not be included in the calculation. It is worth noting that eqs. (4.61)–(4.65) define finite renormalization in the  $\{G_F, M_W, M_Z\}$  input parameter set.

We show few explicit examples of finite renormalization, i.e. how to fix finite counterterms. From the H propagator and the definition of on-shell H mass one obtains

$$d\mathcal{L}_{M_H}^{(n)} = \frac{M_{W;OS}^2}{M_{H;OS}^2} \text{Re} \Delta_{HH;fin}^{(n)} \left( M_{H;OS}^2 \right) + \text{Re} \Pi_{HH;fin}^{(n)} \left( M_{H;OS}^2 \right), \quad (4.68)$$

where  $M_H$  is the renormalized H mass and  $M_{H;OS}$  is the on-shell H mass. From the W propagator we have

$$d\mathcal{L}_{M_W}^{(n)} = \text{Re} \Delta_{WW;fin}^{(n)} \left( M_{W;OS}^2 \right) + \text{Re} \Pi_{WW;fin}^{(n)} \left( M_{W;OS}^2 \right). \quad (4.69)$$

From the Z propagator and the definition of on-shell Z mass we have

$$d\mathcal{L}_{c_\theta}^{(n)} = \frac{1}{2} \text{Re} \left[ d\mathcal{L}_{M_W}^{(n)} - c_w^2 \Delta_{ZZ;fin}^{(n)} \left( M_{Z;OS}^2 \right) - \Pi_{ZZ;fin}^{(n)} \left( M_{Z;OS}^2 \right) \right], \quad (4.70)$$

with  $c_w^2 = M_{W;OS}^2/M_{Z;OS}^2$ . All quantities in eqs. (4.68)–(4.70) are the renormalized ones.

### 4.12.2 $\alpha$ renormalization scheme

This scheme uses the fine structure constant  $\alpha$ . The new renormalization equation is

$$g^2 s_\theta^2 = 4\pi\alpha \left[ 1 - \frac{\alpha}{4\pi} \frac{\Pi_{AA}(0)}{s_\theta^2} \right], \quad (4.71)$$

where  $\alpha = \alpha_{\text{QED}}(0)$ . Therefore, in this scheme, the finite counterterms are

$$g_{\text{ren}}^2 = g_A^2 \left[ 1 + \frac{\alpha}{4\pi} d\mathcal{L}_g \right], \quad c_\theta^{\text{ren}} = \hat{c}_\theta \left[ 1 + \frac{\alpha}{4\pi} d\mathcal{L}_{c_\theta} \right], \quad M_{\text{ren}} = M_{Z;\text{OS}} \hat{c}_\theta^2 \left[ 1 + \frac{\alpha}{8\pi} d\mathcal{L}_{M_W} \right], \quad (4.72)$$

where the parameters  $\hat{c}_\theta$  and  $g_A$  are defined by

$$g_A^2 = \frac{4\pi\alpha}{\hat{s}_\theta^2} \quad \hat{s}_\theta^2 = \frac{1}{2} \left[ 1 - \sqrt{1 - 4 \frac{\pi\alpha}{\sqrt{2} G_F M_{Z;\text{OS}}^2}} \right]. \quad (4.73)$$

The reason for introducing this scheme is that the S, T and U parameters (see ref. [76]) have been originally given in the  $\{\alpha, G_F, M_Z\}$  scheme while, for the rest of the calculations we have adopted the more convenient  $\{G_F, M_W, M_Z\}$  scheme. In this scheme, after requiring that  $M_{Z;\text{OS}}^2$  is a zero of the real part of the inverse Z propagator, we are left with one finite counterterm,  $d\mathcal{L}_g$ . The latter is fixed by using  $G_F$  and requiring that

$$\frac{1}{\sqrt{2}} G_F = \frac{g^2}{8M^2} \left\{ 1 + \frac{g^2}{16\pi^2} \left[ \delta_G + \frac{1}{M^2} \Delta_{\text{WW}}(0) - (dZ_W + dZ_{M_W}) \Delta_{\text{UV}} \right] \right\}, \quad (4.74)$$

where we use

$$g = g_{\text{ren}} \left( 1 + \frac{g_{\text{ren}}^2}{16\pi^2} dZ_g \Delta_{\text{UV}} \right), \quad g_{\text{ren}} = g_A \left( 1 + \frac{\alpha}{8\pi} d\mathcal{L}_g \right), \quad (4.75)$$

for UV and finite renormalization.

### 4.13 Wave function renormalization

Let us summarize the various steps in renormalization. Consider the V propagator, assuming that V couples to conserved currents (in the following we will drop the label V). We have

$$\bar{\Delta}_{\mu\nu} = - \frac{\delta_{\mu\nu}}{s - M^2 + \frac{g^2}{16\pi^2} D} = - \delta_{\mu\nu} \Delta^{-1}(s), \quad (4.76)$$

where  $M$  is the V bare mass. The procedure is as follows: we introduce UV counterterms for the field and its mass,

$$\Delta(s) \Big|_{\text{ren}} = Z_V (s - Z_M M_{\text{ren}}^2) + \frac{g^2}{16\pi^2} D(s) = s - M_{\text{ren}}^2 + \frac{g^2}{16\pi^2} D(s) \Big|_{\text{ren}} \quad (4.77)$$

and write the (finite) renormalization equation

$$M_{\text{ren}}^2 = M_{\text{OS}}^2 + \frac{g^2}{16\pi^2} \text{Re} D(M_{\text{OS}}^2) \Big|_{\text{ren}}, \quad (4.78)$$

where  $M_{\text{OS}}$  is the (on-shell) physical mass. After UV and finite renormalization we can write the following Taylor expansion:

$$\Delta(s)\Big|_{\text{ren}} = (s - M_{\text{OS}}^2) \left( 1 + \frac{g_{\text{exp}}^2}{16\pi^2} \mathbb{W} \right) + \mathcal{O}((s - M_{\text{OS}}^2)^2), \quad (4.79)$$

where  $g_{\text{exp}}$  is defined in eq. (4.63). The wave-function renormalization factor for the field  $\Phi$  will be denoted by

$$Z_{\text{WF};\Phi}^{-1/2} = \left( 1 + \frac{g_{\text{exp}}^2}{16\pi^2} \mathbb{W}_{\Phi} \right)^{-1/2}. \quad (4.80)$$

For fermion fields we use eq. (4.41) and introduce

$$V_f'(s) = -\frac{d}{ds} V_f(s). \quad (4.81)$$

Next we multiply spinors by the appropriate factors, i.e.

$$u_f(p) \rightarrow (1 + \mathbb{W}_{fV} + \mathbb{W}_{fA} \gamma^5) u_f(p) \quad \bar{u}_f(p) \rightarrow \bar{u}_f(p) (1 + \mathbb{W}_{fV} - \mathbb{W}_{fA} \gamma^5), \quad (4.82)$$

where the wave-function renormalization factors are obtained from eq. (4.22)

$$\mathbb{W}_{fV} = \frac{1}{2} \left[ V_f + 2M_f \Delta_f' - 2M_f^2 V_f' \right] \Big|_{s=M_f^2}, \quad \mathbb{W}_{fA} = -\frac{1}{2} A_f \Big|_{s=M_f^2}. \quad (4.83)$$

For illustration we present the H wave-function factor

$$\begin{aligned} \mathbb{W}_H &= \text{Re} \left( \mathbb{W}_H^{(4)} + g_6 \mathbb{W}_H^{(6)} \right), \\ \mathbb{W}_H^{(n)} &= d\Pi_{\text{HH}}^{(n)} \left( M_{\text{H};\text{OS}}^2 \right) M_{\text{H};\text{OS}}^2 + d\Delta_{\text{HH}}^{(n)} \left( M_{\text{H};\text{OS}}^2 \right) M_{\text{W};\text{OS}}^2 + \Pi_{\text{HH}}^{(n)} \left( M_{\text{H};\text{OS}}^2 \right) - 2d\mathcal{L}_g^{(n)} \end{aligned} \quad (4.84)$$

and we expand any function of  $s$  as follows:

$$f(s) = f(M_{\text{OS}}^2) + (s - M_{\text{OS}}^2) df(M_{\text{OS}}^2) + \mathcal{O}((s - M_{\text{OS}}^2)^2), \quad (4.85)$$

with  $f = \Pi^{(n)}, \Delta^{(n)}$ . For the W,Z wave-function factor we obtain

$$\begin{aligned} \mathbb{W}_W^{(n)} &= d\Pi_{\text{WW}}^{(n)} \left( M_{\text{W};\text{OS}}^2 \right) M_{\text{W};\text{OS}}^2 + d\Delta_{\text{WW}}^{(n)} \left( M_{\text{W};\text{OS}}^2 \right) M_{\text{W};\text{OS}}^2 + \Pi_{\text{WW}}^{(n)} \left( M_{\text{W};\text{OS}}^2 \right) - 2d\mathcal{L}_g^{(n)}, \\ \mathbb{W}_Z^{(n)} &= d\Pi_{\text{ZZ}}^{(n)} \left( M_{\text{Z};\text{OS}}^2 \right) M_{\text{Z};\text{OS}}^2 + d\Delta_{\text{ZZ}}^{(n)} \left( M_{\text{Z};\text{OS}}^2 \right) M_{\text{W};\text{OS}}^2 + \Pi_{\text{ZZ}}^{(n)} \left( M_{\text{Z};\text{OS}}^2 \right) - 2d\mathcal{L}_g^{(n)}. \end{aligned} \quad (4.86)$$

Explicit expressions for the wave-function factors are given in appendix F.

#### 4.14 Life and death of renormalization scale

Consider the A bare propagator

$$\bar{\Delta}_{\text{AA}}^{-1} = s + \frac{g^2}{16\pi^2} \Sigma_{\text{AA}}(s) \quad \Sigma_{\text{AA}}(s) = \left( \mathbb{R}^{(4)} + g_6 \mathbb{R}^{(6)} \right) \frac{1}{\varepsilon} + \sum_{x \in \mathcal{L}} \left( \mathbb{L}_x^{(4)} + g_6 \mathbb{L}_x^{(6)} \right) \ln \frac{x}{\mu_{\text{R}}^2} + \Sigma_{\text{AA}}^{\text{rest}}, \quad (4.87)$$



where  $R^{(n)}$  are the residues of the UV poles and  $L^{(n)}$  are arbitrary coefficients of the scale-dependent logarithms. Furthermore,

$$\{\mathcal{X}\} = \{s, m^2, m_0^2, m_H^2, m_t^2, m_b^2\}. \quad (4.88)$$

The renormalized propagator is

$$\overline{\Delta}_{AA}^{-1} \Big|_{\text{ren}} = Z_A s + \frac{g^2}{16\pi^2} \Sigma_{AA}(s) = s + \frac{g^2}{16\pi^2} \Sigma_{AA}^{\text{ren}}(s). \quad (4.89)$$

Furthermore, we can write

$$\Sigma_{AA}^{\text{ren}}(s) = \sum_{x \in \mathcal{X}} \left( L_x^{(4)} + g_6 L_x^{(6)} \right) \ln \frac{x}{\mu_R^2} + \Sigma_{AA}^{\text{rest}}. \quad (4.90)$$

Finite renormalization amounts to write  $\Sigma_{AA}^{\text{ren}}(s) = \Pi_{AA}^{\text{ren}}(s)s$  and to use  $s = 0$  as subtraction point. Therefore, one can easily prove that

$$\frac{\partial}{\partial \mu_R} \left[ \Pi_{AA}^{\text{ren}}(s) - \Pi_{AA}^{\text{ren}}(0) \right] = 0, \quad (4.91)$$

including  $\mathcal{O}^{(6)}$  contribution. Therefore we may conclude that there is no  $\mu_R$  problem when a subtraction point is available. After discussing decays of the Higgs boson in section 5 we will see that an additional step is needed in the renormalization procedure, i.e. mixing of the Wilson coefficients. At this point the scale dependence problem will surface again and renormalized Wilson coefficients become scale dependent.

## 5 Decays of the Higgs boson

In this section we will present results for two-body decays of the Higgs boson while four-body decays will be included in a forthcoming publication. Our approach is based on the fact that renormalizing a theory must be a fully general procedure; only when this step is completed one may consider making approximations, e.g. neglecting the lepton masses, keeping only PTG terms etc. In particular, neglecting LG Wilson coefficients sensibly reduces the number of terms in any amplitude.

It is useful to introduce a more compact notation for Wilson coefficients, given in table 4 and to use the following definition:

**Definition** The PTG scenario: any amplitude computed at  $\mathcal{O}(g^n g_6)$  has a SM component of  $\mathcal{O}(g^n)$  and two dim = 6 components: at  $\mathcal{O}(g^{n-2} g_6)$  we allow both PTG and LG operator while at  $\mathcal{O}(g^n g_6)$  only PTG operators are included.

### 5.1 Loop-induced processes: $H \rightarrow \gamma\gamma$

The amplitude for the process  $H(P) \rightarrow A_\mu(p_1)A_\nu(p_2)$  can be written as

$$A_{HAA}^{\mu\nu} = \mathcal{T}_{HAA} T^{\mu\nu}, \quad M_H^2 T^{\mu\nu} = p_2^\mu p_1^\nu - p_1 \cdot p_2 \delta^{\mu\nu}. \quad (5.1)$$

The  $S$ -matrix element follows from eq. (5.1) when we multiply the amplitude by the photon polarizations  $e_\mu(p_1)e_\nu(p_2)$ ; in writing eq. (5.1) we have used  $p \cdot e(p) = 0$ .

$a_{AA} = W_1$	$a_{ZZ} = W_2$	$a_{AZ} = W_3$
$a_{\phi D} = W_4$	$a_{\phi\Box} = W_5$	$a_{\phi} = W_6$
$a_{1WB} = W_7$	$a_{1BW} = W_8$	$a_{dWB} = W_9$
$a_{dBW} = W_{10}$	$a_{uWB} = W_{11}$	$a_{uBW} = W_{12}$
$a_{1\phi} = W_{13}$	$a_{d\phi} = W_{14}$	$a_{u\phi} = W_{15}$
$a_{\phi 1A} = W_{16}$	$a_{\phi 1V} = W_{17}$	$a_{\phi V} = W_{18}$
$a_{\phi dA} = W_{19}$	$a_{\phi dV} = W_{20}$	$a_{\phi uA} = W_{21}$
$a_{\phi uV} = W_{22}$	$a_{\phi LIdQ} = W_{23}$	$a_{QuQd}^{(1)} = W_{24}$

**Table 4.** Vector-like notation for Wilson coefficients.

Next we introduce dim = 4, LO, sub-amplitudes for t, b loops and for the bosonic loops,

$$\begin{aligned}
 \frac{3}{8} \frac{M_W}{M_t^2} \mathcal{F}_{HAA;LO}^t &= 2 + \left( M_H^2 - 4M_t^2 \right) C_0 \left( -M_H^2, 0, 0; M_t, M_t, M_t \right), \\
 \frac{9}{2} \frac{M_W}{M_b^2} \mathcal{F}_{HAA;LO}^b &= 2 + \left( M_H^2 - 4M_b^2 \right) C_0 \left( -M_H^2, 0, 0; M_b, M_b, M_b \right), \\
 \frac{1}{M_W} \mathcal{F}_{HAA;LO}^W &= -6 - 6 \left( M_H^2 - 2M_W^2 \right) C_0 \left( -M_H^2, 0, 0; M_W, M_W, M_W \right),
 \end{aligned} \tag{5.2}$$

where  $C_0$  is the scalar three-point function. The following result is obtained:

$$\mathcal{F}_{HAA} = i \frac{g^3}{16\pi^2} \left( \mathcal{F}_{HAA}^{(4)} + g_6 \mathcal{F}_{HAA}^{(6),b} \right) + i g g_6 \mathcal{F}_{HAA}^{(6),a}, \tag{5.3}$$

where the dim = 4 part of the amplitude

$$\mathcal{F}_{HAA}^{(4)} = 2s_\theta^2 \left( \sum_{\text{gen}} \sum_f \mathcal{F}_{HAA;LO}^f + \mathcal{F}_{HAA;LO}^W \right) \tag{5.4}$$

is UV finite, as well as  $\mathcal{F}^{(6),a}$  which is given by

$$\mathcal{F}_{HAA}^{(6),a} = 2 \frac{M_H^2}{M} \left( s_\theta^2 a_{\phi W} + c_\theta^2 a_{\phi B} + s_\theta c_\theta a_{\phi WB} \right). \tag{5.5}$$

The  $\mathcal{F}^{(6),b}$  component contains an UV-divergent part. UV renormalization requires

$$\mathcal{F}_{HAA}^{\text{ren}} = \mathcal{F}_{HAA} \left[ 1 + \frac{g^2}{16\pi^2} \left( dZ_A + \frac{1}{2} dZ_H + 3 dZ_g \right) \Delta_{UV} \right], \tag{5.6}$$

$$c_\theta = c_\theta^{\text{ren}} \left( 1 + \frac{g^2}{16\pi^2} dZ_{c_\theta} \Delta_{UV} \right), \quad g = g_{\text{ren}} \left( 1 + \frac{g_{\text{ren}}^2}{16\pi^2} dZ_g \Delta_{UV} \right) \tag{5.7}$$

and we obtain the renormalized version of the amplitude

$$\begin{aligned}
 \mathcal{T}_{\text{HAA}}^{\text{ren}} &= i \frac{g_{\text{ren}}^3}{16\pi^2} \left( \mathcal{T}_{\text{HAA}}^{(4)} + g_6 \mathcal{T}_{\text{HAA};\text{fin}}^{(6),b} \right) + i g_{\text{ren}} g_6 \mathcal{T}_{\text{HAA};\text{ren}}^{(6),a} + i \frac{g_{\text{ren}}^3}{16\pi^2} g_6 \mathcal{T}_{\text{HAA};\text{div}}^{(6)}, \\
 \mathcal{T}_{\text{HAA};\text{div}}^{(6)} &= \mathcal{T}_{\text{HAA};\text{div}}^{(6),b} \Delta_{\text{UV}} \left( M_{\text{W}}^2 \right) + \frac{M_{\text{Hren}}^2}{M_{\text{ren}}} \left\{ \left[ dZ_{\text{H}}^{(4)} - dZ_{M_{\text{W}}}^{(4)} + 2dZ_{\text{A}}^{(4)} - 2dZ_{g_6}^{(4)} \right] a_{\text{AA}} - 2 \frac{c_{\theta}^{\text{ren}}}{s_{\theta}^{\text{ren}}} dZ_{c_{\theta}}^{(4)} a_{\text{AZ}} \right\} \Delta_{\text{UV}}, \\
 \mathcal{T}_{\text{HAA};\text{ren}}^{(6),a} &= 2 \frac{M_{\text{Hren}}^2}{M_{\text{ren}}} a_{\text{AA}}, \tag{5.8}
 \end{aligned}$$

where  $a_{\text{AA}} = s_{\theta} c_{\theta} a_{\phi_{\text{WB}}} + c_{\theta}^2 a_{\phi_{\text{B}}} + s_{\theta}^2 a_{\phi_{\text{W}}}$  and  $c_{\theta} = c_{\theta}^{\text{ren}}$  etc. The last step in the UV-renormalization procedure requires a mixing among Wilson coefficients which cancels the remaining (dim = 6) parts. To this purpose we define

$$\mathbf{W}_i = \sum_j Z_{ij}^{\text{W}} \mathbf{W}_j^{\text{ren}}, \quad Z_{ij}^{\text{W}} = \delta_{ij} + \frac{g^2}{16\pi^2} dZ_{ij}^{\text{W}} \Delta_{\text{UV}}. \tag{5.9}$$

The matrix  $dZ^{\text{W}}$  is fixed by requiring cancellation of the residual UV poles and we obtain

$$\mathcal{T}_{\text{HAA};\text{div}}^{(6)} \rightarrow \mathcal{T}_{\text{HAA}}^{(6),\text{R}} \ln \frac{\mu_{\text{R}}^2}{M^2}. \tag{5.10}$$

Elements of the mixing matrix derived from  $\text{H} \rightarrow \text{AA}$  are given in appendix G. The result of eq. (5.8) becomes

$$\mathcal{T}_{\text{HAA}}^{\text{ren}} = i \frac{g_{\text{ren}}^3}{16\pi^2} \left( \mathcal{T}_{\text{HAA}}^{(4)} + g_6 \mathcal{T}_{\text{HAA};\text{fin}}^{(6),b} + g_6 \mathcal{T}_{\text{HAA}}^{(6),\text{R}} \ln \frac{\mu_{\text{R}}^2}{M^2} \right) + i g_{\text{ren}} g_6 \mathcal{T}_{\text{HAA}}^{(6),a}. \tag{5.11}$$

Inclusion of wave-function renormalization factors and of external leg factors (due to field redefinition described in section 2.4) gives

$$\mathcal{T}_{\text{HAA}}^{\text{ren}} \left[ 1 - \frac{g_{\text{ren}}^2}{16\pi^2} \left( \mathbf{W}_{\text{A}} + \frac{1}{2} \mathbf{W}_{\text{H}} \right) \right] \left[ 1 + g_6 \left( 2a_{\text{AA}} + a_{\phi_{\square}} - \frac{1}{4} a_{\phi_{\text{D}}} \right) \right]. \tag{5.12}$$

Finite renormalization requires writing

$$\begin{aligned}
 M_{\text{ren}}^2 &= M_{\text{W}}^2 \left( 1 + \frac{g_{\text{ren}}^2}{16\pi^2} d\mathcal{L}_{M_{\text{W}}} \right), \\
 c_{\theta}^{\text{ren}} &= c_{\text{W}} \left( 1 + \frac{g_{\text{ren}}^2}{16\pi^2} d\mathcal{L}_{c_{\theta}} \right), \\
 g_{\text{ren}} &= g_{\text{F}} \left( 1 + \frac{g_{\text{F}}^2}{16\pi^2} d\mathcal{L}_g \right), \tag{5.13}
 \end{aligned}$$

where  $g_{\text{F}}^2 = 4\sqrt{2}G_{\text{F}}M_{\text{W}}^2$  and  $c_{\text{W}} = M_{\text{W}}/M_{\text{Z}}$ . Another convenient way for writing the answer is the following: after renormalization we neglect all fermion masses but t, b and write

$$\begin{aligned}
 \mathcal{T}_{\text{HAA}} &= i \frac{g_{\text{F}}^3 s_{\text{W}}^2}{8\pi^2} \sum_{\text{I=W,t,b}} \kappa_{\text{I}}^{\text{HAA}} \mathcal{T}_{\text{HAA};\text{LO}}^{\text{I}} + i g_{\text{F}} g_6 \frac{M_{\text{H}}^2}{M_{\text{W}}} \mathbf{W}_1^{\text{ren}} \\
 &+ i \frac{g_{\text{F}}^3 g_6}{\pi^2} \left[ \sum_{i=1,3} \mathcal{A}_{\text{W},i}^{\text{nfc}} \mathbf{W}_i^{\text{ren}} + \mathcal{T}_{\text{HAA};\text{b}}^{\text{nfc}} \mathbf{W}_9^{\text{ren}} + \sum_{i=1,2} \mathcal{T}_{\text{HAA};\text{t},i}^{\text{nfc}} \mathbf{W}_{10+i}^{\text{ren}} \right], \tag{5.14}
 \end{aligned}$$

where Wilson coefficients are those in table 4. The  $\kappa$ -factors are given by

$$\kappa_{\Gamma}^{\text{proc}} = 1 + g_6 \Delta\kappa_{\Gamma}^{\text{proc}} \quad (5.15)$$

and there are additional, non-factorizable, contributions. The  $\kappa$  factors are

$$\begin{aligned} \Delta\kappa_{\text{t}}^{\text{HAA}} &= \frac{3}{16} \frac{M_{\text{H}}^2}{s_{\text{W}} M_{\text{W}}^2} a_{\text{tWB}} + (2 - s_{\text{W}}^2) \frac{c_{\text{W}}}{s_{\text{W}}} a_{\text{AZ}} + (6 - s_{\text{W}}^2) a_{\text{AA}} \\ &\quad - \frac{1}{2} \left[ a_{\phi\text{D}} + 2s_{\text{W}}^2 (c_{\text{W}}^2 a_{\text{ZZ}} - a_{\text{t}\phi} - 2a_{\phi\Box}) \right] \frac{1}{s_{\text{W}}^2}, \\ \Delta\kappa_{\text{b}}^{\text{HAA}} &= -\frac{3}{8} \frac{M_{\text{H}}^2}{s_{\text{W}} M_{\text{W}}^2} a_{\text{bWB}} + (2 - s_{\text{W}}^2) \frac{c_{\text{W}}}{s_{\text{W}}} a_{\text{AZ}} + (6 - s_{\text{W}}^2) a_{\text{AA}} \\ &\quad - \frac{1}{2} \left[ a_{\phi\text{D}} + 2s_{\text{W}}^2 (c_{\text{W}}^2 a_{\text{ZZ}} + a_{\text{b}\phi} - 2a_{\phi\Box}) \right] \frac{1}{s_{\text{W}}^2}, \\ \Delta\kappa_{\text{W}}^{\text{HAA}} &= (2 + s_{\text{W}}^2) \frac{c_{\text{W}}}{s_{\text{W}}} a_{\text{AZ}} + (6 + s_{\text{W}}^2) a_{\text{AA}} \\ &\quad - \frac{1}{2} \left[ a_{\phi\text{D}} - 2s_{\text{W}}^2 (2a_{\phi\Box} + c_{\text{W}}^2 a_{\text{ZZ}}) \right] \frac{1}{s_{\text{W}}^2}, \end{aligned} \quad (5.16)$$

where Wilson coefficients are the renormalized ones. In the PTG scenario we only keep  $a_{\text{t}\phi}$ ,  $a_{\text{b}\phi}$ ,  $a_{\phi\text{D}}$  and  $a_{\phi\Box}$  in eq. (5.16).

The advantage of eq. (5.14) is to establish a link between EFT and  $\kappa$ -language of ref. [4], which has a validity restricted to LO. As a matter of fact eq. (5.14) tells you that  $\kappa$ -factors can be introduced also at NLO level; they are combinations of Wilson coefficients but we have to extend the scheme with the inclusion of process dependent, non-factorizable, contributions.

Returning to the original convention for Wilson coefficients we derive the following result for the non-factorizable part of the amplitude:

$$\mathcal{T}_{\text{HAA}}^{\text{nc}} = M_{\text{W}} \sum_{a \in \{A\}} \mathcal{T}_{\text{HAA}}^{\text{nc}}(a) a, \quad (5.17)$$

where  $\{A\} = \{a_{\text{tWB}}, a_{\text{bWB}}, a_{\text{AA}}, a_{\text{AZ}}, a_{\text{ZZ}}\}$ . Finite counterterms,  $d\mathcal{L}_{M_{\text{H}}}$ ,  $d\mathcal{L}_g$  and  $d\mathcal{L}_{M_{\text{W}}}$  are defined in eq. (4.61) and in eq. (4.63). The results are reported on appendix H. In the PTG scenario all non-factorizable amplitudes for  $\text{H} \rightarrow \text{AA}$  vanish.

## 5.2 $\text{H} \rightarrow \text{Z}\gamma$

The amplitude for  $\text{H}(P) \rightarrow \text{A}_{\mu}(p_1)\text{Z}_{\nu}(p_2)$  can be written as

$$A_{\text{HAZ}}^{\mu\nu} = M_{\text{H}}^2 \mathcal{D}_{\text{HAZ}} \delta^{\mu\nu} + \mathcal{P}_{\text{HAZ}}^{11} p_1^{\mu} p_1^{\nu} + \mathcal{P}_{\text{HAZ}}^{12} p_1^{\mu} p_2^{\nu} + \mathcal{P}_{\text{HAZ}}^{21} p_2^{\mu} p_1^{\nu} + \mathcal{P}_{\text{HAZ}}^{22} p_2^{\mu} p_2^{\nu}. \quad (5.18)$$

The result of eq. (5.18) is fully general and can be used to prove WST identities. As far as the partial decay width is concerned only  $\mathcal{P}_{\text{HAZ}}^{21}$  will be relevant, due to  $p \cdot e(p) = 0$  where  $e$  is the polarization vector. We start by considering the 1PI component of the amplitude and obtain

$$A_{\text{HAZ}}^{\mu\nu} \Big|_{\text{1PI}} = M_{\text{H}}^2 \mathcal{D}_{\text{HAZ}}^{(\text{1PI})} \delta^{\mu\nu} + \mathcal{T}_{\text{HAZ}}^{(\text{1PI})} T^{\mu\nu}, \quad (5.19)$$

where  $T$  is given by

$$M_H^2 T^{\mu\nu} = p_2^\mu p_1^\nu - p_1 \cdot p_2 \delta^{\mu\nu}. \quad (5.20)$$

Furthermore we can write the following decomposition:

$$\mathcal{F}_{\text{HAZ}}^{(1\text{PI})} = i \frac{g^3}{16\pi^2} \left( \mathcal{F}_{\text{HAZ}}^{(4)} + g_6 \mathcal{F}_{\text{HAZ}}^{(6),b} \right) + i g g_6 \mathcal{F}_{\text{HAZ}}^{(6),a}, \quad \mathcal{D}_{\text{HAZ}}^{(1\text{PI})} = i \frac{g^3 g_6}{16\pi^2} \mathcal{D}_{\text{HAZ}}^{(6)}, \quad (5.21)$$

$$\mathcal{D}_{\text{HAZ}}^{(6)} = 3 c_\theta s_\theta^2 \frac{M_0^3}{M_H^2} C_0 \left( -M_H^2, 0, -M_0^2; M, M, M \right) a_{\text{AZ}},$$

$$\mathcal{F}_{\text{HAZ}}^{(6),a} = \frac{M_H^2}{M} \left[ 2 s_\theta c_\theta (a_{\phi_W} - a_{\phi_B}) + (2 c_\theta^2 - 1) a_{\phi_{WB}} \right]. \quad (5.22)$$

Explicit expressions for  $\mathcal{F}_{\text{HAZ}}^{(4)}$ ,  $\mathcal{F}_{\text{HAZ}}^{(6),b}$  will not be reported here. The 1PR component of the amplitude is given by

$$A_{\text{HAZ}}^{\mu\nu} \Big|_{1\text{PR}} = -\frac{1}{M_0^2} A_{\text{HAA}}^{\mu\alpha\text{off}}(p_1, p_2) S_{\text{AZ}}^{\alpha\nu}(p_2) = M_H^2 \mathcal{D}_{\text{HAZ}}^{(1\text{PR})} \delta^{\mu\nu} + \mathcal{F}_{\text{HAZ}}^{(1\text{PR})} T^{\mu\nu}, \quad (5.23)$$

where  $A_{\text{HAA}}^{\mu\alpha\text{off}}$  denotes the off-shell  $\text{H} \rightarrow \text{AA}$  amplitude. It is straightforward to derive

$$\mathcal{D}_{\text{HAZ}} = \mathcal{D}_{\text{HAZ}}^{(1\text{PI})} + \mathcal{D}_{\text{HAZ}}^{(1\text{PR})} = 0 \quad (5.24)$$

i.e. the complete amplitude for  $\text{H} \rightarrow \text{AZ}$  is proportional to  $T^{\mu\nu}$  and, therefore, is transverse. UV renormalization requires the introduction of counterterms,

$$\mathcal{F}_{\text{HAZ}}^{\text{ren}} = \mathcal{F}_{\text{HAZ}} \left[ 1 + \frac{1}{2} \frac{g^2}{16\pi^2} (dZ_{\text{H}} + dZ_{\text{A}} + dZ_{\text{Z}} - 6 dZ_{\text{g}}) \Delta_{\text{UV}} \right], \quad (5.25)$$

$$c_\theta = c_\theta^{\text{ren}} \left( 1 + \frac{g_{\text{ren}}^2}{16\pi^2} dZ_{c_\theta} \Delta_{\text{UV}} \right), \quad g = g_{\text{ren}} \left( 1 + \frac{g_{\text{ren}}^2}{16\pi^2} dZ_{\text{g}} \Delta_{\text{UV}} \right) \quad (5.26)$$

and we obtain the following result for the renormalized amplitude:

$$\begin{aligned} \mathcal{F}_{\text{HAZ}}^{\text{ren}} &= i \frac{g_{\text{ren}}^3}{16\pi^2} \left( \mathcal{F}_{\text{HAZ}}^{(4)} + g_6 \mathcal{F}_{\text{HAZ};\text{fin}}^{(6),b} \right) + i g_{\text{ren}} g_6 \mathcal{F}_{\text{HAZ};\text{ren}}^{(6),a} + i \frac{g_{\text{ren}}^3}{16\pi^2} g_6 \mathcal{F}_{\text{HAZ};\text{div}}^{(6)}, \\ \mathcal{F}_{\text{HAZ};\text{div}}^{(6)} &= \mathcal{F}_{\text{HAZ};\text{div}}^{(6),b} \Delta_{\text{UV}} \left( M_{\text{W}}^2 \right) + \frac{M_{\text{Hren}}^2}{M_{\text{ren}}} \left\{ \left[ \frac{1}{2} dZ_{\text{H}}^{(4)} + dZ_{\text{A}}^{(4)} - \frac{1}{2} dZ_{M_{\text{W}}}^{(4)} - 2 dZ_{\text{g}}^{(4)} \right] a_{\text{AZ}} \right. \\ &\quad \left. + 2 \frac{c_\theta^{\text{ren}}}{s_\theta^{\text{ren}}} dZ_{c_\theta}^{(4)} (a_{\text{AA}} - a_{\text{ZZ}}) \right\} \Delta_{\text{UV}}, \\ \mathcal{F}_{\text{HAZ};\text{ren}}^{(6),a} &= \frac{M_{\text{Hren}}^2}{M_{\text{ren}}} a_{\text{AZ}}, \end{aligned} \quad (5.27)$$

where  $a_{\text{AA}} = s_\theta c_\theta a_{\phi_{WB}} + c_\theta^2 a_{\phi_B} + s_\theta^2 a_{\phi_W}$  and  $c_\theta = c_\theta^{\text{ren}}$  etc. Once again, the last step in the UV-renormalization procedure requires a mixing among Wilson coefficients, performed according to eq. (5.9). We obtain

$$\mathcal{F}_{\text{HAZ};\text{div}}^{(6)} \rightarrow \mathcal{F}_{\text{HAZ}}^{(6),\text{R}} \ln \frac{\mu_{\text{R}}^2}{M^2}. \quad (5.28)$$

Elements of the mixing matrix derived from the process  $H \rightarrow AZ$  are given in appendix G. After mixing the result of eq. (5.27) becomes

$$\mathcal{T}_{HAZ}^{\text{ren}} = i \frac{g_{\text{ren}}^3}{16\pi^2} \left( \mathcal{T}_{HAZ}^{(4)} + g_6 \mathcal{T}_{HAZ;\text{fin}}^{(6),b} + g_6 \mathcal{T}_{HAZ}^{(6),R} \ln \frac{\mu_R^2}{M^2} \right) + i g_{\text{ren}} g_6 \mathcal{T}_{HAZ}^{(6),a}. \quad (5.29)$$

Inclusion of wave-function renormalization factors and of external leg factors (due to field redefinition, defined in section 2.4) gives

$$\mathcal{T}_{HAZ}^{\text{ren}} \left[ 1 - \frac{1}{2} \frac{g_{\text{ren}}^2}{16\pi^2} (\mathbf{W}_H + \mathbf{W}_A + \mathbf{W}_Z) \right] \left[ 1 + g_6 \left( a_{AA} + a_{ZZ} + a_{\phi\Box} - \frac{1}{4} a_{\phi D} \right) \right]. \quad (5.30)$$

Finite renormalization is performed by using eq. (5.13). To write the final answer it is convenient to define  $\text{dim} = 4$  sub-amplitudes  $\mathcal{T}_{HAZ;LO}^I$  ( $I = W, t, b$ ): they are given in appendix I. Another convenient way for writing  $\mathcal{T}_{HAZ}^{\text{ren}}$  is the following:

$$\begin{aligned} \mathcal{T}_{HAZ}^{\text{ren}} = & i \frac{g_F^3}{\pi^2 M_Z} \sum_{I=W,t,b} \kappa_I^{\text{HAZ}} \mathcal{T}_{HAZ;LO}^I + i g_F g_6 \frac{M_H^2}{M} \mathbf{W}_3^{\text{ren}} \\ & + i \frac{g_F^3 g_6}{\pi^2} \left[ \sum_{i=1,4} \mathcal{T}_{HAZ;W,i}^{\text{nfc}} \mathbf{W}_i^{\text{ren}} + \sum_{i=11,12,22} \mathcal{T}_{HAZ;t,i}^{\text{nfc}} \mathbf{W}_i^{\text{ren}} + \sum_{i=9,10,20} \mathcal{T}_{HAZ;b,i}^{\text{nfc}} \mathbf{W}_i^{\text{ren}} \right]. \end{aligned} \quad (5.31)$$

The factorizable part is defined in terms of  $\kappa$ -factors, see eq. (5.15)

$$\begin{aligned} \Delta \kappa_t^{\text{HAZ}} &= \frac{1}{2} (2a_{t\phi} + 4a_{\phi\Box} - a_{\phi D} + 6a_{AA} + 2a_{ZZ}), \\ \Delta \kappa_b^{\text{HAZ}} &= -\frac{1}{2} (2a_{b\phi} - 4a_{\phi\Box} + a_{\phi D} - 6a_{AA} - 2a_{ZZ}), \\ \Delta \kappa_W^{\text{HAZ}} &= \frac{1 + 6c_W^2}{c_W^2} a_{\phi\Box} - \frac{1}{4} \frac{1 + 4c_W^2}{c_W^2} a_{\phi D} - \frac{1}{2} \frac{1 + c_W^2 - 24c_W^4}{c_W^2} a_{AA}, \\ &+ \frac{1}{4} (1 + 12c_W^2 - 48c_W^4) \frac{s_W}{c_W^3} a_{AZ} + \frac{1}{2} \frac{1 + 15c_W^2 - 24c_W^4}{c_W^2} a_{ZZ}. \end{aligned} \quad (5.32)$$

In the PTG scenario we only keep  $a_{t\phi}, a_{b\phi}, a_{\phi D}$  and  $a_{\phi\Box}$  in eq. (5.32).

Returning to the original convention for Wilson coefficients we derive the following result for the non-factorizable part of the amplitude:

$$\mathcal{T}_{HAZ}^{\text{nfc}} = \sum_{a \in \{A\}} \mathcal{T}_{HAZ}^{\text{nfc}}(a) a, \quad (5.33)$$

where  $\{A\} = \{a_{\phi tV}, a_{tBW}, a_{tWB}, a_{\phi bV}, a_{bWB}, a_{bBW}, a_{\phi D}, a_{AZ}, a_{AA}, a_{ZZ}\}$ . In the PTG scenario there are only 3 non-factorizable amplitudes for  $H \rightarrow AZ$ , those proportional to  $a_{\phi tV}, a_{\phi bV}$  and  $a_{\phi D}$ . The full results are reported on appendix H where

$$\lambda_{AZ} = \frac{M_H^2}{M_W^2} - \frac{M_Z^2}{M_W^2}. \quad (5.34)$$

### 5.3 $H \rightarrow ZZ$

The amplitude for  $H(P) \rightarrow Z_\mu(p_1)Z_\nu(p_2)$  can be written as

$$A_{\text{HZZ}}^{\mu\nu} = \mathcal{D}_{\text{HZZ}} \delta^{\mu\nu} + \mathcal{P}_{\text{HZZ}}^{11} p_1^\mu p_1^\nu + \mathcal{P}_{\text{HZZ}}^{12} p_1^\mu p_2^\nu + \mathcal{P}_{\text{HZZ}}^{21} p_2^\mu p_1^\nu + \mathcal{P}_{\text{HZZ}}^{22} p_2^\mu p_2^\nu. \quad (5.35)$$

The result in eq. (5.35) is fully general and can be used to prove WST identities. As far as the partial decay width is concerned only  $\mathcal{P}_{\text{HZZ}}^{21} \equiv \mathcal{P}_{\text{HZZ}}$  will be relevant, due to  $p \cdot e(p) = 0$  where  $e$  is the polarization vector. Note that computing WST identities requires additional amplitudes, i.e.  $H \rightarrow \phi^0 \gamma$  and  $H \rightarrow \phi^0 \phi^0$ .

We discuss first the 1PI component of the process: as done before the form factors in eq. (5.35) are decomposed as follows:

$$\begin{aligned} \mathcal{D}_{\text{HZZ}}^{\text{1PI}} &= -i g \frac{M}{c_\theta^2} + i \frac{g^3}{16\pi^2} \left( \mathcal{D}_{\text{HZZ}}^{(4)\text{1PI}} + g_6 \mathcal{D}_{\text{HZZ}}^{(6)\text{1PI},b} \right) + i g g_6 \mathcal{D}_{\text{HZZ}}^{(6)\text{1PI},a}, \\ \mathcal{P}_{\text{HZZ}}^{\text{1PI}} &= i \frac{g^3}{16\pi^2} \left( \mathcal{P}_{\text{HZZ}}^{(4)\text{1PI}} + g_6 \mathcal{P}_{\text{HZZ}}^{(6)\text{1PI},b} \right) + i g g_6 \mathcal{P}_{\text{HZZ}}^{(6)\text{1PI},a}. \end{aligned} \quad (5.36)$$

It is easily seen that only  $\mathcal{D}$  contains  $\text{dim} = 4$  UV divergences. The 1PR component of the process involves the A–Z transition and it is given by

$$A_{\text{HZZ}}^{\mu\nu} \Big|_{\text{1PR}} = \frac{i}{M_0^2} \left[ \mathcal{T}_{\text{HAA}}^{\alpha\nu}(p_1, p_2) \Sigma_{\text{AZ}}^{\alpha\mu}(p_1) + \mathcal{T}_{\text{HAA}}^{\mu\alpha}(p_1, p_2) \Sigma_{\text{AZ}}^{\alpha\nu}(p_2) \right], \quad (5.37)$$

where the  $H \rightarrow AZ$  component is computed with off-shell A. The r.h.s. of eq. (5.37) is expanded up to  $\mathcal{O}(g^3 g_6)$  and we will use

$$\mathcal{D}_{\text{HZZ}} = \mathcal{D}_{\text{HZZ}}^{\text{1PI}} + \mathcal{D}_{\text{HZZ}}^{\text{1PR}}, \quad \mathcal{P}_{\text{HZZ}} = \mathcal{P}_{\text{HZZ}}^{\text{1PI}} + \mathcal{P}_{\text{HZZ}}^{\text{1PR}}. \quad (5.38)$$

Complete, bare, amplitudes are constructed

$$\begin{aligned} \mathcal{D}_{\text{HZZ}} &= -i g \frac{M}{c_\theta^2} + i \frac{g^3}{16\pi^2} \left( \mathcal{D}_{\text{HZZ}}^{(4)} + g_6 \mathcal{D}_{\text{HZZ}}^{(6),b} \right) + i g g_6 \mathcal{D}_{\text{HZZ}}^{(6),a}, \\ \mathcal{P}_{\text{HZZ}} &= i \frac{g^3}{16\pi^2} \left( \mathcal{P}_{\text{HZZ}}^{(4)} + g_6 \mathcal{P}_{\text{HZZ}}^{(6),b} \right) + i g g_6 \mathcal{P}_{\text{HZZ}}^{(6),a}, \end{aligned} \quad (5.39)$$

where the  $\mathcal{O}(g g_6)$  components are:

$$\begin{aligned} \mathcal{D}_{\text{HZZ}}^{(6),a} &= -\frac{M}{c_\theta^2} \left( a_{\phi\Box} + \frac{1}{4} a_{\phi\text{D}} \right) + \left( \frac{M_{\text{H}}^2 - 2M_{\text{Z}}^2}{M} - M \right) a_{\text{ZZ}} - M \frac{s_\theta^2}{c_\theta^2} a_{\text{AA}} - M \frac{s_\theta}{c_\theta} a_{\text{AZ}}, \\ \mathcal{P}_{\text{HZZ}}^{(6),a} &= 2 \frac{1}{M} a_{\text{ZZ}}. \end{aligned} \quad (5.40)$$

UV renormalization requires introduction of counterterms,

$$\mathcal{F}_{\text{HZZ}}^{\text{ren}} = \mathcal{D}_{\text{HZZ}} \left[ 1 + \frac{g^2}{16\pi^2} \left( \text{dZ}_{\text{Z}} + \frac{1}{2} \text{dZ}_{\text{H}} - 3 \text{dZ}_{\text{g}} \right) \right], \quad (5.41)$$

where  $\mathcal{F} = \mathcal{D}, \mathcal{P}$  and  $\text{dZ}_i = \text{dZ}_i^{(4)} + g_6 \text{dZ}_i^{(6)}$ . We obtain

$$\mathcal{D}_{\text{HZZ}} = -i g_{\text{ren}} \frac{M_{\text{ren}}}{(c_\theta^{\text{ren}})^2} + i \frac{g_{\text{ren}}^3}{16\pi^2} \left( \mathcal{D}_{\text{HZZ}}^{(4)} + g_6 \mathcal{D}_{\text{HZZ};\text{fin}}^{(6)} \right)$$

$$\begin{aligned}
& + i g_{\text{ren}} g_6 \mathcal{D}_{\text{HZZ}}^{(6),\text{ren},a} + i \frac{g_{\text{ren}}^3}{16\pi^2} g_6 \mathcal{D}_{\text{HZZ};\text{div}}^{(6)}, \\
\mathcal{P}_{\text{HZZ}} & = i \frac{g_{\text{ren}}^3}{16\pi^2} \left( \mathcal{P}_{\text{HZZ}}^{(4)} + g_6 \mathcal{P}_{\text{HZZ};\text{fin}}^{(6)} \right) \\
& + i g_{\text{ren}} g_6 \mathcal{P}_{\text{HZZ}}^{(6),\text{ren},a} + i \frac{g_{\text{ren}}^3}{16\pi^2} g_6 \mathcal{P}_{\text{HZZ};\text{div}}^{(6)}. \tag{5.42}
\end{aligned}$$

The explicit expressions for  $\mathcal{F}_{\text{HZZ};\text{div}}^{(6)}$  will not be reported here. The last step in UV-renormalization requires a mixing among Wilson coefficients, performed according to eq. (5.9). After the removal of the remaining (dim = 6) UV parts we obtain

$$\mathcal{F}_{\text{HZZ};\text{div}}^{(6)} \rightarrow \mathcal{F}_{\text{HZZ}}^{(6),\text{R}} \ln \frac{\mu_{\text{R}}^2}{M^2}. \tag{5.43}$$

Elements of the mixing matrix derived from  $\text{H} \rightarrow \text{ZZ}$  are given in appendix G. Inclusion of wave-function renormalization factors and of external leg factors (due to field redefinition, introduced in section 2.4) gives

$$\mathcal{F}_{\text{HZZ}}^{\text{ren}} \left[ 1 - \frac{1}{2} \frac{g_{\text{ren}}^2}{16\pi^2} (\text{W}_\text{H} + 2\text{W}_\text{Z}) \right] \left[ 1 + g_6 \left( 2a_{\text{ZZ}} + a_{\phi\Box} - \frac{1}{4} a_{\phi\text{D}} \right) \right]. \tag{5.44}$$

Finite renormalization is performed by using eq. (5.13). The process  $\text{H} \rightarrow \text{ZZ}$  starts at  $\mathcal{O}(g)$ , therefore, the full set of counterterms must be included, not only the dim = 4 part, as we have done for the loop induced processes.

It is convenient to define NLO sub-amplitudes; however, to respect a factorization into t, b and bosonic components, we have to introduce the following quantities:

$$\begin{aligned}
\text{W}_\text{H} & = \text{W}_\text{H W} + \text{W}_\text{H t} + \text{W}_\text{H b} & \text{W}_\text{Z} & = \text{W}_\text{Z W} + \text{W}_\text{Z t} + \text{W}_\text{Z b} + \overline{\Sigma}_{\text{gen}} \text{W}_{\text{Z};\text{f}} \\
\text{d}\mathcal{L}_g & = \text{d}\mathcal{L}_{g;\text{W}} + \overline{\Sigma}_{\text{gen}} \text{d}\mathcal{L}_{g;\text{f}} & \text{d}\mathcal{L}_{c_\theta} & = \text{d}\mathcal{L}_{c_\theta;\text{W}} + \text{d}\mathcal{L}_{c_\theta;\text{t}} + \text{d}\mathcal{L}_{c_\theta;\text{b}} + \overline{\Sigma}_{\text{gen}} \text{d}\mathcal{L}_{c_\theta;\text{f}} \\
\text{d}\mathcal{L}_{M_\text{W}} & = \text{d}\mathcal{L}_{M_\text{W};\text{W}} + \overline{\Sigma}_{\text{gen}} \text{d}\mathcal{L}_{M_\text{W};\text{f}}
\end{aligned} \tag{5.45}$$

where  $\text{W}_{\Phi;\phi}$  denotes the  $\phi$  component of the  $\Phi$  wave-function factor etc. Furthermore,  $\Sigma_{\text{gen}}$  implies summing over all fermions and all generations, while  $\overline{\Sigma}_{\text{gen}}$  excludes t and b from the sum. We can now define  $\kappa$ -factors for the process, see eq. (5.15):

$$\begin{aligned}
\Delta\kappa_{\text{D};\text{LO}}^{\text{HZZ}} & = s_{\text{W}}^2 a_{\text{AA}} + \left[ 4 + c_{\text{W}}^2 \left( 1 - \frac{M_{\text{H}}^2}{M_{\text{W}}^2} \right) \right] a_{\text{ZZ}} + c_{\text{W}} s_{\text{W}} a_{\text{AZ}} + 2a_{\phi\Box}, \\
\Delta\kappa_{\text{t};\text{D};\text{NLO}}^{\text{HZZ}} & = a_{\text{t}\phi} + 2a_{\phi\Box} - \frac{1}{2} a_{\phi\text{D}} + 2a_{\text{ZZ}} + s_{\text{W}}^2 a_{\text{AA}}, \\
\Delta\kappa_{\text{b};\text{D};\text{NLO}}^{\text{HZZ}} & = -a_{\text{b}\phi} + 2a_{\phi\Box} - \frac{1}{2} a_{\phi\text{D}} + 2a_{\text{ZZ}} + s_{\text{W}}^2 a_{\text{AA}}, \\
\Delta\kappa_{\text{W};\text{D};\text{NLO}}^{\text{HZZ}} & = \frac{1}{12} \left( 4 + \frac{1}{c_{\text{W}}^2} \right) a_{\phi\text{D}} + 2a_{\phi\Box} + s_{\text{W}}^2 a_{\text{AA}} \\
& + s_{\text{W}}^2 \left( 3c_{\text{W}} + \frac{5}{3} \frac{1}{c_{\text{W}}} \right) a_{\text{AZ}} + \left( 4 + c_{\text{W}}^2 \right) a_{\text{ZZ}},
\end{aligned} \tag{5.46}$$



$$\begin{aligned}
 \Delta\kappa_{t;P;NLO}^{\text{HZZ}} &= a_{t\phi} + 2a_{\phi\Box} - \frac{1}{2}a_{\phi D} + 2a_{ZZ} + s_W^2 a_{AA}, \\
 \Delta\kappa_{b;P;NLO}^{\text{HZZ}} &= -a_{b\phi} + 2a_{\phi\Box} - \frac{1}{2}a_{\phi D} + 2a_{ZZ} + s_W^2 a_{AA}, \\
 \Delta\kappa_{W;P;NLO}^{\text{HZZ}} &= 4a_{\phi\Box} + \frac{5}{2}a_{\phi D} + 12a_{ZZ} + 3s_W^2 a_{AA}
 \end{aligned} \tag{5.47}$$

and obtain the final result for the amplitudes

$$\begin{aligned}
 \mathcal{D}_{\text{HZZ}} &= -i g_F \frac{M_W}{c_W^2} \kappa_{D;LO}^{\text{HZZ}} + i \frac{g_F^3}{\pi^2} \sum_{I=t,b,W} \kappa_{I;D;NLO}^{\text{HZZ}} \mathcal{D}_{\text{HZZ};NLO}^I \\
 &\quad + i \frac{g_F^3}{\pi^2} \mathcal{D}_{\text{HZZ}}^{(4);nfc} + i \frac{g_F^3}{\pi^2} g_6 \sum_{\{a\}} \mathcal{D}_{\text{HZZ}}^{(6);nfc}(a), \\
 \mathcal{P}_{\text{HZZ}} &= 2i g_F g_6 \frac{a_{ZZ}}{M_W} + i \frac{g_F^3}{\pi^2} \sum_{I=t,b,W} \kappa_{I;P;NLO}^{\text{HZZ}} \mathcal{P}_{\text{HZZ};NLO}^I \\
 &\quad + i \frac{g_F^3}{\pi^2} g_6 \sum_{\{a\}} \mathcal{P}_{\text{HZZ}}^{(6);nfc}(a).
 \end{aligned} \tag{5.48}$$

Here we have introduced

$$\mathcal{D}_{\text{HZZ}}^{(4);nfc} = \frac{1}{32} \frac{M_W}{c_W^2} \left( 2 \sum_{\text{gen}} \overline{W_{Z;f}} - \sum_{\text{gen}} dZ_{M_W;f} + 4 \sum_{\text{gen}} \overline{dZ_{c_\theta;f}} - 2 \sum_{\text{gen}} dZ_{g;f} \right). \tag{5.49}$$

Dimension 4 sub-amplitudes  $\mathcal{D}_{\text{HZZ};NLO}^I$ ,  $\mathcal{P}_{\text{HZZ};NLO}^I$  ( $I = W, t, b$ ) are defined by using

$$\lambda_Z = M_H^2 - 4M_Z^2, \quad \lambda_{ZZ} = \frac{M_H^2}{M_W^2} - 4 \frac{M_Z^2}{M_W^2}, \tag{5.50}$$

and are given in appendix I. Non-factorizable dim = 6 amplitudes are reported in appendix H, using again eq. (5.50).

#### 5.4 H → WW

The derivation of the amplitude for H → WW follows closely the one for H → ZZ. There are two main differences, there are only 1PI contributions for H → WW and the process shows an infrared (IR) component.

The IR part originates from two different sources. Vertex diagrams generate an IR  $C_0$  function:

$$C_0 \left( -M_H^2, M_W^2, M_W^2; M_W, 0, M_W \right) = \frac{1}{\beta_W M_H^2} \ln \frac{\beta_W - 1}{\beta_W + 1} \Delta_{\text{IR}} + C_0^{\text{fin}} \left( -M_H^2, M_W^2, M_W^2; M_W, 0, M_W \right), \tag{5.51}$$

where we have introduced

$$\beta_W^2 = 1 - 4 \frac{M_W^2}{M_H^2}, \quad \Delta_{\text{IR}} = \frac{1}{\hat{\epsilon}} + \ln \frac{M_W^2}{\mu^2}. \tag{5.52}$$

The second source of IR behavior is found in the W wave-function factor:

$$W_W = -2s_\theta^2 \Delta_{\text{IR}} + W_W \Big|_{\text{fin}}. \tag{5.53}$$

The lowest-order part of the amplitude is

$$\mathcal{D}_{\text{HWW}}^{\text{LO}} = -gM. \quad (5.54)$$

The  $\mathcal{O}(gg_6)$  components of  $\text{H} \rightarrow \text{WW}$  are:

$$\begin{aligned} \mathcal{D}_{\text{HWW}}^{(6),a} &= \left( \frac{M_{\text{H}}^2 - 2M_{\text{W}}^2}{M} + M \right) a_{\phi\text{W}} - M a_{\phi\Box} + \frac{1}{4} M a_{\phi\text{D}} - M \frac{s_{\theta}^2}{c_{\theta}^2} a_{\text{AA}} - M \frac{s_{\theta}}{c_{\theta}} a_{\text{AZ}}, \\ \mathcal{D}_{\text{HWW}}^{(6),a} &= 2 \frac{1}{M} a_{\phi\text{W}}. \end{aligned} \quad (5.55)$$

With their help we can isolate the IR part of the  $\text{H} \rightarrow \text{WW}$  amplitude

$$\begin{aligned} \mathcal{D}_{\text{HWW}} \Big|_{\text{IR}} &= \text{I}_{\text{HWW}}^{\text{IR}} \frac{1}{\beta_{\text{W}} M_{\text{H}}^2} \ln \frac{\beta_{\text{W}} - 1}{\beta_{\text{W}} + 1} \Delta_{\text{IR}}, \\ \text{I}_{\text{HWW}}^{\text{IR}} &= \frac{1}{8} i \frac{g_{\text{F}}^2}{\pi^2} s_{\text{W}}^2 M_{\text{H}}^2 \left( \mathcal{D}_{\text{HWW}}^{\text{LO}} + g_6 \mathcal{D}_{\text{HWW}}^{(6),a} \right) \\ &\quad + \frac{1}{16} i \frac{g_{\text{F}}^2}{\pi^2} g_6 \mathcal{D}_{\text{HWW}}^{\text{LO}} \left[ 2 \left( M_{\text{H}}^2 + M_{\text{W}}^2 \right) \frac{M_{\text{H}}^2}{M_{\text{W}}^2} s_{\theta}^2 a_{\phi\text{W}} - 4 s_{\theta}^2 M_{\text{H}}^2 a_{\phi\text{W}} - 2 s_{\theta}^2 M_{\text{H}}^2 \right]. \end{aligned} \quad (5.56)$$

Having isolated the IR part of the amplitude we can repeat, step by step, the procedure developed in the previous sections. There is a non trivial aspect in the mixing of Wilson coefficients: the  $\text{dim} = 6$  parts of  $\text{H} \rightarrow \text{AA}, \text{AZ}$  and  $\mathcal{D}_{\text{HZZ,HWW}}^{(6)}$  contain UV divergences proportional to  $W_{1,2,3}$ ; once renormalization is completed for  $\text{H} \rightarrow \text{AA}, \text{AZ}$  and  $\text{ZZ}$  there is no freedom left and UV finiteness of  $\text{H} \rightarrow \text{WW}$  must follow, proving closure of the  $\text{dim} = 6$  basis with respect to renormalization.

We can now define  $\kappa$ -factors for  $\text{H} \rightarrow \text{WW}$ , see eq. (5.15). They are as follows:

$$\begin{aligned} \Delta\kappa_{\text{D};\text{LO}}^{\text{HWW}} &= s_{\text{W}}^2 \left( \frac{M_{\text{H}}^2}{M_{\text{W}}} - 5M_{\text{W}} \right) (a_{\text{AA}} + a_{\text{AZ}} + a_{\text{ZZ}}) + \frac{1}{2} M_{\text{W}} a_{\phi\text{D}} - 2M_{\text{W}} a_{\phi\Box}, \\ \Delta\kappa_{\text{q};\text{D};\text{NLO}}^{\text{HWW}} &= a_{\phi\text{tV}} + a_{\phi\text{tA}} + a_{\phi\text{bV}} + a_{\phi\text{bA}} - a_{\text{d}\phi} + 2a_{\phi\Box} - \frac{1}{2} a_{\phi\text{D}} \\ &\quad + s_{\text{W}} a_{\text{bWB}} + c_{\text{W}} a_{\text{bBW}} + 5s_{\text{W}}^2 a_{\text{AA}} + 5c_{\text{W}} s_{\text{W}} a_{\text{AZ}} + 5c_{\text{W}}^2 a_{\text{ZZ}}, \\ \Delta\kappa_{\text{W};\text{D};\text{NLO}}^{\text{HWW}} &= \frac{1}{96} \frac{s_{\text{W}}^2}{c_{\text{W}}^2} a_{\phi\text{D}} + \frac{23}{12} a_{\phi\Box} - \frac{35}{96} a_{\phi\text{D}} \\ &\quad + 4s_{\text{W}}^2 a_{\text{AA}} + \frac{1}{12} s_{\theta} \left( 3 \frac{1}{c_{\text{W}}} + 49c_{\text{W}} \right) a_{\text{AZ}} + \frac{1}{2} \left( 9c_{\text{W}}^2 + s_{\text{W}}^2 \right) a_{\text{ZZ}}, \\ \Delta\kappa_{\text{Q};\text{P};\text{NLO}}^{\text{HWW}} &= a_{\phi\text{tV}} + a_{\phi\text{tA}} + a_{\phi\text{bV}} + a_{\phi\text{bA}} - a_{\text{d}\phi} + 2a_{\phi\Box} - \frac{1}{2} a_{\phi\text{D}} \\ &\quad + s_{\text{W}} a_{\text{dWB}} + c_{\text{W}} a_{\text{dBW}} + 5s_{\text{W}}^2 a_{\text{AA}} + 5c_{\text{W}} s_{\text{W}} a_{\text{AZ}} + 5c_{\text{W}}^2 a_{\text{ZZ}}, \\ \Delta\kappa_{\text{W};\text{P};\text{NLO}}^{\text{HWW}} &= 7a_{\phi\Box} - 2a_{\phi\text{D}} + 5s_{\text{W}}^2 a_{\text{AA}} + 5c_{\text{W}} s_{\text{W}} a_{\text{AZ}} + 5c_{\text{W}}^2 a_{\text{ZZ}}. \end{aligned} \quad (5.59)$$

Next we obtain the final result for the amplitudes

$$\mathcal{D}_{\text{HWW}} = -i g_{\text{F}} M_{\text{W}} \kappa_{\text{D};\text{LO}}^{\text{HWW}} + i \frac{g_{\text{F}}^3}{\pi^2} \sum_{\text{I=q,W}} \kappa_{\text{I};\text{D};\text{NLO}}^{\text{HWW}} \mathcal{D}_{\text{HWW};\text{NLO}}^{\text{I}}$$

$$\begin{aligned}
& +i\frac{g_F^3}{\pi^2}\mathcal{D}_{\text{HWW}}^{(4);\text{nfc}} + i\frac{g_F^3}{\pi^2}g_6\sum_{\{a\}}\mathcal{D}_{\text{HWW}}^{(6);\text{nfc}}(a), \\
\mathcal{P}_{\text{HWW}} & = 2ig_Fg_6\frac{1}{M_W}\left(s_W^2a_{AA} + c_W^2a_{ZZ} + c_Ws_Wa_{AZ}\right) + i\frac{g_F^3}{\pi^2}\sum_{I=q}\kappa_{I;P;\text{NLO}}^{\text{HWW}}\mathcal{P}_{\text{HWW};\text{NLO}}^I \\
& + i\frac{g_F^3}{\pi^2}g_6\sum_{\{a\}}\mathcal{P}_{\text{HWW}}^{(6);\text{nfc}}(a), \tag{5.60}
\end{aligned}$$

where we have introduced

$$\mathcal{D}_{\text{HWW}}^{(4);\text{nfc}} = \frac{1}{32}M_W\overline{\sum}_{\text{gen}}\left(2W_{W;f} - d\mathcal{L}_{M_W;f} - 2d\mathcal{L}_{g;f}\right). \tag{5.61}$$

Dimension 4 sub-amplitudes  $\mathcal{P}_{\text{HWW};\text{NLO}}^I$ ,  $\mathcal{P}_{\text{HWW};\text{NLO}}^I$  ( $I = W, t, b$ ) are defined by introducing

$$\lambda_W = M_H^2 - 4M_W^2, \quad \lambda_{ww} = \frac{M_H^2}{M_W^2} - 4, \tag{5.62}$$

and are given in appendix I. Non-factorizable  $\text{dim} = 6$  amplitudes are reported in appendix H, using again eq. (5.62).

### 5.5 $H \rightarrow \bar{b}b(\tau^+\tau^-)$ and $H \rightarrow 4l$

These processes share the same level of complexity of  $H \rightarrow ZZ(WW)$ , including the presence of IR singularities. They will be discussed in details in a forthcoming publication.

## 6 ElectroWeak precision data

EFT is not confined to describe Higgs couplings and their SM deviations. It can be used to reformulate the constraints coming from electroweak precision data (EWPD), starting from the S, T and U parameters of ref. [76] and including the full list of LEP pseudo-observables (PO).

There are several ways for incorporating EWPD: the preferred option, so far, is reducing (a priori) the number of  $\text{dim} = 6$  operators. More generally, one could proceed by imposing penalty functions  $\omega$  on the global LHC fit, that is functions defining an  $\omega$ -penalized LS estimator for a set of global penalty parameters (perhaps using *merit functions* and the *homotopy method*). One could also consider using a Bayesian approach [78], with a flat prior for the parameters. Open questions are: one  $\kappa$  at the time? Fit first to the EWPD and then to H observables? Combination of both?

In the following we give a brief description of our procedure: from eq. (4.12) and eq. (4.48) we obtain

$$\Sigma_{AA}^{\text{1PI+1PR}}(s) = -\left[\Pi_{AA}^{\text{1PI}}(0) + g_6\frac{s_\theta}{c_\theta}\frac{1}{M_0^2}a_{AZ}\Pi_{ZA}^{\text{1PI}}(0)\right]s + \mathcal{O}(s^2) = \Pi_{AA}^c s + \mathcal{O}(s^2). \tag{6.1}$$

From eq. (4.20) and eq. (4.48) we obtain

$$\Sigma_{ZA}^{\text{1PI+1PR}}(s) = -\left[\Pi_{ZA}^{\text{1PI}}(0) + g_6s_\theta^2a_{AZ}\Pi_{AA}^{\text{1PI}}(0)\right]s + \mathcal{O}(s^2) = \Pi_{ZA}^c s + \mathcal{O}(s^2). \tag{6.2}$$

From eq. (4.16) and eq. (4.48) we derive

$$D_{ZZ}^{\text{1PI+1PR}}(s) = \Delta_{ZZ}(0) + \left[ \Omega_{ZZ}(0) - g_6 \frac{s_\theta}{c_\theta} a_{AZ} \Pi_{ZA}^{\text{1PI}}(0) \right] s + \mathcal{O}(s^2) = \Delta_{ZZ}(0) + \Omega_{ZZ}^c(0) s + \mathcal{O}(s^2). \quad (6.3)$$

Similarly, we obtain

$$D_{WW}^{\text{1PI}}(s) = \Delta_{WW}(0) + \Omega_{WW}(0) s + \mathcal{O}(s^2). \quad (6.4)$$

The S, T and U parameters are defined in terms of (complete) self-energies at  $s = 0$  and of their (first) derivatives. However, one has to be careful because the corresponding definition (see ref. [76]) is given in the  $\{\alpha, G_F, M_Z\}$  scheme, while we have adopted the more convenient  $\{G_F, M_W, M_Z\}$  scheme. Working in the  $\alpha$ -scheme has one advantage, the possibility of predicting the W (on-shell) mass. After UV renormalization and finite renormalization in the  $\alpha$ -scheme we define  $M_{W;\text{OS}}^2$  as the zero of the real part of the inverse W propagator and derive the effect of  $\text{dim} = 6$  operators.

### 6.1 W mass

Working in the  $\alpha$ -scheme we can predict  $M_W$ . The solution is

$$\begin{aligned} \frac{M_W^2}{M_Z^2} = \hat{c}_\theta^2 + \frac{\alpha}{\pi} \text{Re} \left\{ \left( 1 - \frac{1}{2} g_6 a_{\phi D} \right) \Delta_B^{(4)} M_W + \sum_{\text{gen}} \left[ \left( 1 + 4 g_6 a_{\phi 1}^{(3)} \right) \Delta_1^{(4)} M_W + \left( 1 + 4 g_6 a_{\phi q}^{(3)} \right) \Delta_q^{(4)} M_W \right] \right. \\ \left. + g_6 \left[ \Delta_B^{(6)} M_W + \sum_{\text{gen}} \left( \Delta_1^{(6)} M_W + \Delta_q^{(6)} M_W \right) \right] \right\}. \end{aligned} \quad (6.5)$$

where  $\hat{c}_\theta^2$  is defined in eq. (4.73) and we drop the subscript OS (on-shell). Corrections are given in appendix J. The expansion in eq. (6.5) can be improved when working within the SM ( $\text{dim} = 4$ ), see ref. [44]: for instance, the expansion parameter is set to  $\alpha(M_Z)$  instead of  $\alpha(0)$ , etc. Any equation that gives  $\text{dim} = 6$  corrections to the SM result will always be understood as

$$\mathcal{O} = \mathcal{O}^{\text{SM}} \Big|_{\text{imp}} + \frac{\alpha}{\pi} g_6 \mathcal{O}^{(6)} \quad (6.6)$$

in order to match the TOPAZ0/Zfitter SM results when  $g_6 \rightarrow 0$ , see refs. [79–81] and refs. [82, 83].

### 6.2 S, T and U parameters

The S, U and T (the original  $\rho$ -parameter of Veltman [84]) are defined as follows:

$$\begin{aligned} \alpha T &= \frac{1}{M_W^2} D_{WW}(0) - \frac{1}{M_Z^2} D_{ZZ}(0), \\ \frac{\alpha}{4 \hat{s}_\theta^2 \hat{c}_\theta^2} S &= \Omega_{ZZ}(0) - \frac{\hat{c}_\theta^2 - \hat{s}_\theta^2}{\hat{s}_\theta \hat{c}_\theta} \Pi_{ZA}(0) - \Pi_{AA}(0), \\ \frac{\alpha}{4 \hat{s}_\theta^2} U &= \Omega_{WW}(0) - \hat{c}_\theta^2 \Omega_{ZZ} \Pi_{ZA}(0) - \hat{s}_\theta^2 \Pi_{AA}(0), \end{aligned} \quad (6.7)$$

where all the self-energies are renormalized and  $\hat{s}_\theta$  is defined in eq. (4.73). One of the interesting properties of these parameters is that, within the SM, they are UV finite, i.e. all UV divergences

cancel in eq. (6.7) if they are written in terms of bare parameters and bare self-energies. When  $\dim = 6$  operators are inserted we obtain the following results:

$$\begin{aligned}
 \alpha T &= \alpha \mathcal{T}^{(4)} + \alpha g_6 \mathcal{T}^{(6)}, \\
 \alpha S &= \alpha \mathcal{S}^{(4)} - 4g_6 \frac{1 - 2\hat{s}_\theta^2}{\hat{s}_\theta \hat{c}_\theta} a_{AZ} + \alpha g_6 \mathcal{S}^{(6)}, \\
 \alpha U &= \alpha \mathcal{U}^{(4)} - 4g_6 \frac{1 - 2\hat{s}_\theta^2}{\hat{s}_\theta \hat{c}_\theta^3} a_{AZ} + \alpha g_6 \mathcal{U}^{(6)}
 \end{aligned} \tag{6.8}$$

and the introduction of counterterms is crucial to obtain an UV finite results. Explicit results for the T parameter are given in appendix K where, for simplicity, we only include PTG operators in loops.

## 7 Conclusions

In this paper we have developed a theory for Standard Model deviations based on the Effective Field Theory approach. In particular, we have considered the introduction of  $\dim = 6$  operators and extended their application at the NLO level (for a very recent development see ref. [85]).

The main result is represented by a consistent generalization of the LO  $\kappa$ -framework, currently used by ATLAS and CMS. We believe that the generalized  $\kappa$ -framework provides a useful technical tool to decompose amplitudes at NLO accuracy into a sum of well defined gauge-invariant sub components.

This step forward is better understood when comparing the present situation with the one at LEP; there the dynamics was fully described within the SM, with  $M_H \alpha_s(M_Z), \dots$  as unknowns. Today, post the LHC discovery of a H-candidate, unknowns are SM-deviations. This fact poses precise questions on the next level of dynamics. A specific BSM model is certainly a choice but one would like to try a more model independent approach.

The aim of this paper is to propose a decomposition where dynamics is controlled by  $\dim = 4$  amplitudes (with known analytical properties) and deviations (with a direct link to UV completions) are (constant) combinations of Wilson coefficients for  $\dim = 6$  operators.

Generalized  $\kappa$ -parameters form hyperplanes in the space of Wilson coefficients; each  $\kappa$ -plane describes (tangent) flat directions while normal directions are blind. Finally,  $\kappa$ -planes intersect, providing correlations among different processes.

There are many alternatives for extracting informations on Higgs couplings from the data, all of them allowing a theoretically robust matching between theory and experiments; one can start from direct extraction of Wilson coefficients or use combinations of Wilson coefficients. We do not claim any particular advantage in selecting generalized  $\kappa$ -parameters as LHC observables. Our NLO expressions have a general validity and can be used to construct any set of independent combinations of Wilson coefficients at NLO accuracy.

## 8 Note added in proof

In order to compare our results with those in ref. [85] we have to take into account the different schemes adopted, in particular the different treatment for the  $Z$ – $A$  transition. Therefore, we define

$$\mathcal{L}_{\text{EFT}}^{\text{WB}} = \mathcal{L}_4 + \left(\frac{g_0}{g}\right)^2 a_{\phi\text{W}} \mathcal{O}_{\phi\text{W}} + a_{\phi\text{B}} \mathcal{O}_{\phi\text{B}} + \frac{g_0}{g} a_{\phi\text{WB}} \mathcal{O}_{\phi\text{WB}}, \quad (8.1)$$

where  $g_0$  is given in eq. (2.14). Furthermore, we define

$$a_{\phi\text{B}} = \frac{s_\theta^2}{c_\theta^2} C_{\text{HB}}, \quad a_{\phi\text{W}} = C_{\text{HW}}, \quad a_{\phi\text{WB}} = -\frac{s_\theta}{c_\theta} C_{\text{HWB}}. \quad (8.2)$$

When our results are translated in terms of the  $C$ -coefficients of ref. [85] we find perfect agreement for the universal part of the running (the  $L_{\text{R}}$ -terms in our language), i.e. for those logarithms that do not depend on the renormalization scheme ( $G_{\text{F}}$ -scheme in our case). We would like to thank Christine Hartmann and Michael Trott for their help in clarifying the issue.

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## A $\bar{\beta}_{\text{h}}$ and $\Gamma$

In this appendix we present the full result for  $\bar{\beta}_{\text{h}}$ , defined in eq. (4.7). We have introduced ratios of masses

$$x_{\text{H}} = \frac{M_{\text{H}}}{M_{\text{W}}}, \quad x_{\text{f}} = \frac{M_{\text{f}}}{M_{\text{W}}} \quad (A.1)$$

etc. The various components are given by

$$\begin{aligned} \beta_{-1}^{(4)} &= -\sum_{\text{gen}} (x_{\text{t}}^2 + 3x_{\text{d}}^2 + 3x_{\text{u}}^2) + \frac{1}{8} (12 + 2x_{\text{H}} + 3x_{\text{H}}^2) + \frac{1}{8} \frac{6 + c_\theta^2 x_{\text{H}}}{c_\theta^4} \\ \beta_0^{(4)} &= -\frac{1}{2} \frac{1 + 2c_\theta^4}{c_\theta^4} \\ \beta_{\text{fin}}^{(4)} &= -\frac{1}{4} a_0^{\text{fin}}(M_{\text{W}}) (6 + x_{\text{H}}) - \frac{3}{8} a_0^{\text{fin}}(M_{\text{H}}) x_{\text{H}}^2 - \frac{1}{8} \frac{6 + c_\theta^2 x_{\text{H}}}{c_\theta^4} a_0^{\text{fin}}(M_{\text{Z}}) \\ &\quad + \sum_{\text{gen}} \left[ 3a_0^{\text{fin}}(M_{\text{u}}) x_{\text{u}}^2 + 3a_0^{\text{fin}}(M_{\text{d}}) x_{\text{d}}^2 + a_0^{\text{fin}}(M_{\text{l}}) x_{\text{l}}^2 \right] \end{aligned} \quad (A.2)$$

$$\beta_{-1}^{(6)} = \frac{1}{8} \frac{12 + x_{\text{H}}}{c_\theta^2} a_{\phi\text{W}} + \frac{1}{8} (36 + 2x_{\text{H}} + 3x_{\text{H}}^2) a_{\phi\text{W}} - \frac{1}{4} \sum_{\text{gen}} \left[ 3(4a_{\text{u}\phi} + 4a_{\phi\text{d}} + 4a_{\phi\text{W}} - a_{\phi\text{D}}) x_{\text{u}}^2 \right]$$

$$\begin{aligned}
 & -3(4a_{d\phi} - 4a_{\phi\Box} - 4a_{\phi W} + a_{\phi D})x_d^2 + (4a_{\phi\Box} + 4a_{\phi W} - a_{\phi D} - 4a_{L\phi})x_1^2] \\
 & + \frac{3}{16}(4a_{\phi\Box} + 4a_{\phi W} + a_{\phi D} + 8s_\theta^2 a_{\phi B} - 8c_\theta s_\theta a_{\phi WB})\frac{1}{c_\theta^4} + \frac{1}{32}(4a_{\phi\Box} - a_{\phi D})(12 - 2x_H + 7x_H^2) \\
 & - \frac{1}{32}(4a_{\phi\Box} + a_{\phi D} + 96c_\theta^2 a_\phi)\frac{x_H}{c_\theta^2} \\
 \beta_0^{(6)} & = -\frac{1}{8}\left[8s_\theta^2 a_{\phi B} + 4(1 + 2c_\theta^4)a_{\phi\Box} + 4(1 + 2c_\theta^2 + 6c_\theta^4)a_{\phi W} + (1 - 2c_\theta^4)a_{\phi D} - 8c_\theta s_\theta a_{\phi WB}\right]\frac{1}{c_\theta^4} \\
 \beta_{\text{fin}}^{(6)} & = 3a_0^{\text{fin}}(M_H) a_\phi x_H - \frac{1}{4}a_0^{\text{fin}}(M_W)(18 + x_H)a_{\phi W} - \frac{1}{8}a_0^{\text{fin}}(M_Z)\frac{12 + x_H}{c_\theta^2}a_{\phi W} \\
 & + \frac{1}{4}\sum_{\text{gen}}\left[3(4a_{u\phi} + 4a_{\phi\Box} + 4a_{\phi W} - a_{\phi D})a_0^{\text{fin}}(M_u)x_u^2 - 3(4a_{d\phi} - 4a_{\phi\Box} - 4a_{\phi W} + a_{\phi D})a_0^{\text{fin}}(M_d)x_d^2 \right. \\
 & \left. + (4a_{\phi\Box} + 4a_{\phi W} - a_{\phi D} - 4a_{L\phi})a_0^{\text{fin}}(M_l)x_l^2\right] \\
 & - \frac{3}{16}(4a_{\phi\Box} + 4a_{\phi W} + a_{\phi D} + 8s_\theta^2 a_{\phi B} - 8c_\theta s_\theta a_{\phi WB})\frac{1}{c_\theta^4}a_0^{\text{fin}}(M_Z) - \frac{1}{16}(4a_{\phi\Box} - a_{\phi D})a_0^{\text{fin}}(M_W)(6 - x_H) \\
 & + \frac{1}{32}(4a_{\phi\Box} + a_{\phi D})a_0^{\text{fin}}(M_Z)\frac{x_H}{c_\theta^2} - \frac{1}{32}(28a_{\phi\Box} + 12a_{\phi W} - 7a_{\phi D})a_0^{\text{fin}}(M_H)x_H^2 \tag{A.3}
 \end{aligned}$$

We also present the full result for  $\Gamma$ , defined in eq. (2.14). We have

$$\Gamma_{-1}^{(4)} = -\frac{1}{8} \quad \Gamma_0^{(4)} = \frac{1}{8} \quad \Gamma_{\text{fin}}^{(4)} = \frac{1}{8}a_0^{\text{fin}}(M) \tag{A.4}$$

$$\Gamma_{-1}^{(6)} = -\frac{1}{4}a_{\phi W} \quad \Gamma_0^{(6)} = \frac{1}{4}a_{\phi W} \quad \Gamma_{\text{fin}}^{(6)} = \frac{1}{4}a_0^{\text{fin}}(M)a_{\phi W} \tag{A.5}$$

i.e.  $\Gamma = \Gamma^{(4)}(1 + 2g_6 a_{\phi W})$ .

## B Renormalized self-energies

In this appendix we present the full set of renormalized self-energies. To keep the notation as compact as possible a number of auxiliary quantities has been introduced.

### B.1 Notations

First we define the following set of polynomials:

$\boxed{F}$  where  $s = s_\theta$  and  $c = c_\theta$

$$\begin{aligned}
 F_0^s &= 1 - 6c & F_1^s &= 4 - 9c & F_2^s &= 1 - c \\
 F_3^s &= 2 - 15c & F_4^s &= 2 + 3c & F_5^s &= 11 - 3s \\
 F_6^s &= 8 - 3s
 \end{aligned}$$

$F_0^b = 1 + 2c$	$F_1^b = 1 + 3c$	$F_2^b = 1 + 18c$
$F_3^b = 1 + c$	$F_4^b = 1 + 4c$	$F_5^b = 1 - 2s$
$F_6^b = 1 + 24s^2c$	$F_7^b = 1 + F_0^a c$	$F_8^b = 1 + 4F_1^a c$
$F_9^b = 3 + 4c$	$F_{10}^b = 5 + 8c$	$F_{11}^b = 1 - 40c + 36sc$
$F_{12}^b = 1 + 20c - 12sc$	$F_{13}^b = 5 - 20c + 12sc$	$F_{14}^b = 5 - 3s$
$F_{15}^b = 1 - 12F_2^a c$	$F_{16}^b = 3 + 8sc$	$F_{17}^b = 13 + 4F_3^a c$
$F_{18}^b = 19 - 18s$	$F_{19}^b = 1 - 12c^2$	$F_{20}^b = 1 - 24c^3$
$F_{21}^b = 1 + 4F_4^a c$	$F_{22}^b = 1 + 8F_4^a c^2$	$F_{23}^b = 4 - 3s$
$F_{24}^b = 5 + 12c^2$	$F_{25}^b = 5 - 4F_4^a c$	$F_{26}^b = 13 - F_5^a s$
$F_{27}^b = 21 - 4F_6^a s$	$F_{28}^b = 39 - 40s$	$F_{29}^b = 1 - 4sc$
$F_{30}^b = 3 - 10c$	$F_{31}^b = 3 - 2s$	$F_{32}^b = c + 2s$
$F_{33}^b = 4 - 7c$	$F_{34}^b = 2 + c$	

G where we have introduced

$$v_f = 1 - 2 \frac{Q_f}{I_f^3} s_\theta^2$$

where  $Q_1 = -1$ ,  $Q_u = \frac{2}{3}$  and  $Q_d = -\frac{1}{3}$ ;  $I_f^3$  is the third component of isospin. Furthermore

$$v_{\text{gen}}^{(1)} = v_1^2 + 3 \left( v_u^2 + v_d \right) \quad v_{\text{gen}}^{(2)} = v_1^2 + 2v_u^2 + v_d$$

$G_0 = 1 - 3v_d$	$G_1 = 3 - v_1$	$G_2 = 5 - 3v_u$
$G_3 = 20 - 3v_{\text{gen}}^{(2)}$	$G_4 = 1 + v_u^2$	$G_5 = 1 + v_d^2$
$G_6 = 1 + v_1^2$	$G_7 = 9 + v_{\text{gen}}^{(1)}$	$G_8 = 2 - v_u^2$
$G_9 = 2 - v_d^2$	$G_{10} = 2 - v_1^2$	$G_{11} = 1 - v_1$
$G_{12} = 1 - v_u$	$G_{13} = 1 + v_u$	$G_{14} = 1 - v_d$
$G_{15} = 1 + v_d$	$G_{16} = 1 - 3v_1$	$G_{17} = 1 + v_1$
$G_{18} = 2 + v_u + v_d$	$G_{19} = 3 - 5v_u$	$G_{20} = 3 - v_d$
$G_{21} = 3 + v_1$	$G_{22} = 9 - 5v_u - v_d - 3v_1$	$G_{23} = v_u - v_d$
$G_{24} = 2 - v_u$	$G_{25} = 2 + v_u$	$G_{26} = 2 - v_d$
$G_{27} = 2 + v_d$	$G_{28} = 2 - v_1$	$G_{29} = 2 + v_1$
$G_{30} = 2 + 3v_1$	$G_{31} = 6 + 5v_u$	$G_{32} = 6 + v_d$
$G_{33} = 1 - v_1^2$	$G_{34} = 1 - 2v_1$	$G_{35} = 1 + 2v_1$
$G_{36} = 1 + 7v_1$	$G_{37} = 3 - 4v_1$	$G_{38} = 3 + 4v_1$
$G_{39} = 8 + 3v_1$	$G_{40} = 1 - 7v_1$	$G_{41} = 7 - v_1$
$G_{42} = 1 - v_u^2$	$G_{43} = 1 - 2v_u$	$G_{44} = 1 + 2v_u$
$G_{45} = 3 - 4v_u$	$G_{46} = 3 + 4v_u$	$G_{47} = 3 + 13v_u$
$G_{48} = 4 + 3v_u$	$G_{49} = 16 + 9v_u$	$G_{50} = 3 - 13v_u$
$G_{51} = 13 - 3v_u$	$G_{52} = 1 - v_d^2$	$G_{53} = 1 - 2v_d$
$G_{54} = 1 + 2v_d$	$G_{55} = 2 + 3v_d$	$G_{56} = 3 - 4v_d$
$G_{57} = 3 + 4v_d$	$G_{58} = 3 + 5v_d$	$G_{59} = 8 + 9v_d$
$G_{60} = 3 - 5v_d$	$G_{61} = 5 - 3v_d$	



**H** where we have introduced

$$x_f = \frac{M_f}{M} \quad \text{etc.}$$

$H_0 = 3x_u^2 + 3x_d^2 + x_1^2$	$H_1 = 3x_u^4 + 3x_d^4 + x_1^4$	$H_2 = 4x_u^2 + x_d^2 + 3x_1^2$
$H_3 = -2x_u^2 + x_d^2$	$H_4 = 2x_u^2 + x_d^2$	$H_5 = 10x_u^2 + x_d^2 + 9x_1^2$
$H_6 = -x_u^2 + x_d^2$	$H_7 = x_u^2 + x_d^2$	$H_8 = x_u^4 - 2x_d^2 x_u^2 + x_d^4$
$H_9 = 1 + x_d^2$	$H_{10} = 1 - x_d^2$	$H_{11} = 2 - x_d^2 - x_d^4$
$H_{12} = 2 + x_d^2$	$H_{13} = 2 - x_d^2$	$H_{14} = 1 + x_u^2$
$H_{15} = 1 - x_u^2$	$H_{16} = 2 - x_u^2 - x_u^4$	$H_{17} = 2 + x_u^2$
$H_{18} = 2 - x_u^2$		

**I** where we have introduced

$$x_F = \frac{M_F}{M} \quad \text{etc.}$$

$I_0 = 2 + x_L^2$	$I_1 = 2 - x_L^2$	$I_2 = 1 + 2x_L^2$
$I_3 = 2 + 3x_L^2$	$I_4 = 4 - x_L^2$	$I_5 = 4 + x_L^2$
$I_6 = 1 + 2x_U^2$	$I_7 = 1 + x_U^2$	$I_8 = 2 + 3x_U^2$
$I_9 = 1 + 2x_D^2$	$I_{10} = 1 + x_D^2$	$I_{11} = 2 + 3x_D^2$

**J** where we have introduced

$$x_H = \frac{M_H}{M} \quad x_S = \frac{s}{M^2}$$

$J_0 = 2 + x_H^4$	$J_1 = 12 + x_H^4$	$J_2 = 2 - x_H^2$
$J_3 = 4 - 7x_H^2$	$J_4 = 4 - 5x_H^2$	$J_5 = 12 - x_H^4$
$J_6 = 2 + 11x_H^2$	$J_7 = 3 + 4x_H^4$	$J_8 = 4 + 5x_H^4$
$J_9 = 10 + x_H^4$	$J_{10} = 36 + x_H^4$	$J_{11} = 4 + 3x_S$
$J_{12} = 2 + x_H^2$	$J_{13} = 3 + x_H^2$	$J_{14} = 4 + x_S$
$J_{15} = 6 - x_H^2$	$J_{16} = 8 - x_S$	$J_{17} = 8 + x_S$
$J_{18} = 10 - x_H^2$	$J_{19} = 12 + x_S$	$J_{20} = 2 - 3x_H^2$
$J_{21} = 12 - x_S$	$J_{22} = 12 + 5x_S$	$J_{23} = 12 - x_H^2$
$J_{24} = 32 - 3x_S$	$J_{25} = 32 + x_S$	$J_{26} = 48 + x_S$
$J_{27} = 2x_S - x_H^2$	$J_{28} = 2x_S + x_H^2$	$J_{29} = 5x_S - x_H^2$
$J_{30} = 19 - 3x_H^2$	$J_{31} = 58 - 3x_H^2$	$J_{32} = 70 - 3x_H^2$
$J_{33} = 106 - 3x_H^2$	$J_{34} = x_S - x_H^2$	$J_{35} = 1 + x_H^2$
$J_{36} = 8 + 3x_H^2$	$J_{37} = 11x_S - x_H^2$	$J_{38} = 12x_S - 2x_H^2 x_S + x_H^4$

$$\begin{array}{lll}
 J_{39} = 1 - 2x_S & J_{40} = 1 + 2x_S & J_{41} = 1 + 2x_S - x_H^2 \\
 J_{42} = 1 + 10x_S & J_{43} = 1 + 10x_S - 2x_H^2 - 2x_H^2x_S + x_H^4 & J_{44} = 1 + 18x_S \\
 J_{45} = 2 - 68x_S - x_H^2 & J_{46} = 3 + 5x_S & J_{47} = 42 + x_H^2 \\
 J_{48} = 9 - 5x_S & J_{49} = 1 + 5x_S & J_{50} = 3 + x_S \\
 J_{51} = 7 - 5x_S & J_{52} = 8 - 5x_S & J_{53} = 13 + 5x_S \\
 J_{54} = 20 - 3x_H^2 & J_{55} = 23 - 15x_S & J_{56} = 1 + 3x_S \\
 J_{57} = 1 - 7x_S & J_{58} = 1 - 9x_S & J_{59} = 1 - 5x_S \\
 J_{60} = 1 - 3x_S & J_{61} = 1 - x_S & J_{62} = 1 + x_S - x_H^2 \\
 J_{63} = 1 + 4x_S & J_{64} = 1 + 9x_S & J_{65} = 1 + 12x_S \\
 J_{66} = 1 + 14x_S & J_{67} = 1 + 14x_S - x_H^2 & J_{68} = 1 + 22x_S - 2x_H^2 - 14x_H^2x_S + x_H^4 \\
 J_{69} = 2 - 56x_S - x_H^2 & J_{70} = 2 - x_S & J_{71} = 2 + 3x_S \\
 J_{72} = 2 + 5x_S & J_{73} = 3 + 14x_S & J_{74} = 8 + 24x_S - x_H^2 \\
 J_{75} = 20 - x_H^2 & J_{76} = 1 + x_S &
 \end{array}$$

**K**

$$\begin{array}{lll}
 K_0 = x_S + 2x_U^2 & K_1 = x_S + 2x_D^2 & K_2 = x_S + 2x_1^2 \\
 K_3 = x_S - 6x_1^2 & K_4 = x_S + x_1^2 & K_5 = 2x_S - x_1^2 \\
 K_6 = 2 - x_S & K_7 = 2 + x_S &
 \end{array}$$

**L**

$$\begin{array}{lll}
 L_0 = -x_L^2 + x_S & L_1 = -2x_L^2 + x_S & L_2 = x_L^2 + x_S \\
 L_3 = -x_L^2 + 2x_S & L_4 = x_L^2 + 2x_S & L_5 = xL^4 - 2x_Sx_L^2 + x_S^2 \\
 L_6 = -x_U^2 + x_S & L_7 = -2x_U^2 + x_S & L_8 = x_U^2 + x_S \\
 L_9 = -x_U^2 + 2x_S & L_{10} = x_U^4 - 2x_Sx_U^2 + x_S^2 & L_{11} = -x_D^2 + x_S \\
 L_{12} = -2x_D^2 + x_S & L_{13} = x_D^2 + x_S & L_{14} = -x_D^2 + 2x_S \\
 L_{15} = x_D^4 - 2x_Sx_D^2 + x_S^2 & &
 \end{array}$$

## B.2 Renormalized self-energies

The (renormalized) bosonic self-energies are decomposed according to

$$D_{ij}(s) = \frac{g^2}{16\pi^2} \left[ \Delta_{ij}^{(4)}(s) M_W^2 + \Pi_{ij}^{(4)}(s) s + g_6 \left( \Delta_{ij}^{(6)}(s) M_W^2 + \Pi_{ij}^{(6)}(s) s \right) \right] \quad (\text{B.1})$$

while the (renormalized) fermionic self-energies are decomposed as

$$S_f = \frac{g^2}{16\pi^2} \left[ V_{ff}^{(4)} i \not{p} + A_{ff}^{(4)} i \not{p} \gamma^5 + \Sigma_{ff}^{(4)} M_W + g_6 \left( V_{ff}^{(6)} i \not{p} + A_{ff}^{(6)} i \not{p} \gamma^5 + \Sigma_{ff}^{(6)} M_W \right) \right] \quad (\text{B.2})$$

In the following list we introduce a shorthand notation:

$$B_0^{\text{fin}}(m_1, m_2) = B_0^{\text{fin}}(-s; m_1, m_2) \quad (\text{B.3})$$

and several linear combinations of Wilson coefficients

$$\begin{aligned}
 a_{\phi W} &= c_\theta^2 a_{ZZ} + s_\theta^2 a_{AA} + c_\theta s_\theta a_{AZ} & a_{\phi B} &= c_\theta^2 a_{AA} + \hat{s}_\theta^2 a_{ZZ} - c_\theta s_\theta a_{AZ} \\
 a_{\phi WB} &= (1 - 2s_\theta^2) a_{AZ} + 2c_\theta s_\theta (a_{AA} - a_{ZZ}) & a_{\phi WB} &= c_\theta a_{\phi WA} - s_\theta a_{\phi WZ} \\
 a_{\phi W} &= s_\theta a_{\phi WA} + c_\theta a_{\phi WZ} & a_{\phi WB}^{(a)} &= a_{\phi B} - a_{\phi W} \\
 a_{\phi d} &= \frac{1}{2} (a_{\phi dA} - a_{\phi dV}) & a_{\phi u} &= \frac{1}{2} (a_{\phi uV} - a_{\phi uA}) \\
 a_{\phi q}^{(1)} &= \frac{1}{4} (a_{\phi uV} + a_{\phi uA} - a_{\phi dV} - a_{\phi dA}) & a_{\phi q}^{(3)} &= \frac{1}{4} (a_{\phi dV} + a_{\phi dA} + a_{\phi uV} + a_{\phi uA}) \\
 a_{\phi uVA} &= 2 (a_{\phi q}^{(3)} + a_{\phi q}^{(1)}) & a_{\phi dVA} &= 2 (a_{\phi q}^{(3)} - a_{\phi q}^{(1)}) \\
 \\ 
 a_{\phi l} &= \frac{1}{2} (a_{\phi lA} - a_{\phi lV}) & a_{\phi l}^{(1)} &= a_{\phi l}^{(3)} - \frac{1}{2} (a_{\phi lV} + a_{\phi lA}) \\
 a_{\phi l}^{(3)} &= \frac{1}{4} (a_{\phi lV} + a_{\phi lA} + a_{\phi lV}) & a_{\phi lVA} &= 2 (a_{\phi l}^{(3)} - a_{\phi l}^{(1)}) \\
 a_{lW} &= s_\theta a_{lWB} + c_\theta a_{lBW} & a_{lB} &= -c_\theta a_{lWB} + s_\theta a_{lBW} \\
 a_{uW} &= s_\theta a_{uWB} + c_\theta a_{uBW} & a_{uB} &= c_\theta a_{uWB} - s_\theta a_{uBW} \\
 a_{dW} &= s_\theta a_{dWB} + c_\theta a_{dBW} & a_{dB} &= -c_\theta a_{dWB} + s_\theta a_{dBW} \\
 a_{\phi lW}^{(3)} &= 4a_{\phi l}^{(3)} + 2a_{\phi W} & a_{\phi qW}^{(3)} &= 4a_{\phi q}^{(3)} + 2a_{\phi W} \\
 a_{\phi WD}^{(+)} + a_{\phi WD}^{(-)} &= 8a_{\phi W} & a_{\phi WD}^{(+)} - a_{\phi WD}^{(-)} &= 2a_{\phi D} \\
 a_{\phi WAB} &= a_{\phi WA} - 2s_\theta a_{\phi B} & a_{\phi WAD} &= 4s_\theta a_{\phi WA} - c_\theta^2 a_{\phi D} \\
 a_{l\phi\Box} &= a_{l\phi} - a_{\phi\Box} & a_{u\phi\Box} &= a_{u\phi} + a_{\phi\Box} \\
 a_{d\phi\Box} &= a_{d\phi} - a_{\phi\Box} & & 
 \end{aligned} \tag{B.4}$$

All functions in the following list are decomposed according to

$$F = \sum_{n=0}^2 F_n \tag{B.5}$$

where  $F_0$  is the constant part (containing a dependence on  $\mu_R$ ),  $F_1$  contains (finite) one-point functions and  $F_2$  the (finite) two-point functions. Capital letters (U etc.) denote a specific fermion, small letters (u etc.) are used when summing over fermions.

- H self-energy

$$\Pi_{HH;0}^{(4)} = \frac{1}{2} F_0^b \frac{L_R}{c^2} - \frac{1}{2} \sum_{\text{gen}} H_0 L_R$$

$$\Pi_{HH;1}^{(4)} = 0$$

$$\begin{aligned}
 \Pi_{HH;2}^{(4)} &= \frac{1}{2} \frac{1}{c^2} B_0^{\text{fin}}(M_0, M_0) + B_0^{\text{fin}}(M, M) - \frac{1}{2} \sum_{\text{gen}} x_1^2 B_0^{\text{fin}}(M_1, M_1) \\
 &\quad - \frac{3}{2} \sum_{\text{gen}} x_u^2 B_0^{\text{fin}}(M_u, M_u) - \frac{3}{2} \sum_{\text{gen}} x_d^2 B_0^{\text{fin}}(M_d, M_d)
 \end{aligned}$$

$$\Delta_{HH;0}^{(4)} = 2 + \frac{1}{2} \frac{1}{c^4} (2 - 3L_R) - \frac{3}{2} J_0 L_R + 2 \sum_{\text{gen}} H_1 L_R$$

$$\Delta_{\text{HH};1}^{(4)} = 0$$

$$\begin{aligned} \Delta_{\text{HH};2}^{(4)} = & -\frac{1}{4} J_1 B_0^{\text{fin}}(M, M) - \frac{9}{8} x_{\text{H}}^4 B_0^{\text{fin}}(M_{\text{H}}, M_{\text{H}}) + 2 \sum_{\text{gen}} x_1^4 B_0^{\text{fin}}(M_1, M_1) \\ & + 6 \sum_{\text{gen}} x_{\text{u}}^4 B_0^{\text{fin}}(M_{\text{u}}, M_{\text{u}}) + 6 \sum_{\text{gen}} x_{\text{d}}^4 B_0^{\text{fin}}(M_{\text{d}}, M_{\text{d}}) - \frac{1}{8} (12 + c^4 x_{\text{H}}^4) \frac{1}{c^4} B_0^{\text{fin}}(M_0, M_0) \end{aligned}$$

$$\begin{aligned} \Pi_{\text{HH};0}^{(6)} = & -4 a_{\phi\text{w}} (1 - 2 L_{\text{R}}) - \frac{1}{c^2} a_{\text{zz}} (2 - 3 L_{\text{R}}) \\ & + \frac{1}{8} \left[ (8 a_{\phi\text{w}} + a_{\phi\text{D}} + 8 a_{\phi\Box}) + (4 J_3 a_{\phi\Box} - J_4 a_{\phi\text{D}}) c^2 \right] \frac{L_{\text{R}}}{c^2} \\ & - \frac{1}{4} \sum_{\text{gen}} \left[ (a_{\phi\text{wD}}^{(-)} + 4 a_{\phi\Box}) H_0 + 4 (-3 x_{\text{d}}^2 a_{\text{d}\phi} + 3 x_{\text{u}}^2 a_{\text{u}\phi} - x_1^2 a_{1\phi}) \right] L_{\text{R}} \end{aligned}$$

$$\Pi_{\text{HH};1}^{(6)} = -\frac{1}{8} \frac{1}{c^2} a_{\phi\text{D}} a_0^{\text{fin}}(M_0) - \frac{1}{8} (a_{\phi\text{D}} - 4 a_{\phi\Box}) x_{\text{H}}^2 a_0^{\text{fin}}(M_{\text{H}})$$

$$\begin{aligned} \Pi_{\text{HH};2}^{(6)} = & \frac{1}{4} \left[ 32 a_{\phi\text{w}} - (a_{\phi\text{D}} - 4 a_{\phi\Box}) J_2 \right] B_0^{\text{fin}}(M, M) \\ & - \frac{1}{8} \left[ -8 (3 a_{\text{zz}} + a_{\phi\text{w}} + a_{\phi\Box}) + (a_{\phi\text{D}} + 4 a_{\phi\Box}) c^2 x_{\text{H}}^2 \right] \frac{1}{c^2} B_0^{\text{fin}}(M_0, M_0) \\ & + \frac{3}{8} (a_{\phi\text{D}} - 4 a_{\phi\Box}) x_{\text{H}}^2 B_0^{\text{fin}}(M_{\text{H}}, M_{\text{H}}) \\ & - \frac{1}{4} \sum_{\text{gen}} (a_{\phi\text{wD}}^{(-)} - 4 a_{1\phi\Box}) x_1^2 B_0^{\text{fin}}(M_1, M_1) - \frac{3}{4} \sum_{\text{gen}} (a_{\phi\text{wD}}^{(-)} - 4 a_{\text{d}\phi\Box}) x_{\text{d}}^2 B_0^{\text{fin}}(M_{\text{d}}, M_{\text{d}}) \\ & - \frac{3}{4} \sum_{\text{gen}} (a_{\phi\text{wD}}^{(-)} + 4 a_{\text{u}\phi\Box}) x_{\text{u}}^2 B_0^{\text{fin}}(M_{\text{u}}, M_{\text{u}}) \end{aligned}$$

$$\begin{aligned} \Delta_{\text{HH};0}^{(6)} = & \frac{1}{2} \left[ (6 a_{\phi} + a_{\phi\text{D}} x_{\text{H}}^2) + (6 J_6 a_{\phi} + J_7 a_{\phi\text{D}} - 3 J_8 a_{\phi\Box} - 6 J_9 a_{\phi\text{w}}) c^2 \right] \frac{L_{\text{R}}}{c^2} \\ & + \frac{1}{4} (16 a_{\text{zz}} + 4 a_{\phi\text{w}} + 3 a_{\phi\text{D}} + 4 a_{\phi\Box}) \frac{1}{c^4} (2 - 3 L_{\text{R}}) + (20 a_{\phi\text{w}} - a_{\phi\text{D}} + 4 a_{\phi\Box}) \\ & + \sum_{\text{gen}} \left[ (a_{\phi\text{wD}}^{(-)} + 4 a_{\phi\Box}) H_1 + 8 (-3 x_{\text{d}}^4 a_{\text{d}\phi} + 3 x_{\text{u}}^4 a_{\text{u}\phi} - x_1^4 a_{1\phi}) \right] L_{\text{R}} \end{aligned}$$

$$\begin{aligned} \Delta_{\text{HH};1}^{(6)} = & 4 \sum_{\text{gen}} x_1^4 a_{1\phi} a_0^{\text{fin}}(M_1) - 12 \sum_{\text{gen}} x_{\text{u}}^4 a_{\text{u}\phi} a_0^{\text{fin}}(M_{\text{u}}) + 12 \sum_{\text{gen}} x_{\text{d}}^4 a_{\text{d}\phi} a_0^{\text{fin}}(M_{\text{d}}) \\ & - \frac{3}{4} \left[ 20 a_{\phi} + (a_{\phi\text{D}} - 4 a_{\phi\Box}) x_{\text{H}}^2 \right] x_{\text{H}}^2 a_0^{\text{fin}}(M_{\text{H}}) \\ & - \frac{1}{4} \left[ c^2 a_{\phi\text{D}} x_{\text{H}}^2 - 6 (4 a_{\text{zz}} + a_{\phi\text{D}} - 2 c^2 a_{\phi}) \right] \frac{1}{c^4} a_0^{\text{fin}}(M_0) \\ & - 6 (a_{\phi} - 2 a_{\phi\text{w}}) a_0^{\text{fin}}(M) \end{aligned}$$

$$\begin{aligned} \Delta_{\text{HH};2}^{(6)} = & \frac{3}{16} \left[ 96 a_{\phi} + (-12 a_{\phi\text{w}} + 7 a_{\phi\text{D}} - 28 a_{\phi\Box}) x_{\text{H}}^2 \right] x_{\text{H}}^2 B_0^{\text{fin}}(M_{\text{H}}, M_{\text{H}}) \\ & + \frac{1}{16} \left[ 4 c^2 a_{\phi\text{D}} x_{\text{H}}^2 - 12 (a_{\phi\text{wD}}^{(+)} + 8 a_{\text{zz}} + 4 a_{\phi\Box}) - (a_{\phi\text{wD}}^{(-)} - 4 a_{\phi\Box}) c^4 x_{\text{H}}^4 \right] \frac{1}{c^4} B_0^{\text{fin}}(M_0, M_0) \\ & - \frac{1}{8} \left[ 4 J_{10} a_{\phi\text{w}} - (a_{\phi\text{D}} - 4 a_{\phi\Box}) J_5 \right] B_0^{\text{fin}}(M, M) \\ & + \sum_{\text{gen}} (a_{\phi\text{wD}}^{(-)} - 4 a_{1\phi\Box}) x_1^4 B_0^{\text{fin}}(M_1, M_1) \\ & + 3 \sum_{\text{gen}} (a_{\phi\text{wD}}^{(-)} - 4 a_{\text{d}\phi\Box}) x_{\text{d}}^4 B_0^{\text{fin}}(M_{\text{d}}, M_{\text{d}}) \end{aligned}$$

$$+3 \sum_{\text{gen}} (a_{\phi_{\text{WD}}}^{(-)} + 4a_{\text{u}\phi_{\square}}) x_{\text{u}}^4 B_0^{\text{fin}}(M_{\text{u}}, M_{\text{u}})$$

• A self-energy

$$\Pi_{\text{AA};0}^{(4)} = 3s^2 L_{\text{R}} + \frac{32}{27} s^2 N_{\text{gen}} (1 - 3L_{\text{R}}) + 4 \frac{s^2}{x_{\text{s}}} - \frac{8}{9} \sum_{\text{gen}} \frac{s^2}{x_{\text{s}}} H_2$$

$$\begin{aligned} \Pi_{\text{AA};1}^{(4)} &= 4 \frac{s^2}{x_{\text{s}}} a_0^{\text{fin}}(M) - \frac{8}{3} \sum_{\text{gen}} \frac{x_{\text{I}}^2}{x_{\text{s}}} s^2 a_0^{\text{fin}}(M_{\text{I}}) \\ &\quad - \frac{32}{9} \sum_{\text{gen}} \frac{x_{\text{u}}^2}{x_{\text{s}}} s^2 a_0^{\text{fin}}(M_{\text{u}}) - \frac{8}{9} \sum_{\text{gen}} \frac{x_{\text{d}}^2}{x_{\text{s}}} s^2 a_0^{\text{fin}}(M_{\text{d}}) \end{aligned}$$

$$\begin{aligned} \Pi_{\text{AA};2}^{(4)} &= \frac{s^2}{x_{\text{s}}} J_{11} B_0^{\text{fin}}(M, M) - \frac{4}{3} \sum_{\text{gen}} \frac{s^2}{x_{\text{s}}} K_2 B_0^{\text{fin}}(M_{\text{I}}, M_{\text{I}}) \\ &\quad - \frac{16}{9} \sum_{\text{gen}} \frac{s^2}{x_{\text{s}}} K_0 B_0^{\text{fin}}(M_{\text{u}}, M_{\text{u}}) - \frac{4}{9} \sum_{\text{gen}} \frac{s^2}{x_{\text{s}}} K_1 B_0^{\text{fin}}(M_{\text{d}}, M_{\text{d}}) \end{aligned}$$

$$\begin{aligned} \Pi_{\text{AA};0}^{(6)} &= \frac{16}{27} N_{\text{gen}} a_{\phi_{\text{WAD}}} (1 - 3L_{\text{R}}) + 2 \frac{1}{x_{\text{s}}} a_{\phi_{\text{WAD}}} - \frac{4}{9} \sum_{\text{gen}} \frac{1}{x_{\text{s}}} H_2 a_{\phi_{\text{WAD}}} \\ &\quad - \frac{1}{2} \left[ J_{13} c^2 a_{\phi_{\text{B}}} - J_{15} s c^3 a_{\phi_{\text{WB}}} + (3c^4 a_{\phi_{\text{D}}} + s a_{\phi_{\text{WA}}}) - (J_{12} a_{\phi_{\text{B}}} + J_{18} a_{\phi_{\text{W}}}) s^2 c^2 \right] \frac{L_{\text{R}}}{c^2} \\ &\quad + 2 \sum_{\text{gen}} (-x_{\text{d}}^2 a_{\text{dWB}} + 2x_{\text{u}}^2 a_{\text{uWB}} - x_{\text{I}}^2 a_{\text{IWB}}) s L_{\text{R}} \end{aligned}$$

$$\begin{aligned} \Pi_{\text{AA};1}^{(6)} &= \frac{1}{2} \frac{1}{c^2} a_{\text{AA}} a_0^{\text{fin}}(M_0) + \frac{1}{2} x_{\text{H}}^2 a_{\text{AA}} a_0^{\text{fin}}(M_{\text{H}}) - \frac{4}{3} \sum_{\text{gen}} \frac{x_{\text{I}}^2}{x_{\text{s}}} a_{\phi_{\text{WAD}}} a_0^{\text{fin}}(M_{\text{I}}) \\ &\quad - \frac{16}{9} \sum_{\text{gen}} \frac{x_{\text{u}}^2}{x_{\text{s}}} a_{\phi_{\text{WAD}}} a_0^{\text{fin}}(M_{\text{u}}) - \frac{4}{9} \sum_{\text{gen}} \frac{x_{\text{d}}^2}{x_{\text{s}}} a_{\phi_{\text{WAD}}} a_0^{\text{fin}}(M_{\text{d}}) \\ &\quad + \left[ J_{16} s c a_{\phi_{\text{WB}}} + J_{17} s^2 a_{\phi_{\text{W}}} + (x_{\text{s}} a_{\phi_{\text{B}}} - 2a_{\phi_{\text{D}}}) c^2 \right] \frac{1}{x_{\text{s}}} a_0^{\text{fin}}(M) \end{aligned}$$

$$\begin{aligned} \Pi_{\text{AA};2}^{(6)} &= \frac{1}{2} \left[ 4J_{14} s c a_{\phi_{\text{WB}}} + (-c^2 a_{\phi_{\text{D}}} + 4s^2 a_{\phi_{\text{W}}}) J_{11} \right] \frac{1}{x_{\text{s}}} B_0^{\text{fin}}(M, M) \\ &\quad - \frac{2}{9} \sum_{\text{gen}} (9s x_{\text{d}}^2 x_{\text{s}} a_{\text{dWB}} + K_1 a_{\phi_{\text{WAD}}}) \frac{1}{x_{\text{s}}} B_0^{\text{fin}}(M_{\text{d}}, M_{\text{d}}) \\ &\quad + \frac{4}{9} \sum_{\text{gen}} (9s x_{\text{u}}^2 x_{\text{s}} a_{\text{uWB}} - 2K_0 a_{\phi_{\text{WAD}}}) \frac{1}{x_{\text{s}}} B_0^{\text{fin}}(M_{\text{u}}, M_{\text{u}}) \\ &\quad - \frac{2}{3} \sum_{\text{gen}} (3s x_{\text{I}}^2 x_{\text{s}} a_{\text{IWB}} + K_2 a_{\phi_{\text{WAD}}}) \frac{1}{x_{\text{s}}} B_0^{\text{fin}}(M_{\text{I}}, M_{\text{I}}) \end{aligned}$$

• Z-A transition

$$\begin{aligned} \Pi_{\text{ZA};0}^{(4)} &= -\frac{1}{9} \frac{s}{c} N_{\text{gen}} v_{\text{gen}}^{(2)} (1 - 3L_{\text{R}}) - \frac{1}{6} \frac{s}{c} F_2^b L_{\text{R}} \\ &\quad + \frac{2}{3} \sum_{\text{gen}} (x_{\text{I}}^2 v_{\text{I}} + v_{\text{d}} x_{\text{d}}^2 + 2v_{\text{u}} x_{\text{u}}^2) \frac{s}{c x_{\text{s}}} - \frac{1}{9} (36c^2 + J_{19}) \frac{s}{c x_{\text{s}}} \end{aligned}$$

$$\begin{aligned}\Pi_{ZA;1}^{(4)} &= -\frac{4}{3} \frac{s}{c x_S} F_1^b a_0^{\text{fin}}(M) + \frac{2}{3} \sum_{\text{gen}} \frac{s x_d^2}{c x_S} v_d a_0^{\text{fin}}(M_d) \\ &\quad + \frac{4}{3} \sum_{\text{gen}} \frac{s x_u^2}{c x_S} v_u a_0^{\text{fin}}(M_u) + \frac{2}{3} \sum_{\text{gen}} \frac{s x_1^2}{c x_S} v_1 a_0^{\text{fin}}(M_1)\end{aligned}$$

$$\begin{aligned}\Pi_{ZA;2}^{(4)} &= \frac{1}{3} \sum_{\text{gen}} \frac{s}{c x_S} K_2 v_1 B_0^{\text{fin}}(M_1, M_1) + \frac{2}{3} \sum_{\text{gen}} \frac{s}{c x_S} K_0 v_u B_0^{\text{fin}}(M_u, M_u) \\ &\quad + \frac{1}{3} \sum_{\text{gen}} \frac{s}{c x_S} K_1 v_d B_0^{\text{fin}}(M_d, M_d) - \frac{1}{6} (6J_{11} c^2 + J_{17}) \frac{s}{c x_S} B_0^{\text{fin}}(M, M)\end{aligned}$$

$$\begin{aligned}\Pi_{ZA;0}^{(6)} &= \frac{1}{36} \left[ 144 s^3 c^2 a_{\phi WAB} - 4 J_{26} s c a_{\phi WB} - 144 F_3^b s^2 c a_{\phi WZ} + 36 F_5^b c^2 a_{\phi D} \right. \\ &\quad \left. + (a_{\phi D} - 2 s^2 a_{\phi WD}^{(+)} J_{19}) \frac{1}{s c x_S} \right. \\ &\quad - \frac{1}{24} \left[ F_8^b c a_{\phi D} + 4 (3 a_{\phi B} - a_{\phi W} + 36 s^2 c^2 a_{\phi WB}^{(a)}) s^2 c + 12 (J_{12} a_{\phi B} + J_{18} a_{\phi W}) s^2 c^3 \right. \\ &\quad \left. + 2 (J_{20} c^2 - 6 J_{23} s^2 c^2 + 3 F_6^b) s a_{\phi WB} \right] \frac{L_R}{s c^2} \\ &\quad + \frac{1}{108} \left\{ \left[ 3 c a_{\phi D} v_{\text{gen}}^{(2)} - 4 (3 a_{\phi WB} v_{\text{gen}}^{(2)} - 32 (-c a_{\phi WZ} + s a_{\phi WAB}) s c) s \right] c \right. \\ &\quad \left. + 4 (G_3 a_{\phi W} - F_{10}^b a_{\phi D}) s^2 \right\} \frac{N_{\text{gen}}}{s c} (1 - 3 L_R) \\ &\quad + \frac{1}{18} \sum_{\text{gen}} \left[ 4 (a_{\phi D} + 4 c a_{\phi WZ} - 4 s a_{\phi WAB}) H_2 s^2 c^2 + (-24 x_d^2 a_{\phi D} + 48 x_u^2 a_{\phi U} - 4 G_0 x_d^2 a_{\phi W} \right. \\ &\quad \left. - 12 G_1 x_1^2 a_{\phi W} - 8 G_2 x_u^2 a_{\phi W} - 24 H_3 a_{\phi Q}^{(1)} + 24 H_4 a_{\phi Q}^{(3)} + H_5 a_{\phi D} - 4 K_3 a_{\phi 1V}) s^2 \right. \\ &\quad \left. + 3 (-c a_{\phi D} + 4 s a_{\phi WB}) (x_1^2 v_1 + v_d x_d^2 + 2 v_u x_u^2) c \right] \frac{1}{s c x_S} \\ &\quad + \frac{1}{12} \sum_{\text{gen}} \left\{ 8 s a_{\phi 1V} - 3 \left[ -(a_{1WB} v_1 + 4 s c a_{1BW}) x_1^2 - (3 v_d a_{dWB} + 4 s c a_{dWB}) x_d^2 \right. \right. \\ &\quad \left. \left. + (3 v_u a_{uWB} + 8 s c a_{uBW}) x_u^2 \right] \right\} \frac{L_R}{c} \\ &\quad - \frac{2}{9} \sum_{\text{gen}} (a_{\phi dV} + 2 a_{\phi uV}) (1 - 3 L_R) \frac{s}{c}\end{aligned}$$

$$\begin{aligned}\Pi_{ZA;1}^{(6)} &= -\frac{1}{4} \frac{1}{c^2} a_{AZ} a_0^{\text{fin}}(M_0) - \frac{1}{4} x_H^2 a_{AZ} a_0^{\text{fin}}(M_H) \\ &\quad - \frac{1}{6} \left\{ 2 F_7^b a_{\phi D} + 16 \left[ a_{\phi W} + 3 (c a_{\phi WB}^{(a)} + s a_{\phi WB}) s^2 c \right] s^2 - 6 (x_S a_{\phi B} - J_{17} a_{\phi W}) s^2 c^2 \right. \\ &\quad \left. - (6 J_{21} s^2 - J_{24}) s c a_{\phi WB} \right\} \frac{1}{s c x_S} a_0^{\text{fin}}(M) \\ &\quad - \frac{1}{6} \sum_{\text{gen}} \left\{ \left[ c^2 a_{\phi D} v_1 - 4 (c a_{\phi WB} v_1 + 2 (a_{\phi 1V} + 2 c^3 a_{\phi WZ} - 2 s c^2 a_{\phi WAB}) s) s \right] \right. \\ &\quad \left. + (4 G_1 a_{\phi W} - F_9^b a_{\phi D}) s^2 \right\} \frac{x_1^2}{s c x_S} a_0^{\text{fin}}(M_1) \\ &\quad - \frac{1}{18} \sum_{\text{gen}} \left\{ \left[ 3 c^2 v_d a_{\phi D} - 4 (3 c v_d a_{\phi WB} + 2 (3 a_{\phi dV} + 2 c^3 a_{\phi WZ} - 2 s c^2 a_{\phi WAB}) s) s \right] \right. \\ &\quad \left. + (4 G_0 a_{\phi W} - F_4^b a_{\phi D}) s^2 \right\} \frac{x_d^2}{s c x_S} a_0^{\text{fin}}(M_d) \\ &\quad - \frac{1}{9} \sum_{\text{gen}} \left\{ \left[ 3 c^2 v_u a_{\phi D} - 4 (3 c v_u a_{\phi WB} + 2 (3 a_{\phi uV} + 4 c^3 a_{\phi WZ} - 4 s c^2 a_{\phi WAB}) s) s \right] \right.\end{aligned}$$

$$+(4G_2 a_{\phi W} - F_{10}^b a_{\phi D}) s^2 \left\} \frac{x_u^2}{s c x_s} a_0^{\text{fin}}(M_u)$$

$$\begin{aligned} \Pi_{ZA;2}^{(6)} &= \frac{1}{24} \left\{ -48 \left[ c^3 a_{\phi W} + (c a_{\phi B} + s a_{\phi WB}) s^2 \right] J_{11} s^2 c + (a_{\phi D} - 2s^2 a_{\phi WD}^{(+)}) J_{17} + 6(J_{11} F_5^b) c^2 a_{\phi D} \right. \\ &\quad \left. + 4(6J_{22} s^2 - J_{25}) s c a_{\phi WB} \right\} \frac{1}{s c x_s} B_0^{\text{fin}}(M, M) \\ &\quad + \frac{1}{12} \sum_{\text{gen}} \left\{ - \left[ c^2 a_{\phi D} v_1 - 4(c a_{\phi WB} v_1 + 2(a_{\phi 1V} + 2c^3 a_{\phi WZ} - 2s c^2 a_{\phi WAB}) s) \right] K_2 \right. \\ &\quad \left. + 3(a_{1WB} v_1 + 4s c a_{1BW}) s x_1^2 x_s - (4G_1 K_2 a_{\phi W} - K_2 F_9^b a_{\phi D}) s^2 \right\} \frac{1}{s c x_s} B_0^{\text{fin}}(M_1, M_1) \\ &\quad + \frac{1}{36} \sum_{\text{gen}} \left\{ - \left[ 3c^2 v_d a_{\phi D} - 4(3c v_d a_{\phi WB} + 2(3a_{\phi dV} + 2c^3 a_{\phi WZ} - 2s c^2 a_{\phi WAB}) s) \right] K_1 \right. \\ &\quad \left. + 9(3v_d a_{dWB} + 4s c a_{dBW}) s x_d^2 x_s - (4G_0 K_1 a_{\phi W} - K_1 F_4^b a_{\phi D}) s^2 \right\} \frac{1}{s c x_s} B_0^{\text{fin}}(M_d, M_d) \\ &\quad - \frac{1}{36} \sum_{\text{gen}} \left\{ 2 \left[ 3c^2 v_u a_{\phi D} - 4(3c v_u a_{\phi WB} + 2(3a_{\phi uV} + 4c^3 a_{\phi WZ} - 4s c^2 a_{\phi WAB}) s) \right] K_0 \right. \\ &\quad \left. + 9(3v_u a_{uWB} + 8s c a_{uBW}) s x_u^2 x_s + 2(4G_2 K_0 a_{\phi W} - K_0 F_{10}^b a_{\phi D}) s^2 \right\} \frac{1}{s c x_s} B_0^{\text{fin}}(M_u, M_u) \end{aligned}$$

• Z self-energy

$$\Pi_{ZZ;0}^{(4)} = -3s^2 L_R + \frac{1}{36} \frac{N_{\text{gen}}}{c^2} G_7 (1 - 3L_R) + \frac{2}{9} (1 + 15L_R) - \frac{1}{18} \frac{1}{c^2} (2 + 3L_R)$$

$$\Pi_{ZZ;1}^{(4)} = 0$$

$$\begin{aligned} \Pi_{ZZ;2}^{(4)} &= -\frac{1}{12} \frac{1}{c^2} F_{11}^b B_0^{\text{fin}}(M, M) - \frac{1}{12} \frac{1}{c^2} B_0^{\text{fin}}(M_H, M_0) - \frac{1}{12} \sum_{\text{gen}} \frac{1}{c^2} G_6 B_0^{\text{fin}}(M_1, M_1) \\ &\quad - \frac{1}{4} \sum_{\text{gen}} \frac{1}{c^2} G_4 B_0^{\text{fin}}(M_u, M_u) - \frac{1}{4} \sum_{\text{gen}} \frac{1}{c^2} G_5 B_0^{\text{fin}}(M_d, M_d) - \frac{1}{6} \sum_{\text{gen}} \frac{1}{c^2} B_0^{\text{fin}}(0, 0) \end{aligned}$$

$$\begin{aligned} \Delta_{ZZ;0}^{(4)} &= \frac{1}{6} \frac{1}{c^4} (1 - 6L_R) - 2 \frac{L_R}{c^2} + \frac{1}{2} \sum_{\text{gen}} H_0 \frac{L_R}{c^2} \\ &\quad + \frac{1}{6} (J_{12} + 8F_{14}^b c^2) \frac{1}{c^2} - \frac{1}{6} \sum_{\text{gen}} (3G_4 x_u^2 + 3G_5 x_d^2 + G_6 x_1^2) \frac{1}{c^2} \end{aligned}$$

$$\begin{aligned} \Delta_{ZZ;1}^{(4)} &= \frac{1}{3} \frac{1}{c^2} F_{12}^b a_0^{\text{fin}}(M) - \frac{1}{6} \sum_{\text{gen}} \frac{x_1^2}{c^2} G_6 a_0^{\text{fin}}(M_1) - \frac{1}{2} \sum_{\text{gen}} \frac{x_u^2}{c^2} G_4 a_0^{\text{fin}}(M_u) \\ &\quad - \frac{1}{2} \sum_{\text{gen}} \frac{x_d^2}{c^2} G_5 a_0^{\text{fin}}(M_d) + \frac{1}{12} (1 + J_{27} c^2) \frac{x_H^2}{c^4 x_s} a_0^{\text{fin}}(M_H) - \frac{1}{12} (1 - J_{28} c^2) \frac{1}{c^6 x_s} a_0^{\text{fin}}(M_0) \end{aligned}$$

$$\begin{aligned} \Delta_{ZZ;2}^{(4)} &= -\frac{1}{3} \frac{1}{c^2} F_{13}^b B_0^{\text{fin}}(M, M) + \frac{1}{6} \sum_{\text{gen}} \frac{x_1^2}{c^2} G_{10} B_0^{\text{fin}}(M_1, M_1) + \frac{1}{2} \sum_{\text{gen}} \frac{x_u^2}{c^2} G_8 B_0^{\text{fin}}(M_u, M_u) \\ &\quad + \frac{1}{2} \sum_{\text{gen}} \frac{x_d^2}{c^2} G_9 B_0^{\text{fin}}(M_d, M_d) - \frac{1}{12} (1 - J_{27} c^4 x_H^2 + 2J_{29} c^2) \frac{1}{c^6 x_s} B_0^{\text{fin}}(M_H, M_0) \end{aligned}$$

$$\begin{aligned}
 \Pi_{ZZ;0}^{(6)} &= \frac{1}{9} a_{\phi D} (1 + 15 L_R) - \frac{1}{18} \frac{1}{c^2} a_{\phi \square} (2 + 3 L_R) \\
 &\quad - \frac{1}{18} \left[ a_{\phi D} - 4 c^2 a_{ZZ} + 4 (c a_{AZ} + s a_{AA}) s \right] \frac{1}{c^2} \\
 &\quad + \frac{1}{72} \left[ G_{22} a_{\phi WD}^{(-)} + 4 v_{\text{gen}}^{(2)} (a_{\phi D} + 4 c a_{\phi WZ} - 4 s a_{\phi WAB}) c^2 \right] \frac{N_{\text{gen}}}{c^2} (1 - 3 L_R) \\
 &\quad + \frac{1}{12} \left\{ 2 J_{30} c^2 a_{\phi W} - \left[ a_{\phi WD}^{(+)} + 6 (5 a_{\phi B} + 3 c^2 a_{\phi D} + 24 s^2 c^2 a_{\phi WB}^{(a)}) s^2 \right] - 2 (J_{31} c^2 - 9 F_{16}^b) s c a_{\phi WB} \right. \\
 &\quad \left. + 2 (J_{32} a_{\phi B} - J_{33} a_{\phi W}) s^2 c^2 \right\} \frac{L_R}{c^2} \\
 &\quad - \frac{1}{9} \sum_{\text{gen}} \left[ 3 G_{12} a_{\phi U} - 3 G_{14} a_{\phi D} - 3 G_{18} a_{\phi Q}^{(3)} - G_{21} a_{\phi 1}^{(3)} - 3 G_{23} a_{\phi Q}^{(1)} - (a_{\phi 1}^{(3)} - a_{\phi 1V}) G_{11} \right] \frac{1}{c^2} (1 - 3 L_R) \\
 &\quad + \frac{1}{2} \sum_{\text{gen}} (-x_1^2 v_1 a_{1BW} - 3 v_d x_d^2 a_{dBW} + 3 v_u x_u^2 a_{uBW}) \frac{L_R}{c}
 \end{aligned}$$

$$\Pi_{ZZ;1}^{(6)} = \frac{1}{2} \frac{1}{c^2} a_{ZZ} a_0^{\text{fin}}(M_0) + \frac{1}{2} a_{ZZ} x_H^2 a_0^{\text{fin}}(M_H) + \left[ c^2 a_{\phi W} + (c a_{\phi WB} + s a_{\phi B}) s \right] a_0^{\text{fin}}(M)$$

$$\begin{aligned}
 \Pi_{ZZ;2}^{(6)} &= -\frac{1}{24} (a_{\phi WD}^{(+)} + 48 a_{ZZ} + 4 a_{\phi \square}) \frac{1}{c^2} B_0^{\text{fin}}(M_H, M_0) \\
 &\quad - \frac{1}{24} (F_{11}^b a_{\phi D} + 4 F_{15}^b s a_{\phi WA} + 4 F_{17}^b c a_{\phi WZ} - 16 F_{18}^b s^2 c^2 a_{\phi B}) \frac{1}{c^2} B_0^{\text{fin}}(M, M) \\
 &\quad - \frac{1}{24} \sum_{\text{gen}} \left[ 12 c x_1^2 v_1 a_{1BW} + 4 (a_{\phi D} + 4 c a_{\phi WZ} - 4 s a_{\phi WAB}) c^2 v_1 + (8 G_{11} a_{\phi 1} + G_{16} a_{\phi WD}^{(-)} \right. \\
 &\quad \left. + 4 G_{17} a_{\phi 1VA}) \right] \frac{1}{c^2} B_0^{\text{fin}}(M_1, M_1) \\
 &\quad - \frac{1}{24} \sum_{\text{gen}} \left[ 36 c v_d x_d^2 a_{dBW} + 4 (a_{\phi D} + 4 c a_{\phi WZ} - 4 s a_{\phi WAB}) c^2 v_d \right. \\
 &\quad \left. + (24 G_{14} a_{\phi D} + 12 G_{15} a_{\phi DV_A} + G_{20} a_{\phi WD}^{(-)}) \right] \frac{1}{c^2} B_0^{\text{fin}}(M_d, M_d) \\
 &\quad + \frac{1}{24} \sum_{\text{gen}} \left[ 36 c v_u x_u^2 a_{uBW} - 8 (a_{\phi D} + 4 c a_{\phi WZ} - 4 s a_{\phi WAB}) c^2 v_u \right. \\
 &\quad \left. + (24 G_{12} a_{\phi U} - 12 G_{13} a_{\phi UV_A} - G_{19} a_{\phi WD}^{(-)}) \right] \frac{1}{c^2} B_0^{\text{fin}}(M_u, M_u) \\
 &\quad - \frac{1}{12} \sum_{\text{gen}} (4 a_{\phi V} + a_{\phi WD}^{(-)}) \frac{1}{c^2} B_0^{\text{fin}}(0, 0)
 \end{aligned}$$

$$\begin{aligned}
 \Delta_{ZZ;0}^{(6)} &= \frac{1}{6} \frac{1}{c^4} a_{\phi \square} (2 - 9 L_R) + \frac{1}{24} \frac{1}{c^4} (82 - 9 L_R) a_{\phi D} \\
 &\quad + \frac{1}{12} \left[ 4 F_{20}^b s a_{\phi WA} + 4 F_{22}^b c a_{\phi WZ} + 64 F_{23}^b s^2 c^4 a_{\phi B} - 8 F_{26}^b s^2 a_{\phi D} + (4 a_{\phi \square} x_H^2 + J_{12} a_{\phi WD}^{(+)}) c^2 \right] \frac{1}{c^4} \\
 &\quad - \frac{1}{8} (J_{36} c^2 a_{\phi D} + 16 F_0^b a_{\phi W}) \frac{L_R}{c^4} \\
 &\quad - \frac{1}{12} \sum_{\text{gen}} \left[ 4 (a_{\phi D} + 4 c a_{\phi WZ} - 4 s a_{\phi WAB}) (x_1^2 v_1 + v_d x_d^2 + 2 v_u x_u^2) c^2 \right. \\
 &\quad \left. + (8 G_{11} x_1^2 a_{\phi 1} - 24 G_{12} x_u^2 a_{\phi U} + 12 G_{13} x_u^2 a_{\phi UV_A} + 24 G_{14} x_d^2 a_{\phi D} + 12 G_{15} x_d^2 a_{\phi DV_A} \right. \\
 &\quad \left. + G_{16} x_1^2 a_{\phi WD}^{(-)} + 4 G_{17} x_1^2 a_{\phi 1VA} + G_{19} x_u^2 a_{\phi WD}^{(-)} + G_{20} x_d^2 a_{\phi WD}^{(-)}) \right] \frac{1}{c^2} \\
 &\quad - \frac{1}{4} \sum_{\text{gen}} (-24 x_d^2 a_{\phi D} + 24 x_u^2 a_{\phi U} - 8 x_1^2 a_{\phi 1A} - H_0 a_{\phi WD}^{(-)} + 24 H_6 a_{\phi Q}^{(1)} - 24 H_7 a_{\phi Q}^{(3)}) \frac{L_R}{c^2}
 \end{aligned}$$

$$\Delta_{ZZ;1}^{(6)} = -\frac{1}{24} \left\{ \left[ 48 a_{ZZ} x_S - 4 J_{28} a_{\phi W} + (a_{\phi D} + 4 a_{\phi \square}) J_{34} \right] c^2 + (a_{\phi WD}^{(+)} + 4 a_{\phi \square}) \right\} \frac{1}{c^6 x_S} a_0^{\text{fin}}(M_0)$$



$$\begin{aligned}
 & + \frac{1}{24} \left\{ \left[ 48 a_{ZZ} x_S + J_{37} a_{\phi D} + 4 (a_{\phi W} + a_{\phi \square}) J_{27} \right] c^2 + (a_{\phi WD}^{(+)} + 4 a_{\phi \square}) \right\} \frac{x_H^2}{c^4 x_S} a_0^{\text{fin}}(M_H) \\
 & + \frac{1}{6} (4 F_{19}^b s a_{\phi WA} + 4 F_{21}^b c a_{\phi WZ} + 32 F_{23}^b s^2 c^2 a_{\phi B} + F_{27}^b a_{\phi D}) \frac{1}{c^2} a_0^{\text{fin}}(M) \\
 & - \frac{1}{12} \sum_{\text{gen}} \left[ 4 (a_{\phi D} + 4 c a_{\phi WZ} - 4 s a_{\phi WAB}) c^2 v_1 \right. \\
 & \left. + (8 G_{11} a_{\phi 1} + G_{16} a_{\phi WD}^{(-)} + 4 G_{17} a_{\phi 1VA}) \right] \frac{x_1^2}{c^2} a_0^{\text{fin}}(M_1) \\
 & - \frac{1}{12} \sum_{\text{gen}} \left[ 4 (a_{\phi D} + 4 c a_{\phi WZ} - 4 s a_{\phi WAB}) c^2 v_d \right. \\
 & \left. + (24 G_{14} a_{\phi d} + 12 G_{15} a_{\phi dVA} + G_{20} a_{\phi WD}^{(-)}) \right] \frac{x_d^2}{c^2} a_0^{\text{fin}}(M_d) \\
 & - \frac{1}{12} \sum_{\text{gen}} \left[ 8 (a_{\phi D} + 4 c a_{\phi WZ} - 4 s a_{\phi WAB}) c^2 v_u \right. \\
 & \left. - (24 G_{12} a_{\phi u} - 12 G_{13} a_{\phi uVA} - G_{19} a_{\phi WD}^{(-)}) \right] \frac{x_u^2}{c^2} a_0^{\text{fin}}(M_u) \\
 \\
 \Delta_{ZZ;2}^{(6)} & = \frac{1}{24} \left\{ \left[ 48 J_{35} x_S a_{\phi B} - 4 J_{38} a_{\phi W} + (a_{\phi D} + 4 a_{\phi \square}) J_{27} x_H^2 \right] c^4 \right. \\
 & \quad - 24 \left[ (a_{\phi B} - s c a_{\phi WB}) x_S + (a_{\phi WD}^{(+)} + 4 a_{\phi \square}) J_{29} \right] c^2 \\
 & \quad \left. + 48 (a_{\phi WZ} - c a_{\phi B}) c^5 x_H^2 x_S - (a_{\phi WD}^{(+)} + 4 a_{\phi \square}) \right\} \frac{1}{c^6 x_S} B_0^{\text{fin}}(M_H, M_0) \\
 & - \frac{1}{6} (F_{13}^b a_{\phi D} - 32 F_{23}^b s^2 c^2 a_{\phi B} + 4 F_{24}^b s a_{\phi WA} + 4 F_{25}^b c a_{\phi WZ}) \frac{1}{c^2} B_0^{\text{fin}}(M, M) \\
 & - \frac{1}{12} \sum_{\text{gen}} \left[ 4 (a_{\phi D} + 4 c a_{\phi WZ} - 4 s a_{\phi WAB}) c^2 v_1 \right. \\
 & \quad \left. - (4 G_{28} a_{\phi 1VA} + 8 G_{29} a_{\phi 1} + G_{30} a_{\phi WD}^{(-)}) \right] \frac{x_1^2}{c^2} B_0^{\text{fin}}(M_1, M_1) \\
 & - \frac{1}{12} \sum_{\text{gen}} \left[ 4 (a_{\phi D} + 4 c a_{\phi WZ} - 4 s a_{\phi WAB}) c^2 v_d \right. \\
 & \quad \left. - (12 G_{26} a_{\phi dVA} + 24 G_{27} a_{\phi d} + G_{32} a_{\phi WD}^{(-)}) \right] \frac{x_d^2}{c^2} B_0^{\text{fin}}(M_d, M_d) \\
 & - \frac{1}{12} \sum_{\text{gen}} \left[ 8 (a_{\phi D} + 4 c a_{\phi WZ} - 4 s a_{\phi WAB}) c^2 v_u \right. \\
 & \quad \left. - (12 G_{24} a_{\phi uVA} - 24 G_{25} a_{\phi u} + G_{31} a_{\phi WD}^{(-)}) \right] \frac{x_u^2}{c^2} B_0^{\text{fin}}(M_u, M_u)
 \end{aligned}$$

- W self-energy

$$\Pi_{\text{WW};0}^{(4)} = \frac{4}{9} (1 - 3 L_R) N_{\text{gen}} + \frac{1}{18} (2 + 57 L_R)$$

$$\Pi_{\text{WW};1}^{(4)} = 0$$

$$\begin{aligned}
 \Pi_{\text{WW};2}^{(4)} & = -\frac{1}{12} B_0^{\text{fin}}(M, M_H) + \frac{1}{12} B_0^{\text{fin}}(M, M_0) F_{28}^b \\
 & \quad + \frac{10}{3} B_0^{\text{fin}}(0, M) s^2 - \sum_{\text{gen}} B_0^{\text{fin}}(M_u, M_d) - \frac{1}{3} \sum_{\text{gen}} B_0^{\text{fin}}(0, M_1)
 \end{aligned}$$

$$\Delta_{\text{WW};0}^{(4)} = -2L_R + \frac{1}{6} \frac{1}{c^2} (1 - 6L_R) + \frac{1}{6} J_{47} - \frac{1}{6} \sum_{\text{gen}} H_0 (2 - 3L_R)$$

$$\begin{aligned} \Delta_{\text{WW};1}^{(4)} &= \frac{1}{12} \frac{x_H^2}{x_S} J_{41} a_0^{\text{fin}}(M_H) - \frac{1}{6} \sum_{\text{gen}} \frac{x_1^2}{x_S} K_5 a_0^{\text{fin}}(M_1) \\ &\quad - \frac{1}{12} (1 + 8J_{40} s^2 c^2 - J_{44} c^2) \frac{1}{c^4 x_S} a_0^{\text{fin}}(M_0) + \frac{1}{12} (1 - J_{45} c^2) \frac{1}{c^2 x_S} a_0^{\text{fin}}(M) \\ &\quad - \frac{1}{2} \sum_{\text{gen}} (2x_S - H_6) \frac{x_d^2}{x_S} a_0^{\text{fin}}(M_d) - \frac{1}{2} \sum_{\text{gen}} (2x_S + H_6) \frac{x_u^2}{x_S} a_0^{\text{fin}}(M_u) \end{aligned}$$

$$\begin{aligned} \Delta_{\text{WW};2}^{(4)} &= -\frac{2}{3} \frac{s^2}{x_S} J_{39} B_0^{\text{fin}}(0, M) - \frac{1}{12} \frac{1}{x_S} J_{43} B_0^{\text{fin}}(M, M_H) \\ &\quad + \frac{1}{6} \sum_{\text{gen}} \frac{x_1^2}{x_S} K_4 B_0^{\text{fin}}(0, M_1) + \frac{1}{2} \sum_{\text{gen}} (H_7 x_S + H_8) \frac{1}{x_S} B_0^{\text{fin}}(M_u, M_d) \\ &\quad - \frac{1}{12} (1 - 8J_{39} s^2 c^4 - 7J_{42} c^4 + 2J_{46} c^2) \frac{1}{c^4 x_S} B_0^{\text{fin}}(M, M_0) \end{aligned}$$

$$\begin{aligned} \Pi_{\text{WW};0}^{(6)} &= \frac{2}{9} s^2 a_{AA} (1 - 9L_R) + \frac{8}{9} N_{\text{gen}} a_{\phi W} (1 - 3L_R) - \frac{1}{18} a_{\phi \square} (2 + 3L_R) \\ &\quad + \frac{1}{6} \left[ 6F_3^b s c a_{\phi WB} + (J_{54} c^2 - 3F_{29}^b) a_{\phi W} \right] \frac{L_R}{c^2} - \frac{2}{9} (-c a_{ZZ} + 2s a_{AZ}) c \\ &\quad + \frac{4}{9} \sum_{\text{gen}} (a_{\phi 1}^{(3)} + 3a_{\phi q}^{(3)}) (1 - 3L_R) + \frac{1}{2} \sum_{\text{gen}} (-3x_d^2 a_{dW} + 3x_u^2 a_{uW} - x_1^2 a_{1W}) L_R \end{aligned}$$

$$\begin{aligned} \Pi_{\text{WW};1}^{(6)} &= a_{\phi W} a_0^{\text{fin}}(M) + \frac{1}{2} a_{\phi W} x_H^2 a_0^{\text{fin}}(M_H) \\ &\quad + \frac{1}{6} \left[ F_{30}^b s c a_{AZ} + 3(c^2 a_{ZZ} + s^2 a_{AA}) \right] \frac{1}{c^2} a_0^{\text{fin}}(M_0) \end{aligned}$$

$$\begin{aligned} \Pi_{\text{WW};2}^{(6)} &= -\frac{1}{3} \left\{ 5c^2 a_{\phi D} - J_{53} s c a_{\phi WB} - \left[ 2J_{49} a_{\phi W} + (2a_{AA} + s c a_{AZ}) J_{48} \right] s^2 \right\} B_0^{\text{fin}}(0, M) \\ &\quad + \frac{1}{24} \left\{ 32J_{52} s^4 c a_{\phi WB}^{(a)} + \left[ 39c a_{\phi WD}^{(+)} + 8(3a_{\phi WB} - 5s c a_{\phi D}) s \right] \right. \\ &\quad + 16(a_{\phi WA} - s a_{\phi B}) J_{48} s^5 c - 16(J_{46} a_{\phi W} + J_{51} a_{\phi B}) s^2 c \\ &\quad \left. - 8(5J_{50} + J_{55} s^2) s c^2 a_{\phi WB} \right\} \frac{1}{c} B_0^{\text{fin}}(M, M_0) \\ &\quad + \frac{1}{24} (-52a_{\phi W} + a_{\phi D} - 4a_{\phi \square}) B_0^{\text{fin}}(M, M_H) \\ &\quad - \frac{1}{2} \sum_{\text{gen}} (2a_{\phi qW}^{(3)} + 3x_d^2 a_{dW} - 3x_u^2 a_{uW}) B_0^{\text{fin}}(M_u, M_d) \\ &\quad - \frac{1}{6} \sum_{\text{gen}} (2a_{\phi 1W}^{(3)} + 3x_1^2 a_{1W}) B_0^{\text{fin}}(0, M_1) \end{aligned}$$

$$\begin{aligned} \Delta_{\text{WW};0}^{(6)} &= \frac{1}{3} \frac{1}{c^2} a_{\phi W} (1 - 6L_R) - \frac{1}{12} \frac{1}{c^2} a_{\phi D} (11 - 15L_R) \\ &\quad + \frac{1}{6} \left[ 2c^3 a_{\phi WB} + (3a_{\phi D} - 4c^2 a_{AA} + 4c^2 a_{\phi W}) s \right] \frac{s}{c^2} (2 - 3L_R) \\ &\quad - \frac{1}{12} \left[ 40s a_{\phi WB} - (4J_{35} a_{\phi \square} + 4J_{47} a_{\phi W} + J_{75} a_{\phi D}) c \right] \frac{1}{c} \\ &\quad - \frac{1}{2} (8a_{\phi W} + 3a_{\phi \square}) L_R - \frac{1}{3} \sum_{\text{gen}} (2x_1^2 a_{\phi 1}^{(3)} + H_0 a_{\phi W} + 6H_7 a_{\phi q}^{(3)}) (2 - 3L_R) \end{aligned}$$

$$\begin{aligned}
 \Delta_{\text{WW};1}^{(6)} &= \frac{1}{24} \left[ a_{\phi\text{WD}}^{(+)} + 8J_{61} s c^3 a_{\text{AZ}} - 8J_{71} s c a_{\phi\text{WB}} \right. \\
 &\quad \left. - (12a_{\phi\text{D}} x_s + 4J_{62} a_{\phi\text{D}} + 4J_{69} a_{\phi\text{W}} - J_{74} a_{\phi\text{D}}) c^2 \right] \frac{1}{c^2 x_s} a_0^{\text{fin}}(M) \\
 &\quad + \frac{1}{24} \left[ 16J_{63} s^5 c^2 a_{\phi\text{WA}} - 16J_{64} s^4 c^2 a_{\phi\text{W}} - 16F_4^b s^2 c^4 x_s a_{\phi\text{B}} - (a_{\phi\text{WD}}^{(+)} - 16s^4 c^4 a_{\phi\text{B}}) \right. \\
 &\quad \left. - 8(J_{40} a_{\phi\text{D}} + 2J_{70} a_{\phi\text{W}}) s^2 c^2 + (4J_{44} a_{\phi\text{W}} + J_{65} a_{\phi\text{D}}) c^2 \right. \\
 &\quad \left. - 8(J_{66} s^2 c^2 - J_{71} + J_{72} c^2) s c a_{\phi\text{WB}} \right] \frac{1}{c^4 x_s} a_0^{\text{fin}}(M_0) \\
 &\quad + \frac{1}{24} \left[ 4J_{67} a_{\phi\text{W}} - (a_{\phi\text{D}} - 4a_{\phi\text{D}}) J_{41} \right] \frac{x_{\text{H}}^2}{x_s} a_0^{\text{fin}}(M_{\text{H}}) \\
 &\quad - \frac{1}{2} \sum_{\text{gen}} (2x_s a_{\phi\text{qW}}^{(3)} - 3x_{\text{d}}^2 x_s a_{\text{dW}} - 3x_{\text{u}}^2 x_s a_{\text{uW}} - H_6 a_{\phi\text{qW}}^{(3)}) \frac{x_{\text{d}}^2}{x_s} a_0^{\text{fin}}(M_{\text{d}}) \\
 &\quad - \frac{1}{2} \sum_{\text{gen}} (2x_s a_{\phi\text{qW}}^{(3)} + 3x_{\text{d}}^2 x_s a_{\text{dW}} + 3x_{\text{u}}^2 x_s a_{\text{uW}} + H_6 a_{\phi\text{qW}}^{(3)}) \frac{x_{\text{u}}^2}{x_s} a_0^{\text{fin}}(M_{\text{u}}) \\
 &\quad + \frac{1}{6} \sum_{\text{gen}} (3x_{\text{t}}^2 x_s a_{\text{lW}} - K_5 a_{\phi\text{lW}}^{(3)}) \frac{x_{\text{t}}^2}{x_s} a_0^{\text{fin}}(M_{\text{l}})
 \end{aligned}$$

$$\begin{aligned}
 \Delta_{\text{WW};2}^{(6)} &= \frac{1}{3} \left[ J_{39} c^2 a_{\phi\text{D}} - (2a_{\text{AA}} + s c a_{\text{AZ}}) J_{56} s^2 - (c a_{\phi\text{WB}} + 2s a_{\phi\text{W}}) J_{57} s \right] \frac{1}{x_s} B_0^{\text{fin}}(0, M) \\
 &\quad - \frac{1}{24} \left[ 4J_{68} a_{\phi\text{W}} - (a_{\phi\text{D}} - 4a_{\phi\text{D}}) J_{43} \right] \frac{1}{x_s} B_0^{\text{fin}}(M, M_{\text{H}}) \\
 &\quad - \frac{1}{24} \left\{ -8 \left[ J_{39} a_{\phi\text{D}} + 4J_{60} a_{\phi\text{W}} + 4(a_{\phi\text{B}} - 3s^2 a_{\phi\text{WB}}^{(a)}) x_s \right] s^2 c^4 + (1 - 7J_{42} c^4 + 2J_{46} c^2) a_{\phi\text{WD}}^{(+)} \right. \\
 &\quad \left. + 16(a_{\phi\text{WA}} - s a_{\phi\text{B}}) J_{56} s^5 c^4 + 8(J_{58} s^2 c^4 - J_{59} c^4 - J_{71} + J_{73} c^2) s c a_{\phi\text{WB}} \right\} \frac{1}{c^4 x_s} B_0^{\text{fin}}(M, M_0) \\
 &\quad + \frac{1}{6} \sum_{\text{gen}} (3x_{\text{t}}^2 x_s a_{\text{lW}} + K_4 a_{\phi\text{lW}}^{(3)}) \frac{x_{\text{t}}^2}{x_s} B_0^{\text{fin}}(0, M_{\text{l}}) \\
 &\quad + \frac{1}{2} \sum_{\text{gen}} (3H_6 x_{\text{d}}^2 x_s a_{\text{dW}} + 3H_6 x_{\text{u}}^2 x_s a_{\text{uW}} + H_7 x_s a_{\phi\text{qW}}^{(3)} + H_8 a_{\phi\text{qW}}^{(3)}) \frac{1}{x_s} B_0^{\text{fin}}(M_{\text{u}}, M_{\text{d}})
 \end{aligned}$$

- neutrino (N) self-energy

$$V_{\text{NN};0}^{(4)} = -\frac{1}{4} - \frac{1}{8} \frac{1}{c^2} (1 - L_{\text{R}}) + \frac{1}{8} I_0 L_{\text{R}}$$

$$V_{\text{NN};1}^{(4)} = -\frac{1}{8} \frac{1}{c^4 x_s} a_0^{\text{fin}}(M_0) - \frac{1}{8} \frac{1}{x_s} I_0 a_0^{\text{fin}}(M)$$

$$V_{\text{NN};2}^{(4)} = -\frac{1}{8} \frac{1}{x_s} I_0 J_{61} B_0^{\text{fin}}(0, M) - \frac{1}{8} (1 - c^2 x_s) \frac{1}{c^4 x_s} B_0^{\text{fin}}(0, M_0)$$

$$\begin{aligned}
 V_{\text{NN};0}^{(6)} &= -\frac{1}{4} a_{\phi\text{LW}}^{(3)} + \frac{1}{8} x_{\text{L}}^2 a_{\text{LW}} (2 - 3L_{\text{R}}) \\
 &\quad - \frac{1}{16} (4a_{\phi\text{N}} + a_{\phi\text{WD}}^{(-)}) \frac{1}{c^2} (1 - L_{\text{R}}) + \frac{1}{4} (I_0 a_{\phi\text{W}} + 2I_1 a_{\phi\text{L}}^{(3)}) L_{\text{R}}
 \end{aligned}$$

$$V_{\text{NN};1}^{(6)} = -\frac{1}{16} (4a_{\phi\text{N}} + a_{\phi\text{WD}}^{(-)}) \frac{1}{c^4 x_s} a_0^{\text{fin}}(M_0)$$

$$-\frac{1}{8}(3x_L^2 a_{LW} + I_0 a_{\phi LW}^{(3)}) \frac{1}{x_S} a_0^{\text{fin}}(M)$$

$$V_{\text{NN};2}^{(6)} = -\frac{1}{16}(4a_{\phi N} + a_{\phi \text{WD}}^{(-)})(1 - c^2 x_S) \frac{1}{c^4 x_S} B_0^{\text{fin}}(0, M_0) \\ -\frac{1}{8}(4I_0 a_{\phi L}^{(3)} + 2I_0 J_{61} a_{\phi W} - 4I_1 x_S a_{\phi L}^{(3)} + 3J_{76} x_L^2 a_{LW}) \frac{1}{x_S} B_0^{\text{fin}}(0, M)$$

$$A_{\text{NN};0}^{(4)} = -\frac{1}{4} - \frac{1}{8} \frac{1}{c^2} (1 - L_R) + \frac{1}{8} I_0 L_R$$

$$A_{\text{NN};1}^{(4)} = -\frac{1}{8} \frac{1}{c^4 x_S} a_0^{\text{fin}}(M_0) - \frac{1}{8} \frac{1}{x_S} I_0 a_0^{\text{fin}}(M)$$

$$A_{\text{NN};2}^{(4)} = -\frac{1}{8} \frac{1}{x_S} I_0 J_{61} B_0^{\text{fin}}(0, M) - \frac{1}{8} (1 - c^2 x_S) \frac{1}{c^4 x_S} B_0^{\text{fin}}(0, M_0)$$

$$A_{\text{NN};0}^{(6)} = -\frac{1}{4} a_{\phi LW}^{(3)} + \frac{1}{8} x_L^2 a_{LW} (2 - 3L_R) \\ -\frac{1}{16}(4a_{\phi N} + a_{\phi \text{WD}}^{(-)}) \frac{1}{c^2} (1 - L_R) \\ +\frac{1}{4}(I_0 a_{\phi W} + 2I_1 a_{\phi L}^{(3)}) L_R$$

$$A_{\text{NN};1}^{(6)} = -\frac{1}{16}(4a_{\phi N} + a_{\phi \text{WD}}^{(-)}) \frac{1}{c^4 x_S} a_0^{\text{fin}}(M_0) \\ -\frac{1}{8}(3x_L^2 a_{LW} + I_0 a_{\phi LW}^{(3)}) \frac{1}{x_S} a_0^{\text{fin}}(M)$$

$$A_{\text{NN};2}^{(6)} = -\frac{1}{16}(4a_{\phi N} + a_{\phi \text{WD}}^{(-)})(1 - c^2 x_S) \frac{1}{c^4 x_S} B_0^{\text{fin}}(0, M_0) \\ -\frac{1}{8}(4I_0 a_{\phi L}^{(3)} + 2I_0 J_{61} a_{\phi W} - 4I_1 x_S a_{\phi L}^{(3)} + 3J_{76} x_L^2 a_{LW}) \frac{1}{x_S} B_0^{\text{fin}}(0, M)$$

- lepton (L) self-energy

$$\Sigma_{\text{LL};0}^{(4)} = -\frac{1}{8}(16s^2 c^2 - G_{33}) \frac{1}{c^2} x_L (1 - 2L_R)$$

$$\Sigma_{\text{LL};1}^{(4)} = 0$$

$$\Sigma_{\text{LL};2}^{(4)} = -\frac{1}{4} x_L^3 B_0^{\text{fin}}(M_H, M_L) + 4s^2 x_L B_0^{\text{fin}}(0, M_L) \\ +\frac{1}{4}(c^2 x_L^2 - G_{33}) \frac{1}{c^2} x_L B_0^{\text{fin}}(M_0, M_L)$$

$$\Sigma_{\text{LL};0}^{(6)} = -\frac{1}{8} \left[ c^2 v_1 x_L^2 a_{LWB} + 4F_5^b s c^2 a_{\phi WB} v_1 + (v_1 a_{LWB} + 2c^3 a_{LW}) \right] \frac{1}{c^3} x_L (2 - 3L_R) \\ +\frac{1}{8} \left\{ s c x_S v_1 x_L a_{LB} - 16G_{29} s^2 c^2 a_{\phi W} x_L + 4L_1 s c^2 x_L a_{LWB} \right. \\ \left. + \left[ G_{29} a_{LW} x_L + 128(M \Delta_{UV} C_{UV}^L) \right] c^2 x_S + 2(G_{11} a_{\phi LVA} + 2G_{17} a_{\phi L}) x_L \right\} \frac{1}{c^2}$$

$$\begin{aligned}
 & -\frac{1}{16} \left\{ 32G_{28} s c^3 a_{\phi_{WB}} - 16F_{31}^b s^2 a_{\phi_W} v_1 \right. \\
 & \left. - \left[ 4G_{11} a_{\phi_W} - G_{36} a_{\phi_D} + 4(4c^4 + s^2 v_1) a_{\phi_D} \right] \right\} \frac{1}{c^2} x_L (1 - 2L_R) \\
 & + \frac{1}{4} \left\{ 4G_{39} s^2 c^2 a_{\phi_W} + \left[ 3x_H^2 a_{L\phi} + 2I_0 a_{\phi_L}^{(3)} + I_2 a_{L\phi} - 2(-a_{\phi_L} + a_{\phi_L}^{(1)} + a_{\phi_{\square}} - 3s a_{LWB}) x_L^2 \right] c^2 \right. \\
 & \left. - \left[ G_{34} a_{\phi_{LVA}} + 2G_{35} a_{\phi_L} - (a_{L\phi} + 4s^2 c^2 a_{\phi_B} v_1) \right] \right\} \frac{L_R}{c^2} x_L \\
 & + \frac{3}{2} \sum_{\text{gen}} (-x_d^2 x_d a_{L1dQ} + x_u^2 x_u a_{L1Qu}^{(1)}) L_R
 \end{aligned}$$

$$\begin{aligned}
 \Sigma_{LL;1}^{(6)} = & -\frac{3}{4} x_H^2 x_L a_{L\phi} a_0^{\text{fin}}(M_H) - \frac{3}{2} \sum_{\text{gen}} x_u^2 x_u a_{L1Qu}^{(1)} a_0^{\text{fin}}(M_u) \\
 & + \frac{3}{2} \sum_{\text{gen}} x_d^2 x_d a_{L1dQ} a_0^{\text{fin}}(M_d) - \frac{1}{8} (2a_{L\phi} + 3a_{LW} + 4a_{\phi_L}^{(3)}) x_L a_0^{\text{fin}}(M) \\
 & - \frac{1}{16} (3v_1 a_{LBW} + 4c a_{\phi_{LA}} + 4c a_{L\phi}) \frac{1}{c^3} x_L a_0^{\text{fin}}(M_0) \\
 & - \frac{1}{16} (3v_1 a_{LBW} + 4c a_{\phi_{LA}} + 12s c a_{LWB}) \frac{1}{c} x_L^3 a_0^{\text{fin}}(M_L)
 \end{aligned}$$

$$\begin{aligned}
 \Sigma_{LL;2}^{(6)} = & \frac{1}{16} \left\{ 2c^3 a_{\phi_{WD}}^{(-)} x_L^2 - 32F_3^b s^2 c a_{AA} v_1 - 8F_4^b s c^2 v_1 a_{AZ} \right. \\
 & \left. + \left[ 3a_{LBW} - 8(a_{\phi_D} - 4c^2 a_{ZZ}) s^2 c \right] v_1 - (3v_1 a_{LBW} + 4c a_{\phi_{LA}}) L_0 c^2 \right. \\
 & \left. - 2(4G_{11} a_{\phi_W} - G_{36} a_{\phi_D} + G_{37} a_{\phi_{LVA}} + 2G_{38} a_{\phi_L}) c \right\} \frac{1}{c^3} x_L B_0^{\text{fin}}(M_0, M_L) \\
 & + \frac{1}{8} (3a_{LW} + 4a_{\phi_L}^{(3)}) J_{61} x_L B_0^{\text{fin}}(0, M) \\
 & + \frac{1}{4} (8a_{\phi_{WAD}} - 3L_0 s a_{LWB}) x_L B_0^{\text{fin}}(0, M_L) \\
 & - \frac{1}{8} (a_{\phi_{WD}}^{(-)} - 4a_{\phi_{\square}}) x_L^3 B_0^{\text{fin}}(M_H, M_L)
 \end{aligned}$$

$$V_{LL;0}^{(4)} = -\frac{1}{4} + \frac{1}{8} I_3 L_R - \frac{1}{16} (16s^2 c^2 + G_6) \frac{1}{c^2} (1 - L_R)$$

$$\begin{aligned}
 V_{LL;1}^{(4)} = & -\frac{1}{8} \frac{x_L^2}{x_S} x_H^2 a_0^{\text{fin}}(M_H) - \frac{1}{8} \frac{1}{x_S} I_0 a_0^{\text{fin}}(M) \\
 & - \frac{1}{16} (2c^2 x_L^2 + G_6) \frac{1}{c^4 x_S} a_0^{\text{fin}}(M_0) + \frac{1}{16} (4c^2 x_L^2 + 16s^2 c^2 + G_6) \frac{x_L^2}{c^2 x_S} a_0^{\text{fin}}(M_L)
 \end{aligned}$$

$$\begin{aligned}
 V_{LL;2}^{(4)} = & \frac{s^2}{x_S} L_2 B_0^{\text{fin}}(0, M_L) - \frac{1}{8} \frac{1}{x_S} I_0 J_{61} B_0^{\text{fin}}(0, M) \\
 & - \frac{1}{16} \left[ (G_6 - 2L_2 c^4 x_L^2) - (G_6 x_S - G_{33} x_L^2) c^2 \right] \frac{1}{c^4 x_S} B_0^{\text{fin}}(M_0, M_L) \\
 & + \frac{1}{8} (x_L^2 + J_{34}) \frac{x_L^2}{x_S} B_0^{\text{fin}}(M_H, M_L)
 \end{aligned}$$

$$\begin{aligned}
 V_{LL;0}^{(6)} = & -\frac{1}{32} \left\{ 32G_{28} s c^3 a_{\phi_{WB}} - 16F_{31}^b s^2 a_{\phi_W} v_1 \right. \\
 & \left. + \left[ 8G_{11} a_{\phi_L} - G_{40} a_{\phi_D} + 4(a_{\phi_{LVA}} + a_{\phi_W}) G_{17} - 4(4c^4 + s^2 v_1) a_{\phi_D} \right] \right\} \frac{1}{c^2} (1 - L_R) \\
 & + \frac{1}{8} \left\{ \left[ 2I_3 a_{\phi_W} + 2I_4 a_{\phi_L}^{(3)} + (-2a_{\phi_L} + 2a_{\phi_L}^{(1)} - a_{\phi_D} - 2a_{\phi_{\square}}) x_L^2 \right] + 8(a_{\phi_B} v_1 + 2a_{\phi_W}) s^2 \right\} L_R
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{16} (v_1 a_{L\text{BW}} + 4 s c a_{L\text{WB}}) \frac{1}{c} x_L^2 (2 - 3 L_R) \\
 & - \frac{1}{4} (c a_{\phi L W}^{(3)} + 4 G_{29} s^2 c a_{\phi W} + 2 F_5^b s a_{\phi W B} v_1) \frac{1}{c}
 \end{aligned}$$

$$\begin{aligned}
 V_{LL;1}^{(6)} = & \frac{1}{32} \left\{ 4 s^2 a_{\phi W D}^{(+)} v_1 - 16 F_5^b s c a_{\phi W B} v_1 \right. \\
 & - 2 \left[ 3 v_1 a_{L\text{BW}} + 4 c a_{\phi L A} + c a_{\phi W D}^{(-)} + 8 (a_{\phi W Z} - c a_{\phi B}) s^2 c^2 v_1 \right] c x_L^2 \\
 & - \left[ 8 G_{11} a_{\phi L} - G_{40} a_{\phi D} + 4 (a_{\phi L V A} + a_{\phi W}) G_{17} \right] + 8 (x_S a_{A Z} + I_4 a_{\phi W B}) s c^3 v_1 \left. \right\} \frac{1}{c^4 x_S} a_0^{\text{fin}}(M_0) \\
 & + \frac{1}{32} \left\{ \left[ 8 G_{11} a_{\phi L} - G_{40} a_{\phi D} + 4 (a_{\phi L V A} + a_{\phi W}) G_{17} - 4 (4 c^4 + s^2 v_1) a_{\phi D} \right] \right. \\
 & + 2 (3 v_1 a_{L\text{BW}} + 4 c a_{\phi L A} + 2 c a_{\phi W D}^{(-)} - 4 c a_{1\phi\Box} + 12 s c a_{L\text{WB}}) c x_L^2 + 16 (2 G_{28} c^2 + F_5^b v_1) s c a_{\phi W B} \\
 & \left. + 16 (2 G_{29} c^2 - F_{31}^b v_1) s^2 a_{\phi W} \right\} \frac{x_L^2}{c^2 x_S} a_0^{\text{fin}}(M_L) \\
 & - \frac{1}{4} (4 a_{\phi L}^{(3)} + I_0 a_{\phi W}) \frac{1}{x_S} a_0^{\text{fin}}(M) \\
 & - \frac{1}{16} (a_{\phi W D}^{(-)} - 4 a_{1\phi\Box}) \frac{x_L^2}{x_S} x_H^2 a_0^{\text{fin}}(M_H)
 \end{aligned}$$

$$\begin{aligned}
 V_{LL;2}^{(6)} = & \frac{1}{16} \left[ (a_{\phi W D}^{(-)} - 4 a_{1\phi\Box}) x_L^2 + (a_{\phi W D}^{(-)} - 4 a_{1\phi\Box}) J_{34} \right] \frac{x_L^2}{x_S} B_0^{\text{fin}}(M_H, M_L) \\
 & + \frac{1}{32} \left\{ 2 L_2 c^4 a_{\phi W D}^{(-)} x_L^2 - \left[ 8 G_{11} a_{\phi L} - G_{40} a_{\phi D} + 4 (a_{\phi L V A} + a_{\phi W}) G_{17} \right] \right. \\
 & + \left[ 8 G_{11} x_S a_{\phi L} - 4 G_{11} a_{\phi W} x_L^2 + G_{36} a_{\phi D} x_L^2 - G_{40} a_{\phi D} x_S + 4 (a_{\phi L V A} + a_{\phi W}) G_{17} x_S \right] c^2 \\
 & - 2 (3 a_{L\text{BW}} - 4 c a_{\phi L V} - 8 s^2 c^3 a_{\phi B}) c v_1 x_L^2 + 8 (a_{A Z} x_S^2 - 2 I_0 x_S a_{\phi W B} - I_4 a_{\phi W B} x_L^2) s c^5 v_1 \\
 & - 16 (a_{\phi W Z} - c a_{\phi B}) L_3 s^2 c^5 v_1 x_L^2 \\
 & - 2 (3 v_1 a_{L\text{BW}} + 4 c a_{\phi L A}) L_0 c^3 x_L^2 + 4 (a_{\phi D} + 4 F_{31}^b a_{\phi W}) s^2 v_1 \\
 & - 8 (-c a_{\phi W B} - 2 s a_{A A} + 4 s a_{\phi W}) s c^2 x_S v_1 \\
 & - 4 (4 I_3 a_{\phi W} + L_2 a_{\phi D} - 4 F_{32}^b a_{\phi W} x_L^2) s^2 c^2 v_1 \\
 & \left. + 8 (I_5 c^2 - 2 L_4 s^2 c^2 - 2 F_5^b) s c a_{\phi W B} v_1 \right\} \frac{1}{c^4 x_S} B_0^{\text{fin}}(M_0, M_L) \\
 & - \frac{1}{4} (I_0 J_{61} a_{\phi W} + 4 J_{61} a_{\phi L}^{(3)}) \frac{1}{x_S} B_0^{\text{fin}}(0, M) \\
 & - \frac{1}{4} (3 L_0 s x_L^2 a_{L\text{WB}} - 2 L_2 a_{\phi W A D} + L_5 s c v_1 a_{A Z}) \frac{1}{x_S} B_0^{\text{fin}}(0, M_L)
 \end{aligned}$$

$$A_{LL;0}^{(4)} = -\frac{1}{4} - \frac{1}{8} \frac{1}{c^2} v_1 (1 - L_R) + \frac{1}{8} I_1 L_R$$

$$A_{LL;1}^{(4)} = \frac{1}{8} \frac{x_L^2}{c^2 x_S} v_1 a_0^{\text{fin}}(M_L) - \frac{1}{8} \frac{1}{c^4 x_S} v_1 a_0^{\text{fin}}(M_0) - \frac{1}{8} \frac{1}{x_S} I_1 a_0^{\text{fin}}(M)$$

$$A_{LL;2}^{(4)} = -\frac{1}{8} \frac{1}{x_S} I_1 J_{61} B_0^{\text{fin}}(0, M) - \frac{1}{8} (1 - L_2 c^2) \frac{1}{c^4 x_S} v_1 B_0^{\text{fin}}(M_0, M_L)$$

$$\begin{aligned}
 A_{LL;0}^{(6)} = & \frac{1}{16} \frac{1}{c} x_L^2 a_{L\text{BW}} (2 - 3 L_R) \\
 & + \frac{1}{32} \left\{ 4 s^2 a_{\phi D} + \left[ 8 G_{11} a_{\phi L} - G_{41} a_{\phi D} - 4 (a_{\phi L V A} + a_{\phi W}) G_{17} \right] \right\} \frac{1}{c^2} (1 - L_R)
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{1}{4} \left\{ 2F_{31}^b s^2 a_{\phi W} - \left[ I_1 a_{\phi W} + I_4 a_{\phi L}^{(3)} + (-a_{\phi LV} + a_{\phi L}^{(3)}) x_L^2 \right] c^2 - 4(-c a_{\phi WB} + s a_{\phi B}) s c^2 \right\} \frac{L_R}{c^2} \\
 & + \frac{1}{2} (-2c^2 a_{\phi L}^{(3)} - c^2 a_{ZZ} + s^2 a_{AA}) \frac{1}{c^2}
 \end{aligned}$$

$$\begin{aligned}
 A_{LL;1}^{(6)} &= \frac{1}{32} \left\{ 4s^2 a_{\phi WD}^{(+)} - 16F_5^b s c a_{\phi WB} - 2 \left[ 3a_{LBW} + 4c a_{\phi LV} + 8(a_{\phi WZ} - c a_{\phi B}) s^2 c^2 \right] c x_L^2 \right. \\
 & \quad \left. + \left[ 8G_{11} a_{\phi L} - G_{41} a_{\phi D} - 4(a_{\phi LVA} + a_{\phi W}) G_{17} \right] + 8(x_S a_{AZ} + I_4 a_{\phi WB}) s c^3 \right\} \frac{1}{c^4 x_S} a_0^{\text{fin}}(M_0) \\
 & - \frac{1}{32} \left\{ 16F_3^b s^2 a_{AA} + 4 \left[ 4c^3 a_{AZ} + (a_{\phi D} - 4c^2 a_{ZZ}) s \right] s \right. \\
 & \quad \left. + \left[ 8G_{11} a_{\phi L} - G_{41} a_{\phi D} - 4(a_{\phi LVA} + a_{\phi W}) G_{17} \right] - 2(3a_{LBW} + 4c a_{\phi LV}) c x_L^2 \right\} \frac{x_L^2}{c^2 x_S} a_0^{\text{fin}}(M_L) \\
 & - \frac{1}{4} (4a_{\phi L}^{(3)} + I_1 a_{\phi W}) \frac{1}{x_S} a_0^{\text{fin}}(M)
 \end{aligned}$$

$$\begin{aligned}
 A_{LL;2}^{(6)} &= -\frac{1}{4} \frac{1}{x_S} L_5 s c a_{AZ} B_0^{\text{fin}}(0, M_L) \\
 & - \frac{1}{32} \left\{ - \left[ 8G_{11} a_{\phi L} - G_{41} a_{\phi D} - 4(a_{\phi LVA} + a_{\phi W}) G_{17} \right] + 2(3a_{LBW} + 4c a_{\phi LV}) L_0 c^3 x_L^2 \right. \\
 & \quad \left. + 2(3a_{LBW} - 4c v_1 a_{\phi LA} - 8s^2 c^3 a_{\phi B}) c x_L^2 - 8(a_{AZ} x_S^2 - 2I_0 x_S a_{\phi WB} - I_4 a_{\phi WB} x_L^2) s c^5 \right. \\
 & \quad \left. + 16(a_{\phi WZ} - c a_{\phi B}) L_3 s^2 c^5 x_L^2 \right. \\
 & \quad \left. - 4(a_{\phi D} + 4F_{31}^b a_{\phi W}) s^2 + 8(-c a_{\phi WB} - 2s a_{AA} + 4s a_{\phi W}) s c^2 x_S \right. \\
 & \quad \left. + (8G_{11} x_S a_{\phi L} - 4G_{17} x_S a_{\phi LVA} - 4G_{17} L_2 a_{\phi W} - G_{41} L_2 a_{\phi D}) c^2 \right. \\
 & \quad \left. + 4(4I_3 a_{\phi W} + L_2 a_{\phi D} - 4F_{32}^b a_{\phi W} x_L^2) s^2 c^2 \right. \\
 & \quad \left. - 8(I_5 c^2 - 2L_4 s^2 c^2 - 2F_5^b) s c a_{\phi WB} \right\} \frac{1}{c^4 x_S} B_0^{\text{fin}}(M_0, M_L) \\
 & - \frac{1}{4} (I_1 J_{61} a_{\phi W} + 4J_{61} a_{\phi L}^{(3)}) \frac{1}{x_S} B_0^{\text{fin}}(0, M)
 \end{aligned}$$

We have introduced

$$\begin{aligned}
 C_{UV}^L &= -\frac{1}{256} \frac{M_L^2}{M} \left[ 3(s a_{LB} + c a_{LW}) v_1 \right. \\
 & \quad \left. + 4c(a_{\phi L} - a_{\phi L}^{(1)} + 3a_{\phi L}^{(3)} + 3c s a_{LB}) + 6(1 + 2s^2) c a_{LW} \right] \frac{1}{c}
 \end{aligned}$$

- U quark self-energy

$$\Sigma_{UU;0}^{(4)} = \frac{1}{2} L_R x_D^2 x_U - \frac{1}{72} (64s^2 c^2 - 9G_{42}) \frac{1}{c^2} x_U (1 - 2L_R)$$

$$\Sigma_{UU;1}^{(4)} = 0$$

$$\begin{aligned}
 \Sigma_{UU;2}^{(4)} &= \frac{1}{2} x_D^2 x_U B_0^{\text{fin}}(M, M_D) - \frac{1}{4} x_U x_U^2 B_0^{\text{fin}}(M_H, M_U) \\
 & \quad + \frac{16}{9} s^2 x_U B_0^{\text{fin}}(0, M_U) + \frac{1}{4} (c^2 x_U^2 - G_{42}) \frac{1}{c^2} x_U B_0^{\text{fin}}(M_0, M_U)
 \end{aligned}$$

$$\Sigma_{UU;0}^{(6)} = \frac{1}{3} \frac{s}{c} v_U a_{\phi WB} x_U (2 - 5L_R)$$

$$\begin{aligned}
 & + \frac{1}{8} \left[ 2\text{H}_9 c^3 a_{\text{UW}} + (1 + c^2 x_{\text{U}}^2) v_{\text{U}} a_{\text{UBW}} \right] \frac{1}{c^3} x_{\text{U}} (2 - 3\text{L}_R) \\
 & + \frac{1}{72} \left\{ 9s c v_{\text{U}} x_{\text{S}} x_{\text{U}} a_{\text{UB}} - 32\text{G}_{48} s^2 c^2 a_{\phi\text{W}} x_{\text{U}} - 24\text{L}_7 s c^2 x_{\text{U}} a_{\text{UWB}} \right. \\
 & \left. - 9 \left[ \text{G}_{25} x_{\text{U}} a_{\text{UW}} - 128 (M \Delta_{\text{UV}} C_{\text{UV}}^{\text{U}}) \right] c^2 x_{\text{S}} + 18 (\text{G}_{12} a_{\phi\text{UVA}} - 2\text{G}_{13} a_{\phi\text{U}}) x_{\text{U}} \right\} \frac{1}{c^2} \\
 & + \frac{1}{36} \left\{ 8\text{G}_{49} s^2 c^2 a_{\phi\text{W}} - 9 \left[ 3x_{\text{H}}^2 a_{\text{U}\phi} - 4x_{\text{D}}^2 a_{\phi\text{W}} - 4\text{H}_9 a_{\phi\text{Q}}^{(3)} + \text{I}_6 a_{\text{U}\phi} - 2(a_{\phi\text{UA}} - a_{\phi\text{Q}} - 2s a_{\text{UWB}}) x_{\text{U}}^2 \right] c^2 \right. \\
 & \left. - 3 \left[ 3\text{G}_{43} a_{\phi\text{UVA}} - 6\text{G}_{44} a_{\phi\text{U}} + (3a_{\text{U}\phi} - 8(c a_{\phi\text{B}} + s a_{\phi\text{WB}}) s^2 c v_{\text{U}}) \right] \right\} \frac{\text{L}_R}{c^2} x_{\text{U}} \\
 & + \frac{1}{144} \left\{ 96\text{F}_{31}^b s^2 v_{\text{U}} a_{\phi\text{W}} + \left[ 36\text{G}_{12} a_{\phi\text{W}} - 3\text{G}_{47} a_{\phi\text{D}} + 8(8c^4 a_{\phi\text{D}} \right. \right. \\
 & \left. \left. - (-32c^3 a_{\phi\text{WB}} + 3s v_{\text{U}} a_{\phi\text{D}}) s \right] \right\} \frac{1}{c^2} x_{\text{U}} (1 - 2\text{L}_R) \\
 & + \frac{1}{2} \sum_{\text{gen}} (-3x_{\text{d}}^3 a_{\text{QUQd}}^{(1)} + x_{\text{I}}^2 x_{\text{I}} a_{\text{L1Qu}}^{(1)}) \text{L}_R
 \end{aligned}$$

$$\begin{aligned}
 \Sigma_{\text{UU};1}^{(6)} &= \frac{3}{4} x_{\text{H}}^2 x_{\text{U}} a_{\text{U}\phi} a_0^{\text{fin}}(M_{\text{H}}) - \frac{1}{2} \sum_{\text{gen}} x_{\text{I}}^3 a_{\text{L1Qu}}^{(1)} a_0^{\text{fin}}(M_{\text{I}}) + \frac{3}{2} \sum_{\text{gen}} x_{\text{d}}^3 a_{\text{QUQd}}^{(1)} a_0^{\text{fin}}(M_{\text{d}}) \\
 & + \frac{1}{8} (3a_{\text{UW}} - 4a_{\phi\text{Q}}^{(3)}) x_{\text{D}}^2 x_{\text{U}} a_0^{\text{fin}}(M_{\text{D}}) + \frac{1}{8} (3a_{\text{UW}} - 4a_{\phi\text{Q}}^{(3)} + 2a_{\text{U}\phi}) x_{\text{U}} a_0^{\text{fin}}(M) \\
 & + \frac{1}{16} (3v_{\text{U}} a_{\text{UBW}} - 4c a_{\phi\text{UA}} + 4c a_{\text{U}\phi}) \frac{1}{c^3} x_{\text{U}} a_0^{\text{fin}}(M_0) \\
 & + \frac{1}{16} (3v_{\text{U}} a_{\text{UBW}} - 4c a_{\phi\text{UA}} + 8s c a_{\text{UWB}}) \frac{1}{c} x_{\text{U}} x_{\text{U}}^2 a_0^{\text{fin}}(M_{\text{U}})
 \end{aligned}$$

$$\begin{aligned}
 \Sigma_{\text{UU};2}^{(6)} &= \frac{1}{8} \left[ 8x_{\text{D}}^2 a_{\phi\text{W}} + (3a_{\text{UW}} - 4a_{\phi\text{Q}}^{(3)}) x_{\text{S}} - (3a_{\text{UW}} - 4a_{\phi\text{Q}}^{(3)}) \text{H}_9 \right] x_{\text{U}} \text{B}_0^{\text{fin}}(M, M_{\text{D}}) \\
 & + \frac{1}{48} \left\{ 6c^3 a_{\phi\text{WD}}^{(-)} x_{\text{U}}^2 - 64\text{F}_3^b s^2 c v_{\text{U}} a_{\text{AA}} - 16\text{F}_4^b s c^2 v_{\text{U}} a_{\text{AZ}} \right. \\
 & \left. - \left[ 9a_{\text{UBW}} + 16(a_{\phi\text{D}} - 4c^2 a_{\text{ZZ}}) s^2 c \right] v_{\text{U}} + 3(3v_{\text{U}} a_{\text{UBW}} - 4c a_{\phi\text{UA}}) \text{L}_6 c^2 \right. \\
 & \left. - 2(12\text{G}_{12} a_{\phi\text{W}} + 3\text{G}_{45} a_{\phi\text{UVA}} - 6\text{G}_{46} a_{\phi\text{U}} - \text{G}_{47} a_{\phi\text{D}}) c \right\} \frac{1}{c^3} x_{\text{U}} \text{B}_0^{\text{fin}}(M_0, M_{\text{U}}) \\
 & + \frac{1}{18} (16a_{\phi\text{WAD}} + 9\text{L}_6 s a_{\text{UWB}}) x_{\text{U}} \text{B}_0^{\text{fin}}(0, M_{\text{U}}) \\
 & - \frac{1}{8} (a_{\phi\text{WD}}^{(-)} + 4a_{\text{U}\phi\text{Q}}) x_{\text{U}} x_{\text{U}}^2 \text{B}_0^{\text{fin}}(M_{\text{H}}, M_{\text{U}})
 \end{aligned}$$

$$V_{\text{UU};0}^{(4)} = -\frac{1}{4} + \frac{1}{8} (3x_{\text{U}}^2 + \text{H}_{12}) \text{L}_R - \frac{1}{144} (64s^2 c^2 + 9\text{G}_4) \frac{1}{c^2} (1 - \text{L}_R)$$

$$\begin{aligned}
 V_{\text{UU};1}^{(4)} &= -\frac{1}{8} \frac{x_{\text{U}}^2}{x_{\text{S}}} x_{\text{H}}^2 a_0^{\text{fin}}(M_{\text{H}}) + \frac{1}{8} (x_{\text{U}}^2 + \text{H}_{12}) \frac{x_{\text{D}}^2}{x_{\text{S}}} a_0^{\text{fin}}(M_{\text{D}}) \\
 & - \frac{1}{8} (x_{\text{U}}^2 + \text{H}_{12}) \frac{1}{x_{\text{S}}} a_0^{\text{fin}}(M) - \frac{1}{16} (2c^2 x_{\text{U}}^2 + \text{G}_4) \frac{1}{c^4 x_{\text{S}}} a_0^{\text{fin}}(M_0) \\
 & + \frac{1}{144} (36c^2 x_{\text{U}}^2 + 64s^2 c^2 + 9\text{G}_4) \frac{x_{\text{U}}^2}{c^2 x_{\text{S}}} a_0^{\text{fin}}(M_{\text{U}})
 \end{aligned}$$

$$\begin{aligned}
 V_{\text{UU};2}^{(4)} &= \frac{4}{9} \frac{s^2}{x_{\text{S}}} \text{L}_8 \text{B}_0^{\text{fin}}(0, M_{\text{U}}) \\
 & - \frac{1}{16} \left[ (\text{G}_4 - 2\text{L}_8 c^4 x_{\text{U}}^2) - (\text{G}_4 x_{\text{S}} - \text{G}_{42} x_{\text{U}}^2) c^2 \right] \frac{1}{c^4 x_{\text{S}}} \text{B}_0^{\text{fin}}(M_0, M_{\text{U}}) \\
 & + \frac{1}{8} (x_{\text{U}}^2 + \text{J}_{34}) \frac{x_{\text{U}}^2}{x_{\text{S}}} \text{B}_0^{\text{fin}}(M_{\text{H}}, M_{\text{U}})
 \end{aligned}$$



$$+\frac{1}{8}(x_s x_U^2 - H_{10} x_U^2 - H_{11} + H_{12} x_s) \frac{1}{x_s} B_0^{\text{fin}}(M, M_D)$$

$$\begin{aligned} V_{\text{UU};0}^{(6)} &= \frac{1}{48} \left[ 6c x_D^2 a_{D\text{W}} - (3v_u a_{\text{UBW}} + 8sc a_{\text{UWB}}) x_U^2 \right] \frac{1}{c} (2 - 3L_R) \\ &+ \frac{1}{96} \left\{ 8s^2 v_u a_{\phi D} + \left[ 24G_{12} a_{\phi U} + G_{50} a_{\phi D} - 12(a_{\phi UVA} + a_{\phi W}) G_{13} \right] \right\} \frac{1}{c^2} (1 - L_R) \\ &- \frac{1}{72} \left\{ - \left[ 18H_{12} a_{\phi W} + 36H_{13} a_{\phi Q}^{(3)} - 9(2a_{\phi UA} - 6a_{\phi W} + a_{\phi D} - 2a_{\text{U}\phi\text{Q}}) x_U^2 \right. \right. \\ &\left. \left. + 8(2a_{\phi WAD} + 3s^2 v_u a_{ZZ}) \right] c^2 + 24(c a_{AZ} + s a_{AA}) F_3^b s v_u \right\} \frac{L_R}{c^2} \\ &- \frac{1}{36} \left\{ \left[ (9a_{\phi QW}^{(3)} - 8c^2 a_{\phi D}) c - 4(3v_u - 8c^2) s a_{\phi WB} \right] c + 4(2G_{48} c^2 - 3F_{31}^b v_u) s^2 a_{\phi W} \right\} \frac{1}{c^2} \end{aligned}$$

$$\begin{aligned} V_{\text{UU};1}^{(6)} &= \frac{1}{96} \left\{ 16K_7 s c^3 v_u a_{AZ} + \left[ 24G_{12} a_{\phi U} + G_{50} a_{\phi D} - 12(a_{\phi UVA} + a_{\phi W}) G_{13} \right] \right. \\ &\left. + 8 \left[ 4F_3^b a_{AA} + (a_{\phi D} - 4c^2 a_{ZZ}) \right] s^2 v_u + 2(9v_u a_{\text{UBW}} - 12c a_{\phi UA}) \right. \\ &\left. - 3c a_{\phi WD}^{(-)} - 8s c^2 v_u a_{AZ} \right\} c x_U^2 \frac{1}{c^4 x_s} a_0^{\text{fin}}(M_0) \\ &- \frac{1}{288} \left\{ \left[ 72G_{12} a_{\phi U} + 3G_{50} a_{\phi D} - 36(a_{\phi UVA} + a_{\phi W}) G_{13} + 8(8c^4 a_{\phi D} \right. \right. \\ &\left. \left. + (4c a_{\phi WB} v_u + 3s v_u a_{\phi D}) s \right] + 18(3v_u a_{\text{UBW}} - 4c a_{\phi UA} - 2c a_{\phi WD}^{(-)} - 4c a_{\text{U}\phi\text{Q}} + 8s c a_{\text{UWB}}) c x_U^2 \right. \\ &\left. - 32(2G_{48} c^2 - 3F_{31}^b v_u) s^2 a_{\phi W} \right\} \frac{x_U^2}{c^2 x_s} a_0^{\text{fin}}(M_U) \\ &- \frac{1}{16} (a_{\phi WD}^{(-)} + 4a_{\text{U}\phi\text{Q}}) \frac{x_U^2}{x_s} x_H^2 a_0^{\text{fin}}(M_H) \\ &+ \frac{1}{8} (2a_{\phi W} x_U^2 + 3x_D^2 a_{D\text{W}} + H_{12} a_{\phi QW}^{(3)}) \frac{x_D^2}{x_s} a_0^{\text{fin}}(M_D) \\ &- \frac{1}{8} (2a_{\phi W} x_U^2 + 3x_D^2 a_{D\text{W}} + H_{12} a_{\phi QW}^{(3)}) \frac{1}{x_s} a_0^{\text{fin}}(M) \end{aligned}$$

$$\begin{aligned} V_{\text{UU};2}^{(6)} &= \frac{1}{16} \left[ (a_{\phi WD}^{(-)} + 4a_{\text{U}\phi\text{Q}}) x_U^2 + (a_{\phi WD}^{(-)} + 4a_{\text{U}\phi\text{Q}}) J_{34} \right] \frac{x_U^2}{x_s} B_0^{\text{fin}}(M_H, M_U) \\ &+ \frac{1}{96} \left\{ 6L_8 c^4 a_{\phi WD}^{(-)} x_U^2 + 32F_{31}^b s^2 v_u a_{\phi W} + 16F_{33}^b s c^2 v_u a_{\phi WA} x_U^2 \right. \\ &\left. + 8 \left[ 4c a_{\phi WB} + (a_{\phi D} - 8c^2 a_{\phi B}) s \right] s v_u + \left[ 24G_{12} a_{\phi U} + G_{50} a_{\phi D} - 12(a_{\phi UVA} + a_{\phi W}) G_{13} \right] \right. \\ &\left. - \left[ 24G_{12} x_s a_{\phi U} + 12G_{12} a_{\phi W} x_U^2 - G_{47} a_{\phi D} x_U^2 + G_{50} a_{\phi D} x_s - 12(a_{\phi UVA} + a_{\phi W}) G_{13} x_s \right] c^2 \right. \\ &\left. + 2(9a_{\text{UBW}} + 12c a_{\phi UV} + 40s^2 c^2 a_{\phi WZ}) c v_u x_U^2 - 16(a_{AZ} - 2sc a_{ZZ}) s c^3 v_u x_s \right. \\ &\left. + 6(3v_u a_{\text{UBW}} - 4c a_{\phi UA}) L_6 c^3 x_U^2 + 8(8I_7 a_{\phi B} - 4I_8 a_{\phi W} - L_8 a_{\phi D} - 4F_3^b x_s a_{AA} - 4F_{32}^b a_{\phi B} x_U^2) s^2 c^2 v_u \right. \\ &\left. - 16(K_6 x_s + L_9 x_U^2) s c^5 v_u a_{AZ} \right\} \frac{1}{c^4 x_s} B_0^{\text{fin}}(M_0, M_U) \\ &+ \frac{1}{8} (2a_{\phi W} x_s x_U^2 - 3x_D^2 x_s a_{D\text{W}} - 2H_{10} a_{\phi W} x_U^2 - 3H_{10} x_D^2 a_{D\text{W}} - H_{11} a_{\phi QW}^{(3)} + 2H_{12} a_{\phi W} x_s \\ &+ 4H_{13} x_s a_{\phi Q}^{(3)}) \frac{1}{x_s} B_0^{\text{fin}}(M, M_D) \\ &+ \frac{1}{18} (9L_6 s x_U^2 a_{\text{UWB}} + 4L_8 a_{\phi WAD} - 3L_{10} s c v_u a_{AZ}) \frac{1}{x_s} B_0^{\text{fin}}(0, M_U) \end{aligned}$$

$$A_{\text{UU};0}^{(4)} = -\frac{1}{4} - \frac{1}{8} \frac{1}{c^2} v_u (1 - L_R) - \frac{1}{8} (x_U^2 - H_{12}) L_R$$

$$A_{UU;1}^{(4)} = \frac{1}{8} \frac{x_U^2}{c^2 x_S} v_u a_0^{\text{fin}}(M_U) - \frac{1}{8} \frac{1}{c^4 x_S} v_u a_0^{\text{fin}}(M_0) \\ - \frac{1}{8} (x_U^2 - H_{12}) \frac{x_U^2}{x_S} a_0^{\text{fin}}(M_D) + \frac{1}{8} (x_U^2 - H_{12}) \frac{1}{x_S} a_0^{\text{fin}}(M)$$

$$A_{UU;2}^{(4)} = -\frac{1}{8} (1 - L_8 c^2) \frac{1}{c^4 x_S} v_u B_0^{\text{fin}}(M_0, M_U) \\ - \frac{1}{8} (x_S x_U^2 - H_{10} x_U^2 + H_{11} - H_{12} x_S) \frac{1}{x_S} B_0^{\text{fin}}(M, M_D)$$

$$A_{UU;0}^{(6)} = \frac{1}{6} \frac{s^2}{c^2} a_{AA} (1 - 2L_R) - \frac{1}{6} s c a_{AZ} (1 + 2L_R) \\ - \frac{1}{6} \left[ F_{34}^b c^2 a_{ZZ} - (-6c^2 a_{\phi Q}^{(3)} + s^4 a_{AA}) \right] \frac{1}{c^2} \\ + \frac{1}{96} \left\{ 8s^2 a_{\phi D} - \left[ 24G_{12} a_{\phi U} + G_{51} a_{\phi D} + 12(a_{\phi UVA} + a_{\phi W}) G_{13} \right] \right\} \frac{1}{c^2} (1 - L_R) \\ - \frac{1}{12} \left\{ -3 \left[ H_{12} a_{\phi W} + 2H_{13} a_{\phi Q}^{(3)} - (a_{\phi UV} + a_{\phi W}) x_U^2 \right] c + 4(a_{AZ} + s c a_{AA} - s c a_{ZZ}) s \right\} \frac{L_R}{c} \\ - \frac{1}{16} (x_U^2 a_{UBW} - 2c x_D^2 a_{DW}) (2 - 3L_R) \frac{1}{c}$$

$$A_{UU;1}^{(6)} = \frac{1}{96} \left\{ 16K_7 s c^3 a_{AZ} - \left[ 24G_{12} a_{\phi U} + G_{51} a_{\phi D} + 12(a_{\phi UVA} + a_{\phi W}) G_{13} \right] \right. \\ \left. + 8 \left[ 4F_3^b a_{AA} + (a_{\phi D} - 4c^2 a_{ZZ}) \right] s^2 + 2(9a_{UBW} - 12c a_{\phi UV} - 8s c^2 a_{AZ}) c x_U^2 \right\} \frac{1}{c^4 x_S} a_0^{\text{fin}}(M_0) \\ - \frac{1}{96} \left\{ 32F_3^b s^2 a_{AA} + 8 \left[ 4c^3 a_{AZ} + (a_{\phi D} - 4c^2 a_{ZZ}) s \right] s \right. \\ \left. - \left[ 24G_{12} a_{\phi U} + G_{51} a_{\phi D} + 12(a_{\phi UVA} + a_{\phi W}) G_{13} \right] + 6(3a_{UBW} - 4c a_{\phi UV}) c x_U^2 \right\} \frac{x_U^2}{c^2 x_S} a_0^{\text{fin}}(M_U) \\ - \frac{1}{8} (2a_{\phi W} x_U^2 - 3x_D^2 a_{DW} - H_{12} a_{\phi QW}^{(3)}) \frac{x_D^2}{x_S} a_0^{\text{fin}}(M_D) \\ + \frac{1}{8} (2a_{\phi W} x_U^2 - 3x_D^2 a_{DW} - H_{12} a_{\phi QW}^{(3)}) \frac{1}{x_S} a_0^{\text{fin}}(M)$$

$$A_{UU;2}^{(6)} = -\frac{1}{6} \frac{1}{x_S} L_{10} s c a_{AZ} B_0^{\text{fin}}(0, M_U) \\ + \frac{1}{96} \left\{ 32F_{31}^b s^2 a_{\phi W} + 16F_{33}^b s c^2 a_{\phi WA} x_U^2 + 8 \left[ 4c a_{\phi WB} + (a_{\phi D} - 8c^2 a_{\phi B}) s \right] s \right. \\ \left. - \left[ 24G_{12} a_{\phi U} + G_{51} a_{\phi D} + 12(a_{\phi UVA} + a_{\phi W}) G_{13} \right] + 6(3a_{UBW} - 4c a_{\phi UV}) L_6 c^3 x_U^2 \right. \\ \left. + 2(9a_{UBW} + 12c v_u a_{\phi UA} + 40s^2 c^2 a_{\phi WZ}) c x_U^2 - 16(a_{AZ} - 2s c a_{ZZ}) s c^3 x_S \right. \\ \left. + (24G_{12} x_S a_{\phi U} + 12G_{13} x_S a_{\phi UVA} + 12G_{13} L_8 a_{\phi W} + G_{51} L_8 a_{\phi D}) c^2 \right. \\ \left. + 8(8I_7 a_{\phi B} - 4I_8 a_{\phi W} - L_8 a_{\phi D} - 4F_3^b x_S a_{AA} - 4F_{32}^b a_{\phi B} x_U^2) s^2 c^2 \right. \\ \left. - 16(K_6 x_S + L_9 x_U^2) s c^5 a_{AZ} \right\} \frac{1}{c^4 x_S} B_0^{\text{fin}}(M_0, M_U) \\ - \frac{1}{8} (2a_{\phi W} x_S x_U^2 + 3x_D^2 x_S a_{DW} - 2H_{10} a_{\phi W} x_U^2 + 3H_{10} x_D^2 a_{DW} + H_{11} a_{\phi QW}^{(3)} - 2H_{12} a_{\phi W} x_S \\ - 4H_{13} x_S a_{\phi Q}^{(3)}) \frac{1}{x_S} B_0^{\text{fin}}(M, M_D)$$

We have introduced

$$C_{UV}^U = -\frac{1}{256} \frac{M_U^2}{M} \left[ 3(s a_{UB} - c a_{UW}) v_u \right]$$

$$+4c(a_{\phi Q}^{(1)} - a_{\phi U} + 3a_{\phi Q}^{(3)} + 2csa_{UB}) - 2(3+4s^2)ca_{UW} \Big] \frac{1}{c}$$

- D quark self-energy

$$\Sigma_{DD;0}^{(4)} = \frac{1}{2} L_R x_U^2 x_D - \frac{1}{72} (16s^2 c^2 - 9G_{52}) \frac{1}{c^2} x_D (1 - 2L_R)$$

$$\Sigma_{DD;1}^{(4)} = 0$$

$$\begin{aligned} \Sigma_{DD;2}^{(4)} &= \frac{1}{2} x_U^2 x_D B_0^{\text{fin}}(M, M_U) - \frac{1}{4} x_D x_D^2 B_0^{\text{fin}}(M_H, M_D) \\ &\quad + \frac{4}{9} s^2 x_D B_0^{\text{fin}}(0, M_D) + \frac{1}{4} (c^2 x_D^2 - G_{52}) \frac{1}{c^2} x_D B_0^{\text{fin}}(M_0, M_D) \end{aligned}$$

$$\begin{aligned} \Sigma_{DD;0}^{(6)} &= \frac{1}{6} \frac{s}{c} v_d a_{\phi WB} x_D (2 - 5L_R) \\ &\quad - \frac{1}{8} \left[ 2H_{14} c^3 a_{DW} + (1 + c^2 x_D^2) v_d a_{DBW} \right] \frac{1}{c^3} x_D (2 - 3L_R) \\ &\quad + \frac{1}{72} \left\{ 9scv_d x_S x_D a_{DB} - 16G_{55} s^2 c^2 a_{\phi W} x_D + 12L_{12} s c^2 x_D a_{DWB} \right. \\ &\quad \left. + 9 \left[ G_{27} a_{DW} x_D + 128(M\Delta_{UV} C_{UV}^D) \right] c^2 x_S + 18(G_{14} a_{\phi DVA} + 2G_{15} a_{\phi D}) x_D \right\} \frac{1}{c^2} \\ &\quad + \frac{1}{36} \left\{ 4G_{59} s^2 c^2 a_{\phi W} + 9 \left[ 3x_H^2 a_{D\phi} + 4x_U^2 a_{\phi W} + 4H_{14} a_{\phi Q}^{(3)} + I_9 a_{D\phi} \right. \right. \\ &\quad \left. \left. + 2(a_{\phi DA} - a_{\phi Q} + s a_{DWB}) x_D^2 \right] c^2 - 3 \left[ 3G_{53} a_{\phi DVA} + 6G_{54} a_{\phi D} \right. \right. \\ &\quad \left. \left. - (3a_{D\phi} + 4(ca_{\phi B} + s a_{\phi WB}) s^2 c v_d) \right] \right\} \frac{L_R}{c^2} x_D \\ &\quad + \frac{1}{144} \left\{ 48F_{31}^b s^2 v_d a_{\phi W} + \left[ 36G_{14} a_{\phi W} - 3G_{58} a_{\phi D} + 4(4c^4 a_{\phi D} \right. \right. \\ &\quad \left. \left. + (-16c^3 a_{\phi WB} + 3s v_d a_{\phi D}) s \right] \right\} \frac{1}{c^2} x_D (1 - 2L_R) \\ &\quad - \frac{1}{2} \sum_{\text{gen}} (3x_U^3 a_{QUQd}^{(1)} + x_1^2 x_1 a_{L1dQ}) L_R \end{aligned}$$

$$\begin{aligned} \Sigma_{DD;1}^{(6)} &= -\frac{3}{4} x_H^2 x_D a_{D\phi} a_0^{\text{fin}}(M_H) + \frac{1}{2} \sum_{\text{gen}} x_1^3 a_{L1dQ} a_0^{\text{fin}}(M_1) + \frac{3}{2} \sum_{\text{gen}} x_U^3 a_{QUQd}^{(1)} a_0^{\text{fin}}(M_U) \\ &\quad - \frac{1}{8} (2a_{D\phi} + 3a_{DW} + 4a_{\phi Q}^{(3)}) x_D a_0^{\text{fin}}(M) - \frac{1}{8} (3a_{DW} + 4a_{\phi Q}^{(3)}) x_U^2 x_D a_0^{\text{fin}}(M_U) \\ &\quad - \frac{1}{16} (3v_d a_{DBW} + 4ca_{\phi DA} + 4ca_{D\phi}) \frac{1}{c^3} x_D a_0^{\text{fin}}(M_0) \\ &\quad - \frac{1}{16} (3v_d a_{DBW} + 4ca_{\phi DA} + 4sca_{DWB}) \frac{1}{c} x_D x_D^2 a_0^{\text{fin}}(M_D) \end{aligned}$$

$$\begin{aligned} \Sigma_{DD;2}^{(6)} &= \frac{1}{8} \left[ 8x_U^2 a_{\phi W} - (3a_{DW} + 4a_{\phi Q}^{(3)}) x_S + (3a_{DW} + 4a_{\phi Q}^{(3)}) H_{14} \right] x_D B_0^{\text{fin}}(M, M_U) \\ &\quad + \frac{1}{48} \left\{ 6c^3 a_{\phi WD}^{(-)} x_D^2 - 32F_3^b s^2 c v_d a_{AA} - 8F_4^b s c^2 v_d a_{AZ} \right. \\ &\quad \left. + \left[ 9a_{DBW} - 8(a_{\phi D} - 4c^2 a_{ZZ}) s^2 c \right] v_d - 3(3v_d a_{DBW} + 4ca_{\phi DA}) L_{11} c^2 \right. \\ &\quad \left. - 2(12G_{14} a_{\phi W} + 3G_{56} a_{\phi DVA} + 6G_{57} a_{\phi D} - G_{58} a_{\phi D}) c \right\} \frac{1}{c^3} x_D B_0^{\text{fin}}(M_0, M_D) \\ &\quad + \frac{1}{36} (8a_{\phi WAD} - 9L_{11} s a_{DWB}) x_D B_0^{\text{fin}}(0, M_D) \end{aligned}$$

$$\begin{aligned}
 & -\frac{1}{8} (a_{\phi_{\text{WD}}}^{(-)} - 4a_{\text{d}\phi_{\square}}) x_{\text{D}} x_{\text{D}}^2 \text{B}_0^{\text{fin}}(M_{\text{H}}, M_{\text{D}}) \\
 V_{\text{DD};0}^{(4)} &= -\frac{1}{4} + \frac{1}{8} (3x_{\text{D}}^2 + \text{H}_{17}) \text{L}_{\text{R}} - \frac{1}{144} (16s^2 c^2 + 9\text{G}_5) \frac{1}{c^2} (1 - \text{L}_{\text{R}}) \\
 V_{\text{DD};1}^{(4)} &= -\frac{1}{8} \frac{x_{\text{D}}^2}{x_{\text{S}}} x_{\text{H}}^2 a_0^{\text{fin}}(M_{\text{H}}) + \frac{1}{8} (x_{\text{D}}^2 + \text{H}_{17}) \frac{x_{\text{U}}^2}{x_{\text{S}}} a_0^{\text{fin}}(M_{\text{U}}) \\
 & -\frac{1}{8} (x_{\text{D}}^2 + \text{H}_{17}) \frac{1}{x_{\text{S}}} a_0^{\text{fin}}(M) - \frac{1}{16} (2c^2 x_{\text{D}}^2 + \text{G}_5) \frac{1}{c^4 x_{\text{S}}} a_0^{\text{fin}}(M_0) \\
 & + \frac{1}{144} (36c^2 x_{\text{D}}^2 + 16s^2 c^2 + 9\text{G}_5) \frac{x_{\text{D}}^2}{c^2 x_{\text{S}}} a_0^{\text{fin}}(M_{\text{D}}) \\
 V_{\text{DD};2}^{(4)} &= \frac{1}{9} \frac{s^2}{x_{\text{S}}} \text{L}_{13} \text{B}_0^{\text{fin}}(0, M_{\text{D}}) \\
 & -\frac{1}{16} \left[ (\text{G}_5 - 2\text{L}_{13} c^4 x_{\text{D}}^2) - (\text{G}_5 x_{\text{S}} - \text{G}_{52} x_{\text{D}}^2) c^2 \right] \frac{1}{c^4 x_{\text{S}}} \text{B}_0^{\text{fin}}(M_0, M_{\text{D}}) \\
 & + \frac{1}{8} (x_{\text{D}}^2 + \text{J}_{34}) \frac{x_{\text{D}}^2}{x_{\text{S}}} \text{B}_0^{\text{fin}}(M_{\text{H}}, M_{\text{D}}) \\
 & + \frac{1}{8} (x_{\text{S}} x_{\text{D}}^2 - \text{H}_{15} x_{\text{D}}^2 - \text{H}_{16} + \text{H}_{17} x_{\text{S}}) \frac{1}{x_{\text{S}}} \text{B}_0^{\text{fin}}(M, M_{\text{U}}) \\
 V_{\text{DD};0}^{(6)} &= -\frac{1}{48} \left[ 6c x_{\text{U}}^2 a_{\text{UW}} - (3v_{\text{d}} a_{\text{DBW}} + 4sc a_{\text{DWB}}) x_{\text{D}}^2 \right] \frac{1}{c} (2 - 3\text{L}_{\text{R}}) \\
 & + \frac{1}{96} \left\{ 4s^2 v_{\text{d}} a_{\phi_{\text{D}}} - \left[ 24\text{G}_{14} a_{\phi_{\text{D}}} - \text{G}_{60} a_{\phi_{\text{D}}} + 12(a_{\phi_{\text{DVA}}} + a_{\phi_{\text{W}}}) \text{G}_{15} \right] \right\} \frac{1}{c^2} (1 - \text{L}_{\text{R}}) \\
 & -\frac{1}{72} \left\{ -\left[ 18\text{H}_{17} a_{\phi_{\text{W}}} + 36\text{H}_{18} a_{\phi_{\text{Q}}}^{(3)} - 9(2a_{\phi_{\text{DA}}} - 6a_{\phi_{\text{W}}} + a_{\phi_{\text{D}}} + 2a_{\text{d}\phi_{\square}}) x_{\text{D}}^2 \right. \right. \\
 & \left. \left. + 4(a_{\phi_{\text{WAD}}} + 3s^2 v_{\text{d}} a_{\text{ZZ}}) \right] c^2 + 12(c a_{\text{AZ}} + s a_{\text{AA}}) \text{F}_3^b s v_{\text{d}} \right\} \frac{\text{L}_{\text{R}}}{c^2} \\
 & -\frac{1}{36} \left\{ \left[ (9a_{\phi_{\text{QW}}}^{(3)} - 2c^2 a_{\phi_{\text{D}}}) c - 2(3v_{\text{d}} - 4c^2) s a_{\phi_{\text{WB}}} \right] c + 2(2\text{G}_{55} c^2 - 3\text{F}_{31}^b v_{\text{d}}) s^2 a_{\phi_{\text{W}}} \right\} \frac{1}{c^2} \\
 V_{\text{DD};1}^{(6)} &= \frac{1}{96} \left\{ 8\text{K}_7 s c^3 v_{\text{d}} a_{\text{AZ}} - \left[ 24\text{G}_{14} a_{\phi_{\text{D}}} - \text{G}_{60} a_{\phi_{\text{D}}} + 12(a_{\phi_{\text{DVA}}} + a_{\phi_{\text{W}}}) \text{G}_{15} \right] \right. \\
 & \left. + 4 \left[ 4\text{F}_3^b a_{\text{AA}} + (a_{\phi_{\text{D}}} - 4c^2 a_{\text{ZZ}}) \right] s^2 v_{\text{d}} - 2(9v_{\text{d}} a_{\text{DBW}} + 12c a_{\phi_{\text{DA}}} + 3c a_{\phi_{\text{WD}}}^{(-)} \right. \\
 & \left. + 4s c^2 v_{\text{d}} a_{\text{AZ}}) c x_{\text{D}}^2 \right\} \frac{1}{c^4 x_{\text{S}}} a_0^{\text{fin}}(M_0) \\
 & + \frac{1}{288} \left\{ \left[ 72\text{G}_{14} a_{\phi_{\text{D}}} - 3\text{G}_{60} a_{\phi_{\text{D}}} + 36(a_{\phi_{\text{DVA}}} + a_{\phi_{\text{W}}}) \text{G}_{15} - 4(4c^4 a_{\phi_{\text{D}}} \right. \right. \\
 & \left. \left. + (4c a_{\phi_{\text{WB}}} v_{\text{d}} + 3s v_{\text{d}} a_{\phi_{\text{D}}}) s \right] + 18(3v_{\text{d}} a_{\text{DBW}} + 4c a_{\phi_{\text{DA}}} + 2c a_{\phi_{\text{WD}}}^{(-)} - 4c a_{\text{d}\phi_{\square}} + 4s c a_{\text{DWB}}) c x_{\text{D}}^2 \right. \\
 & \left. + 16(2\text{G}_{55} c^2 - 3\text{F}_{31}^b v_{\text{d}}) s^2 a_{\phi_{\text{W}}} \right\} \frac{x_{\text{D}}^2}{c^2 x_{\text{S}}} a_0^{\text{fin}}(M_{\text{D}}) \\
 & -\frac{1}{16} (a_{\phi_{\text{WD}}}^{(-)} - 4a_{\text{d}\phi_{\square}}) \frac{x_{\text{D}}^2}{x_{\text{S}}} x_{\text{H}}^2 a_0^{\text{fin}}(M_{\text{H}}) \\
 & + \frac{1}{8} (2a_{\phi_{\text{W}}} x_{\text{D}}^2 - 3x_{\text{U}}^2 a_{\text{UW}} + \text{H}_{17} a_{\phi_{\text{QW}}}^{(3)}) \frac{x_{\text{U}}^2}{x_{\text{S}}} a_0^{\text{fin}}(M_{\text{U}}) \\
 & -\frac{1}{8} (2a_{\phi_{\text{W}}} x_{\text{D}}^2 - 3x_{\text{U}}^2 a_{\text{UW}} + \text{H}_{17} a_{\phi_{\text{QW}}}^{(3)}) \frac{1}{x_{\text{S}}} a_0^{\text{fin}}(M) \\
 V_{\text{DD};2}^{(6)} &= \frac{1}{16} \left[ (a_{\phi_{\text{WD}}}^{(-)} - 4a_{\text{d}\phi_{\square}}) x_{\text{D}}^2 + (a_{\phi_{\text{WD}}}^{(-)} - 4a_{\text{d}\phi_{\square}}) \text{J}_{34} \right] \frac{x_{\text{D}}^2}{x_{\text{S}}} \text{B}_0^{\text{fin}}(M_{\text{H}}, M_{\text{D}})
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{96} \left\{ 6L_{13}c^4 a_{\phi_{\text{WD}}}^{(-)} x_{\text{D}}^2 + 16F_{31}^b s^2 v_{\text{d}} a_{\phi_{\text{W}}} + 8F_{33}^b s c^2 v_{\text{d}} a_{\phi_{\text{WA}}} x_{\text{D}}^2 \right. \\
 & + 4 \left[ 4c a_{\phi_{\text{WB}}} + (a_{\phi_{\text{D}}} - 8c^2 a_{\phi_{\text{B}}}) s \right] s v_{\text{d}} + \left[ -12G_{14} a_{\phi_{\text{W}}} x_{\text{D}}^2 + 24G_{14} a_{\phi_{\text{D}}} x_{\text{S}} + G_{58} a_{\phi_{\text{D}}} x_{\text{D}}^2 - G_{60} a_{\phi_{\text{D}}} x_{\text{S}} \right. \\
 & \left. \left. + 12(a_{\phi_{\text{DVA}}} + a_{\phi_{\text{W}}}) G_{15} x_{\text{S}} \right] c^2 \right. \\
 & - \left[ 24G_{14} a_{\phi_{\text{D}}} - G_{60} a_{\phi_{\text{D}}} + 12(a_{\phi_{\text{DVA}}} + a_{\phi_{\text{W}}}) G_{15} \right] - 2(9a_{\text{DBW}} - 12c a_{\phi_{\text{DV}}} - 20s^2 c^2 a_{\phi_{\text{WZ}}}) c v_{\text{d}} x_{\text{D}}^2 \\
 & - 8(a_{\text{AZ}} - 2s c a_{\text{ZZ}}) s c^3 v_{\text{d}} x_{\text{S}} - 6(3v_{\text{d}} a_{\text{DBW}} + 4c a_{\phi_{\text{DA}}}) L_{11} c^3 x_{\text{D}}^2 \\
 & + 4(8I_{10} a_{\phi_{\text{B}}} - 4I_{11} a_{\phi_{\text{W}}} - L_{13} a_{\phi_{\text{D}}} - 4F_3^b x_{\text{S}} a_{\text{AA}} \\
 & - 4F_{32}^b a_{\phi_{\text{B}}} x_{\text{D}}^2) s^2 c^2 v_{\text{d}} - 8(K_6 x_{\text{S}} + L_{14} x_{\text{D}}^2) s c^5 v_{\text{d}} a_{\text{AZ}} \left. \right\} \frac{1}{c^4 x_{\text{S}}} B_0^{\text{fin}}(M_0, M_{\text{D}}) \\
 & + \frac{1}{8} (2a_{\phi_{\text{W}}} x_{\text{S}} x_{\text{D}}^2 + 3x_{\text{U}}^2 x_{\text{S}} a_{\text{UW}} - 2H_{15} a_{\phi_{\text{W}}} x_{\text{D}}^2 + 3H_{15} x_{\text{U}}^2 a_{\text{UW}} - H_{16} a_{\phi_{\text{QW}}}^{(3)}) \\
 & + 2H_{17} a_{\phi_{\text{W}}} x_{\text{S}} + 4H_{18} x_{\text{S}} a_{\phi_{\text{Q}}}^{(3)} \frac{1}{x_{\text{S}}} B_0^{\text{fin}}(M, M_{\text{U}}) \\
 & - \frac{1}{36} (9L_{11} s x_{\text{D}}^2 a_{\text{DWB}} - 2L_{13} a_{\phi_{\text{WAD}}} + 3L_{15} s c v_{\text{d}} a_{\text{AZ}}) \frac{1}{x_{\text{S}}} B_0^{\text{fin}}(0, M_{\text{D}})
 \end{aligned}$$

$$A_{\text{DD};0}^{(4)} = -\frac{1}{4} - \frac{1}{8} \frac{1}{c^2} v_{\text{d}} (1 - L_{\text{R}}) - \frac{1}{8} (x_{\text{D}}^2 - H_{17}) L_{\text{R}}$$

$$\begin{aligned}
 A_{\text{DD};1}^{(4)} &= \frac{1}{8} \frac{x_{\text{D}}^2}{c^2 x_{\text{S}}} v_{\text{d}} a_0^{\text{fin}}(M_{\text{D}}) - \frac{1}{8} \frac{1}{c^4 x_{\text{S}}} v_{\text{d}} a_0^{\text{fin}}(M_0) \\
 &\quad - \frac{1}{8} (x_{\text{D}}^2 - H_{17}) \frac{x_{\text{U}}^2}{x_{\text{S}}} a_0^{\text{fin}}(M_{\text{U}}) + \frac{1}{8} (x_{\text{D}}^2 - H_{17}) \frac{1}{x_{\text{S}}} a_0^{\text{fin}}(M)
 \end{aligned}$$

$$\begin{aligned}
 A_{\text{DD};2}^{(4)} &= -\frac{1}{8} (1 - L_{13} c^2) \frac{1}{c^4 x_{\text{S}}} v_{\text{d}} B_0^{\text{fin}}(M_0, M_{\text{D}}) \\
 &\quad - \frac{1}{8} (x_{\text{S}} x_{\text{D}}^2 - H_{15} x_{\text{D}}^2 + H_{16} - H_{17} x_{\text{S}}) \frac{1}{x_{\text{S}}} B_0^{\text{fin}}(M, M_{\text{U}})
 \end{aligned}$$

$$\begin{aligned}
 A_{\text{DD};0}^{(6)} &= -\frac{1}{6} \frac{s^2}{c^2} a_{\text{AA}} (1 + L_{\text{R}}) \\
 &\quad - \frac{1}{6} s c a_{\text{AZ}} (2 + L_{\text{R}}) \\
 &\quad - \frac{1}{6} \left[ F_0^b c^2 a_{\text{ZZ}} - 2(-3c^2 a_{\phi_{\text{Q}}}^{(3)} + s^4 a_{\text{AA}}) \right] \frac{1}{c^2} \\
 &\quad + \frac{1}{96} \left\{ 4s^2 a_{\phi_{\text{D}}} + \left[ 24G_{14} a_{\phi_{\text{D}}} - G_{61} a_{\phi_{\text{D}}} - 12(a_{\phi_{\text{DVA}}} + a_{\phi_{\text{W}}}) G_{15} \right] \right\} \frac{1}{c^2} (1 - L_{\text{R}}) \\
 &\quad - \frac{1}{12} \left\{ -3 \left[ H_{17} a_{\phi_{\text{W}}} + 2H_{18} a_{\phi_{\text{Q}}}^{(3)} - (a_{\phi_{\text{DV}}} + a_{\phi_{\text{W}}}) x_{\text{D}}^2 \right] c + 2(a_{\text{AZ}} + s c a_{\text{AA}} - s c a_{\text{ZZ}}) s \right\} \frac{L_{\text{R}}}{c} \\
 &\quad + \frac{1}{16} (x_{\text{D}}^2 a_{\text{DBW}} - 2c x_{\text{U}}^2 a_{\text{UW}}) (2 - 3L_{\text{R}}) \frac{1}{c}
 \end{aligned}$$

$$\begin{aligned}
 A_{\text{DD};1}^{(6)} &= \frac{1}{96} \left\{ 8K_7 s c^3 a_{\text{AZ}} + \left[ 24G_{14} a_{\phi_{\text{D}}} - G_{61} a_{\phi_{\text{D}}} - 12(a_{\phi_{\text{DVA}}} + a_{\phi_{\text{W}}}) G_{15} \right] \right. \\
 &\quad \left. + 4 \left[ 4F_3^b a_{\text{AA}} + (a_{\phi_{\text{D}}} - 4c^2 a_{\text{ZZ}}) \right] s^2 - 2(9a_{\text{DBW}} + 12c a_{\phi_{\text{DV}}} + 4s c^2 a_{\text{AZ}}) c x_{\text{D}}^2 \right\} \frac{1}{c^4 x_{\text{S}}} a_0^{\text{fin}}(M_0) \\
 &\quad - \frac{1}{96} \left\{ 16F_3^b s^2 a_{\text{AA}} + 4 \left[ 4c^3 a_{\text{AZ}} + (a_{\phi_{\text{D}}} - 4c^2 a_{\text{ZZ}}) s \right] s \right. \\
 &\quad \left. + \left[ 24G_{14} a_{\phi_{\text{D}}} - G_{61} a_{\phi_{\text{D}}} - 12(a_{\phi_{\text{DVA}}} + a_{\phi_{\text{W}}}) G_{15} \right] - 6(3a_{\text{DBW}} + 4c a_{\phi_{\text{DV}}}) c x_{\text{D}}^2 \right\} \frac{x_{\text{D}}^2}{c^2 x_{\text{S}}} a_0^{\text{fin}}(M_{\text{D}}) \\
 &\quad - \frac{1}{8} (2a_{\phi_{\text{W}}} x_{\text{D}}^2 + 3x_{\text{U}}^2 a_{\text{UW}} - H_{17} a_{\phi_{\text{QW}}}^{(3)}) \frac{x_{\text{U}}^2}{x_{\text{S}}} a_0^{\text{fin}}(M_{\text{U}})
 \end{aligned}$$

$$+\frac{1}{8}(2a_{\phi W}x_D^2+3x_U^2a_{UW}-H_{17}a_{\phi QW}^{(3)})\frac{1}{x_S}a_0^{\text{fin}}(M)$$

$$\begin{aligned} A_{\text{DD};2}^{(6)} = & -\frac{1}{12}\frac{1}{x_S}L_{15}sc a_{AZ}B_0^{\text{fin}}(0, M_D) \\ & +\frac{1}{96}\left\{16F_{31}^b s^2 a_{\phi W}+8F_{33}^b s c^2 a_{\phi WA}x_D^2+4\left[4ca_{\phi WB}+(a_{\phi D}-8c^2 a_{\phi B})s\right]s\right. \\ & +\left[24G_{14}a_{\phi D}-G_{61}a_{\phi D}-12(a_{\phi DV A}+a_{\phi W})G_{15}\right]-6(3a_{DBW}+4ca_{\phi DV})L_{11}c^3x_D^2 \\ & -2(9a_{DBW}-12c v_d a_{\phi DA}-20s^2c^2 a_{\phi WZ})cx_D^2-8(a_{AZ}-2sc a_{ZZ})sc^3x_S \\ & -(24G_{14}a_{\phi D}x_S-12G_{15}x_S a_{\phi DV A}-12G_{15}L_{13}a_{\phi W}-G_{61}L_{13}a_{\phi D})c^2 \\ & +4(8I_{10}a_{\phi B}-4I_{11}a_{\phi W}-L_{13}a_{\phi D}-4F_3^b x_S a_{AA}-4F_{32}^b a_{\phi B}x_D^2)s^2c^2 \\ & \left.-8(K_6x_S+L_{14}x_D^2)sc^5 a_{AZ}\right\}\frac{1}{c^4x_S}B_0^{\text{fin}}(M_0, M_D) \\ & -\frac{1}{8}(2a_{\phi W}x_Sx_D^2-3x_U^2x_S a_{UW}-2H_{15}a_{\phi W}x_D^2-3H_{15}x_U^2a_{UW}+H_{16}a_{\phi QW}^{(3)}) \\ & -2H_{17}a_{\phi W}x_S-4H_{18}x_S a_{\phi Q}^{(3)}\frac{1}{x_S}B_0^{\text{fin}}(M, M_U) \end{aligned}$$

We have introduced

$$\begin{aligned} C_{UV}^D = & -\frac{1}{256}\frac{M_D^2}{M}\left[3(sa_{DB}+ca_{DW})v_d\right. \\ & \left.+4c(a_{\phi D}-a_{\phi Q}^{(1)}+3a_{\phi Q}^{(3)}+csa_{DB})+2(3+2s^2)ca_{DW}\right]\frac{1}{c} \end{aligned}$$

## C Listing the counterterms

In this appendix we give the full list of counterterms, dropping the  $\text{ren}$ -subscript for the parameters,  $s_\theta = s_{\theta\text{ren}}$  etc. To keep the notation as compact as possible a number of auxiliary quantities are introduced. First we define the following set of polynomials:

$A(x)$  where  $x = s_\theta^2$ :

$$\begin{aligned} A_0^a &= 1-x & A_1^a &= 55-36x & A_2^a &= 10-9x \\ A_3^a &= 5-2x & A_4^a &= 7-4x & A_5^a &= 3-2x \end{aligned}$$

$$\begin{aligned} A_0^b &= 19-72x & A_1^b &= 19+9A_0^a x & A_2^b &= 29-2A_1^a x \\ A_3^b &= 2-x & A_4^b &= 53-26x & A_5^b &= 19-36x \\ A_6^b &= 61+36A_0^a x & A_7^b &= 56-29x & A_8^b &= 13+12A_0^a x \\ A_9^b &= 3+2A_0^a x & A_{10}^b &= 14-9x & A_{11}^b &= 2-3x \\ A_{12}^b &= 1-4A_0^a x & A_{13}^b &= 29+4A_2^a x & A_{14}^b &= 50-23x \\ A_{15}^b &= 29+18A_0^a x & A_{16}^b &= 1-x & A_{17}^b &= 3-A_3^a x \\ A_{18}^b &= 3-A_4^a x & A_{19}^b &= 1-A_5^a x \end{aligned}$$

$$\begin{array}{lll}
 A_0^c = 10 - 13x & A_1^c = 3 - 8x & A_2^c = 19 - 18x \\
 A_3^c = 1 - 7x & A_4^c = 1 - 4x & A_5^c = 1 - 2x \\
 A_6^c = 1 - A_0^b x & A_7^c = 1 - 2A_1^b x & A_8^c = 2 - A_2^b x \\
 A_9^c = 3 - 2x & A_{10}^c = 4 + 3x & A_{11}^c = 19 - 24A_3^b x \\
 A_{12}^c = 24 - A_4^b x & A_{13}^c = 1 + 3x & A_{14}^c = 1 - A_5^b x \\
 A_{15}^c = 2 - A_6^b x & A_{16}^c = 3 - 4x & A_{17}^c = 3 + x \\
 A_{18}^c = 3 + 2x & A_{19}^c = 41 - 48A_3^b x & A_{20}^c = 97 - 4A_7^b x \\
 A_{21}^c = 1 - 2A_8^b x & A_{22}^c = 3 - A_9^b x & A_{23}^c = 97 - 4A_4^b x \\
 A_{24}^c = 5 - 6x & A_{25}^c = 1 + 2x & A_{26}^c = 9 - 4x \\
 A_{27}^c = 11 - 6x & A_{28}^c = 13 - 8x & A_{29}^c = 19 - 4A_{10}^b x \\
 A_{30}^c = 17 + 12A_{11}^b x & A_{31}^c = 23 - 4x & A_{32}^c = 29 - 24x \\
 A_{33}^c = 73 - 72x & A_{34}^c = 1 - 2A_{12}^b x & A_{35}^c = 2 - A_{13}^b x \\
 A_{36}^c = 19 - 18A_{12}^b x & A_{37}^c = 83 - 96A_3^b x & A_{38}^c = 293 - 12A_{14}^b x \\
 A_{39}^c = 1 - A_{15}^b x & A_{40}^c = 7 - 6x & A_{41}^c = 1 + 2A_{16}^b x^2 \\
 A_{42}^c = 1 - 4A_{17}^b x & A_{43}^c = 1 - 8A_{18}^b x & A_{44}^c = 1 - 4A_{19}^b x
 \end{array}$$

**B**( $x$ ) where  $x = x_H = M_H/M_W$ :

$$\begin{array}{lll}
 B_0^g = 2 - 3x^2 & B_1^g = 11 + 15x^2 & B_2^g = 22 + 9x^2 \\
 B_3^g = 106 + 9x^2 & B_4^g = 74 + 9x^2 & B_5^g = 2 - 11x^2 \\
 B_6^g = 4 - 21x^2 & B_7^g = 11 - 3x^2 & \\
 \\
 B_0^b = 6 - B_0^g x^2 & B_1^b = 2 + x^2 & B_2^b = 7 - x^2 \\
 B_3^b = 10 - 3x^2 & B_4^b = 10 - x^2 & B_5^b = 11 + 6x^2 \\
 B_6^b = 31 + 15x^2 & B_7^b = 32 - 3x^2 & B_8^b = 96 - B_1^g x^2 \\
 B_9^b = 132 - B_2^g x^2 & B_{10}^b = 132 - B_3^g x^2 & B_{11}^b = 196 - B_4^g x^2 \\
 B_{12}^b = 2 + 11x^2 & B_{13}^b = 4 + x^2 & B_{14}^b = 6 - B_5^g x^2 \\
 B_{15}^b = 12 - x^2 & B_{16}^b = 12 - B_6^g x^2 & B_{17}^b = 30 - B_7^g x^2 \\
 B_{18}^b = 1 - 9x^2 & B_{19}^b = 3 - 4x^2 & B_{20}^b = 7 + 8x^2 \\
 B_{21}^b = 15 + 4x^2 & B_{22}^b = 20 + 9x^2 & B_{23}^b = 55 - 3x^2 \\
 B_{24}^b = 80 + 27x^2 & B_{25}^b = 1 - 6x^2 & B_{26}^b = 2 - x^2 \\
 B_{27}^b = 10 - 11x^2 & B_{28}^b = 12 + 11x^2 & B_{29}^b = 23 - 3x^2 \\
 B_{30}^b = 34 + 15x^2 & B_{31}^b = 56 + 9x^2 & B_{32}^b = 144 - B_2^g x^2 \\
 B_{33}^b = 4 + 3x^2 & B_{34}^b = 3 - x^2 & B_{35}^b = 6 - x^2 \\
 B_{36}^b = 9 - x^2 & B_{37}^b = 10 + 3x^2 & B_{38}^b = 22 - 3x^2 \\
 B_{39}^b = 82 - 9x^2 & B_{40}^b = 8 + 9x^2 & B_{41}^b = 22 - x^2 \\
 B_{42}^b = 34 - x^2 & B_{43}^b = 9 - 11x^2 & B_{44}^b = 31 - 6x^2 \\
 B_{45}^b = 32 + 3x^2 & B_{46}^b = 33 - 7x^2 & B_{47}^b = 40 + 33x^2 \\
 B_{48}^b = 51 - 7x^2 & B_{49}^b = 204 - B_2^g x^2 & B_{50}^b = 204 - B_3^g x^2 \\
 B_{51}^b = 5 + 4x^2 & & 
 \end{array}$$

**C** where we have introduced  $v_f = 1 - 2\frac{Q_f}{I_f^2} s_\theta^2$  and

$$v_{\text{gen}}^{(1)} = v_1^2 + 3(v_u^2 + v_d^2) \quad v_{\text{gen}}^{(2)} = v_1^2 + 2v_u^2 + v_d^2 \quad v_f^\pm = 1 \pm v_f \quad (\text{C.1})$$

$$\begin{array}{lll}
 C_0^a = 9 + v_{\text{gen}}^{(1)} & C_1^a = 17 + v_{\text{gen}}^{(3)} & C_2^a = v_u^+ - v_d^+ \\
 C_3^a = v_u^+ + v_d^+ & C_4^a = 8 - v_{\text{gen}}^{(2)} & C_5^a = 16 - 3v_{\text{gen}}^{(2)} \\
 C_6^a = 41 - 4v_{\text{gen}}^{(2)} + v_{\text{gen}}^{(1)} & C_7^a = 53 - 12v_{\text{gen}}^{(2)} - 3v_{\text{gen}}^{(1)} & C_8^a = 3 + v_l \\
 C_9^a = 4 - v_{\text{gen}}^{(2)} & C_{10}^a = 5v_u^- + v_d^- + 3v_L^- & C_{11}^a = 2 + v_{\text{gen}}^{(2)} \\
 C_{12}^a = 9 + 4v_{\text{gen}}^{(2)} + v_{\text{gen}}^{(1)} & C_{13}^a = 17 + v_{\text{gen}}^{(3)} - 2v_{\text{gen}}^{(2)} & 
 \end{array}$$

**D** where we have introduced

$$x_f = \frac{M_f}{M_W} \quad x_{\text{gen}}^{(1)} = x_1^2 + 3(x_u^2 + x_d^2) \quad x_{\text{gen}}^{(2)} = x_1^4 + 3(x_u^4 + x_d^4) \quad (\text{C.2})$$

$$\begin{array}{lll}
 D_0^a = 2 - 9x_1^2 & D_1^a = 2 - 9x_u^2 - 9x_d^2 & D_2^a = -x_u^2 + x_d^2 \\
 D_3^a = x_{\text{gen}}^{(1)} - 2x_{\text{gen}}^{(2)} & D_4^a = x_u^2 + x_d^2 & D_5^a = 2 - 3x_1^2 \\
 D_6^a = 2 - 3x_u^2 - 3x_d^2 & & 
 \end{array}$$

**E**

$$\begin{array}{lll}
 E_0^a = 2 - 9x_U^2 + 9x_D^2 & E_1^a = 2 - 9x_U^2 & E_2^a = 26 + 9x_U^2 + 9x_D^2 \\
 E_3^a = 14 + 9x_U^2 - 9x_D^2 & E_4^a = 8 - 9x_U^2 & E_5^a = 14 + 3x_U^2 \\
 E_6^a = 2 - x_L^2 & E_7^a = 2 + x_L^2 & E_8^a = 1 + 3x_D^2 \\
 E_9^a = 4 + 4x_U^2 + 3x_D^2 & E_{10}^a = 4 + 6x_U^2 + 3x_D^2 & E_{11}^a = 4 - 3x_D^2 \\
 E_{12}^a = 4 + 3x_D^2 & E_{13}^a = 8 - 3x_D^2 & E_{14}^a = 16 - 6x_U^2 - 3x_D^2 \\
 E_{15}^a = 1 + 3x_U^2 & E_{16}^a = 4 + 3x_U^2 + 4x_D^2 & E_{17}^a = 8 - 3x_U^2 \\
 E_{18}^a = 8 + 3x_U^2 & E_{19}^a = 8 + 3x_U^2 + 6x_D^2 & E_{20}^a = 16 - 15x_U^2 \\
 E_{21}^a = 20 - 3x_U^2 - 6x_D^2 & E_{22}^a = 1 + 3x_L^2 & E_{23}^a = 4 - x_L^2 \\
 E_{24}^a = 4 + x_L^2 & E_{25}^a = 4 + 3x_L^2 & E_{26}^a = 8 - 9x_L^2 \\
 E_{27}^a = 8 - x_L^2 & & 
 \end{array}$$

### C.1 dim = 4 counterterms

First we list the SM counterterms. In the following we use  $s = s_\theta$  and  $c = c_\theta$ .

$$dZ_{\text{H}}^{(4)} = \frac{4}{3} N_{\text{gen}} - \frac{1}{2} \sum_{\text{gen}} x_{\text{gen}}^{(1)} - \frac{1}{6} \frac{A_0^c}{c^2} \quad (\text{C.3})$$

$$dZ_{M_{\text{H}}}^{(4)} = -\frac{1}{2} \left\{ \sum_{\text{gen}} (4x_{\text{gen}}^{(2)} - x_{\text{gen}}^{(1)} x_{\text{H}}^2) - \left[ 3 - (x_{\text{H}}^2 - c^2 B_0^b) c^2 \right] \frac{1}{c^4} \right\} \frac{1}{x_{\text{H}}^2} \quad (\text{C.4})$$

$$dZ_{\text{A}}^{(4)} = -\frac{1}{6} A_2^c + \frac{4}{9} N_{\text{gen}} A_1^c \quad dZ_{\text{AZ}}^{(4)} = \frac{1}{6} \frac{s}{c} A_2^c - \frac{1}{3} \frac{s}{c} N_{\text{gen}} v_{\text{gen}}^{(2)} \quad (\text{C.5})$$

$$dZ_{\text{W}}^{(4)} = 0 \quad dZ_{M_{\text{W}}}^{(4)} = \frac{4}{3} N_{\text{gen}} - \frac{1}{2} \sum_{\text{gen}} x_{\text{gen}}^{(1)} - \frac{1}{6} \frac{A_3^c}{c^2} \quad (\text{C.6})$$

$$dZ_{c_\theta}^{(4)} = -\frac{1}{12} \frac{s^2}{c^2} A_2^c - \frac{1}{24} (C^a - 16c^2) \frac{N_{\text{gen}}}{c^2} \quad dZ_{\text{Z}}^{(4)} = -\frac{1}{6} \frac{s^2}{c^2} A_2^c - \frac{1}{12} (C^a - 16c^2) \frac{N_{\text{gen}}}{c^2} \quad (\text{C.7})$$

$$dZ_{\text{g}}^{(4)} = -\frac{19}{12} + \frac{2}{3} N_{\text{gen}} \quad (\text{C.8})$$



$$dZ_{M_D}^{(4)} = \frac{1}{48} [3C_1^a + 2(3E_0^a - 8s^2)c^2] \frac{1}{c^2} \quad (C.9)$$

$$dZ_{R_D}^{(4)} = \frac{1}{144} [9C_2^a + 8(9x_d^2 + 2s^2)c^2] \frac{1}{c^2} \quad dZ_{L_D}^{(4)} = \frac{1}{144} [9C^a + 4(9E_1^a + 4s^2)c^2] \frac{1}{c^2} \quad (C.10)$$

$$dZ_{M_U}^{(4)} = -\frac{1}{24} (5 - c^2 E_2^a) \frac{1}{c^2} \quad (C.11)$$

$$dZ_{R_U}^{(4)} = \frac{1}{18} (8 - c^2 E_3^a) \frac{1}{c^2} \quad dZ_{L_U}^{(4)} = \frac{1}{36} (1 + c^2 E_4^a) \frac{1}{c^2} \quad (C.12)$$

$$dZ_{M_L}^{(4)} = -\frac{1}{8} (11 - c^2 E_5^a) \frac{1}{c^2} \quad (C.13)$$

$$dZ_{R_L}^{(4)} = \frac{1}{2} (2 - c^2 E_6^a) \frac{1}{c^2} \quad dZ_{L_L}^{(4)} = \frac{1}{4} (1 + c^2 E_7^a) \frac{1}{c^2} \quad (C.14)$$

$$dZ_{R_V}^{(4)} = 0 \quad dZ_{L_V}^{(4)} = \frac{1}{4} (1 + c^2 E_7^a) \frac{1}{c^2} \quad (C.15)$$

## C.2 dim = 6 counterterms

To present dim = 6 counterterms we define vectors; for counterterms

$$\begin{aligned} CT_1 &= dZ_H^{(6)} & CT_2 &= dZ_{M_H}^{(6)} & CT_3 &= dZ_A^{(6)} & CT_4 &= dZ_W^{(6)} & CT_5 &= dZ_{M_W}^{(6)} \\ CT_6 &= dZ_{AZ}^{(6)} & CT_7 &= dZ_{\hat{c}_\theta}^{(6)} & CT_8 &= dZ_Z^{(6)} & CT_9 &= dZ_g^{(6)} & CT_{10} &= dZ_{M_D}^{(6)} \\ CT_{11} &= dZ_{R_D}^{(6)} & CT_{12} &= dZ_{L_D}^{(6)} & CT_{13} &= dZ_{M_U}^{(6)} & CT_{14} &= dZ_{R_U}^{(6)} & CT_{15} &= dZ_{L_U}^{(6)} \\ CT_{16} &= dZ_{M_L}^{(6)} & CT_{17} &= dZ_{R_L}^{(6)} & CT_{18} &= dZ_{L_L}^{(6)} & CT_{19} &= dZ_{R_V}^{(6)} & CT_{20} &= dZ_{L_V}^{(6)} \end{aligned} \quad (C.16)$$

and for Wilson coefficients

$$\begin{aligned} a_\phi &= W_1 & a_{\phi\Box} &= W_2 & a_{\phi D} &= W_3 & a_{\phi W} &= W_4 \\ a_{\phi B} &= W_5 & a_{\phi WB} &= W_6 & a_{u\phi} &= W_7(q, u) & a_{d\phi} &= W_8(q, d) \\ a_{L\phi} &= W_9(\lambda, 1) & a_{\phi q}^{(1)} &= W_{10}(q) & a_{\phi 1}^{(1)} &= W_{11}(1) & a_{\phi u} &= W_{12}(u) \\ a_{\phi d} &= W_{13}(d) & a_{\phi 1} &= W_{14}(1) & a_{\phi q}^{(3)} &= W_{15}(q) & a_{\phi 1}^{(3)} &= W_{16}(1) \\ a_{uW} &= W_{17}(q, u) & a_{dW} &= W_{18}(q, d) & a_{1W} &= W_{19}(\lambda, 1) & a_{uB} &= W_{20}(q, u) \\ a_{dB} &= W_{21}(q, d) & a_{1B} &= W_{22}(\Lambda, 1) & a_{L1dQ} &= W_{23}(\Lambda, 1, d, Q) & a_{QuQd}^{(1)} &= W_{24}(Q, u, Q, d) \\ a_{L1Qu}^{(1)} &= W_{25}(\Lambda, 1, Q, u) \end{aligned} \quad (C.17)$$

The result is given by introducing a matrix

$$CT_i = \sum_{j=1,6} M_{ij}^{ct} W_j + \sum_{j=7,25} \sum_{\text{gen}} M_{ij}^{ct}(\text{label}) W_j(\text{label}) \quad (C.18)$$

where, without assuming universality, we have

$$\begin{aligned} \sum_{\text{gen}} M_{i,10}^{ct}(\text{label}) W_{10}(\text{label}) &= M_{i,10}^{ct}(u, d) W_{10}(u, d) + M_{i,10}^{ct}(c, s) W_{10}(c, s) + M_{i,10}^{ct}(t, b) W_{10}(t, b) \\ \sum_{\text{gen}} M_{i,12}^{ct}(\text{label}) W_{12}(\text{label}) &= M_{i,12}^{ct}(u) W_{12}(u) + M_{i,12}^{ct}(c) W_{12}(c) + M_{i,12}^{ct}(t) W_{12}(t) \end{aligned} \quad (C.19)$$

etc. In the following we introduce all non-zero entries of the matrix  $M^{ct}$ .

•  $M_{1,i}^{\text{ct}}$  entries

$$M_{1,2}^{\text{ct}} = -\frac{1}{6} \left\{ 4A_{11}^c - B_3^b c^2 \right\} \frac{1}{c^2}$$

$$M_{1,3}^{\text{ct}} = \frac{1}{2} \sum_{\text{gen}} x_{\text{gen}}^{(1)} - \frac{1}{6} \left\{ 3A_{12}^c - B_5^b c^2 \right\} \frac{1}{c^2}$$

$$\begin{aligned} M_{1,4}^{\text{ct}} = & -\sum_{\text{gen}} \left\{ x_{\text{gen}}^{(1)} - \left[ c^2 x_{\text{gen}}^{(1)} x_{\text{H}}^2 + 2(2s^2 x_{\text{gen}}^{(2)} + D_3^a) \right] c^2 \right\} \\ & + \frac{1}{6} \left\{ 9A_9^c - \left[ -(A_8^c x_{\text{H}}^2 + 8B_2^b) + (-B_{10}^b s^2 + B_{11}^b) c^2 \right] c^2 \right\} \frac{1}{c^2} \\ & + \frac{1}{3} \left\{ C_1^a + \left[ 8c^2 x_{\text{H}}^2 - (16 + C_0^a x_{\text{H}}^2) \right] c^2 \right\} N_{\text{gen}} \end{aligned}$$

$$\begin{aligned} M_{1,5}^{\text{ct}} = & \sum_{\text{gen}} \left\{ 2x_{\text{gen}}^{(1)} - \left[ 4x_{\text{gen}}^{(2)} - x_{\text{gen}}^{(1)} x_{\text{H}}^2 \right] c^2 \right\} s^2 \\ & + \frac{1}{6} \left\{ 6A_{10}^c - \left[ (A_8^c x_{\text{H}}^2 + 2B_7^b) - (4B_4^b - B_9^b s^2) c^2 \right] c^2 \right\} \frac{1}{c^2} \\ & - \frac{1}{6} \left\{ -16 \left[ 2 - c^2 x_{\text{H}}^2 \right] c^2 + \left[ 2 - A_5^c x_{\text{H}}^2 \right] C_0^a \right\} N_{\text{gen}} \end{aligned}$$

$$\begin{aligned} M_{1,6}^{\text{ct}} = & -\sum_{\text{gen}} \left\{ 2x_{\text{gen}}^{(1)} - \left[ 4x_{\text{gen}}^{(2)} - x_{\text{gen}}^{(1)} x_{\text{H}}^2 \right] c^2 \right\} s c \\ & + \frac{1}{12} \left\{ 2A_5^c C_0^a + \left[ 16A_5^c c^2 x_{\text{H}}^2 - (A_4^c C_0^a x_{\text{H}}^2 + 32A_5^c) \right] c^2 \right\} \frac{1}{s c} N_{\text{gen}} \\ & + \frac{1}{6} \left\{ 2A_7^c + \left[ 2B_6^b s^2 - (-A_6^c x_{\text{H}}^2 + B_1^b - B_8^b s^2) c^2 \right] c^2 \right\} \frac{1}{s c} \end{aligned}$$

$$M_{1,7}^{\text{ct}}(q, u) = -3x_u^2 \quad M_{1,8}^{\text{ct}}(q, d) = 3x_d^2 \quad M_{1,9}^{\text{ct}}(\lambda, l) = x_l^2$$

$$M_{1,10}^{\text{ct}}(q) = 2 \left\{ C_2^a + 6D_2^a c^2 \right\} \frac{1}{c^2} \quad M_{1,11}^{\text{ct}}(l) = -\frac{2}{3} \left\{ v_1^+ - 6c^2 x_l^2 \right\} \frac{1}{c^2}$$

$$M_{1,12}^{\text{ct}}(u) = -2 \left\{ v_u^- - 6c^2 x_u^2 \right\} \frac{1}{c^2} \quad M_{1,13}^{\text{ct}}(d) = 2 \left\{ v_d^- - 6c^2 x_d^2 \right\} \frac{1}{c^2}$$

$$M_{1,14}^{\text{ct}}(l) = \frac{2}{3} \left\{ v_l^- - 6c^2 x_l^2 \right\} \frac{1}{c^2} \quad M_{1,15}^{\text{ct}}(q) = 2 \left\{ C_3^a + D_1^a c^2 \right\} \frac{1}{c^2}$$

$$M_{1,16}^{\text{ct}}(l) = \frac{2}{3} \left\{ v_l^+ + D_0^a c^2 \right\} \frac{1}{c^2}$$

$$M_{1,17}^{\text{ct}}(q, u) = -\frac{3}{2} x_u^2 \quad M_{1,18}^{\text{ct}}(q, d) = \frac{3}{2} x_d^2 \quad M_{1,19}^{\text{ct}}(\lambda, l) = \frac{1}{2} x_l^2$$

•  $M_{2,i}^{\text{ct}}$  entries

$$M_{2,1}^{\text{ct}} = -3 \left\{ 1 + B_{12}^b c^2 \right\} \frac{1}{c^2 x_{\text{H}}^2}$$

$$M_{2,2}^{\text{ct}} = \left\{ -\sum_{\text{gen}} \left[ 4x_{\text{gen}}^{(2)} - x_{\text{gen}}^{(1)} x_{\text{H}}^2 \right] + \left[ 3 - (x_{\text{H}}^2 - B_{14}^b c^2) c^2 \right] \frac{1}{c^4} \right\} \frac{1}{x_{\text{H}}^2}$$

$$M_{2,3}^{\text{ct}} = \frac{1}{8} \left\{ 2 \sum_{\text{gen}} \left[ 4x_{\text{gen}}^{(2)} - x_{\text{gen}}^{(1)} x_{\text{H}}^2 \right] + \left[ 18 - (5x_{\text{H}}^2 + \mathbf{B}_{16}^b c^2) c^2 \right] \frac{1}{c^4} \right\} \frac{1}{x_{\text{H}}^2}$$

$$M_{2,4}^{\text{ct}} = \left\{ - \sum_{\text{gen}} \left[ 4x_{\text{gen}}^{(2)} - x_{\text{gen}}^{(1)} x_{\text{H}}^2 \right] + \left[ 3 + (\mathbf{B}_{15}^b + \mathbf{B}_{17}^b c^2) c^2 \right] \frac{1}{c^4} \right\} \frac{1}{x_{\text{H}}^2}$$

$$M_{2,5}^{\text{ct}} = 3 \left\{ 4 - \left[ -c^2 x_{\text{H}}^2 + \mathbf{B}_{13}^b \right] c^2 \right\} \frac{1}{c^4 x_{\text{H}}^2}$$

$$M_{2,6}^{\text{ct}} = -3 \left\{ 4 - c^2 x_{\text{H}}^2 \right\} \frac{1}{x_{\text{H}}^2 c^3} \quad M_{2,7}^{\text{ct}}(\mathbf{q}, \mathbf{u}) = -3 \left\{ 8x_{\text{u}}^2 - x_{\text{H}}^2 \right\} \frac{1}{x_{\text{H}}^2} x_{\text{u}}^2$$

$$M_{2,8}^{\text{ct}}(\mathbf{q}, \mathbf{d}) = 3 \left\{ 8x_{\text{d}}^2 - x_{\text{H}}^2 \right\} \frac{1}{x_{\text{H}}^2} x_{\text{d}}^2 \quad M_{2,9}^{\text{ct}}(\lambda, 1) = \left\{ 8x_{\text{1}}^2 - x_{\text{H}}^2 \right\} \frac{1}{x_{\text{H}}^2} x_{\text{1}}^2$$

•  $M_{3,i}^{\text{ct}}$  entries

$$M_{3,2}^{\text{ct}} = \sum_{\text{gen}} x_{\text{gen}}^{(1)} - \frac{1}{3} \left\{ \mathbf{A}_{19}^c + \mathbf{B}_{18}^b c^2 \right\} \frac{1}{c^2}$$

$$M_{3,3}^{\text{ct}} = \frac{16}{9} N_{\text{gen}} c^2 + \frac{1}{4} \sum_{\text{gen}} x_{\text{gen}}^{(1)} - \frac{1}{24} \left\{ 3\mathbf{A}_{20}^c - \mathbf{B}_{22}^b c^2 \right\} \frac{1}{c^2}$$

$$\begin{aligned} M_{3,4}^{\text{ct}} &= \sum_{\text{gen}} \left\{ c^2 x_{\text{gen}}^{(1)} x_{\text{H}}^2 + 2 \left[ 2s^2 x_{\text{gen}}^{(2)} + \mathbf{D}_3^a \right] \right\} c^2 \\ &\quad + \frac{1}{6} \left\{ \left[ 18 - \mathbf{B}_{20}^b + 2\mathbf{B}_{21}^b s^2 \right] - \left[ -2\mathbf{A}_{14}^c x_{\text{H}}^2 - \mathbf{B}_{10}^b s^2 + \mathbf{B}_{11}^b \right] c^2 \right\} \\ &\quad + \frac{1}{9} \left\{ 3 \left[ 8c^2 - (\mathbf{C}_0^a) \right] c^2 x_{\text{H}}^2 + \left[ -16\mathbf{A}_{17}^c + 3\mathbf{C}_1^a \right] \right\} N_{\text{gen}} \end{aligned}$$

$$\begin{aligned} M_{3,5}^{\text{ct}} &= \sum_{\text{gen}} \left\{ 2x_{\text{gen}}^{(1)} - \left[ 4x_{\text{gen}}^{(2)} - x_{\text{gen}}^{(1)} x_{\text{H}}^2 \right] c^2 \right\} s^2 \\ &\quad + \frac{1}{6} \left\{ 6\mathbf{A}_{13}^c - \left[ -(-2\mathbf{A}_{14}^c x_{\text{H}}^2 + 4\mathbf{B}_4^b - \mathbf{B}_9^b s^2) c^2 + (-2\mathbf{B}_{19}^b s^2 + \mathbf{B}_{23}^b) \right] c^2 \right\} \frac{1}{c^2} \\ &\quad - \frac{1}{6} \left\{ -16 \left[ 2 - c^2 x_{\text{H}}^2 \right] c^2 + \left[ 2 - \mathbf{A}_5^c x_{\text{H}}^2 \right] \mathbf{C}_0^a \right\} N_{\text{gen}} \end{aligned}$$

$$\begin{aligned} M_{3,6}^{\text{ct}} &= - \sum_{\text{gen}} \left\{ 2x_{\text{gen}}^{(1)} - \left[ 4x_{\text{gen}}^{(2)} - x_{\text{gen}}^{(1)} x_{\text{H}}^2 \right] c^2 \right\} s c \\ &\quad + \frac{1}{36} \left\{ 6\mathbf{A}_5^c \mathbf{C}_0^a + \left[ 48\mathbf{A}_5^c c^2 x_{\text{H}}^2 - (3\mathbf{A}_4^c \mathbf{C}_0^a x_{\text{H}}^2 + 32\mathbf{A}_{18}^c) \right] c^2 \right\} \frac{1}{s c} N_{\text{gen}} \\ &\quad + \frac{1}{6} \left\{ \mathbf{A}_{15}^c + \left[ \mathbf{B}_{24}^b s^2 - (-\mathbf{A}_6^c x_{\text{H}}^2 + \mathbf{B}_1^b - \mathbf{B}_8^b s^2) c^2 \right] c^2 \right\} \frac{1}{s c} \end{aligned}$$

$$M_{3,10}^{\text{ct}}(\mathbf{q}) = 2 \left\{ \mathbf{C}_2^a + 6\mathbf{D}_2^a c^2 \right\} \frac{1}{c^2} \quad M_{3,11}^{\text{ct}}(\mathbf{l}) = -\frac{2}{3} \left\{ v_1^+ - 6c^2 x_1^2 \right\} \frac{1}{c^2}$$

$$M_{3,12}^{\text{ct}}(\mathbf{u}) = -2 \left\{ v_{\text{u}}^- - 6c^2 x_{\text{u}}^2 \right\} \frac{1}{c^2} \quad M_{3,13}^{\text{ct}}(\mathbf{d}) = 2 \left\{ v_{\text{d}}^- - 6c^2 x_{\text{d}}^2 \right\} \frac{1}{c^2}$$

$$M_{3,14}^{\text{ct}}(\mathbf{l}) = \frac{2}{3} \left\{ v_1^- - 6c^2 x_1^2 \right\} \frac{1}{c^2} \quad M_{3,15}^{\text{ct}}(\mathbf{q}) = 2 \left\{ \mathbf{C}_3^a + \mathbf{D}_1^a c^2 \right\} \frac{1}{c^2} \quad M_{3,16}^{\text{ct}}(\mathbf{l}) = \frac{2}{3} \left\{ v_1^+ + \mathbf{D}_0^a c^2 \right\} \frac{1}{c^2}$$

$$M_{3,17}^{\text{ct}}(\mathbf{q}, \mathbf{u}) = -\frac{1}{2} \mathbf{A}_1^c x_{\text{u}}^2 \quad M_{3,18}^{\text{ct}}(\mathbf{q}, \mathbf{d}) = \frac{1}{2} \mathbf{A}_{16}^c x_{\text{d}}^2 \quad M_{3,19}^{\text{ct}}(\lambda, 1) = \frac{1}{2} \mathbf{A}_4^c x_1^2$$

$$M_{3,20}^{\text{ct}}(\mathbf{q}, \mathbf{u}) = 4s c x_{\text{u}}^2 \quad M_{3,21}^{\text{ct}}(\mathbf{q}, \mathbf{d}) = 2s c x_{\text{d}}^2 \quad M_{3,22}^{\text{ct}}(\lambda, 1) = 2s c x_1^2$$

•  $M_{4,i}^{\text{ct}}$  entries

$$M_{4,2}^{\text{ct}} = \sum_{\text{gen}} x_{\text{gen}}^{(1)} - \frac{1}{6} \left\{ 2A_{19}^c + 3B_{25}^b c^2 \right\} \frac{1}{c^2}$$

$$M_{4,3}^{\text{ct}} = \frac{1}{4} \sum_{\text{gen}} x_{\text{gen}}^{(1)} - \frac{1}{24} \left\{ 3A_{23}^c - B_{31}^b c^2 \right\} \frac{1}{c^2}$$

$$\begin{aligned} M_{4,4}^{\text{ct}} &= \sum_{\text{gen}} \left\{ c^2 x_{\text{gen}}^{(1)} x_{\text{H}}^2 + 2 \left[ 2s^2 x_{\text{gen}}^{(2)} + D_3^a \right] \right\} c^2 \\ &+ \frac{1}{6} \left\{ 6A_{22}^c + \left[ -(-2A_{14}^c x_{\text{H}}^2 - B_{10}^b s^2 + B_{11}^b) c^2 + (B_{27}^b + B_{28}^b s^2) \right] \right\} \frac{1}{c^2} \\ &- \frac{1}{6} \left\{ 16 \left[ 2 - c^2 x_{\text{H}}^2 \right] c^2 - \left[ -A_5^c x_{\text{H}}^2 + B_{26}^b \right] C_0^a \right\} N_{\text{gen}} \end{aligned}$$

$$\begin{aligned} M_{4,5}^{\text{ct}} &= \sum_{\text{gen}} \left\{ 2x_{\text{gen}}^{(1)} - \left[ 4x_{\text{gen}}^{(2)} - x_{\text{gen}}^{(1)} x_{\text{H}}^2 \right] \right\} c^2 s^2 \\ &+ \frac{1}{6} \left\{ 6A_{13}^c - \left[ (A_8^c x_{\text{H}}^2 + 2B_{29}^b) - (4B_4^b - B_{32}^b s^2) c^2 \right] \right\} \frac{1}{c^2} \\ &- \frac{1}{6} \left\{ -16 \left[ 2 - c^2 x_{\text{H}}^2 \right] c^2 + \left[ 2 - A_5^c x_{\text{H}}^2 \right] C_0^a \right\} N_{\text{gen}} \end{aligned}$$

$$\begin{aligned} M_{4,6}^{\text{ct}} &= - \sum_{\text{gen}} \left\{ 2x_{\text{gen}}^{(1)} - \left[ 4x_{\text{gen}}^{(2)} - x_{\text{gen}}^{(1)} x_{\text{H}}^2 \right] \right\} s c \\ &+ \frac{1}{12} \left\{ 2A_5^c C_0^a + \left[ 16A_5^c c^2 x_{\text{H}}^2 - (A_4^c C_0^a x_{\text{H}}^2 + 32A_5^c) \right] \right\} \frac{1}{s c} N_{\text{gen}} \\ &+ \frac{1}{6} \left\{ 2A_{21}^c + \left[ 2B_{30}^b s^2 - (-A_6^c x_{\text{H}}^2 + B_1^b - B_8^b s^2) c^2 \right] \right\} \frac{1}{s c} \end{aligned}$$

$$M_{4,10}^{\text{ct}}(q) = 2 \left\{ C_2^a + 6D_2^a c^2 \right\} \frac{1}{c^2} \quad M_{4,11}^{\text{ct}}(l) = -\frac{2}{3} \left\{ v_1^+ - 6c^2 x_1^2 \right\} \frac{1}{c^2}$$

$$M_{4,12}^{\text{ct}}(u) = -2 \left\{ v_u^- - 6c^2 x_u^2 \right\} \frac{1}{c^2} \quad M_{4,13}^{\text{ct}}(d) = 2 \left\{ v_d^- - 6c^2 x_d^2 \right\} \frac{1}{c^2}$$

$$M_{4,14}^{\text{ct}}(l) = \frac{2}{3} \left\{ v_1^- - 6c^2 x_1^2 \right\} \frac{1}{c^2} \quad M_{4,15}^{\text{ct}}(q) = 2 \left\{ C_3^a - 9D_4^a c^2 \right\} \frac{1}{c^2} \quad M_{4,16}^{\text{ct}}(l) = \frac{2}{3} \left\{ v_1^+ - 9c^2 x_1^2 \right\} \frac{1}{c^2}$$

•  $M_{5,i}^{\text{ct}}$  entries

$$M_{5,2}^{\text{ct}} = \frac{5}{3} \quad M_{5,3}^{\text{ct}} = -\frac{1}{4} A_{24}^c \frac{1}{c^2}$$

$$M_{5,4}^{\text{ct}} = \frac{8}{3} N_{\text{gen}} - \sum_{\text{gen}} x_{\text{gen}}^{(1)} + \frac{1}{6} \left\{ 15 + B_{33}^b c^2 \right\} \frac{1}{c^2}$$

$$M_{5,6}^{\text{ct}} = -\frac{s}{c} \quad M_{5,15}^{\text{ct}}(q) = 2D_6^a \quad M_{5,16}^{\text{ct}}(l) = \frac{2}{3} D_6^a$$

$$M_{5,17}^{\text{ct}}(q, u) = -\frac{3}{2} x_u^2 \quad M_{5,18}^{\text{ct}}(q, d) = \frac{3}{2} x_d^2 \quad M_{5,19}^{\text{ct}}(\lambda, l) = \frac{1}{2} x_l^2$$

•  $M_{6,i}^{\text{ct}}$  entries

$$M_{6,3}^{\text{ct}} = -\frac{1}{24} A_{29}^c \frac{1}{s c} - \frac{1}{36} \left\{ -3 c^2 v_{\text{gen}}^{(2)} + 4 A_{28}^c s^2 \right\} \frac{1}{s c} N_{\text{gen}}$$

$$M_{6,4}^{\text{ct}} = \frac{1}{36} \left\{ 64 A_{25}^c c^2 + \left[ 24 s^2 v_{\text{gen}}^{(2)} + C_7^a \right] \right\} \frac{s}{c} N_{\text{gen}} \\ + \frac{1}{6} \left\{ A_{27}^c - 6 \left[ A_{25}^c B_{15}^b - 2 B_{35}^b s^2 \right] c^2 \right\} \frac{s}{c}$$

$$M_{6,5}^{\text{ct}} = \frac{1}{6} \left\{ 5 A_{24}^c - 6 \left[ B_1^b c^2 - (-2 B_{35}^b + B_{38}^b) s^2 \right] c^2 \right\} \frac{s}{c} \\ + \frac{1}{36} \left\{ 3 C_0^a - 8 \left[ 16 s^2 + 3 C_4^a \right] c^2 \right\} \frac{s}{c} N_{\text{gen}}$$

$$M_{6,6}^{\text{ct}} = -\frac{1}{72} \left\{ 3 A_{16}^c C_0^a - 2 \left[ 128 s^2 c^2 + (-8 C_5^a s^2 + 3 C_6^a) \right] c^2 \right\} \frac{1}{c^2} N_{\text{gen}} \\ - \frac{1}{12} \left\{ A_{26}^c - 2 \left[ -(3 B_{13}^b + 12 B_{34}^b s^4 - B_{39}^b s^2) + (-12 B_{36}^b s^2 + B_{37}^b) c^2 \right] c^2 \right\} \frac{1}{c^2}$$

$$M_{6,10}^{\text{ct}}(q) = -\frac{2}{3} \frac{s}{c} \quad M_{6,11}^{\text{ct}}(l) = \frac{2}{3} \frac{s}{c} \quad M_{6,12}^{\text{ct}}(u) = -\frac{4}{3} \frac{s}{c} \quad M_{6,13}^{\text{ct}}(d) = \frac{2}{3} \frac{s}{c}$$

$$M_{6,14}^{\text{ct}}(l) = \frac{2}{3} \frac{s}{c} \quad M_{6,15}^{\text{ct}}(q) = -2 \frac{s}{c} \quad M_{6,16}^{\text{ct}}(l) = -\frac{2}{3} \frac{s}{c}$$

$$M_{6,17}^{\text{ct}}(q, u) = \frac{1}{4} \left\{ 8 c^2 + 3 v_u \right\} \frac{s}{c} x_u^2 \quad M_{6,18}^{\text{ct}}(q, d) = -\frac{1}{4} \left\{ 4 c^2 + 3 v_d \right\} \frac{s}{c} x_d^2$$

$$M_{6,19}^{\text{ct}}(\lambda, l) = -\frac{1}{4} \left\{ 4 c^2 + v_l \right\} \frac{s}{c} x_l^2 \quad M_{6,20}^{\text{ct}}(q, u) = \frac{1}{4} \left\{ 3 v_u - 8 s^2 \right\} x_u^2$$

$$M_{6,21}^{\text{ct}}(q, d) = \frac{1}{4} \left\{ 3 v_d - 4 s^2 \right\} x_d^2 \quad M_{6,22}^{\text{ct}}(\lambda, l) = \frac{1}{4} \left\{ v_l - 4 s^2 \right\} x_l^2$$

•  $M_{7,i}^{\text{ct}}$  entries

$$M_{7,2}^{\text{ct}} = -\frac{5}{6} \frac{s^2}{c^2}$$

$$M_{7,3}^{\text{ct}} = -\frac{1}{8} \sum_{\text{gen}} x_{\text{gen}}^{(1)} + \frac{1}{48} \left\{ -4 c^2 v_{\text{gen}}^{(2)} + C_{10}^a \right\} \frac{1}{c^2} N_{\text{gen}} - \frac{1}{48} \left\{ A_{30}^c + B_{40}^b c^2 \right\} \frac{1}{c^2}$$

$$M_{7,4}^{\text{ct}} = \frac{1}{12} \left\{ A_{33}^c - 3 B_{42}^b c^2 \right\} \frac{s^2}{c^2} - \frac{1}{12} \left\{ -4 C_9^a c^2 + C_{10}^a \right\} \frac{1}{c^2} N_{\text{gen}}$$

$$M_{7,5}^{\text{ct}} = -\frac{1}{4} \left\{ A_{32}^c - B_{41}^b c^2 \right\} \frac{s^2}{c^2} \quad M_{7,6}^{\text{ct}} = \frac{1}{3} \frac{s}{c} N_{\text{gen}} v_{\text{gen}}^{(2)} + \frac{1}{12} \left\{ A_{31}^c - B_{38}^b c^2 \right\} \frac{s}{c}$$

$$M_{7,10}^{\text{ct}}(q) = -\frac{1}{2} \left\{ C_2^a + 6 D_2^a c^2 \right\} \frac{1}{c^2} \quad M_{7,11}^{\text{ct}}(l) = -\frac{1}{6} \left\{ v_l^- + 6 c^2 x_l^2 \right\} \frac{1}{c^2}$$

$$M_{7,12}^{\text{ct}}(u) = \frac{1}{2} \left\{ v_u^- - 6 c^2 x_u^2 \right\} \frac{1}{c^2} \quad M_{7,13}^{\text{ct}}(d) = -\frac{1}{2} \left\{ v_d^- - 6 c^2 x_d^2 \right\} \frac{1}{c^2}$$

$$M_{7,14}^{\text{ct}}(l) = -\frac{1}{6} \left\{ v_l^- - 6 c^2 x_l^2 \right\} \frac{1}{c^2} \quad M_{7,15}^{\text{ct}}(q) = -\frac{1}{2} \left\{ -4 c^2 + C_3^a \right\} \frac{1}{c^2}$$

$$M_{7,16}^{\text{ct}}(l) = -\frac{1}{6} \left\{ -4c^2 + C_8^a \right\} \frac{1}{c^2} \quad M_{7,17}^{\text{ct}}(q, u) = -\frac{3}{4} x_u^2 v_u^-$$

$$M_{7,18}^{\text{ct}}(q, d) = \frac{3}{4} x_d^2 v_d^- \quad M_{7,19}^{\text{ct}}(\lambda, l) = \frac{1}{4} x_l^2 v_l^-$$

$$M_{7,20}^{\text{ct}}(q, u) = -\frac{3}{4} \frac{s}{c} v_u x_u^2 \quad M_{7,21}^{\text{ct}}(q, d) = -\frac{3}{4} \frac{s}{c} v_d x_d^2$$

$$M_{7,22}^{\text{ct}}(\lambda, l) = -\frac{1}{4} \frac{s}{c} v_l x_l^2$$

•  $M_{8,i}^{\text{ct}}$  entries

$$M_{8,2}^{\text{ct}} = \sum_{\text{gen}} x_{\text{gen}}^{(1)} - \frac{1}{6} \left\{ A_{37}^c + 2B_{18}^b c^2 \right\} \frac{1}{c^2}$$

$$M_{8,3}^{\text{ct}} = \frac{1}{4} \sum_{\text{gen}} x_{\text{gen}}^{(1)} + \frac{1}{24} \left\{ -4c^2 v_{\text{gen}}^{(2)} + C_{10}^a \right\} \frac{1}{c^2} N_{\text{gen}} - \frac{1}{24} \left\{ A_{38}^c - 3B_{45}^b c^2 \right\} \frac{1}{c^2}$$

$$M_{8,4}^{\text{ct}} = \sum_{\text{gen}} \left\{ c^2 x_{\text{gen}}^{(1)} x_{\text{H}}^2 + 2 \left[ 2s^2 x_{\text{gen}}^{(2)} + D_3^a \right] \right\} c^2 \\ + \frac{1}{6} \left\{ A_{36}^c + \left[ -(-2A_{14}^c x_{\text{H}}^2 + B_{11}^b - B_{50}^b s^2) c^2 + (B_{43}^b - 2B_{48}^b s^2) \right] \right\} \frac{1}{c^2} \\ - \frac{1}{6} \left\{ C_{10}^a + 2 \left[ (16 - 8c^2 x_{\text{H}}^2 + C_0^a x_{\text{H}}^2) c^2 - (C_{13}^a) \right] \right\} \frac{1}{c^2} N_{\text{gen}}$$

$$M_{8,5}^{\text{ct}} = \sum_{\text{gen}} \left\{ 2x_{\text{gen}}^{(1)} - \left[ 4x_{\text{gen}}^{(2)} - x_{\text{gen}}^{(1)} x_{\text{H}}^2 \right] \right\} c^2 s^2 \\ - \frac{1}{6} \left\{ 9A_{34}^c + \left[ -(-2A_{14}^c x_{\text{H}}^2 + 4B_4^b - B_{49}^b s^2) c^2 + (B_{44}^b - 2B_{46}^b s^2) \right] \right\} \frac{1}{c^2} \\ - \frac{1}{6} \left\{ -16 \left[ 2 - c^2 x_{\text{H}}^2 \right] c^2 + \left[ 2 - A_5^c x_{\text{H}}^2 \right] C_0^a \right\} N_{\text{gen}}$$

$$M_{8,6}^{\text{ct}} = -\sum_{\text{gen}} \left\{ 2x_{\text{gen}}^{(1)} - \left[ 4x_{\text{gen}}^{(2)} - x_{\text{gen}}^{(1)} x_{\text{H}}^2 \right] \right\} s c \\ + \frac{1}{6} \left\{ A_{35}^c + \left[ B_{47}^b s^2 - (-A_6^c x_{\text{H}}^2 + B_1^b - B_8^b s^2) c^2 \right] \right\} \frac{1}{s c} \\ - \frac{1}{12} \left\{ \left[ 16(2 - A_5^c x_{\text{H}}^2) c^2 - (-A_4^c C_0^a x_{\text{H}}^2 + 2C_0^a + 16C_{11}^a s^2) \right] \right\} c^2 \\ + 2 \left\{ (-8s^2 v_{\text{gen}}^{(2)} + C_{12}^a) \right\} \frac{1}{s c} N_{\text{gen}}$$

$$M_{8,10}^{\text{ct}}(q) = \left\{ C_2^a + 12D_2^a c^2 \right\} \frac{1}{c^2} \quad M_{8,11}^{\text{ct}}(l) = -\frac{1}{3} \left\{ -12c^2 x_l^2 + C_8^a \right\} \frac{1}{c^2}$$

$$M_{8,12}^{\text{ct}}(u) = -\left\{ v_u^- - 12c^2 x_u^2 \right\} \frac{1}{c^2} \quad M_{8,13}^{\text{ct}}(d) = \left\{ v_d^- - 12c^2 x_d^2 \right\} \frac{1}{c^2}$$

$$M_{8,14}^{\text{ct}}(l) = \frac{1}{3} \left\{ v_l^- - 12c^2 x_l^2 \right\} \frac{1}{c^2} \quad M_{8,15}^{\text{ct}}(q) = \left\{ C_3^a + 2D_1^a c^2 \right\} \frac{1}{c^2}$$

$$M_{8,16}^{\text{ct}}(l) = -\frac{1}{3} \left\{ v_l^- - 2D_0^a c^2 \right\} \frac{1}{c^2} \quad M_{8,17}^{\text{ct}}(q, u) = -\frac{3}{2} x_u^2 v_u^-$$

$$M_{8,18}^{\text{ct}}(\mathbf{q}, \mathbf{d}) = \frac{3}{2} x_{\mathbf{d}}^2 v_{\mathbf{d}}^- \quad M_{8,19}^{\text{ct}}(\lambda, 1) = \frac{1}{2} x_1^2 v_1^-$$

$$M_{8,20}^{\text{ct}}(\mathbf{q}, \mathbf{u}) = -\frac{3}{2} \frac{s}{c} v_{\mathbf{u}} x_{\mathbf{u}}^2 \quad M_{8,21}^{\text{ct}}(\mathbf{q}, \mathbf{d}) = -\frac{3}{2} \frac{s}{c} v_{\mathbf{d}} x_{\mathbf{d}}^2$$

$$M_{8,22}^{\text{ct}}(\lambda, 1) = -\frac{1}{2} \frac{s}{c} v_1 x_1^2$$

•  $M_{9,i}^{\text{ct}}$  entries

$$M_{9,2}^{\text{ct}} = \frac{1}{2} \sum_{\text{gen}} x_{\text{gen}}^{(1)} - \frac{1}{6} \left\{ A_{19}^c + B_{18}^b c^2 \right\} \frac{1}{c^2}$$

$$M_{9,3}^{\text{ct}} = \frac{1}{8} \sum_{\text{gen}} x_{\text{gen}}^{(1)} - \frac{1}{48} \left\{ 3A_{23}^c - B_{31}^b c^2 \right\} \frac{1}{c^2}$$

$$\begin{aligned} M_{9,4}^{\text{ct}} &= \frac{1}{2} \sum_{\text{gen}} \left\{ c^2 x_{\text{gen}}^{(1)} x_{\mathbf{H}}^2 + 2 \left[ 2s^2 x_{\text{gen}}^{(2)} + D_3^a \right] \right\} c^2 \\ &+ \frac{1}{12} \left\{ 3A_{40}^c - \left[ (-A_8^c x_{\mathbf{H}}^2 + 2B_{51}^b) + (-B_{10}^b s^2 + B_{11}^b) c^2 \right] \right\} \frac{1}{c^2} \\ &+ \frac{1}{6} \left\{ C_1^a + \left[ 8c^2 x_{\mathbf{H}}^2 - (16 + C_0^a x_{\mathbf{H}}^2) \right] \right\} N_{\text{gen}} \end{aligned}$$

$$\begin{aligned} M_{9,5}^{\text{ct}} &= \frac{1}{2} \sum_{\text{gen}} \left\{ 2x_{\text{gen}}^{(1)} - \left[ 4x_{\text{gen}}^{(2)} - x_{\text{gen}}^{(1)} x_{\mathbf{H}}^2 \right] c^2 \right\} s^2 \\ &+ \frac{1}{12} \left\{ 6A_{13}^c - \left[ (A_8^c x_{\mathbf{H}}^2 + 2B_{29}^b) - (4B_4^b - B_9^b s^2) c^2 \right] \right\} \frac{1}{c^2} \\ &- \frac{1}{12} \left\{ -16 \left[ 2 - c^2 x_{\mathbf{H}}^2 \right] c^2 + \left[ 2 - A_5^c x_{\mathbf{H}}^2 \right] C_0^a \right\} N_{\text{gen}} \end{aligned}$$

$$\begin{aligned} M_{9,6}^{\text{ct}} &= -\frac{1}{2} \sum_{\text{gen}} \left\{ 2x_{\text{gen}}^{(1)} - \left[ 4x_{\text{gen}}^{(2)} - x_{\text{gen}}^{(1)} x_{\mathbf{H}}^2 \right] c^2 \right\} s c \\ &+ \frac{1}{24} \left\{ 2A_5^c C_0^a + \left[ 16A_5^c c^2 x_{\mathbf{H}}^2 - (A_4^c C_0^a x_{\mathbf{H}}^2 + 32A_5^c) \right] \right\} \frac{1}{s c} N_{\text{gen}} \\ &+ \frac{1}{12} \left\{ 2A_{39}^c + \left[ 2B_6^b s^2 - (-A_6^c x_{\mathbf{H}}^2 + B_1^b - B_8^b s^2) c^2 \right] \right\} \frac{1}{s c} \end{aligned}$$

$$M_{9,10}^{\text{ct}}(\mathbf{q}) = \left\{ C_2^a + 6D_2^a c^2 \right\} \frac{1}{c^2} \quad M_{9,11}^{\text{ct}}(1) = -\frac{1}{3} \left\{ v_1^+ - 6c^2 x_1^2 \right\} \frac{1}{c^2}$$

$$M_{9,12}^{\text{ct}}(\mathbf{u}) = -\left\{ v_{\mathbf{u}}^- - 6c^2 x_{\mathbf{u}}^2 \right\} \frac{1}{c^2} \quad M_{9,13}^{\text{ct}}(\mathbf{d}) = \left\{ v_{\mathbf{d}}^- - 6c^2 x_{\mathbf{d}}^2 \right\} \frac{1}{c^2}$$

$$M_{9,14}^{\text{ct}}(1) = \frac{1}{3} \left\{ v_1^- - 6c^2 x_1^2 \right\} \frac{1}{c^2} \quad M_{9,15}^{\text{ct}}(\mathbf{q}) = \left\{ C_3^a + D_1^a c^2 \right\} \frac{1}{c^2}$$

$$M_{9,16}^{\text{ct}}(1) = \frac{1}{3} \left\{ v_1^+ + D_0^a c^2 \right\} \frac{1}{c^2} \quad M_{9,17}^{\text{ct}}(\mathbf{q}, \mathbf{u}) = -\frac{3}{4} x_{\mathbf{u}}^2$$

$$M_{9,18}^{\text{ct}}(\mathbf{q}, \mathbf{d}) = \frac{3}{4} x_{\mathbf{d}}^2 \quad M_{9,19}^{\text{ct}}(\lambda, 1) = \frac{1}{4} x_1^2$$

•  $M_{10,i}^{\text{ct}}$  entries

$$M_{10,2}^{\text{ct}} = \frac{3}{4} x_{\text{D}}^2 \quad M_{10,3}^{\text{ct}} = -\frac{1}{48} \left\{ 7 + 6c^2 x_{\text{D}}^2 \right\} \frac{1}{c^2}$$

$$M_{10,4}^{\text{ct}} = \frac{1}{12} \left\{ 7 + E_0^a c^2 \right\} \frac{1}{c^2} \quad M_{10,6}^{\text{ct}} = -\frac{1}{6} \frac{s}{c}$$

$$M_{10,8}^{\text{ct}}(\text{Q}, \text{D}) = -\frac{1}{4} \left\{ 1 + \left[ 3x_{\text{H}}^2 + E_8^a \right] c^2 \right\} \frac{1}{c^2} \quad M_{10,10}^{\text{ct}}(\text{Q}) = -\frac{1}{4} \left\{ 4 - E_{12}^a c^2 \right\} \frac{1}{c^2}$$

$$M_{10,13}^{\text{ct}}(\text{D}) = \frac{1}{4} \left\{ 2 + E_{11}^a c^2 \right\} \frac{1}{c^2} \quad M_{10,15}^{\text{ct}}(\text{Q}) = \frac{1}{4} \left\{ 4 - E_{10}^a c^2 \right\} \frac{1}{c^2}$$

$$M_{10,17}^{\text{ct}}(\text{Q}, \text{U}) = \frac{3}{8} x_{\text{U}}^2 \quad M_{10,18}^{\text{ct}}(\text{Q}, \text{D}) = -\frac{1}{16} \left\{ 2A_{16}^c + 3E_9^a c^2 \right\} \frac{1}{c^2}$$

$$M_{10,21}^{\text{ct}}(\text{Q}, \text{D}) = \frac{1}{16} \left\{ 2 - E_{13}^a c^2 \right\} \frac{s}{c^3} \quad M_{10,23}^{\text{ct}}(\lambda, 1, \text{D}, \text{Q}) = \frac{1}{2} \sum_{\text{gen}} \frac{x_{\text{D}}^3}{x_{\text{D}}}$$

$$M_{10,24}^{\text{ct}}(\text{q}, \text{u}, \text{Q}, \text{D}) = \frac{3}{2} \sum_{\text{gen}} \frac{x_{\text{U}}^3}{x_{\text{D}}}$$

•  $M_{11,i}^{\text{ct}}$  entries

$$M_{11,2}^{\text{ct}} = \frac{1}{4} x_{\text{D}}^2 \quad M_{11,3}^{\text{ct}} = -\frac{1}{72} \left\{ 4 + 9c^2 x_{\text{D}}^2 \right\} \frac{1}{c^2}$$

$$M_{11,4}^{\text{ct}} = \frac{1}{9} \left\{ 2A_{41}^c - E_1^a c^2 \right\} \frac{1}{c^2} \quad M_{11,5}^{\text{ct}} = -\frac{4}{9} s^4$$

$$M_{11,6}^{\text{ct}} = \frac{2}{9} A_{25}^c s c \quad M_{11,8}^{\text{ct}}(\text{Q}, \text{D}) = -\frac{1}{4} x_{\text{D}}^2$$

$$M_{11,13}^{\text{ct}}(\text{D}) = \frac{1}{6} \left\{ 4 - E_{12}^a c^2 \right\} \frac{1}{c^2} \quad M_{11,21}^{\text{ct}}(\text{Q}, \text{D}) = \frac{1}{4} \frac{s}{c} x_{\text{D}}^2$$

•  $M_{12,i}^{\text{ct}}$  entries

$$M_{12,2}^{\text{ct}} = \frac{1}{4} x_{\text{D}}^2 \quad M_{12,3}^{\text{ct}} = -\frac{1}{72} \left\{ 1 + 9c^2 x_{\text{D}}^2 \right\} \frac{1}{c^2}$$

$$M_{12,4}^{\text{ct}} = \frac{1}{18} \left\{ A_{42}^c + E_2^a c^2 \right\} \frac{1}{c^2} \quad M_{12,5}^{\text{ct}} = \frac{2}{9} A_9^c s^2$$

$$M_{12,6}^{\text{ct}} = -\frac{4}{9} s c^3 \quad M_{12,8}^{\text{ct}}(\text{Q}, \text{D}) = -\frac{1}{4} x_{\text{D}}^2$$

$$M_{12,10}^{\text{ct}}(\text{Q}) = -\frac{1}{6} \left\{ 2 + E_{11}^a c^2 \right\} \frac{1}{c^2} \quad M_{12,15}^{\text{ct}}(\text{Q}) = \frac{1}{6} \left\{ 2 + E_{14}^a c^2 \right\} \frac{1}{c^2}$$

$$M_{12,17}^{\text{ct}}(\text{Q}, \text{U}) = \frac{3}{4} x_{\text{U}}^2 \quad M_{12,18}^{\text{ct}}(\text{Q}, \text{D}) = -\frac{3}{8} x_{\text{D}}^2$$

$$M_{12,21}^{\text{ct}}(\text{Q}, \text{D}) = -\frac{1}{8} \frac{s}{c} x_{\text{D}}^2$$

•  $M_{13,i}^{\text{ct}}$  entries



$$M_{13,2}^{\text{ct}} = \frac{3}{4} x_u^2 \quad M_{13,3}^{\text{ct}} = \frac{1}{48} \left\{ 5 - 6c^2 x_u^2 \right\} \frac{1}{c^2}$$

$$M_{13,4}^{\text{ct}} = -\frac{1}{12} \left\{ 5 - E_3^a c^2 \right\} \frac{1}{c^2} \quad M_{13,6}^{\text{ct}} = -\frac{5}{3} \frac{s}{c}$$

$$M_{13,7}^{\text{ct}}(Q, U) = \frac{1}{4} \left\{ 1 + \left[ 3x_H^2 + E_{15}^a \right] c^2 \right\} \frac{1}{c^2} \quad M_{13,10}^{\text{ct}}(Q) = \frac{1}{4} \left\{ 8 - E_{18}^a c^2 \right\} \frac{1}{c^2}$$

$$M_{13,12}^{\text{ct}}(U) = \frac{1}{4} \left\{ 2 - E_{17}^a c^2 \right\} \frac{1}{c^2} \quad M_{13,15}^{\text{ct}}(Q) = \frac{1}{4} \left\{ 8 - E_{19}^a c^2 \right\} \frac{1}{c^2}$$

$$M_{13,17}^{\text{ct}}(Q, U) = \frac{1}{16} \left\{ 2A_1^c + 3E_{16}^a c^2 \right\} \frac{1}{c^2} \quad M_{13,18}^{\text{ct}}(Q, D) = -\frac{3}{8} x_D^2$$

$$M_{13,20}^{\text{ct}}(Q, U) = \frac{1}{16} \left\{ 10 - E_{20}^a c^2 \right\} \frac{s}{c^3} \quad M_{13,24}^{\text{ct}}(Q, U, q, d) = \frac{3}{2} \sum_{\text{gen}} \frac{x_u^3}{x_u}$$

$$M_{13,25}^{\text{ct}}(\lambda, 1, Q, U) = -\frac{1}{2} \sum_{\text{gen}} \frac{x_1^3}{x_u}$$

•  $M_{14,i}^{\text{ct}}$  entries

$$M_{14,2}^{\text{ct}} = \frac{1}{4} x_u^2 \quad M_{14,3}^{\text{ct}} = -\frac{1}{72} \left\{ 16 + 9c^2 x_u^2 \right\} \frac{1}{c^2}$$

$$M_{14,4}^{\text{ct}} = \frac{1}{9} \left\{ 8A_{41}^c - E_4^a c^2 \right\} \frac{1}{c^2} \quad M_{14,5}^{\text{ct}} = -\frac{16}{9} s^4$$

$$M_{14,6}^{\text{ct}} = \frac{8}{9} A_{25}^c s c \quad M_{14,7}^{\text{ct}}(Q, U) = \frac{1}{4} x_u^2$$

$$M_{14,12}^{\text{ct}}(U) = -\frac{1}{6} \left\{ 8 - E_{18}^a c^2 \right\} \frac{1}{c^2} \quad M_{14,20}^{\text{ct}}(Q, U) = \frac{1}{2} \frac{s}{c} x_u^2$$

•  $M_{15,i}^{\text{ct}}$  entries

$$M_{15,2}^{\text{ct}} = \frac{1}{4} x_u^2 \quad M_{15,3}^{\text{ct}} = -\frac{1}{72} \left\{ 1 + 9c^2 x_u^2 \right\} \frac{1}{c^2}$$

$$M_{15,4}^{\text{ct}} = \frac{1}{18} \left\{ A_{43}^c + E_2^a c^2 \right\} \frac{1}{c^2} \quad M_{15,5}^{\text{ct}} = \frac{4}{9} A_{16}^c s^2$$

$$M_{15,6}^{\text{ct}} = -\frac{4}{9} A_4^c s c \quad M_{15,7}^{\text{ct}}(Q, U) = \frac{1}{4} x_u^2$$

$$M_{15,10}^{\text{ct}}(Q) = -\frac{1}{6} \left\{ 2 - E_{17}^a c^2 \right\} \frac{1}{c^2} \quad M_{15,15}^{\text{ct}}(Q) = -\frac{1}{6} \left\{ 2 - E_{21}^a c^2 \right\} \frac{1}{c^2}$$

$$M_{15,17}^{\text{ct}}(Q, U) = \frac{3}{8} x_u^2 \quad M_{15,18}^{\text{ct}}(Q, D) = -\frac{3}{4} x_D^2$$

$$M_{15,20}^{\text{ct}}(Q, U) = \frac{1}{8} \frac{s}{c} x_u^2$$

•  $M_{16,i}^{\text{ct}}$  entries

$$\begin{aligned}
 M_{16,2}^{\text{ct}} &= \frac{3}{4} x_L^2 & M_{16,3}^{\text{ct}} &= \frac{1}{16} \left\{ 11 - 2c^2 x_L^2 \right\} \frac{1}{c^2} \\
 M_{16,4}^{\text{ct}} &= -\frac{1}{4} \left\{ 11 - E_5^a c^2 \right\} \frac{1}{c^2} & M_{16,6}^{\text{ct}} &= -\frac{9}{2} \frac{s}{c} \\
 M_{16,9}^{\text{ct}}(\Lambda, L) &= -\frac{1}{4} \left\{ 1 + \left[ 3x_H^2 + E_{22}^a \right] c^2 \right\} \frac{1}{c^2} & M_{16,11}^{\text{ct}}(L) &= -\frac{3}{4} \left\{ 4 - E_{24}^a c^2 \right\} \frac{1}{c^2} \\
 M_{16,14}^{\text{ct}}(L) &= -\frac{3}{4} \left\{ 2 - E_{23}^a c^2 \right\} \frac{1}{c^2} & M_{16,16}^{\text{ct}}(L) &= \frac{3}{4} \left\{ 4 - E_{24}^a c^2 \right\} \frac{1}{c^2} \\
 M_{16,19}^{\text{ct}}(\Lambda, L) &= -\frac{3}{16} \left\{ 2A_4^c + E_{25}^a c^2 \right\} \frac{1}{c^2} & M_{16,22}^{\text{ct}}(\Lambda, L) &= \frac{3}{16} \left\{ 6 - E_{26}^a c^2 \right\} \frac{s}{c^3} \\
 M_{16,23}^{\text{ct}}(\Lambda, L, d, q) &= \frac{3}{2} \sum_{\text{gen}} \frac{x_d^3}{x_L} & M_{16,25}^{\text{ct}}(\Lambda, L, q, u) &= -\frac{3}{2} \sum_{\text{gen}} \frac{x_u^3}{x_L}
 \end{aligned}$$

•  $M_{17,i}^{\text{ct}}$  entries

$$\begin{aligned}
 M_{17,2}^{\text{ct}} &= \frac{1}{4} x_L^2 & M_{17,3}^{\text{ct}} &= -\frac{1}{8} \left\{ 4 + c^2 x_L^2 \right\} \frac{1}{c^2} \\
 M_{17,4}^{\text{ct}} &= \left\{ 2A_{41}^c - E_6^a c^2 \right\} \frac{1}{c^2} & M_{17,5}^{\text{ct}} &= -4s^4 \\
 M_{17,6}^{\text{ct}} &= 2A_{25}^c s c & M_{17,9}^{\text{ct}}(\Lambda, L) &= -\frac{1}{4} x_L^2 \\
 M_{17,14}^{\text{ct}}(L) &= \frac{1}{2} \left\{ 4 - E_{24}^a c^2 \right\} \frac{1}{c^2} & M_{17,22}^{\text{ct}}(\Lambda, L) &= \frac{3}{4} \frac{s}{c} x_L^2
 \end{aligned}$$

•  $M_{18,i}^{\text{ct}}$  entries

$$\begin{aligned}
 M_{18,2}^{\text{ct}} &= \frac{1}{4} x_L^2 & M_{18,3}^{\text{ct}} &= -\frac{1}{8} \left\{ 1 + c^2 x_L^2 \right\} \frac{1}{c^2} \\
 M_{18,4}^{\text{ct}} &= \frac{1}{2} \left\{ A_{44}^c + E_7^a c^2 \right\} \frac{1}{c^2} & M_{18,5}^{\text{ct}} &= 2A_5^c s^2 \\
 M_{18,6}^{\text{ct}} &= 4s^3 c & M_{18,9}^{\text{ct}}(\Lambda, L) &= -\frac{1}{4} x_L^2 \\
 M_{18,11}^{\text{ct}}(L) &= \frac{1}{2} \left\{ 2 - E_{23}^a c^2 \right\} \frac{1}{c^2} & M_{18,16}^{\text{ct}}(L) &= -\frac{1}{2} \left\{ 2 - E_{27}^a c^2 \right\} \frac{1}{c^2} \\
 M_{18,19}^{\text{ct}}(\Lambda, L) &= -\frac{3}{8} x_L^2 & M_{18,22}^{\text{ct}}(\Lambda, L) &= \frac{3}{8} \frac{s}{c} x_L^2
 \end{aligned}$$

•  $M_{20,i}^{\text{ct}}$  entries

$$\begin{aligned}
 M_{20,3}^{\text{ct}} &= -\frac{1}{8} \frac{1}{c^2} & M_{20,4}^{\text{ct}} &= \frac{1}{2} \left\{ 1 + E_7^a c^2 \right\} \frac{1}{c^2} \\
 M_{20,11}^{\text{ct}}(L) &= \frac{1}{c^2} & M_{20,16}^{\text{ct}}(L) &= \left\{ 1 + E_6^a c^2 \right\} \frac{1}{c^2} \\
 M_{20,19}^{\text{ct}}(\Lambda, L) &= -\frac{3}{4} x_L^2 & &
 \end{aligned}$$

(C.20)

## D Self-energies at $s = 0$

In this appendix we present the full list of bosonic self energies evaluated at  $s = 0$ . We have introduced a simplified notation,

$$B_{0p}^{\text{fin}}(m_1, m_1) = B_{0p}^{\text{fin}}(0; m_1, m_2) \quad (\text{D.1})$$

etc. We introduce the following polynomials:

M where  $s = s_\theta$ ,  $c = c_\theta$  and

$$\begin{aligned}
 M_0^a &= 5 - 3c \\
 M_0^b &= 1 - 2c & M_1^b &= 4 - 9c & M_2^b &= 7 - 10c \\
 M_3^b &= 5 + 4c & M_4^b &= 1 - 3s & M_5^b &= 4 - c \\
 M_6^b &= 17 + 16c & M_7^b &= 31 - 11c & M_8^b &= 4 - s \\
 M_9^b &= 1 + s & M_{10}^b &= 3 + c & M_{11}^b &= 15 + M_0^a c \\
 M_{12}^b &= 17 - 8c & M_{13}^b &= 8 - 5c & M_{14}^b &= 41 - 24s \\
 \\ 
 M_0^c &= 1 + 18c & M_1^c &= 1 + 4c & M_2^c &= 1 + 6c^2 \\
 M_3^c &= 1 + 24s^2 c & M_4^c &= 1 + 3M_0^b c & M_5^c &= 1 + 4M_1^b c \\
 M_6^c &= 2 - M_2^b c & M_7^c &= 3 - c & M_8^c &= 3 + 4c \\
 M_9^c &= 5 + 8c & M_{10}^c &= 7 - 38c & M_{11}^c &= c - 2 \\
 M_{12}^c &= 1 - 40c + 36sc & M_{13}^c &= 3 - 2M_3^b c & M_{14}^c &= 9 - 8s \\
 M_{15}^c &= 11 + 4c & M_{16}^c &= 1 + 10s & M_{17}^c &= 1 + 2M_4^b s \\
 M_{18}^c &= 1 - 2M_5^b c & M_{19}^c &= 2 - c & M_{20}^c &= 3 - M_6^b c \\
 \\ 
 M_{21}^c &= 4 + c & M_{22}^c &= 5 - 2c & M_{23}^c &= 6 - M_7^b c \\
 M_{24}^c &= 7 - 2M_8^b s & M_{25}^c &= 9 - 2M_9^b s & M_{26}^c &= 18 - 11c \\
 M_{27}^c &= 5 - 35c + 8sc & M_{28}^c &= 39 - 40s & M_{29}^c &= 1 + c \\
 M_{30}^c &= 1 - 4sc & M_{31}^c &= 2 - 3M_{10}^b c & M_{32}^c &= 3 - M_{11}^b c \\
 M_{33}^c &= 4 + 3c & M_{34}^c &= 5 - 2M_{12}^b c & M_{35}^c &= 9 - 8M_{13}^b c \\
 M_{36}^c &= 29 - 16c & M_{37}^c &= 37 - 48s & M_{38}^c &= 37 - 2M_{14}^b s \\
 M_{39}^c &= 49 - 34c & M_{40}^c &= 79 - 40c
 \end{aligned}$$

N

$$\begin{aligned}
 N_0 &= 1 - 3 * v_d & N_1 &= 3 - v_1 & N_2 &= 5 - 3 * v_u \\
 N_3 &= 20 - 3 * v_{\text{gen}}^{(2)} & N_4 &= 1 + v_u^2 & N_5 &= 1 + v_d^2 \\
 N_6 &= 1 + v_1^2 & N_7 &= 9 + v_{\text{gen}}^{(1)} & N_8 &= 38 + 3 v_{\text{gen}}^{(1)} \\
 N_9 &= 1 - v_1 & N_{10} &= 1 - v_u & N_{11} &= 1 + v_u \\
 N_{12} &= 1 - v_d & N_{13} &= 1 + v_d & N_{14} &= 1 - 3 v_1 \\
 N_{15} &= 1 + v_1 & N_{16} &= 2 - v_u & N_{17} &= 2 + v_u + v_d \\
 N_{18} &= 2 - v_d & N_{19} &= 2 - v_1 & N_{20} &= 2 + 3 v_1 \\
 N_{21} &= 3 - 5 v_u & N_{22} &= 3 - v_d & N_{23} &= 3 + v_1 \\
 N_{24} &= 4 + v_u + v_d & N_{25} &= 9 - 5 v_u - v_d - 3 v_1 & N_{26} &= 10 + 3 v_1 \\
 N_{27} &= 38 - 15 v_u - 3 v_d - 9 v_1 & N_{28} &= v_u - v_d
 \end{aligned}$$

O

$$\begin{aligned} O_0 &= x_1^2 + 3x_d^2 + 3x_u^2 & O_1 &= x_d^2 + x_u^2 & O_2 &= x_d^2 - 3x_u^2 \\ O_3 &= 3x_d^2 - x_u^2 & O_4 &= (x_u^2 - x_d^2)^2 \end{aligned}$$

P

$$\begin{aligned} P_0 &= 1 - x_H^2 & P_1 &= 2 - x_H^2 - x_H^4 & P_2 &= 3 - 2x_H^2 - x_H^4 \\ P_3 &= 6 - 7x_H^2 + x_H^4 & P_4 &= 10 - 11x_H^2 + x_H^4 & P_5 &= 2 - 5x_H^2 + 3x_H^4 \\ P_6 &= 12 - 13x_H^2 + x_H^4 & P_7 &= 12 - 11x_H^2 - x_H^4 & P_8 &= 8 - 5x_H^2 - 3x_H^4 \\ P_9 &= 1 - 3x_H^2 + 3x_H^4 - x_H^6 & P_{10} &= 42 - 41x_H^2 - x_H^4 & P_{11} &= 1 - x_H^4 \\ P_{12} &= 7 - 6x_H^2 & P_{13} &= 9 - x_H^2 & P_{14} &= 20 - 21x_H^2 + x_H^4 \\ P_{15} &= 5 - 6x_H^2 + x_H^4 & P_{16} &= 78 - 79x_H^2 & P_{17} &= 10 + 3x_H^2 \\ P_{18} &= 11 - 18x_H^2 + 7x_H^4 & P_{19} &= 20 - 23x_H^2 + 3x_H^4 & P_{20} &= 32 - 45x_H^2 \\ P_{21} &= 80 - 79x_H^2 \end{aligned}$$

With their help we derive the vector-vector transitions at  $s = 0$

- A self-energy

$$\Pi_{AA;0}^{(4)}(0) = -\frac{32}{9} s^2 N_{\text{gen}} (1 - L_R) + \frac{1}{3} s^2 (7 - 9L_R)$$

$$\begin{aligned} \Pi_{AA;1}^{(4)}(0) &= 3s^2 a_0^{\text{fin}}(M) - \frac{4}{3} \sum_{\text{gen}} s^2 a_0^{\text{fin}}(M_1) \\ &\quad - \frac{16}{9} \sum_{\text{gen}} s^2 a_0^{\text{fin}}(M_u) - \frac{4}{9} \sum_{\text{gen}} s^2 a_0^{\text{fin}}(M_d) \end{aligned}$$

$$\Pi_{AA;2}^{(4)}(0) = 0$$

$$\begin{aligned} \Pi_{AA;0}^{(6)}(0) &= -\frac{16}{9} N_{\text{gen}} a_{\phi_{\text{WAD}}} (1 - L_R) - \frac{1}{6} c^2 a_{\phi_{\text{D}}} (7 - 9L_R) \\ &\quad + \frac{1}{2} \left[ s a_{\phi_{\text{WA}}} - \frac{c^2}{x_H^2 - 1} P_2 a_{\phi_{\text{B}}} + \frac{s c^3}{x_H^2 - 1} P_3 a_{\phi_{\text{WB}}} + (P_1 a_{\phi_{\text{B}}} + P_4 a_{\phi_{\text{W}}}) \frac{s^2 c^2}{x_H^2 - 1} \right] \frac{L_R}{c^2} \\ &\quad + \frac{2}{3} (c a_{\phi_{\text{WB}}} + 7 s a_{\phi_{\text{W}}}) s - 2 \sum_{\text{gen}} (a_{\text{dWB}} x_d^2 - 2 a_{\text{uWB}} x_u^2 + a_{\text{lWB}} x_l^2) s (1 - L_R) \end{aligned}$$

$$\begin{aligned} \Pi_{AA;1}^{(6)}(0) &= \frac{1}{2} \frac{x_H^2}{x_H^2 - 1} P_0 a_{AA} a_0^{\text{fin}}(M_H) - \frac{1}{2} \frac{1}{c^2} a_{AA} a_0^{\text{fin}}(M_0) \\ &\quad - \frac{1}{2} \left[ 2c^2 a_{\phi_{\text{B}}} + 3c^2 a_{\phi_{\text{D}}} - 2(3c a_{\phi_{\text{WB}}} + 5s a_{\phi_{\text{W}}}) s \right] a_0^{\text{fin}}(M) - \frac{2}{9} \sum_{\text{gen}} (a_{\phi_{\text{WAD}}} + 9s a_{\text{dWB}} x_d^2) a_0^{\text{fin}}(M_d) \\ &\quad - \frac{2}{3} \sum_{\text{gen}} (a_{\phi_{\text{WAD}}} + 3s a_{\text{lWB}} x_l^2) a_0^{\text{fin}}(M_1) - \frac{4}{9} \sum_{\text{gen}} (2a_{\phi_{\text{WAD}}} - 9s a_{\text{uWB}} x_u^2) a_0^{\text{fin}}(M_u) \end{aligned}$$

$$\Pi_{AA;2}^{(6)}(0) = 0$$

- Z–A transition

$$\Pi_{ZA;0}^{(4)}(0) = -\frac{1}{3} \frac{s}{c} N_{\text{gen}} v_{\text{gen}}^{(2)} (1 - L_R) - \frac{1}{6} \frac{s}{c} (1 + L_R) + \frac{1}{3} s c (7 - 9L_R)$$

$$\begin{aligned} \Pi_{ZA;1}^{(4)}(0) &= \frac{1}{6} \frac{s}{c} M_0^c a_0^{\text{fin}}(M) - \frac{1}{3} \sum_{\text{gen}} \frac{s}{c} v_1 a_0^{\text{fin}}(M_1) \\ &\quad - \frac{2}{3} \sum_{\text{gen}} \frac{s}{c} v_u a_0^{\text{fin}}(M_u) - \frac{1}{3} \sum_{\text{gen}} \frac{s}{c} v_d a_0^{\text{fin}}(M_d) \end{aligned}$$

$$\Pi_{ZA;2}^{(4)}(0) = 0$$

$$\begin{aligned} \Pi_{ZA;0}^{(6)}(0) &= \frac{2}{3} \frac{1}{s} c a_{\phi D} (1 - L_R) - \frac{1}{24} \frac{1}{s c} a_{\phi D} (1 + L_R) - \frac{1}{6} \frac{1}{s} c^3 a_{\phi D} (7 - 9L_R) \\ &\quad - \frac{1}{12} \left[ 3M_3^c a_{\phi WB} - 2(-3a_{\phi B} + a_{\phi W} - 36s^2 c^2 a_{\phi WB}^{(a)}) s c - 6(P_1 a_{\phi B} + P_4 a_{\phi W}) \frac{s c^3}{x_H^2 - 1} \right. \\ &\quad \left. - (P_5 - 6P_6 s^2) \frac{c^2}{x_H^2 - 1} a_{\phi WB} \right] \frac{L_R}{c^2} \\ &\quad - \frac{1}{6} \left[ M_6^c a_{\phi WA} + M_{10}^c s c a_{\phi WZ} - 28s^3 c a_{\phi B} c \right] \frac{1}{c} \\ &\quad + \frac{1}{36} \left\{ \left[ 3c a_{\phi D} v_{\text{gen}}^{(2)} - 4(3a_{\phi WB} v_{\text{gen}}^{(2)} - 32(-c a_{\phi WZ} + s a_{\phi WAB}) s c) s \right] c \right. \\ &\quad \left. + 4(N_3 a_{\phi W} - M_5^c a_{\phi D}) s^2 \right\} \frac{N_{\text{gen}}}{s c} (1 - L_R) \\ &\quad - \frac{1}{12} \sum_{\text{gen}} \left\{ -3 \left[ -(a_{1WB} v_1 + 4s c a_{1BW}) x_1^2 - (3v_d a_{dWB} + 4s c a_{dBW}) x_d^2 + (3v_u a_{uWB} \right. \right. \\ &\quad \left. \left. + 8s c a_{uBW}) x_u^2 \right] + 8(a_{\phi 1V} + a_{\phi dV} + 2a_{\phi uV}) s \right\} \frac{1}{c} (1 - L_R) \end{aligned}$$

$$\begin{aligned} \Pi_{ZA;1}^{(6)}(0) &= \frac{1}{4} \frac{x_H^2}{x_H^2 - 1} P_0 a_{AZ} a_0^{\text{fin}}(M_H) - \frac{1}{4} \frac{1}{c^2} a_{AZ} a_0^{\text{fin}}(M_0) \\ &\quad + \frac{1}{24} (8M_2^c s^2 a_{AA} - 8M_4^c s c a_{AZ} + M_5^c a_{\phi D} + 48M_7^c s^2 c^2 a_{ZZ}) \frac{1}{s c} a_0^{\text{fin}}(M) \\ &\quad - \frac{1}{12} \sum_{\text{gen}} \left\{ - \left[ c^2 a_{\phi D} v_1 - 4(c a_{\phi WB} v_1 + 2(a_{\phi 1V} + 2c^3 a_{\phi WZ} - 2s c^2 a_{\phi WAB}) s) s \right] \right. \\ &\quad \left. + 3(a_{1WB} v_1 + 4s c a_{1BW}) s x_1^2 - (4N_1 a_{\phi W} - M_8^c a_{\phi D}) s^2 \right\} \frac{1}{s c} a_0^{\text{fin}}(M_1) \\ &\quad - \frac{1}{36} \sum_{\text{gen}} \left\{ - \left[ 3c^2 v_d a_{\phi D} - 4(3c v_d a_{\phi WB} + 2(3a_{\phi dV} + 2c^3 a_{\phi WZ} - 2s c^2 a_{\phi WAB}) s) s \right] \right. \\ &\quad \left. + 9(3v_d a_{dWB} + 4s c a_{dBW}) s x_d^2 - (4N_0 a_{\phi W} - M_1^c a_{\phi D}) s^2 \right\} \frac{1}{s c} a_0^{\text{fin}}(M_d) \\ &\quad + \frac{1}{36} \sum_{\text{gen}} \left\{ 2 \left[ 3c^2 v_u a_{\phi D} - 4(3c v_u a_{\phi WB} + 2(3a_{\phi uV} + 4c^3 a_{\phi WZ} - 4s c^2 a_{\phi WAB}) s) s \right] \right. \\ &\quad \left. + 9(3v_u a_{uWB} + 8s c a_{uBW}) s x_u^2 + 2(4N_2 a_{\phi W} - M_9^c a_{\phi D}) s^2 \right\} \frac{1}{s c} a_0^{\text{fin}}(M_u) \end{aligned}$$

$$\Pi_{ZA;2}^{(6)}(0) = 0$$

## • Z self-energy

$$\begin{aligned}\Delta_{ZZ;0}^{(4)}(0) &= -\frac{1}{6} \frac{1}{c^4} (1 - 6L_R) + 2 \frac{L_R}{c^2} \\ &\quad + \frac{1}{6} \frac{1}{c^2} \frac{1}{x_H^2 - 1} P_7 + \frac{1}{2} \sum_{\text{gen}} \frac{1}{c^2} O_0 (1 - L_R)\end{aligned}$$

$$\begin{aligned}\Delta_{ZZ;1}^{(4)}(0) &= -2 \frac{1}{c^2} a_0^{\text{fin}}(M) + \frac{1}{2} \sum_{\text{gen}} \frac{x_1^2}{c^2} a_0^{\text{fin}}(M_1) + \frac{3}{2} \sum_{\text{gen}} \frac{x_u^2}{c^2} a_0^{\text{fin}}(M_u) \\ &\quad + \frac{3}{2} \sum_{\text{gen}} \frac{x_d^2}{c^2} a_0^{\text{fin}}(M_d) \\ &\quad - \frac{2}{3} \left( c^2 - \frac{1}{1 - c^2 x_H^2} M_{11}^c \right) \frac{1}{c^6} a_0^{\text{fin}}(M_H) - \frac{1}{3} \left( c^2 + 2 \frac{1}{1 - c^2 x_H^2} M_{11}^c \right) \frac{1}{c^6} a_0^{\text{fin}}(M_0)\end{aligned}$$

$$\Delta_{ZZ;2}^{(4)}(0) = -\frac{1}{12} \left[ 1 + (2P_0 - P_0 c^2 x_H^2) \frac{c^2}{x_H^2 - 1} x_H^2 \right] \frac{1}{c^6} B_{0p}^{\text{fin}}(M_H, M_0)$$

$$\begin{aligned}\Delta_{ZZ;0}^{(6)}(0) &= -\frac{1}{8} \frac{1}{x_H^2 - 1} P_8 a_{\phi D} \frac{L_R}{c^2} - \frac{1}{24} (4a_{\phi\Box} + a_{\phi D}) \frac{1}{c^4} (2 - 9L_R) \\ &\quad + \frac{1}{12} (4P_0 x_H^2 a_{\phi\Box} + P_7 a_{\phi D}) \frac{1}{c^2} \frac{1}{x_H^2 - 1} \\ &\quad + \frac{1}{4} \sum_{\text{gen}} (24x_d^2 a_{\phi d} - 24x_u^2 a_{\phi u} + 8x_1^2 a_{\phi 1A} - O_0 a_{\phi D} + 12O_1 a_{\phi u \text{VA}}) \frac{1}{c^2} (1 - L_R)\end{aligned}$$

$$\begin{aligned}\Delta_{ZZ;1}^{(6)}(0) &= -\frac{1}{c^2} a_{\phi D} a_0^{\text{fin}}(M) \\ &\quad + \frac{1}{24} \left[ 9 \frac{c^4}{x_H^2 - 1} P_0 a_{\phi D} x_H^2 - 8(4a_{\phi\Box} + a_{\phi D}) c^2 + 8(4a_{\phi\Box} + a_{\phi D}) \frac{1}{1 - c^2 x_H^2} M_{11}^c \right] \frac{1}{c^6} a_0^{\text{fin}}(M_H) \\ &\quad - \frac{1}{24} \left[ (4a_{\phi\Box} + a_{\phi D}) c^2 + 8(4a_{\phi\Box} + a_{\phi D}) \frac{1}{1 - c^2 x_H^2} M_{11}^c \right] \frac{1}{c^6} a_0^{\text{fin}}(M_0) \\ &\quad + \frac{1}{4} \sum_{\text{gen}} (8a_{\phi 1A} - a_{\phi D}) \frac{x_1^2}{c^2} a_0^{\text{fin}}(M_1) + \frac{3}{4} \sum_{\text{gen}} (8a_{\phi dA} - a_{\phi D}) \frac{x_d^2}{c^2} a_0^{\text{fin}}(M_d) \\ &\quad + \frac{3}{4} \sum_{\text{gen}} (8a_{\phi uA} - a_{\phi D}) \frac{x_u^2}{c^2} a_0^{\text{fin}}(M_u)\end{aligned}$$

$$\Delta_{ZZ;2}^{(6)}(0) = -\frac{1}{24} \left[ (4a_{\phi\Box} + a_{\phi D}) + (4a_{\phi\Box} + a_{\phi D}) (2P_0 - P_0 c^2 x_H^2) \frac{c^2}{x_H^2 - 1} x_H^2 \right] \frac{1}{c^6} B_{0p}^{\text{fin}}(M_H, M_0)$$

$$\begin{aligned}\Omega_{ZZ;0}^{(4)}(0) &= \frac{2}{3} (3 - 5L_R) - \frac{1}{3} s^2 (7 - 9L_R) + \frac{1}{36} \frac{1}{c^2} (11 + 6L_R) \\ &\quad + \frac{1}{12} \frac{N_{\text{gen}}}{c^2} N_7 L_R - \frac{1}{36} (N_8 + 6L_{\text{ir}}) \frac{N_{\text{gen}}}{c^2}\end{aligned}$$

$$\begin{aligned}\Omega_{ZZ;1}^{(4)}(0) &= -\frac{1}{12} \frac{1}{c^2} M_{12}^c a_0^{\text{fin}}(M) - \frac{1}{12} \frac{1}{c^6} \frac{1}{1 - c^2 x_H^2} a_0^{\text{fin}}(M_0) \\ &\quad - \frac{1}{12} \sum_{\text{gen}} \frac{1}{c^2} N_6 a_0^{\text{fin}}(M_1) - \frac{1}{4} \sum_{\text{gen}} \frac{1}{c^2} N_4 a_0^{\text{fin}}(M_u) \\ &\quad - \frac{1}{4} \sum_{\text{gen}} \frac{1}{c^2} N_5 a_0^{\text{fin}}(M_d) + \frac{1}{12} \left( -c^4 + \frac{1}{1 - c^2 x_H^2} \right) \frac{1}{c^6} a_0^{\text{fin}}(M_H)\end{aligned}$$

$$\begin{aligned}\Omega_{ZZ;2}^{(4)}(0) &= \frac{1}{24} \left[ 1 + (2P_0 - P_0 c^2 x_H^2) \frac{c^2}{x_H^2 - 1} x_H^2 \right] \frac{1}{c^6} B_{0s}^{\text{fin}}(M_H, M_0) \\ &\quad - \frac{1}{6} \left( 5 + \frac{c^2}{x_H^2 - 1} P_0 x_H^2 \right) \frac{1}{c^4} B_{0p}^{\text{fin}}(M_H, M_0)\end{aligned}$$

$$\begin{aligned}\Omega_{ZZ;0}^{(6)}(0) &= -\frac{1}{6} N_{\text{gen}} a_{\phi D} v_{\text{gen}}^{(2)} (1 - L_R) + \frac{1}{18} \frac{1}{c^2} a_{\phi \square} (2 + 3L_R) \\ &\quad + \frac{1}{3} a_{\phi D} (3 - 5L_R) - \frac{1}{6} s^2 a_{\phi D} (7 - 9L_R) \\ &\quad + \frac{1}{72} \frac{1}{c^2} a_{\phi D} (11 + 6L_R) - \frac{1}{24} \frac{N_{\text{gen}}}{c^2} N_{25} L_R a_{\phi D} \\ &\quad - \sum_{\text{gen}} (1 - L_R) \frac{1}{c^2} N_{28} a_{\phi q}^{(1)} + \frac{1}{72} (N_{27} + 6L_{\text{ir}}) \frac{N_{\text{gen}}}{c^2} a_{\phi D} \\ &\quad - \frac{1}{3} \sum_{\text{gen}} \left[ 3N_{10} a_{\phi u} - 3N_{12} a_{\phi d} - 3N_{17} a_{\phi q}^{(3)} - N_{23} a_{\phi 1}^{(3)} - (a_{\phi 1}^{(3)} - a_{\phi 1v}) N_9 \right] \frac{L_R}{c^2} \\ &\quad + \frac{1}{9} \sum_{\text{gen}} \left[ 9N_{16} a_{\phi u} - 9N_{18} a_{\phi d} - 3N_{19} a_{\phi 1} - N_{20} a_{\phi 1}^{(3)} - 9N_{24} a_{\phi q}^{(3)} + N_{26} a_{\phi 1}^{(1)} - 3L_{\text{ir}} a_{\phi v} \right] \frac{1}{c^2}\end{aligned}$$

$$\begin{aligned}\Omega_{ZZ;1}^{(6)}(0) &= -\frac{1}{24} \frac{1}{c^2} M_{12}^c a_{\phi D} a_0^{\text{fin}}(M) \\ &\quad + \frac{1}{24} \left[ -(4a_{\phi \square} + a_{\phi D}) c^4 + (4a_{\phi \square} + a_{\phi D}) \frac{1}{1 - c^2 x_H^2} \right] \frac{1}{c^6} a_0^{\text{fin}}(M_H) \\ &\quad - \frac{1}{24} (4a_{\phi \square} + a_{\phi D}) \frac{1}{c^6} \frac{1}{1 - c^2 x_H^2} a_0^{\text{fin}}(M_0) \\ &\quad - \frac{1}{24} \sum_{\text{gen}} \left[ 4c^2 a_{\phi D} v_1 + (8N_9 a_{\phi 1} - N_{14} a_{\phi D} + 4N_{15} a_{\phi 1v\lambda}) \right] \frac{1}{c^2} a_0^{\text{fin}}(M_1) \\ &\quad - \frac{1}{24} \sum_{\text{gen}} \left[ 4c^2 v_d a_{\phi D} + (24N_{12} a_{\phi d} + 12N_{13} a_{\phi d v\lambda} - N_{22} a_{\phi D}) \right] \frac{1}{c^2} a_0^{\text{fin}}(M_d) \\ &\quad - \frac{1}{24} \sum_{\text{gen}} \left[ 8c^2 v_u a_{\phi D} - (24N_{10} a_{\phi u} - 12N_{11} a_{\phi u v\lambda} + N_{21} a_{\phi D}) \right] \frac{1}{c^2} a_0^{\text{fin}}(M_u)\end{aligned}$$

$$\begin{aligned}\Omega_{ZZ;2}^{(6)}(0) &= \frac{1}{48} \left[ (4a_{\phi \square} + a_{\phi D}) + (4a_{\phi \square} + a_{\phi D}) (2P_0 - P_0 c^2 x_H^2) \frac{c^2}{x_H^2 - 1} x_H^2 \right] \frac{1}{c^6} B_{0s}^{\text{fin}}(M_H, M_0) \\ &\quad - \frac{1}{12} \left[ 5(4a_{\phi \square} + a_{\phi D}) + (4a_{\phi \square} + a_{\phi D}) \frac{c^2}{x_H^2 - 1} P_0 x_H^2 \right] \frac{1}{c^4} B_{0p}^{\text{fin}}(M_H, M_0)\end{aligned}$$

• W self-energy

$$\Delta_{\text{WW};0}^{(4)}(0) = 2L_R - \frac{1}{6} \frac{1}{c^2} (1 - 6L_R) + \frac{1}{6} \frac{1}{x_H^2 - 1} P_{10} + \frac{1}{6} \sum_{\text{gen}} O_0 (2 - 3L_R)$$

$$\begin{aligned}\Delta_{\text{WW};1}^{(4)}(0) &= -\frac{2}{3} \frac{x_H^2}{x_H^2 - 1} a_0^{\text{fin}}(M_H) - \frac{1}{3} \frac{1}{s^2 c^2} M_{13}^c a_0^{\text{fin}}(M_0) \\ &\quad - \frac{1}{2} \sum_{\text{gen}} \frac{x_u^2}{x_u^2 - x_d^2} O_2 a_0^{\text{fin}}(M_u) - \frac{1}{2} \sum_{\text{gen}} \frac{x_d^2}{x_u^2 - x_d^2} O_3 a_0^{\text{fin}}(M_d) \\ &\quad + \frac{1}{2} \sum_{\text{gen}} x_1^2 a_0^{\text{fin}}(M_1) + \frac{1}{3} \left( 2 \frac{s^2}{x_H^2 - 1} - M_{15}^c \right) \frac{1}{s^2} a_0^{\text{fin}}(M)\end{aligned}$$

$$\begin{aligned}\Delta_{\text{WW};2}^{(4)}(0) &= -\frac{1}{12} \frac{s^4}{c^4} M_{14}^c B_{0p}^{\text{fin}}(M, M_0) + \frac{1}{12} \frac{1}{x_H^2 - 1} P_9 B_{0p}^{\text{fin}}(M, M_H) \\ &\quad - \frac{2}{3} s^2 B_{0p}^{\text{fin}}(0, M) + \frac{1}{2} \sum_{\text{gen}} O_4 B_{0p}^{\text{fin}}(M_u, M_d) + \frac{1}{6} \sum_{\text{gen}} x_1^4 B_{0p}^{\text{fin}}(0, M_1)\end{aligned}$$

$$\begin{aligned}\Delta_{\text{WW};0}^{(6)} &= -\frac{1}{3} \frac{1}{c^2} a_{\phi W} (1 - 6L_R) + \frac{1}{12} \frac{1}{c^2} a_{\phi D} (11 - 15L_R) \\ &\quad - \frac{1}{6} \left[ 2c^3 a_{\phi WB} + (3a_{\phi D} - 4c^2 a_{AA} + 4c^2 a_{\phi W}) s \right] \frac{s}{c^2} (2 - 3L_R) \\ &\quad + \frac{1}{12} \left[ 40s a_{\phi WB} + (4P_{10} a_{\phi W} + 4P_{11} a_{\phi \square} + P_{14} a_{\phi D}) \frac{c}{x_H^2 - 1} \right] \frac{1}{c} \\ &\quad + \frac{1}{2} (3a_{\phi \square} + 8a_{\phi W}) L_R + \frac{1}{3} \sum_{\text{gen}} (2x_1^2 a_{\phi 1}^{(3)} + O_0 a_{\phi W} + 6O_1 a_{\phi \square}^{(3)}) (2 - 3L_R)\end{aligned}$$

$$\begin{aligned}\Delta_{\text{WW};1}^{(6)}(0) &= -\frac{1}{2} \sum_{\text{gen}} \frac{x_u^2}{x_u^2 - x_d^2} O_2 a_{\phi qW}^{(3)} a_0^{\text{fin}}(M_u) \\ &\quad - \frac{1}{2} \sum_{\text{gen}} \frac{x_d^2}{x_u^2 - x_d^2} O_3 a_{\phi qW}^{(3)} a_0^{\text{fin}}(M_d) + \frac{1}{2} \sum_{\text{gen}} x_1^2 a_{\phi 1W}^{(3)} a_0^{\text{fin}}(M_1) \\ &\quad - \frac{1}{6} \left[ 4M_{16}^c s^2 a_{AA} + 3M_{21}^c a_{\phi D} + 20M_{22}^c c^2 a_{ZZ} + 4M_{26}^c s c a_{AZ} \right. \\ &\quad \left. - (8a_{\phi W} - 2P_{12} a_{\phi D} + P_{13} a_{\phi \square}) \frac{s^2}{x_H^2 - 1} \right] \frac{1}{s^2} a_0^{\text{fin}}(M) \\ &\quad - \frac{1}{3} (a_{\phi WD}^{(-)} + 4a_{\phi \square}) \frac{x_H^2}{x_H^2 - 1} a_0^{\text{fin}}(M_H) \\ &\quad + \frac{1}{12} (8M_{17}^c s^2 a_{AA} - 24M_{18}^c c^2 a_{ZZ} - M_{20}^c a_{\phi D} - 4M_{23}^c s c a_{AZ}) \frac{1}{s^2 c^2} a_0^{\text{fin}}(M_0)\end{aligned}$$

$$\begin{aligned}\Delta_{\text{WW};2}^{(6)}(0) &= \frac{1}{2} \sum_{\text{gen}} O_4 a_{\phi qW}^{(3)} B_{0p}^{\text{fin}}(M_u, M_d) + \frac{1}{6} \sum_{\text{gen}} x_1^4 a_{\phi 1W}^{(3)} B_{0p}^{\text{fin}}(0, M_1) \\ &\quad - \frac{1}{3} \left[ M_{19}^c s c a_{AZ} + (-c^2 a_{\phi D} + 4s^2 a_{AA}) \right] B_{0p}^{\text{fin}}(0, M) \\ &\quad + \frac{1}{24} (a_{\phi WD}^{(-)} + 4a_{\phi \square}) \frac{1}{x_H^2 - 1} P_9 B_{0p}^{\text{fin}}(M, M_H) \\ &\quad + \frac{1}{24} (16s^2 c^4 a_{\phi B} - M_{14}^c a_{\phi D} - 4M_{24}^c s a_{\phi WA} - 4M_{25}^c c a_{\phi WZ}) \frac{s^4}{c^4} B_{0p}^{\text{fin}}(M, M_0)\end{aligned}$$

$$\Omega_{\text{WW};0}^{(4)}(0) = -\frac{4}{9} N_{\text{gen}} (1 - 3L_R) - \frac{1}{18} (2 + 57L_R)$$

$$\begin{aligned}\Omega_{\text{WW};1}^{(4)}(0) &= -\frac{1}{12} \frac{x_H^2}{x_H^2 - 1} a_0^{\text{fin}}(M_H) + \frac{1}{12} \frac{1}{s^2} M_{28}^c a_0^{\text{fin}}(M_0) \\ &\quad - \sum_{\text{gen}} \frac{x_u^2}{x_u^2 - x_d^2} a_0^{\text{fin}}(M_u) + \sum_{\text{gen}} \frac{x_d^2}{x_u^2 - x_d^2} a_0^{\text{fin}}(M_d) \\ &\quad - \frac{1}{3} \sum_{\text{gen}} a_0^{\text{fin}}(M_1) - \frac{1}{12} (39 + \frac{s^2}{x_H^2 - 1} P_{16}) \frac{1}{s^2} a_0^{\text{fin}}(M)\end{aligned}$$

$$\begin{aligned}\Omega_{\text{WW};2}^{(4)}(0) &= \frac{1}{24} \frac{s^4}{c^4} M_{14}^c B_{0s}^{\text{fin}}(M, M_0) + \frac{1}{6} \frac{1}{x_H^2 - 1} P_{15} B_{0p}^{\text{fin}}(M, M_H) \\ &\quad - \frac{1}{24} \frac{1}{x_H^2 - 1} P_9 B_{0s}^{\text{fin}}(M, M_H) - \frac{1}{6} \frac{1}{c^2} M_{27}^c B_{0p}^{\text{fin}}(M, M_0)\end{aligned}$$



$$\begin{aligned}
 & + \frac{4}{3} s^2 B_{0p}^{\text{fin}}(0, M) + \frac{1}{3} s^2 B_{0s}^{\text{fin}}(0, M) \\
 & + \frac{1}{2} \sum_{\text{gen}} O_1 B_{0p}^{\text{fin}}(M_u, M_d) + \frac{1}{6} \sum_{\text{gen}} x_1^2 B_{0p}^{\text{fin}}(0, M_1) \\
 & - \frac{1}{4} \sum_{\text{gen}} O_4 B_{0s}^{\text{fin}}(M_u, M_d) - \frac{1}{12} \sum_{\text{gen}} x_1^4 B_{0s}^{\text{fin}}(0, M_1)
 \end{aligned}$$

$$\begin{aligned}
 \Omega_{\text{WW};0}^{(6)}(0) &= -\frac{2}{9} s^2 a_{AA} (1 - 9L_R) - \frac{8}{9} N_{\text{gen}} a_{\phi W} (1 - 3L_R) \\
 & + \frac{1}{18} a_{\phi\Box} (2 + 3L_R) + \frac{2}{9} (-c a_{ZZ} + 2s a_{AZ}) c \\
 & + \frac{1}{6} \left( \frac{c^2}{x_H^2 - 1} P_{19} a_{\phi W} - 6M_{29}^c s c a_{\phi WB} + 3M_{30}^c a_{\phi W} \right) \frac{L_R}{c^2} \\
 & - \frac{4}{9} \sum_{\text{gen}} (a_{\phi 1}^{(3)} + 3a_{\phi q}^{(3)}) (1 - 3L_R) + \frac{1}{2} \sum_{\text{gen}} (3x_d^2 a_{dW} - 3x_u^2 a_{uW} + x_1^2 a_{1W}) L_R
 \end{aligned}$$

$$\begin{aligned}
 \Omega_{\text{WW};1}^{(6)}(0) &= -\frac{1}{24} \left[ 4P_{17} a_{\phi W} + (4a_{\phi\Box} - a_{\phi D}) \right] \frac{x_H^2}{x_H^2 - 1} a_0^{\text{fin}}(M_H) \\
 & - \frac{1}{24} \left[ 4M_{35}^c s c a_{AZ} + 4M_{38}^c s^2 a_{AA} - (M_{28}^c a_{\phi D} - 12M_{34}^c a_{ZZ}) c^2 \right] \frac{1}{s^2 c^2} a_0^{\text{fin}}(M_0) \\
 & - \frac{1}{24} \left[ 12M_{36}^c c^2 a_{ZZ} - 4M_{37}^c s^2 a_{AA} + 4M_{39}^c s c a_{AZ} + M_{40}^c a_{\phi D} \right. \\
 & \left. - (4a_{\phi\Box} - 4P_{20} a_{\phi W} - P_{21} a_{\phi D}) \frac{s^2}{x_H^2 - 1} \right] \frac{1}{s^2} a_0^{\text{fin}}(M) \\
 & - \frac{1}{2} \sum_{\text{gen}} (2a_{\phi qW}^{(3)} + 3x_d^2 a_{dW} - 3x_u^2 a_{uW}) \frac{x_u^2}{x_u^2 - x_d^2} a_0^{\text{fin}}(M_u) \\
 & + \frac{1}{2} \sum_{\text{gen}} (2a_{\phi qW}^{(3)} + 3x_d^2 a_{dW} - 3x_u^2 a_{uW}) \frac{x_d^2}{x_u^2 - x_d^2} a_0^{\text{fin}}(M_d) \\
 & - \frac{1}{6} \sum_{\text{gen}} (2a_{\phi 1W}^{(3)} + 3x_1^2 a_{1W}) a_0^{\text{fin}}(M_1)
 \end{aligned}$$

$$\begin{aligned}
 \Omega_{\text{WW};2}^{(6)}(0) &= -\frac{1}{4} \sum_{\text{gen}} O_4 a_{\phi qW}^{(3)} B_{0s}^{\text{fin}}(M_u, M_d) - \frac{1}{12} \sum_{\text{gen}} x_1^4 a_{\phi 1W}^{(3)} B_{0s}^{\text{fin}}(0, M_1) \\
 & + \frac{1}{12} \left[ 4s^2 c a_{AA} - 4M_{32}^c s a_{AZ} - (M_{27}^c a_{\phi D} + 12M_{31}^c a_{ZZ}) c \right] \frac{1}{c^3} B_{0p}^{\text{fin}}(M, M_0) \\
 & + \frac{1}{12} \left[ 4P_{18} a_{\phi W} + (4a_{\phi\Box} - a_{\phi D}) P_{15} \right] \frac{1}{x_H^2 - 1} B_{0p}^{\text{fin}}(M, M_H) \\
 & + \frac{1}{6} \left[ M_{19}^c s c a_{AZ} + (-c^2 a_{\phi D} + 4s^2 a_{AA}) \right] B_{0s}^{\text{fin}}(0, M) \\
 & + \frac{1}{3} \left[ M_{33}^c s c a_{AZ} + 2(-c^2 a_{\phi D} + 4s^2 a_{AA}) \right] B_{0p}^{\text{fin}}(0, M) \\
 & - \frac{1}{48} (a_{\phi WD}^{(-)} + 4a_{\phi\Box}) \frac{1}{x_H^2 - 1} P_9 B_{0s}^{\text{fin}}(M, M_H) \\
 & - \frac{1}{48} (16s^2 c^4 a_{\phi B} - M_{14}^c a_{\phi D} - 4M_{24}^c s a_{\phi WA} - 4M_{25}^c c a_{\phi WZ}) \frac{s^4}{c^4} B_{0s}^{\text{fin}}(M, M_0) \\
 & + \frac{1}{6} \sum_{\text{gen}} (a_{\phi 1W}^{(3)} + 3x_1^2 a_{1W}) x_1^2 B_{0p}^{\text{fin}}(0, M_1) \\
 & + \frac{1}{2} \sum_{\text{gen}} (O_1 a_{\phi qW}^{(3)} - 3O_1 x_d^2 a_{dW} - 3O_1 x_u^2 a_{uW}) B_{0p}^{\text{fin}}(M_u, M_d)
 \end{aligned}$$

## E Finite counterterms

In this appendix we present the list of finite counterterms for fields and parameters, as defined in section 4.12. It should be understood that only the real part of the loop functions has to be included, i.e.  $B_0 \equiv \text{Re } B_0$  etc.

$$\begin{aligned}
 d\mathcal{L}_{M_W}^{(4)} = & -\frac{1}{18} \left[ 3c^2 x_H^2 + (3 + 128c^2) \right] \frac{1}{c^2} - \frac{4}{9} (1 - 3L_R) N_{\text{gen}} \\
 & + \frac{1}{6} (6 - 7c^2) \frac{1}{c^2} L_R + \frac{1}{6} \sum_{\text{gen}} (3x_d^2 + 3x_u^2 + x_1^2) (2 - 3L_R) \\
 & + \frac{1}{2} \sum_{\text{gen}} (2 - x_d^2 + x_u^2) x_d^2 a_0^{\text{fin}}(M_b) + \frac{1}{2} \sum_{\text{gen}} (2 + x_d^2 - x_u^2) x_u^2 a_0^{\text{fin}}(M_t) \\
 & + \frac{1}{6} \sum_{\text{gen}} (2 - x_1^2) x_1^2 a_0^{\text{fin}}(M_1) - \frac{1}{12} \left[ c^2 x_H^2 + (1 + 66c^2) \right] \frac{1}{c^2} a_0^{\text{fin}}(M_W) \\
 & - \frac{1}{12} (3 - x_H^2) x_H^2 a_0^{\text{fin}}(M_H) + \frac{1}{12} (1 - 19c^2 + 24s^2 c^2) \frac{1}{c^4} a_0^{\text{fin}}(M_Z) \\
 & - 4s^2 B_0^{\text{fin}}(-M_W^2; 0, M_W) \\
 & + \frac{1}{6} \sum_{\text{gen}} \left[ 2 - (1 + x_1^2) x_1^2 \right] B_0^{\text{fin}}(-M_W^2; 0, M_1) \\
 & + \frac{1}{2} \sum_{\text{gen}} \left[ 2 - x_d^2 - x_u^2 - (x_u^2 - x_d^2)^2 \right] B_0^{\text{fin}}(-M_W^2; M_t, M_b) \\
 & + \frac{1}{12} \left[ 1 + 48s^2 c^4 + 4(4 - 29c^2) c^2 \right] \frac{1}{c^4} B_0^{\text{fin}}(-M_W^2; M_W, M_Z) \\
 & + \frac{1}{12} \left[ 12 - (4 - x_H^2) x_H^2 \right] B_0^{\text{fin}}(-M_W^2; M_W, M_H) \\
 \\
 d\mathcal{L}_{c_\theta}^{(4)} = & -\frac{1}{36} \left\{ -3 \sum_{\text{gen}} (1 - v_1^2) c^2 x_1^2 - 9 \sum_{\text{gen}} (1 - v_d^2) c^2 x_d^2 - 9 \sum_{\text{gen}} (1 - v_u^2) c^2 x_u^2 \right. \\
 & \left. + 2 \left[ 97 - 12(11 - 3s^2) s^2 \right] s^2 \right\} \frac{1}{c^2} \\
 & - \frac{1}{12} (19 - 18s^2) \frac{s^2}{c^2} L_R + \frac{1}{72} \left[ (9 - 16c^2) + (v_1^2 + 3v_d^2 + 3v_u^2) \right] \frac{1}{c^2} (1 - 3L_R) N_{\text{gen}} \\
 & - \frac{1}{12} \sum_{\text{gen}} \left[ x_1^2 - (1 - v_1^2) \right] x_1^2 a_0^{\text{fin}}(M_1) \\
 & + \frac{1}{4} \sum_{\text{gen}} \left[ (1 - v_d^2) - (x_d^2 - x_u^2) \right] x_d^2 a_0^{\text{fin}}(M_b) \\
 & + \frac{1}{4} \sum_{\text{gen}} \left[ (1 - v_u^2) + (x_d^2 - x_u^2) \right] x_u^2 a_0^{\text{fin}}(M_t) \\
 & + \frac{1}{24} \left[ c^4 x_H^2 + (1 - 18c^2 + 24s^2 c^2) \right] \frac{1}{c^4} a_0^{\text{fin}}(M_Z) \\
 & - \frac{1}{24} \left\{ c^2 x_H^2 + \left[ 1 + 48s^2 c^4 + 2(31 - 40c^2) c^2 \right] \right\} \frac{1}{c^2} a_0^{\text{fin}}(M_W) \\
 & - \frac{1}{24} (3 - x_H^2) x_H^2 a_0^{\text{fin}}(M_H) + \frac{1}{24} (3 - c^2 x_H^2) x_H^2 a_0^{\text{fin}}(M_H) \\
 & - 2s^2 B_0^{\text{fin}}(-M_W^2; 0, M_W) - \frac{1}{12} \frac{1}{c^2} \sum_{\text{gen}} B_0^{\text{fin}}(-M_Z^2; 0, 0) \\
 & + \frac{1}{12} \sum_{\text{gen}} \left[ 2 - (1 + x_1^2) x_1^2 \right] B_0^{\text{fin}}(-M_W^2; 0, M_1) \\
 & - \frac{1}{24} \sum_{\text{gen}} \left[ (1 + v_1^2) - 2(2 - v_1^2) c^2 x_1^2 \right] \frac{1}{c^2} B_0^{\text{fin}}(-M_Z^2; M_1, M_1)
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{1}{8} \sum_{\text{gen}} \left[ (1 + v_d^2) - 2(2 - v_d^2) c^2 x_d^2 \right] \frac{1}{c^2} \mathbf{B}_0^{\text{fin}} \left( -M_Z^2; M_b, M_b \right) \\
 & -\frac{1}{8} \sum_{\text{gen}} \left[ (1 + v_u^2) - 2(2 - v_u^2) c^2 x_u^2 \right] \frac{1}{c^2} \mathbf{B}_0^{\text{fin}} \left( -M_Z^2; M_t, M_t \right) \\
 & +\frac{1}{4} \sum_{\text{gen}} \left[ 2 - x_d^2 - x_u^2 - (x_u^2 - x_d^2)^2 \right] \mathbf{B}_0^{\text{fin}} \left( -M_W^2; M_t, M_b \right) \\
 & +\frac{1}{24} \left[ 1 + 48 s^2 c^4 + 4(4 - 29 c^2) c^2 \right] \frac{1}{c^4} \mathbf{B}_0^{\text{fin}} \left( -M_W^2; M_W, M_Z \right) \\
 & +\frac{1}{24} \left[ 12 - (4 - x_H^2) x_H^2 \right] \mathbf{B}_0^{\text{fin}} \left( -M_W^2; M_W, M_H \right) \\
 & -\frac{1}{24} \left\{ 1 + 4 \left[ 4 - (17 + 12 c^2) c^2 \right] c^2 \right\} \frac{1}{c^2} \mathbf{B}_0^{\text{fin}} \left( -M_Z^2; M_W, M_W \right) \\
 & -\frac{1}{24} (12 - 4 c^2 x_H^2 + c^4 x_H^4) \frac{1}{c^2} \mathbf{B}_0^{\text{fin}} \left( -M_Z^2; M_H, M_Z \right) \\
 d\mathcal{L}_g^{(4)} = & -\frac{1}{2} \delta_G^{(4)} - \frac{2}{9} (1 - 3 L_R) N_{\text{gen}} - \frac{1}{36} (2 + 57 L_R) \\
 & +\frac{1}{4} \sum_{\text{gen}} \left[ 2 \frac{x_d^2}{x_u^2 - x_d^2} + (1 - x_d^2 + x_u^2) \right] x_d^2 a_0^{\text{fin}}(M_b) \\
 & -\frac{1}{4} \sum_{\text{gen}} \left\{ 2 \frac{x_d^4}{x_u^2 - x_d^2} + \left[ 2 x_d^2 + (1 - x_d^2 + x_u^2) x_u^2 \right] \right\} a_0^{\text{fin}}(M_t) \\
 & -\frac{1}{12} \sum_{\text{gen}} (1 + x_1^2) x_1^2 a_0^{\text{fin}}(M_1) \\
 & +\frac{1}{24} \left[ 8 \frac{x_H^2}{x_H^2 - 1} - (11 - x_H^2) \right] x_H^2 a_0^{\text{fin}}(M_H) \\
 & +\frac{1}{24} \left\{ 1 + \left[ 16 - (69 + 8 c^2) c^2 \right] c^2 \right\} \frac{1}{s^2 c^4} a_0^{\text{fin}}(M_Z) \\
 & -\frac{1}{24} \left\{ -7 s^2 c^2 x_H^2 + 8 \frac{x_H^4}{x_H^2 - 1} s^2 c^2 + \left[ 1 + (13 - 74 c^2) c^2 \right] \right\} \frac{1}{s^2 c^2} a_0^{\text{fin}}(M_W) \\
 & -2 s^2 \mathbf{B}_0^{\text{fin}} \left( -M_W^2; 0, M_W \right) \\
 & +\frac{1}{12} \sum_{\text{gen}} \left[ 2 - (1 + x_1^2) x_1^2 \right] \mathbf{B}_0^{\text{fin}} \left( -M_W^2; 0, M_1 \right) \\
 & +\frac{1}{4} \sum_{\text{gen}} \left[ 2 - x_d^2 - x_u^2 - (x_u^2 - x_d^2)^2 \right] \mathbf{B}_0^{\text{fin}} \left( -M_W^2; M_t, M_b \right) \\
 & +\frac{1}{24} \left[ 1 + 48 s^2 c^4 + 4(4 - 29 c^2) c^2 \right] \frac{1}{c^4} \mathbf{B}_0^{\text{fin}} \left( -M_W^2; M_W, M_Z \right) \\
 & +\frac{1}{24} \left[ 12 - (4 - x_H^2) x_H^2 \right] \mathbf{B}_0^{\text{fin}} \left( -M_W^2; M_W, M_H \right) \\
 & -\frac{1}{12} \sum_{\text{gen}} x_1^4 \mathbf{B}_{0p}^{\text{fin}}(0; 0, M_1) + \frac{1}{3} s^2 \mathbf{B}_{0p}^{\text{fin}}(0; 0, M_W) \\
 & -\frac{1}{4} \sum_{\text{gen}} (-x_d^2 + x_u^2)^2 \mathbf{B}_{0p}^{\text{fin}}(0; M_t, M_b) \\
 & +\frac{1}{24} (-1 + x_H^2)^2 \mathbf{B}_{0p}^{\text{fin}}(0; M_W, M_H) \\
 & +\frac{1}{24} (9 - 8 s^2) \frac{s^4}{c^4} \mathbf{B}_{0p}^{\text{fin}}(0; M_W, M_Z) \\
 d\mathcal{L}_{M_H}^{(4)} = & \frac{1}{2} \left\{ 3 c^2 x_H^2 - \sum_{\text{gen}} \left[ 3(-x_H^2 + 4 x_d^2) x_d^2 + 3(-x_H^2 + 4 x_u^2) x_u^2 \right. \right. \\
 & \left. \left. + (-x_H^2 + 4 x_1^2) x_1^2 \right] \frac{1}{x_H^2} c^2 - (1 + 2 c^2) \right\} \frac{1}{c^2} L_R
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{1}{2}(1+2c^4)\frac{1}{c^4}\frac{1}{x_H^2}(2-3L_R) \\
 & +\frac{9}{8}x_H^2B_0^{\text{fin}}\left(-M_H^2;M_H,M_H\right) \\
 & -\frac{3}{2}\sum_{\text{gen}}(-x_H^2+4x_d^2)\frac{x_d^2}{x_H^2}B_0^{\text{fin}}\left(-M_H^2;M_b,M_b\right) \\
 & -\frac{3}{2}\sum_{\text{gen}}(-x_H^2+4x_u^2)\frac{x_u^2}{x_H^2}B_0^{\text{fin}}\left(-M_H^2;M_t,M_t\right) \\
 & -\frac{1}{2}\sum_{\text{gen}}(-x_H^2+4x_l^2)\frac{x_l^2}{x_H^2}B_0^{\text{fin}}\left(-M_H^2;M_l,M_l\right) \\
 & +\frac{1}{4}\left[12-(4-x_H^2)x_H^2\right]\frac{1}{x_H^2}B_0^{\text{fin}}\left(-M_H^2;M_W,M_W\right) \\
 & +\frac{1}{8}(-4c^2+c^4x_H^2+12\frac{1}{x_H^2})\frac{1}{c^4}B_0^{\text{fin}}\left(-M_H^2;M_Z,M_Z\right)
 \end{aligned}$$

$$\begin{aligned}
 d\mathcal{L}_{M_W}^{(6)} & =\frac{8}{9}a_{\phi W}(1-3L_R)N_{\text{gen}} \\
 & -\frac{1}{36}\left[3c^2a_{\phi W D}^{(-)}x_H^2+4(2+3x_H^2)c^2a_{\phi\Box}+4(3+128c^2)a_{\phi W}+3(9-8s^2)a_{\phi D}\right]\frac{1}{c^2} \\
 & +\frac{1}{12}\left\{20c^2a_{\phi\Box}+6\sum_{\text{gen}}c^2a_{1W}x_l^2+18\sum_{\text{gen}}c^2a_{dW}x_d^2-18\sum_{\text{gen}}c^2a_{uW}x_u^2\right. \\
 & \left.+2\left[3c^2x_H^2+(15+4c^2)\right]a_{\phi W}-3(5-6s^2)a_{\phi D}\right\}\frac{1}{c^2}L_R+\frac{1}{3}\frac{s}{c}a_{\phi WB}(10-3L_R) \\
 & +\frac{1}{6}\sum_{\text{gen}}\left[a_{\phi 1W}^{(3)}x_l^2+3(x_d^2+x_u^2)a_{\phi qW}^{(3)}\right](2-3L_R)-\frac{4}{9}\sum_{\text{gen}}(a_{\phi l}^{(3)}+3a_{\phi q}^{(3)})(1-3L_R) \\
 & -\frac{1}{6}\sum_{\text{gen}}\left[3a_{1W}x_l^2-(2-x_l^2)a_{\phi 1W}^{(3)}\right]x_l^2a_0^{\text{fin}}(M_l) \\
 & -\frac{1}{2}\sum_{\text{gen}}\left[3a_{dW}x_d^2+3a_{uW}x_u^2-(2-x_d^2+x_u^2)a_{\phi qW}^{(3)}\right]x_d^2a_0^{\text{fin}}(M_b) \\
 & +\frac{1}{2}\sum_{\text{gen}}\left[3a_{dW}x_d^2+3a_{uW}x_u^2+(2+x_d^2-x_u^2)a_{\phi qW}^{(3)}\right]x_u^2a_0^{\text{fin}}(M_t) \\
 & +\frac{1}{24}\left[a_{\phi W D}^{(-)}x_H^2-72a_{\phi W}+3a_{\phi D}-4(3-x_H^2)a_{\phi\Box}\right]x_H^2a_0^{\text{fin}}(M_H) \\
 & +\frac{1}{24}\left[-c^2a_{\phi W D}^{(-)}x_H^2+40sc a_{\phi WB}-4(1+60c^2)a_{\phi W}+4(2-x_H^2)c^2a_{\phi\Box}-(21-20s^2)a_{\phi D}\right]\frac{1}{c^2}a_0^{\text{fin}}(M_W) \\
 & +\frac{1}{24}\left[(1-13c^2+24s^2c^2)a_{\phi D}+4(1-8c^2)sa_{\phi WA}+4(11-32c^2)ca_{\phi WZ}\right]\frac{1}{c^4}a_0^{\text{fin}}(M_Z) \\
 & +\frac{1}{2}\sum_{\text{gen}}\left[3(1-x_d^2+x_u^2)a_{dW}x_d^2-3(1+x_d^2-x_u^2)a_{uW}x_u^2+(2-x_d^2-x_u^2-(x_u^2-x_d^2)^2)a_{\phi qW}^{(3)}\right] \\
 & \times B_0^{\text{fin}}\left(-M_W^2;M_t,M_b\right) \\
 & +\frac{1}{6}\sum_{\text{gen}}\left\{\left[2-(1+x_l^2)x_l^2\right]a_{\phi 1W}^{(3)}+3(1-x_l^2)a_{1W}x_l^2\right\}B_0^{\text{fin}}\left(-M_W^2;0,M_l\right) \\
 & +\frac{1}{24}\left\{a_{\phi W D}^{(-)}x_H^4+4\left[12-(4-x_H^2)x_H^2\right]a_{\phi\Box}-4(3-x_H^2)a_{\phi D}+16(9-4x_H^2)a_{\phi W}\right\}B_0^{\text{fin}}\left(-M_W^2;M_W,M_H\right) \\
 & +\frac{1}{24}\left\{\left[1+48s^2c^4+4(4-29c^2)c^2\right]a_{\phi D}+44\left[1-2(1+4c^2)c^2\right]ca_{\phi WZ}\right. \\
 & \left.+4\left[1+2(3-20c^2)c^2\right]sa_{\phi WA}\right\}\frac{1}{c^4}B_0^{\text{fin}}\left(-M_W^2;M_W,M_Z\right) \\
 & -2a_{\phi W AD}B_0^{\text{fin}}\left(-M_W^2;0,M_W\right)
 \end{aligned}$$

$$\begin{aligned}
 d\mathcal{L}_{c_\theta}^{(6)} = & \frac{1}{18} \frac{s^2}{c^2} a_{\phi\Box} (2 - 15L_R) \\
 & + \frac{1}{144} \left\{ 9a_{\phi\text{WD}}^{(-)} - 64c^2 a_{\phi\text{W}} + \left[ (1+4c^2)v_d + (3+4c^2)v_1 + (5+8c^2)v_u \right] a_{\phi\text{D}} \right. \\
 & - 4 \left[ (1+8c^2)v_d + (3+8c^2)v_1 + (5+16c^2)v_u \right] s a_{\phi\text{WA}} - 4 \left[ (5-8c^2)v_d + (7-8c^2)v_1 \right. \\
 & \left. + (13-16c^2)v_u \right] c a_{\phi\text{WZ}} + 64 \left[ (v_1 + v_d + 2v_u) \right] s^2 c^2 a_{\phi\text{B}} \left. \right\} \frac{1}{c^2} (1-3L_R) N_{\text{gen}} \\
 & + \frac{1}{72} \left\{ 3c^2 a_{\phi\text{WD}}^{(+)} x_{\text{H}}^2 - 3c^2 a_{\phi\text{WD}}^{(-)} x_{\text{H}}^2 + 6 \sum_{\text{gen}} c^2 a_{\phi\text{1W}}^{(3)} x_1^2 - 24 \sum_{\text{gen}} c^2 a_{\phi\text{1V}} x_1^2 v_1 - 72 \sum_{\text{gen}} c^2 v_d a_{\phi\text{dV}} x_{\text{d}}^2 \right. \\
 & - 72 \sum_{\text{gen}} c^2 v_u a_{\phi\text{uV}} x_{\text{u}}^2 + 12 \sum_{\text{gen}} \left[ (1+8c^2)v_d x_{\text{d}}^2 + (3+8c^2)x_1^2 v_1 + (5+16c^2)v_u x_{\text{u}}^2 \right] s^2 c^2 a_{\phi\text{WA}} \\
 & + 12 \sum_{\text{gen}} \left[ (5-8c^2)v_d x_{\text{d}}^2 + (7-8c^2)x_1^2 v_1 + (13-16c^2)v_u x_{\text{u}}^2 \right] c^3 a_{\phi\text{WZ}} + 18 \sum_{\text{gen}} (x_{\text{d}}^2 + x_{\text{u}}^2) c^2 a_{\phi\text{qW}}^{(3)} \\
 & - 192 \sum_{\text{gen}} (x_1^2 v_1 + v_d x_{\text{d}}^2 + 2v_u x_{\text{u}}^2) s^2 c^4 a_{\phi\text{B}} - 8 \left[ 1 + (31+36c^4)c^2 \right] s^2 a_{\text{AA}} \\
 & - 8 \left[ 17 + (43+36(1+c^2)c^2)c^2 \right] s c a_{\text{AZ}} - 8 \left[ 29 + 36(1-s^2)s^2 \right] s^2 c^2 a_{\text{ZZ}} \\
 & - \left[ 3 \sum_{\text{gen}} (1+4c^2)c^2 v_d x_{\text{d}}^2 + 3 \sum_{\text{gen}} (3+4c^2)c^2 x_1^2 v_1 \right. \\
 & \left. + 3 \sum_{\text{gen}} (5+8c^2)c^2 v_u x_{\text{u}}^2 - 2(52 - (149 - 12(11-3s^2)s^2)s^2) \right] a_{\phi\text{D}} \left. \right\} \frac{1}{c^2} \\
 & + \frac{1}{72} \sum_{\text{gen}} \left[ 4a_{\phi\text{V}} + 4a_{\phi\text{1A}} + 4a_{\phi\text{1V}} v_1 + 12a_{\phi\text{dA}} + 12a_{\phi\text{uA}} + 12v_d a_{\phi\text{dV}} + 12v_u a_{\phi\text{uV}} \right. \\
 & - 16c^2 a_{\phi\text{1}}^{(3)} - 48c^2 a_{\phi\text{q}}^{(3)} - 24c^2 a_{\phi\text{1}} x_1^2 - 72c^2 a_{\phi\text{d}} x_{\text{d}}^2 + 72c^2 a_{\phi\text{u}} x_{\text{u}}^2 + 24c^2 a_{\phi\text{1}}^{(1)} x_1^2 \\
 & \left. + 72(x_{\text{d}}^2 - x_{\text{u}}^2)c^2 a_{\phi\text{q}}^{(1)} + 3(3x_{\text{d}}^2 + 3x_{\text{u}}^2 + x_1^2)c^2 a_{\phi\text{D}} \right] \frac{1}{c^2} (1-3L_R) \\
 & + \frac{1}{48} \left\{ 12c^2 a_{\phi\text{W}} x_{\text{H}}^2 - 12 \sum_{\text{gen}} c a_{\text{1BW}} x_1^2 v_1 - 36 \sum_{\text{gen}} c v_d a_{\text{dBW}} x_{\text{d}}^2 + 36 \sum_{\text{gen}} c v_u a_{\text{uBW}} x_{\text{u}}^2 \right. \\
 & + 12 \sum_{\text{gen}} c^2 a_{\text{1W}} x_1^2 + 36 \sum_{\text{gen}} c^2 a_{\text{dW}} x_{\text{d}}^2 - 36 \sum_{\text{gen}} c^2 a_{\text{uW}} x_{\text{u}}^2 - 4 \left[ 7 + 4(14+3c^2)c^2 \right] s c a_{\text{AZ}} \\
 & - 4 \left[ 19 - 4(8-3s^2)s^2 \right] s^2 a_{\text{AA}} - 4 \left[ 3c^2 x_{\text{H}}^2 + (31-4(1+3s^2)s^2)s^2 \right] a_{\text{ZZ}} \\
 & \left. - \left[ 9c^2 x_{\text{H}}^2 + (25+4(4-9s^2)s^2) \right] a_{\phi\text{D}} \right\} \frac{1}{c^2} L_R \\
 & + \frac{1}{24} \sum_{\text{gen}} \left\{ -8a_{\phi\text{1V}} v_1 - 6a_{\text{1W}} x_1^2 - 8a_{\phi\text{1}} + 8a_{\phi\text{1}}^{(1)} - 64s^2 c^2 a_{\phi\text{B}} v_1 + \left[ 1 - (3+4c^2)v_1 \right] a_{\phi\text{D}} \right. \\
 & + 2(1-x_1^2)a_{\phi\text{1W}}^{(3)} + 4(3+8c^2)s a_{\phi\text{WA}} v_1 + 4(7-8c^2)c a_{\phi\text{WZ}} v_1 \left. \right\} x_1^2 a_0^{\text{fin}}(M_1) \\
 & + \frac{1}{24} \sum_{\text{gen}} \left\{ -18a_{\text{dW}} x_{\text{d}}^2 - 18a_{\text{uW}} x_{\text{u}}^2 - 24a_{\phi\text{d}} + 24a_{\phi\text{q}}^{(1)} - 24v_d a_{\phi\text{dV}} - 64s^2 c^2 v_d a_{\phi\text{B}} \right. \\
 & + \left[ 3 - (1+4c^2)v_d \right] a_{\phi\text{D}} + 6(1-x_{\text{d}}^2 + x_{\text{u}}^2)a_{\phi\text{qW}}^{(3)} + 4(1+8c^2)s v_d a_{\phi\text{WA}} \\
 & \left. + 4(5-8c^2)c v_d a_{\phi\text{WZ}} \right\} x_{\text{d}}^2 a_0^{\text{fin}}(M_b) \\
 & - \frac{1}{24} \sum_{\text{gen}} \left\{ -18a_{\text{dW}} x_{\text{d}}^2 - 18a_{\text{uW}} x_{\text{u}}^2 - 24a_{\phi\text{u}} + 24a_{\phi\text{q}}^{(1)} + 24v_u a_{\phi\text{uV}} + 128s^2 c^2 v_u a_{\phi\text{B}} \right. \\
 & - \left[ 3 - (5+8c^2)v_u \right] a_{\phi\text{D}} - 6(1+x_{\text{d}}^2 - x_{\text{u}}^2)a_{\phi\text{qW}}^{(3)} - 4(5+16c^2)s v_u a_{\phi\text{WA}} \\
 & \left. - 4(13-16c^2)c v_u a_{\phi\text{WZ}} \right\} x_{\text{u}}^2 a_0^{\text{fin}}(M_t) \\
 & + \frac{1}{48} \left[ a_{\phi\text{WD}}^{(-)} x_{\text{H}}^2 - 72a_{\phi\text{W}} + 3a_{\phi\text{D}} - 4(3-x_{\text{H}}^2)a_{\phi\Box} \right] x_{\text{H}}^2 a_0^{\text{fin}}(M_{\text{H}})
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{48} \left[ 12a_{\phi_D} - c^2 a_{\phi_{WD}}^{(+)} x_H^2 + 12sca_{AZ} + 12s^2 a_{AA} + 4(3 - c^2 x_H^2) a_{\phi_{\square}} + 12(5 + c^2) a_{ZZ} \right] x_H^2 a_0^{\text{fin}}(M_H) \\
 & - \frac{1}{48} \left\{ c^2 a_{\phi_{WD}}^{(-)} x_H^2 + 4 \left[ 11 + 8(8 + 3(1 + 2c^2)c^2)c^2 \right] sca_{AZ} \right. \\
 & + 4 \left[ 15 + 4(7 - 6(5 - 2c^2)c^2)c^2 \right] c^2 a_{ZZ} + 4 \left[ 61 - 12(11 - 2(5 - 2s^2)s^2)s^2 \right] s^2 a_{AA} \\
 & \left. - \left[ 63 - 16(12 - (11 - 3s^2)s^2)s^2 \right] a_{\phi_D} - 4(2 - x_H^2)c^2 a_{\phi_{\square}} \right\} \frac{1}{c^2} a_0^{\text{fin}}(M_W) \\
 & + \frac{1}{48} \left\{ c^4 a_{\phi_{WD}}^{(+)} x_H^2 + 4 \left[ 8 - (31 - 24s^2)s^2 \right] s^2 a_{AA} + 4 \left[ 11 - (41 - 24c^2)c^2 \right] sca_{AZ} \right. \\
 & + 4 \left[ 12 - (65 - 24c^2)c^2 \right] c^2 a_{ZZ} + (1 - 15c^2 + 24s^2c^2) a_{\phi_D} - 4(2 - c^2 x_H^2)c^2 a_{\phi_{\square}} \left. \right\} \frac{1}{c^4} a_0^{\text{fin}}(M_Z) \\
 & + \frac{1}{4} \sum_{\text{gen}} \left[ 3(1 - x_d^2 + x_u^2) a_{dW} x_d^2 - 3(1 + x_d^2 - x_u^2) a_{uW} x_u^2 + (2 - x_d^2 - x_u^2 - (x_u^2 - x_d^2)^2) a_{\phi_{qW}}^{(3)} \right] \\
 & \times B_0^{\text{fin}}(-M_W^2; M_t, M_b) \\
 & - \frac{1}{48} \sum_{\text{gen}} \left\{ 12ca_{1BW} x_1^2 v_1 + \left[ (3 + 4c^2) + 2(3 + 4c^2)c^2 x_1^2 \right] a_{\phi_D} v_1 \right. \\
 & - 4 \left[ (3 + 8c^2) + 2(3 + 8c^2)c^2 x_1^2 \right] sa_{\phi_{WA}} v_1 - 4 \left[ (7 - 8c^2) + 2(7 - 8c^2)c^2 x_1^2 \right] ca_{\phi_{WZ}} v_1 \\
 & + (1 - 4c^2 x_1^2) a_{\phi_{WD}}^{(-)} + 8(1 - 4c^2 x_1^2) a_{\phi_{1A}} + 8(1 + 2c^2 x_1^2) a_{\phi_{1V}} v_1 \\
 & \left. + 64(1 + 2c^2 x_1^2) s^2 c^2 a_{\phi_B} v_1 \right\} \frac{1}{c^2} B_0^{\text{fin}}(-M_Z^2; M_1, M_1) \\
 & - \frac{1}{48} \sum_{\text{gen}} \left\{ 36cv_d a_{dBW} x_d^2 + \left[ (1 + 4c^2) + 2(1 + 4c^2)c^2 x_d^2 \right] v_d a_{\phi_D} - 4 \left[ (1 + 8c^2) \right. \right. \\
 & \left. \left. + 2(1 + 8c^2)c^2 x_d^2 \right] sv_d a_{\phi_{WA}} - 4 \left[ (5 - 8c^2) + 2(5 - 8c^2)c^2 x_d^2 \right] cv_d a_{\phi_{WZ}} + 3(1 - 4c^2 x_d^2) a_{\phi_{WD}}^{(-)} \right. \\
 & \left. + 24(1 - 4c^2 x_d^2) a_{\phi_{dA}} + 24(1 + 2c^2 x_d^2) v_d a_{\phi_{dV}} + 64(1 + 2c^2 x_d^2) s^2 c^2 v_d a_{\phi_B} \right\} \frac{1}{c^2} \\
 & \times B_0^{\text{fin}}(-M_Z^2; M_b, M_b) \\
 & + \frac{1}{48} \sum_{\text{gen}} \left\{ 36cv_u a_{uBW} x_u^2 - \left[ (5 + 8c^2) + 2(5 + 8c^2)c^2 x_u^2 \right] v_u a_{\phi_D} + 4 \left[ (5 + 16c^2) \right. \right. \\
 & \left. \left. + 2(5 + 16c^2)c^2 x_u^2 \right] sv_u a_{\phi_{WA}} + 4 \left[ (13 - 16c^2) + 2(13 - 16c^2)c^2 x_u^2 \right] cv_u a_{\phi_{WZ}} \right. \\
 & \left. - 3(1 - 4c^2 x_u^2) a_{\phi_{WD}}^{(-)} - 24(1 - 4c^2 x_u^2) a_{\phi_{uA}} - 24(1 + 2c^2 x_u^2) v_u a_{\phi_{uV}} - 128(1 + 2c^2 x_u^2) s^2 c^2 v_u a_{\phi_B} \right\} \frac{1}{c^2} \\
 & \times B_0^{\text{fin}}(-M_Z^2; M_t, M_t) \\
 & + \frac{1}{12} \sum_{\text{gen}} \left\{ \left[ 2 - (1 + x_1^2)x_1^2 \right] a_{\phi_{1W}}^{(3)} + 3(1 - x_1^2) a_{1W} x_1^2 \right\} B_0^{\text{fin}}(-M_W^2; 0, M_1) \\
 & - \frac{1}{24} \sum_{\text{gen}} (a_{\phi_{WD}}^{(-)} + 4a_{\phi_V}) \frac{1}{c^2} B_0^{\text{fin}}(-M_Z^2; 0, 0) \\
 & - \frac{1}{48} \left[ 48(2 - c^2 x_H^2) a_{ZZ} + (12 - 4c^2 x_H^2 + c^4 x_H^4) a_{\phi_{WD}}^{(+)} + 4(12 - 4c^2 x_H^2 + c^4 x_H^4) a_{\phi_{\square}} \right] \frac{1}{c^2} \\
 & \times B_0^{\text{fin}}(-M_Z^2; M_H, M_Z) \\
 & + \frac{1}{48} \left\{ a_{\phi_{WD}}^{(-)} x_H^4 + 4 \left[ 12 - (4 - x_H^2)x_H^2 \right] a_{\phi_{\square}} - 4(3 - x_H^2) a_{\phi_D} + 16(9 - 4x_H^2) a_{\phi_W} \right\} B_0^{\text{fin}}(-M_W^2; M_W, M_H) \\
 & + \frac{1}{48} \left\{ \left[ 1 + 48s^2c^4 + 4(4 - 29c^2)c^2 \right] a_{\phi_D} + 44 \left[ 1 - 2(1 + 4c^2)c^2 \right] ca_{\phi_{WZ}} \right. \\
 & \left. + 4 \left[ 1 + 2(3 - 20c^2)c^2 \right] sa_{\phi_{WA}} \right\} \frac{1}{c^4} B_0^{\text{fin}}(-M_W^2; M_W, M_Z) \\
 & - \frac{1}{48} \left\{ \left[ 1 + 4(4 - (17 + 12c^2)c^2)c^2 \right] a_{\phi_D} + 12 \left[ 5 + 4(6 + (3 + 4c^2)c^2)c^2 \right] sca_{AZ} \right.
 \end{aligned}$$

$$\begin{aligned}
 & +12 \left[ 7 - 4(5 + (9 - 4c^2)c^2) \right] c^2 a_{ZZ} + 4 \left[ 33 - 4(29 - 3(11 - 4s^2)s^2) \right] s^2 a_{AA} \left\} \frac{1}{c^2} \right. \\
 & \times \mathbf{B}_0^{\text{fin}} \left( -M_Z^2; M_W, M_W \right) \\
 & - a_{\phi_{\text{WAD}}} \mathbf{B}_0^{\text{fin}} \left( -M_W^2; 0, M_W \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{d}\mathcal{Z}_g^{(6)} = & -\frac{1}{9} (c^2 a_{ZZ} - 2sc a_{AZ} + s^2 a_{AA}) + \frac{1}{36} a_{\phi_{\square}} (2 + 3L_R) \\
 & -\frac{4}{9} a_{\phi_W} (1 - 3L_R) N_{\text{gen}} - \frac{2}{9} \sum_{\text{gen}} (a_{\phi_1}^{(3)} + 3a_{\phi_q}^{(3)}) (1 - 3L_R) \\
 & + \frac{1}{12} \left[ 3c^2 a_{\phi_W} x_H^2 + 3 \sum_{\text{gen}} c^2 a_{1W} x_1^2 + 9 \sum_{\text{gen}} c^2 a_{dW} x_d^2 - 9 \sum_{\text{gen}} c^2 a_{uW} x_u^2 + (9 - 38c^2) sc a_{AZ} \right. \\
 & \left. + (15 - 32c^2) c^2 a_{ZZ} - (29 - 32s^2) s^2 a_{AA} \right] \frac{1}{c^2} L_R \\
 & - \frac{1}{12} \sum_{\text{gen}} \left[ 3a_{1W} x_1^2 + (1 + x_1^2) a_{\phi_{1W}}^{(3)} \right] x_1^2 a_0^{\text{fin}} (M_1) \\
 & + \frac{1}{4} \sum_{\text{gen}} \left[ 3(1 - x_d^2 + x_u^2) a_{dW} x_d^2 - 3(1 + x_d^2 - x_u^2) a_{uW} x_u^2 + (2 - x_d^2 - x_u^2 - (x_u^2 - x_d^2)^2) a_{\phi_{qW}}^{(3)} \right] \\
 & \times \mathbf{B}_0^{\text{fin}} \left( -M_W^2; M_t, M_b \right) \\
 & - \frac{1}{4} \sum_{\text{gen}} \left\{ 3a_{dW} x_d^2 + 3a_{uW} x_u^2 - \left[ 2 \frac{x_d^2}{x_u^2 - x_d^2} + (1 - x_d^2 + x_u^2) \right] a_{\phi_{qW}}^{(3)} \right\} x_d^2 a_0^{\text{fin}} (M_b) \\
 & + \frac{1}{4} \sum_{\text{gen}} \left\{ 3a_{dW} x_u^2 x_d^2 + 3a_{uW} x_u^4 - \left[ 2 \frac{x_d^4}{x_u^2 - x_d^2} + (2x_d^2 + (1 - x_d^2 + x_u^2)x_u^2) \right] a_{\phi_{qW}}^{(3)} \right\} a_0^{\text{fin}} (M_t) \\
 & + \frac{1}{48} \left\{ -104a_{\phi_W} + 11a_{\phi_D} + 4 \left[ 8 \frac{x_H^2}{x_H^2 - 1} - (11 - x_H^2) \right] a_{\phi_{\square}} + (1 + 8 \frac{1}{x_H^2 - 1}) a_{\phi_{\text{WD}}}^{(-)} x_H^2 \right\} x_H^2 a_0^{\text{fin}} (M_H) \\
 & - \frac{1}{48} \left\{ \left[ 1 + (27 - 88c^2)c^2 \right] a_{\phi_D} - 4 \left[ 4 + (3 - 8s^2)s^2 \right] s^2 a_{AA} + 4 \left[ 11 - 3(17 - 4c^2)c^2 \right] sc a_{AZ} \right. \\
 & \left. + 4 \left[ 21 - (89 - 8c^2)c^2 \right] c^2 a_{ZZ} + 4 \left[ 8 \frac{x_H^4}{x_H^2 - 1} - (11 + 7x_H^2) \right] s^2 c^2 a_{\phi_{\square}} \right. \\
 & \left. - (7 - 8 \frac{x_H^2}{x_H^2 - 1}) s^2 c^2 a_{\phi_{\text{WD}}}^{(-)} x_H^2 \right\} \frac{1}{s^2 c^2} a_0^{\text{fin}} (M_W) \\
 & + \frac{1}{48} \left\{ \left[ 1 + (16 - (69 + 8c^2)c^2)c^2 \right] a_{\phi_D} - 4 \left[ 4 - (3 + 2s^2)s^2 \right] s^2 a_{AA} + 12 \left[ 7 - (25 + 2c^2)c^2 \right] c^2 a_{ZZ} \right. \\
 & \left. + 4 \left[ 11 - (41 - 2(2 - c^2)c^2)c^2 \right] sc a_{AZ} \right\} \frac{1}{s^2 c^4} a_0^{\text{fin}} (M_Z) \\
 & + \frac{1}{12} \sum_{\text{gen}} \left\{ \left[ 2 - (1 + x_1^2)x_1^2 \right] a_{\phi_{1W}}^{(3)} + 3(1 - x_1^2) a_{1W} x_1^2 \right\} \mathbf{B}_0^{\text{fin}} \left( -M_W^2; 0, M_1 \right) \\
 & + \frac{1}{48} \left\{ a_{\phi_{\text{WD}}}^{(-)} x_H^4 + 4 \left[ 12 - (4 - x_H^2)x_H^2 \right] a_{\phi_{\square}} - 4(3 - x_H^2) a_{\phi_D} + 16(9 - 4x_H^2) a_{\phi_W} \right\} \mathbf{B}_0^{\text{fin}} \left( -M_W^2; M_W, M_H \right) \\
 & + \frac{1}{48} \left\{ \left[ 1 + 48s^2 c^4 + 4(4 - 29c^2)c^2 \right] a_{\phi_D} + 44 \left[ 1 - 2(1 + 4c^2)c^2 \right] c a_{\phi_{\text{WZ}}} \right. \\
 & \left. + 4 \left[ 1 + 2(3 - 20c^2)c^2 \right] s a_{\phi_{\text{WA}}} \right\} \frac{1}{c^4} \mathbf{B}_0^{\text{fin}} \left( -M_W^2; M_W, M_Z \right) \\
 & - a_{\phi_{\text{WAD}}} \mathbf{B}_0^{\text{fin}} \left( -M_W^2; 0, M_W \right) \\
 & - \frac{1}{4} \sum_{\text{gen}} (-x_d^2 + x_u^2)^2 a_{\phi_{qW}}^{(3)} \mathbf{B}_{0p}^{\text{fin}} (0; M_t, M_b) \\
 & - \frac{1}{12} \sum_{\text{gen}} a_{\phi_{1W}}^{(3)} x_1^4 \mathbf{B}_{0p}^{\text{fin}} (0; 0, M_1)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{6} \left[ -c^2 a_{\phi D} + 4s^2 a_{\Lambda\Lambda} + (2-c^2) s c a_{\Lambda Z} \right] B_{0p}^{\text{fin}}(0; 0, M_W) \\
 & + \frac{1}{48} (x_H^2 - 1)^2 (a_{\phi WD}^{(-)} + 4a_{\phi\Box}) B_{0p}^{\text{fin}}(0; M_W, M_H) \\
 & - \frac{1}{48} \left\{ 16s^2 c^4 a_{\phi B} - 4 \left[ 7 - 2(4-s^2)s^2 \right] s a_{\phi WA} - 4 \left[ 9 - 2(1+s^2)s^2 \right] c a_{\phi WZ} \right. \\
 & \left. - (9-8s^2) a_{\phi D} \right\} \frac{s^4}{c^4} B_{0p}^{\text{fin}}(0; M_W, M_Z)
 \end{aligned}$$

$$\begin{aligned}
 d\mathcal{Z}_{M_H}^{(6)} &= 4a_{\phi W} \\
 & - \frac{1}{4} \left[ 4(1+2c^4) \frac{1}{x_H^2} a_{\phi\Box} + 4(1+10c^4) \frac{1}{x_H^2} a_{\phi W} + (3-2c^4) \frac{1}{x_H^2} a_{\phi D} + 4(-c^2+4\frac{1}{x_H^2}) a_{ZZ} \right] \frac{1}{c^4} (2-3L_R) \\
 & + \frac{1}{8} \left\{ 192 \frac{x_d^2}{x_H^2} \sum_{\text{gen}} c^2 a_{d\phi} x_d^2 - 192 \frac{x_u^2}{x_H^2} \sum_{\text{gen}} c^2 a_{u\phi} x_u^2 + 64 \frac{x_1^2}{x_H^2} \sum_{\text{gen}} c^2 a_{L\phi} x_1^2 \right. \\
 & - 2 \sum_{\text{gen}} \left[ 3(-x_H^2+4x_d^2) x_d^2 + 3(-x_H^2+4x_u^2) x_u^2 + (-x_H^2+4x_1^2) x_1^2 \right] \frac{1}{x_H^2} c^2 a_{\phi WD}^{(-)} \\
 & - 8 \sum_{\text{gen}} (a_{1\phi\Box} x_1^2 + 3a_{d\phi\Box} x_d^2 - 3a_{u\phi\Box} x_u^2) c^2 - 24 \left[ 11c^2 + (1+2c^2) \frac{1}{x_H^2} \right] a_\phi + 8 \left[ 3c^2 x_H^2 - (1+8c^2) \right] a_{\phi W} \\
 & + 8 \left[ 11c^2 x_H^2 - 4 \sum_{\text{gen}} (3x_d^4 + 3x_u^4 + x_1^4) \frac{1}{x_H^2} c^2 - (1+2c^2) \right] a_{\phi\Box} \\
 & - \left[ 21c^2 x_H^2 + (5-4c^2) \right] a_{\phi D} \left. \right\} \frac{1}{c^2} L_R \\
 & - 12 \frac{x_d^2}{x_H^2} \sum_{\text{gen}} a_{d\phi} x_d^2 a_0^{\text{fin}}(M_b) + 12 \frac{x_u^2}{x_H^2} \sum_{\text{gen}} a_{u\phi} x_u^2 a_0^{\text{fin}}(M_t) - 4 \frac{x_1^2}{x_H^2} \sum_{\text{gen}} a_{L\phi} x_1^2 a_0^{\text{fin}}(M_1) \\
 & - \frac{3}{8} \left[ 16 \frac{1}{x_H^2} a_{ZZ} - 8 \frac{1}{x_H^2} c^2 a_\phi + (-c^2+4\frac{1}{x_H^2}) a_{\phi D} \right] \frac{1}{c^4} a_0^{\text{fin}}(M_Z) \\
 & + 6(-2a_{\phi W} + a_\phi) \frac{1}{x_H^2} a_0^{\text{fin}}(M_W) + \frac{1}{8} (7a_{\phi D} x_H^2 - 28a_{\phi\Box} x_H^2 + 120a_\phi) a_0^{\text{fin}}(M_H) \\
 & - \frac{3}{4} \sum_{\text{gen}} \left\{ -4 \left[ (-x_H^2+4x_d^2) a_{d\phi\Box} \right] + (-x_H^2+4x_d^2) a_{\phi WD}^{(-)} \right\} \frac{x_d^2}{x_H^2} B_0^{\text{fin}}(-M_H^2; M_b, M_b) \\
 & - \frac{3}{4} \sum_{\text{gen}} \left\{ 4 \left[ (-x_H^2+4x_u^2) a_{u\phi\Box} \right] + (-x_H^2+4x_u^2) a_{\phi WD}^{(-)} \right\} \frac{x_u^2}{x_H^2} B_0^{\text{fin}}(-M_H^2; M_t, M_t) \\
 & - \frac{1}{4} \sum_{\text{gen}} \left\{ -4 \left[ (-x_H^2+4x_1^2) a_{1\phi\Box} \right] + (-x_H^2+4x_1^2) a_{\phi WD}^{(-)} \right\} \frac{x_1^2}{x_H^2} B_0^{\text{fin}}(-M_H^2; M_1, M_1) \\
 & + \frac{1}{16} \left[ (-4c^2+c^4 x_H^2 + 12\frac{1}{x_H^2}) a_{\phi WD}^{(+)} + 4(-4c^2+c^4 x_H^2 + 12\frac{1}{x_H^2}) a_{\phi\Box} + 48(-c^2+2\frac{1}{x_H^2}) a_{ZZ} \right] \frac{1}{c^4} \\
 & \times B_0^{\text{fin}}(-M_H^2; M_Z, M_Z) \\
 & + \frac{1}{8} \left\{ a_{\phi WD}^{(-)} x_H^2 + 4 \left[ 12 - (4-x_H^2) x_H^2 \right] \frac{1}{x_H^2} a_{\phi\Box} - 4(3-x_H^2) \frac{1}{x_H^2} a_{\phi D} + 16(9-4x_H^2) \frac{1}{x_H^2} a_{\phi W} \right\} \\
 & \times B_0^{\text{fin}}(-M_H^2; M_W, M_W) \\
 & - \frac{9}{16} (-4a_{\phi W} x_H^2 + 3a_{\phi D} x_H^2 - 12a_{\phi\Box} x_H^2 + 32a_\phi) B_0^{\text{fin}}(-M_H^2; M_H, M_H)
 \end{aligned}$$

## F Wave-function factors

In this appendix we present the full list of wave-function renormalization factors for H,Z and W fields. For the W factor we present only the IR finite part.



$$\begin{aligned}
 W_H^{(4)} = & -\frac{1}{2} \left[ -\sum_{\text{gen}} (3x_d^2 + 3x_u^2 + x_l^2) c^2 + (1 + 2c^2) \right] \frac{1}{c^2} L_R \\
 & + \frac{3}{2} \sum_{\text{gen}} x_d^2 B_0^{\text{fin}}(-M_H^2; M_b, M_b) + \frac{3}{2} \sum_{\text{gen}} x_u^2 B_0^{\text{fin}}(-M_H^2; M_t, M_t) \\
 & + \frac{1}{2} \sum_{\text{gen}} x_l^2 B_0^{\text{fin}}(-M_H^2; M_1, M_1) - \frac{1}{2} \frac{1}{c^2} B_0^{\text{fin}}(-M_H^2; M_Z, M_Z) \\
 & - B_0^{\text{fin}}(-M_H^2; M_W, M_W) + \frac{9}{8} x_H^4 B_{0p}^{\text{fin}}(-M_H^2; M_H, M_H) \\
 & - \frac{3}{2} \sum_{\text{gen}} (-x_H^2 + 4x_d^2) x_d^2 B_{0p}^{\text{fin}}(-M_H^2; M_b, M_b) \\
 & - \frac{3}{2} \sum_{\text{gen}} (-x_H^2 + 4x_u^2) x_u^2 B_{0p}^{\text{fin}}(-M_H^2; M_t, M_t) \\
 & - \frac{1}{2} \sum_{\text{gen}} (-x_H^2 + 4x_l^2) x_l^2 B_{0p}^{\text{fin}}(-M_H^2; M_1, M_1) \\
 & + \frac{1}{4} [12 - (4 - x_H^2) x_H^2] B_{0p}^{\text{fin}}(-M_H^2; M_W, M_W) \\
 & + \frac{1}{8} (12 - 4c^2 x_H^2 + c^4 x_H^4) \frac{1}{c^4} B_{0p}^{\text{fin}}(-M_H^2; M_Z, M_Z)
 \end{aligned}$$

$$\begin{aligned}
 W_Z^{(4)} = & \frac{1}{9} (1 - 2c^2) \frac{1}{c^2} - \frac{1}{36} [(9 + v_l^2 + 3v_d^2 + 3v_u^2)] \frac{1}{c^2} (1 - 3L_R) N_{\text{gen}} \\
 & + \frac{1}{6} (1 - 20c^2 + 18s^2 c^2) \frac{1}{c^2} L_R \\
 & + \frac{1}{12} (1 - c^2 x_H^2) x_H^2 a_0^{\text{fin}}(M_H) - \frac{1}{12} (1 - c^2 x_H^2) \frac{1}{c^2} a_0^{\text{fin}}(M_Z) \\
 & + \frac{1}{12} \sum_{\text{gen}} [(1 + v_l^2)] \frac{1}{c^2} B_0^{\text{fin}}(-M_Z^2; M_1, M_1) \\
 & + \frac{1}{6} \frac{1}{c^2} \sum_{\text{gen}} B_0^{\text{fin}}(-M_Z^2; 0, 0) \\
 & + \frac{1}{4} \sum_{\text{gen}} [(1 + v_d^2)] \frac{1}{c^2} B_0^{\text{fin}}(-M_Z^2; M_b, M_b) \\
 & + \frac{1}{4} \sum_{\text{gen}} [(1 + v_u^2)] \frac{1}{c^2} B_0^{\text{fin}}(-M_Z^2; M_t, M_t) \\
 & + \frac{1}{12} (1 - 40c^2 + 36s^2 c^2) \frac{1}{c^2} B_0^{\text{fin}}(-M_Z^2; M_W, M_W) \\
 & + \frac{1}{6} \frac{1}{c^4} \sum_{\text{gen}} B_{0p}^{\text{fin}}(-M_Z^2; 0, 0) \\
 & + \frac{1}{12} \sum_{\text{gen}} [(1 + v_l^2) - 2(2 - v_l^2) c^2 x_l^2] \frac{1}{c^4} B_{0p}^{\text{fin}}(-M_Z^2; M_1, M_1) \\
 & + \frac{1}{4} \sum_{\text{gen}} [(1 + v_d^2) - 2(2 - v_d^2) c^2 x_d^2] \frac{1}{c^4} B_{0p}^{\text{fin}}(-M_Z^2; M_b, M_b) \\
 & + \frac{1}{4} \sum_{\text{gen}} [(1 + v_u^2) - 2(2 - v_u^2) c^2 x_u^2] \frac{1}{c^4} B_{0p}^{\text{fin}}(-M_Z^2; M_t, M_t) \\
 & + \frac{1}{12} \left\{ 1 + 4 [4 - (17 + 12c^2) c^2] c^2 \right\} \frac{1}{c^4} B_{0p}^{\text{fin}}(-M_Z^2; M_W, M_W) \\
 & + \frac{1}{12} (2 - c^2 x_H^2) x_H^2 B_0^{\text{fin}}(-M_Z^2; M_H, M_Z) \\
 & + \frac{1}{12} (12 - 4c^2 x_H^2 + c^4 x_H^4) \frac{1}{c^4} B_{0p}^{\text{fin}}(-M_Z^2; M_H, M_Z)
 \end{aligned}$$

$$\begin{aligned}
 W_W^{(4)} = & -\frac{1}{9}(1-36s^2) - \frac{19}{6}L_R - \frac{4}{9}(1-3L_R)N_{\text{gen}} \\
 & + \frac{1}{6} \sum_{\text{gen}} x_1^4 a_0^{\text{fin}}(M_1) + \frac{1}{2} \sum_{\text{gen}} (x_d^2 - x_u^2) x_d^2 a_0^{\text{fin}}(M_b) \\
 & - \frac{1}{2} \sum_{\text{gen}} (x_d^2 - x_u^2) x_u^2 a_0^{\text{fin}}(M_t) + \frac{1}{12} [c^2 x_H^2 + (1-2c^2)] \frac{1}{c^2} a_0^{\text{fin}}(M_W) \\
 & + \frac{1}{12} (1-x_H^2) x_H^2 a_0^{\text{fin}}(M_H) - \frac{1}{12} (9-8s^2) \frac{s^2}{c^4} a_0^{\text{fin}}(M_Z) \\
 & - 4s^2 B_0^{\text{fin}}(-M_W^2; 0, M_W) \\
 & + \frac{1}{6} \sum_{\text{gen}} (2+x_1^4) B_0^{\text{fin}}(-M_W^2; 0, M_1) \\
 & + \frac{1}{2} \sum_{\text{gen}} (2+(x_u^2-x_d^2)^2) B_0^{\text{fin}}(-M_W^2; M_t, M_b) \\
 & - \frac{1}{12} [1-48s^2c^4+2(3+16c^2)c^2] \frac{1}{c^4} B_0^{\text{fin}}(-M_W^2; M_W, M_Z) \\
 & + \frac{1}{12} (2-x_H^2) x_H^2 B_0^{\text{fin}}(-M_W^2; M_W, M_H) \\
 & + \frac{1}{6} \sum_{\text{gen}} [2-(1+x_1^2)x_1^2] B_{0p}^{\text{fin}}(-M_W^2; 0, M_1) \\
 & + \frac{1}{2} \sum_{\text{gen}} (2-x_d^2-x_u^2-(x_u^2-x_d^2)^2) B_{0p}^{\text{fin}}(-M_W^2; M_t, M_b) \\
 & + \frac{1}{12} [1+48s^2c^4+4(4-29c^2)c^2] \frac{1}{c^4} B_{0p}^{\text{fin}}(-M_W^2; M_W, M_Z) \\
 & + \frac{1}{12} [12-(4-x_H^2)x_H^2] B_{0p}^{\text{fin}}(-M_W^2; M_W, M_H)
 \end{aligned}$$

$$\begin{aligned}
 W_H^{(6)} = & 4a_{\phi_W} + \frac{1}{c^2} a_{ZZ} (2-3L_R) \\
 & - \frac{1}{8} \left\{ -2 \sum_{\text{gen}} (3x_d^2 + 3x_u^2 + x_1^2) c^2 a_{\phi_{WD}}^{(-)} + 8 \sum_{\text{gen}} (a_{1\phi_{\square}} x_1^2 + 3a_{d\phi_{\square}} x_d^2 - 3a_{u\phi_{\square}} x_u^2) c^2 \right. \\
 & \left. + [5c^2 x_H^2 + (1-4c^2)] a_{\phi_D} - 4 [7c^2 x_H^2 - 2(1+2c^2)] a_{\phi_{\square}} + 8(1+8c^2) a_{\phi_W} \right\} \frac{1}{c^2} L_R \\
 & + \frac{1}{8} \frac{1}{c^2} a_{\phi_D} a_0^{\text{fin}}(M_Z) + \frac{1}{8} (a_{\phi_D} - 4a_{\phi_{\square}}) x_H^2 a_0^{\text{fin}}(M_H) \\
 & + \frac{3}{4} \sum_{\text{gen}} [a_{\phi_{WD}}^{(-)} + 4a_{u\phi_{\square}}] x_u^2 B_0^{\text{fin}}(-M_H^2; M_t, M_t) + \frac{3}{4} \sum_{\text{gen}} [a_{\phi_{WD}}^{(-)} - 4a_{d\phi_{\square}}] x_d^2 B_0^{\text{fin}}(-M_H^2; M_b, M_b) \\
 & + \frac{1}{4} \sum_{\text{gen}} [a_{\phi_{WD}}^{(-)} - 4a_{1\phi_{\square}}] x_1^2 B_0^{\text{fin}}(-M_H^2; M_1, M_1) \\
 & - \frac{1}{8} [24a_{ZZ} + 8a_{\phi_W} - c^2 a_{\phi_D} x_H^2 + 4(2-c^2 x_H^2) a_{\phi_{\square}}] \frac{1}{c^2} B_0^{\text{fin}}(-M_H^2; M_Z, M_Z) \\
 & - \frac{1}{4} [32a_{\phi_W} - (2-x_H^2) a_{\phi_D} + 4(2-x_H^2) a_{\phi_{\square}}] B_0^{\text{fin}}(-M_H^2; M_W, M_W) \\
 & - \frac{3}{8} (a_{\phi_D} - 4a_{\phi_{\square}}) x_H^2 B_0^{\text{fin}}(-M_H^2; M_H, M_H) \\
 & - \frac{3}{4} \sum_{\text{gen}} \left\{ -4 [(-x_H^2 + 4x_d^2) a_{d\phi_{\square}}] + (-x_H^2 + 4x_d^2) a_{\phi_{WD}}^{(-)} \right\} x_d^2 B_{0p}^{\text{fin}}(-M_H^2; M_b, M_b) \\
 & - \frac{3}{4} \sum_{\text{gen}} \left\{ 4 [(-x_H^2 + 4x_u^2) a_{u\phi_{\square}}] + (-x_H^2 + 4x_u^2) a_{\phi_{WD}}^{(-)} \right\} x_u^2 B_{0p}^{\text{fin}}(-M_H^2; M_t, M_t) \\
 & - \frac{1}{4} \sum_{\text{gen}} \left\{ -4 [(-x_H^2 + 4x_1^2) a_{1\phi_{\square}}] + (-x_H^2 + 4x_1^2) a_{\phi_{WD}}^{(-)} \right\} x_1^2 B_{0p}^{\text{fin}}(-M_H^2; M_1, M_1) \\
 & + \frac{1}{16} [48(2-c^2 x_H^2) a_{ZZ} + (12-4c^2 x_H^2 + c^4 x_H^4) a_{\phi_{WD}}^{(+)} + 4(12-4c^2 x_H^2 + c^4 x_H^4) a_{\phi_{\square}}] \frac{1}{c^4}
 \end{aligned}$$

$$\begin{aligned}
 & \times \mathbf{B}_{0p}^{\text{fin}} \left( -M_{\text{H}}^2; M_Z, M_Z \right) \\
 & + \frac{1}{8} \left\{ a_{\phi_{\text{WD}}}^{(-)} x_{\text{H}}^4 + 4 \left[ 12 - (4 - x_{\text{H}}^2) x_{\text{H}}^2 \right] a_{\phi_{\square}} - 4 (3 - x_{\text{H}}^2) a_{\phi_{\text{D}}} + 16 (9 - 4 x_{\text{H}}^2) a_{\phi_{\text{W}}} \right\} \mathbf{B}_{0p}^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{W}}, M_{\text{W}} \right) \\
 & - \frac{9}{16} (-4 a_{\phi_{\text{W}}} x_{\text{H}}^2 + 3 a_{\phi_{\text{D}}} x_{\text{H}}^2 - 12 a_{\phi_{\square}} x_{\text{H}}^2 + 32 a_{\phi}) x_{\text{H}}^2 \mathbf{B}_{0p}^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{H}}, M_{\text{H}} \right) \\
 \mathbf{W}_Z^{(6)} = & \frac{1}{18} \frac{1}{c^2} a_{\phi_{\square}} (2 + 3 \mathbf{L}_{\text{R}}) \\
 & - \frac{1}{9} \sum_{\text{gen}} (a_{\phi_{\text{V}}} + a_{\phi_{1\text{A}}} + a_{\phi_{1\text{V}}} v_1 + 3 a_{\phi_{\text{dA}}} + 3 a_{\phi_{\text{UA}}} + 3 v_{\text{d}} a_{\phi_{\text{dV}}} + 3 v_{\text{u}} a_{\phi_{\text{UV}}}) \frac{1}{c^2} (1 - 3 \mathbf{L}_{\text{R}}) \\
 & - \frac{1}{72} \left\{ 9 a_{\phi_{\text{WD}}}^{(-)} + \left[ (1 + 4 c^2) v_{\text{d}} + (3 + 4 c^2) v_1 + (5 + 8 c^2) v_{\text{u}} \right] a_{\phi_{\text{D}}} \right. \\
 & - 4 \left[ (1 + 8 c^2) v_{\text{d}} + (3 + 8 c^2) v_1 + (5 + 16 c^2) v_{\text{u}} \right] s a_{\phi_{\text{WA}}} \\
 & - 4 \left[ (5 - 8 c^2) v_{\text{d}} + (7 - 8 c^2) v_1 + (13 - 16 c^2) v_{\text{u}} \right] c a_{\phi_{\text{WZ}}} \\
 & + 64 \left[ (v_1 + v_{\text{d}} + 2 v_{\text{u}}) \right] s^2 c^2 a_{\phi_{\text{B}}} \left. \right\} \frac{1}{c^2} (1 - 3 \mathbf{L}_{\text{R}}) \mathbf{N}_{\text{gen}} \\
 & - \frac{1}{12} \left\{ -6 \sum_{\text{gen}} c a_{1\text{BW}} x_{\text{I}}^2 v_1 - 18 \sum_{\text{gen}} c v_{\text{d}} a_{\text{dBW}} x_{\text{d}}^2 + 18 \sum_{\text{gen}} c v_{\text{u}} a_{\text{uBW}} x_{\text{u}}^2 \right. \\
 & - 8 \left[ 4 + 3 (2 + c^2) c^2 \right] s c a_{\text{AZ}} - 2 \left[ 3 c^2 x_{\text{H}}^2 + (15 + 4 (7 - 3 (6 - c^2) c^2) c^2) \right] a_{\text{ZZ}} \\
 & - (1 - 20 c^2 + 18 s^2 c^2) a_{\phi_{\text{D}}} + 4 (5 - 6 s^4) s^2 a_{\text{AA}} \left. \right\} \frac{1}{c^2} \mathbf{L}_{\text{R}} \\
 & + \frac{1}{18} \left[ -4 c^2 a_{\text{ZZ}} + 8 s c a_{\text{AZ}} + 4 s^2 a_{\text{AA}} + (1 - 2 c^2) a_{\phi_{\text{D}}} \right] \frac{1}{c^2} \\
 & + \frac{1}{24} \left[ a_{\phi_{\text{D}}} - 8 c a_{\phi_{\text{WZ}}} - c^2 a_{\phi_{\text{WD}}}^{(+)} x_{\text{H}}^2 + 4 s a_{\phi_{\text{WA}}} - 12 s^2 a_{\phi_{\text{B}}} + 4 (1 - c^2 x_{\text{H}}^2) a_{\phi_{\square}} \right] x_{\text{H}}^2 a_0^{\text{fin}} (M_{\text{H}}) \\
 & - \frac{1}{24} \left[ a_{\phi_{\text{D}}} + 16 c a_{\phi_{\text{WZ}}} - c^2 a_{\phi_{\text{WD}}}^{(+)} x_{\text{H}}^2 + 4 s a_{\phi_{\text{WA}}} + 12 s^2 a_{\phi_{\text{B}}} + 4 (1 - c^2 x_{\text{H}}^2) a_{\phi_{\square}} \right] \frac{1}{c^2} a_0^{\text{fin}} (M_Z) \\
 & - (c^2 a_{\phi_{\text{W}}} + s c a_{\phi_{\text{WB}}} + s^2 a_{\phi_{\text{B}}}) a_0^{\text{fin}} (M_{\text{W}}) \\
 & + \frac{1}{24} \sum_{\text{gen}} \left[ a_{\phi_{\text{WD}}}^{(-)} + 8 a_{\phi_{1\text{A}}} + 8 a_{\phi_{1\text{V}}} v_1 + 12 c a_{1\text{BW}} x_{\text{I}}^2 v_1 + 64 s^2 c^2 a_{\phi_{\text{B}}} v_1 + (3 + 4 c^2) a_{\phi_{\text{D}}} v_1 \right. \\
 & - 4 (3 + 8 c^2) s a_{\phi_{\text{WA}}} v_1 - 4 (7 - 8 c^2) c a_{\phi_{\text{WZ}}} v_1 \left. \right] \frac{1}{c^2} \mathbf{B}_0^{\text{fin}} \left( -M_Z^2; M_1, M_1 \right) \\
 & + \frac{1}{24} \sum_{\text{gen}} \left[ 3 a_{\phi_{\text{WD}}}^{(-)} + 24 a_{\phi_{\text{dA}}} + 24 v_{\text{d}} a_{\phi_{\text{dV}}} + 36 c v_{\text{d}} a_{\text{dBW}} x_{\text{d}}^2 + 64 s^2 c^2 v_{\text{d}} a_{\phi_{\text{B}}} + (1 + 4 c^2) v_{\text{d}} a_{\phi_{\text{D}}} \right. \\
 & - 4 (1 + 8 c^2) s v_{\text{d}} a_{\phi_{\text{WA}}} - 4 (5 - 8 c^2) c v_{\text{d}} a_{\phi_{\text{WZ}}} \left. \right] \frac{1}{c^2} \mathbf{B}_0^{\text{fin}} \left( -M_Z^2; M_{\text{b}}, M_{\text{b}} \right) \\
 & + \frac{1}{24} \sum_{\text{gen}} \left[ 3 a_{\phi_{\text{WD}}}^{(-)} + 24 a_{\phi_{\text{UA}}} + 24 v_{\text{u}} a_{\phi_{\text{UV}}} - 36 c v_{\text{u}} a_{\text{uBW}} x_{\text{u}}^2 + 128 s^2 c^2 v_{\text{u}} a_{\phi_{\text{B}}} + (5 + 8 c^2) v_{\text{u}} a_{\phi_{\text{D}}} \right. \\
 & - 4 (5 + 16 c^2) s v_{\text{u}} a_{\phi_{\text{WA}}} - 4 (13 - 16 c^2) c v_{\text{u}} a_{\phi_{\text{WZ}}} \left. \right] \frac{1}{c^2} \mathbf{B}_0^{\text{fin}} \left( -M_Z^2; M_{\text{t}}, M_{\text{t}} \right) \\
 & + \frac{1}{12} \sum_{\text{gen}} (a_{\phi_{\text{WD}}}^{(-)} + 4 a_{\phi_{\text{V}}}) \frac{1}{c^2} \mathbf{B}_0^{\text{fin}} \left( -M_Z^2; 0, 0 \right) \\
 & + \frac{1}{24} \left[ 48 a_{\text{ZZ}} + (2 - c^2 x_{\text{H}}^2) c^2 a_{\phi_{\text{WD}}}^{(+)} x_{\text{H}}^2 + 4 (2 - c^2 x_{\text{H}}^2) c^2 a_{\phi_{\square}} x_{\text{H}}^2 \right] \frac{1}{c^2} \mathbf{B}_0^{\text{fin}} \left( -M_Z^2; M_{\text{H}}, M_Z \right) \\
 & + \frac{1}{24} \left\{ 12 \left[ 5 + 4 (3 - c^2) c^2 \right] s c a_{\text{AZ}} + 12 (7 - 4 (4 + c^2) c^2) c^2 a_{\text{ZZ}} \right. \\
 & - 4 \left[ 35 - 12 (4 - s^2) s^2 \right] s^2 a_{\text{AA}} + (1 - 40 c^2 + 36 s^2 c^2) a_{\phi_{\text{D}}} \left. \right\} \frac{1}{c^2} \mathbf{B}_0^{\text{fin}} \left( -M_Z^2; M_{\text{W}}, M_{\text{W}} \right) \\
 & + \frac{1}{24} \sum_{\text{gen}} \left\{ 12 c a_{1\text{BW}} x_{\text{I}}^2 v_1 + \left[ (3 + 4 c^2) + 2 (3 + 4 c^2) c^2 x_{\text{I}}^2 \right] a_{\phi_{\text{D}}} v_1 \right. \\
 & - 4 \left[ (3 + 8 c^2) + 2 (3 + 8 c^2) c^2 x_{\text{I}}^2 \right] s a_{\phi_{\text{WA}}} v_1 - 4 \left[ (7 - 8 c^2, r p) + 2 (7 - 8 c^2) c^2 x_{\text{I}}^2 \right] c a_{\phi_{\text{WZ}}} v_1
 \end{aligned}$$

$$\begin{aligned}
 & + (1 - 4c^2 x_1^2) a_{\phi_{\text{WD}}}^{(-)} + 8(1 - 4c^2 x_1^2) a_{\phi_{1\Lambda}} + 8(1 + 2c^2 x_1^2) a_{\phi_{1\nu}} v_1 \\
 & + 64(1 + 2c^2 x_1^2) s^2 c^2 a_{\phi_{\text{B}}} v_1 \left. \right\} \frac{1}{c^4} \text{B}_{0\text{p}}^{\text{fin}} \left( -M_Z^2; M_1, M_1 \right) \\
 & + \frac{1}{24} \sum_{\text{gen}} \left\{ 36c v_{\text{d}} a_{\text{dBW}} x_{\text{d}}^2 + \left[ (1 + 4c^2) + 2(1 + 4c^2) c^2 x_{\text{d}}^2 \right] v_{\text{d}} a_{\phi_{\text{D}}} \right. \\
 & - 4 \left[ (1 + 8c^2) + 2(1 + 8c^2) c^2 x_{\text{d}}^2 \right] s v_{\text{d}} a_{\phi_{\text{WA}}} - 4 \left[ (5 - 8c^2, rp) + 2(5 - 8c^2) c^2 x_{\text{d}}^2 \right] c v_{\text{d}} a_{\phi_{\text{WZ}}} \\
 & + 3(1 - 4c^2 x_{\text{d}}^2) a_{\phi_{\text{WD}}}^{(-)} + 24(1 - 4c^2 x_{\text{d}}^2) a_{\phi_{\text{d}\Lambda}} + 24(1 + 2c^2 x_{\text{d}}^2) v_{\text{d}} a_{\phi_{\text{d}\nu}} \\
 & \left. + 64(1 + 2c^2 x_{\text{d}}^2) s^2 c^2 v_{\text{d}} a_{\phi_{\text{B}}} \right\} \frac{1}{c^4} \text{B}_{0\text{p}}^{\text{fin}} \left( -M_Z^2; M_{\text{b}}, M_{\text{b}} \right) \\
 & - \frac{1}{24} \sum_{\text{gen}} \left\{ 36c v_{\text{u}} a_{\text{uBW}} x_{\text{u}}^2 - \left[ (5 + 8c^2) + 2(5 + 8c^2) c^2 x_{\text{u}}^2 \right] v_{\text{u}} a_{\phi_{\text{D}}} \right. \\
 & + 4 \left[ (5 + 16c^2) + 2(5 + 16c^2) c^2 x_{\text{u}}^2 \right] s v_{\text{u}} a_{\phi_{\text{WA}}} + 4 \left[ (13 - 16c^2) + 2(13 - 16c^2) c^2 x_{\text{u}}^2 \right] c v_{\text{u}} a_{\phi_{\text{WZ}}} \\
 & - 3(1 - 4c^2 x_{\text{u}}^2) a_{\phi_{\text{WD}}}^{(-)} - 24(1 - 4c^2 x_{\text{u}}^2) a_{\phi_{\text{u}\Lambda}} - 24(1 + 2c^2 x_{\text{u}}^2) v_{\text{u}} a_{\phi_{\text{u}\nu}} \\
 & \left. - 128(1 + 2c^2 x_{\text{u}}^2) s^2 c^2 v_{\text{u}} a_{\phi_{\text{B}}} \right\} \frac{1}{c^4} \text{B}_{0\text{p}}^{\text{fin}} \left( -M_Z^2; M_{\text{t}}, M_{\text{t}} \right) \\
 & + \frac{1}{12} \sum_{\text{gen}} (a_{\phi_{\text{WD}}}^{(-)} + 4a_{\phi_{\nu}}) \frac{1}{c^4} \text{B}_{0\text{p}}^{\text{fin}} \left( -M_Z^2; 0, 0 \right) \\
 & + \frac{1}{24} \left[ 48(2 - c^2 x_{\text{H}}^2) a_{\text{ZZ}} + (12 - 4c^2 x_{\text{H}}^2 + c^4 x_{\text{H}}^4) a_{\phi_{\text{WD}}}^{(+)} + 4(12 - 4c^2 x_{\text{H}}^2 + c^4 x_{\text{H}}^4) a_{\phi_{\square}} \right] \frac{1}{c^4} \\
 & \times \text{B}_{0\text{p}}^{\text{fin}} \left( -M_Z^2; M_{\text{H}}, M_{\text{Z}} \right) \\
 & + \frac{1}{24} \left\{ \left[ 1 + 4(4 - (17 + 12c^2) c^2) c^2 \right] a_{\phi_{\text{D}}} + 12 \left[ 5 + 4(6 + (3 + 4c^2) c^2) c^2 \right] s c a_{\text{AZ}} \right. \\
 & \left. + 12 \left[ 7 - 4(5 + (9 - 4c^2) c^2) c^2 \right] c^2 a_{\text{ZZ}} + 4 \left[ 33 - 4(29 - 3(11 - 4s^2) s^2) s^2 \right] s^2 a_{\text{AA}} \right\} \frac{1}{c^4} \\
 & \times \text{B}_{0\text{p}}^{\text{fin}} \left( -M_Z^2; M_{\text{W}}, M_{\text{W}} \right)
 \end{aligned}$$

$$\begin{aligned}
 \text{W}_{\text{W}}^{(6)} & = -\frac{1}{9} (3a_{\phi_{\text{qW}}}^{(3)} + a_{\phi_{1\text{W}}}^{(3)}) (1 - 3\text{L}_{\text{R}}) \text{N}_{\text{gen}} + \frac{1}{18} a_{\phi_{\square}} (2 + 3\text{L}_{\text{R}}) \\
 & + \frac{1}{6} \left[ 3c^2 a_{\phi_{\text{W}}} x_{\text{H}}^2 + 3 \sum_{\text{gen}} c^2 a_{1\text{W}} x_1^2 + 9 \sum_{\text{gen}} c^2 a_{\text{dW}} x_{\text{d}}^2 - 9 \sum_{\text{gen}} c^2 a_{\text{uW}} x_{\text{u}}^2 + (9 - 38c^2) s c a_{\text{AZ}} \right. \\
 & \left. + (15 - 32c^2) c^2 a_{\text{ZZ}} - (29 - 32s^2) s^2 a_{\text{AA}} \right] \frac{1}{c^2} \text{L}_{\text{R}} \\
 & - \frac{2}{9} \left[ 9c^2 a_{\phi_{\text{D}}} - 39s c a_{\phi_{\text{WB}}} + 6s^2 a_{\text{AA}} + (1 - 42s^2) a_{\phi_{\text{W}}} \right] \\
 & + \frac{1}{6} \sum_{\text{gen}} a_{\phi_{1\text{W}}}^{(3)} x_1^4 a_0^{\text{fin}} (M_1) \\
 & + \frac{1}{2} \sum_{\text{gen}} (x_{\text{d}}^2 - x_{\text{u}}^2) a_{\phi_{\text{qW}}}^{(3)} x_{\text{d}}^2 a_0^{\text{fin}} (M_{\text{b}}) \\
 & - \frac{1}{2} \sum_{\text{gen}} (x_{\text{d}}^2 - x_{\text{u}}^2) a_{\phi_{\text{qW}}}^{(3)} x_{\text{u}}^2 a_0^{\text{fin}} (M_{\text{t}}) \\
 & - \frac{1}{24} \left[ a_{\phi_{\text{WD}}}^{(-)} x_{\text{H}}^2 + 8a_{\phi_{\text{W}}} + a_{\phi_{\text{D}}} - 4(1 - x_{\text{H}}^2) a_{\phi_{\square}} \right] x_{\text{H}}^2 a_0^{\text{fin}} (M_{\text{H}}) \\
 & + \frac{1}{24} \left[ c^2 a_{\phi_{\text{WD}}}^{(-)} x_{\text{H}}^2 - 4(1 - x_{\text{H}}^2) c^2 a_{\phi_{\square}} + (1 + 8c^2) a_{\phi_{\text{D}}} + 4(5 - 14c^2) s c a_{\text{AZ}} \right. \\
 & \left. + 4(9 - 16c^2) c^2 a_{\text{ZZ}} - 4(15 - 16s^2) s^2 a_{\text{AA}} \right] \frac{1}{c^2} a_0^{\text{fin}} (M_{\text{W}}) \\
 & + \frac{1}{6} \sum_{\text{gen}} \left[ 3a_{1\text{W}} x_1^2 + (2 + x_1^4) a_{\phi_{1\text{W}}}^{(3)} \right] \text{B}_0^{\text{fin}} \left( -M_{\text{W}}^2; 0, M_1 \right)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{2} \sum_{\text{gen}} \left[ 3 a_{\text{dW}} x_{\text{d}}^2 - 3 a_{\text{uW}} x_{\text{u}}^2 + (2 + (x_{\text{u}}^2 - x_{\text{d}}^2)^2) a_{\phi_{\text{qW}}}^{(3)} \right] \mathbf{B}_0^{\text{fin}} \left( -M_{\text{W}}^2; M_{\text{t}}, M_{\text{b}} \right) \\
 & + \frac{1}{24} \left[ 48 a_{\phi_{\text{W}}} + (2 - x_{\text{H}}^2) a_{\phi_{\text{WD}}}^{(-)} x_{\text{H}}^2 + 4(2 - x_{\text{H}}^2) a_{\phi_{\square} x_{\text{H}}^2} \right] \mathbf{B}_0^{\text{fin}} \left( -M_{\text{W}}^2; M_{\text{W}}, M_{\text{H}} \right) \\
 & - \frac{1}{24} \left\{ \left[ 1 - 48 s^2 c^4 + 2(3 + 16 c^2) c^2 \right] a_{\phi_{\text{D}}} + 4 \left[ 1 + 2(1 - 4 c^4) c^2 \right] s a_{\phi_{\text{WA}}} \right. \\
 & + 4 \left[ 5 - 2(5 - 2(9 + 2 c^2) c^2) c^2 \right] c a_{\phi_{\text{WZ}}} + 16(3 - 4 s^2) s^2 c^4 a_{\phi_{\text{B}}} \left. \right\} \frac{1}{c^4} \mathbf{B}_0^{\text{fin}} \left( -M_{\text{W}}^2; M_{\text{W}}, M_{\text{Z}} \right) \\
 & - 2 a_{\phi_{\text{WAD}}} \mathbf{B}_0^{\text{fin}} \left( -M_{\text{W}}^2; 0, M_{\text{W}} \right) \\
 & + \frac{1}{2} \sum_{\text{gen}} \left[ 3(1 - x_{\text{d}}^2 + x_{\text{u}}^2) a_{\text{dW}} x_{\text{d}}^2 - 3(1 + x_{\text{d}}^2 - x_{\text{u}}^2) a_{\text{uW}} x_{\text{u}}^2 + (2 - x_{\text{d}}^2 - x_{\text{u}}^2 - (x_{\text{u}}^2 - x_{\text{d}}^2)^2) a_{\phi_{\text{qW}}}^{(3)} \right] \\
 & \times \mathbf{B}_{0\text{p}}^{\text{fin}} \left( -M_{\text{W}}^2; M_{\text{t}}, M_{\text{b}} \right) \\
 & + \frac{1}{6} \sum_{\text{gen}} \left\{ \left[ 2 - (1 + x_{\text{t}}^2) x_{\text{t}}^2 \right] a_{\phi_{1\text{W}}}^{(3)} + 3(1 - x_{\text{t}}^2) a_{1\text{W}} x_{\text{t}}^2 \right\} \mathbf{B}_{0\text{p}}^{\text{fin}} \left( -M_{\text{W}}^2; 0, M_{\text{t}} \right) \\
 & - \frac{1}{24} \left[ 12(3 - 2 c^2) c^2 a_{\text{ZZ}} + 4(3 - 2 s^2) s^2 a_{\text{AA}} + 4(5 - 12 c^4) s c a_{\text{AZ}} + (9 - 8 s^2) s^2 a_{\phi_{\text{D}}} \right] \frac{1}{c^4} a_0^{\text{fin}}(M_{\text{Z}}) \\
 & + \frac{1}{24} \left\{ a_{\phi_{\text{WD}}}^{(-)} x_{\text{H}}^4 + 4 \left[ 12 - (4 - x_{\text{H}}^2) x_{\text{H}}^2 \right] a_{\phi_{\square}} - 4(3 - x_{\text{H}}^2) a_{\phi_{\text{D}}} + 16(9 - 4 x_{\text{H}}^2) a_{\phi_{\text{W}}} \right\} \mathbf{B}_{0\text{p}}^{\text{fin}} \left( -M_{\text{W}}^2; M_{\text{W}}, M_{\text{H}} \right) \\
 & + \frac{1}{24} \left\{ \left[ 1 + 48 s^2 c^4 + 4(4 - 29 c^2) c^2 \right] a_{\phi_{\text{D}}} + 44 \left[ 1 - 2(1 + 4 c^2) c^2 \right] c a_{\phi_{\text{WZ}}} \right. \\
 & \left. + 4 \left[ 1 + 2(3 - 20 c^2) c^2 \right] s a_{\phi_{\text{WA}}} \right\} \frac{1}{c^4} \mathbf{B}_{0\text{p}}^{\text{fin}} \left( -M_{\text{W}}^2; M_{\text{W}}, M_{\text{Z}} \right)
 \end{aligned}$$

## G Mixing of Wilson coefficients

In this appendix we present the entries of the mixing matrix, eq. (5.9), that can be derived from the renormalization of  $\text{H} \rightarrow \text{V}\text{V}$ .

### G.1 Notations

First we define

R

$$\begin{aligned}
 \mathbf{R}_0^a &= 1 + 2c^2 & \mathbf{R}_1^a &= 7 - 12c^2 & \mathbf{R}_2^a &= 23 - 12c^2 \\
 \mathbf{R}_3^a &= 13 - 9c^2 & \mathbf{R}_4^a &= 1 - 12c^2 & \mathbf{R}_5^a &= 7 + 6c^2 \\
 \mathbf{R}_6^a &= 77 + 48c^2 & \mathbf{R}_7^a &= 5 - 12c^2 & \mathbf{R}_8^a &= 11 - 4c^2 \\
 \mathbf{R}_9^a &= 5 + 48c^2
 \end{aligned}$$

$$\begin{aligned}
 \mathbf{R}_0^b &= 1 + 6s^2 & \mathbf{R}_1^b &= 1 - s^2 & \mathbf{R}_2^b &= 11 - 12s^2 \\
 \mathbf{R}_3^b &= 1 - c^2 & \mathbf{R}_4^b &= 13 - 18\mathbf{R}_0^a c^2 & \mathbf{R}_5^b &= 41 + 6\mathbf{R}_1^a c^2 \\
 \mathbf{R}_6^b &= 65 - 6\mathbf{R}_2^a c^2 & \mathbf{R}_7^b &= 9 - 4s^2 & \mathbf{R}_8^b &= 7 - 6s^2 \\
 \mathbf{R}_9^b &= 11 - 9s^2 & \mathbf{R}_{10}^b &= 2 - c^2 & \mathbf{R}_{11}^b &= 43 - 18s^2 \\
 \mathbf{R}_{12}^b &= 31 - 8\mathbf{R}_5^a c^2 & \mathbf{R}_{13}^b &= 7 - 8c^4 & \mathbf{R}_{14}^b &= 179 + 16\mathbf{R}_4^a c^2 \\
 \mathbf{R}_{15}^b &= 77 - 12\mathbf{R}_5^a c^2 & \mathbf{R}_{16}^b &= 79 - 4\mathbf{R}_6^a c^2 & \mathbf{R}_{17}^b &= 7c^2 + 3s^2 \\
 \mathbf{R}_{18}^b &= 35 + 3\mathbf{R}_7^a c^2 & \mathbf{R}_{19}^b &= 107 - 32\mathbf{R}_8^a c^2 & \mathbf{R}_{20}^b &= 267 - 4\mathbf{R}_3^a c^2 \\
 \mathbf{R}_{21}^b &= 7 - 78c^2
 \end{aligned}$$

$$\begin{array}{lll}
 R_0^c = 1 + 2c^2 - 4R_0^b s^2 c^2 & R_1^c = 3 - 4R_1^b s^2 & R_2^c = 1 - 2s^2 \\
 R_3^c = 7 + 6R_2^b s^2 & R_4^c = 9 - 16c^2 & R_5^c = 4 - 3s^2 \\
 R_6^c = 3 - 4R_3^b c^2 & R_7^c = 17 + 2R_4^b c^2 & R_8^c = 1 - 2c^2 \\
 R_9^c = 11 - R_5^b c^2 & R_{10}^c = 1 + R_6^b c^2 & R_{11}^c = 91 - 18R_7^b s^2 \\
 R_{12}^c = 1 + 8R_1^b s^2 & R_{13}^c = 5 + 5c^2 - 12R_8^b s^2 c^2 & R_{14}^c = 27 + 128s^2 c^2 \\
 R_{15}^c = 1 - 4s^2 & R_{16}^c = 19 - 8R_9^b s^2 & R_{17}^c = 1 - R_{10}^b c^2 \\
 R_{18}^c = 117 - 4R_{11}^b s^2 & R_{19}^c = 1 + 4s^2 & R_{20}^c = 1 + R_{12}^b c^2 \\
 R_{21}^c = 1 + 8c^2 & R_{22}^c = 1 + c^2 & R_{23}^c = 3 - 5c^2 \\
 R_{24}^c = 1 - 7c^2 & R_{25}^c = 5 - 11c^2 & R_{26}^c = 1 + R_{13}^b c^2 \\
 R_{27}^c = 3 - R_{14}^b c^2 & R_{28}^c = 27 - 48c^2 + 128s^4 & R_{29}^c = 9 + 8c^4 \\
 R_{30}^c = 13 - R_{15}^b c^2 & R_{31}^c = 58 + R_{16}^b c^2 & R_{32}^c = 3c^2 + R_{17}^b s^2 \\
 R_{33}^c = 2 + c^2 & R_{34}^c = 14 - R_{18}^b c^2 & R_{35}^c = 54 + R_{19}^b c^2 \\
 R_{36}^c = 55 - R_{20}^b c^2 & R_{37}^c = 7 - 2R_{21}^b c^2 &
 \end{array}$$

S

$$\begin{array}{lll}
 S_0 = 3x_d^2 + 3x_u^2 + x_l^2 & S_1 = 5x_d^2 + x_u^2 & S_2 = x_d^2 + x_u^2 \\
 S_3 = x_d^2 + 5x_u^2 & S_4 = 3x_d^4 + 3x_u^4 + x_l^4 &
 \end{array}$$

With their help we derive the relevant elements of the mixing matrix.

## G.2 Mixing matrix

$$\begin{aligned}
 dZ_{1,1}^W &= \frac{32}{9} s^2 N_{\text{gen}} + \frac{1}{4} (R_0^c + R_1^c c^2 x_H^2) \frac{1}{c^2} \\
 dZ_{1,2}^W &= -(6 - x_H^2) s^2 c^2 \\
 dZ_{1,3}^W &= -\frac{1}{24} (v_{\text{gen}}^{(1)} + R_4^c) \frac{1}{s c} N_{\text{gen}} - \frac{1}{12} (6R_2^c c^2 x_H^2 + R_3^c) \frac{s}{c} \\
 dZ_{2,1}^W &= (c^2 x_H^2 - 2R_5^c) s^2 \\
 dZ_{2,2}^W &= \frac{1}{12} (9 + v_{\text{gen}}^{(1)}) \frac{1}{c^2} N_{\text{gen}} + \frac{1}{12} (3R_6^c c^2 x_H^2 + R_7^c) \frac{1}{c^2} \\
 dZ_{2,3}^W &= \frac{1}{24} (v_{\text{gen}}^{(1)} + 8s^2 v_{\text{gen}}^{(2)} + R_4^c) \frac{1}{s c} N_{\text{gen}} - \frac{1}{12} (6R_8^c s^2 c^2 x_H^2 - R_9^c) \frac{1}{s c} \\
 dZ_{3,1}^W &= \frac{1}{12} (v_{\text{gen}}^{(1)} + 8s^2 v_{\text{gen}}^{(2)} + R_4^c) \frac{1}{s c} N_{\text{gen}} - \frac{1}{6} (6R_2^c s^2 c^2 x_H^2 + R_{10}^c) \frac{1}{s c} \\
 dZ_{3,2}^W &= -\frac{1}{12} (v_{\text{gen}}^{(1)} + R_4^c) \frac{1}{s c} N_{\text{gen}} + \frac{1}{6} (6R_2^c c^2 x_H^2 - R_{11}^c) \frac{s}{c} \\
 dZ_{3,3}^W &= \frac{1}{72} (3v_{\text{gen}}^{(1)} + R_{14}^c) \frac{1}{c^2} N_{\text{gen}} + \frac{1}{12} (3R_{12}^c c^2 x_H^2 + 2R_{13}^c) \frac{1}{c^2} \\
 dZ_{4,1}^W &= \frac{16}{3} s^2 N_{\text{gen}} - \frac{1}{3} (3R_{15}^c c^2 x_H^2 - R_{16}^c) \frac{s^2}{c^2}
 \end{aligned}$$

$$dZ_{4,2}^W = -\frac{80}{9} \frac{1}{c^2} R_{17}^c N_{\text{gen}} + \frac{1}{3} (3R_{15}^c c^2 x_H^2 - R_{18}^c) \frac{s^2}{c^2}$$

$$dZ_{4,3}^W = \frac{80}{9} \frac{s^3}{c} N_{\text{gen}} + \frac{1}{3} (3R_{19}^c s^2 c^2 x_H^2 - R_{20}^c) \frac{1}{sc}$$

$$dZ_{4,4}^W = \frac{1}{2} \sum_{\text{gen}} S_0 + \frac{1}{12} (9c^2 x_H^2 + 7R_{21}^c) \frac{1}{c^2}$$

$$dZ_{4,5}^W = \frac{10}{3} \frac{s^2}{c^2}$$

$$dZ_{4,16}^W = -5 \sum_{\text{gen}} x_1^2 + \frac{2}{3} \frac{1}{c^2} R_{22}^c N_{\text{gen}}$$

$$dZ_{4,17}^W = -\sum_{\text{gen}} x_1^2 - \frac{2}{3} \frac{1}{c^2} R_{23}^c N_{\text{gen}}$$

$$dZ_{4,18}^W = \frac{2}{3} N_{\text{gen}} - \sum_{\text{gen}} x_1^2$$

$$dZ_{4,19}^W = 2 \frac{1}{c^2} R_{22}^c N_{\text{gen}} - 3 \sum_{\text{gen}} S_1$$

$$dZ_{4,20}^W = -\frac{2}{3} \frac{1}{c^2} R_{24}^c N_{\text{gen}} - 3 \sum_{\text{gen}} S_2$$

$$dZ_{4,21}^W = 2 \frac{1}{c^2} R_{22}^c N_{\text{gen}} - 3 \sum_{\text{gen}} S_3$$

$$dZ_{4,22}^W = -\frac{2}{3} \frac{1}{c^2} R_{25}^c N_{\text{gen}} - 3 \sum_{\text{gen}} S_2$$

$$dZ_{5,1}^W = -\frac{1}{72} (3v_{\text{gen}}^{(1)} + 24s^2 v_{\text{gen}}^{(2)} + R_{28}^c) N_{\text{gen}} + \frac{1}{24} (12s^2 c^6 x_H^4 - 9R_{26}^c c^2 x_H^2 - R_{27}^c) \frac{1}{c^2}$$

$$\begin{aligned} dZ_{5,2}^W &= -\frac{1}{24} (2v_{\text{gen}}^{(1)} - 9c^2 x_H^2 - c^2 v_{\text{gen}}^{(1)} x_H^2 + 2R_{29}^c) \frac{1}{c^2} N_{\text{gen}} \\ &\quad - \frac{1}{24} (R_{30}^c c^2 x_H^2 + R_{31}^c + 3R_{32}^c c^4 x_H^4) \frac{1}{c^2} \\ &\quad - \frac{1}{4} \sum_{\text{gen}} (2S_0 + S_0 c^2 x_H^2 - 4S_4 c^2) \end{aligned}$$

$$\begin{aligned} dZ_{5,3}^W &= \frac{1}{144} (3c^2 v_{\text{gen}}^{(1)} x_H^2 + 3R_4^c c^2 x_H^2 - 3R_{33}^c v_{\text{gen}}^{(1)} - 24R_{33}^c s^2 v_{\text{gen}}^{(2)} - R_{35}^c) \frac{1}{sc} N_{\text{gen}} \\ &\quad + \frac{1}{24} (6R_2^c s^2 c^4 x_H^4 + 2R_{34}^c c^2 x_H^2 - R_{36}^c) \frac{1}{sc} \end{aligned}$$

$$dZ_{5,4}^W = \frac{1}{48} \frac{1}{c^2} R_{37}^c$$

$$dZ_{5,5}^W = 4c^2$$

## H Non-factorizable amplitudes

In this appendix we present the explicit expressions for the non-factorizable part of the  $H \rightarrow AA, AZ, ZZ$  and  $WW$  amplitudes.

### H.1 Notations

It is useful to introduce the following sets of polynomials:

$\boxed{T}$  where  $s = s_\theta$  and  $c = c_\theta$

$$\begin{array}{llll}
 T_0^a = 81 - 56c^2 & T_1^a = 53 - 39c^2 & T_2^a = 35 - 26c^2 & T_3^a = 35 - 18c^2 \\
 T_4^a = 19 - 10c^2 & T_5^a = 21 - 11c^2 & T_6^a = 5 + 4c^2 & T_7^a = 1 - 6c^2 \\
 T_8^a = 7 - 12c^2 & T_9^a = 47 + 12c^2 & T_{10}^a = 7 - c^2 & T_{11}^a = 37 + 4c^2 \\
 T_{12}^a = 213 - 68c^2 & T_{13}^a = 29 - 4c^2 & T_{14}^a = 49 - 12c^2 & T_{15}^a = 173 - 192c^2 \\
 T_{16}^a = 331 - 200c^2 & T_{17}^a = 33 - 16c^2 & T_{18}^a = 53 - 16c^2 & T_{19}^a = 8 - 3c^2 \\
 T_{20}^a = 16 - 9c^2 & T_{21}^a = 3 - c^2 & & 
 \end{array}$$

$$\begin{array}{llll}
 T_0^b = 7 - 18c^2 & T_1^b = 2 - 27c^2 & T_2^b = 49 - 2T_0^a c^2 & T_3^b = 11 - T_1^a c^2 \\
 T_4^b = 41 - 6T_2^a c^2 & T_5^b = 37 - 2T_3^a c^2 & T_6^b = 39 - 4T_4^a c^2 & T_7^b = 11 - T_5^a c^2 \\
 T_8^b = 33 - 56c^2 & T_9^b = 37 - 78c^2 & T_{10}^b = 6 - 13c^2 & T_{11}^b = 4 - 5c^2 \\
 T_{12}^b = 7 - 9c^2 & T_{13}^b = 19 - 22c^2 & T_{14}^b = 1 - c^2 & T_{15}^b = 23 - 22c^2 \\
 T_{16}^b = 1 + T_6^a c^2 & T_{17}^b = 1 + 4c^2 & T_{18}^b = 11 + 12c^2 & T_{19}^b = 1 + 2T_7^a c^2 \\
 T_{20}^b = 7 - 12c^2 & T_{21}^b = 1 - 6c^2 & T_{22}^b = 3 - 2c^2 & T_{23}^b = 25 + 6T_8^a c^2 \\
 T_{24}^b = 1 + 2c^2 & T_{25}^b = 7 - 6c^2 & T_{26}^b = 1 - 2c^2 & T_{27}^b = 1 + 3c^2 \\
 T_{28}^b = 1 + 12c^2 & T_{29}^b = 7 + 6c^2 & T_{30}^b = 1 + c^2 & T_{31}^b = 3 - 4c^2
 \end{array}$$

$$\begin{array}{llll}
 T_{32}^b = 13 - 34c^2 & T_{33}^b = 1 - 5c^2 & T_{34}^b = 3 - 5c^2 & T_{35}^b = 1 + 20c^2 \\
 T_{36}^b = 3 + 4c^2 & T_{37}^b = 27 + 68c^2 & T_{38}^b = 11 + 3c^2 & T_{39}^b = 13 - 3c^2 \\
 T_{40}^b = 5 + 4c^2 & T_{41}^b = 12 + T_9^a c^2 & T_{42}^b = 1 - 4c^2 & T_{43}^b = 7 - 3c^2 \\
 T_{44}^b = 13 - 12c^2 & T_{45}^b = 37 - 12T_{10}^a c^2 & T_{46}^b = 11 - 23c^2 & T_{47}^b = 7 + c^2 \\
 T_{48}^b = 7 - c^2 & T_{49}^b = 44 - T_{11}^a c^2 & T_{50}^b = 11 + c^2 & T_{51}^b = 23 - 7c^2 \\
 T_{52}^b = 78 - T_{12}^a c^2 & T_{53}^b = 58 - 3T_{13}^a c^2 & T_{54}^b = 37 - 3c^2 & T_{55}^b = 8 - T_{14}^a c^2 \\
 T_{56}^b = 55 - 56c^2 & T_{57}^b = 7 + 4c^2 & T_{58}^b = 1 - 36c^2 & T_{59}^b = 115 - 4T_{15}^a c^2 \\
 T_{60}^b = 77 - 192c^2 & T_{61}^b = 3 - 8c^2 & T_{62}^b = 119 + 360c^2 & T_{63}^b = 31 + 18c^2
 \end{array}$$

$$\begin{array}{llll}
 T_{64}^b = 37c^2 - s^2 & T_{65}^b = 14 - 11c^2 & T_{66}^b = 231 - 200c^2 & T_{67}^b = 184 - T_{16}^a c^2 \\
 T_{68}^b = 809 - 552c^2 & T_{69}^b = 365 - 496c^2 & T_{70}^b = 103 - 96c^2 & T_{71}^b = 185 - 52c^2 \\
 T_{72}^b = 51 + 11c^2 & T_{73}^b = 22 - T_{17}^a c^2 & T_{74}^b = 23 - 4c^2 & T_{75}^b = 45 - 16c^2 \\
 T_{76}^b = 79 - 2T_{18}^a c^2 & T_{77}^b = 121 - 40c^2 & T_{78}^b = 6 + c^2 & T_{79}^b = 11 - 8c^2 \\
 T_{80}^b = 87 - 40c^2 & T_{81}^b = 3c^2 - s^2 & T_{82}^b = 31 - 36c^2 & T_{83}^b = 13 + 30c^2 \\
 T_{84}^b = 47 - 40c^2 & T_{85}^b = 37 - 42c^2 & T_{86}^b = 1 + 45c^2 & T_{87}^b = 37 - 32c^2 \\
 T_{88}^b = 1 + 16c^2 & T_{89}^b = 9 + 16c^2 & T_{90}^b = 2 - c^2 & T_{91}^b = 11 - 2T_{19}^a c^2 \\
 T_{92}^b = 5 - 2T_{20}^a c^2 & T_{93}^b = 9 - 4c^2 & T_{94}^b = 13 - 6c^2 & T_{95}^b = 49 - 16c^2 \\
 T_{96}^b = 69 - 32c^2 & T_{97}^b = 97 - 32T_{21}^a c^2 & T_{98}^b = 1 - 2s^2 & 
 \end{array}$$



$T_0^c = 1 - s^2$	$T_1^c = 3 + 2T_0^b c^2$	$T_2^c = 3 + 2T_1^b c^2$	$T_3^c = 3 - T_2^b c^2$
$T_4^c = 5 - 6c^2$	$T_5^c = 5 + 8T_3^b c^2$	$T_6^c = 6 - 7s^2$	$T_7^c = 7 + 2T_4^b c^2$
$T_8^c = 9 - 2T_5^b c^2$	$T_9^c = 9 - 2T_6^b c^2$	$T_{10}^c = 11 - 8T_7^b c^2$	$T_{11}^c = 2 - c^2$
$T_{12}^c = 4 - 3s^2$	$T_{13}^c = 49 - 78s^2$	$T_{14}^c = 15 - 8c^2$	$T_{15}^c = 2 - T_8^b c^2$
$T_{16}^c = 3 + 2T_9^b c^2$	$T_{17}^c = 5 + 12T_{10}^b c^2$	$T_{18}^c = 7 - 8T_{11}^b c^2$	$T_{19}^c = 7 - 4T_{12}^b c^2$
$T_{20}^c = 9 - 2T_{13}^b c^2$	$T_{21}^c = 35 + 6c^2$	$T_{22}^c = 25 - 66c^2$	$T_{23}^c = 5 - 6s^2$
$T_{24}^c = 4 - T_8^b c^2$	$T_{25}^c = 11 - 40T_{14}^b c^2$	$T_{26}^c = 11 - 36T_{14}^b c^2$	$T_{27}^c = 13 - 2T_{15}^b c^2$
$T_{28}^c = 1 - 2c^2$	$T_{29}^c = 1 - 12c^2$	$T_{30}^c = 5 - 4T_{16}^b c^2$	$T_{31}^c = 1 + 4c^2$
$T_{32}^c = 2 - 9T_{17}^b c^2$	$T_{33}^c = 11 + 16c^2$	$T_{34}^c = 12 - T_{18}^b c^2$	$T_{35}^c = 1 + c^2$
$T_{36}^c = 3 - 4c^2$	$T_{37}^c = 5 - 2T_{19}^b c^2$	$T_{38}^c = 11 - 36c^2$	$T_{39}^c = 11 + 12c^2$
$T_{40}^c = 2 + T_{20}^b c^2$	$T_{41}^c = 5 + T_{21}^b c^2$	$T_{42}^c = 29 - 18T_{22}^b c^2$	$T_{43}^c = 47 - 36c^4$
$T_{44}^c = 5 - 2T_{23}^b c^2$	$T_{45}^c = 19 - 36T_{24}^b c^2$	$T_{46}^c = 1 + 6c^2$	$T_{47}^c = 1 - 2T_{14}^b c^2$
$T_{48}^c = 1 - 3c^2$	$T_{49}^c = 1 - 4T_{25}^b c^2$	$T_{50}^c = 5 + 6c^2$	$T_{51}^c = 5 - 2T_{22}^b c^2$
$T_{52}^c = 4 + c^2$	$T_{53}^c = 1 - 4c^2$	$T_{54}^c = 1 - c^2$	$T_{55}^c = 1 + 4T_{14}^b c^2$
$T_{56}^c = 1 + T_{26}^b c^2$	$T_{57}^c = 2 - T_{27}^b c^2$	$T_{58}^c = 3 - T_{28}^b c^2$	$T_{59}^c = 5 - 2T_{29}^b c^2$
$T_{60}^c = 3 + 4c^2$	$T_{61}^c = 5 + 12T_{26}^b c^2$	$T_{62}^c = 1 + 16c^2$	$T_{63}^c = 2 - T_{20}^b c^2$
$T_{64}^c = 3 - 4T_{14}^b c^2$	$T_{65}^c = 31 - 12T_{30}^b c^2$	$T_{66}^c = 41 - 12T_{31}^b c^4$	$T_{67}^c = 5 + 2T_{32}^b c^2$
$T_{68}^c = 1 + 8c^2$	$T_{69}^c = 1 + 4T_{33}^b c^2$	$T_{70}^c = 59 + 12T_{34}^b c^2$	$T_{71}^c = 61 + 12T_{35}^b c^4$
$T_{72}^c = 2 - 3T_{36}^b c^2$	$T_{73}^c = 22 - T_{37}^b c^2$	$T_{74}^c = 1 + 12T_{30}^b c^2$	$T_{75}^c = 23 - 4T_{38}^b c^2$
$T_{76}^c = 3 + c^2$	$T_{77}^c = 11 - 7c^2$	$T_{78}^c = 15 + 4T_{39}^b c^2$	$T_{79}^c = 12 - T_{40}^b c^2$
$T_{80}^c = 55 + 117c^2$	$T_{81}^c = 15 - 4T_{41}^b c^2$	$T_{82}^c = 8 - 3s^2$	$T_{83}^c = 23 + 3c^2$
$T_{84}^c = 1 + 12T_{14}^b c^2$	$T_{85}^c = 1 + 3T_{42}^b c^2$	$T_{86}^c = 8 + 3c^2$	$T_{87}^c = 4 - 3c^2$
$T_{88}^c = 5 - 4c^2$	$T_{89}^c = 19 - 4T_{43}^b c^2$	$T_{90}^c = 13 + T_{44}^b c^2$	$T_{91}^c = 1 - 3c^4$
$T_{92}^c = 3 + 4T_{39}^b c^2$	$T_{93}^c = 17 - 2T_{45}^b c^2$	$T_{94}^c = 19 - 4T_{46}^b c^2$	$T_{95}^c = 35 - 4T_{47}^b c^2$
$T_{96}^c = 37 - 12T_{48}^b c^2$	$T_{97}^c = 37 - 4T_{49}^b c^2$	$T_{98}^c = 51 - 4T_{50}^b c^2$	$T_{99}^c = 67 - 4T_{51}^b c^2$
$T_{100}^c = 75 + 4T_{52}^b c^2$	$T_{101}^c = 77 - 4T_{53}^b c^2$	$T_{102}^c = 5 - 7c^2$	$T_{103}^c = 41 - 4T_{47}^b c^2$
$T_{104}^c = 53 - 4T_{54}^b c^2$	$T_{105}^c = 67 - 4T_{55}^b c^2$	$T_{106}^c = 1 - 24c^2$	$T_{107}^c = 13 - T_{56}^b c^2$
$T_{108}^c = 27 - 56c^2$	$T_{109}^c = 41 - 64c^2$	$T_{110}^c = 1 - 2T_{57}^b c^2$	$T_{111}^c = 2c^2 - s^2$
$T_{112}^c = 7 + 8c^2$	$T_{113}^c = 9 - 2T_{58}^b c^2$	$T_{114}^c = 11 + 36c^2$	$T_{115}^c = 17 - 72c^2$
$T_{116}^c = 20 + T_{59}^b c^2$	$T_{117}^c = 39 + 4T_{60}^b c^2$	$T_{118}^c = 41 - 156c^2$	$T_{119}^c = 49 - 204c^2$
$T_{120}^c = 79 + 228T_{61}^b c^2$	$T_{121}^c = 151 - 4T_{62}^b c^2$	$T_{122}^c = 5 - 12c^2$	$T_{123}^c = 25 - 8T_{63}^b c^2$
$T_{124}^c = 35c^2 - s^2$	$T_{125}^c = 36c^2 - s^2$	$T_{126}^c = 71c^4 - T_{64}^b s^2$	$T_{127}^c = 8 - 7T_{22}^b c^2$
$T_{128}^c = 12 - 11c^2$	$T_{129}^c = 11 - 6c^2$	$T_{130}^c = 13 - 24c^2$	$T_{131}^c = 17 - 22c^2$
$T_{132}^c = 19 - 286c^2$	$T_{133}^c = 23 - 4T_{65}^b c^2$	$T_{134}^c = 137 - 2T_{66}^b c^2$	$T_{135}^c = 143 - 4T_{67}^b c^2$
$T_{136}^c = 164 - T_{68}^b c^2$	$T_{137}^c = 251 - 152c^2$	$T_{138}^c = 19 - T_{69}^b c^2$	$T_{139}^c = 18 + T_{70}^b c^2$
$T_{140}^c = 55 + 2T_{71}^b c^2$	$T_{141}^c = 85 - 96c^2$	$T_{142}^c = 85 - 8T_{72}^b c^2$	$T_{143}^c = 3 - 11c^2$
$T_{144}^c = 3 + 13c^2$	$T_{145}^c = 9 - 16c^2$	$T_{146}^c = 21 - 40c^2$	$T_{147}^c = 21 - 4T_{73}^b c^2$
$T_{148}^c = 3 - 8c^2$	$T_{149}^c = 5 + 2c^2$	$T_{150}^c = 9 - 4c^2$	$T_{151}^c = 17 - 14c^2$
$T_{152}^c = 7 + T_{74}^b c^2$	$T_{153}^c = 31 - 8c^2$	$T_{154}^c = 17 - T_{75}^b c^2$	$T_{155}^c = 31 - 2T_{76}^b c^2$
$T_{156}^c = 69 - 2T_{77}^b c^2$	$T_{157}^c = 11 - 8T_{78}^b c^2$	$T_{158}^c = 19 + 2c^2$	$T_{159}^c = 4 - 7c^2$
$T_{160}^c = 13 + 2T_{79}^b c^2$	$T_{161}^c = 21 + 2T_{80}^b c^2$	$T_{162}^c = 11 - 8c^2$	$T_{163}^c = 3c^4 - T_{81}^b s^2$
$T_{164}^c = 1 - 8c^2$	$T_{165}^c = 5 - 24c^2$	$T_{166}^c = 4 - T_{82}^b c^2$	$T_{167}^c = 7 - 3c^2$
$T_{168}^c = 13 - 36c^2$	$T_{169}^c = 19 - 2T_{83}^b c^2$	$T_{170}^c = 25 - 112c^2$	$T_{171}^c = 105 - 8T_{84}^b c^2$
$T_{172}^c = 49 - 120c^2$	$T_{173}^c = 2 - 47c^2$	$T_{174}^c = 2 - 7c^2$	$T_{175}^c = 2 + 313c^2$
$T_{176}^c = 19 - 48c^2$	$T_{177}^c = 8 - 21c^2$	$T_{178}^c = 9 - 8c^2$	$T_{179}^c = 11 - 2T_{85}^b c^2$
$T_{180}^c = 13 - 56c^2$	$T_{181}^c = 13 - 2T_{86}^b c^2$	$T_{182}^c = 15 - 58c^2$	$T_{183}^c = 49 - 4T_{87}^b c^2$
$T_{184}^c = 2 - 77c^2$	$T_{185}^c = 2 - 251c^2$	$T_{186}^c = 1 + 2c^2$	$T_{187}^c = 3 - 2T_{22}^b c^2$
$T_{188}^c = 21 + 4T_{88}^b c^2$	$T_{189}^c = 7 - 4c^2$	$T_{190}^c = 17 - 18c^2$	$T_{191}^c = 15 + T_{89}^b c^2$

$$\begin{array}{llll}
 T_{192}^c = 41 - 16c^2 & T_{193}^c = 23 - 16c^2 & T_{194}^c = 39 - 34c^2 & T_{195}^c = 23 - 10c^2 \\
 T_{196}^c = 53 - 4T_{79}^b c^2 & T_{197}^c = 3 + 16c^2 & T_{198}^c = 5 - 4T_{90}^b c^2 & T_{199}^c = 1 - 2s^2 \\
 T_{200}^c = 1 + 3T_{14}^b c^2 & T_{201}^c = 2 - T_{91}^b c^2 & T_{202}^c = 3 + 2T_{92}^b c^2 & T_{203}^c = 3 - 2T_{93}^b c^2 \\
 T_{204}^c = 8 - 9c^2 & T_{205}^c = 8 + c^2 & T_{206}^c = 9 - 2T_{94}^b c^2 & T_{207}^c = 10 - 13c^2 \\
 T_{208}^c = 10 + c^2 & T_{209}^c = 13 - 5c^2 & T_{210}^c = 14 - T_{95}^b c^2 & T_{211}^c = 18 - T_{96}^b c^2 \\
 T_{212}^c = 21 - 8c^2 & T_{213}^c = 46 - T_{97}^b c^2 & T_{214}^c = 37 - 16c^2 & T_{215}^c = c^2 - T_{98}^b s^2
 \end{array}$$

$$\begin{array}{llll}
 T_0^d = 1 - 6s^2 & T_1^d = 1 - 3s^2 & T_2^d = 1 - 2s^2 & T_3^d = 1 - 4T_0^c s^2 \\
 T_4^d = 2 - 3s^2 & T_5^d = 3 - 4T_0^c s^2 & T_6^d = 1 - 6c^2 - 72s^2 c^2 & T_7^d = 1 - 2c^2 \\
 T_8^d = 1 + 4c^2 & T_9^d = 5 + 8c^2 & T_{10}^d = c^2 - T_1^c s^2 & T_{11}^d = 3c^4 - T_2^c s^2 \\
 T_{12}^d = 1 - 4s^2 & T_{13}^d = 1 - 8T_0^c s^2 & T_{14}^d = 1 + 8T_0^c s^2 & T_{15}^d = 1 + 2T_3^c c^2 \\
 T_{16}^d = 1 - 4T_4^c c^4 & T_{17}^d = 1 - T_5^c c^2 & T_{18}^d = 1 + 8T_6^c s^2 c^2 & T_{19}^d = 1 - T_7^c c^2 \\
 T_{20}^d = 1 + T_8^c c^2 & T_{21}^d = 1 + T_9^c c^2 & T_{22}^d = 1 + T_{10}^c c^2 & T_{23}^d = 3 - 4T_{11}^c c^2 \\
 T_{24}^d = 3 - 4T_{12}^c s^2 & T_{25}^d = 5 - 8s^2 & T_{26}^d = 5 - 6s^2 & T_{27}^d = 5 + T_{13}^c s^2 \\
 T_{28}^d = 11 - 2T_{14}^c c^2 & T_{29}^d = 1 - c^2 & T_{30}^d = 1 + 2T_{15}^c c^2 & T_{31}^d = 1 - T_{16}^c c^2
 \end{array}$$

$$\begin{array}{llll}
 T_{32}^d = 1 - T_{17}^c c^2 & T_{33}^d = 1 + T_{18}^c c^2 & T_{34}^d = 1 + T_{19}^c c^2 & T_{35}^d = 1 + T_{20}^c c^2 \\
 T_{36}^d = 1 + T_{21}^c c^2 & T_{37}^d = 2 - 3T_{22}^c c^2 & T_{38}^d = 3 - 2c^2 & T_{39}^d = 5 - 21c^2 \\
 T_{40}^d = 5 - 4s^2 & T_{41}^d = 7 + 6T_{23}^c s^2 & T_{42}^d = 11 - 8c^2 & T_{43}^d = 13 - 8s^2 \\
 T_{44}^d = 20 - 39s^2 & T_{45}^d = 1 + 2T_{24}^c c^2 & T_{46}^d = 1 + T_{25}^c c^2 & T_{47}^d = 1 + T_{26}^c c^2 \\
 T_{48}^d = 1 + T_{27}^c c^2 & T_{49}^d = 3 + 4s^2 & T_{50}^d = 3 + 8s^2 & T_{51}^d = 7 - 6s^2 \\
 T_{52}^d = 1 - 4s^2 c^2 & T_{53}^d = 1 - 14c^2 & T_{54}^d = 1 - 11c^2 & T_{55}^d = 1 - 8c^2 \\
 T_{56}^d = 1 - 2c^2 - 4T_{28}^c s^2 c^2 & T_{57}^d = 1 + 6c^2 & T_{58}^d = 1 + 8c^2 & T_{59}^d = 1 + 10c^2 \\
 T_{60}^d = 1 + 18c^2 & T_{61}^d = 1 - 4T_{29}^c c^4 & T_{62}^d = 1 - T_{30}^c c^2 & T_{63}^d = 2 - 7c^2 - 6s^2 c^2
 \end{array}$$

$$\begin{array}{llll}
 T_{64}^d = 2 - c^2 & T_{65}^d = 3 + 10c^2 - 4T_{31}^c s^2 c^2 & T_{66}^d = 3 + 192c^6 + 8T_{32}^c s^2 c^2 & T_{67}^d = 3 - 4T_{33}^c c^2 \\
 T_{68}^d = 7 + 4c^2 - 24s^2 c^2 & T_{69}^d = 13 - 4T_{34}^c c^2 & T_{70}^d = 1 + 3c^2 & T_{71}^d = 1 - 4c^4 \\
 T_{72}^d = 1 + 12T_{35}^c c^2 & T_{73}^d = 1 - 6T_{36}^c c^2 & T_{74}^d = 1 - T_{37}^c c^2 & T_{75}^d = 1 - 2T_{38}^c c^2 \\
 T_{76}^d = 2 - T_{39}^c c^2 & T_{77}^d = 3 - 8c^2 & T_{78}^d = 3 - 4c^2 & T_{79}^d = 5 - 4T_{40}^c c^2 \\
 T_{80}^d = 5 - 24T_{41}^c c^4 & T_{81}^d = 7 - 2T_{42}^c c^2 & T_{82}^d = 11 - T_{43}^c c^2 & T_{83}^d = 15 - T_{44}^c c^2 \\
 T_{84}^d = 17 - 12c^2 & T_{85}^d = 25 - 24c^2 & T_{86}^d = 45 - 2T_{45}^c c^2 & T_{87}^d = 1 - 6c^2 \\
 T_{88}^d = 1 - 4c^2 & T_{89}^d = 1 - 2T_{46}^c c^2 & T_{90}^d = 1 + 4T_{47}^c c^2 & T_{91}^d = 2 + T_{48}^c c^2 \\
 T_{92}^d = 3 + 4c^2 & T_{93}^d = 3 - 2s^2 & T_{94}^d = 3 + T_{49}^c c^2 & T_{95}^d = 3 - 2T_{50}^c c^2
 \end{array}$$

$$\begin{array}{llll}
 T_{96}^d = 3 - T_{51}^c c^2 & T_{97}^d = 1 - 3c^6 - T_{52}^c s^2 c^2 & T_{98}^d = 1 - 2T_{56}^c c^2 & T_{99}^d = 1 + T_{53}^c c^2 \\
 T_{100}^d = 1 - 4T_{54}^c c^2 & T_{101}^d = 1 - T_{55}^c c^2 & T_{102}^d = 1 - 2T_{56}^c c^2 & T_{103}^d = 1 - 4T_{57}^c c^2 \\
 T_{104}^d = 1 - 2T_{58}^c c^2 & T_{105}^d = 1 - T_{59}^c c^2 & T_{106}^d = 2 + c^2 & T_{107}^d = 3 + c^2 \\
 T_{108}^d = 3 - 4T_{54}^c c^2 & T_{109}^d = 3 - T_{60}^c c^2 & T_{110}^d = 4 + c^2 & T_{111}^d = 4 + T_{61}^c c^2 \\
 T_{112}^d = 7 - 4c^2 & T_{113}^d = 1 + 8c^2 - 12s^2 c^2 & T_{114}^d = 1 + 32c^4 - 4T_{62}^c s^2 c^2 & T_{115}^d = 1 - 4T_{36}^c c^4 \\
 T_{116}^d = 1 - 4T_{63}^c c^2 & T_{117}^d = 2 - 3T_{64}^c c^2 & T_{118}^d = 3 - 16s^2 c^2 & T_{119}^d = 4 - 3T_{64}^c c^2 \\
 T_{120}^d = 4 - T_{65}^c c^2 & T_{121}^d = 8 - T_{66}^c c^2 & T_{122}^d = 1 - 2T_{67}^c c^2 & T_{123}^d = 2 - 60s^2 c^4 + 5T_{68}^c c^2 \\
 T_{124}^d = 2 - 3T_{69}^c c^2 & T_{125}^d = 4 - 3T_{69}^c c^2 & T_{126}^d = 4 + T_{70}^c c^2 & T_{127}^d = 8 + T_{71}^c c^2
 \end{array}$$

$$\begin{array}{llll}
 T_{128}^d = 15 - 16s^2 c^2 & T_{129}^d = 17 - 24s^2 c^2 & T_{130}^d = 1 + 4T_{72}^c c^2 & T_{131}^d = 1 + 4T_{73}^c c^2 \\
 T_{132}^d = 3 + 2T_{74}^c c^2 & T_{133}^d = 4 - T_{75}^c c^2 & T_{134}^d = 5 + 4T_{54}^c c^2 & T_{135}^d = 7 - 4T_{76}^c c^2 \\
 T_{136}^d = 11 - 2T_{75}^c c^2 & T_{137}^d = 15 - 4T_{77}^c c^2 & T_{138}^d = 16 + T_{78}^c c^2 & T_{139}^d = 17 - 4T_{79}^c c^2 \\
 T_{140}^d = 19 - 4T_{80}^c c^2 & T_{141}^d = 24 - T_{81}^c c^2 & T_{142}^d = 55 - 12T_{82}^c s^2 & T_{143}^d = 73 - 12T_{83}^c c^2 \\
 T_{144}^d = 1 + 2c^2 & T_{145}^d = 1 - 12c^4 & T_{146}^d = 1 - T_{84}^c c^2 & T_{147}^d = 1 - 4T_{85}^c c^2 \\
 T_{148}^d = 1 - 4T_{86}^c c^2 & T_{149}^d = 3 + 10c^2 & T_{150}^d = 3 - 4T_{87}^c c^2 & T_{151}^d = 3 - 2T_{88}^c c^2 \\
 T_{152}^d = 3 - T_{89}^c c^2 & T_{153}^d = 4 - 3s^2 & T_{154}^d = 5 - 2T_{31}^c c^2 & T_{155}^d = 7 - 4T_{90}^c c^2 \\
 T_{156}^d = 1 + 2T_{91}^c c^2 & T_{157}^d = 1 + 2T_{92}^c c^2 & T_{158}^d = 2 + T_{92}^c c^2 & T_{159}^d = 4 - 3c^2
 \end{array}$$

$T_{160}^d = 4 + 3c^2$	$T_{161}^d = 5 - 2c^2$	$T_{162}^d = 5 + 6c^2$	$T_{163}^d = 6 - c^2$
$T_{164}^d = 8 - 3c^2$	$T_{165}^d = 8 + T_{93}^c c^2$	$T_{166}^d = 9 - 2c^2$	$T_{167}^d = 10 - 3c^2$
$T_{168}^d = 10 - T_{94}^c c^2$	$T_{169}^d = 10 - T_{95}^c c^2$	$T_{170}^d = 10 - T_{96}^c c^2$	$T_{171}^d = 10 + T_{97}^c c^2$
$T_{172}^d = 10 - T_{98}^c c^2$	$T_{173}^d = 10 - T_{99}^c c^2$	$T_{174}^d = 10 - T_{100}^c c^2$	$T_{175}^d = 10 - T_{101}^c c^2$
$T_{176}^d = 13 - 24s^2 c^2 + 2T_{102}^c c^4$	$T_{177}^d = 14 - 3c^2$	$T_{178}^d = 16 - 15c^2$	$T_{179}^d = 30 - T_{103}^c c^2$
$T_{180}^d = 32 - 21c^2$	$T_{181}^d = 118 + T_{104}^c c^2$	$T_{182}^d = 122 + T_{105}^c c^2$	$T_{183}^d = 1 + 9c^2$
$T_{184}^d = 1 + 8c^4$	$T_{185}^d = 1 + 5T_{106}^c c^2$	$T_{186}^d = 1 + 4T_{107}^c c^2$	$T_{187}^d = 1 + 2T_{108}^c c^2$
$T_{188}^d = 3 + 158c^2$	$T_{189}^d = 10 - T_{109}^c c^2$	$T_{190}^d = 12 + 13c^2$	$T_{191}^d = 12 - T_{110}^c c^2$
$T_{192}^d = 23 - 146c^2$	$T_{193}^d = 23 - 48c^2$	$T_{194}^d = 49 + 8c^2$	$T_{195}^d = c^2 - s^2$
$T_{196}^d = c^4 - T_{111}^c s^2$	$T_{197}^d = 1 - 84c^2$	$T_{198}^d = 1 - 12T_{112}^c c^2$	$T_{199}^d = 1 - 4T_{113}^c c^2$
$T_{200}^d = 1 + T_{114}^c c^2$	$T_{201}^d = 1 + 4T_{115}^c c^2$	$T_{202}^d = 1 - 4T_{116}^c c^2$	$T_{203}^d = 1 - 2T_{117}^c c^2$
$T_{204}^d = 1 - 2T_{118}^c c^2$	$T_{205}^d = 1 - 2T_{119}^c c^2$	$T_{206}^d = 1 - T_{120}^c c^2$	$T_{207}^d = 2 - T_{121}^c c^2$
$T_{208}^d = 3 + 19T_{122}^c c^2$	$T_{209}^d = 23 + 2T_{123}^c c^2$	$T_{210}^d = 25 + 108c^2$	$T_{211}^d = 4c^2 + s^2$
$T_{212}^d = 13c^2 + s^2$	$T_{213}^d = 29c^2 + s^2$	$T_{214}^d = 48c^2 - T_{124}^c s^2$	$T_{215}^d = 59c^2 + 23s^2$
$T_{216}^d = 61c^2 + s^2$	$T_{217}^d = 77c^2 + s^2$	$T_{218}^d = 83c^2 - s^2$	$T_{219}^d = 85c^2 + s^2$
$T_{220}^d = 93c^2 + s^2$	$T_{221}^d = 109c^2 + s^2$	$T_{222}^d = 115c^2 - s^2$	$T_{223}^d = 133c^2 + s^2$
$T_{224}^d = 157c^2 + s^2$	$T_{225}^d = 167c^2 + 3s^2$	$T_{226}^d = 173c^2 + s^2$	$T_{227}^d = 35c^4 - T_{125}^c s^2$
$T_{228}^d = 35c^6 - T_{126}^c s^2$	$T_{229}^d = 1 - 3c^2$	$T_{230}^d = 1 + c^2$	$T_{231}^d = 1 - 8T_{28}^c c^2$
$T_{232}^d = 1 - 2T_{28}^c c^2$	$T_{233}^d = 1 - 2T_{31}^c c^2$	$T_{234}^d = 1 - T_{127}^c c^2$	$T_{235}^d = 1 - T_{128}^c c^2$
$T_{236}^d = 3 - 131c^2$	$T_{237}^d = 3 - 107c^2$	$T_{238}^d = 3 - 83c^2$	$T_{239}^d = 3 - 41c^2$
$T_{240}^d = 3 - 11c^2$	$T_{241}^d = 3 - T_{129}^c c^2$	$T_{242}^d = 3 + T_{130}^c c^2$	$T_{243}^d = 3 - T_{131}^c c^2$
$T_{244}^d = 3 + T_{132}^c c^2$	$T_{245}^d = 3 - T_{133}^c c^2$	$T_{246}^d = 3 - T_{134}^c c^2$	$T_{247}^d = 3 - T_{135}^c c^2$
$T_{248}^d = 3 - T_{136}^c c^2$	$T_{249}^d = 3 - T_{137}^c c^2$	$T_{250}^d = 6 + T_{138}^c c^2$	$T_{251}^d = 9 + 71c^2$
$T_{252}^d = 15 - 23c^2$	$T_{253}^d = 15 - T_{139}^c c^2$	$T_{254}^d = 15 + T_{140}^c c^2$	$T_{255}^d = 21 - T_{141}^c c^2$
$T_{256}^d = 21 - T_{142}^c c^2$	$T_{257}^d = 31 - 47c^2$	$T_{258}^d = 24c^2 - T_{143}^c s^2$	$T_{259}^d = 24c^2 - T_{144}^c s^2$
$T_{260}^d = 1 + 22c^2$	$T_{261}^d = 1 - 2T_{145}^c c^2$	$T_{262}^d = 1 - T_{146}^c c^2$	$T_{263}^d = 1 - T_{147}^c c^2$
$T_{264}^d = 2 + T_{148}^c c^2$	$T_{265}^d = 3 - 4T_{149}^c c^2$	$T_{266}^d = 3 - 2T_{150}^c c^2$	$T_{267}^d = 3 - T_{151}^c c^2$
$T_{268}^d = 7 - 40c^2$	$T_{269}^d = 151 - 150s^2$	$T_{270}^d = 4c^2 + 5s^2$	$T_{271}^d = 1 - 44c^4$
$T_{272}^d = 1 - T_{68}^c c^2$	$T_{273}^d = 1 + 4T_{152}^c c^2$	$T_{274}^d = 1 - 2T_{153}^c c^2$	$T_{275}^d = 2 - 11c^2$
$T_{276}^d = 3 + 130c^2$	$T_{277}^d = 3 + 4T_{154}^c c^2$	$T_{278}^d = 3 + 2T_{155}^c c^2$	$T_{279}^d = 3 + T_{156}^c c^2$
$T_{280}^d = 7 - 2T_{157}^c c^2$	$T_{281}^d = 8 - 5T_{158}^c c^2$	$T_{282}^d = 9 + 4T_{159}^c c^2$	$T_{283}^d = 9 - T_{160}^c c^2$
$T_{284}^d = 10 - T_{161}^c c^2$	$T_{285}^d = 11 - 2T_{162}^c c^2$	$T_{286}^d = c^2 - 2s^2$	$T_{287}^d = 3c^2 + s^2$
$T_{288}^d = 6c^2 - s^2$	$T_{289}^d = 13c^2 - s^2$	$T_{290}^d = 17c^2 - s^2$	$T_{291}^d = 21c^2 - s^2$
$T_{292}^d = c^6 - T_{163}^c s^2$	$T_{293}^d = 1 + 40c^2$	$T_{294}^d = 1 + 4T_{164}^c c^2$	$T_{295}^d = 1 + T_{165}^c c^2$
$T_{296}^d = 5 - 56c^2$	$T_{297}^d = 5 - 8c^2$	$T_{298}^d = 5 + 72c^2$	$T_{299}^d = 5 + 88c^2$
$T_{300}^d = 5 + 4T_{166}^c c^2$	$T_{301}^d = 5 - 8T_{167}^c c^2$	$T_{302}^d = 5 + 2T_{168}^c c^2$	$T_{303}^d = 5 - 4T_{169}^c c^2$
$T_{304}^d = 5 + T_{170}^c c^2$	$T_{305}^d = 5 + T_{171}^c c^2$	$T_{306}^d = 7 - 50c^2$	$T_{307}^d = 7 - 18c^2$
$T_{308}^d = 7 - 2c^2$	$T_{309}^d = 9 - 8s^2$	$T_{310}^d = 11 + 4T_{172}^c c^2$	$T_{311}^d = s^4 - T_{173}^c c^2$
$T_{312}^d = s^4 - T_{174}^c c^2$	$T_{313}^d = 7s^4 + T_{175}^c c^2$	$T_{314}^d = 1 - 7c^2$	$T_{315}^d = 1 + 14c^2$
$T_{316}^d = 1 + 16c^2$	$T_{317}^d = 1 + 2T_{176}^c c^2$	$T_{318}^d = 5 - 28c^2$	$T_{319}^d = 5 - 12c^2$
$T_{320}^d = 5 + 14c^2$	$T_{321}^d = 5 + 66c^2$	$T_{322}^d = 5 - 4T_{86}^c c^2$	$T_{323}^d = 5 + 8T_{88}^c c^2$
$T_{324}^d = 5 + 4T_{177}^c c^2$	$T_{325}^d = 5 + 4T_{178}^c c^2$	$T_{326}^d = 5 + 2T_{179}^c c^2$	$T_{327}^d = 5 + 2T_{180}^c c^2$
$T_{328}^d = 5 - 4T_{181}^c c^2$	$T_{329}^d = 5 + 2T_{182}^c c^2$	$T_{330}^d = 5 + 2T_{183}^c c^2$	$T_{331}^d = 9c^2 + s^2$
$T_{332}^d = s^4 - T_{184}^c c^2$	$T_{333}^d = 5s^4 - T_{185}^c c^2$	$T_{334}^d = 1 - 20c^2$	$T_{335}^d = 1 + 5c^2$
$T_{336}^d = 1 - T_4^c c^2$	$T_{337}^d = 1 - T_{178}^c c^2$	$T_{338}^d = 1 - 4T_{186}^c c^2$	$T_{339}^d = 1 - 2T_{187}^c c^2$
$T_{340}^d = 1 - T_{188}^c c^2$	$T_{341}^d = 2 - 7T_{54}^c c^2$	$T_{342}^d = 3 + 2s^2$	$T_{343}^d = 3 - T_{189}^c c^2$
$T_{344}^d = 3 - 4T_{190}^c c^2$	$T_{345}^d = 4 - 9c^2$	$T_{346}^d = 4 - T_{191}^c c^2$	$T_{347}^d = 5 - 13c^2$
$T_{348}^d = 5 + c^2$	$T_{349}^d = 6 - 13c^2$	$T_{350}^d = 6 - T_{192}^c c^2$	$T_{351}^d = 7 - T_{193}^c c^2$

$$\begin{array}{llll}
 T_{352}^d = 7 - T_{194}^c c^2 & T_{353}^d = 8 - T_{195}^c c^2 & T_{354}^d = 9 - 35 c^2 & T_{355}^d = 11 - 10 c^2 \\
 T_{356}^d = 14 - T_{196}^c c^2 & T_{357}^d = 1 + 68 c^2 & T_{358}^d = 1 - T_{87}^c c^2 & T_{359}^d = 1 - T_{88}^c c^2 \\
 T_{360}^d = 1 - 4 T_{197}^c c^2 & T_{361}^d = 1 - T_{198}^c c^2 & T_{362}^d = 4 - c^2 & T_{363}^d = 9 - 16 c^2 \\
 T_{364}^d = 13 - 32 c^2 & T_{365}^d = 13 - 8 c^2 & T_{366}^d = 25 + 32 c^4 & T_{367}^d = c^2 - T_{199}^c s^2 \\
 T_{368}^d = 1 + 17 c^2 & T_{369}^d = 1 + 8 T_{11}^c c^2 & T_{370}^d = 1 - 8 T_{200}^c c^2 & T_{371}^d = 1 + 8 T_{201}^c c^2 \\
 T_{372}^d = 1 - 4 T_{202}^c c^2 & T_{373}^d = 1 + 4 T_{203}^c c^2 & T_{374}^d = 1 + 2 T_{204}^c c^2 & T_{375}^d = 1 + 2 T_{205}^c c^2 \\
 T_{376}^d = 1 + 2 T_{206}^c c^2 & T_{377}^d = 1 + 2 T_{207}^c c^2 & T_{378}^d = 1 + 2 T_{208}^c c^2 & T_{379}^d = 1 - T_{209}^c c^2 \\
 T_{380}^d = 1 + T_{210}^c c^2 & T_{381}^d = 1 + T_{211}^c c^2 & T_{382}^d = 1 + T_{212}^c c^2 & T_{383}^d = 1 + T_{213}^c c^2 \\
 \\ 
 T_{384}^d = 3 + 4 c^4 & T_{385}^d = 9 - 20 c^2 & T_{386}^d = 17 - 8 c^2 & T_{387}^d = 23 - 68 c^2 \\
 T_{388}^d = 23 - 4 T_{214}^c c^2 & T_{389}^d = 9 c^2 - 8 s^2 & T_{390}^d = 8 c^4 - T_{215}^c s^2 & 
 \end{array}$$

U

$$\begin{array}{llll}
 U_0 = 2 v_t + v_b + v_l & U_1 = 3 - v_t^2 & U_2 = 3 + v_t^2 & U_3 = 3 - v_b^2 \\
 U_4 = 3 + v_b^2 & U_5 = 27 - 16 v_t & U_6 = 27 - 8 v_b & U_7 = 21 - v_t^2 \\
 U_8 = 21 - v_b^2 & U_9 = 27 + v_t^2 & U_{10} = 27 + v_b^2 & 
 \end{array}$$

V

$$\begin{array}{llll}
 V_0 = 2 - x_H^2 & V_1 = 3 - x_H^2 & V_2 = 6 - x_H^2 & \\
 V_3 = 4 x_t^2 - x_H^2 & V_4 = 4 x_b^2 - x_H^2 & V_5 = 8 + x_H^2 & \\
 V_6 = x_b^2 + x_t^2 & V_7 = 6 x_b^2 + 6 x_t^2 + 7 x_H^2 & & \\
 V_8 = -(2 x_t^2 - x_H^2) \frac{1}{x_H} & V_9 = \frac{1}{x_H} & & \\
 V_{10} = (2 x_t^2 + x_H^2) \frac{1}{x_H} & V_{11} = -(2 x_b^2 - x_H^2) \frac{1}{x_H} & & \\
 V_{12} = (2 x_b^2 + x_H^2) \frac{1}{x_H} & V_{13} = (x_b^2 + x_t^2) \frac{1}{x_H} & & \\
 V_{14} = 2 - x_b^2 - x_t^2 & V_{15} = 2 - x_b^2 + x_t^2 & V_{16} = 2 + x_b^2 - x_t^2 & \\
 V_{17} = \left[ 6 - (1 - 2 x_b^2 + x_t^2) x_t^2 - (1 + x_b^2) x_b^2 \right] & V_{18} = 1 + x_b^2 - x_t^2 & & \\
 V_{19} = \left[ 1 + (2 + x_b^2) x_b^2 - (2 + 2 x_b^2 - x_t^2) x_t^2 \right] & V_{20} = \left[ 1 - x_t^4 + (2 + x_b^2) x_b^2 \right] & & \\
 V_{21} = \left[ 2 - 2 x_b^2 + 2 x_t^2 - (1 + x_b^2 - x_t^2) x_H^2 \right] & V_{22} = 2 + 2 x_b^2 - 2 x_t^2 - x_H^2 & & \\
 V_{23} = 2 + 2 x_b^2 + 2 x_t^2 - x_H^2 & V_{24} = \left[ 1 - x_t^4 - (2 - x_b^2) x_b^2 \right] & & \\
 V_{25} = \left[ 2 + 2 x_b^4 + x_H^2 x_b^2 - 2(1 + x_b^2) x_t^2 \right] & V_{26} = \left\{ \left[ 2 - 2 x_b^2 + (1 - x_b^2) x_H^2 \right] x_b^2 + (4 + 2 x_b^2 + x_H^2 x_b^2) x_t^2 \right\} & & \\
 V_{27} = \left[ 4 x_t^2 + (4 - x_H^2) x_b^2 \right] & V_{28} = \left[ 4 - (4 - x_H^2) x_H^2 \right] & & \\
 V_{29} = \left[ 32 - (81 - 8 x_H^2) x_H^2 \right] & V_{30} = \left[ 61 - 2(4 - x_H^2) x_H^2 \right] & & \\
 V_{31} = \left[ 64 + (50 - 41 x_H^2) x_H^2 \right] & V_{32} = \left[ 96 + (148 - 61 x_H^2) x_H^2 \right] & & \\
 V_{33} = \left[ 100 - (50 + 49 x_H^2) x_H^2 \right] & V_{34} = \left\{ 128 + \left[ 36 - (132 - 41 x_H^2) x_H^2 \right] x_H^2 \right\} & & \\
 V_{35} = 146 + x_H^2 & V_{36} = 4 - x_H^2 & V_{37} = 4 + x_H^4 & 
 \end{array}$$

$$\begin{aligned}
 V_{38} &= 2 + x_H^2 & V_{39} &= 5 - x_H^2 & V_{40} &= 10 - x_H^2 \\
 V_{41} &= 9 - 4x_H^2 & V_{42} &= 14 - x_H^2 \\
 V_{43} &= [14 + (8 + x_H^2)x_H^2] & V_{44} &= [18 - (10 - x_H^2)x_H^2] \\
 V_{45} &= 22 + x_H^2 & V_{46} &= [28 - 9(2 + x_H^2)x_H^2] \\
 V_{47} &= 3 - x_b^2 + x_t^2 & V_{48} &= 3 + x_b^2 - x_t^2 \\
 V_{49} &= [5 - (6 + 2x_b^2 - x_t^2)x_t^2 + (10 + x_b^2)x_b^2] & V_{50} &= [1 - (2 - x_b^2)x_b^2 - (2 + 2x_b^2 - x_t^2)x_t^2] \frac{1}{x_H^2} \\
 V_{51} &= (1 - x_b^2 + x_t^2) \frac{1}{x_H^2} & V_{52} &= (1 + x_b^2 - x_t^2) \frac{1}{x_H^2} \\
 V_{53} &= 1 - x_b^2 + x_t^2 & V_{54} &= [1 - x_t^4 + (4 + x_b^2)x_b^2] \\
 V_{55} &= x_b^2 - x_t^2 & V_{56} &= (1 + x_b^2 - x_t^2 + x_H^2) \frac{1}{x_H^2} \\
 V_{57} &= [3 + (2 - 2x_b^2 - x_t^2)x_t^2 - 3(2 - x_b^2)x_b^2] & V_{58} &= 3 - 3x_b^2 - x_t^2 \\
 V_{59} &= 3 + 5x_b^2 - x_t^2 & V_{60} &= [5 - (6 + 6x_b^2 - x_t^2)x_t^2 + (14 + 5x_b^2)x_b^2] \\
 V_{61} &= [1 - 2x_b^2x_b^2 - (2 - x_b^2)x_b^2 - (2 + 2x_b^2 - x_t^2)x_t^2] \frac{1}{x_H^2} & V_{62} &= \left\{ [1 - x_b^2 + (1 - x_b^2)x_b^2]x_b^2 + (1 + 2x_b^2 - x_t^2 + x_H^2x_b^2)x_t^2 \right\} \frac{1}{x_H^2} \\
 V_{63} &= -[2x_t^2 - (2 + x_H^2)x_b^2] \frac{1}{x_H^2} & V_{64} &= [2x_t^2 + (2 - x_H^2)x_b^2] \frac{1}{x_H^2} \\
 V_{65} &= [3(1 - x_b^2)x_b^2 + (3 + 2x_b^2 + x_t^2)x_t^2] & V_{66} &= -(2 - x_H^2) \frac{1}{x_H^2} \\
 V_{67} &= [8 - (4 - x_H^2)x_H^2] & V_{68} &= 1 - x_H^2 \\
 V_{69} &= 2 - 5x_H^2 & V_{70} &= [16 - 5(4 - x_H^2)x_H^2] \\
 V_{71} &= [16 - (18 - 5x_H^2)x_H^2] & V_{72} &= [24 - (2 + 5x_H^2)x_H^2] \\
 V_{73} &= \left\{ 32 - [52 - (28 - 5x_H^2)x_H^2]x_H^2 \right\} & V_{74} &= [48 - (32 - 5x_H^2)x_H^2] \\
 V_{75} &= [88 - (6 - 5x_H^2)x_H^2] & V_{76} &= \left\{ 160 - [168 - (52 - 5x_H^2)x_H^2]x_H^2 \right\} \\
 V_{77} &= [2 - (4 - x_H^2)x_H^2] & V_{78} &= [2 - (3 - x_H^2)x_H^2] \\
 V_{79} &= \left\{ 4 - [8 - (5 - x_H^2)x_H^2]x_H^2 \right\} & V_{80} &= [6 - (4 - x_H^2)x_H^2] \\
 V_{81} &= [8 - (12 - x_H^2)x_H^2] & V_{82} &= 12 - x_H^2 \\
 V_{83} &= 16 - 3x_H^4 & V_{84} &= \left\{ 20 - [24 - (8 - x_H^2)x_H^2]x_H^2 \right\} \\
 V_{85} &= [48 - (12 - x_H^2)x_H^2] & V_{86} &= 64 + 3x_H^4 & V_{87} &= 1 + 2x_H^2 & V_{88} &= 7 - 4x_H^2 \\
 V_{89} &= 12 + 7x_H^2 & V_{90} &= 48 - 25x_H^2 & V_{91} &= 68 + 9x_H^2 \\
 V_{92} &= \frac{1}{x_H^4} & V_{93} &= (9 + 2x_H^2) \frac{1}{x_H^2} \\
 V_{94} &= 3 + 2x_H^2 & V_{95} &= 4 + x_H^2
 \end{aligned}$$

W

$$\begin{aligned}
 W_0 &= 1 - 4 \frac{x_t^2}{\lambda_{AZ}} & W_1 &= 1 - 4 \frac{x_b^2}{\lambda_{AZ}} & W_2 &= 1 + \frac{x_H^2}{\lambda_{AZ}} & W_3 &= 1 + 2 \frac{x_H^2}{\lambda_{AZ}} \\
 W_4 &= 2 + \frac{x_H^2}{\lambda_{AZ}} & W_5 &= 3 + 2 \frac{x_H^2}{\lambda_{AZ}} & W_6 &= 1 - 6 \frac{x_t^2}{\lambda_{ZZ}} & W_7 &= 1 - 6 \frac{x_b^2}{\lambda_{ZZ}} \\
 W_8 &= 1 + 4 \frac{x_H^2}{\lambda_{ZZ}} & W_9 &= 1 - \frac{x_H^2}{\lambda_{ZZ}} & W_{10} &= 1 + \frac{x_H^2}{\lambda_{ZZ}} & W_{11} &= 1 + 3 \frac{x_H^2}{\lambda_{ZZ}} \\
 W_{12} &= 1 + 5 \frac{x_H^2}{\lambda_{ZZ}} & W_{13} &= 1 + 6 \frac{x_H^2}{\lambda_{ZZ}} & W_{14} &= 4 + 15 \frac{x_H^2}{\lambda_{ZZ}} & W_{15} &= 5 + 6 \frac{x_H^2}{\lambda_{ZZ}} \\
 W_{16} &= 5 + 8 \frac{x_H^2}{\lambda_{ZZ}} & W_{17} &= 1 + 2 \frac{x_H^2}{\lambda_{ZZ}} & W_{18} &= \frac{1}{\lambda_{ZZ}} - \mathcal{R}_9 & W_{19} &= \frac{1}{\lambda_{ZZ}} + \mathcal{R}_9 \\
 W_{20} &= 2 \frac{x_t^2}{\lambda_{ZZ}} - \mathcal{R}_8 & W_{21} &= 7 \frac{1}{\lambda_{ZZ}} + \mathcal{R}_9 & W_{22} &= 9 \frac{1}{\lambda_{ZZ}} - \mathcal{R}_9 & W_{23} &= 2 \frac{x_t^2}{\lambda_{ZZ}} + \mathcal{R}_{10} \\
 W_{24} &= 2 \frac{x_b^2}{\lambda_{ZZ}} - \mathcal{R}_{11} & W_{25} &= 2 \frac{x_b^2}{\lambda_{ZZ}} + \mathcal{R}_{12} & W_{26} &= 1 + 3 \frac{x_H^4}{\lambda_{ZZ}^2} - \frac{x_H^2}{\lambda_{ZZ}} & W_{27} &= 1 - 6 \frac{x_H^2}{\lambda_{ZZ}} \\
 W_{28} &= 6 \frac{x_H^2}{\lambda_{ZZ}^2} + \frac{1}{\lambda_{ZZ}} - \mathcal{R}_9 & W_{29} &= 1 - 9 \frac{x_H^4}{\lambda_{ZZ}^2} - \frac{x_H^2}{\lambda_{ZZ}} & W_{30} &= 1 - 6 \frac{x_H^4}{\lambda_{ZZ}^2} - 6 \frac{x_H^2}{\lambda_{ZZ}} & W_{31} &= 1 - 3 \frac{x_H^4}{\lambda_{ZZ}^2} + 3 \frac{x_H^2}{\lambda_{ZZ}}
 \end{aligned}$$

$$\begin{aligned}
 W_{32} &= 1 + 3 \frac{x_H^4}{\lambda_{ZZ}^2} + \frac{x_H^2}{\lambda_{ZZ}} & W_{33} &= 1 + 6 \frac{x_H^4}{\lambda_{ZZ}^2} - 9 \frac{x_H^2}{\lambda_{ZZ}} & W_{34} &= 2 + 3 \frac{x_H^2}{\lambda_{ZZ}} & W_{35} &= 5 - 6 \frac{x_H^2}{\lambda_{ZZ}} \\
 W_{36} &= 1 - 96 \frac{x_H^4}{\lambda_{ZZ}^2} + 9 \frac{x_H^2}{\lambda_{ZZ}} & W_{37} &= 1 + 15 \frac{x_H^4}{\lambda_{ZZ}^2} + 3 \frac{x_H^2}{\lambda_{ZZ}} & W_{38} &= 1 + 30 \frac{x_H^4}{\lambda_{ZZ}^2} - 9 \frac{x_H^2}{\lambda_{ZZ}} & W_{39} &= 1 + 15 \frac{x_H^2}{\lambda_{ZZ}} \\
 W_{40} &= 2 + 3 \frac{x_H^4}{\lambda_{ZZ}^2} + 9 \frac{x_H^2}{\lambda_{ZZ}} & W_{41} &= 4 + 51 \frac{x_H^2}{\lambda_{ZZ}} & W_{42} &= 5 + 39 \frac{x_H^2}{\lambda_{ZZ}} & W_{43} &= 6 + 17 \frac{x_H^2}{\lambda_{ZZ}} \\
 W_{44} &= 17 + 51 \frac{x_H^4}{\lambda_{ZZ}^2} - 13 \frac{x_H^2}{\lambda_{ZZ}} & W_{45} &= 21 - 72 \frac{x_H^4}{\lambda_{ZZ}^2} - 29 \frac{x_H^2}{\lambda_{ZZ}} & W_{46} &= 6 \frac{x_H^2}{\lambda_{ZZ}^2} + 15 \frac{1}{\lambda_{ZZ}} + 2 \mathcal{R}_9 & W_{47} &= \mathcal{R}_6 \frac{1}{\lambda_{ZZ}} + \mathcal{R}_{13} \\
 W_{48} &= 1 - 3 \frac{x_H^4}{\lambda_{ZZ}^2} - \frac{x_H^2}{\lambda_{ZZ}} & W_{49} &= 1 + 3 \frac{x_H^4}{\lambda_{ZZ}^2} + 5 \frac{x_H^2}{\lambda_{ZZ}} & W_{50} &= 1 + 6 \frac{x_H^4}{\lambda_{ZZ}^2} + 3 \frac{x_H^2}{\lambda_{ZZ}} & W_{51} &= 6 \frac{x_H^2}{\lambda_{ZZ}^2} + 7 \frac{1}{\lambda_{ZZ}} + \mathcal{R}_9 \\
 W_{52} &= 1 + 6 \frac{x_H^4}{\lambda_{ZZ}^2} + 3 \frac{x_H^2}{\lambda_{ZZ}} & W_{53} &= 1 + 3 \frac{x_H^4}{\lambda_{ZZ}^2} & W_{54} &= 1 + 2 \mathcal{R}_{18} \frac{1}{\lambda_{WW}} & W_{55} &= \mathcal{R}_{18} + 2 \mathcal{R}_{19} \frac{1}{\lambda_{WW}} \\
 W_{56} &= 8 \mathcal{R}_{18} \frac{1}{\lambda_{WW}} + \mathcal{R}_{21} & W_{57} &= 8 \mathcal{R}_{18} \frac{1}{\lambda_{WW}} + \mathcal{R}_{23} & W_{58} &= 4 \mathcal{R}_{19} \frac{1}{\lambda_{WW}} + \mathcal{R}_{20} & W_{59} &= x_b^2 + 2 \mathcal{R}_{18} \frac{1}{\lambda_{WW}} \\
 W_{60} &= 8 \mathcal{R}_{18} \frac{x_t^2}{\lambda_{WW}} + \mathcal{R}_{26} & W_{61} &= 4 \mathcal{R}_{19} \frac{1}{\lambda_{WW}} + \mathcal{R}_{25} & W_{62} &= 1 - \mathcal{R}_{28} \frac{1}{\lambda_{WW}} & W_{63} &= 36 + \frac{x_H^2}{\lambda_{WW}}
 \end{aligned}$$

$$\begin{aligned}
 W_{64} &= 8 \mathcal{R}_{30} + \mathcal{R}_{31} \frac{1}{\lambda_{WW}} & W_{65} &= \mathcal{R}_{29} + \mathcal{R}_{34} \frac{1}{\lambda_{WW}} & W_{66} &= 2 \mathcal{R}_{30} + \mathcal{R}_{32} \frac{1}{\lambda_{WW}} \\
 W_{67} &= \mathcal{R}_{33} - \mathcal{R}_{35} \frac{x_H^4}{\lambda_{WW}} & W_{68} &= 2 - \frac{x_H^4}{\lambda_{WW}} & W_{69} &= 8 \frac{1}{\lambda_{WW}} + \mathcal{R}_0 \\
 W_{70} &= 16 \frac{1}{\lambda_{WW}} + \mathcal{R}_{37} & W_{71} &= 24 \frac{1}{\lambda_{WW}} - \mathcal{R}_{36} x_H^2 & W_{72} &= \frac{x_H^4}{\lambda_{WW}} + \mathcal{R}_{43} \\
 W_{73} &= 2 \mathcal{R}_0 \frac{1}{\lambda_{WW}} - \mathcal{R}_{40} & W_{74} &= \mathcal{R}_{28} \frac{1}{\lambda_{WW}} - \mathcal{R}_{41} & W_{75} &= \mathcal{R}_{42} \frac{x_H^2}{\lambda_{WW}} + \mathcal{R}_{44} \\
 W_{76} &= \mathcal{R}_{45} \frac{x_H^4}{\lambda_{WW}} - \mathcal{R}_{46} & W_{77} &= 8 + \frac{x_H^2}{\lambda_{WW}} & W_{78} &= \frac{1}{\lambda_{WW}} - \mathcal{R}_9 \\
 W_{79} &= \frac{1}{\lambda_{WW}} + \mathcal{R}_9 & W_{80} &= 12 \mathcal{R}_{18} \frac{1}{\lambda_{WW}^2} + \mathcal{R}_{47} \frac{1}{\lambda_{WW}} + \mathcal{R}_{52} & W_{81} &= 12 \mathcal{R}_{18} \frac{1}{\lambda_{WW}^2} + \mathcal{R}_{48} \frac{1}{\lambda_{WW}} + \mathcal{R}_{51} \\
 W_{82} &= 12 \mathcal{R}_{19} \frac{1}{\lambda_{WW}^2} + \mathcal{R}_{49} \frac{1}{\lambda_{WW}} - \mathcal{R}_{50} & W_{83} &= 1 + 2 \frac{1}{\lambda_{WW}} & W_{84} &= 1 - 2 \mathcal{R}_6 \frac{1}{\lambda_{WW}} - 12 \mathcal{R}_{18} \frac{1}{\lambda_{WW}^2} \\
 W_{85} &= 1 + 2 \mathcal{R}_{53} \frac{1}{\lambda_{WW}} & W_{86} &= \mathcal{R}_{18} - 12 \mathcal{R}_{18} \frac{1}{\lambda_{WW}^2} + 4 \mathcal{R}_{55} \frac{1}{\lambda_{WW}} & W_{87} &= 6 \mathcal{R}_{19} \frac{1}{\lambda_{WW}} + \mathcal{R}_{54} \\
 W_{88} &= 2 \frac{x_t^2}{\lambda_{WW}} + \mathcal{R}_{63} & W_{89} &= 2 \frac{x_t^2}{\lambda_{WW}} + \mathcal{R}_{64} & W_{90} &= 12 \mathcal{R}_{18} \frac{1}{\lambda_{WW}^2} + \mathcal{R}_{51} + \mathcal{R}_{59} \frac{1}{\lambda_{WW}} \\
 W_{91} &= 12 \mathcal{R}_{18} \frac{x_t^2}{\lambda_{WW}^2} + \mathcal{R}_{62} + \mathcal{R}_{65} \frac{1}{\lambda_{WW}} & W_{92} &= 12 \mathcal{R}_{19} \frac{1}{\lambda_{WW}^2} + \mathcal{R}_{60} \frac{1}{\lambda_{WW}} - \mathcal{R}_{61} & W_{93} &= \mathcal{R}_{50} - \mathcal{R}_{57} \frac{1}{\lambda_{WW}} \\
 W_{94} &= \mathcal{R}_{56} + \mathcal{R}_{58} \frac{1}{\lambda_{WW}} & W_{95} &= 1 - \frac{x_H^2}{\lambda_{WW}}
 \end{aligned}$$

$$\begin{aligned}
 W_{96} &= 1 - \frac{x_H^2}{\lambda_{WW}} + 12 \mathcal{X}_0 \frac{1}{\lambda_{WW}} & W_{97} &= 12 \mathcal{X}_0 \frac{1}{\lambda_{WW}} - \mathcal{X}_0 \frac{1}{\lambda_{WW}} - \mathcal{X}_{66} & W_{98} &= 12 \mathcal{X}_{28} \frac{1}{\lambda_{WW}} + \mathcal{X}_{36} + \mathcal{X}_{67} \frac{1}{\lambda_{WW}} \\
 W_{99} &= 1 - 12 \frac{x_H^2}{\lambda_{WW}^2} - \frac{x_H^2}{\lambda_{WW}} & W_{100} &= 1 + 12 \frac{x_H^2}{\lambda_{WW}^2} - \frac{x_H^2}{\lambda_{WW}} & W_{101} &= 1 + 3 \frac{x_H^2}{\lambda_{WW}} \\
 W_{102} &= 1 + \mathcal{X}_{36} \frac{1}{\lambda_{WW}} - 12 \mathcal{X}_{68} \frac{1}{\lambda_{WW}^2} & W_{103} &= 5x_H^2 - 12 \mathcal{X}_{71} \frac{1}{\lambda_{WW}^2} - \mathcal{X}_{74} \frac{1}{\lambda_{WW}} & W_{104} &= 12 \mathcal{X}_{69} \frac{x_H^4}{\lambda_{WW}^2} - \mathcal{X}_{72} - \mathcal{X}_{75} \frac{x_H^2}{\lambda_{WW}} \\
 W_{105} &= \mathcal{X}_{70} + 12 \mathcal{X}_{73} \frac{1}{\lambda_{WW}^2} + \mathcal{X}_{76} \frac{1}{\lambda_{WW}} & W_{106} &= 3 + 4 \frac{x_H^2}{\lambda_{WW}} & W_{107} &= 36 \frac{x_H^6}{\lambda_{WW}^2} + \mathcal{X}_{83} + \mathcal{X}_{86} \frac{x_H^2}{\lambda_{WW}} \\
 W_{108} &= x_H^2 - 12 \mathcal{X}_{78} \frac{1}{\lambda_{WW}^2} - \mathcal{X}_{80} \frac{1}{\lambda_{WW}} & W_{109} &= \mathcal{X}_{36} - \mathcal{X}_{36} \frac{x_H^2}{\lambda_{WW}} & W_{110} &= \mathcal{X}_{77} + 12 \mathcal{X}_{79} \frac{1}{\lambda_{WW}^2} + \mathcal{X}_{84} \frac{1}{\lambda_{WW}} \\
 W_{111} &= 12 \mathcal{X}_{81} \frac{1}{\lambda_{WW}^2} - \mathcal{X}_{82} - \mathcal{X}_{85} \frac{1}{\lambda_{WW}} & W_{112} &= 1 - 8 \frac{1}{\lambda_{WW}} & W_{113} &= 60 \frac{x_H^2}{\lambda_{WW}^2} - \mathcal{X}_{88} + \mathcal{X}_{90} \frac{1}{\lambda_{WW}} \\
 W_{114} &= 60 \frac{x_H^4}{\lambda_{WW}^2} + \mathcal{X}_{89} + \mathcal{X}_{91} \frac{x_H^2}{\lambda_{WW}} & W_{115} &= 5 \frac{x_H^2}{\lambda_{WW}} - \mathcal{X}_{87} & W_{116} &= 60 \frac{x_H^2}{\lambda_{WW}^2} + 17 \frac{x_H^2}{\lambda_{WW}} + \mathcal{X}_{94} \\
 W_{117} &= 13 \frac{1}{\lambda_{WW}} - \mathcal{X}_{93} & W_{118} &= 24 \frac{1}{\lambda_{WW}} + \mathcal{X}_{36} & W_{119} &= 1 - \mathcal{X}_{95} \frac{1}{\lambda_{WW}} \\
 W_{120} &= 60 \frac{x_H^2}{\lambda_{WW}^2} + \mathcal{X}_{94}
 \end{aligned}$$

**X**

Here  $W_{\Phi|\phi}$  denotes the  $\phi$  component of the  $\Phi$  wave-function factor etc. Furthermore,  $\sum_{\text{gen}}$  implies summing over all fermions and all generations, while  $\overline{\sum}_{\text{gen}}$  excludes t and b from the sum.

$$\begin{aligned}
 X_0 &= W_H^{(4)} + 2W_A^{(4)} - 2d\mathcal{Z}_g^{(4)} + d\mathcal{Z}_{M_W}^{(4)} & X_1 &= 3 - W_Z^{(4)} - W_H^{(4)} - W_A^{(4)} + 2d\mathcal{Z}_g^{(4)} - d\mathcal{Z}_{M_W}^{(4)} \\
 X_2 &= 2W_{Z|t}^{(4)} + W_{H|t}^{(4)} + 4d\mathcal{Z}_{c_\theta|t}^{(4)} & X_3 &= 2W_{Z|b}^{(4)} + W_{H|b}^{(4)} + 4d\mathcal{Z}_{c_\theta|b}^{(4)} \\
 X_4 &= 2 \sum_{\text{gen}} d\mathcal{Z}_{g|f}^{(4)} - 4 \overline{\sum}_{\text{gen}} d\mathcal{Z}_{c_\theta|f}^{(4)} + \sum_{\text{gen}} d\mathcal{Z}_{M|f}^{(4)} - 2 \overline{\sum}_{\text{gen}} W_{Z|f}^{(4)} & X_5 &= 4 - 2d\mathcal{Z}_{g|W}^{(4)} + 2W_{Z|W}^{(4)} + W_{H|W}^{(4)} - d\mathcal{Z}_{M|W}^{(4)} + 4d\mathcal{Z}_{c_\theta|W}^{(4)} \\
 X_6 &= 6 + 4d\mathcal{Z}_{g|W}^{(4)} - 4W_{Z|W}^{(4)} - 2W_{H|W}^{(4)} + 6W_{Z|b}^{(4)} + 3W_{H|b}^{(4)} + 6W_{Z|t}^{(4)} + 3W_{H|t}^{(4)} + 2d\mathcal{Z}_{M|W}^{(4)} - 8d\mathcal{Z}_{c_\theta|W}^{(4)} + 12d\mathcal{Z}_{c_\theta|b}^{(4)} + 12d\mathcal{Z}_{c_\theta|t}^{(4)} \\
 X_7 &= 11 + 30d\mathcal{Z}_{g|W}^{(4)} - 30W_{Z|W}^{(4)} - 15W_{H|W}^{(4)} + 15d\mathcal{Z}_{M|W}^{(4)} - 60d\mathcal{Z}_{c_\theta|W}^{(4)} \\
 X_8 &= 46 + 12d\mathcal{Z}_{g|W}^{(4)} - 12W_{Z|W}^{(4)} - 6W_{H|W}^{(4)} - 6 \sum_{\text{gen}} d\mathcal{Z}_{g|f}^{(4)} + 6 \overline{\sum}_{\text{gen}} d\mathcal{Z}_{c_\theta|f}^{(4)} - 3 \sum_{\text{gen}} d\mathcal{Z}_{M|f}^{(4)} + 6 \overline{\sum}_{\text{gen}} W_{Z|f}^{(4)} + 6W_{Z|b}^{(4)} + 3W_{H|b}^{(4)} \\
 &\quad + 6W_{Z|t}^{(4)} + 3W_{H|t}^{(4)} + 6d\mathcal{Z}_{M|W}^{(4)} - 30d\mathcal{Z}_{c_\theta|W}^{(4)} + 6d\mathcal{Z}_{c_\theta|b}^{(4)} + 6d\mathcal{Z}_{c_\theta|t}^{(4)} \\
 X_9 &= \overline{\sum}_{\text{gen}} d\mathcal{Z}_{c_\theta|f}^{(4)} + d\mathcal{Z}_{c_\theta|W}^{(4)} + d\mathcal{Z}_{c_\theta|b}^{(4)} + d\mathcal{Z}_{c_\theta|t}^{(4)} & X_{10} &= 1 - 2 \sum_{\text{gen}} d\mathcal{Z}_{g|f}^{(4)} + 4 \overline{\sum}_{\text{gen}} d\mathcal{Z}_{c_\theta|f}^{(4)} - \sum_{\text{gen}} d\mathcal{Z}_{M|f}^{(4)} + 2 \overline{\sum}_{\text{gen}} W_{Z|f}^{(4)} \\
 X_{11} &= 5 + 4 \overline{\sum}_{\text{gen}} d\mathcal{Z}_{c_\theta|f}^{(4)} + 4d\mathcal{Z}_{c_\theta|W}^{(4)} + 4d\mathcal{Z}_{c_\theta|b}^{(4)} + 4d\mathcal{Z}_{c_\theta|t}^{(4)} \\
 X_{12} &= 1 - 8 \sum_{\text{gen}} d\mathcal{Z}_{g|f}^{(4)} + 16 \overline{\sum}_{\text{gen}} d\mathcal{Z}_{c_\theta|f}^{(4)} + 8 \overline{\sum}_{\text{gen}} W_{Z|f}^{(4)} + 4W_{Z|b}^{(4)} + 2W_{H|b}^{(4)} + 4W_{Z|t}^{(4)} + 2W_{H|t}^{(4)} + 4d\mathcal{Z}_{M|W}^{(4)} + 8d\mathcal{Z}_{c_\theta|b}^{(4)} + 8d\mathcal{Z}_{c_\theta|t}^{(4)} \\
 X_{13} &= 3 + 2d\mathcal{Z}_{g|W}^{(4)} - 2W_{Z|W}^{(4)} - W_{H|W}^{(4)} + 2 \sum_{\text{gen}} d\mathcal{Z}_{g|f}^{(4)} - \sum_{\text{gen}} d\mathcal{Z}_{M|f}^{(4)} - 2 \overline{\sum}_{\text{gen}} W_{Z|f}^{(4)} - 2W_{Z|b}^{(4)} - W_{H|b}^{(4)} - 2W_{Z|t}^{(4)} - W_{H|t}^{(4)} - d\mathcal{Z}_{M|W}^{(4)} \\
 X_{14} &= 44 - 2 \sum_{\text{gen}} d\mathcal{Z}_{g|f}^{(4)} - \sum_{\text{gen}} d\mathcal{Z}_{M|f}^{(4)} + 2 \overline{\sum}_{\text{gen}} W_{Z|f}^{(4)} + 2W_{Z|b}^{(4)} + W_{H|b}^{(4)} + 2W_{Z|t}^{(4)} + W_{H|t}^{(4)} - 4d\mathcal{Z}_{c_\theta|W}^{(4)} \\
 X_{15} &= W_H^{(4)} 6 + 2W_Z^{(4)} 6 + 4d\mathcal{Z}_c^{(4)} 6 - d\mathcal{Z}_{M_W}^{(4)} 6 - 2d\mathcal{Z}_g^{(4)} 6 \\
 X_{16} &= 1 - 8d\mathcal{Z}_{g|W}^{(4)} + 8W_{Z|W}^{(4)} + 4W_{H|W}^{(4)} - 8 \sum_{\text{gen}} d\mathcal{Z}_{g|f}^{(4)} + 4 \sum_{\text{gen}} d\mathcal{Z}_{M|f}^{(4)} + 8 \overline{\sum}_{\text{gen}} W_{Z|f}^{(4)} + 8W_{Z|b}^{(4)} + 4W_{H|b}^{(4)} + 8W_{Z|t}^{(4)} + 4W_{H|t}^{(4)} \\
 &\quad + 4d\mathcal{Z}_{M|W}^{(4)} \\
 X_{17} &= 2 + 2d\mathcal{Z}_{g|t,b}^{(4)} + d\mathcal{Z}_{M|t,b}^{(4)} - 2W_{W|t,b}^{(4)} - W_{H|t,b}^{(4)} & X_{18} &= 2d\mathcal{Z}_{g|t,b}^{(4)} + d\mathcal{Z}_{M|t,b}^{(4)} - 2W_{W|t,b}^{(4)} - W_{H|t,b}^{(4)} \\
 X_{19} &= 103 + 144 \sum_{\text{gen}} d\mathcal{Z}_{g|f}^{(4)} + 72 \overline{\sum}_{\text{gen}} d\mathcal{Z}_{M|f}^{(4)} - 6W_{W|W}^{(4)} - 144 \overline{\sum}_{\text{gen}} W_{W|f}^{(4)} + 6d\mathcal{Z}_{g|W}^{(4)} - 3W_{H|W}^{(4)} + 3d\mathcal{Z}_{M|W}^{(4)}
 \end{aligned}$$

$$\begin{aligned}
 X_{20} &= 17 - 2W_{W|W}^{(4)} + 2d_{g|W}^{(4)} - W_{H|W}^{(4)} + d_{M|W}^{(4)} \\
 X_{21} &= 163 - 96 \sum_{\text{gen}} d_{g|f}^{(4)} - 48 \overline{\sum_{\text{gen}}} d_{M|f}^{(4)} + 26W_{W|W}^{(4)} + 96 \overline{\sum_{\text{gen}}} W_{W|f}^{(4)} - 26d_{g|W}^{(4)} + 13W_{H|W}^{(4)} - 13d_{M|W}^{(4)} \\
 X_{22} &= 11 - 16 \sum_{\text{gen}} d_{g|f}^{(4)} + 8 \overline{\sum_{\text{gen}}} d_{M|f}^{(4)} + 16W_{W|W}^{(4)} + 16 \overline{\sum_{\text{gen}}} W_{W|f}^{(4)} - 16d_{g|t,b}^{(4)} + 8d_{M|t,b}^{(4)} + 16W_{W|t,b}^{(4)} + 8W_{H|t,b}^{(4)} \\
 &\quad - 16d_{g|W}^{(4)} + 8W_{H|W}^{(4)} + 8d_{M|W}^{(4)} - 16d_{\mathcal{C}}^{(4)} \\
 X_{23} &= 35 + 120 \sum_{\text{gen}} d_{g|f}^{(4)} + 12 \overline{\sum_{\text{gen}}} d_{M|f}^{(4)} - 22W_{W|W}^{(4)} - 120 \overline{\sum_{\text{gen}}} W_{W|f}^{(4)} - 48d_{M|t,b}^{(4)} + 22d_{g|W}^{(4)} - 11W_{H|W}^{(4)} - 37d_{M|W}^{(4)} \\
 &\quad + 120d_{\mathcal{C}}^{(4)} \\
 X_{24} &= 293 + 12W_{W|W}^{(4)} - 12d_{g|W}^{(4)} + 6W_{H|W}^{(4)} - 6d_{M|W}^{(4)} \quad X_{25} = d_{\mathcal{C}}^{(4)} \\
 X_{26} &= 3 - 4 \sum_{\text{gen}} d_{g|f}^{(4)} + 2 \overline{\sum_{\text{gen}}} d_{M|f}^{(4)} + 4W_{W|W}^{(4)} + 4 \overline{\sum_{\text{gen}}} W_{W|f}^{(4)} - 4d_{g|t,b}^{(4)} + 2d_{M|t,b}^{(4)} + 4W_{W|t,b}^{(4)} + 2W_{H|t,b}^{(4)} - 4d_{g|W}^{(4)} \\
 &\quad + 2W_{H|W}^{(4)} + 2d_{M|W}^{(4)} \\
 X_{27} &= 72 - 10 \sum_{\text{gen}} d_{g|f}^{(4)} - \overline{\sum_{\text{gen}}} d_{M|f}^{(4)} + 2W_{W|W}^{(4)} + 10 \overline{\sum_{\text{gen}}} W_{W|f}^{(4)} + 4d_{M|t,b}^{(4)} - 2d_{g|W}^{(4)} + W_{H|W}^{(4)} + 3d_{M|W}^{(4)} \\
 X_{28} &= 3 - 8 \sum_{\text{gen}} d_{g|f}^{(4)} + 4 \overline{\sum_{\text{gen}}} d_{M|f}^{(4)} + 8W_{W|W}^{(4)} + 8 \overline{\sum_{\text{gen}}} W_{W|f}^{(4)} - 8d_{g|t,b}^{(4)} + 4d_{M|t,b}^{(4)} + 8W_{W|t,b}^{(4)} + 4W_{H|t,b}^{(4)} \\
 &\quad - 8d_{g|W}^{(4)} + 4W_{H|W}^{(4)} + 4d_{M|W}^{(4)} - 16d_{\mathcal{C}}^{(4)} \\
 X_{29} &= 11 - 20 \sum_{\text{gen}} d_{g|f}^{(4)} - 2 \overline{\sum_{\text{gen}}} d_{M|f}^{(4)} + 2W_{W|W}^{(4)} + 20 \overline{\sum_{\text{gen}}} W_{W|f}^{(4)} + 8d_{M|t,b}^{(4)} - 2d_{g|W}^{(4)} + W_{H|W}^{(4)} + 7d_{M|W}^{(4)} - 40d_{\mathcal{C}}^{(4)} \\
 X_{30} &= 2W_W^{(6)} + W_H^{(6)} - d_{M_W}^{(6)} - 2d_g^{(6)} \quad X_{31} = 2W_W^{(4)} + W_H^{(4)} - 2d_g^{(4)} - 2d_{\mathcal{C}}^{(4)} + d_{M_W}^{(4)} \\
 X_{32} &= 2W_W^{(4)} + W_H^{(4)} - 2d_g^{(4)} + d_{M_W}^{(4)} \quad X_{33} = 2W_W^{(4)} + W_H^{(4)} - 2d_g^{(4)} - 4d_{\mathcal{C}}^{(4)} + d_{M_W}^{(4)} \tag{H.1}
 \end{aligned}$$

## H.2 Amplitudes

We use the following notation:  $\mathcal{T}_{AA}^{\text{nfc}}(a_{uWB})$  is the non-factorizable part of the  $\mathcal{T}_{AA}$  amplitude that is proportional to the Wilson coefficient  $a_{uWB}$  etc.

$$\mathcal{D}_{\text{HVV}}^{\text{nfc}} = M_W \mathcal{T}_{D;VV}^{\text{nfc}}, \quad \mathcal{P}_{\text{HVV}}^{\text{nfc}} = \frac{1}{M_W} \mathcal{T}_{P;VV}^{\text{nfc}}, \tag{H.2}$$

with  $V = Z, W$ , while  $\mathcal{T}_{\text{HAA,HAZ}}^{\text{nfc}}$  should be multiplied by  $M_W$  to restore its dimensionality. Furthermore,  $\lambda_{AZ}$  is defined in eq. (5.34),  $\lambda_{ZZ}$  in eq. (5.50) and  $\lambda_{WW}$  in eq. (5.62). The function  $C_0$  in this appendix is the scalar three-point function, scaled with  $M_W$ . The amplitudes are listed in the following equations:

- HAA Amplitudes

$$\begin{aligned}
 \mathcal{T}_{AA}^{\text{nfc}}(a_{uWB}) &= -\frac{1}{8} s x_H^2 x_t^2 + \frac{1}{8} s x_H^2 x_t^2 a_0^{\text{fin}}(M_t) \\
 &\quad + \frac{1}{8} s x_H^2 x_t^2 B_0^{\text{fin}}(-M_H^2; M_t, M_t) \\
 &\quad - \frac{1}{16} s x_H^4 x_t^2 C_0(-M_H^2, 0, 0; M_t, M_t, M_t)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{T}_{AA}^{\text{nfc}}(a_{dWB}) &= \frac{1}{16} s x_H^2 x_b^2 - \frac{1}{16} s x_H^2 x_b^2 a_0^{\text{fin}}(M_b) \\
 &\quad - \frac{1}{16} s x_H^2 x_b^2 B_0^{\text{fin}}(-M_H^2; M_b, M_b) \\
 &\quad + \frac{1}{32} s x_H^4 x_b^2 C_0(-M_H^2, 0, 0; M_b, M_b, M_b)
 \end{aligned}$$



$$\begin{aligned}
 \mathcal{F}_{AA}^{\text{nfc}}(a_{AZ}) &= -\frac{1}{2} s^3 c x_H^2 a_0^{\text{fin}}(M_W) + \frac{1}{4} T_1^d s c x_H^2 \\
 &\quad - \frac{1}{16} T_2^d V_0 s c x_H^2 B_0^{\text{fin}}(-M_H^2; M_W, M_W) \\
 &\quad + \frac{1}{2} (s^2 x_H^2 + T_1^d) s c x_H^2 C_0(-M_H^2, 0, 0; M_W, M_W, M_W) \\
 &\quad - \frac{1}{16} (2T_0^d - T_2^d x_H^2) s c x_H^2 L_R
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{AA}^{\text{nfc}}(a_{AA}) &= -\frac{1}{16} x_H^2 X_0 + \frac{1}{4} T_4^d s^2 x_H^2 - \frac{1}{2} s^4 x_H^2 a_0^{\text{fin}}(M_W) \\
 &\quad - \frac{1}{64} x_H^4 B_0^{\text{fin}}(-M_H^2; M_Z, M_Z) \\
 &\quad - \frac{3}{64} x_H^4 B_0^{\text{fin}}(-M_H^2; M_H, M_H) \\
 &\quad + \frac{1}{2} (s^2 x_H^2 + T_4^d) s^2 x_H^2 C_0(-M_H^2, 0, 0; M_W, M_W, M_W) \\
 &\quad - \frac{1}{32} (8s^2 c^2 + T_3^d x_H^2) x_H^2 B_0^{\text{fin}}(-M_H^2; M_W, M_W) \\
 &\quad - \frac{1}{32} (8T_1^d s^2 + T_5^d x_H^2) x_H^2 L_R
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{AA}^{\text{nfc}}(a_{ZZ}) &= -\frac{3}{4} s^2 c^2 x_H^2 - \frac{1}{2} s^2 c^2 x_H^2 a_0^{\text{fin}}(M_W) \\
 &\quad + \frac{1}{8} V_0 s^2 c^2 x_H^2 B_0^{\text{fin}}(-M_H^2; M_W, M_W) \\
 &\quad - \frac{1}{2} V_1 s^2 c^2 x_H^2 C_0(-M_H^2, 0, 0; M_W, M_W, M_W) \\
 &\quad + \frac{1}{8} V_2 s^2 c^2 x_H^2 L_R
 \end{aligned}$$

• HAZ Amplitudes

$$\begin{aligned}
 \mathcal{F}_{AZ}^{\text{nfc}}(a_{\phi t v}) &= \frac{1}{2} \frac{s}{c^3} \frac{x_H^2}{\lambda_{AZ}^2} x_t^2 B_0^{\text{fin}}(-M_H^2; M_t, M_t) \\
 &\quad - \frac{1}{2} \frac{s}{c^3} \frac{x_H^2}{\lambda_{AZ}^2} x_t^2 B_0^{\text{fin}}(-M_Z^2; M_t, M_t) \\
 &\quad + \frac{1}{2} \frac{s}{c} \frac{x_H^2}{\lambda_{AZ}} x_t^2 \\
 &\quad + \frac{1}{4} \frac{s}{c} W_0 x_H^2 x_t^2 C_0(-M_H^2, 0, -M_Z^2; M_t, M_t, M_t)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{AZ}^{\text{nfc}}(a_{\phi b v}) &= \frac{1}{4} \frac{s}{c^3} \frac{x_H^2}{\lambda_{AZ}^2} x_b^2 B_0^{\text{fin}}(-M_H^2; M_b, M_b) \\
 &\quad - \frac{1}{4} \frac{s}{c^3} \frac{x_H^2}{\lambda_{AZ}^2} x_b^2 B_0^{\text{fin}}(-M_Z^2; M_b, M_b) \\
 &\quad + \frac{1}{4} \frac{s}{c} \frac{x_H^2}{\lambda_{AZ}} x_b^2 \\
 &\quad + \frac{1}{8} \frac{s}{c} W_1 x_H^2 x_b^2 C_0(-M_H^2, 0, -M_Z^2; M_b, M_b, M_b)
 \end{aligned}$$

$$\mathcal{F}_{AZ}^{\text{nfc}}(a_{tBW}) = \frac{1}{16} s x_H^2 x_t^2 a_0^{\text{fin}}(M_t) - \frac{1}{8} \frac{s}{c^2} \frac{x_H^2}{\lambda_{AZ}} x_t^2$$

$$\begin{aligned}
 & -\frac{1}{16} \left[ 2 \frac{1}{\lambda_{AZ}^2} + (-c^2 + \frac{1}{\lambda_{AZ}}) c^2 \right] \frac{s}{c^4} x_H^2 x_t^2 B_0^{\text{fin}}(-M_H^2; M_t, M_t) \\
 & + \frac{1}{8} (-c^2 + 2 \frac{1}{\lambda_{AZ}}) \frac{s}{c^2} x_H^2 x_t^4 C_0(-M_H^2, 0, -M_Z^2; M_t, M_t, M_t) \\
 & + \frac{1}{16} (c^2 + 2 \frac{1}{\lambda_{AZ}}) \frac{s}{c^4} \frac{x_H^2}{\lambda_{AZ}} x_t^2 B_0^{\text{fin}}(-M_Z^2; M_t, M_t)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{AZ}^{\text{nfc}}(a_{\text{tWB}}) = & -\frac{3}{128} \frac{v_t}{c} x_H^2 x_t^2 \\
 & -\frac{3}{64} \frac{v_t}{c} x_H^2 x_t^4 C_0(-M_H^2, 0, -M_Z^2; M_t, M_t, M_t) \\
 & + \frac{3}{128} (c^2 + \frac{1}{\lambda_{AZ}}) \frac{v_t}{c^3} x_H^2 x_t^2 B_0^{\text{fin}}(-M_H^2; M_t, M_t) \\
 & -\frac{3}{128} (c^2 + \frac{1}{\lambda_{AZ}}) \frac{v_t}{c^3} x_H^2 x_t^2 B_0^{\text{fin}}(-M_Z^2; M_t, M_t)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{AZ}^{\text{nfc}}(a_{\text{bBW}}) = & -\frac{1}{32} s x_H^2 x_b^2 a_0^{\text{fin}}(M_b) + \frac{1}{16} \frac{s}{c^2} \frac{x_H^2}{\lambda_{AZ}} x_b^2 \\
 & + \frac{1}{32} \left[ 2 \frac{1}{\lambda_{AZ}^2} + (-c^2 + \frac{1}{\lambda_{AZ}}) c^2 \right] \frac{s}{c^4} x_H^2 x_b^2 B_0^{\text{fin}}(-M_H^2; M_b, M_b) \\
 & -\frac{1}{16} (-c^2 + 2 \frac{1}{\lambda_{AZ}}) \frac{s}{c^2} x_H^2 x_b^4 C_0(-M_H^2, 0, -M_Z^2; M_b, M_b, M_b) \\
 & -\frac{1}{32} (c^2 + 2 \frac{1}{\lambda_{AZ}}) \frac{s}{c^4} \frac{x_H^2}{\lambda_{AZ}} x_b^2 B_0^{\text{fin}}(-M_Z^2; M_b, M_b)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{AZ}^{\text{nfc}}(a_{\text{bWB}}) = & \frac{3}{128} \frac{v_b}{c} x_H^2 x_b^2 \\
 & + \frac{3}{64} \frac{v_b}{c} x_H^2 x_b^4 C_0(-M_H^2, 0, -M_Z^2; M_b, M_b, M_b) \\
 & -\frac{3}{128} (c^2 + \frac{1}{\lambda_{AZ}}) \frac{v_b}{c^3} x_H^2 x_b^2 B_0^{\text{fin}}(-M_H^2; M_b, M_b) \\
 & + \frac{3}{128} (c^2 + \frac{1}{\lambda_{AZ}}) \frac{v_b}{c^3} x_H^2 x_b^2 B_0^{\text{fin}}(-M_Z^2; M_b, M_b)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{AZ}^{\text{nfc}}(a_{\phi D}) = & \frac{1}{32} \left[ 2 \frac{1}{\lambda_{AZ}} T_6^d c^2 - (\frac{x_H^2}{\lambda_{AZ}} T_{10}^d - 2 T_{11}^d) \right] \frac{1}{s c^3} x_H^2 C_0(-M_H^2, 0, -M_Z^2; M_W, M_W, M_W) \\
 & + \frac{1}{192} \left\{ 3 \frac{x_H^2}{\lambda_{AZ}} T_{10}^d - 6 \left[ T_6^d + (x_b^2 v_b + 2 x_t^2 v_t) c^2 \right] \right\} \frac{1}{\lambda_{AZ}} c^2 \\
 & + 2 \left[ (\frac{x_b^2}{\lambda_{AZ}} T_8^d + 2 \frac{x_t^2}{\lambda_{AZ}} T_9^d + 9 T_7^d) \right] s^2 c^2 \left\{ \frac{1}{s c^3} x_H^2 \right. \\
 & -\frac{1}{192} (3 c^2 v_b - T_8^d s^2) \frac{1}{s c} W_1 x_H^2 x_b^2 C_0(-M_H^2, 0, -M_Z^2; M_b, M_b, M_b) \\
 & -\frac{1}{96} (3 c^2 v_b - T_8^d s^2) \frac{1}{s c^3} \frac{x_H^2}{\lambda_{AZ}^2} x_b^2 B_0^{\text{fin}}(-M_H^2; M_b, M_b) \\
 & + \frac{1}{96} (3 c^2 v_b - T_8^d s^2) \frac{1}{s c^3} \frac{x_H^2}{\lambda_{AZ}^2} x_b^2 B_0^{\text{fin}}(-M_Z^2; M_b, M_b) \\
 & -\frac{1}{96} (3 c^2 v_t - T_9^d s^2) \frac{1}{s c} W_0 x_H^2 x_t^2 C_0(-M_H^2, 0, -M_Z^2; M_t, M_t, M_t) \\
 & \left. -\frac{1}{48} (3 c^2 v_t - T_9^d s^2) \frac{1}{s c^3} \frac{x_H^2}{\lambda_{AZ}^2} x_t^2 B_0^{\text{fin}}(-M_H^2; M_t, M_t) \right\}
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{48} (3c^2 v_t - T_9^d s^2) \frac{1}{s c^3} \frac{x_H^2}{\lambda_{AZ}^2} x_t^2 B_0^{\text{fin}}(-M_Z^2; M_t, M_t) \\
 & - \frac{1}{64} \left( -\frac{x_H^2}{\lambda_{AZ}} T_{10}^d + 2 \frac{1}{\lambda_{AZ}} T_6^d c^2 - 6 T_7^d s^2 c^2 \right) \frac{1}{s c^5} \frac{x_H^2}{\lambda_{AZ}} B_0^{\text{fin}}(-M_H^2; M_W, M_W) \\
 & + \frac{1}{64} \left( -\frac{x_H^2}{\lambda_{AZ}} T_{10}^d + 2 \frac{1}{\lambda_{AZ}} T_6^d c^2 - 6 T_7^d s^2 c^2 \right) \frac{1}{s c^5} \frac{x_H^2}{\lambda_{AZ}} B_0^{\text{fin}}(-M_Z^2; M_W, M_W)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{G}_{AZ}^{\text{nfc}}(a_{AZ}) = & \frac{1}{32} x_H^2 X_1 - \frac{1}{64} x_H^4 a_0^{\text{fin}}(M_H) + \frac{1}{64} \frac{1}{c^2} x_H^2 a_0^{\text{fin}}(M_Z) \\
 & - \frac{1}{64} \left[ 2 \frac{1}{\lambda_{AZ}} - (c^2 x_H^2 + 2W_4) c^2 \right] \frac{1}{c^6} \frac{x_H^2}{\lambda_{AZ}} B_0^{\text{fin}}(-M_Z^2; M_H, M_Z) \\
 & + \frac{1}{16} \left[ \frac{1}{\lambda_{AZ}} T_7^d + \left( \frac{x_H^2}{\lambda_{AZ}} T_2^d - T_{26}^d \right) c^2 \right] \frac{s^2}{c^2} x_H^2 a_0^{\text{fin}}(M_W) \\
 & + \frac{1}{32} \left\{ 16 s^2 c^6 x_H^2 + 8 \frac{1}{\lambda_{AZ}} T_{27}^d c^4 - \left[ -\frac{x_H^2}{\lambda_{AZ}} T_{22}^d + 2 T_{15}^d \right] \right\} \frac{1}{c^4} x_H^2 \\
 & \times C_0(-M_H^2, 0, -M_Z^2; M_W, M_W, M_W) \\
 & - \frac{1}{64} \left\{ 4 \frac{x_H^2}{\lambda_{AZ}} T_2^d s^2 c^6 - \left[ (2 T_{19}^d + T_{20}^d x_H^2) \right] \frac{1}{\lambda_{AZ}^2} + 2 \left( 3 \frac{1}{\lambda_{AZ}} T_{23}^d + T_{24}^d c^2 \right) c^4 \right\} \frac{1}{c^6} x_H^2 \\
 & \times B_0^{\text{fin}}(-M_Z^2; M_W, M_W) \\
 & + \frac{1}{64} \left\{ 8 \frac{x_H^2}{\lambda_{AZ}} T_2^d s^2 c^6 + \left[ 2 \frac{1}{\lambda_{AZ}} T_{28}^d - (2 T_2^d - T_{13}^d x_H^2) c^2 \right] c^4 - \left[ (2 T_{19}^d + T_{20}^d x_H^2) \right] \frac{1}{\lambda_{AZ}^2} \right\} \frac{1}{c^6} x_H^2 \\
 & \times B_0^{\text{fin}}(-M_H^2; M_W, M_W) \\
 & + \frac{1}{32} \left\{ \frac{1}{\lambda_{AZ}^2} - \left[ 2 \frac{1}{\lambda_{AZ}} W_2 - \left( \frac{1}{\lambda_{AZ}} W_2 + W_2 c^2 \right) c^2 x_H^2 \right] c^2 \right\} \frac{1}{c^8} x_H^2 C_0(-M_H^2, 0, -M_Z^2; M_Z, M_H, M_Z) \\
 & + \frac{1}{128} \left\{ 4 \frac{1}{\lambda_{AZ}^2} - \left[ 2 \frac{1}{\lambda_{AZ}} W_5 + (c^2 x_H^2 + 2W_3) c^2 \right] c^2 \right\} \frac{1}{c^6} x_H^2 B_0^{\text{fin}}(-M_H^2; M_Z, M_Z) \\
 & - \frac{1}{192} \left\{ 3 \left[ T_{18}^d - 8(x_b^2 v_b + 2x_t^2 v_t) \right] \frac{1}{\lambda_{AZ}} c^2 \right\} c^2 + 3 \left[ (2 T_{17}^d + T_{21}^d x_H^2) \right] \frac{1}{\lambda_{AZ}} \\
 & + 8 \left( T_{12}^d x_b^2 + 2 T_{25}^d x_t^2 \right) \frac{1}{\lambda_{AZ}} s^2 c^4 \left\} \frac{1}{c^4} x_H^2 \\
 & - \frac{1}{64} (1 - c^2 x_H^2) \frac{1}{c^4} \frac{x_H^2}{\lambda_{AZ}} B_0^{\text{fin}}(0; M_Z, M_H) \\
 & - \frac{3}{32} (1 - c^2 x_H^2) \frac{1}{c^4} \frac{x_H^2}{\lambda_{AZ}} x_H^2 C_0(-M_H^2, 0, -M_Z^2; M_H, M_Z, M_H) \\
 & + \frac{1}{24} (3v_b - T_{12}^d s^2) \frac{1}{c^2} \frac{x_H^2}{\lambda_{AZ}^2} x_b^2 B_0^{\text{fin}}(-M_H^2; M_b, M_b) \\
 & - \frac{1}{24} (3v_b - T_{12}^d s^2) \frac{1}{c^2} \frac{x_H^2}{\lambda_{AZ}^2} x_b^2 B_0^{\text{fin}}(-M_Z^2; M_b, M_b) \\
 & + \frac{1}{48} (3v_b - T_{12}^d s^2) W_1 x_H^2 x_b^2 C_0(-M_H^2, 0, -M_Z^2; M_b, M_b, M_b) \\
 & + \frac{1}{12} (3v_t - T_{25}^d s^2) \frac{1}{c^2} \frac{x_H^2}{\lambda_{AZ}^2} x_t^2 B_0^{\text{fin}}(-M_H^2; M_t, M_t) \\
 & - \frac{1}{12} (3v_t - T_{25}^d s^2) \frac{1}{c^2} \frac{x_H^2}{\lambda_{AZ}^2} x_t^2 B_0^{\text{fin}}(-M_Z^2; M_t, M_t) \\
 & + \frac{1}{24} (3v_t - T_{25}^d s^2) W_0 x_H^2 x_t^2 C_0(-M_H^2, 0, -M_Z^2; M_t, M_t, M_t)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{3}{128} (-c^2 + 4 \frac{1}{\lambda_{AZ}}) \frac{1}{c^2} x_H^4 B_0^{\text{fin}} (-M_H^2; M_H, M_H) \\
 & - \frac{1}{64} (c^2 + 6 \frac{1}{\lambda_{AZ}}) \frac{1}{c^2} x_H^4 B_0^{\text{fin}} (-M_Z^2; M_Z, M_H) \\
 & - \frac{1}{64} (T_{14}^d c^2 x_H^2 + 2 T_{16}^d) \frac{1}{c^2} x_H^2 L_R
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{AZ}^{\text{nfc}}(a_{AA}) = & - \frac{1}{12} s c x_H^2 x_b^2 v_b a_0^{\text{fin}}(M_b) - \frac{1}{6} s c x_H^2 x_t^2 v_t a_0^{\text{fin}}(M_t) \\
 & - \frac{1}{24} \frac{s^3}{c^3} \frac{x_H^2}{\lambda_{AZ}^2} T_{40}^d x_b^2 B_0^{\text{fin}} (-M_H^2; M_b, M_b) \\
 & - \frac{1}{12} \frac{s^3}{c^3} \frac{x_H^2}{\lambda_{AZ}^2} T_{43}^d x_t^2 B_0^{\text{fin}} (-M_H^2; M_t, M_t) \\
 & - \frac{1}{48} \frac{s^3}{c} T_{40}^d W_1 x_H^2 x_b^2 C_0 (-M_H^2, 0, -M_Z^2; M_b, M_b, M_b) \\
 & - \frac{1}{24} \frac{s^3}{c} T_{43}^d W_0 x_H^2 x_t^2 C_0 (-M_H^2, 0, -M_Z^2; M_t, M_t, M_t) \\
 & - \frac{1}{24} \frac{s}{c} x_H^2 v_1 B_0^{\text{fin}} (-M_Z^2; 0, 0) \\
 & + \frac{1}{24} \frac{s}{c} U_0 x_H^2 (1 - 3 L_R) \\
 & + \frac{1}{24} \left[ \frac{x_b^2}{\lambda_{AZ}^2} T_{40}^d s^2 - (1 + 2c^2 x_b^2) c^2 v_b \right] \frac{s}{c^3} x_H^2 B_0^{\text{fin}} (-M_Z^2; M_b, M_b) \\
 & + \frac{1}{12} \left[ \frac{x_t^2}{\lambda_{AZ}^2} T_{43}^d s^2 - (1 + 2c^2 x_t^2) c^2 v_t \right] \frac{s}{c^3} x_H^2 B_0^{\text{fin}} (-M_Z^2; M_t, M_t) \\
 & + \frac{1}{16} \left\{ 8s^2 c^4 x_H^2 - 8 \frac{1}{\lambda_{AZ}} T_{44}^d c^4 + \left[ -\frac{x_H^2}{\lambda_{AZ}} T_{35}^d + 2 T_{30}^d \right] \right\} \frac{s}{c^3} x_H^2 \\
 & \times C_0 (-M_H^2, 0, -M_Z^2; M_W, M_W, M_W) \\
 & + \frac{1}{32} \left\{ 8 \frac{x_H^2}{\lambda_{AZ}} s^2 c^6 - 2 \left[ \frac{1}{\lambda_{AZ}} T_{42}^d + (1 - T_2^d x_H^2) c^2 \right] c^4 + \left[ (2 T_{32}^d + T_{34}^d x_H^2) \right] \frac{1}{\lambda_{AZ}^2} \right\} \frac{s}{c^5} x_H^2 \\
 & \times B_0^{\text{fin}} (-M_H^2; M_W, M_W) \\
 & - \frac{1}{96} \left\{ 12 \frac{x_H^2}{\lambda_{AZ}} s^2 c^6 + 3 \left[ (2 T_{32}^d + T_{34}^d x_H^2) \right] \frac{1}{\lambda_{AZ}^2} - 2 \left( 9 \frac{1}{\lambda_{AZ}} T_{38}^d + T_{36}^d \right) c^4 \right\} \frac{s}{c^5} x_H^2 \\
 & \times B_0^{\text{fin}} (-M_Z^2; M_W, M_W) \\
 & + \frac{1}{288} \left\{ 2 \left[ T_{37}^d - 12 (x_b^2 v_b + 2 x_t^2 v_t) c^2 \right] c^2 + 9 \left[ (2 T_{31}^d + T_{33}^d x_H^2) \right] \frac{1}{\lambda_{AZ}} \right. \\
 & \left. - 12 (T_{40}^d x_b^2 + 2 T_{43}^d x_t^2) \frac{1}{\lambda_{AZ}} s^2 c^2 \right\} \frac{s}{c^3} x_H^2 \\
 & + \frac{1}{24} \left( 3 \frac{x_H^2}{\lambda_{AZ}} s^2 c^2 - 3 \frac{1}{\lambda_{AZ}} T_{29}^d - T_{39}^d c^2 \right) \frac{s}{c} x_H^2 a_0^{\text{fin}}(M_W) \\
 & + \frac{1}{48} (3 T_2^d c^2 x_H^2 + T_{41}^d) \frac{s}{c} x_H^2 L_R
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{AZ}^{\text{nfc}}(a_{zz}) = & \frac{1}{24} \frac{s}{c} \frac{x_H^2}{\lambda_{AZ}^2} T_{49}^d x_b^2 B_0^{\text{fin}} (-M_H^2; M_b, M_b) \\
 & - \frac{1}{24} \frac{s}{c} \frac{x_H^2}{\lambda_{AZ}^2} T_{49}^d x_b^2 B_0^{\text{fin}} (-M_Z^2; M_b, M_b) \\
 & + \frac{1}{12} \frac{s}{c} \frac{x_H^2}{\lambda_{AZ}^2} T_{50}^d x_t^2 B_0^{\text{fin}} (-M_H^2; M_t, M_t)
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{1}{12} \frac{s}{c} \frac{x_H^2}{\lambda_{AZ}^2} T_{50}^d x_t^2 B_0^{\text{fin}}(-M_Z^2; M_t, M_t) \\
 & + \frac{1}{48} T_{49}^d W_1 s c x_H^2 x_b^2 C_0(-M_H^2, 0, -M_Z^2; M_b, M_b, M_b) \\
 & + \frac{1}{24} T_{50}^d W_0 s c x_H^2 x_t^2 C_0(-M_H^2, 0, -M_Z^2; M_t, M_t, M_t) \\
 & + \frac{1}{32} \left\{ 4 \frac{x_H^2}{\lambda_{AZ}} s^2 c^6 + \left[ (2T_{32}^d + T_{47}^d x_H^2) \right] \frac{1}{\lambda_{AZ}^2} - 2 \left( 3 \frac{1}{\lambda_{AZ}} T_{38}^d - T_{51}^d c^2 \right) c^4 \right\} \frac{s}{c^5} x_H^2 \\
 & \times B_0^{\text{fin}}(-M_Z^2; M_W, M_W) \\
 & - \frac{1}{32} \left\{ 8 \frac{x_H^2}{\lambda_{AZ}} s^2 c^6 - 2 \left[ \frac{1}{\lambda_{AZ}} T_{42}^d + (1 - T_2^d x_H^2) c^2 \right] c^4 + \left[ (2T_{32}^d + T_{47}^d x_H^2) \right] \frac{1}{\lambda_{AZ}^2} \right\} \frac{s}{c^5} x_H^2 \\
 & \times B_0^{\text{fin}}(-M_H^2; M_W, M_W) \\
 & - \frac{1}{96} \left\{ 2 \left[ \left( -2 \frac{x_b^2}{\lambda_{AZ}} T_{49}^d - 4 \frac{x_t^2}{\lambda_{AZ}} T_{50}^d + 21 T_2^d \right) \right] c^4 + 3 \left[ (2T_{31}^d + T_{46}^d x_H^2) \right] \frac{1}{\lambda_{AZ}} \right\} \frac{s}{c^3} x_H^2 \\
 & + \frac{1}{16} \left\{ - \left[ \left( -\frac{x_H^2}{\lambda_{AZ}} T_{48}^d + 2 T_{45}^d \right) \right] + 8 \left( c^2 x_H^2 + \frac{1}{\lambda_{AZ}} T_{44}^d \right) c^4 \right\} \frac{s}{c^3} x_H^2 \\
 & \times C_0(-M_H^2, 0, -M_Z^2; M_W, M_W, M_W) \\
 & + \frac{1}{16} (12c^2 - T_2^d x_H^2) s c x_H^2 L_R \\
 & - \frac{1}{8} \left( \frac{x_H^2}{\lambda_{AZ}} s^2 c^2 - \frac{1}{\lambda_{AZ}} T_{29}^d + T_4^d c^2 \right) \frac{s}{c} x_H^2 a_0^{\text{fin}}(M_W)
 \end{aligned}$$

• HZZ Amplitudes

$$\begin{aligned}
 \mathcal{F}_{D;ZZ}^{\text{nfc}}(a_{\phi t v}) &= \frac{5}{16} \frac{v_t}{c^2} x_t^2 + \frac{1}{8} \frac{v_t}{c^2} x_t^2 a_0^{\text{fin}}(M_t) - \frac{1}{48} \frac{v_t}{c^4} (1 - 3L_R) \\
 & + \frac{3}{8} \frac{v_t}{c^4} \frac{x_t^2}{\lambda_{ZZ}} B_0^{\text{fin}}(-M_H^2; M_t, M_t) \\
 & - \frac{3}{32} \left[ 4 \frac{1}{\lambda_{ZZ}} + (2 + V_3 c^2) c^2 \right] \frac{v_t}{c^6} x_t^2 C_0(-M_H^2, -M_Z^2, -M_Z^2; M_t, M_t, M_t) \\
 & + \frac{1}{16} (sW_6 + 2c^2 x_t^2) \frac{v_t}{c^4} B_0^{\text{fin}}(-M_Z^2; M_t, M_t)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{D;ZZ}^{\text{nfc}}(a_{\phi t A}) &= \frac{1}{8} \frac{1}{c^2} x_t^2 a_0^{\text{fin}}(M_t) + \frac{1}{16} \frac{1}{c^2} x_t^2 (5 - 12L_R) - \frac{1}{48} \frac{1}{c^4} (1 - 3L_R) \\
 & + \frac{3}{8} \frac{1}{c^4} \frac{x_t^2}{\lambda_{ZZ}} B_0^{\text{fin}}(-M_H^2; M_t, M_t) \\
 & - \frac{3}{32} \left[ 4 \frac{1}{\lambda_{ZZ}} + (2 - V_3 c^2) c^2 \right] \frac{1}{c^6} x_t^2 C_0(-M_H^2, -M_Z^2, -M_Z^2; M_t, M_t, M_t) \\
 & + \frac{1}{16} (sW_6 - 10c^2 x_t^2) \frac{1}{c^4} B_0^{\text{fin}}(-M_Z^2; M_t, M_t)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{D;ZZ}^{\text{nfc}}(a_{\phi b v}) &= \frac{5}{16} \frac{v_b}{c^2} x_b^2 + \frac{1}{8} \frac{v_b}{c^2} x_b^2 a_0^{\text{fin}}(M_b) - \frac{1}{48} \frac{v_b}{c^4} (1 - 3L_R) \\
 & + \frac{3}{8} \frac{v_b}{c^4} \frac{x_b^2}{\lambda_{ZZ}} B_0^{\text{fin}}(-M_H^2; M_b, M_b) \\
 & - \frac{3}{32} \left[ 4 \frac{1}{\lambda_{ZZ}} + (2 + V_4 c^2) c^2 \right] \frac{v_b}{c^6} x_b^2 C_0(-M_H^2, -M_Z^2, -M_Z^2; M_b, M_b, M_b)
 \end{aligned}$$

$$+ \frac{1}{16} (sW_7 + 2c^2 x_b^2) \frac{v_b}{c^4} B_0^{\text{fin}}(-M_Z^2; M_b, M_b)$$

$$\begin{aligned} \mathcal{F}_{D;ZZ}^{\text{nfc}}(a_{\phi b_A}) &= \frac{1}{8} \frac{1}{c^2} x_b^2 a_0^{\text{fin}}(M_b) + \frac{1}{16} \frac{1}{c^2} x_b^2 (5 - 12L_R) - \frac{1}{48} \frac{1}{c^4} (1 - 3L_R) \\ &+ \frac{3}{8} \frac{1}{c^4} \frac{x_b^2}{\lambda_{ZZ}} B_0^{\text{fin}}(-M_H^2; M_b, M_b) \\ &- \frac{3}{32} \left[ 4 \frac{1}{\lambda_{ZZ}} + (2 - V_4 c^2) c^2 \right] \frac{1}{c^6} x_b^2 C_0(-M_H^2, -M_Z^2, -M_Z^2; M_b, M_b, M_b) \\ &+ \frac{1}{16} (sW_7 - 10c^2 x_b^2) \frac{1}{c^4} B_0^{\text{fin}}(-M_Z^2; M_b, M_b) \end{aligned}$$

$$\mathcal{F}_{D;ZZ}^{\text{nfc}}(a_{\phi 1_V}) = \frac{1}{48} \frac{1}{c^4} v_1 B_0^{\text{fin}}(-M_Z^2; 0, 0) - \frac{1}{144} \frac{1}{c^4} v_1 (1 - 3L_R)$$

$$\mathcal{F}_{D;ZZ}^{\text{nfc}}(a_{\phi 1_A}) = \frac{1}{48} \frac{1}{c^4} B_0^{\text{fin}}(-M_Z^2; 0, 0) - \frac{1}{144} \frac{1}{c^4} (1 - 3L_R)$$

$$\mathcal{F}_{D;ZZ}^{\text{nfc}}(a_{t\phi}) = -\frac{1}{32} \frac{1}{c^2} X_2$$

$$\mathcal{F}_{D;ZZ}^{\text{nfc}}(a_{b\phi}) = \frac{1}{32} \frac{1}{c^2} X_3$$

$$\begin{aligned} \mathcal{F}_{D;ZZ}^{\text{nfc}}(a_{tBW}) &= -\frac{3}{128} \frac{v_t}{c} x_H^2 x_t^2 \\ &- \frac{3}{128} \left[ 8 \frac{1}{\lambda_{ZZ}} + (2 - c^2 x_H^2) c^2 \right] \frac{v_t}{c^5} x_t^2 B_0^{\text{fin}}(-M_H^2; M_t, M_t) \\ &+ \frac{3}{128} \left[ 8 \frac{1}{\lambda_{ZZ}} + (2 - c^2 x_H^2) c^2 \right] \frac{v_t}{c^5} x_t^2 B_0^{\text{fin}}(-M_Z^2; M_t, M_t) \\ &+ \frac{3}{64} \left\{ 4 \frac{1}{\lambda_{ZZ}} + \left[ 1 + (4 - c^2 x_H^2) c^2 x_t^2 \right] c^2 \right\} \frac{1}{c^7} x_t^2 v_t C_0(-M_H^2, -M_Z^2, -M_Z^2; M_t, M_t, M_t) \end{aligned}$$

$$\begin{aligned} \mathcal{F}_{D;ZZ}^{\text{nfc}}(a_{bBW}) &= \frac{3}{128} \frac{v_b}{c} x_H^2 x_b^2 \\ &+ \frac{3}{128} \left[ 8 \frac{1}{\lambda_{ZZ}} + (2 - c^2 x_H^2) c^2 \right] \frac{v_b}{c^5} x_b^2 B_0^{\text{fin}}(-M_H^2; M_b, M_b) \\ &- \frac{3}{128} \left[ 8 \frac{1}{\lambda_{ZZ}} + (2 - c^2 x_H^2) c^2 \right] \frac{v_b}{c^5} x_b^2 B_0^{\text{fin}}(-M_Z^2; M_b, M_b) \\ &- \frac{3}{64} \left\{ 4 \frac{1}{\lambda_{ZZ}} + \left[ 1 + (4 - c^2 x_H^2) c^2 x_b^2 \right] c^2 \right\} \frac{1}{c^7} x_b^2 v_b C_0(-M_H^2, -M_Z^2, -M_Z^2; M_b, M_b, M_b) \end{aligned}$$

$$\begin{aligned} \mathcal{F}_{D;ZZ}^{\text{nfc}}(a_\phi) &= \frac{3}{16} \frac{1}{c^2} \\ &- \frac{3}{8} \left[ 4 \frac{1}{\lambda_{ZZ}} + \left( \frac{x_H^2}{\lambda_{ZZ}} c^2 x_H^2 - W_8 \right) c^2 \right] \frac{1}{c^6} C_0(-M_H^2, -M_Z^2, -M_Z^2; M_H, M_Z, M_H) \\ &+ \frac{3}{8} (2 - c^2 x_H^2) \frac{1}{c^4 \lambda_{ZZ}} B_0^{\text{fin}}(-M_H^2; M_H, M_H) \\ &- \frac{3}{8} (2 - c^2 x_H^2) \frac{1}{c^4 \lambda_{ZZ}} B_0^{\text{fin}}(-M_Z^2; M_H, M_Z) \end{aligned}$$

$$\mathcal{F}_{D;ZZ}^{\text{nfc}}(a_{\phi\Box}) = -\frac{1}{16} \frac{1}{c^2} X_4 - \frac{1}{192} \frac{1}{c^2} x_H^2 (25 + 3L_R) - \frac{1}{288} \frac{1}{c^4} (13 - 57L_R)$$

$$\begin{aligned}
 & -\frac{1}{16} \left[ 2 - (c^2 x_H^2 + W_{11}) c^2 x_H^2 \right] \frac{1}{c^6} C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_Z, M_H, M_Z \right) \\
 & -\frac{1}{32} \left[ 8 \frac{1}{\lambda_{ZZ}} + (2W_{11} - W_{12} c^2 x_H^2) c^2 \right] \frac{1}{c^6} B_0^{\text{fin}} \left( -M_H^2; M_H, M_H \right) \\
 & +\frac{1}{32} \left\{ 16 \frac{1}{\lambda_{ZZ}} + \left[ 4W_{10} + (W_{12} c^2 x_H^2 - 2W_{16}) c^2 x_H^2 \right] c^2 \right\} \frac{1}{c^8} \\
 & \times C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_H, M_Z, M_H \right) \\
 & +\frac{1}{96} \left\{ 24 \frac{1}{\lambda_{ZZ}} + \left[ 6W_{15} + (c^2 x_H^2 - W_{14}) c^2 x_H^2 \right] c^2 \right\} \frac{1}{c^6} B_0^{\text{fin}} \left( -M_Z^2; M_H, M_Z \right) \\
 & +\frac{1}{96} (2 - c^2 x_H^2) \frac{1}{c^4} a_0^{\text{fin}} (M_Z) \\
 & -\frac{1}{96} (3 - c^2 x_H^2) \frac{1}{c^2} x_H^2 a_0^{\text{fin}} (M_H) \\
 & -\frac{1}{64} (2sW_{13} + 3c^2 x_H^2) \frac{1}{c^4} B_0^{\text{fin}} \left( -M_H^2; M_Z, M_Z \right) \\
 & -\frac{1}{16} \left( 4 \frac{1}{\lambda_{ZZ}} T_{52}^d + T_{52}^d c^2 sW_9 \right) \frac{1}{c^6} B_0^{\text{fin}} \left( -M_H^2; M_W, M_W \right) \\
 & +\frac{1}{16} \left( 4 \frac{1}{\lambda_{ZZ}} T_{52}^d + T_{52}^d c^2 sW_9 \right) \frac{1}{c^6} B_0^{\text{fin}} \left( -M_Z^2; M_W, M_W \right) \\
 & +\frac{1}{16} \left( 4 \frac{1}{\lambda_{ZZ}} T_{52}^d + T_{52}^d c^2 sW_9 \right) \frac{1}{c^8} C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_W, M_W, mw \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{D;ZZ}^{\text{nfc}}(a_{\phi D}) &= \frac{1}{192} \frac{1}{c^2} X_6 - \frac{1}{384} \frac{1}{c^4} X_5 \\
 & -\frac{1}{384} \left[ 4T_{63}^d c^2 + (T_{56}^d x_H^2 + 2T_{69}^d) \frac{1}{\lambda_{ZZ}} \right] \frac{1}{c^6} B_0^{\text{fin}} \left( -M_Z^2; M_W, M_W \right) \\
 & +\frac{1}{384} \left[ T_{68}^d c^2 + (T_{56}^d x_H^2 + 2T_{69}^d) \frac{1}{\lambda_{ZZ}} \right] \frac{1}{c^6} B_0^{\text{fin}} \left( -M_H^2; M_W, M_W \right) \\
 & -\frac{1}{384} \left[ (T_{56}^d x_H^2 + 2T_{69}^d) \frac{1}{\lambda_{ZZ}} - (T_{61}^d c^2 x_H^2 - 6T_{62}^d) c^2 \right] \frac{1}{c^8} \\
 & \times C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_W, M_W, M_W \right) \\
 & +\frac{1}{64} \left\{ 3c^2 + \left[ (3 - T_8^d v_b) \right] \frac{1}{\lambda_{ZZ}} \right\} \frac{1}{c^4} x_b^2 B_0^{\text{fin}} \left( -M_Z^2; M_b, M_b \right) \\
 & +\frac{1}{64} \left\{ 3c^2 + \left[ (3 - T_9^d v_t) \right] \frac{1}{\lambda_{ZZ}} \right\} \frac{1}{c^4} x_t^2 B_0^{\text{fin}} \left( -M_Z^2; M_t, M_t \right) \\
 & -\frac{1}{256} \left\{ 4 \frac{1}{\lambda_{ZZ}} V_5 + \left[ (8 - 4 \frac{x_H^2}{\lambda_{ZZ}} T_{57}^d x_H^2 - T_{60}^d x_H^2) + (2c^2 + \frac{x_H^2}{\lambda_{ZZ}} T_{58}^d) c^2 x_H^4 \right] c^2 \right\} \frac{1}{c^8} \\
 & \times C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_H, M_Z, M_H \right) \\
 & +\frac{1}{768} \left\{ 4T_7^d - \left[ \left( \frac{x_H^2}{\lambda_{ZZ}} \right) T_8^d + T_{55}^d \right] c^2 x_H^2 + 2 \left( \frac{x_H^2}{\lambda_{ZZ}} T_{53}^d + T_7^d \right) c^2 x_H^2 \right\} \frac{1}{c^8} \\
 & \times C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_Z, M_H, M_Z \right) \\
 & +\frac{1}{2304} \left\{ T_{66}^d + 3 \left[ T_{65}^d x_H^2 + 6(T_8^d x_b^2 v_b + T_9^d x_t^2 v_t - 3V_6) c^2 \right] c^2 \right\} \frac{1}{c^6} \\
 & +\frac{1}{256} \left\{ \left[ 7c^2 x_H^2 + \left( 4 - \frac{x_H^2}{\lambda_{ZZ}} T_{58}^d x_H^2 \right) c^2 + 2 \left[ (8 + T_8^d x_H^2) \right] \frac{1}{\lambda_{ZZ}} \right] \frac{1}{c^6} B_0^{\text{fin}} \left( -M_H^2; M_H, M_H \right) \right. \\
 & \left. +\frac{1}{384} \left\{ \left[ \frac{x_H^2}{\lambda_{ZZ}} T_{59}^d - 2(1 - c^2 x_H^2) c^2 \right] c^2 x_H^2 - 2 \left[ \left( \frac{x_H^2}{\lambda_{ZZ}} T_{64}^d + 12 \frac{1}{\lambda_{ZZ}} + T_{54}^d \right) \right] \right\} \frac{1}{c^6} \right. \\
 & \left. \times B_0^{\text{fin}} \left( -M_Z^2; M_H, M_Z \right) \right.
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{256} \left\{ 4 \left[ (3 - T_8^d v_b) \right] \frac{1}{\lambda_{ZZ}} + \left[ 2(3 - T_8^d v_b) - (T_8^d V_4 v_b + 3 V_4) c^2 \right] c^2 \right\} \frac{1}{c^6} x_b^2 \\
 & \times C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_b, M_b, M_b \right) \\
 & + \frac{1}{256} \left\{ 4 \left[ (3 - T_9^d v_t) \right] \frac{1}{\lambda_{ZZ}} + \left[ 2(3 - T_9^d v_t) - (T_9^d V_3 v_t + 3 V_3) c^2 \right] c^2 \right\} \frac{1}{c^6} x_t^2 \\
 & \times C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_t, M_t, M_t \right) \\
 & - \frac{1}{768} \left\{ - \left[ \left( 2 \frac{x_H^2}{\lambda_{ZZ}} T_{53}^d + T_{55}^d \right) \right] + (9c^2 - \frac{x_H^2}{\lambda_{ZZ}}) T_8^d c^2 x_H^2 \right\} \frac{1}{c^6} B_0^{\text{fin}} \left( -M_H^2; M_Z, M_Z \right) \\
 & + \frac{1}{384} (1 - 2c^2 x_H^2) \frac{1}{c^4} a_0^{\text{fin}}(M_Z) \\
 & - \frac{1}{64} (3 - T_8^d v_b) \frac{1}{c^4} \frac{x_b^2}{\lambda_{ZZ}} B_0^{\text{fin}} \left( -M_H^2; M_b, M_b \right) \\
 & - \frac{1}{64} (3 - T_9^d v_t) \frac{1}{c^4} \frac{x_t^2}{\lambda_{ZZ}} B_0^{\text{fin}} \left( -M_H^2; M_t, M_t \right) \\
 & - \frac{1}{384} (15 - 2c^2 x_H^2) \frac{1}{c^2} x_H^2 a_0^{\text{fin}}(M_H) \\
 & - \frac{1}{768} (T_{67}^d - 6V_7 c^4) \frac{1}{c^6} L_R \\
 \mathcal{F}_{D:ZZ}^{\text{nfc}}(a_{AZ}) = & \frac{1}{288} \frac{s}{c^3} X_7 + \frac{1}{96} \frac{s}{c} X_8 + \frac{1}{16} \frac{c}{s} X_9 + \frac{1}{12} T_{70}^d s c x_H^2 a_0^{\text{fin}}(M_W) \\
 & + \frac{5}{192} \left[ 4 - (2W_{10} + W_{10} c^2 x_H^2) c^2 x_H^2 \right] \frac{s}{c^7} C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_Z, M_H, M_Z \right) \\
 & - \frac{5}{64} \left[ 4 \frac{1}{\lambda_{ZZ}} + \left( \frac{x_H^2}{\lambda_{ZZ}} c^2 x_H^2 - W_8 \right) c^2 \right] \frac{s}{c^7} x_H^2 C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_H, M_Z, M_H \right) \\
 & - \frac{1}{192} \left[ 2T_{82}^d c^4 x_H^2 + T_{83}^d - 6(T_7^d c^2 x_H^4 - 9V_6) s^2 c^4 \right] \frac{1}{s c^5} L_R \\
 & - \frac{1}{96} \left[ 4(T_{72}^d + T_{76}^d c^2 x_H^2) c^2 + (T_{79}^d x_H^2 - 2T_{81}^d) \frac{1}{\lambda_{ZZ}} \right] \frac{s}{c^5} B_0^{\text{fin}} \left( -M_Z^2; M_W, M_W \right) \\
 & + \frac{1}{576} \left\{ 3T_{80}^d + \left[ T_{86}^d x_H^2 - 3(8c^2 x_H^2 x_b^2 v_b + 16c^2 x_H^2 x_t^2 v_t - 2T_{84}^d x_b^2 v_b - 2T_{85}^d x_t^2 v_t \right. \right. \\
 & \left. \left. - 9U_2 x_t^2 - 9U_4 x_b^2) c^2 \right] c^2 \right\} \frac{s}{c^5} \\
 & - \frac{1}{96} \left\{ \left[ 4c^2 x_H^2 x_b^2 v_b + (2x_H^2 v_b + U_6 x_b^2) \right] c^2 - \left[ (4v_b - 6 \frac{x_b^2}{\lambda_{ZZ}} T_{78}^d v_b - 9 \frac{x_b^2}{\lambda_{ZZ}} U_4) \right] \right\} \frac{s}{c^3} \\
 & \times B_0^{\text{fin}} \left( -M_Z^2; M_b, M_b \right) \\
 & - \frac{1}{96} \left\{ \left[ 8c^2 x_H^2 x_t^2 v_t + (4x_H^2 v_t + U_5 x_t^2) \right] c^2 - \left[ (8v_t - 6 \frac{x_t^2}{\lambda_{ZZ}} T_{77}^d v_t - 9 \frac{x_t^2}{\lambda_{ZZ}} U_2) \right] \right\} \frac{s}{c^3} \\
 & \times B_0^{\text{fin}} \left( -M_Z^2; M_t, M_t \right) \\
 & - \frac{1}{96} \left\{ \left[ T_{73}^d - 3(T_7^d c^2 x_H^2 - T_{71}^d) c^2 x_H^2 \right] c^2 - (T_{79}^d x_H^2 - 2T_{81}^d) \frac{1}{\lambda_{ZZ}} \right\} \frac{s}{c^5} \\
 & \times B_0^{\text{fin}} \left( -M_H^2; M_W, M_W \right) \\
 & + \frac{1}{96} \left\{ \left[ 6T_{74}^d + (24c^6 x_H^2 - T_{75}^d) c^2 x_H^2 \right] c^2 - (T_{79}^d x_H^2 - 2T_{81}^d) \frac{1}{\lambda_{ZZ}} \right\} \frac{s}{c^7} \\
 & \times C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_W, M_W, M_W \right) \\
 & - \frac{1}{128} \left\{ 4 \left[ (2T_{77}^d v_t + 3U_2) \right] \frac{1}{\lambda_{ZZ}} + \left[ 2(2T_{77}^d v_t + 3U_2) + (2T_{77}^d V_3 v_t - 3V_3 U_1) c^2 \right] c^2 \right\} \frac{s}{c^5} x_t^2 \\
 & \times C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_t, M_t, M_t \right)
 \end{aligned}$$



$$\begin{aligned}
& -\frac{1}{128} \left\{ 4 \left[ (2T_{78}^d v_b + 3U_4) \right] \frac{1}{\lambda_{zz}} + \left[ 2(2T_{78}^d v_b + 3U_4) + (2T_{78}^d V_4 v_b - 3V_4 U_3) c^2 \right] c^2 \right\} \frac{s}{c^5} x_b^2 \\
& \times C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_b, M_b, M_b \right) \\
& + \frac{1}{48} (2 - c^2 x_H^2) \frac{s}{c^3} v_1 B_0^{\text{fin}} \left( -M_Z^2; 0, 0 \right) \\
& + \frac{5}{64} (2 - c^2 x_H^2) \frac{s}{c^5} \frac{x_H^2}{\lambda_{zz}} B_0^{\text{fin}} \left( -M_H^2; M_H, M_H \right) \\
& + \frac{1}{24} (2 - c^2 x_H^2) \frac{s}{c} x_b^2 v_b a_0^{\text{fin}} \left( M_b \right) \\
& + \frac{1}{12} (2 - c^2 x_H^2) \frac{s}{c} x_t^2 v_t a_0^{\text{fin}} \left( M_t \right) \\
& + \frac{5}{192} (sW_{17} + \frac{x_H^2}{\lambda_{zz}} c^2 x_H^2) \frac{s}{c^5} B_0^{\text{fin}} \left( -M_H^2; M_Z, M_Z \right) \\
& - \frac{5}{96} (2sW_{17} - \frac{x_H^2}{\lambda_{zz}} c^2 x_H^2) \frac{s}{c^5} B_0^{\text{fin}} \left( -M_Z^2; M_H, M_Z \right) \\
& + \frac{1}{32} (2T_{77}^d v_t + 3U_2) \frac{s}{c^3} \frac{x_t^2}{\lambda_{zz}} B_0^{\text{fin}} \left( -M_H^2; M_t, M_t \right) \\
& + \frac{1}{32} (2T_{78}^d v_b + 3U_4) \frac{s}{c^3} \frac{x_b^2}{\lambda_{zz}} B_0^{\text{fin}} \left( -M_H^2; M_b, M_b \right) \\
& - \frac{1}{48} (2U_0 - U_0 c^2 x_H^2) \frac{s}{c^3} (1 - 3L_R)
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_{D;ZZ}^{\text{nfc}}(a_{AA}) &= \frac{1}{32} X_{11} + \frac{1}{32} \frac{s^2}{c^2} X_{10} \\
& + \frac{3}{32} \left[ 4 \frac{1}{\lambda_{zz}} + \left( \frac{x_H^2}{\lambda_{zz}} c^2 x_H^2 - W_8 \right) c^2 \right] \frac{s^2}{c^6} x_H^2 C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_H, M_Z, M_H \right) \\
& - \frac{1}{32} \left[ 4T_{29}^d - (2W_{10} + W_{10} c^2 x_H^2) s^2 c^2 x_H^2 \right] \frac{1}{c^6} C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_Z, M_H, M_Z \right) \\
& - \frac{1}{32} \left[ T_{90}^d + T_{96}^d c^2 x_H^2 + (T_{88}^d x_b^2 v_b + T_9^d x_t^2 v_t) s^2 c^2 \right] \frac{1}{c^4} \\
& + \frac{1}{16} \left[ (T_{71}^d x_H^2 - 4T_{87}^d) \frac{1}{\lambda_{zz}} + (2T_{88}^d + T_{92}^d c^2 x_H^2) c^2 \right] \frac{s^2}{c^4} B_0^{\text{fin}} \left( -M_Z^2; M_W, M_W \right) \\
& - \frac{1}{16} \left\{ 3c^2 + \left[ (3 - T_8^d v_b) \right] \frac{1}{\lambda_{zz}} \right\} \frac{s^2}{c^4} x_b^2 B_0^{\text{fin}} \left( -M_Z^2; M_b, M_b \right) \\
& - \frac{1}{16} \left\{ 3c^2 + \left[ (3 - T_9^d v_t) \right] \frac{1}{\lambda_{zz}} \right\} \frac{s^2}{c^4} x_t^2 B_0^{\text{fin}} \left( -M_Z^2; M_t, M_t \right) \\
& - \frac{1}{16} \left\{ \left[ c^2 x_H^4 + (4 - T_{93}^d x_H^2) \right] c^4 + (T_{71}^d x_H^2 - 4T_{87}^d) \frac{1}{\lambda_{zz}} \right\} \frac{s^2}{c^4} B_0^{\text{fin}} \left( -M_H^2; M_W, M_W \right) \\
& + \frac{1}{16} \left\{ - \left[ 2T_{89}^d - (4c^4 x_H^2 + T_{95}^d) c^2 x_H^2 \right] c^2 + (T_{71}^d x_H^2 - 4T_{87}^d) \frac{1}{\lambda_{zz}} \right\} \frac{s^2}{c^6} \\
& \times C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_W, M_W, M_W \right) \\
& - \frac{1}{64} \left\{ 4 \left[ (3 - T_8^d v_b) \right] \frac{1}{\lambda_{zz}} + \left[ 2(3 - T_8^d v_b) - (T_8^d V_4 v_b + 3V_4) c^2 \right] c^2 \right\} \frac{s^2}{c^6} x_b^2 \\
& \times C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_b, M_b, M_b \right) \\
& - \frac{1}{64} \left\{ 4 \left[ (3 - T_9^d v_t) \right] \frac{1}{\lambda_{zz}} + \left[ 2(3 - T_9^d v_t) - (T_9^d V_3 v_t + 3V_3) c^2 \right] c^2 \right\} \frac{s^2}{c^6} x_t^2 \\
& \times C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_t, M_t, M_t \right)
\end{aligned}$$

$$\begin{aligned}
& -\frac{3}{32} (2 - c^2 x_H^2) \frac{s^2}{c^4} \frac{x_H^2}{\lambda_{zz}} B_0^{\text{fin}}(-M_H^2; M_H, M_H) \\
& + \frac{1}{16} (3 - T_8^d v_b) \frac{s^2}{c^4} \frac{x_b^2}{\lambda_{zz}} B_0^{\text{fin}}(-M_H^2; M_b, M_b) \\
& + \frac{1}{16} (3 - T_9^d v_t) \frac{s^2}{c^4} \frac{x_t^2}{\lambda_{zz}} B_0^{\text{fin}}(-M_H^2; M_t, M_t) \\
& - \frac{1}{32} (sW_{17} + \frac{x_H^2}{\lambda_{zz}} c^2 x_H^2) \frac{s^2}{c^4} B_0^{\text{fin}}(-M_H^2; M_Z, M_Z) \\
& + \frac{1}{16} (2sW_{17} - \frac{x_H^2}{\lambda_{zz}} c^2 x_H^2) \frac{s^2}{c^4} B_0^{\text{fin}}(-M_Z^2; M_H, M_Z) \\
& - \frac{1}{32} (2s^2 c^6 x_H^4 - 4T_{91}^d c^4 x_H^2 - T_{94}^d) \frac{1}{c^4} L_R
\end{aligned}$$

$$\begin{aligned}
\mathcal{F}_{D:ZZ}^{\text{nfc}}(a_{zz}) = & \frac{1}{32} X_{14} + \frac{1}{32} x_H^2 X_{13} + \frac{1}{32} \frac{1}{c^2} X_{12} \\
& + \frac{1}{32} \left[ 2 \frac{x_H^2}{\lambda_{zz}} s^2 c^2 x_H^2 - (10 - c^2 x_H^2) c^2 x_H^2 + 2 \left( \frac{x_H^2}{\lambda_{zz}} T_8^d + 2T_{107}^d \right) \right] \frac{1}{c^4} B_0^{\text{fin}}(-M_Z^2; M_H, M_Z) \\
& + \frac{1}{128} \left[ 4 \frac{x_H^2}{\lambda_{zz}} s^2 c^2 x_H^2 - 4T_{110}^d sW_{17} - (10 + c^2 x_H^2) c^2 x_H^2 \right] \frac{1}{c^4} B_0^{\text{fin}}(-M_H^2; M_Z, M_Z) \\
& - \frac{3}{32} \left[ 4 \frac{1}{\lambda_{zz}} T_{29}^d - (4 - c^2 x_H^2) \frac{x_H^2}{\lambda_{zz}} s^2 c^2 - (2c^2 x_H^2 - T_{107}^d) c^2 \right] \frac{1}{c^6} x_H^2 \\
& \times C_0(-M_H^2, -M_Z^2, -M_Z^2; M_H, M_Z, M_H) \\
& + \frac{3}{128} \left[ 4(2 - c^2 x_H^2) \frac{1}{\lambda_{zz}} s^2 + (10 - c^2 x_H^2) c^2 \right] \frac{1}{c^4} x_H^2 B_0^{\text{fin}}(-M_H^2; M_H, M_H) \\
& - \frac{1}{32} \left\{ 9c^2 + \left[ (2T_{42}^d v_t + 3U_2) \right] \frac{1}{\lambda_{zz}} \right\} \frac{1}{c^2} x_t^2 B_0^{\text{fin}}(-M_Z^2; M_t, M_t) \\
& - \frac{1}{32} \left\{ 9c^2 + \left[ (2T_{112}^d v_b + 3U_4) \right] \frac{1}{\lambda_{zz}} \right\} \frac{1}{c^2} x_b^2 B_0^{\text{fin}}(-M_Z^2; M_b, M_b) \\
& - \frac{1}{32} \left\{ \frac{x_H^2}{\lambda_{zz}} s^2 c^4 x_H^4 + 4T_{106}^d - \left[ T_{107}^d c^2 x_H^2 + 2 \left( \frac{x_H^2}{\lambda_{zz}} T_{110}^d + T_{107}^d \right) \right] c^2 x_H^2 \right\} \frac{1}{c^6} \\
& \times C_0(-M_H^2, -M_Z^2, -M_Z^2; M_Z, M_H, M_Z) \\
& + \frac{1}{64} \left\{ 4T_{97}^d x_H^2 + \left[ (2T_{42}^d x_t^2 v_t - 32T_{107}^d + 2T_{112}^d x_b^2 v_b + 3U_2 x_t^2 + 3U_4 x_b^2) \right] c^2 \right\} \frac{1}{c^2} \\
& + \frac{1}{64} \left\{ 2T_{111}^d + \left[ 2T_{103}^d x_H^2 - (T_{108}^d x_H^4 + 18V_6) c^2 \right] c^2 \right\} \frac{1}{c^4} L_R \\
& - \frac{1}{64} \left\{ \left[ T_{100}^d c^2 x_H^4 + 2(2T_{100}^d - T_{102}^d x_H^2) \right] c^2 + 4 \left[ (6T_{98}^d - T_{101}^d x_H^2) \right] \frac{1}{\lambda_{zz}} \right\} \frac{1}{c^4} \\
& \times B_0^{\text{fin}}(-M_H^2; M_W, M_W) \\
& + \frac{1}{16} \left\{ \left[ 2T_{104}^d + (4c^6 x_H^2 - T_{105}^d) c^2 x_H^2 \right] c^2 + \left[ (6T_{98}^d - T_{101}^d x_H^2) \right] \frac{1}{\lambda_{zz}} \right\} \frac{1}{c^6} \\
& \times C_0(-M_H^2, -M_Z^2, -M_Z^2; M_W, M_W, M_W) \\
& - \frac{1}{128} \left\{ 4 \left[ (2T_{42}^d v_t + 3U_2) \right] \frac{1}{\lambda_{zz}} + \left[ 2(2T_{42}^d v_t + 3U_2) + (2T_{42}^d V_3 v_t - 3V_3 U_1) c^2 \right] c^2 \right\} \frac{1}{c^4} x_t^2 \\
& \times C_0(-M_H^2, -M_Z^2, -M_Z^2; M_t, M_t, M_t) \\
& + \frac{1}{16} \left\{ \left[ (6T_{98}^d - T_{101}^d x_H^2) \right] \frac{1}{\lambda_{zz}} + (2T_{99}^d - T_{109}^d c^2 x_H^2) c^2 \right\} \frac{1}{c^4} B_0^{\text{fin}}(-M_Z^2; M_W, M_W)
\end{aligned}$$

$$\begin{aligned}
 & -\frac{1}{128} \left\{ 4 \left[ (2T_{112}^d v_b + 3U_4) \right] \frac{1}{\lambda_{zz}} + \left[ 2(2T_{112}^d v_b + 3U_4) + (2T_{112}^d V_4 v_b - 3V_4 U_3) c^2 \right] c^2 \right\} \frac{1}{c^4} x_b^2 \\
 & \times C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_b, M_b, M_b \right) \\
 & -\frac{1}{32} (4 - c^2 x_H^2) \frac{1}{c^2} x_H^2 a_0^{\text{fin}}(M_H) \\
 & +\frac{1}{32} (4 - c^2 x_H^2) \frac{1}{c^4} a_0^{\text{fin}}(M_Z) \\
 & +\frac{1}{32} (2T_{42}^d v_t + 3U_2) \frac{1}{c^2} \frac{x_t^2}{\lambda_{zz}} B_0^{\text{fin}} \left( -M_H^2; M_t, M_t \right) \\
 & +\frac{1}{32} (2T_{112}^d v_b + 3U_4) \frac{1}{c^2} \frac{x_b^2}{\lambda_{zz}} B_0^{\text{fin}} \left( -M_H^2; M_b, M_b \right)
 \end{aligned}$$

$$\mathcal{F}_{D:ZZ}^{\text{nfc}}(\text{ren}) = \frac{1}{32} \frac{1}{c^2} X_{15}$$

$$\begin{aligned}
 \mathcal{F}_{P:ZZ}^{\text{nfc}}(a_{\phi t v}) &= \frac{3}{16} \frac{v_t}{c^2} W_{19} x_t^2 \\
 & -\frac{3}{32} \left[ 12 \frac{1}{\lambda_{zz}^2} + (W_{18} + 2W_{20} c^2) c^2 \right] \frac{v_t}{c^6} x_t^2 C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_t, M_t, M_t \right) \\
 & +\frac{3}{32} \left( -c^2 sW_{18} + 12 \frac{1}{\lambda_{zz}^2} \right) \frac{v_t}{c^4} x_t^2 B_0^{\text{fin}} \left( -M_H^2; M_t, M_t \right) \\
 & -\frac{3}{32} \left( -c^2 sW_{18} + 12 \frac{1}{\lambda_{zz}^2} \right) \frac{v_t}{c^4} x_t^2 B_0^{\text{fin}} \left( -M_Z^2; M_t, M_t \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{P:ZZ}^{\text{nfc}}(a_{\phi t \lambda}) &= \frac{3}{16} \frac{1}{c^2} W_{19} x_t^2 \\
 & -\frac{3}{32} \left[ 12 \frac{1}{\lambda_{zz}^2} + (W_{22} + 2W_{23} c^2) c^2 \right] \frac{1}{c^6} x_t^2 C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_t, M_t, M_t \right) \\
 & +\frac{3}{32} \left( c^2 sW_{21} + 12 \frac{1}{\lambda_{zz}^2} \right) \frac{1}{c^4} x_t^2 B_0^{\text{fin}} \left( -M_H^2; M_t, M_t \right) \\
 & -\frac{3}{32} \left( c^2 sW_{21} + 12 \frac{1}{\lambda_{zz}^2} \right) \frac{1}{c^4} x_t^2 B_0^{\text{fin}} \left( -M_Z^2; M_t, M_t \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{P:ZZ}^{\text{nfc}}(a_{\phi b v}) &= \frac{3}{16} \frac{v_b}{c^2} W_{19} x_b^2 \\
 & -\frac{3}{32} \left[ 12 \frac{1}{\lambda_{zz}^2} + (W_{18} + 2W_{24} c^2) c^2 \right] \frac{v_b}{c^6} x_b^2 C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_b, M_b, M_b \right) \\
 & +\frac{3}{32} \left( -c^2 sW_{18} + 12 \frac{1}{\lambda_{zz}^2} \right) \frac{v_b}{c^4} x_b^2 B_0^{\text{fin}} \left( -M_H^2; M_b, M_b \right) \\
 & -\frac{3}{32} \left( -c^2 sW_{18} + 12 \frac{1}{\lambda_{zz}^2} \right) \frac{v_b}{c^4} x_b^2 B_0^{\text{fin}} \left( -M_Z^2; M_b, M_b \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{P:ZZ}^{\text{nfc}}(a_{\phi b \lambda}) &= \frac{3}{16} \frac{1}{c^2} W_{19} x_b^2 \\
 & -\frac{3}{32} \left[ 12 \frac{1}{\lambda_{zz}^2} + (W_{22} + 2W_{25} c^2) c^2 \right] \frac{1}{c^6} x_b^2 C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_b, M_b, M_b \right) \\
 & +\frac{3}{32} \left( 12 \frac{1}{\lambda_{zz}^2} + W_{21} c^2 \right) \frac{1}{c^4} x_b^2 B_0^{\text{fin}} \left( -M_H^2; M_b, M_b \right) \\
 & -\frac{3}{32} \left( 12 \frac{1}{\lambda_{zz}^2} + W_{21} c^2 \right) \frac{1}{c^4} x_b^2 B_0^{\text{fin}} \left( -M_Z^2; M_b, M_b \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{\text{P};\text{ZZ}}^{\text{nfc}}(a_{\text{tBW}}) &= \frac{3}{32} \left\{ 6 \frac{1}{\lambda_{\text{ZZ}}^2} + \left[ \frac{1}{\lambda_{\text{ZZ}}} + (-c^2 + 2 \frac{1}{\lambda_{\text{ZZ}}}) c^2 x_{\text{t}}^2 \right] c^2 \right\} \frac{1}{c^7} x_{\text{t}}^2 v_{\text{t}} \\
 &\quad \times \text{C}_0 \left( -M_{\text{H}}^2, -M_{\text{Z}}^2, -M_{\text{Z}}^2; M_{\text{t}}, M_{\text{t}}, M_{\text{t}} \right) \\
 &\quad - \frac{3}{64} \left( c^2 + 2 \frac{1}{\lambda_{\text{ZZ}}} \right) \frac{v_{\text{t}}}{c^3} x_{\text{t}}^2 \\
 &\quad - \frac{3}{64} \left( -c^4 + 12 \frac{1}{\lambda_{\text{ZZ}}} \right) \frac{v_{\text{t}}}{c^5} x_{\text{t}}^2 \text{B}_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{t}}, M_{\text{t}} \right) \\
 &\quad + \frac{3}{64} \left( -c^4 + 12 \frac{1}{\lambda_{\text{ZZ}}} \right) \frac{v_{\text{t}}}{c^5} x_{\text{t}}^2 \text{B}_0^{\text{fin}} \left( -M_{\text{Z}}^2; M_{\text{t}}, M_{\text{t}} \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{\text{P};\text{ZZ}}^{\text{nfc}}(a_{\text{bBW}}) &= -\frac{3}{32} \left\{ 6 \frac{1}{\lambda_{\text{ZZ}}^2} + \left[ \frac{1}{\lambda_{\text{ZZ}}} + (-c^2 + 2 \frac{1}{\lambda_{\text{ZZ}}}) c^2 x_{\text{b}}^2 \right] c^2 \right\} \frac{1}{c^7} x_{\text{b}}^2 v_{\text{b}} \\
 &\quad \times \text{C}_0 \left( -M_{\text{H}}^2, -M_{\text{Z}}^2, -M_{\text{Z}}^2; M_{\text{b}}, M_{\text{b}}, M_{\text{b}} \right) \\
 &\quad + \frac{3}{64} \left( c^2 + 2 \frac{1}{\lambda_{\text{ZZ}}} \right) \frac{v_{\text{b}}}{c^3} x_{\text{b}}^2 \\
 &\quad + \frac{3}{64} \left( -c^4 + 12 \frac{1}{\lambda_{\text{ZZ}}} \right) \frac{v_{\text{b}}}{c^5} x_{\text{b}}^2 \text{B}_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{b}}, M_{\text{b}} \right) \\
 &\quad - \frac{3}{64} \left( -c^4 + 12 \frac{1}{\lambda_{\text{ZZ}}} \right) \frac{v_{\text{b}}}{c^5} x_{\text{b}}^2 \text{B}_0^{\text{fin}} \left( -M_{\text{Z}}^2; M_{\text{b}}, M_{\text{b}} \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{\text{P};\text{ZZ}}^{\text{nfc}}(a_{\phi}) &= \frac{3}{16} \frac{1}{c^2} \text{W}_{18} a_0^{\text{fin}}(M_{\text{Z}}) + \frac{3}{16} \frac{1}{c^2} \text{W}_{19} + \frac{3}{16} \text{W}_9 a_0^{\text{fin}}(M_{\text{H}}) \\
 &\quad + \frac{3}{32} \left[ 24 \frac{1}{\lambda_{\text{ZZ}}^2} - (12 \frac{x_{\text{H}}^2}{\lambda_{\text{ZZ}}^2} - \text{W}_9 c^2) c^2 \right] \frac{1}{c^4} \text{B}_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{H}}, M_{\text{H}} \right) \\
 &\quad - \frac{3}{32} \left[ 24 \frac{1}{\lambda_{\text{ZZ}}^2} - (\text{W}_9 c^2 + 2 \text{W}_{27}) c^2 \right] \frac{1}{c^4} \text{B}_0^{\text{fin}} \left( -M_{\text{Z}}^2; M_{\text{H}}, M_{\text{Z}} \right) \\
 &\quad - \frac{3}{32} \left\{ 48 \frac{1}{\lambda_{\text{ZZ}}^2} + \left[ 8 \frac{1}{\lambda_{\text{ZZ}}} \text{W}_{26} - (\text{W}_9 c^2 x_{\text{H}}^2 - 4 \text{W}_{25}) c^2 \right] c^2 \right\} \frac{1}{c^6} \text{C}_0 \left( -M_{\text{H}}^2, -M_{\text{Z}}^2, -M_{\text{Z}}^2; M_{\text{H}}, M_{\text{Z}}, M_{\text{H}} \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{\text{P};\text{ZZ}}^{\text{nfc}}(a_{\phi\Box}) &= \frac{3}{64} \frac{1}{c^2} \text{W}_9 a_0^{\text{fin}}(M_{\text{Z}}) - \frac{3}{64} \text{W}_9 x_{\text{H}}^2 a_0^{\text{fin}}(M_{\text{H}}) \\
 &\quad + \frac{1}{128} \left[ 8 \frac{1}{\lambda_{\text{ZZ}}} s \text{W}_{26} - (\text{W}_9 c^2 x_{\text{H}}^2 - 2 \text{W}_{32}) c^2 \right] \frac{1}{c^4} \text{B}_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{Z}}, M_{\text{Z}} \right) \\
 &\quad + \frac{1}{128} \left[ 16 \frac{1}{\lambda_{\text{ZZ}}} s \text{W}_{11} + (\text{W}_9 c^2 x_{\text{H}}^2 + 4 \text{W}_{30}) c^2 \right] \frac{1}{c^4} x_{\text{H}}^2 \text{C}_0 \left( -M_{\text{H}}^2, -M_{\text{Z}}^2, -M_{\text{Z}}^2; M_{\text{Z}}, M_{\text{H}}, M_{\text{Z}} \right) \\
 &\quad - \frac{1}{64} \left\{ 48 \frac{1}{\lambda_{\text{ZZ}}^2} + \left[ 12 \frac{1}{\lambda_{\text{ZZ}}} + \left( -4 \frac{1}{\lambda_{\text{ZZ}}} \text{W}_{33} + \text{W}_9 c^2 \right) c^2 x_{\text{H}}^2 \right] c^2 \right\} \frac{1}{c^6} \text{B}_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{H}}, M_{\text{H}} \right) \\
 &\quad + \frac{1}{128} \left\{ 96 \frac{1}{\lambda_{\text{ZZ}}^2} + \left[ 16 \frac{1}{\lambda_{\text{ZZ}}} \text{W}_{11} - (3 \text{W}_9 c^2 x_{\text{H}}^2 - 4 \text{W}_{28}) c^2 \right] c^2 \right\} \frac{1}{c^6} \text{B}_0^{\text{fin}} \left( -M_{\text{Z}}^2; M_{\text{H}}, M_{\text{Z}} \right) \\
 &\quad + \frac{1}{64} \left\{ 96 \frac{1}{\lambda_{\text{ZZ}}^2} + \left[ 8 \frac{1}{\lambda_{\text{ZZ}}} \text{W}_{34} - (\text{W}_9 c^4 x_{\text{H}}^4 - 4 \text{W}_{29} - 4 \text{W}_{31} c^2 x_{\text{H}}^2) c^2 \right] c^2 \right\} \frac{1}{c^8} \\
 &\quad \times \text{C}_0 \left( -M_{\text{H}}^2, -M_{\text{Z}}^2, -M_{\text{Z}}^2; M_{\text{H}}, M_{\text{Z}}, M_{\text{H}} \right) \\
 &\quad - \frac{1}{32} \left\{ 12 \frac{1}{\lambda_{\text{ZZ}}^2} \text{T}_{117}^d - \left[ (12 \frac{x_{\text{H}}^2}{\lambda_{\text{ZZ}}^2} \text{T}_{52}^d - \text{T}_{120}^d) \frac{1}{\lambda_{\text{ZZ}}} + (\text{T}_{52}^d \text{W}_9 + \text{T}_{113}^d \text{V}_9) c^2 \right] c^2 \right\} \frac{1}{c^6} \\
 &\quad \times \text{B}_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{W}}, M_{\text{W}} \right) \\
 &\quad + \frac{1}{32} \left\{ 12 \frac{1}{\lambda_{\text{ZZ}}^2} \text{T}_{117}^d - \left[ (12 \frac{x_{\text{H}}^2}{\lambda_{\text{ZZ}}^2} \text{T}_{52}^d - \text{T}_{120}^d) \frac{1}{\lambda_{\text{ZZ}}} + (\text{T}_{52}^d \text{W}_9 + \text{T}_{113}^d \text{V}_9) c^2 \right] c^2 \right\} \frac{1}{c^6}
 \end{aligned}$$

$$\begin{aligned}
 & \times B_0^{\text{fin}}(-M_Z^2; M_W, M_W) \\
 & + \frac{1}{32} \left\{ 12 \frac{1}{\lambda_{ZZ}^2} T_{117}^d - \left[ (12 \frac{x_H^2}{\lambda_{ZZ}} T_{52}^d - T_{121}^d) \frac{1}{\lambda_{ZZ}} - (-\frac{x_H^2}{\lambda_{ZZ}} T_{115}^d + T_{114}^d + T_{116}^d V_9) c^2 \right] c^2 \right\} \frac{1}{c^8} \\
 & \times C_0(-M_H^2, -M_Z^2, -M_Z^2; M_W, M_W, M_W) \\
 & - \frac{1}{64} \left\{ 4 \frac{1}{\lambda_{ZZ}} T_{119}^d + \left[ (5 - \frac{x_H^2}{\lambda_{ZZ}} T_{118}^d - 4 T_{113}^d V_9) \right] c^2 \right\} \frac{1}{c^4} \\
 \\
 \mathcal{F}_{P:ZZ}^{\text{nfc}}(a_{\text{0D}}) = & \frac{1}{512} \left[ 8sW_{45} - (5W_9 c^2 x_H^2 - 2W_{37}) c^2 \right] \frac{1}{c^4} B_0^{\text{fin}}(-M_H^2; M_Z, M_Z) \\
 & - \frac{1}{512} \left[ 16sW_{39} - (5W_9 c^2 x_H^2 - 4W_{36}) c^2 x_H^2 \right] \frac{1}{c^4} C_0(-M_H^2, -M_Z^2, -M_Z^2; M_Z, M_H, M_Z) \\
 & + \frac{1}{128} \left[ (\frac{x_H^2}{\lambda_{ZZ}} T_{128}^d + T_8^d W_{19} x_b^2 v_b + T_9^d W_{19} x_t^2 v_t + T_{129}^d - 3W_{47}) c^2 \right. \\
 & \left. 2 \left( \frac{1}{\lambda_{ZZ}} T_{125}^d + T_{123}^d V_9 \right) \right] \frac{1}{c^4} \\
 & - \frac{1}{256} \left\{ 48 \frac{1}{\lambda_{ZZ}^2} + \left[ 12 \frac{1}{\lambda_{ZZ}} W_{42} - (6W_9 c^2 x_H^2 - W_{44}) c^2 \right] c^2 \right\} \frac{1}{c^6} B_0^{\text{fin}}(-M_Z^2; M_H, M_Z) \\
 & + \frac{1}{512} \left\{ 96 \frac{1}{\lambda_{ZZ}^2} + \left[ 24 \frac{1}{\lambda_{ZZ}} W_{38} + (-4 \frac{1}{\lambda_{ZZ}} W_{40} + 17W_9 c^2) c^2 x_H^2 \right] c^2 \right\} \frac{1}{c^6} B_0^{\text{fin}}(-M_H^2; M_H, M_H) \\
 & - \frac{1}{512} \left\{ 192 \frac{1}{\lambda_{ZZ}^2} + \left[ 16 \frac{1}{\lambda_{ZZ}} W_{41} - (17W_9 c^4 x_H^4 - 8W_{35} - 4W_{43} c^2 x_H^2) c^2 \right] c^2 \right\} \frac{1}{c^8} \\
 & \times C_0(-M_H^2, -M_Z^2, -M_Z^2; M_H, M_Z, M_H) \\
 & + \frac{1}{128} \left\{ 12 \frac{1}{\lambda_{ZZ}^2} T_{124}^d + \left[ (24 \frac{x_H^2}{\lambda_{ZZ}} T_{52}^d + T_{126}^d) \frac{1}{\lambda_{ZZ}} + (2T_{52}^d W_9 + 5T_{113}^d V_9) c^2 \right] c^2 \right\} \frac{1}{c^6} \\
 & \times B_0^{\text{fin}}(-M_H^2; M_W, M_W) \\
 & - \frac{1}{128} \left\{ 12 \frac{1}{\lambda_{ZZ}^2} T_{124}^d + \left[ (24 \frac{x_H^2}{\lambda_{ZZ}} T_{52}^d + T_{126}^d) \frac{1}{\lambda_{ZZ}} + (2T_{52}^d W_9 + 5T_{113}^d V_9) c^2 \right] c^2 \right\} \frac{1}{c^6} \\
 & \times B_0^{\text{fin}}(-M_Z^2; M_W, M_W) \\
 & - \frac{1}{128} \left\{ 12 \frac{1}{\lambda_{ZZ}^2} T_{124}^d + \left[ (24 \frac{x_H^2}{\lambda_{ZZ}} T_{52}^d + T_{127}^d) \frac{1}{\lambda_{ZZ}} + (2 \frac{x_H^2}{\lambda_{ZZ}} T_{115}^d - 5T_{116}^d V_9 - 2T_{122}^d) c^2 \right] c^2 \right\} \frac{1}{c^8} \\
 & \times C_0(-M_H^2, -M_Z^2, -M_Z^2; M_W, M_W, M_W) \\
 & + \frac{1}{256} \left\{ 12 \left[ (3 - T_8^d v_b) \right] \frac{1}{\lambda_{ZZ}^2} - \left[ (T_8^d W_{18} v_b - 3W_{22}) + 2(T_8^d W_{24} v_b - 3W_{46}) c^2 \right] c^2 \right\} \frac{1}{c^6} x_b^2 \\
 & \times C_0(-M_H^2, -M_Z^2, -M_Z^2; M_b, M_b, M_b) \\
 & - \frac{1}{256} \left\{ 12 \left[ (3 - T_8^d v_b) \right] \frac{1}{\lambda_{ZZ}^2} + (T_8^d W_{18} v_b + 3W_{21}) c^2 \right\} \frac{1}{c^4} x_b^2 B_0^{\text{fin}}(-M_H^2; M_b, M_b) \\
 & + \frac{1}{256} \left\{ 12 \left[ (3 - T_8^d v_b) \right] \frac{1}{\lambda_{ZZ}^2} + (T_8^d W_{18} v_b + 3W_{21}) c^2 \right\} \frac{1}{c^4} x_b^2 B_0^{\text{fin}}(-M_Z^2; M_b, M_b) \\
 & + \frac{1}{256} \left\{ 12 \left[ (3 - T_9^d v_t) \right] \frac{1}{\lambda_{ZZ}^2} - \left[ (T_9^d W_{18} v_t - 3W_{22}) + 2(T_9^d W_{20} v_t - 3W_{23}) c^2 \right] c^2 \right\} \frac{1}{c^6} x_t^2 \\
 & \times C_0(-M_H^2, -M_Z^2, -M_Z^2; M_t, M_t, M_t) \\
 & - \frac{1}{256} \left\{ 12 \left[ (3 - T_9^d v_t) \right] \frac{1}{\lambda_{ZZ}^2} + (T_9^d W_{18} v_t + 3W_{21}) c^2 \right\} \frac{1}{c^4} x_t^2 B_0^{\text{fin}}(-M_H^2; M_t, M_t)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{256} \left\{ 12 \left[ (3 - T_9^d v_t) \right] \frac{1}{\lambda_{zz}^2} + (T_9^d W_{18} v_t + 3 W_{21}) c^2 \right\} \frac{1}{c^4} x_t^2 B_0^{\text{fin}} \left( -M_Z^2; M_t, M_t \right) \\
 & - \frac{1}{64} (2 - 3 c^2 s W_9 x_H^2) \frac{1}{c^2} a_0^{\text{fin}} (M_H) \\
 & - \frac{1}{64} (3 c^2 s W_9 - 2 V_9) \frac{1}{c^4} a_0^{\text{fin}} (M_Z)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{P}_{P;ZZ}^{\text{nfc}}(a_{Az}) = & - \frac{1}{12} s c x_b^2 v_b a_0^{\text{fin}} (M_b) - \frac{1}{6} s c x_t^2 v_t a_0^{\text{fin}} (M_t) \\
 & - \frac{1}{24} \frac{s}{c} v_t B_0^{\text{fin}} \left( -M_Z^2; 0, 0 \right) + \frac{1}{24} \frac{s}{c} U_0 (1 - 3 L_R) + \frac{1}{6} T_{70}^d s c a_0^{\text{fin}} (M_W) \\
 & - \frac{3}{256} \left[ 4 s W_{51} + (12 \frac{x_H^2}{\lambda_{zz}^2} - W_9 c^2) c^2 x_H^2 \right] \frac{s}{c^3} B_0^{\text{fin}} \left( -M_H^2; M_Z, M_Z \right) \\
 & + \frac{3}{256} \left[ 8 s W_{49} - (W_9 c^2 x_H^2 - 2 W_{50}) c^2 x_H^2 \right] \frac{s}{c^3} C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_Z, M_H, M_Z \right) \\
 & - \frac{9}{256} \left[ 24 \frac{1}{\lambda_{zz}^2} - (12 \frac{x_H^2}{\lambda_{zz}^2} - W_9 c^2) c^2 \right] \frac{s}{c^3} x_H^2 B_0^{\text{fin}} \left( -M_H^2; M_H, M_H \right) \\
 & + \frac{3}{128} \left[ 16 \frac{1}{\lambda_{zz}} s W_{11} - (W_9 c^2 x_H^2 - 4 W_{48}) c^2 \right] \frac{s}{c^3} B_0^{\text{fin}} \left( -M_Z^2; M_H, M_Z \right) \\
 & + \frac{9}{256} \left\{ 48 \frac{1}{\lambda_{zz}^2} + \left[ 8 \frac{1}{\lambda_{zz}} W_{26} - (W_9 c^2 x_H^2 - 4 W_{25}) c^2 \right] c^2 \right\} \frac{s}{c^5} x_H^2 \\
 & \times C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_H, M_Z, M_H \right) \\
 & + \frac{1}{128} \left\{ 24 \frac{1}{\lambda_{zz}^2} T_{133}^d + \left[ (64 c^6 x_H^2 - \frac{x_H^2}{\lambda_{zz}} T_{139}^d - 2 T_{130}^d V_9 + T_{131}^d) c^2 \right. \right. \\
 & \left. \left. - 2 (6 \frac{x_H^2}{\lambda_{zz}} T_{100}^d - T_{141}^d) \frac{1}{\lambda_{zz}} \right] c^2 \right\} \frac{s}{c^7} C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_W, M_W, M_W \right) \\
 & - \frac{1}{128} \left\{ 24 \frac{1}{\lambda_{zz}^2} T_{133}^d - \left[ 2 (6 \frac{x_H^2}{\lambda_{zz}} T_{100}^d - T_{138}^d) \frac{1}{\lambda_{zz}} + ( \frac{x_H^2}{\lambda_{zz}} T_{137}^d + 8 T_7^d c^2 x_H^2 \right. \right. \\
 & \left. \left. - 2 T_{72}^d V_9 - T_{135}^d) c^2 \right] c^2 \right\} \frac{s}{c^5} B_0^{\text{fin}} \left( -M_H^2; M_W, M_W \right) \\
 & + \frac{1}{384} \left\{ 72 \frac{1}{\lambda_{zz}^2} T_{133}^d + \left[ -6 (6 \frac{x_H^2}{\lambda_{zz}} T_{100}^d - T_{138}^d) \frac{1}{\lambda_{zz}} \right. \right. \\
 & \left. \left. + (-3 \frac{x_H^2}{\lambda_{zz}} T_{137}^d + 6 T_{72}^d V_9 - T_{140}^d) c^2 \right] c^2 \right\} \frac{s}{c^5} B_0^{\text{fin}} \left( -M_Z^2; M_W, M_W \right) \\
 & - \frac{1}{384} \left\{ \left[ 32 c^2 x_b^2 v_b + (16 v_b + 9 \frac{x_b^2}{\lambda_{zz}} U_8 - 6 T_{78}^d W_{18} x_b^2 v_b + 9 V_9 U_4 x_b^2) \right] c^2 \right. \\
 & \left. + 36 \left[ (2 T_{78}^d v_b + 3 U_4) \frac{x_b^2}{\lambda_{zz}^2} \right] \frac{s}{c^3} B_0^{\text{fin}} \left( -M_Z^2; M_b, M_b \right) \right. \\
 & \left. - \frac{1}{384} \left\{ \left[ 64 c^2 x_t^2 v_t + (32 v_t + 9 \frac{x_t^2}{\lambda_{zz}} U_7 - 6 T_{77}^d W_{18} x_t^2 v_t + 9 V_9 U_2 x_t^2) \right] c^2 \right. \right. \\
 & \left. \left. + 36 \left[ (2 T_{77}^d v_t + 3 U_2) \frac{x_t^2}{\lambda_{zz}^2} \right] \frac{s}{c^3} B_0^{\text{fin}} \left( -M_Z^2; M_t, M_t \right) \right. \right. \\
 & \left. \left. - \frac{1}{576} \left\{ \left[ 48 (x_b^2 v_b + 2 x_t^2 v_t) c^2 - (-9 \frac{x_H^2}{\lambda_{zz}} T_{134}^d + 18 T_{77}^d W_{19} x_t^2 v_t \right. \right. \right. \right. \\
 & \left. \left. \left. + 18 T_{78}^d W_{19} x_b^2 v_b - T_{143}^d + 27 U_2 W_{19} x_t^2 + 27 U_4 W_{19} x_b^2) \right] c^2 + 9 \left( \frac{1}{\lambda_{zz}} T_{136}^d + T_{132}^d V_9 \right) \right\} \frac{s}{c^3} \right. \\
 & \left. + \frac{1}{128} \left\{ - \left[ (-3 \frac{1}{\lambda_{zz}} U_7 + 2 T_{77}^d W_{18} v_t - 3 V_9 U_2) \right] c^2 + 12 \left[ (2 T_{77}^d v_t + 3 U_2) \right] \frac{1}{\lambda_{zz}^2} \right\} \frac{s}{c^3} x_t^2
 \end{aligned}$$

$$\begin{aligned}
 & \times \mathbf{B}_0^{\text{fin}}(-M_{\text{H}}^2; M_{\text{t}}, M_{\text{t}}) \\
 & + \frac{1}{128} \left\{ - \left[ \left( -3 \frac{1}{\lambda_{\text{ZZ}}} U_8 + 2 T_{78}^d W_{18} v_{\text{b}} - 3 V_9 U_4 \right) c^2 + 12 \left[ (2 T_{78}^d v_{\text{b}} + 3 U_4) \right] \frac{1}{\lambda_{\text{ZZ}}} \right\} \frac{s}{c^3} x_{\text{b}}^2 \\
 & \times \mathbf{B}_0^{\text{fin}}(-M_{\text{H}}^2; M_{\text{b}}, M_{\text{b}}) \\
 & - \frac{1}{128} \left\{ \left[ \left( 3 \frac{1}{\lambda_{\text{ZZ}}} U_9 + 2 T_{77}^d W_{18} v_{\text{t}} - 3 V_9 U_2 \right) + 2 (2 T_{77}^d W_{20} v_{\text{t}} + 3 U_1 + 6 U_2 W_{19} x_{\text{t}}^2) c^2 \right] c^2 \right. \\
 & \left. + 12 \left[ (2 T_{77}^d v_{\text{t}} + 3 U_2) \right] \frac{1}{\lambda_{\text{ZZ}}} \right\} \frac{s}{c^5} x_{\text{t}}^2 C_0(-M_{\text{H}}^2, -M_{\text{Z}}^2, -M_{\text{Z}}^2; M_{\text{t}}, M_{\text{t}}, M_{\text{t}}) \\
 & - \frac{1}{128} \left\{ \left[ \left( 3 \frac{1}{\lambda_{\text{ZZ}}} U_{10} + 2 T_{78}^d W_{18} v_{\text{b}} - 3 V_9 U_4 \right) + 2 (2 T_{78}^d W_{24} v_{\text{b}} + 3 U_3 + 6 U_4 W_{19} x_{\text{b}}^2) c^2 \right] c^2 \right. \\
 & \left. + 12 \left[ (2 T_{78}^d v_{\text{b}} + 3 U_4) \right] \frac{1}{\lambda_{\text{ZZ}}} \right\} \frac{s}{c^5} x_{\text{b}}^2 C_0(-M_{\text{H}}^2, -M_{\text{Z}}^2, -M_{\text{Z}}^2; M_{\text{b}}, M_{\text{b}}, M_{\text{b}}) \\
 & + \frac{3}{64} (1 - c^2 x_{\text{H}}^2) \frac{s}{c} W_9 a_0^{\text{fin}}(M_{\text{H}}) \\
 & + \frac{3}{64} (s W_{18} + c^2 s W_9) \frac{s}{c^3} a_0^{\text{fin}}(M_{\text{Z}}) \\
 & + \frac{1}{48} (3 T_7^d c^2 x_{\text{H}}^2 + T_{142}^d) \frac{s}{c} L_{\text{R}}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{P}_{\text{P};\text{ZZ}}^{\text{nfc}}(a_{\Lambda\Lambda}) &= \frac{1}{32} \left\{ 12 \frac{1}{\lambda_{\text{ZZ}}} T_{150}^d - \left[ 2 (2 c^2 x_{\text{H}}^2 + \frac{x_{\text{H}}^2}{\lambda_{\text{ZZ}}} T_{38}^d - T_{144}^d) c^2 + (24 \frac{x_{\text{H}}^2}{\lambda_{\text{ZZ}}} T_7^d \right. \right. \\
 & \left. \left. - \frac{1}{\lambda_{\text{ZZ}}} T_{148}^d + T_{145}^d V_9 \right] c^2 \right\} \frac{s^2}{c^4} \mathbf{B}_0^{\text{fin}}(-M_{\text{H}}^2; M_{\text{W}}, M_{\text{W}}) \\
 & - \frac{1}{32} \left\{ 12 \frac{1}{\lambda_{\text{ZZ}}} T_{150}^d - \left[ 2 (8 c^4 x_{\text{H}}^2 + \frac{x_{\text{H}}^2}{\lambda_{\text{ZZ}}} T_{154}^d + T_{151}^d) c^2 + (24 \frac{x_{\text{H}}^2}{\lambda_{\text{ZZ}}} T_7^d \right. \right. \\
 & \left. \left. - \frac{1}{\lambda_{\text{ZZ}}} T_{155}^d - T_{147}^d V_9 \right] c^2 \right\} \frac{s^2}{c^6} C_0(-M_{\text{H}}^2, -M_{\text{Z}}^2, -M_{\text{Z}}^2; M_{\text{W}}, M_{\text{W}}, M_{\text{W}}) \\
 & - \frac{1}{32} \left\{ 12 \frac{1}{\lambda_{\text{ZZ}}} T_{150}^d - \left[ (24 \frac{x_{\text{H}}^2}{\lambda_{\text{ZZ}}} T_7^d - \frac{1}{\lambda_{\text{ZZ}}} T_{148}^d + T_{145}^d V_9) + 2 (\frac{x_{\text{H}}^2}{\lambda_{\text{ZZ}}} T_{38}^d + T_{149}^d) c^2 \right] c^2 \right\} \frac{s^2}{c^4} \\
 & \times \mathbf{B}_0^{\text{fin}}(-M_{\text{Z}}^2; M_{\text{W}}, M_{\text{W}}) \\
 & - \frac{1}{32} \left\{ 8 T_{29}^d c^4 + \left[ 4 \frac{x_{\text{H}}^2}{\lambda_{\text{ZZ}}} T_7^d c^2 + (T_8^d W_{19} x_{\text{b}}^2 v_{\text{b}} + T_9^d W_{19} x_{\text{t}}^2 v_{\text{t}} - 3 W_{47}) \right] s^2 \right. \\
 & \left. + 2 \left( -\frac{1}{\lambda_{\text{ZZ}}} T_{152}^d + T_{146}^d V_9 \right) \right\} \frac{1}{c^2} \\
 & - \frac{1}{64} \left\{ 12 \left[ (3 - T_8^d v_{\text{b}}) \right] \frac{1}{\lambda_{\text{ZZ}}} - \left[ (T_8^d W_{18} v_{\text{b}} - 3 W_{22}) + 2 (T_8^d W_{24} v_{\text{b}} - 3 W_{46}) c^2 \right] c^2 \right\} \frac{s^2}{c^6} x_{\text{b}}^2 \\
 & \times C_0(-M_{\text{H}}^2, -M_{\text{Z}}^2, -M_{\text{Z}}^2; M_{\text{b}}, M_{\text{b}}, M_{\text{b}}) \\
 & + \frac{1}{64} \left\{ 12 \left[ (3 - T_8^d v_{\text{b}}) \right] \frac{1}{\lambda_{\text{ZZ}}} + (T_8^d W_{18} v_{\text{b}} + 3 W_{21}) c^2 \right\} \frac{s^2}{c^4} x_{\text{b}}^2 \mathbf{B}_0^{\text{fin}}(-M_{\text{H}}^2; M_{\text{b}}, M_{\text{b}}) \\
 & - \frac{1}{64} \left\{ 12 \left[ (3 - T_8^d v_{\text{b}}) \right] \frac{1}{\lambda_{\text{ZZ}}} + (T_8^d W_{18} v_{\text{b}} + 3 W_{21}) c^2 \right\} \frac{s^2}{c^4} x_{\text{b}}^2 \mathbf{B}_0^{\text{fin}}(-M_{\text{Z}}^2; M_{\text{b}}, M_{\text{b}}) \\
 & - \frac{1}{64} \left\{ 12 \left[ (3 - T_9^d v_{\text{t}}) \right] \frac{1}{\lambda_{\text{ZZ}}} - \left[ (T_9^d W_{18} v_{\text{t}} - 3 W_{22}) + 2 (T_9^d W_{20} v_{\text{t}} - 3 W_{23}) c^2 \right] c^2 \right\} \frac{s^2}{c^6} x_{\text{t}}^2 \\
 & \times C_0(-M_{\text{H}}^2, -M_{\text{Z}}^2, -M_{\text{Z}}^2; M_{\text{t}}, M_{\text{t}}, M_{\text{t}}) \\
 & + \frac{1}{64} \left\{ 12 \left[ (3 - T_9^d v_{\text{t}}) \right] \frac{1}{\lambda_{\text{ZZ}}} + (T_9^d W_{18} v_{\text{t}} + 3 W_{21}) c^2 \right\} \frac{s^2}{c^4} x_{\text{t}}^2 \mathbf{B}_0^{\text{fin}}(-M_{\text{H}}^2; M_{\text{t}}, M_{\text{t}})
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{1}{64} \left\{ 12 \left[ (3 - T_9^d v_t) \right] \frac{1}{\lambda_{zz}^2} + (T_9^d W_{18} v_t + 3 W_{21}) c^2 \right\} \frac{s^2}{c^4} x_t^2 B_0^{\text{fin}} \left( -M_Z^2; M_t, M_t \right) \\
 & -\frac{1}{8} (c^2 x_H^2 - 2 T_{153}^d) s^2 L_R
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{P;ZZ}^{\text{nfc}}(a_{zz}) = & -\frac{1}{64} X_{16} \\
 & +\frac{3}{256} \left\{ 24 \frac{1}{\lambda_{zz}^2} T_{167}^d + \left[ 4(8 - 3 \frac{x_H^2}{\lambda_{zz}} T_{167}^d) \frac{1}{\lambda_{zz}} + (-\frac{x_H^2}{\lambda_{zz}} T_{167}^d + 3 T_{64}^d) c^2 \right] c^2 \right\} \frac{1}{c^4} x_H^2 \\
 & \times B_0^{\text{fin}} \left( -M_H^2; M_H, M_H \right) \\
 & -\frac{3}{256} \left\{ 48 \frac{1}{\lambda_{zz}^2} T_{167}^d + \left[ -(12 \frac{x_H^2}{\lambda_{zz}} T_{163}^d + T_{167}^d W_9 c^2 x_H^2 - 4 T_{167}^d W_{53}) c^2 \right. \right. \\
 & \left. \left. + 24 (-2 \frac{x_H^2}{\lambda_{zz}} T_{167}^d + T_{163}^d) \frac{1}{\lambda_{zz}} \right] c^2 \right\} \frac{1}{c^6} x_H^2 C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_H, M_Z, M_H \right) \\
 & -\frac{1}{64} \left\{ T_{164}^d + \left[ (\frac{x_H^2}{\lambda_{zz}} T_{167}^d + 3 \frac{1}{\lambda_{zz}} - T_{177}^d) \right] c^2 x_H^2 \right\} \frac{1}{c^2} a_0^{\text{fin}}(M_H) \\
 & +\frac{1}{64} \left\{ T_{164}^d V_9 + \left[ (\frac{x_H^2}{\lambda_{zz}} T_{167}^d + 3 \frac{1}{\lambda_{zz}} - T_{177}^d) \right] c^2 \right\} \frac{1}{c^4} a_0^{\text{fin}}(M_Z) \\
 & +\frac{1}{64} \left\{ T_{165}^d V_9 + \left[ (\frac{x_H^2}{\lambda_{zz}} T_{179}^d + \frac{1}{\lambda_{zz}} T_{157}^d + 2 T_{176}^d) + (2 T_{42}^d W_{19} x_t^2 v_t \right. \right. \\
 & \left. \left. + 2 T_{112}^d W_{19} x_b^2 v_b + 3 U_2 W_{19} x_t^2 + 3 U_4 W_{19} x_b^2) c^2 \right] c^2 \right\} \frac{1}{c^4} \\
 & +\frac{1}{128} \left\{ \left[ 64 c^8 x_H^2 - (\frac{x_H^2}{\lambda_{zz}} T_{171}^d + 2 \frac{1}{\lambda_{zz}} T_{182}^d - T_{174}^d - 2 T_{175}^d V_9) \right] c^2 \right. \\
 & \left. - 12 \left[ (2 T_{158}^d + T_{169}^d x_H^2) \frac{1}{\lambda_{zz}^2} \right] \frac{1}{c^6} C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_W, M_W, M_W \right) \right. \\
 & \left. - \frac{1}{128} \left\{ \left[ 4 T_{100}^d c^2 x_H^2 - (\frac{x_H^2}{\lambda_{zz}} T_{173}^d + 2 \frac{1}{\lambda_{zz}} T_{181}^d + 2 T_{170}^d V_9 + T_{172}^d) \right] c^2 \right. \right. \\
 & \left. \left. - 12 \left[ (2 T_{158}^d + T_{169}^d x_H^2) \frac{1}{\lambda_{zz}^2} \right] \frac{1}{c^4} B_0^{\text{fin}} \left( -M_H^2; M_W, M_W \right) \right. \right. \\
 & \left. \left. - \frac{1}{256} \left\{ - \left[ T_{167}^d W_9 c^2 x_H^2 + 2(36 \frac{x_H^2}{\lambda_{zz}^2} - 6 \frac{x_H^2}{\lambda_{zz}^2} T_{167}^d x_H^2 - 3 \frac{x_H^2}{\lambda_{zz}^2} T_{159}^d + T_{160}^d) \right] c^2 x_H^2 \right. \right. \right. \\
 & \left. \left. + 8 \left( \frac{x_H^2}{\lambda_{zz}^2} T_{178}^d + T_{159}^d \right) \right\} \frac{1}{c^4} C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_Z, M_H, M_Z \right) \right. \\
 & \left. \left. - \frac{1}{128} \left\{ \left[ 2(-6 \frac{x_H^2}{\lambda_{zz}^2} T_{167}^d x_H^2 + \frac{x_H^2}{\lambda_{zz}^2} T_{162}^d + 3 T_{166}^d) - (\frac{x_H^2}{\lambda_{zz}^2} T_{167}^d + 3 T_{163}^d) c^2 x_H^2 \right] c^2 \right. \right. \right. \\
 & \left. \left. + 8 \left[ (9 \frac{x_H^2}{\lambda_{zz}^2} T_{161}^d + 2 T_{159}^d) \right] \frac{1}{\lambda_{zz}} \right\} \frac{1}{c^4} B_0^{\text{fin}} \left( -M_Z^2; M_H, M_Z \right) \right. \\
 & \left. \left. + \frac{1}{256} \left\{ \left[ 12 \left( \frac{x_H^2}{\lambda_{zz}^2} T_{167}^d x_H^2 - W_{52} \right) + (\frac{x_H^2}{\lambda_{zz}^2} T_{167}^d - T_{177}^d) c^2 x_H^2 \right] c^2 + 4 \left( \frac{1}{\lambda_{zz}^2} T_{180}^d + T_{164}^d V_9 \right) \right\} \frac{1}{c^4} \right. \right. \\
 & \left. \left. \times B_0^{\text{fin}} \left( -M_H^2; M_Z, M_Z \right) \right. \right. \\
 & \left. \left. + \frac{1}{128} \left\{ - \left[ (-3 \frac{1}{\lambda_{zz}} U_7 + 2 T_{42}^d W_{18} v_t - 3 V_9 U_2) \right] c^2 + 12 \left[ (2 T_{42}^d v_t + 3 U_2) \right] \frac{1}{\lambda_{zz}^2} \right\} \frac{1}{c^2} x_t^2 \right. \right. \\
 & \left. \left. \times B_0^{\text{fin}} \left( -M_H^2; M_t, M_t \right) \right. \right. \\
 & \left. \left. - \frac{1}{128} \left\{ - \left[ (-3 \frac{1}{\lambda_{zz}} U_7 + 2 T_{42}^d W_{18} v_t - 3 V_9 U_2) \right] c^2 + 12 \left[ (2 T_{42}^d v_t + 3 U_2) \right] \frac{1}{\lambda_{zz}^2} \right\} \frac{1}{c^2} x_t^2 \right. \right.
 \end{aligned}$$



$$\begin{aligned}
 & \times \mathbf{B}_0^{\text{fin}}(-M_Z^2; M_t, M_t) \\
 & + \frac{1}{128} \left\{ - \left[ \left( -3 \frac{1}{\lambda_{ZZ}} U_8 + 2 T_{112}^d W_{18} v_b - 3 V_9 U_4 \right) c^2 + 12 \left[ (2 T_{112}^d v_b + 3 U_4) \right] \frac{1}{\lambda_{ZZ}^2} \right\} \frac{1}{c^2} x_b^2 \\
 & \times \mathbf{B}_0^{\text{fin}}(-M_H^2; M_b, M_b) \\
 & - \frac{1}{128} \left\{ - \left[ \left( -3 \frac{1}{\lambda_{ZZ}} U_8 + 2 T_{112}^d W_{18} v_b - 3 V_9 U_4 \right) c^2 + 12 \left[ (2 T_{112}^d v_b + 3 U_4) \right] \frac{1}{\lambda_{ZZ}^2} \right\} \frac{1}{c^2} x_b^2 \\
 & \times \mathbf{B}_0^{\text{fin}}(-M_Z^2; M_b, M_b) \\
 & - \frac{1}{128} \left\{ \left[ \left( 3 \frac{1}{\lambda_{ZZ}} U_9 + 2 T_{42}^d W_{18} v_t - 3 V_9 U_2 \right) + 2 (2 T_{42}^d W_{20} v_t + 3 U_1 + 6 U_2 W_{19} x_t^2) c^2 \right] c^2 \right. \\
 & \left. + 12 \left[ (2 T_{42}^d v_t + 3 U_2) \right] \frac{1}{\lambda_{ZZ}^2} \right\} \frac{1}{c^4} x_t^2 C_0(-M_H^2, -M_Z^2, -M_Z^2; M_t, M_t, M_t) \\
 & - \frac{1}{128} \left\{ \left[ \left( 3 \frac{1}{\lambda_{ZZ}} U_{10} + 2 T_{112}^d W_{18} v_b - 3 V_9 U_4 \right) + 2 (2 T_{112}^d W_{24} v_b + 3 U_3 + 6 U_4 W_{19} x_b^2) c^2 \right] c^2 \right. \\
 & \left. + 12 \left[ (2 T_{112}^d v_b + 3 U_4) \right] \frac{1}{\lambda_{ZZ}^2} \right\} \frac{1}{c^4} x_b^2 C_0(-M_H^2, -M_Z^2, -M_Z^2; M_b, M_b, M_b) \\
 & - \frac{1}{128} \left\{ 12 \left[ (2 T_{158}^d + T_{169}^d x_H^2) \right] \frac{1}{\lambda_{ZZ}^2} + \left( -\frac{x_H^2}{\lambda_{ZZ}} T_{173} + 2 \frac{1}{\lambda_{ZZ}} T_{181} + T_{168}^d + 2 T_{170}^d V_9 \right) c^2, rc \right\} \frac{1}{c^4} \\
 & \times \mathbf{B}_0^{\text{fin}}(-M_Z^2; M_W, M_W) \\
 & - \frac{1}{32} (T_{108}^d c^2 x_H^2 + 4 T_{156}^d) \frac{1}{c^2} L_R
 \end{aligned}$$

• HWW Amplitudes

$$\begin{aligned}
 \mathcal{F}_{D;WW}^{\text{nfc}}(a_{\phi 1\nu}) &= \frac{1}{32} X_{17} - \frac{1}{16} V_6 - \frac{3}{32} V_{14} L_R \\
 & - \frac{1}{32} V_{15} x_b^2 a_0^{\text{fin}}(M_b) - \frac{1}{32} V_{16} x_t^2 a_0^{\text{fin}}(M_t) - \frac{1}{32} V_{17} \mathbf{B}_0^{\text{fin}}(-M_W^2; M_t, M_b)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{D;WW}^{\text{nfc}}(a_{\phi 1\lambda}) &= \frac{1}{32} X_{17} - \frac{1}{16} V_6 - \frac{3}{32} V_{14} L_R \\
 & - \frac{1}{32} V_{15} x_b^2 a_0^{\text{fin}}(M_b) - \frac{1}{32} V_{16} x_t^2 a_0^{\text{fin}}(M_t) - \frac{1}{32} V_{17} \mathbf{B}_0^{\text{fin}}(-M_W^2; M_t, M_b)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{D;WW}^{\text{nfc}}(a_{\phi b\nu}) &= \frac{1}{32} X_{17} - \frac{1}{16} V_6 - \frac{3}{32} V_{14} L_R \\
 & - \frac{1}{32} V_{15} x_b^2 a_0^{\text{fin}}(M_b) - \frac{1}{32} V_{16} x_t^2 a_0^{\text{fin}}(M_t) - \frac{1}{32} V_{17} \mathbf{B}_0^{\text{fin}}(-M_W^2; M_t, M_b)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{D;WW}^{\text{nfc}}(a_{\phi b\lambda}) &= \frac{1}{32} X_{17} - \frac{1}{16} V_6 - \frac{3}{32} V_{14} L_R \\
 & - \frac{1}{32} V_{15} x_b^2 a_0^{\text{fin}}(M_b) - \frac{1}{32} V_{16} x_t^2 a_0^{\text{fin}}(M_t) - \frac{1}{32} V_{17} \mathbf{B}_0^{\text{fin}}(-M_W^2; M_t, M_b)
 \end{aligned}$$

$$\mathcal{F}_{D;WW}^{\text{nfc}}(a_{\phi\nu}) = -\frac{1}{16} \mathbf{B}_0^{\text{fin}}(-M_W^2; 0, 0) + \frac{1}{48} (1 - 3L_R)$$

$$\mathcal{F}_{D;WW}^{\text{nfc}}(a_{\phi 1\nu}) = -\frac{1}{16} \mathbf{B}_0^{\text{fin}}(-M_W^2; 0, 0) + \frac{1}{48} (1 - 3L_R)$$

$$\mathcal{F}_{D;WW}^{\text{nfc}}(a_{\phi 1\lambda}) = -\frac{1}{16} B_0^{\text{fin}}(-M_W^2; 0, 0) + \frac{1}{48} (1 - 3L_R)$$

$$\begin{aligned} \mathcal{F}_{D;WW}^{\text{nfc}}(a_{t\phi}) &= \frac{3}{32} x_t^2 (1 - L_R) \\ &+ \frac{3}{16} \frac{x_t^2}{\lambda_{ww}} V_{18} B_0^{\text{fin}}(-M_H^2; M_t, M_t) \\ &- \frac{3}{32} W_{54} x_t^2 B_0^{\text{fin}}(-M_W^2; M_t, M_b) \\ &- \frac{3}{32} W_{55} x_t^2 C_0(-M_H^2, -M_W^2, -M_W^2; M_t, M_b, M_t) \end{aligned}$$

$$\begin{aligned} \mathcal{F}_{D;WW}^{\text{nfc}}(a_{b\phi}) &= -\frac{1}{32} X_{18} + \frac{3}{32} x_t^2 (1 - L_R) \\ &+ \frac{3}{16} \frac{x_t^2}{\lambda_{ww}} V_{18} B_0^{\text{fin}}(-M_H^2; M_t, M_t) \\ &- \frac{3}{32} W_{54} x_t^2 B_0^{\text{fin}}(-M_W^2; M_t, M_b) \\ &- \frac{3}{32} W_{55} x_t^2 C_0(-M_H^2, -M_W^2, -M_W^2; M_t, M_b, M_t) \end{aligned}$$

$$\begin{aligned} \mathcal{F}_{D;WW}^{\text{nfc}}(a_{tBW}) &= -\frac{3}{128} c x_H^2 x_t^2 \\ &- \frac{3}{128} c x_H^2 x_t^2 x_b^2 a_0^{\text{fin}}(M_b) + \frac{3}{128} c x_H^2 x_t^4 a_0^{\text{fin}}(M_t) \\ &+ \frac{3}{32} c x_t^2 x_b^2 B_0^{\text{fin}}(-M_H^2; M_b, M_b) \\ &+ \frac{3}{64} V_{22} c x_t^2 x_b^2 C_0(-M_H^2, -M_W^2, -M_W^2; M_b, M_t, M_b) \\ &+ \frac{3}{128} W_{56} c x_t^2 B_0^{\text{fin}}(-M_W^2; M_t, M_b) \\ &- \frac{3}{128} W_{57} c x_t^2 B_0^{\text{fin}}(-M_H^2; M_t, M_t) \\ &+ \frac{3}{64} W_{58} c x_t^2 C_0(-M_H^2, -M_W^2, -M_W^2; M_t, M_b, M_t) \end{aligned}$$

$$\begin{aligned} \mathcal{F}_{D;WW}^{\text{nfc}}(a_{tWB}) &= -\frac{3}{128} s x_H^2 x_t^2 \\ &- \frac{3}{128} s x_H^2 x_t^2 x_b^2 a_0^{\text{fin}}(M_b) + \frac{3}{128} s x_H^2 x_t^4 a_0^{\text{fin}}(M_t) \\ &+ \frac{3}{32} s x_t^2 x_b^2 B_0^{\text{fin}}(-M_H^2; M_b, M_b) \\ &+ \frac{3}{64} V_{22} s x_t^2 x_b^2 C_0(-M_H^2, -M_W^2, -M_W^2; M_b, M_t, M_b) \\ &+ \frac{3}{128} W_{56} s x_t^2 B_0^{\text{fin}}(-M_W^2; M_t, M_b) \\ &- \frac{3}{128} W_{57} s x_t^2 B_0^{\text{fin}}(-M_H^2; M_t, M_t) \\ &+ \frac{3}{64} W_{58} s x_t^2 C_0(-M_H^2, -M_W^2, -M_W^2; M_t, M_b, M_t) \end{aligned}$$

$$\begin{aligned} \mathcal{F}_{D;WW}^{\text{nfc}}(a_{bBW}) &= \frac{1}{32} c X_{18} + \frac{3}{32} V_6 c L_R \\ &- \frac{3}{128} c x_H^2 x_b^4 a_0^{\text{fin}}(M_b) + \frac{3}{128} c x_H^2 x_t^2 x_b^2 a_0^{\text{fin}}(M_t) \\ &+ \frac{3}{128} V_{23} c x_b^2 B_0^{\text{fin}}(-M_H^2; M_b, M_b) \end{aligned}$$

$$\begin{aligned}
 & + \frac{3}{64} V_{24} c x_b^2 C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_b, M_t, M_b \right) \\
 & - \frac{3}{128} V_{27} c \\
 & - \frac{3}{32} W_{59} c x_t^2 B_0^{\text{fin}} \left( -M_H^2; M_t, M_t \right) \\
 & + \frac{3}{128} W_{60} c B_0^{\text{fin}} \left( -M_W^2; M_t, M_b \right) \\
 & + \frac{3}{64} W_{61} c x_t^2 C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_t, M_b, M_t \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{D;WW}^{\text{nfc}}(a_{b_{WB}}) &= \frac{1}{32} s X_{18} + \frac{3}{32} V_6 s L_R \\
 & - \frac{3}{128} s x_H^2 x_b^4 a_0^{\text{fin}}(M_b) + \frac{3}{128} s x_H^2 x_t^2 x_b^2 a_0^{\text{fin}}(M_t) \\
 & + \frac{3}{128} V_{23} s x_b^2 B_0^{\text{fin}} \left( -M_H^2; M_b, M_b \right) \\
 & + \frac{3}{64} V_{24} s x_b^2 C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_b, M_t, M_b \right) \\
 & - \frac{3}{128} V_{27} s \\
 & - \frac{3}{32} W_{59} s x_t^2 B_0^{\text{fin}} \left( -M_H^2; M_t, M_t \right) \\
 & + \frac{3}{128} W_{60} s B_0^{\text{fin}} \left( -M_W^2; M_t, M_b \right) \\
 & + \frac{3}{64} W_{61} s x_t^2 C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_t, M_b, M_t \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{D;WW}^{\text{nfc}}(a_{\phi_B}) &= \frac{3}{16} \\
 & + \frac{3}{8} \frac{1}{\lambda_{ww}} V_0 B_0^{\text{fin}} \left( -M_H^2; M_H, M_H \right) \\
 & - \frac{3}{8} \frac{1}{\lambda_{ww}} V_0 B_0^{\text{fin}} \left( -M_W^2; M_W, M_H \right) \\
 & + \frac{3}{8} W_{62} C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_H, M_W, M_H \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{D;WW}^{\text{nfc}}(a_{\phi_{\square}}) &= -\frac{1}{1152} X_{19} - \frac{1}{96} x_H^2 a_0^{\text{fin}}(M_W) \\
 & - \frac{1}{768} x_H^2 (103 + 12L_R) - \frac{1}{768} \frac{1}{c^2} + \frac{1}{768} \frac{1}{c^2} T_{188}^d L_R \\
 & + \frac{1}{96} V_0 s^2 C_0^{\text{fin}} \left( -M_H^2, -M_W^2, -M_W^2; M_W, 0, M_W \right) \\
 & - \frac{1}{96} V_1 x_H^2 a_0^{\text{fin}}(M_H) \\
 & - \frac{1}{256} W_{64} B_0^{\text{fin}} \left( -M_H^2; M_H, M_H \right) \\
 & + \frac{1}{256} W_{65} C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_H, M_W, M_H \right) \\
 & + \frac{1}{384} W_{66} B_0^{\text{fin}} \left( -M_W^2; M_W, M_H \right) \\
 & - \frac{1}{768} W_{67} C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_W, M_H, M_W \right) \\
 & - \frac{1}{384} \left[ \frac{1}{\lambda_{ww}} T_{185}^d + (23 \frac{x_H^2}{\lambda_{ww}} c^2 - 2 T_{183}^d) c^2 \right] \frac{1}{c^4} B_0^{\text{fin}} \left( -M_W^2; M_W, M_Z \right) \\
 & + \frac{1}{768} \left[ 2 \frac{1}{\lambda_{ww}} T_{187}^d + (23 \frac{x_H^2}{\lambda_{ww}} T_{195}^d + T_{193}^d) c^2 \right] \frac{1}{c^4} B_0^{\text{fin}} \left( -M_H^2; M_Z, M_Z \right)
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{1}{768} \left\{ c^2 s W_{63} x_H^2 + \left[ \left( -\frac{x_H^2}{\lambda_{\text{ww}}} T_{192}^d + 2 \frac{1}{\lambda_{\text{ww}}} T_{194}^d + 2 T_{190}^d \right) \right] \right\} \frac{1}{c^2} B_0^{\text{fin}} \left( -M_H^2; M_W, M_W \right) \\
 & + \frac{1}{768} \left\{ 2 \frac{1}{\lambda_{\text{ww}}} T_{186}^d - \left[ 9 c^4 x_H^2 - \left( -23 \frac{x_H^2}{\lambda_{\text{ww}}} T_{196}^d + 2 T_{189}^d \right) \right] c^2 \right\} \frac{1}{c^6} \\
 & \times C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_Z, M_W, M_Z \right) \\
 & - \frac{1}{768} \left\{ T_{184}^d c^2 x_H^2 + \left[ \left( 23 \frac{x_H^2}{\lambda_{\text{ww}}} - 2 \frac{1}{\lambda_{\text{ww}}} T_{194}^d - 2 T_{191}^d \right) \right] \right\} \frac{1}{c^4} \\
 & \times C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_W, M_Z, M_W \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{D;WW}^{\text{nfc}}(a_{\phi_D}) = & -\frac{1}{3072} X_{21} + \frac{1}{3072} \frac{s^2}{c^2} X_{20} \\
 & + \frac{1}{384} x_H^2 a_0^{\text{fin}}(M_W) + \frac{1}{384} \frac{1}{c^4} T_{211}^d a_0^{\text{fin}}(M_Z) + \frac{1}{384} V_1 x_H^2 a_0^{\text{fin}}(M_H) \\
 & + \frac{1}{3072} \left[ \frac{1}{\lambda_{\text{ww}}} T_{206}^d + \left( \frac{x_H^2}{\lambda_{\text{ww}}} T_{215}^d c^2 + 2 T_{208}^d \right) c^2 \right] \frac{1}{c^6} B_0^{\text{fin}} \left( -M_W^2; M_W, M_Z \right) \\
 & - \frac{1}{6144} \left[ \left( T_{197}^d x_H^2 + 2 T_{201}^d \right) \frac{1}{\lambda_{\text{ww}}} - \left( T_{199}^d x_H^2 - 2 T_{209}^d \right) c^2 \right] \frac{1}{c^6} \\
 & \times C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_W, M_Z, M_W \right) \\
 & - \frac{1}{3072} \left\{ 8 c^2 s W_{71} + \left[ \left( 4 \frac{x_H^2}{\lambda_{\text{ww}}} T_{219}^d + T_{223}^d W_{68} \right) \right] \right\} \frac{1}{c^2} B_0^{\text{fin}} \left( -M_W^2; M_W, M_H \right) \\
 & - \frac{1}{2048} \left\{ 16 c^2 s W_{70} + \left[ \left( -4 \frac{x_H^2}{\lambda_{\text{ww}}} T_{217}^d + \frac{x_H^2}{\lambda_{\text{ww}}} T_{220}^d x_H^2 + 4 \frac{1}{\lambda_{\text{ww}}} T_{213}^d - T_{226}^d \right) \right] x_H^2 \right\} \frac{1}{c^2} \\
 & \times C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_H, M_W, M_H \right) \\
 & + \frac{1}{2048} \left\{ 16 c^2 s W_{69} + \left[ \left( 2 T_{216}^d - T_{220}^d x_H^2 \right) \frac{x_H^2}{\lambda_{\text{ww}}} \right] \right\} \frac{1}{c^2} B_0^{\text{fin}} \left( -M_H^2; M_H, M_H \right) \\
 & - \frac{1}{6144} \left\{ 2 \frac{1}{\lambda_{\text{ww}}} T_{202}^d + \left[ 9 T_{218}^d c^4 x_H^2 - \left( \frac{x_H^2}{\lambda_{\text{ww}}} T_{228}^d + 2 T_{207}^d \right) \right] c^2 \right\} \frac{1}{c^8} \\
 & \times C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_Z, M_W, M_Z \right) \\
 & - \frac{1}{6144} \left\{ 2 \frac{1}{\lambda_{\text{ww}}} T_{203}^d + \left[ 48 c^4 x_H^2 - \left( -\frac{x_H^2}{\lambda_{\text{ww}}} T_{227}^d + T_{198}^d \right) \right] c^2 \right\} \frac{1}{c^6} B_0^{\text{fin}} \left( -M_H^2; M_Z, M_Z \right) \\
 & + \frac{1}{6144} \left\{ \left[ 144 c^2 x_H^2 + \left( \frac{x_H^2}{\lambda_{\text{ww}}} T_{212}^d x_H^2 + 2 T_{210}^d \right) \right] c^2 + \left[ \left( 2 T_{201}^d + T_{204}^d x_H^2 \right) \right] \frac{1}{\lambda_{\text{ww}}} \right\} \frac{1}{c^4} \\
 & \times B_0^{\text{fin}} \left( -M_H^2; M_W, M_W \right) \\
 & + \frac{1}{2048} \left( 16 c^4 x_H^2 - T_{205}^d \right) \frac{1}{c^4} L_R \\
 & - \frac{1}{6144} \left( \frac{x_H^2}{\lambda_{\text{ww}}} T_{212}^d x_H^4 + 2 \frac{x_H^2}{\lambda_{\text{ww}}} T_{224}^d x_H^2 - 2 T_{216}^d V_0 + T_{221}^d x_H^4 \right) \frac{1}{c^2} \\
 & \times C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_W, M_H, M_W \right) \\
 & + \frac{1}{768} \left( T_{200}^d x_H^2 - 2 T_{214}^d \right) \frac{1}{c^2} C_0^{\text{fin}} \left( -M_H^2, -M_W^2, -M_W^2; M_W, 0, M_W \right) \\
 & - \frac{1}{6144} \left( T_{222}^d - T_{225}^d c^2 x_H^2 \right) \frac{1}{c^4}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{D;WW}^{\text{nfc}}(a_{A_Z}) = & -\frac{1}{384} s c X_{23} - \frac{1}{256} s c x_H^2 X_{22} \\
 & - \frac{1}{768} \frac{s}{c} X_{24} + \frac{1}{16} \frac{c^3}{s} V_{39} X_{25} + \frac{1}{64} \frac{s}{c} T_{88}^d x_H^2 a_0^{\text{fin}}(M_W)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{8} \frac{1}{\lambda_{\text{ww}}} \frac{s}{c^3} T_{88}^d B_0^{\text{fin}} \left( -M_{\text{H}}^2; 0, M_Z \right) \\
 & - \frac{1}{32} V_{36} s c x_{\text{H}}^2 a_0^{\text{fin}} (M_{\text{H}}) \\
 & + \frac{1}{256} \left[ 48 c^2 x_{\text{H}}^2 + \left( -\frac{1}{\lambda_{\text{ww}}} T_{240}^d V_{28} + T_{237}^d \right) \right] \frac{s}{c} x_{\text{H}}^2 C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{H}}, M_{\text{W}}, M_{\text{H}} \right) \\
 & + \frac{1}{96} \left[ 12 s^2 c^2 x_{\text{H}}^4 - (T_{258}^d x_{\text{H}}^2 - 2 T_{259}^d) \right] \frac{s}{c} C_0^{\text{fin}} \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{W}}, 0, M_{\text{W}} \right) \\
 & + \frac{1}{16} \left[ 2 (c^2 x_{\text{H}}^2 - T_{232}^d) c^2 - \left( \frac{1}{\lambda_{\text{ww}}} T_{88}^d - V_9 \right) \right] \frac{s}{c^3} B_0^{\text{fin}} \left( -M_{\text{W}}^2; 0, M_{\text{W}} \right) \\
 & - \frac{1}{768} \left\{ 2 \frac{1}{\lambda_{\text{ww}}} T_{246}^d + \left[ 6 (c^2 x_{\text{H}}^2 + 2 T_{229}^d) c^2 x_{\text{H}}^2 + \left( \frac{x_{\text{H}}^2}{\lambda_{\text{ww}}} T_{243}^d + T_{255}^d \right) \right] c^2 \right\} \frac{s}{c^5} \\
 & \times B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_Z, M_Z \right) \\
 & - \frac{1}{768} \left\{ 2 \frac{1}{\lambda_{\text{ww}}} T_{247}^d + \left[ 3 T_{257}^d c^4 x_{\text{H}}^2 + \left( \frac{x_{\text{H}}^2}{\lambda_{\text{ww}}} T_{245}^d + 2 T_{250}^d \right) \right] c^2 \right\} \frac{s}{c^7} \\
 & \times C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_Z, M_{\text{W}}, M_Z \right) \\
 & + \frac{1}{384} \left\{ \frac{1}{\lambda_{\text{ww}}} T_{248}^d - \left[ \frac{x_{\text{H}}^2}{\lambda_{\text{ww}}} T_{252}^d c^2 + 2 (3 T_{233}^d x_{\text{H}}^2 - T_{253}^d + 12 V_9) \right] c^2 \right\} \frac{s}{c^5} \\
 & \times B_0^{\text{fin}} \left( -M_{\text{W}}^2; M_{\text{W}}, M_Z \right) \\
 & - \frac{1}{384} \left\{ 12 V_{40} c^2 x_{\text{H}}^2 + \left[ 4 \frac{x_{\text{H}}^2}{\lambda_{\text{ww}}} T_{239}^d - \frac{x_{\text{H}}^2}{\lambda_{\text{ww}}} T_{240}^d x_{\text{H}}^2 + 2 T_{237}^d \right] \right\} \frac{s}{c} \\
 & \times B_0^{\text{fin}} \left( -M_{\text{W}}^2; M_{\text{W}}, M_{\text{H}} \right) \\
 & - \frac{1}{768} \left\{ \left[ 12 c^2 x_{\text{H}}^4 - \left( \frac{x_{\text{H}}^2}{\lambda_{\text{ww}}} T_{240}^d x_{\text{H}}^2 + 24 T_{88}^d x_{\text{H}}^2 - 2 T_{251}^d \right) \right] c^2 - (T_{244}^d x_{\text{H}}^2 + 2 T_{249}^d) \frac{1}{\lambda_{\text{ww}}} \right\} \frac{s}{c^3} \\
 & \times B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{W}}, M_{\text{W}} \right) \\
 & + \frac{1}{768} \left\{ \left[ 96 c^6 x_{\text{H}}^4 + (2 T_{254}^d - T_{256}^d x_{\text{H}}^2) \right] c^2 - (T_{242}^d x_{\text{H}}^2 + 2 T_{249}^d) \frac{1}{\lambda_{\text{ww}}} \right\} \frac{s}{c^5} \\
 & \times C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{W}}, M_Z, M_{\text{W}} \right) \\
 & + \frac{1}{32} \left\{ 2 \left[ T_8^d + (c^2 x_{\text{H}}^2 - 2 T_{230}^d) c^2 x_{\text{H}}^2 \right] c^2 + \left( \frac{1}{\lambda_{\text{ww}}} T_{231}^d - V_9 \right) \right\} \frac{s}{c^5} \\
 & \times C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_Z, M_{\text{W}}, 0 \right) \\
 & + \frac{1}{32} \left\{ 2 \left[ T_8^d + (c^2 x_{\text{H}}^2 - 2 T_{230}^d) c^2 x_{\text{H}}^2 \right] c^2 + \left( \frac{1}{\lambda_{\text{ww}}} T_{231}^d - V_9 \right) \right\} \frac{s}{c^5} \\
 & \times C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; 0, M_{\text{W}}, M_Z \right) \\
 & + \frac{1}{256} (1 - c^2 x_{\text{H}}^2) \frac{s}{c^3} \\
 & - \frac{1}{256} (12 s^2 c^4 x_{\text{H}}^4 + 3 T_{234}^d + 8 T_{235}^d c^2 x_{\text{H}}^2) \frac{1}{s c^3} L_{\text{R}} \\
 & - \frac{1}{768} \left( 2 \frac{x_{\text{H}}^2}{\lambda_{\text{ww}}} T_{236}^d x_{\text{H}}^2 + \frac{x_{\text{H}}^2}{\lambda_{\text{ww}}} T_{240}^d x_{\text{H}}^4 + T_{237}^d V_{38} x_{\text{H}}^2 - 4 T_{238}^d \right) \frac{s}{c} \\
 & \times C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{W}}, M_{\text{H}}, M_{\text{W}} \right) \\
 & + \frac{1}{256} \left( \frac{1}{\lambda_{\text{ww}}} T_{240}^d V_0 + 6 V_{40} c^2 \right) \frac{s}{c} x_{\text{H}}^2 B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{H}}, M_{\text{H}} \right) \\
 & - \frac{1}{192} (3 T_{100}^d x_{\text{H}}^2 - 4 T_{241}^d) \frac{s}{c^3} a_0^{\text{fin}} (M_Z)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{\text{D};\text{WW}}^{\text{nc}}(a_{\Lambda\Lambda}) = & \frac{1}{32} s^2 X_{27} - \frac{1}{64} s^2 x_{\text{H}}^2 X_{26} + \frac{1}{8} V_{39} c^2 X_{25} \\
 & - \frac{1}{16} s^2 x_{\text{H}}^2 a_0^{\text{fin}}(M_{\text{W}}) \\
 & - \frac{1}{32} \frac{s^2}{c^2} T_7^d V_{36} a_0^{\text{fin}}(M_{\text{Z}}) \\
 & - \frac{1}{64} \frac{s^2}{c^2} T_{269}^d \\
 & - \frac{1}{8} V_0 s^2 x_{\text{H}}^2 C_0(-M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; 0, M_{\text{W}}, 0) \\
 & - \frac{1}{32} V_{36} s^2 x_{\text{H}}^2 a_0^{\text{fin}}(M_{\text{H}}) \\
 & + \frac{1}{8} V_{39} c^2 d\mathcal{Z}_c^{(4)} \\
 & - \frac{3}{128} W_{73} s^2 x_{\text{H}}^2 B_0^{\text{fin}}(-M_{\text{H}}^2; M_{\text{H}}, M_{\text{H}}) \\
 & + \frac{3}{64} W_{74} s^2 x_{\text{H}}^2 C_0(-M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{H}}, M_{\text{W}}, M_{\text{H}}) \\
 & + \frac{1}{32} W_{75} s^2 B_0^{\text{fin}}(-M_{\text{W}}^2; M_{\text{W}}, M_{\text{H}}) \\
 & + \frac{1}{64} W_{76} s^2 C_0(-M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{W}}, M_{\text{H}}, M_{\text{W}}) \\
 & - \frac{1}{64} \left\{ c^2 s W_{72} + \left[ (2T_{77}^d + T_{260}^d x_{\text{H}}^2) \right] \frac{1}{\lambda_{\text{WW}}} \right\} \frac{s^2}{c^2} B_0^{\text{fin}}(-M_{\text{H}}^2; M_{\text{W}}, M_{\text{W}}) \\
 & + \frac{1}{8} \left\{ s^2 x_{\text{H}}^4 + \left[ (2T_{107}^d - T_{270}^d x_{\text{H}}^2) \right] \right\} s^2 C_0^{\text{fin}}(-M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{W}}, 0, M_{\text{W}}) \\
 & + \frac{1}{128} \left\{ 4 \frac{1}{\lambda_{\text{WW}}} T_{261}^d + \left[ V_2 c^2 x_{\text{H}}^2 + 2 \left( -\frac{x_{\text{H}}^2}{\lambda_{\text{WW}}} T_{195}^d + T_{77}^d \right) \right] c^2 \right\} \frac{s^2}{c^4} B_0^{\text{fin}}(-M_{\text{H}}^2; M_{\text{Z}}, M_{\text{Z}}) \\
 & - \frac{1}{32} \left\{ \frac{1}{\lambda_{\text{WW}}} T_{262}^d - \left[ \frac{x_{\text{H}}^2}{\lambda_{\text{WW}}} c^2 + (x_{\text{H}}^2 - 2T_{264}^d) \right] c^2 \right\} \frac{s^2}{c^4} B_0^{\text{fin}}(-M_{\text{W}}^2; M_{\text{W}}, M_{\text{Z}}) \\
 & + \frac{1}{64} \left\{ 2 \frac{1}{\lambda_{\text{WW}}} T_{263}^d + \left[ \frac{x_{\text{H}}^2}{\lambda_{\text{WW}}} T_{196}^d + (11c^2 x_{\text{H}}^2 + 2T_{268}^d) c^2 \right] s^2 c^2 \right\} \frac{1}{c^6} \\
 & \times C_0(-M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{Z}}, M_{\text{W}}, M_{\text{Z}}) \\
 & + \frac{1}{64} \left\{ \left[ 8c^4 x_{\text{H}}^4 + (T_{265}^d x_{\text{H}}^2 - 2T_{266}^d) \right] c^2 + (x_{\text{H}}^2 + 2T_{77}^d) \frac{1}{\lambda_{\text{WW}}} \right\} \frac{s^2}{c^4} \\
 & \times C_0(-M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{W}}, M_{\text{Z}}, M_{\text{W}}) \\
 & + \frac{1}{4} (x_{\text{H}}^2 - 2c^2) s^2 B_0^{\text{fin}}(-M_{\text{W}}^2; 0, M_{\text{W}}) \\
 & - \frac{1}{64} (3s^2 c^2 x_{\text{H}}^4 - 22T_{29}^d c^2 x_{\text{H}}^2 - T_{267}^d) \frac{1}{c^2} \text{LR}
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{\text{D};\text{WW}}^{\text{nc}}(a_{\text{ZZ}}) = & \frac{1}{64} c^2 X_{29} - \frac{1}{128} c^2 x_{\text{H}}^2 X_{28} + \frac{1}{64} s^2 X_{20} \\
 & + \frac{1}{2} s^2 c^2 B_0^{\text{fin}}(-M_{\text{W}}^2; 0, M_{\text{W}}) \\
 & + \frac{1}{32} \frac{s^2}{c^2} T_7^d V_{36} a_0^{\text{fin}}(M_{\text{Z}}) \\
 & - \frac{1}{32} T_{195}^d x_{\text{H}}^2 a_0^{\text{fin}}(M_{\text{W}}) \\
 & - \frac{1}{32} V_{36} c^2 x_{\text{H}}^2 a_0^{\text{fin}}(M_{\text{H}}) \\
 & + \frac{3}{128} \left[ 8c^2 x_{\text{H}}^2 + \left( \frac{1}{\lambda_{\text{WW}}} T_{195}^d V_{28} - T_{290}^d \right) \right] x_{\text{H}}^2 C_0(-M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{H}}, M_{\text{W}}, M_{\text{H}})
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{16} \left[ 2c^2 x_H^4 - (T_{195}^d x_H^2 + 2T_{287}^d) \right] s^2 C_0^{\text{fin}} \left( -M_H^2, -M_W^2, -M_W^2; M_W, 0, M_W \right) \\
 & + \frac{1}{128} \left[ T_{282}^d - 2(3c^2 x_H^2 + 2T_{275}^d) c^2 x_H^2 \right] \frac{1}{c^2} \text{LR} \\
 & - \frac{1}{64} \left[ 2V_{40} c^2 x_H^2 + \left( \frac{x_H^2}{\lambda_{\text{ww}}} T_{195}^d x_H^2 - 4 \frac{x_H^2}{\lambda_{\text{ww}}} T_{288}^d - 2T_{290}^d \right) \right] B_0^{\text{fin}} \left( -M_W^2; M_W, M_H \right) \\
 & + \frac{1}{128} \left\{ 2 \frac{1}{\lambda_{\text{ww}}} T_{277}^d - \left[ (c^2 x_H^2 - 2T_{286}^d) c^2 x_H^2 + \left( \frac{x_H^2}{\lambda_{\text{ww}}} T_{196}^d + T_{285}^d \right) \right] c^2 \right\} \frac{1}{c^4} B_0^{\text{fin}} \left( -M_H^2; M_Z, M_Z \right) \\
 & + \frac{1}{128} \left\{ 2 \frac{1}{\lambda_{\text{ww}}} T_{278}^d + \left[ (16c^4 x_H^2 + T_{281}^d) c^2 x_H^2 - \left( -\frac{x_H^2}{\lambda_{\text{ww}}} T_{292}^d + 2T_{284}^d \right) \right] c^2 \right\} \frac{1}{c^6} \\
 & \times C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_Z, M_W, M_Z \right) \\
 & - \frac{1}{64} \left\{ \frac{1}{\lambda_{\text{ww}}} T_{279}^d - \left[ \frac{x_H^2}{\lambda_{\text{ww}}} T_{195}^d c^2 - 2(T_{272}^d x_H^2 - T_{283}^d) \right] c^2 \right\} \frac{1}{c^4} B_0^{\text{fin}} \left( -M_W^2; M_W, M_Z \right) \\
 & - \frac{1}{128} \left\{ \left[ 2c^2 x_H^4 + (T_{195}^d W_{77} x_H^2 + 2T_{289}^d) \right] c^2 - (T_{271}^d x_H^2 + 2T_{274}^d) \frac{1}{\lambda_{\text{ww}}} \right\} \frac{1}{c^2} \\
 & \times B_0^{\text{fin}} \left( -M_H^2; M_W, M_W \right) \\
 & + \frac{1}{128} \left\{ \left[ 16c^6 x_H^4 + (2T_{273}^d - T_{280}^d x_H^2) \right] c^2 - (T_7^d x_H^2 + 2T_{274}^d) \frac{1}{\lambda_{\text{ww}}} \right\} \frac{1}{c^4} \\
 & \times C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_W, M_Z, M_W \right) \\
 & + \frac{1}{128} \left( \frac{x_H^2}{\lambda_{\text{ww}}} T_{195}^d x_H^4 + 2 \frac{x_H^2}{\lambda_{\text{ww}}} T_{291}^d x_H^2 - 4T_{289}^d + T_{290}^d V_{38} x_H^2 \right) \\
 & \times C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_W, M_H, M_W \right) \\
 & - \frac{3}{128} \left( \frac{1}{\lambda_{\text{ww}}} T_{195}^d V_0 - V_{40} c^2 \right) x_H^2 B_0^{\text{fin}} \left( -M_H^2; M_H, M_H \right) \\
 & - \frac{1}{128} (T_{153}^d c^2 x_H^2 + T_{276}^d) \frac{1}{c^2}
 \end{aligned}$$

$$\mathcal{F}_{\text{D;WW}}^{\text{nfc}}(\text{ren}) = \frac{1}{32} X_{30}$$

$$\begin{aligned}
 \mathcal{F}_{\text{P;WW}}^{\text{nfc}}(a_{t\phi}) &= \frac{3}{32} W_{78} x_t^2 x_b^2 a_0^{\text{fin}}(M_b) - \frac{3}{32} W_{78} x_t^4 a_0^{\text{fin}}(M_t) + \frac{3}{32} W_{79} x_t^2 \\
 & - \frac{3}{64} W_{80} x_t^2 B_0^{\text{fin}} \left( -M_W^2; M_t, M_b \right) \\
 & + \frac{3}{64} W_{81} x_t^2 B_0^{\text{fin}} \left( -M_H^2; M_t, M_t \right) \\
 & - \frac{3}{64} W_{82} x_t^2 C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_t, M_b, M_t \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{\text{P;WW}}^{\text{nfc}}(a_{b\phi}) &= \frac{3}{32} W_{78} x_t^2 x_b^2 a_0^{\text{fin}}(M_b) - \frac{3}{32} W_{78} x_t^4 a_0^{\text{fin}}(M_t) + \frac{3}{32} W_{79} x_t^2 \\
 & - \frac{3}{64} W_{80} x_t^2 B_0^{\text{fin}} \left( -M_W^2; M_t, M_b \right) \\
 & + \frac{3}{64} W_{81} x_t^2 B_0^{\text{fin}} \left( -M_H^2; M_t, M_t \right) \\
 & - \frac{3}{64} W_{82} x_t^2 C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_t, M_b, M_t \right)
 \end{aligned}$$

$$\mathcal{F}_{\text{P;WW}}^{\text{nfc}}(a_{t\text{BW}}) = \frac{3}{16} \frac{x_t^2}{\lambda_{\text{ww}}} c x_b^2 B_0^{\text{fin}} \left( -M_H^2; M_b, M_b \right)$$

$$\begin{aligned}
& + \frac{3}{32} \frac{x_t^2}{\lambda_{\text{WW}}} W_{87} c C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_t, M_b, M_t \right) \\
& - \frac{3}{64} W_{83} c x_t^2 \\
& - \frac{3}{64} W_{83} c x_t^2 x_b^2 a_0^{\text{fin}}(M_b) \\
& + \frac{3}{64} W_{83} c x_t^4 a_0^{\text{fin}}(M_t) \\
& + \frac{3}{64} W_{84} c x_t^2 B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_t, M_t \right) \\
& - \frac{3}{32} W_{85} c x_t^2 x_b^2 C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_b, M_t, M_b \right) \\
& - \frac{3}{64} W_{86} c x_t^2 B_0^{\text{fin}} \left( -M_{\text{W}}^2; M_t, M_b \right)
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{\text{P};\text{WW}}^{\text{nfc}}(a_{\text{tWB}}) &= \frac{3}{16} \frac{x_t^2}{\lambda_{\text{WW}}} s x_b^2 B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_b, M_b \right) \\
& + \frac{3}{32} \frac{x_t^2}{\lambda_{\text{WW}}} W_{87} s C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_t, M_b, M_t \right) \\
& - \frac{3}{64} W_{83} s x_t^2 \\
& - \frac{3}{64} W_{83} s x_t^2 x_b^2 a_0^{\text{fin}}(M_b) \\
& + \frac{3}{64} W_{83} s x_t^4 a_0^{\text{fin}}(M_t) \\
& + \frac{3}{64} W_{84} s x_t^2 B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_t, M_t \right) \\
& - \frac{3}{32} W_{85} s x_t^2 x_b^2 C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_b, M_t, M_b \right) \\
& - \frac{3}{64} W_{86} s x_t^2 B_0^{\text{fin}} \left( -M_{\text{W}}^2; M_t, M_b \right)
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{\text{P};\text{WW}}^{\text{nfc}}(a_{\text{bBW}}) &= -\frac{3}{64} W_{88} c x_b^2 a_0^{\text{fin}}(M_b) \\
& + \frac{3}{64} W_{88} c x_t^2 a_0^{\text{fin}}(M_t) \\
& - \frac{3}{64} W_{89} c \\
& - \frac{3}{64} W_{90} c x_t^2 B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_t, M_t \right) \\
& + \frac{3}{64} W_{91} c B_0^{\text{fin}} \left( -M_{\text{W}}^2; M_t, M_b \right) \\
& + \frac{3}{64} W_{92} c x_t^2 C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_t, M_b, M_t \right) \\
& - \frac{3}{64} W_{93} c x_b^2 C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_b, M_t, M_b \right) \\
& - \frac{3}{64} W_{94} c x_b^2 B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_b, M_b \right)
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{\text{P};\text{WW}}^{\text{nfc}}(a_{\text{bWB}}) &= -\frac{3}{64} W_{88} s x_b^2 a_0^{\text{fin}}(M_b) \\
& + \frac{3}{64} W_{88} s x_t^2 a_0^{\text{fin}}(M_t) \\
& - \frac{3}{64} W_{89} s \\
& - \frac{3}{64} W_{90} s x_t^2 B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_t, M_t \right)
\end{aligned}$$



$$\begin{aligned}
 & + \frac{3}{64} W_{91} s B_0^{\text{fin}}(-M_W^2; M_t, M_b) \\
 & + \frac{3}{64} W_{92} s x_t^2 C_0(-M_H^2, -M_W^2, -M_W^2; M_t, M_b, M_t) \\
 & - \frac{3}{64} W_{93} s x_b^2 C_0(-M_H^2, -M_W^2, -M_W^2; M_b, M_t, M_b) \\
 & - \frac{3}{64} W_{94} s x_b^2 B_0^{\text{fin}}(-M_H^2; M_b, M_b)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{P;WW}^{\text{nfc}}(a_{\Phi B}) &= \frac{3}{16} W_{78} a_0^{\text{fin}}(M_W) \\
 & + \frac{3}{16} W_{79} \\
 & + \frac{3}{16} W_{95} a_0^{\text{fin}}(M_H) \\
 & + \frac{3}{32} W_{96} B_0^{\text{fin}}(-M_H^2; M_H, M_H) \\
 & - \frac{3}{32} W_{97} B_0^{\text{fin}}(-M_W^2; M_W, M_H) \\
 & - \frac{3}{32} W_{98} C_0(-M_H^2, -M_W^2, -M_W^2; M_H, M_W, M_H)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{P;WW}^{\text{nfc}}(a_{\Phi \square}) &= -\frac{5}{64} \frac{s^2}{c^4} T_{309}^d W_{78} a_0^{\text{fin}}(M_Z) \\
 & + \frac{1}{64} \frac{1}{c^2} T_{297}^d W_{78} a_0^{\text{fin}}(M_W) \\
 & - \frac{5}{2} \frac{1}{\lambda_{ww}} s^2 B_0^{\text{fin}}(-M_W^2; 0, M_W) \\
 & - \frac{3}{64} W_{95} a_0^{\text{fin}}(M_H) \\
 & - \frac{1}{16} W_{102} B_0^{\text{fin}}(-M_W^2; M_W, M_H) \\
 & + \frac{1}{256} W_{103} B_0^{\text{fin}}(-M_H^2; M_H, M_H) \\
 & + \frac{1}{256} W_{104} C_0(-M_H^2, -M_W^2, -M_W^2; M_W, M_H, M_W) \\
 & + \frac{1}{256} W_{105} C_0(-M_H^2, -M_W^2, -M_W^2; M_H, M_W, M_H) \\
 & + \frac{1}{64} \left[ 4c^2 s W_{101} + \left( \frac{1}{\lambda_{ww}} T_{298}^d + T_{299}^d V_9 \right) \right] \frac{1}{c^2} \\
 & - \frac{1}{256} \left\{ 5c^2 s W_{99} x_H^2 + \left[ \left( -12 \frac{x_H^2}{\lambda_{ww}^2} T_{308}^d - \frac{x_H^2}{\lambda_{ww}} T_{306}^d - 24 \frac{1}{\lambda_{ww}^2} T_{293}^d \right. \right. \right. \\
 & \left. \left. \left. + 22 \frac{1}{\lambda_{ww}} T_{55}^d + 2T_{296}^d V_9 + T_{307}^d \right) \right] \right\} \frac{1}{c^2} B_0^{\text{fin}}(-M_H^2; M_W, M_W) \\
 & - \frac{1}{128} \left\{ 7c^4 s W_{100} - \left[ \left( 12 \frac{1}{\lambda_{ww}^2} T_{304}^d - \frac{1}{\lambda_{ww}} T_{305}^d + 5T_{295}^d V_9 \right) \right] \right\} \frac{1}{c^4} B_0^{\text{fin}}(-M_W^2; M_W, M_Z) \\
 & + \frac{1}{256} \left\{ 7 \left[ \left( 12 \frac{x_H^2}{\lambda_{ww}^2} T_{195}^d + W_{95} \right) \right] c^2 + 2 \left[ \left( -12 \frac{1}{\lambda_{ww}^2} T_{302}^d - \frac{1}{\lambda_{ww}} T_{301}^d + 5T_{58}^d V_9 \right) \right] \right\} \frac{1}{c^4} \\
 & \times B_0^{\text{fin}}(-M_H^2; M_Z, M_Z) \\
 & - \frac{1}{256} \left\{ 7 \left[ \left( 12 \frac{x_H^2}{\lambda_{ww}^2} T_{196}^d + \frac{x_H^2}{\lambda_{ww}} T_{312}^d - T_{311}^d \right) \right] c^2 \right. \\
 & \left. - 2 \left[ \left( -12 \frac{1}{\lambda_{ww}^2} T_{300}^d - \frac{1}{\lambda_{ww}} T_{303}^d + 5T_{294}^d V_9 \right) \right] \right\} \frac{1}{c^6} C_0(-M_H^2, -M_W^2, -M_W^2; M_Z, M_W, M_Z)
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{1}{256} \left( 84 \frac{x_H^2}{\lambda_{\text{ww}}^2} + 7 \frac{x_H^2}{\lambda_{\text{ww}}} T_8^d + 24 \frac{1}{\lambda_{\text{ww}}^2} T_{293}^d - 2 \frac{1}{\lambda_{\text{ww}}} T_{310}^d - 10 T_{294}^d V_9 - T_{313}^d \right) \frac{1}{c^4} \\
 & \times C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_W, M_Z, M_W \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{\text{P;WW}}^{\text{nfc}}(a_{\Phi\text{D}}) = & -\frac{1}{4} \frac{1}{\lambda_{\text{ww}}} T_{12}^d B_0^{\text{fin}} \left( -M_W^2; 0, M_W \right) \\
 & + \frac{1}{512} W_{107} C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_W, M_H, M_W \right) \\
 & - \frac{1}{128} W_{108} B_0^{\text{fin}} \left( -M_H^2; M_H, M_H \right) \\
 & + \frac{1}{256} W_{109} a_0^{\text{fin}} \left( M_H \right) \\
 & - \frac{1}{128} W_{110} C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_H, M_W, M_H \right) \\
 & - \frac{1}{512} W_{111} B_0^{\text{fin}} \left( -M_W^2; M_W, M_H \right) \\
 & + \frac{1}{256} \left[ c^2 s W_{95} + 2 \left( \frac{1}{\lambda_{\text{ww}}} T_{323}^d - T_{325}^d V_9 \right) \right] \frac{1}{c^4} a_0^{\text{fin}} \left( M_Z \right) \\
 & + \frac{1}{512} \left[ T_{331}^d c^2 s W_{100} - 2 \left( 12 \frac{1}{\lambda_{\text{ww}}^2} T_{329}^d - \frac{1}{\lambda_{\text{ww}}} T_{330}^d + T_{327}^d V_9 \right) \right] \frac{1}{c^4} B_0^{\text{fin}} \left( -M_W^2; M_W, M_Z \right) \\
 & - \frac{1}{128} \left\{ 2 c^2 s W_{106} + \left[ \left( \frac{1}{\lambda_{\text{ww}}} T_{321}^d + 5 T_{315}^d V_9 \right) \right] \right\} \frac{1}{c^2} \\
 & + \frac{1}{512} \left\{ 3 c^2 s W_{99} x_H^2 - \left[ \left( 60 \frac{x_H^2}{\lambda_{\text{ww}}^2} + \frac{x_H^2}{\lambda_{\text{ww}}} T_{318}^d + 48 \frac{1}{\lambda_{\text{ww}}^2} T_{316}^d - 8 \frac{1}{\lambda_{\text{ww}}} T_{229}^d \right. \right. \right. \\
 & \left. \left. \left. - 8 T_{314}^d V_9 - T_{319}^d \right) \right] \right\} \frac{1}{c^2} B_0^{\text{fin}} \left( -M_H^2; M_W, M_W \right) \\
 & - \frac{1}{256} \left\{ 2 \left[ \left( 12 \frac{x_H^2}{\lambda_{\text{ww}}^2} T_{195}^d + W_{95} \right) \right] c^2 + \left[ \left( -12 \frac{1}{\lambda_{\text{ww}}^2} T_{324}^d - \frac{1}{\lambda_{\text{ww}}} T_{322}^d + 5 T_{58}^d V_9 \right) \right] \right\} \frac{1}{c^4} \\
 & \times B_0^{\text{fin}} \left( -M_H^2; M_Z, M_Z \right) \\
 & + \frac{1}{256} \left\{ 2 \left[ \left( 12 \frac{x_H^2}{\lambda_{\text{ww}}^2} T_{196}^d + \frac{x_H^2}{\lambda_{\text{ww}}} T_{312}^d - T_{332}^d \right) \right] c^2 - \left[ \left( -12 \frac{1}{\lambda_{\text{ww}}^2} T_{326}^d - \frac{1}{\lambda_{\text{ww}}} T_{328}^d \right. \right. \right. \\
 & \left. \left. \left. + 5 T_{294}^d V_9 \right) \right] \right\} \frac{1}{c^6} C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_Z, M_W, M_Z \right) \\
 & + \frac{1}{512} \left( 60 \frac{x_H^2}{\lambda_{\text{ww}}^2} + 5 \frac{x_H^2}{\lambda_{\text{ww}}} T_8^d + 48 \frac{1}{\lambda_{\text{ww}}^2} T_{316}^d - 8 \frac{1}{\lambda_{\text{ww}}} T_{317}^d - 8 T_{294}^d V_9 - T_{333}^d \right) \frac{1}{c^4} \\
 & \times C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_W, M_Z, M_W \right) \\
 & + \frac{1}{128} \left( -\frac{1}{\lambda_{\text{ww}}} T_{320}^d + 5 T_{144}^d V_9 \right) \frac{1}{c^2} a_0^{\text{fin}} \left( M_W \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{\text{P;WW}}^{\text{nfc}}(a_{\text{AZ}}) = & -\frac{1}{16} s c X_{31} - \frac{1}{8} \frac{c^3}{s} X_{25} \\
 & - \frac{3}{8} \frac{x_H^2}{\lambda_{\text{ww}}} V_0 s c C_0 \left( -M_H^2, -M_W^2, -M_W^2; M_H, M_W, M_H \right) \\
 & - \frac{1}{8} \frac{1}{s} c^3 d \mathcal{Z}_c^{(4)} \\
 & - \frac{1}{4} V_0 s^3 c C_0^{\text{fin}} \left( -M_H^2, -M_W^2, -M_W^2; M_W, 0, M_W \right) \\
 & - \frac{3}{64} W_{112} s c x_H^2 B_0^{\text{fin}} \left( -M_H^2; M_H, M_H \right) \\
 & + \frac{1}{64} W_{113} s c B_0^{\text{fin}} \left( -M_W^2; M_W, M_H \right)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{64} W_{114} s c C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{W}}, M_{\text{H}}, M_{\text{W}} \right) \\
 & - \frac{1}{32} W_{115} s c a_0^{\text{fin}} \left( M_{\text{H}} \right) \\
 & - \frac{1}{128} \left[ 12 \frac{x_{\text{H}}^2}{\lambda_{\text{WW}}^2} s^2 c^2 + \frac{x_{\text{H}}^2}{l_{\text{WW}}} T_{351}^d c^2 + \left( 24 \frac{1}{\lambda_{\text{WW}}^2} T_{352}^d - 2 \frac{1}{\lambda_{\text{WW}}} T_{356}^d + T_{346}^d \right. \right. \\
 & \left. \left. + 2 T_{350}^d V_9 + 24 V_{92} \right) \right] \frac{s}{c^3} B_0^{\text{fin}} \left( -M_{\text{W}}^2; M_{\text{W}}, M_{\text{Z}} \right) \\
 & + \frac{1}{64} \left[ \frac{x_{\text{H}}^2}{\lambda_{\text{WW}}} s^2 c^2 - \left( -2 \frac{1}{\lambda_{\text{WW}}} T_{353}^d + T_{341}^d + 2 T_{349}^d V_9 \right) \right] \frac{s}{c^3} a_0^{\text{fin}} \left( M_{\text{Z}} \right) \\
 & - \frac{1}{16} \left[ \left( -18 \frac{1}{\lambda_{\text{WW}}^2} T_{339}^d + T_{98}^d V_9 \right) + \left( 6 \frac{1}{\lambda_{\text{WW}}} T_{336}^d + 5 T_7^d c^2 \right) \right] \frac{s}{c^5} \\
 & \times C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{Z}}, M_{\text{W}}, M_{\text{Z}} \right) \\
 & - \frac{1}{128} \left\{ 4 c^2 x_{\text{H}}^2 - \left[ \left( 12 \frac{x_{\text{H}}^2}{\lambda_{\text{WW}}^2} T_{54}^d + \frac{x_{\text{H}}^2}{\lambda_{\text{WW}}} T_{354}^d - 24 \frac{1}{\lambda_{\text{WW}}^2} T_{355}^d + 6 \frac{1}{\lambda_{\text{WW}}} T_{334}^d \right. \right. \right. \\
 & \left. \left. + 10 T_{88}^d V_9 - T_{335}^d \right) \right] \right\} \frac{s}{c} B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{W}}, M_{\text{W}} \right) \\
 & + \frac{1}{128} \left\{ 32 c^6 x_{\text{H}}^2 - 12 \frac{x_{\text{H}}^2}{\lambda_{\text{WW}}^2} s^2 - \left[ \left( 3 \frac{x_{\text{H}}^2}{\lambda_{\text{WW}}} T_{343}^d - 24 \frac{1}{\lambda_{\text{WW}}^2} T_{355}^d + 2 \frac{1}{\lambda_{\text{WW}}} T_{344}^d \right. \right. \right. \\
 & \left. \left. + 10 T_{88}^d V_9 - T_{340}^d \right) \right] \right\} \frac{s}{c^3} C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{W}}, M_{\text{Z}}, M_{\text{W}} \right) \\
 & - \frac{1}{64} \left\{ \left[ c^2 x_{\text{H}}^2 - 2 \left( -6 \frac{1}{\lambda_{\text{WW}}} T_7^d + T_2^d \right) \right] c^2 + 4 \left( -18 \frac{1}{\lambda_{\text{WW}}^2} T_{100}^d + T_7^d V_9 \right) \right\} \frac{s}{c^3} \\
 & \times B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{Z}}, M_{\text{Z}} \right) \\
 & + \frac{1}{32} \left\{ -2 \left[ 2 \left( 2 - c^2 x_{\text{H}}^2 \right) c^2 + \left( \frac{1}{\lambda_{\text{WW}}} T_{88}^d - 3 V_9 \right) \right] c^2 + 3 \left( \frac{1}{\lambda_{\text{WW}}^2} T_{231}^d - V_{92} \right) \right\} \frac{s}{c^5} \\
 & \times C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{Z}}, M_{\text{W}}, 0 \right) \\
 & + \frac{1}{32} \left\{ -2 \left[ 2 \left( 2 - c^2 x_{\text{H}}^2 \right) c^2 + \left( \frac{1}{\lambda_{\text{WW}}} T_{88}^d - 3 V_9 \right) \right] c^2 + 3 \left( \frac{1}{\lambda_{\text{WW}}^2} T_{231}^d - V_{92} \right) \right\} \frac{s}{c^5} \\
 & \times C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; 0, M_{\text{W}}, M_{\text{Z}} \right) \\
 & - \frac{1}{16} \left\{ - \left[ \left( \frac{x_{\text{H}}^2}{\lambda_{\text{WW}}} T_2^d + T_{342}^d \right) c^2 - \left( \frac{1}{\lambda_{\text{WW}}} T_{338}^d + 3 V_9 \right) \right] c^2 + 3 \left( \frac{1}{\lambda_{\text{WW}}^2} T_{88}^d - V_{92} \right) \right\} \frac{s}{c^3} \\
 & \times B_0^{\text{fin}} \left( -M_{\text{W}}^2; 0, M_{\text{W}} \right) \\
 & - \frac{1}{32} \left( 1 + 2 \frac{1}{\lambda_{\text{WW}}} T_{348}^d + 2 T_{110}^d V_9 \right) \frac{s}{c} \\
 & + \frac{1}{32} \left( s W_{78} + 12 \frac{1}{\lambda_{\text{WW}}^2} T_{88}^d \right) \frac{s}{c^3} B_0^{\text{fin}} \left( -M_{\text{H}}^2; 0, M_{\text{Z}} \right) \\
 & - \frac{1}{32} \left( 3 s^2 c^2 x_{\text{H}}^2 + 2 T_{337}^d \right) \frac{1}{s c} L_{\text{R}} \\
 & + \frac{1}{32} \left( -2 \frac{1}{\lambda_{\text{WW}}} T_{347}^d + T_{88}^d + 2 T_{345}^d V_9 \right) \frac{s}{c} a_0^{\text{fin}} \left( M_{\text{W}} \right) \\
 \\
 \mathcal{P}_{\text{P}; \text{WW}}^{\text{nc}}(a_{\text{AA}}) & = -\frac{1}{4} c^2 X_{25} - \frac{1}{16} s^2 X_{32} \\
 & + \frac{1}{4} s^2 x_{\text{H}}^2 C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; 0, M_{\text{W}}, 0 \right) \\
 & - \frac{3}{8} \frac{x_{\text{H}}^2}{\lambda_{\text{WW}}} V_0 s^2 C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{H}}, M_{\text{W}}, M_{\text{H}} \right)
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{1}{4} V_0 s^4 C_0^{\text{fin}} \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{W}}, 0, M_{\text{W}} \right) \\
 & -\frac{1}{16} W_{79} s^2 \\
 & -\frac{3}{64} W_{112} s^2 x_{\text{H}}^2 B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{H}}, M_{\text{H}} \right) \\
 & +\frac{1}{64} W_{113} s^2 B_0^{\text{fin}} \left( -M_{\text{W}}^2; M_{\text{W}}, M_{\text{H}} \right) \\
 & +\frac{1}{64} W_{114} s^2 C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{W}}, M_{\text{H}}, M_{\text{W}} \right) \\
 & -\frac{1}{32} W_{115} s^2 a_0^{\text{fin}} \left( M_{\text{H}} \right) \\
 & +\frac{1}{16} W_{117} s^2 a_0^{\text{fin}} \left( M_{\text{W}} \right) \\
 & +\frac{1}{8} \left[ 5s^2 c^4 + 6 \frac{1}{\lambda_{\text{ww}}} T_{358}^d c^2 + \left( -18 \frac{1}{\lambda_{\text{ww}}^2} T_{361}^d + T_{359}^d V_9 \right) \right] \frac{1}{c^4} \\
 & \times C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{Z}}, M_{\text{W}}, M_{\text{Z}} \right) \\
 & +\frac{1}{64} \left[ 8 \frac{x_{\text{H}}^2}{\lambda_{\text{ww}}} s^2 c^2 + \left( 12 \frac{1}{\lambda_{\text{ww}}^2} T_{364}^d - \frac{1}{\lambda_{\text{ww}}} T_{366}^d + T_{363}^d V_9 + 4 T_{367}^d \right) \right] \frac{s^2}{c^2} \\
 & \times B_0^{\text{fin}} \left( -M_{\text{W}}^2; M_{\text{W}}, M_{\text{Z}} \right) \\
 & +\frac{1}{64} \left\{ c^2 s W_{118} - 8 \left[ \left( 18 \frac{1}{\lambda_{\text{ww}}^2} T_7^d - V_9 \right) \right] \right\} \frac{s^2}{c^2} B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{Z}}, M_{\text{Z}} \right) \\
 & -\frac{1}{64} \left\{ c^2 s W_{116} + \left[ \left( 12 \frac{1}{\lambda_{\text{ww}}^2} T_{55}^d + \frac{1}{\lambda_{\text{ww}}} T_{357}^d - T_{334}^d V_9 \right) \right] \right\} \frac{s^2}{c^2} B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{W}}, M_{\text{W}} \right) \\
 & +\frac{1}{8} \left\{ \frac{x_{\text{H}}^2}{\lambda_{\text{ww}}} c^2 + \left[ \left( 4 \frac{1}{\lambda_{\text{ww}}} T_{106}^d + T_{362}^d \right) \right] \right\} s^2 B_0^{\text{fin}} \left( -M_{\text{W}}^2; 0, M_{\text{W}} \right) \\
 & +\frac{1}{64} \left\{ 8 \frac{x_{\text{H}}^2}{\lambda_{\text{ww}}} s^2 c^2 - 16 V_0 c^6 + \left[ \left( 12 \frac{1}{\lambda_{\text{ww}}^2} T_{55}^d + \frac{1}{\lambda_{\text{ww}}} T_{360}^d - T_{88}^d V_9 \right) \right] \right\} \frac{s^2}{c^4} \\
 & \times C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{W}}, M_{\text{Z}}, M_{\text{W}} \right) \\
 & -\frac{1}{32} \left( 3s^2 x_{\text{H}}^2 - 16 T_{29}^d \right) L_{\text{R}} \\
 & +\frac{1}{32} \left( -\frac{1}{\lambda_{\text{ww}}} T_{365}^d + 2 T_7^d + 9 V_9 \right) \frac{s^2}{c^2} a_0^{\text{fin}} \left( M_{\text{Z}} \right)
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{P}_{\text{P;WW}}^{\text{nfc}}(a_{\text{ZZ}}) &= -\frac{1}{16} c^2 X_{33} \\
 & -\frac{3}{8} \frac{x_{\text{H}}^2}{\lambda_{\text{ww}}} V_0 c^2 C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{H}}, M_{\text{W}}, M_{\text{H}} \right) \\
 & -\frac{1}{4} V_0 s^2 c^2 C_0^{\text{fin}} \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{W}}, 0, M_{\text{W}} \right) \\
 & -\frac{3}{64} W_{112} c^2 x_{\text{H}}^2 B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{H}}, M_{\text{H}} \right) \\
 & +\frac{1}{64} W_{113} c^2 B_0^{\text{fin}} \left( -M_{\text{W}}^2; M_{\text{W}}, M_{\text{H}} \right) \\
 & +\frac{1}{64} W_{114} c^2 C_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{W}}, M_{\text{H}}, M_{\text{W}} \right) \\
 & -\frac{1}{32} W_{115} c^2 a_0^{\text{fin}} \left( M_{\text{H}} \right) \\
 & +\frac{1}{8} W_{119} s^2 c^2 B_0^{\text{fin}} \left( -M_{\text{W}}^2; 0, M_{\text{W}} \right)
 \end{aligned}$$

$$\begin{aligned}
 & -\frac{1}{32} \left[ 2c^2 + \left( \frac{1}{\lambda_{\text{ww}}} \text{T}_{378}^d + \text{T}_{375}^d \text{V}_9 \right) \right] \frac{1}{c^2} \\
 & -\frac{1}{64} \left[ c^2 s \text{W}_{120} - \left( -\frac{x_{\text{H}}^2}{\lambda_{\text{ww}}} \text{T}_{389}^d - 12 \frac{1}{\lambda_{\text{ww}}^2} \text{T}_{386}^d + \frac{1}{\lambda_{\text{ww}}} \text{T}_{387}^d + \text{T}_{385}^d \text{V}_9 \right) \right] \\
 & \times \text{B}_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{W}}, M_{\text{W}} \right) \\
 & -\frac{1}{64} \left[ c^6 x_{\text{H}}^2 + 4s^2 c^4 + \left( -12 \frac{1}{\lambda_{\text{ww}}^2} \text{T}_{376}^d - \frac{1}{\lambda_{\text{ww}}} \text{T}_{370}^d + \text{T}_{369}^d \text{V}_9 \right) \right] \frac{1}{c^4} \text{B}_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{Z}}, M_{\text{Z}} \right) \\
 & -\frac{1}{64} \left[ 8 \frac{x_{\text{H}}^2}{\lambda_{\text{ww}}} s^4 c^4 - 4 \text{T}_{390}^d c^2 + \left( 12 \frac{1}{\lambda_{\text{ww}}^2} \text{T}_{381}^d - \frac{1}{\lambda_{\text{ww}}} \text{T}_{383}^d + \text{T}_{380}^d \text{V}_9 \right) \right] \frac{1}{c^4} \\
 & \times \text{B}_0^{\text{fin}} \left( -M_{\text{W}}^2; M_{\text{W}}, M_{\text{Z}} \right) \\
 & -\frac{1}{32} \left[ 2 \text{T}_7^d c^2 + \left( -\frac{1}{\lambda_{\text{ww}}} \text{T}_{382}^d + \text{T}_{368}^d \text{V}_9 \right) \right] \frac{s^2}{c^4} a_0^{\text{fin}}(M_{\text{Z}}) \\
 & -\frac{1}{32} \left[ 2 \text{T}_{195}^d c^2 - \left( -\frac{1}{\lambda_{\text{ww}}} \text{T}_{377}^d + \text{T}_{374}^d \text{V}_9 \right) \right] \frac{1}{c^2} a_0^{\text{fin}}(M_{\text{W}}) \\
 & -\frac{1}{64} \left\{ 8 \frac{x_{\text{H}}^2}{\lambda_{\text{ww}}} s^4 + \left[ \left( -12 \frac{1}{\lambda_{\text{ww}}^2} \text{T}_{386}^d + \frac{1}{\lambda_{\text{ww}}} \text{T}_{388}^d + 9 \text{T}_{88}^d \text{V}_9 \right) \right] - 8(2c^4 x_{\text{H}}^2 - \text{T}_{384}^d) c^2 \right\} \frac{1}{c^2} \\
 & \times \text{C}_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{W}}, M_{\text{Z}}, M_{\text{W}} \right) \\
 & + \frac{1}{64} \left\{ \left[ \left( 12 \frac{1}{\lambda_{\text{ww}}^2} \text{T}_{371}^d + \frac{1}{\lambda_{\text{ww}}} \text{T}_{372}^d - \text{T}_{373}^d \text{V}_9 \right) \right] + 8(2c^4 x_{\text{H}}^2 + \text{T}_{379}^d) c^4 \right\} \frac{1}{c^6} \\
 & \times \text{C}_0 \left( -M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{Z}}, M_{\text{W}}, M_{\text{Z}} \right) \\
 & -\frac{1}{32} (3c^2 x_{\text{H}}^2 + 4 \text{T}_{88}^d) \text{L}_{\text{R}}
 \end{aligned}$$

## I dim = 4 sub-amplitudes

In this appendix we list the dim = 4 sub-amplitudes for  $\text{H} \rightarrow \text{AZ}, \text{ZZ}$  and  $\text{WW}$ .

### I.1 $\text{H} \rightarrow \text{AZ}$

$$\begin{aligned}
 \mathcal{F}_{\text{HAZ};\text{LO}}^{\text{t}} &= \frac{1}{4} \frac{M_{\text{H}}^2 M_{\text{t}}^2}{M_{\text{W}} (M_{\text{H}}^2 - M_{\text{Z}}^2)} \frac{s}{c} v_{\text{t}} + \frac{1}{8} \left( 1 - 4 \frac{M_{\text{t}}^2}{M_{\text{H}}^2 - M_{\text{Z}}^2} \right) \frac{M_{\text{H}}^2 M_{\text{t}}^2}{M_{\text{W}}} \frac{s}{c} v_{\text{t}} \text{C}_0 \left( -M_{\text{H}}^2, 0, -M_{\text{Z}}^2; M_{\text{t}}, M_{\text{t}}, M_{\text{t}} \right) \\
 & -\frac{1}{4} \frac{M_{\text{H}}^2 M_{\text{W}}}{M_{\text{H}}^2 - M_{\text{Z}}^2} \frac{M_{\text{t}}^2}{M_{\text{H}}^2 - M_{\text{Z}}^2} \frac{s}{c^3} v_{\text{t}} \text{B}_0^{\text{fin}} \left( -M_{\text{Z}}^2; M_{\text{t}}, M_{\text{t}} \right) \\
 & + \frac{1}{4} \frac{M_{\text{W}} M_{\text{H}}^2}{M_{\text{H}}^2 - M_{\text{Z}}^2} \frac{M_{\text{t}}^2}{M_{\text{H}}^2 - M_{\text{Z}}^2} \frac{s}{c^3} v_{\text{t}} \text{B}_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{t}}, M_{\text{t}} \right)
 \end{aligned} \tag{I.1}$$

$$\begin{aligned}
 \mathcal{F}_{\text{HAZ};\text{LO}}^{\text{b}} &= \frac{1}{8} \frac{M_{\text{H}}^2 M_{\text{b}}^2}{M_{\text{W}} (M_{\text{H}}^2 - M_{\text{Z}}^2)} \frac{s}{c} v_{\text{b}} + \frac{1}{16} \left( 1 - 4 \frac{M_{\text{b}}^2}{M_{\text{H}}^2 - M_{\text{Z}}^2} \right) \frac{M_{\text{H}}^2 M_{\text{b}}^2}{M_{\text{W}}} \frac{s}{c} v_{\text{b}} \text{C}_0 \left( -M_{\text{H}}^2, 0, -M_{\text{Z}}^2; M_{\text{b}}, M_{\text{b}}, M_{\text{b}} \right) \\
 & -\frac{1}{8} \frac{M_{\text{H}}^2 M_{\text{W}}}{M_{\text{H}}^2 - M_{\text{Z}}^2} \frac{M_{\text{b}}^2}{M_{\text{H}}^2 - M_{\text{Z}}^2} \frac{s}{c^3} v_{\text{b}} \text{B}_0^{\text{fin}} \left( -M_{\text{Z}}^2; M_{\text{b}}, M_{\text{b}} \right) \\
 & + \frac{1}{8} \frac{M_{\text{W}} M_{\text{H}}^2}{M_{\text{H}}^2 - M_{\text{Z}}^2} \frac{M_{\text{b}}^2}{M_{\text{H}}^2 - M_{\text{Z}}^2} \frac{s}{c^3} v_{\text{b}} \text{B}_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{b}}, M_{\text{b}} \right)
 \end{aligned} \tag{I.2}$$

$$\mathcal{F}_{\text{HAZ};\text{LO}}^{\text{W}} = \frac{1}{16} \left[ 2(1 - 6c^2) M_{\text{W}} + (1 - 2c^2) \frac{M_{\text{H}}^2}{M_{\text{W}}} \right] \frac{M_{\text{H}}^2}{M_{\text{H}}^2 - M_{\text{Z}}^2} \frac{s}{c}$$

$$\begin{aligned}
 & -\frac{1}{8} \left[ 2(1-6c^2) \frac{M_W^3}{M_H^2 - M_Z^2} - 2(1-4c^2) M_W + (1-2c^2) \frac{M_H^2 M_W}{M_H^2 - M_Z^2} \right] M_H^2 \frac{s}{c} \\
 & \times C_0 \left( -M_H^2, 0, -M_Z^2; M_W, M_W, M_W \right) \\
 & -\frac{1}{16} \left[ 2(1-6c^2) \frac{M_W^3}{M_H^2 - M_Z^2} + (1-2c^2) \frac{M_H^2 M_W}{M_H^2 - M_Z^2} \right] \frac{M_H^2}{M_H^2 - M_Z^2} \frac{s}{c^3} B_0^{\text{fin}} \left( -M_Z^2; M_W, M_W \right) \\
 & +\frac{1}{16} \left[ 2(1-6c^2) \frac{M_W^3}{M_H^2 - M_Z^2} + (1-2c^2) \frac{M_H^2 M_W}{M_H^2 - M_Z^2} \right] \frac{M_H^2}{M_H^2 - M_Z^2} \frac{s}{c^3} B_0^{\text{fin}} \left( -M_H^2; M_W, M_W \right) \quad (\text{I.3})
 \end{aligned}$$

## I.2 H → ZZ

$$\begin{aligned}
 \mathcal{D}_{\text{HZZ};\text{NLO}}^{\text{t}} &= -\frac{3}{64} \left[ 2L_{\text{R}} - (1+v_{\text{t}}^2) \right] \frac{1}{c^2} \frac{M_{\text{t}}^2}{M_{\text{W}}} \\
 &+ \frac{3}{32} \left[ c^2 + (1+v_{\text{t}}^2) \frac{M_{\text{W}}^2}{\lambda_z} \right] \frac{1}{c^4} \frac{M_{\text{t}}^2}{M_{\text{W}}} B_0^{\text{fin}} \left( -M_{\text{Z}}^2; M_{\text{t}}, M_{\text{t}} \right) \\
 &+ \frac{3}{32} \left[ (1+v_{\text{t}}^2) \right] \frac{1}{c^4} \frac{M_{\text{W}} M_{\text{t}}^2}{\lambda_z} B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{t}}, M_{\text{t}} \right) \\
 &+ \frac{3}{128} \left[ -4(1-v_{\text{t}}^2) c^4 M_{\text{t}}^2 + (1-v_{\text{t}}^2) c^4 M_{\text{H}}^2 + 2(1+v_{\text{t}}^2) c^2 M_{\text{W}}^2 + 4(1+v_{\text{t}}^2) \frac{M_{\text{W}}^4}{\lambda_z} \right] \frac{1}{c^6} \frac{M_{\text{t}}^2}{M_{\text{W}}} \\
 &\times C_0 \left( -M_{\text{H}}^2, -M_{\text{Z}}^2, -M_{\text{Z}}^2; M_{\text{t}}, M_{\text{t}}, M_{\text{t}} \right) \\
 &+ \Delta \mathcal{D}_{\text{HZZ};\text{NLO}}^{\text{t}} \quad (\text{I.4})
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{D}_{\text{HZZ};\text{NLO}}^{\text{b}} &= -\frac{3}{64} \left[ 2L_{\text{R}} - (1+v_{\text{b}}^2) \right] \frac{1}{c^2} \frac{M_{\text{b}}^2}{M_{\text{W}}} \\
 &+ \frac{3}{32} \left[ c^2 + (1+v_{\text{b}}^2) \frac{M_{\text{W}}^2}{\lambda_z} \right] \frac{1}{c^4} \frac{M_{\text{b}}^2}{M_{\text{W}}} B_0^{\text{fin}} \left( -M_{\text{Z}}^2; M_{\text{b}}, M_{\text{b}} \right) \\
 &+ \frac{3}{32} \left[ (1+v_{\text{b}}^2) \right] \frac{1}{c^4} \frac{M_{\text{b}}^2 M_{\text{W}}}{\lambda_z} B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{b}}, M_{\text{b}} \right) \\
 &+ \frac{3}{128} \left[ (1-v_{\text{b}}^2) c^4 M_{\text{H}}^2 - 4(1-v_{\text{b}}^2) M_{\text{b}}^2 c^4 + 2(1+v_{\text{b}}^2) c^2 M_{\text{W}}^2 + 4(1+v_{\text{b}}^2) \frac{M_{\text{W}}^4}{\lambda_z} \right] \frac{1}{c^6} \frac{M_{\text{b}}^2}{M_{\text{W}}} \\
 &\times C_0 \left( -M_{\text{H}}^2, -M_{\text{Z}}^2, -M_{\text{Z}}^2; M_{\text{b}}, M_{\text{b}}, M_{\text{b}} \right) \\
 &+ \Delta \mathcal{D}_{\text{HZZ};\text{NLO}}^{\text{b}} \quad (\text{I.5})
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{D}_{\text{HZZ};\text{NLO}}^{\text{W}} &= -\frac{1}{4} a_0^{\text{fin}} (M_{\text{W}}) M_{\text{W}} \\
 &+ \frac{1}{64} \left[ 3(1+2c^2) L_{\text{R}} M_{\text{W}}^2 - (1+2c^2 + 32c^4 - 24s^2 c^4) M_{\text{W}}^2 - (3-4s^2 c^2) c^2 M_{\text{H}}^2 \right] \frac{1}{M_{\text{W}}} \frac{1}{c^4} \\
 &+ \frac{1}{32} \left[ c^2 + 2(1+8c^2 - 12s^2 c^2) \frac{M_{\text{W}}^2}{\lambda_z} + (1-4s^2 c^2) \frac{M_{\text{H}}^2}{\lambda_z} \right] \frac{1}{c^4} M_{\text{W}} B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{W}}, M_{\text{W}} \right) \\
 &+ \frac{1}{32} \left[ 4(1-2c^2) c^2 + 2(1+8c^2 - 12s^2 c^2) \frac{M_{\text{W}}^2}{\lambda_z} + (1-4s^2 c^2) \frac{M_{\text{H}}^2}{\lambda_z} \right] \frac{1}{c^4} M_{\text{W}} B_0^{\text{fin}} \left( -M_{\text{Z}}^2; M_{\text{W}}, M_{\text{W}} \right) \\
 &+ \frac{1}{64} \left( M_{\text{W}}^2 + \frac{M_{\text{H}}^4}{\lambda_z} c^2 + 2 \frac{M_{\text{W}}^2 M_{\text{H}}^2}{\lambda_z} \right) \frac{1}{M_{\text{W}}} \frac{1}{c^4} B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{Z}}, M_{\text{Z}} \right) \\
 &+ \frac{3}{64} (2M_{\text{W}}^2 - c^2 M_{\text{H}}^2) \frac{1}{M_{\text{W}}} \frac{1}{c^4} \frac{M_{\text{H}}^2}{\lambda_z} B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{H}}, M_{\text{H}} \right)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{32} \left( 2M_W^2 - \frac{M_H^4}{\lambda_z} c^2 + 4 \frac{M_W^2 M_H^2}{\lambda_z} \right) \frac{1}{M_W} \frac{1}{c^4} B_0^{\text{fin}} \left( -M_Z^2; M_H, M_Z \right) \\
 & + \frac{1}{32} \left[ 6(1-4c^2-4c^4)c^4 M_W^2 + (1-4c^2+12c^4)c^4 M_H^2 - 2(1+8c^2-12s^2c^2) \frac{M_W^4}{\lambda_z} \right. \\
 & \left. - (1-4s^2c^2) \frac{M_W^2 M_H^2}{\lambda_z} \right] \frac{1}{c^6} M_W C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_W, M_W, M_W \right) \\
 & + \frac{1}{64} \left( 4M_W^4 - 2c^2 M_W^2 M_H^2 - c^4 M_H^4 - \frac{M_H^6}{\lambda_z} c^4 - 2 \frac{M_W^2 M_H^4}{\lambda_z} c^2 \right) \frac{1}{M_W} \frac{1}{c^6} \\
 & \times C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_Z, M_H, M_Z \right) \\
 & + \frac{3}{64} \left( -c^2 M_W^2 + \frac{M_H^4}{\lambda_z} c^4 - 4 \frac{M_W^2 M_H^2}{\lambda_z} c^2 + 4 \frac{M_W^4}{\lambda_z} \right) \frac{1}{c^6} \frac{M_H^2}{M_W} \\
 & \times C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_H, M_Z, M_H \right) \\
 & + \Delta \mathcal{D}_{\text{HZZ};\text{NLO}}^{\text{W}}
 \end{aligned} \tag{I.6}$$

$$\begin{aligned}
 \Delta \mathcal{D}_{\text{HZZ};\text{NLO}}^{\text{t}} &= \frac{1}{32} \frac{M_W}{c^2} \left( W_{\text{H};\text{t}}^{(4)} + 2W_{\text{Z};\text{t}}^{(4)} + 4d\mathcal{Z}_{\text{c};\text{t}}^{(4)} \right) \\
 \Delta \mathcal{D}_{\text{HZZ};\text{NLO}}^{\text{b}} &= \frac{1}{32} \frac{M_W}{c^2} \left( W_{\text{H};\text{b}}^{(4)} + 2W_{\text{Z};\text{b}}^{(4)} + 4d\mathcal{Z}_{\text{c};\text{b}}^{(4)} \right) \\
 \Delta \mathcal{D}_{\text{HZZ};\text{NLO}}^{\text{W}} &= \frac{1}{32} \frac{M_W}{c^2} \left( W_{\text{H};\text{W}}^{(4)} + 2W_{\text{Z};\text{W}}^{(4)} - d\mathcal{Z}_{M_W;\text{W}}^{(4)} - 2d\mathcal{Z}_{\text{g};\text{W}}^{(4)} + 4d\mathcal{Z}_{\text{c};\text{W}}^{(4)} \right)
 \end{aligned} \tag{I.7}$$

$$\begin{aligned}
 \mathcal{P}_{\text{HZZ};\text{NLO}}^{\text{t}} &= \frac{3}{64} \left[ (1+v_t^2) + (1+v_t^2) \frac{M_H^2}{\lambda_z} \right] \frac{1}{c^2} \frac{M_t^2}{M_W M_H^2} \\
 & + \frac{3}{128} \left[ (1+v_t^2)c^2 + 12(1+v_t^2) \frac{M_W^2 M_H^2}{\lambda_z^2} + (7-v_t^2) \frac{M_H^2}{\lambda_z} c^2 \right] \frac{1}{c^4} \frac{M_t^2}{M_W M_H^2} B_0^{\text{fin}} \left( -M_H^2; M_t, M_t \right) \\
 & + \frac{3}{128} \left[ (1+v_t^2)c^2 + 12(1+v_t^2) \frac{M_W^2 M_H^2}{\lambda_z^2} + (7-v_t^2) \frac{M_H^2}{\lambda_z} c^2 \right] \frac{1}{c^4} \frac{M_t^2}{M_W M_H^2} B_0^{\text{fin}} \left( -M_Z^2; M_t, M_t \right) \\
 & + \frac{3}{128} \left[ 2(1-v_t^2)c^4 M_H^2 - (1+v_t^2)c^2 M_W^2 + 4(1+v_t^2)c^4 M_t^2 + 4(1+v_t^2) \frac{M_H^2 M_t^2}{\lambda_z} c^4 \right. \\
 & \left. + 12(1+v_t^2) \frac{M_W^4 M_H^2}{\lambda_z^2} + (9+v_t^2) \frac{M_W^2 M_H^2}{\lambda_z} c^2 \right] \frac{1}{c^6} \frac{M_t^2}{M_W M_H^2} \\
 & \times C_0 \left( -M_H^2, -M_Z^2, -M_Z^2; M_t, M_t, M_t \right)
 \end{aligned} \tag{I.8}$$

$$\begin{aligned}
 \mathcal{P}_{\text{HZZ};\text{NLO}}^{\text{b}} &= \frac{3}{64} \left[ (1+v_b^2) + (1+v_b^2) \frac{M_H^2}{\lambda_z} \right] \frac{1}{c^2} \frac{M_b^2}{M_W M_H^2} \\
 & + \frac{3}{128} \left[ (1+v_b^2)c^2 + 12(1+v_b^2) \frac{M_W^2 M_H^2}{\lambda_z^2} + (7-v_b^2) \frac{M_H^2}{\lambda_z} c^2 \right] \frac{1}{c^4} \frac{M_b^2}{M_W M_H^2} B_0^{\text{fin}} \left( -M_H^2; M_b, M_b \right) \\
 & + \frac{3}{128} \left[ (1+v_b^2)c^2 + 12(1+v_b^2) \frac{M_W^2 M_H^2}{\lambda_z^2} + (7-v_b^2) \frac{M_H^2}{\lambda_z} c^2 \right] \frac{1}{c^4} \frac{M_b^2}{M_W M_H^2} B_0^{\text{fin}} \left( -M_Z^2; M_b, M_b \right) \\
 & + \frac{3}{128} \left[ 2(1-v_b^2)c^4 M_H^2 - (1+v_b^2)c^2 M_W^2 + 4(1+v_b^2)M_b^2 c^4 + 12(1+v_b^2) \frac{M_W^4 M_H^2}{\lambda_z^2} \right. \\
 & \left. + 4(1+v_b^2) \frac{M_b^2 M_H^2}{\lambda_z} c^4 + (9+v_b^2) \frac{M_W^2 M_H^2}{\lambda_z} c^2 \right] \frac{1}{c^6} \frac{M_b^2}{M_W M_H^2}
 \end{aligned}$$

$$\times C_0 \left( -M_{\text{H}}^2, -M_{\text{Z}}^2, -M_{\text{Z}}^2; M_{\text{b}}, M_{\text{b}}, M_{\text{b}} \right) \quad (\text{I.9})$$

$$\begin{aligned}
 \mathcal{P}_{\text{HZZ};\text{NLO}}^{\text{W}} = & -\frac{1}{64} \left[ (1+2c^2+16c^4-24s^2c^4)M_{\text{W}}^2 + (1+2c^2+16c^4-24s^2c^4) \frac{M_{\text{W}}^2 M_{\text{H}}^2}{\lambda_{\text{z}}} \right. \\
 & + (3-4s^2c^2)c^2 M_{\text{H}}^2 + (3-4s^2c^2) \frac{M_{\text{H}}^4}{\lambda_{\text{z}}} c^2 \left. \right] \frac{1}{c^4} \frac{1}{M_{\text{W}} M_{\text{H}}^2} \\
 & + \frac{1}{64} \left( M_{\text{W}}^2 - c^2 M_{\text{H}}^2 + \frac{M_{\text{H}}^4}{\lambda_{\text{z}}} c^2 - \frac{M_{\text{W}}^2 M_{\text{H}}^2}{\lambda_{\text{z}}} \right) \frac{1}{c^2} \frac{1}{M_{\text{W}}^3} a_0^{\text{fin}}(M_{\text{H}}) \\
 & + \frac{1}{64} \left( M_{\text{W}}^2 - c^2 M_{\text{H}}^2 + \frac{M_{\text{H}}^4}{\lambda_{\text{z}}} c^2 - \frac{M_{\text{W}}^2 M_{\text{H}}^2}{\lambda_{\text{z}}} \right) \frac{1}{c^4} \frac{1}{M_{\text{W}} M_{\text{H}}^2} a_0^{\text{fin}}(M_{\text{Z}}) \\
 & + \frac{1}{128} \left[ 2(1+8c^2-12s^2c^2)c^2 M_{\text{W}}^2 + 24(1+8c^2-12s^2c^2) \frac{M_{\text{W}}^4 M_{\text{H}}^2}{\lambda_{\text{z}}^2} \right. \\
 & + (1-4s^2c^2)c^2 M_{\text{H}}^2 - (1-4s^2c^2) \frac{M_{\text{H}}^4}{\lambda_{\text{z}}} c^2 + 12(1-4s^2c^2) \frac{M_{\text{W}}^2 M_{\text{H}}^4}{\lambda_{\text{z}}^2} \\
 & \left. + 2(15-8c^2+12s^2c^2) \frac{M_{\text{W}}^2 M_{\text{H}}^2}{\lambda_{\text{z}}} c^2 \right] \frac{1}{c^4} \frac{1}{M_{\text{W}} M_{\text{H}}^2} B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{W}}, M_{\text{W}} \right) \\
 & + \frac{1}{128} \left[ 2(1+8c^2-12s^2c^2)c^2 M_{\text{W}}^2 + 24(1+8c^2-12s^2c^2) \frac{M_{\text{W}}^4 M_{\text{H}}^2}{\lambda_{\text{z}}^2} \right. \\
 & + (1-4s^2c^2)c^2 M_{\text{H}}^2 - (1-4s^2c^2) \frac{M_{\text{H}}^4}{\lambda_{\text{z}}} c^2 + 12(1-4s^2c^2) \frac{M_{\text{W}}^2 M_{\text{H}}^4}{\lambda_{\text{z}}^2} \\
 & \left. + 2(15-8c^2+12s^2c^2) \frac{M_{\text{W}}^2 M_{\text{H}}^2}{\lambda_{\text{z}}} c^2 \right] \frac{1}{c^4} \frac{1}{M_{\text{W}} M_{\text{H}}^2} B_0^{\text{fin}} \left( -M_{\text{Z}}^2; M_{\text{W}}, M_{\text{W}} \right) \\
 & + \frac{1}{256} \left( 4M_{\text{W}}^4 - c^4 M_{\text{H}}^4 + \frac{M_{\text{H}}^6}{\lambda_{\text{z}}} c^4 + 12 \frac{M_{\text{W}}^2 M_{\text{H}}^6}{\lambda_{\text{z}}^2} c^2 + 28 \frac{M_{\text{W}}^4 M_{\text{H}}^2}{\lambda_{\text{z}}} + 24 \frac{M_{\text{W}}^4 M_{\text{H}}^4}{\lambda_{\text{z}}^2} \right) \frac{1}{c^4} \frac{1}{M_{\text{W}}^3 M_{\text{H}}^2} \\
 & \times B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{Z}}, M_{\text{Z}} \right) \\
 & + \frac{1}{128} \left( 4c^2 M_{\text{W}}^2 - c^4 M_{\text{H}}^2 + \frac{M_{\text{H}}^4}{\lambda_{\text{z}}} c^4 - 4 \frac{M_{\text{W}}^2 M_{\text{H}}^2}{\lambda_{\text{z}}} c^2 - 12 \frac{M_{\text{W}}^2 M_{\text{H}}^4}{\lambda_{\text{z}}^2} c^2 + 16 \frac{M_{\text{W}}^4}{\lambda_{\text{z}}} \right. \\
 & \left. + 48 \frac{M_{\text{W}}^4 M_{\text{H}}^2}{\lambda_{\text{z}}^2} \right) \frac{1}{c^4} \frac{1}{M_{\text{W}}^3} B_0^{\text{fin}} \left( -M_{\text{Z}}^2; M_{\text{H}}, M_{\text{Z}} \right) \\
 & + \frac{3}{256} \left( c^4 - \frac{M_{\text{H}}^2}{\lambda_{\text{z}}} c^4 - 12 \frac{M_{\text{W}}^2 M_{\text{H}}^2}{\lambda_{\text{z}}^2} c^2 + 24 \frac{M_{\text{W}}^4}{\lambda_{\text{z}}^2} \right) \frac{1}{c^4} \frac{M_{\text{H}}^2}{M_{\text{W}}^3} B_0^{\text{fin}} \left( -M_{\text{H}}^2; M_{\text{H}}, M_{\text{H}} \right) \\
 & + \frac{1}{128} \left[ (1-8c^2-44c^4+112c^6)c^2 M_{\text{H}}^2 + 2(1-8c^2+28c^4-48c^6)c^2 M_{\text{W}}^2 \right. \\
 & - 24(1+8c^2-12s^2c^2) \frac{M_{\text{W}}^4 M_{\text{H}}^2}{\lambda_{\text{z}}^2} - (1-12c^4+16c^6) \frac{M_{\text{H}}^4}{\lambda_{\text{z}}} c^2 - 12(1-4s^2c^2) \frac{M_{\text{W}}^2 M_{\text{H}}^4}{\lambda_{\text{z}}^2} \\
 & \left. - 2(17-4c^4+48c^6) \frac{M_{\text{W}}^2 M_{\text{H}}^2}{\lambda_{\text{z}}} c^2 \right] \frac{1}{c^6} \frac{M_{\text{W}}}{M_{\text{H}}^2} C_0 \left( -M_{\text{H}}^2, -M_{\text{Z}}^2, -M_{\text{Z}}^2; M_{\text{W}}, M_{\text{W}}, M_{\text{W}} \right) \\
 & + \frac{1}{256} \left( 8M_{\text{W}}^4 + 2c^2 M_{\text{W}}^2 M_{\text{H}}^2 - c^4 M_{\text{H}}^4 + \frac{M_{\text{H}}^6}{\lambda_{\text{z}}} c^4 + 6 \frac{M_{\text{W}}^2 M_{\text{H}}^4}{\lambda_{\text{z}}} c^2 + 12 \frac{M_{\text{W}}^2 M_{\text{H}}^6}{\lambda_{\text{z}}^2} c^2 + 40 \frac{M_{\text{W}}^4 M_{\text{H}}^2}{\lambda_{\text{z}}} \right. \\
 & \left. + 24 \frac{M_{\text{W}}^4 M_{\text{H}}^4}{\lambda_{\text{z}}^2} \right) \frac{1}{c^4} \frac{1}{M_{\text{W}}^3} C_0 \left( -M_{\text{H}}^2, -M_{\text{Z}}^2, -M_{\text{Z}}^2; M_{\text{Z}}, M_{\text{H}}, M_{\text{Z}} \right)
 \end{aligned}$$



$$\begin{aligned}
 & + \frac{3}{256} (4c^4 M_W^2 - c^6 M_H^2 + \frac{M_H^4}{\lambda_Z} c^6 - 4 \frac{M_W^2 M_H^2}{\lambda_Z} c^4 + 12 \frac{M_W^2 M_H^4}{\lambda_Z^2} c^4 + 8 \frac{M_W^4}{\lambda_Z} c^2 - 48 \frac{M_W^4 M_H^2}{\lambda_Z^2} c^2 \\
 & + 48 \frac{M_W^6}{\lambda_Z^2}) \frac{1}{c^6} \frac{M_H^2}{M_W^3} C_0(-M_H^2, -M_Z^2, -M_Z^2; M_H, M_Z, M_H)
 \end{aligned} \tag{I.10}$$

### I.3 H → WW

$$\begin{aligned}
 \mathcal{D}_{\text{HWW}; \text{NLO}}^q &= \frac{3}{32} (M_b^2 + M_t^2 - L_R M_b^2 - L_R M_t^2) \frac{1}{M_W} \\
 & + \frac{3}{16} (M_b^2 - M_t^2 + M_W^2) \frac{1}{M_W} \frac{M_t^2}{\lambda_w} B_0^{\text{fin}}(-M_H^2; M_t, M_t) \\
 & + \frac{3}{16} (-M_b^2 + M_t^2 + M_W^2) \frac{1}{M_W} \frac{M_b^2}{\lambda_w} B_0^{\text{fin}}(-M_H^2; M_b, M_b) \\
 & - \frac{3}{32} (M_b^2 + M_t^2 - 2 \frac{M_b^4}{\lambda_w} + 4 \frac{M_t^2 M_b^2}{\lambda_w} - 2 \frac{M_t^4}{\lambda_w} + 2 \frac{M_W^2 M_b^2}{\lambda_w} + 2 \frac{M_W^2 M_t^2}{\lambda_w}) \frac{1}{M_W} B_0^{\text{fin}}(-M_W^2; M_t, M_b) \\
 & - \frac{3}{32} (-M_b^2 + M_t^2 + M_W^2 + 2 \frac{M_b^4}{\lambda_w} - 4 \frac{M_t^2 M_b^2}{\lambda_w} + 2 \frac{M_t^4}{\lambda_w} - 4 \frac{M_W^2 M_b^2}{\lambda_w} + 4 \frac{M_W^2 M_t^2}{\lambda_w} + 2 \frac{M_W^4}{\lambda_w}) \frac{M_b^2}{M_W} \\
 & \times C_0(-M_H^2, -M_W^2, -M_W^2; M_b, M_t, M_b) \\
 & - \frac{3}{32} (M_b^2 - M_t^2 + M_W^2 + 2 \frac{M_b^4}{\lambda_w} - 4 \frac{M_t^2 M_b^2}{\lambda_w} + 2 \frac{M_t^4}{\lambda_w} + 4 \frac{M_W^2 M_b^2}{\lambda_w} - 4 \frac{M_W^2 M_t^2}{\lambda_w} + 2 \frac{M_W^4}{\lambda_w}) \frac{M_t^2}{M_W} \\
 & \times C_0(-M_H^2, -M_W^2, -M_W^2; M_t, M_b, M_t) \\
 & + \Delta \mathcal{D}_{\text{HWW}; \text{NLO}}^q
 \end{aligned} \tag{I.11}$$

$$\begin{aligned}
 \mathcal{D}_{\text{HWW}; \text{NLO}}^W &= -\frac{1}{4} a_0^{\text{fin}}(M_W) M_W \\
 & - \frac{1}{64} [3c^2 M_H^2 - 3(1+2c^2) L_R M_W^2 + (1+34c^2) M_W^2] \frac{1}{M_W} \frac{1}{c^2} \\
 & - \frac{1}{64} [c^2 - 2(1+6c^2 - 16c^4) \frac{M_W^2}{\lambda_w} - (-c^2 + s^2) \frac{M_H^2}{\lambda_w} c^2] \frac{1}{c^4} M_W B_0^{\text{fin}}(-M_H^2; M_Z, M_Z) \\
 & - \frac{1}{64} [2c^2 M_W^2 + \frac{M_H^4}{\lambda_w} c^2 + (1+2c^2) \frac{M_W^2 M_H^2}{\lambda_w} + 2(1+8c^2) \frac{M_W^4}{\lambda_w}] \frac{1}{M_W} \frac{1}{c^2} B_0^{\text{fin}}(-M_H^2; M_W, M_W) \\
 & + \frac{1}{32} [\frac{M_H^2}{\lambda_w} c^4 + 2(1-3c^2) c^2 - (1+5c^2 - 24c^4) \frac{M_W^2}{\lambda_w}] \frac{1}{c^4} M_W B_0^{\text{fin}}(-M_W^2; M_W, M_Z) \\
 & + \frac{3}{64} (M_H^2 - 2M_W^2) \frac{1}{M_W} \frac{M_H^2}{\lambda_w} B_0^{\text{fin}}(-M_H^2; M_H, M_H) \\
 & - \frac{1}{32} (-2M_W^2 + \frac{M_H^4}{\lambda_w} - 4 \frac{M_W^2 M_H^2}{\lambda_w}) \frac{1}{M_W} B_0^{\text{fin}}(-M_W^2; M_W, M_H) \\
 & - \frac{1}{64} [9c^6 M_H^2 - 2(1+4c^2 - 28c^4 + 32c^6) \frac{M_W^4}{\lambda_w} + 2(2-7c^2 - 16c^4) c^2 M_W^2 \\
 & - (c^2 - s^2)^2 \frac{M_W^2 M_H^2}{\lambda_w} c^2] \frac{1}{c^6} M_W C_0(-M_H^2, -M_W^2, -M_W^2; M_Z, M_W, M_Z) \\
 & + \frac{1}{64} [\frac{M_W^2 M_H^2}{\lambda_w} - 2(1-22c^2 + 8s^2 c^2) c^2 M_W^2 + 2(1+8c^2) \frac{M_W^4}{\lambda_w} \\
 & - (1+8c^4) c^2 M_H^2] \frac{1}{c^4} M_W C_0(-M_H^2, -M_W^2, -M_W^2; M_W, M_Z, M_W)
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{64} (M_H^4 + 2M_W^2 M_H^2 - 4M_W^4 + \frac{M_H^6}{\lambda_w} + 2 \frac{M_W^2 M_H^4}{\lambda_w}) \frac{1}{M_W} C_0(-M_H^2, -M_W^2, -M_W^2; M_W, M_H, M_W) \\
 & + \frac{3}{64} (-M_W^2 + \frac{M_H^4}{\lambda_w} - 4 \frac{M_W^2 M_H^2}{\lambda_w} + 4 \frac{M_W^4}{\lambda_w}) \frac{M_H^2}{M_W} C_0(-M_H^2, -M_W^2, -M_W^2; M_H, M_W, M_H) \\
 & - \frac{1}{8} (M_H^2 - 2M_W^2) s^2 M_W C_0^{\text{fin}}(-M_H^2, -M_W^2, -M_W^2; M_W, 0, M_W) \\
 & + \Delta \mathcal{D}_{\text{HWW}; \text{NLO}}^{\text{W}}
 \end{aligned} \tag{I.12}$$

$$\begin{aligned}
 \mathcal{D}_{\text{HWW}; \text{NLO}}^{\text{q}} = & \frac{3}{32} (M_b^2 + M_t^2 + \frac{M_H^2 M_b^2}{\lambda_w} + \frac{M_H^2 M_t^2}{\lambda_w}) \frac{1}{M_W M_H^2} \\
 & + \frac{3}{32} (M_b^2 - M_t^2 - \frac{M_H^2 M_b^2}{\lambda_w} + \frac{M_H^2 M_t^2}{\lambda_w}) \frac{1}{M_W^2} \frac{M_b^2}{M_W M_H^2} a_0^{\text{fin}}(M_b) \\
 & - \frac{3}{32} (M_b^2 - M_t^2 - \frac{M_H^2 M_b^2}{\lambda_w} + \frac{M_H^2 M_t^2}{\lambda_w}) \frac{1}{M_W^2} \frac{M_t^2}{M_W M_H^2} a_0^{\text{fin}}(M_t) \\
 & + \frac{3}{64} (-M_b^2 + M_t^2 + M_W^2 + \frac{M_H^2 M_b^2}{\lambda_w} - \frac{M_H^2 M_t^2}{\lambda_w} + 3 \frac{M_W^2 M_H^2}{\lambda_w} + 12 \frac{M_W^2 M_H^2 M_b^2}{\lambda_w^2} \\
 & - 12 \frac{M_W^2 M_H^2 M_t^2}{\lambda_w^2} + 12 \frac{M_W^4 M_H^2}{\lambda_w^2}) \frac{1}{M_W^2} \frac{M_t^2}{M_W M_H^2} B_0^{\text{fin}}(-M_H^2; M_t, M_t) \\
 & + \frac{3}{64} (M_b^2 - M_t^2 + M_W^2 - \frac{M_H^2 M_b^2}{\lambda_w} + \frac{M_H^2 M_t^2}{\lambda_w} + 3 \frac{M_W^2 M_H^2}{\lambda_w} - 12 \frac{M_W^2 M_H^2 M_b^2}{\lambda_w^2} + 12 \frac{M_W^2 M_H^2 M_t^2}{\lambda_w^2} \\
 & + 12 \frac{M_W^4 M_H^2}{\lambda_w^2}) \frac{1}{M_W^2} \frac{M_b^2}{M_W M_H^2} B_0^{\text{fin}}(-M_H^2; M_b, M_b) \\
 & - \frac{3}{64} (-M_b^4 + 2M_t^2 M_b^2 - M_t^4 + M_W^2 M_b^2 + M_W^2 M_t^2 + \frac{M_H^2 M_b^4}{\lambda_w} - 2 \frac{M_H^2 M_t^2 M_b^2}{\lambda_w} + \frac{M_H^2 M_t^4}{\lambda_w} \\
 & + 3 \frac{M_W^2 M_H^2 M_b^2}{\lambda_w} - 12 \frac{M_W^2 M_H^2 M_b^4}{\lambda_w^2} + 3 \frac{M_W^2 M_H^2 M_t^2}{\lambda_w} + 24 \frac{M_W^2 M_H^2 M_t^2 M_b^2}{\lambda_w^2} \\
 & - 12 \frac{M_W^2 M_H^2 M_t^4}{\lambda_w^2} + 12 \frac{M_W^4 M_H^2 M_b^2}{\lambda_w^2} + 12 \frac{M_W^4 M_H^2 M_t^2}{\lambda_w^2}) \frac{1}{M_W^3 M_H^2} B_0^{\text{fin}}(-M_W^2; M_t, M_b) \\
 & + \frac{3}{64} (M_b^4 - 2M_t^2 M_b^2 + M_t^4 - 2M_W^2 M_b^2 - 2M_W^2 M_t^2 + M_W^4 - \frac{M_H^2 M_b^4}{\lambda_w} + 2 \frac{M_H^2 M_t^2 M_b^2}{\lambda_w} - \frac{M_H^2 M_t^4}{\lambda_w} \\
 & - 10 \frac{M_W^2 M_H^2 M_b^2}{\lambda_w} - 12 \frac{M_W^2 M_H^2 M_b^4}{\lambda_w^2} + 6 \frac{M_W^2 M_H^2 M_t^2}{\lambda_w} + 24 \frac{M_W^2 M_H^2 M_t^2 M_b^2}{\lambda_w^2} - 12 \frac{M_W^2 M_H^2 M_t^4}{\lambda_w^2} \\
 & - 5 \frac{M_W^4 M_H^2}{\lambda_w} - 24 \frac{M_W^4 M_H^2 M_b^2}{\lambda_w^2} + 24 \frac{M_W^4 M_H^2 M_t^2}{\lambda_w^2} \\
 & - 12 \frac{M_W^6 M_H^2}{\lambda_w^2}) \frac{1}{M_W^2} \frac{M_t^2}{M_W M_H^2} C_0(-M_H^2, -M_W^2, -M_W^2; M_t, M_b, M_t) \\
 & + \frac{3}{64} (M_b^4 - 2M_t^2 M_b^2 + M_t^4 - 2M_W^2 M_b^2 - 2M_W^2 M_t^2 + M_W^4 - \frac{M_H^2 M_b^4}{\lambda_w} + 2 \frac{M_H^2 M_t^2 M_b^2}{\lambda_w} - \frac{M_H^2 M_t^4}{\lambda_w} \\
 & + 6 \frac{M_W^2 M_H^2 M_b^2}{\lambda_w} - 12 \frac{M_W^2 M_H^2 M_b^4}{\lambda_w^2} - 10 \frac{M_W^2 M_H^2 M_t^2}{\lambda_w} + 24 \frac{M_W^2 M_H^2 M_t^2 M_b^2}{\lambda_w^2} - 12 \frac{M_W^2 M_H^2 M_t^4}{\lambda_w^2} \\
 & - 5 \frac{M_W^4 M_H^2}{\lambda_w} + 24 \frac{M_W^4 M_H^2 M_b^2}{\lambda_w^2} - 24 \frac{M_W^4 M_H^2 M_t^2}{\lambda_w^2} - 12 \frac{M_W^6 M_H^2}{\lambda_w^2}) \frac{1}{M_W^2} \frac{M_b^2}{M_W M_H^2} \\
 & \times C_0(-M_H^2, -M_W^2, -M_W^2; M_b, M_t, M_b)
 \end{aligned} \tag{I.13}$$

$$\begin{aligned}
 \Delta \mathcal{D}_{\text{HWW};\text{NLO}}^{\text{q}} &= \frac{1}{32} M_{\text{W}} \left( \text{W}_{\text{H};\text{t,b}}^{(4)} + 2 \text{W}_{\text{W};\text{t,b}}^{(4)} - \text{d} \mathcal{Z}_{M_{\text{W}};\text{t,b}}^{(4)} - 2 \text{d} \mathcal{Z}_{g;\text{t,b}}^{(4)} \right) \\
 \Delta \mathcal{D}_{\text{HWW};\text{NLO}}^{\text{W}} &= \frac{1}{32} M_{\text{W}} \left( \text{W}_{\text{H};\text{W}}^{(4)} + 2 \text{W}_{\text{W};\text{W}}^{(4)} - \text{d} \mathcal{Z}_{M_{\text{W}};\text{W}}^{(4)} - 2 \text{d} \mathcal{Z}_{g;\text{W}}^{(4)} \right)
 \end{aligned} \tag{I.14}$$

$$\begin{aligned}
 \mathcal{D}_{\text{HWW};\text{NLO}}^{\text{W}} &= -\frac{1}{64} \left[ 3c^2 M_{\text{H}}^2 + 3 \frac{M_{\text{H}}^4}{\lambda_{\text{W}}} c^2 + (1+18c^2) M_{\text{W}}^2 + (1+18c^2) \frac{M_{\text{W}}^2 M_{\text{H}}^2}{\lambda_{\text{W}}} \right] \frac{1}{c^2} \frac{1}{M_{\text{W}} M_{\text{H}}^2} \\
 &+ \frac{1}{64} \left[ c^2 M_{\text{H}}^2 - \frac{M_{\text{H}}^4}{\lambda_{\text{W}}} c^2 + (1-2c^2) M_{\text{W}}^2 - (1-2c^2) \frac{M_{\text{W}}^2 M_{\text{H}}^2}{\lambda_{\text{W}}} \right] \frac{1}{c^2} \frac{1}{M_{\text{W}} M_{\text{H}}^2} a_0^{\text{fin}}(M_{\text{W}}) \\
 &- \frac{1}{64} \left[ (9-8s^2) - (9-8s^2) \frac{M_{\text{H}}^2}{\lambda_{\text{W}}} \right] \frac{M_{\text{W}}}{M_{\text{H}}^2} \frac{s^2}{c^4} a_0^{\text{fin}}(M_{\text{Z}}) \\
 &- \frac{1}{64} \left( M_{\text{H}}^2 - M_{\text{W}}^2 - \frac{M_{\text{H}}^4}{\lambda_{\text{W}}} + \frac{M_{\text{W}}^2 M_{\text{H}}^2}{\lambda_{\text{W}}} \right) \frac{1}{M_{\text{W}}^3} a_0^{\text{fin}}(M_{\text{H}}) \\
 &+ \frac{1}{2} \frac{M_{\text{W}}}{\lambda_{\text{W}}} s^2 \text{B}_0^{\text{fin}}(-M_{\text{W}}^2; 0, M_{\text{W}}) \\
 &- \frac{1}{256} \left[ c^2 M_{\text{H}}^2 - \frac{M_{\text{H}}^4}{\lambda_{\text{W}}} c^2 - 2(1-8c^2) \frac{M_{\text{W}}^2 M_{\text{H}}^2}{\lambda_{\text{W}}} - 24(1+6c^2-16c^4) \frac{M_{\text{W}}^4 M_{\text{H}}^2}{\lambda_{\text{W}}^2} + 2(1+8c^2) M_{\text{W}}^2 \right. \\
 &\left. - 12(-c^2+s^2) \frac{M_{\text{W}}^2 M_{\text{H}}^4}{\lambda_{\text{W}}^2} c^2 \right] \frac{1}{c^4} \frac{1}{M_{\text{W}} M_{\text{H}}^2} \text{B}_0^{\text{fin}}(-M_{\text{H}}^2; M_{\text{Z}}, M_{\text{Z}}) \\
 &+ \frac{1}{256} \left[ c^2 M_{\text{H}}^4 - \frac{M_{\text{H}}^6}{\lambda_{\text{W}}} c^2 - 12 \frac{M_{\text{W}}^2 M_{\text{H}}^6}{\lambda_{\text{W}}^2} c^2 + 2(1-12c^2) M_{\text{W}}^4 + (1-2c^2) M_{\text{W}}^2 M_{\text{H}}^2 \right. \\
 &\left. - (1-2c^2) \frac{M_{\text{W}}^2 M_{\text{H}}^4}{\lambda_{\text{W}}} - 12(1+2c^2) \frac{M_{\text{W}}^4 M_{\text{H}}^4}{\lambda_{\text{W}}^2} - 24(1+8c^2) \frac{M_{\text{W}}^6 M_{\text{H}}^2}{\lambda_{\text{W}}^2} \right. \\
 &\left. - 2(1+20c^2) \frac{M_{\text{W}}^4 M_{\text{H}}^2}{\lambda_{\text{W}}} \right] \frac{1}{c^2} \frac{1}{M_{\text{W}}^3 M_{\text{H}}^2} \text{B}_0^{\text{fin}}(-M_{\text{H}}^2; M_{\text{W}}, M_{\text{W}}) \\
 &+ \frac{1}{128} \left[ c^4 M_{\text{H}}^2 - \frac{M_{\text{H}}^4}{\lambda_{\text{W}}} c^4 + 12 \frac{M_{\text{W}}^2 M_{\text{H}}^4}{\lambda_{\text{W}}^2} c^4 - (1+5c^2-24c^4) M_{\text{W}}^2 \right. \\
 &\left. - 12(1+5c^2-24c^4) \frac{M_{\text{W}}^4 M_{\text{H}}^2}{\lambda_{\text{W}}^2} + (1+21c^2-72c^4+64c^6) \frac{M_{\text{W}}^2 M_{\text{H}}^2}{\lambda_{\text{W}}} \right] \frac{1}{c^4} \frac{1}{M_{\text{W}} M_{\text{H}}^2} \\
 &\times \text{B}_0^{\text{fin}}(-M_{\text{W}}^2; M_{\text{W}}, M_{\text{Z}}) \\
 &- \frac{3}{256} \left( 1 - \frac{M_{\text{H}}^2}{\lambda_{\text{W}}} - 12 \frac{M_{\text{W}}^2 M_{\text{H}}^2}{\lambda_{\text{W}}^2} + 24 \frac{M_{\text{W}}^4}{\lambda_{\text{W}}^2} \frac{M_{\text{H}}^2}{M_{\text{W}}^3} \right) \text{B}_0^{\text{fin}}(-M_{\text{H}}^2; M_{\text{H}}, M_{\text{H}}) \\
 &- \frac{1}{128} \left( M_{\text{H}}^2 - 4M_{\text{W}}^2 - \frac{M_{\text{H}}^4}{\lambda_{\text{W}}} + 4 \frac{M_{\text{W}}^2 M_{\text{H}}^2}{\lambda_{\text{W}}} + 12 \frac{M_{\text{W}}^2 M_{\text{H}}^4}{\lambda_{\text{W}}^2} - 16 \frac{M_{\text{W}}^4}{\lambda_{\text{W}}} - 48 \frac{M_{\text{W}}^4 M_{\text{H}}^2}{\lambda_{\text{W}}^2} \right) \frac{1}{M_{\text{W}}^3} \\
 &\times \text{B}_0^{\text{fin}}(-M_{\text{W}}^2; M_{\text{W}}, M_{\text{H}}) \\
 &+ \frac{1}{256} \left[ 12 \frac{M_{\text{W}}^2 M_{\text{H}}^4}{\lambda_{\text{W}}^2} + 2(1-36c^2+96c^4) \frac{M_{\text{W}}^2 M_{\text{H}}^2}{\lambda_{\text{W}}} + (1+4c^2) \frac{M_{\text{H}}^4}{\lambda_{\text{W}}} \right. \\
 &\left. - 2(1+4c^2-32c^4) M_{\text{W}}^2 + 24(1+8c^2) \frac{M_{\text{W}}^4 M_{\text{H}}^2}{\lambda_{\text{W}}^2} \right. \\
 &\left. - (-2c^2+63c^4+s^4) M_{\text{H}}^2 \right] \frac{1}{c^4} \frac{M_{\text{W}}}{M_{\text{H}}^2} \text{C}_0(-M_{\text{H}}^2, -M_{\text{W}}^2, -M_{\text{W}}^2; M_{\text{W}}, M_{\text{Z}}, M_{\text{W}})
 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{256} \left[ 2(1 - 12c^2 + 8c^4 + 64c^6) \frac{M_W^2 M_H^2}{\lambda_w} - 2(1 + 4c^2 - 32c^4) M_W^2 \right. \\
 & + 24(1 + 4c^2 - 28c^4 + 32c^6) \frac{M_W^4 M_H^2}{\lambda_w^2} + (-2c^2 + 7c^4 + s^4) \frac{M_H^4}{\lambda_w} c^2 - (-2c^2 + 63c^4 + s^4) c^2 M_H^2 \\
 & \left. + 12(c^4 - 2s^2 c^2 + s^4) \frac{M_W^2 M_H^4}{\lambda_w^2} c^2 \right] \frac{1}{c^6} \frac{M_W}{M_H^2} C_0(-M_H^2, -M_W^2, -M_W^2; M_Z, M_W, M_Z) \\
 & - \frac{3}{256} (M_H^2 - 4M_W^2 - \frac{M_H^4}{\lambda_w} + 4 \frac{M_W^2 M_H^2}{\lambda_w} - 12 \frac{M_W^2 M_H^4}{\lambda_w^2} - 8 \frac{M_W^4}{\lambda_w} + 48 \frac{M_W^4 M_H^2}{\lambda_w^2} \\
 & - 48 \frac{M_W^6}{\lambda_w^2}) \frac{M_H^2}{M_W^3} C_0(-M_H^2, -M_W^2, -M_W^2; M_H, M_W, M_H) \\
 & - \frac{1}{256} (M_H^4 - 2M_W^2 M_H^2 - 8M_W^4 - \frac{M_H^6}{\lambda_w} - 6 \frac{M_W^2 M_H^4}{\lambda_w} - 12 \frac{M_W^2 M_H^6}{\lambda_w^2} - 40 \frac{M_W^4 M_H^2}{\lambda_w} \\
 & - 24 \frac{M_W^4 M_H^4}{\lambda_w^2}) \frac{1}{M_W^3} C_0(-M_H^2, -M_W^2, -M_W^2; M_W, M_H, M_W) \tag{I.15}
 \end{aligned}$$

## J Corrections for the W mass

In this appendix we present the full list of corrections for  $M_W$  in the  $\alpha$ -scheme, as given in eq. (6.5). In this appendix we use  $s = \hat{s}_\theta$  and  $c = \hat{c}_\theta$  where  $\hat{c}_\theta^2$  is defined in eq. (4.73). Furthermore, in this appendix, ratios of masses are defined according to

$$x_H = \frac{M_H}{M_Z}, \quad x_f = \frac{M_f}{M_Z} \tag{J.1}$$

etc. We introduce the following polynomials:

$$Q_0^a = 23 - 4c^2 \quad Q_1^a = 77 - 12c^2$$

$$\begin{aligned}
 Q_0^b &= 21 - 4c^2 & Q_1^b &= 65 - 12c^2 & Q_2^b &= 109 - 6Q_0^a c^2 \\
 Q_3^b &= 5 - 2c^2 & Q_4^b &= 62 - Q_1^a c^2
 \end{aligned}$$

$$\begin{aligned}
 Q_0^c &= 2 - c^2 & Q_1^c &= 12c^3 + 29s & Q_2^c &= 19 - Q_0^b c^2 \\
 Q_3^c &= 75 - 16c^2 & Q_4^c &= 22 - Q_0^b c^2 & Q_5^c &= 52 - Q_1^b c^2 \\
 Q_6^c &= 5 - 2c^2 & Q_7^c &= 32 - Q_2^b c^2 & Q_8^c &= 7 - Q_3^b c^2 \\
 Q_9^c &= 1 + 2Q_4^b c^2
 \end{aligned}$$

$$\begin{aligned}
 Q_0^d &= 1 - 2c^2 & Q_1^d &= 119 - 128Q_0^c c^2 & Q_2^d &= 29 - Q_1^c s \\
 Q_3^d &= 43 - 6Q_2^c c^2 & Q_4^d &= 15 - Q_3^c c^2 & Q_5^d &= 61 - 12Q_4^c c^2 \\
 Q_6^d &= 2 + 3Q_5^c c^2 & Q_7^d &= 1 - s^2 & Q_8^d &= 1 - s^2 c^2 \\
 Q_9^d &= 5 - 4c^2 & Q_{10}^d &= 7 - 4c^2 & Q_{11}^d &= 9 - 8c^2 \\
 Q_{12}^d &= 13 - 8c^2 & Q_{13}^d &= 19 + 4Q_6^c c^2 & Q_{14}^d &= 1 - 2Q_7^c c^2 \\
 Q_{15}^d &= 7 - Q_8^c c^2 & Q_{16}^d &= 7 - Q_9^c c^2
 \end{aligned}$$

$$\begin{aligned}
 Q_0^e &= 16Q_0^d s - Q_1^d c & Q_1^e &= 1 + 16c^2 - 4Q_2^d s c & Q_2^e &= 1 + 62Q_0^d s c - 2Q_3^d c^2 \\
 Q_3^e &= 1 + Q_4^d c^2 & Q_4^e &= 9 - 8s^2 & Q_5^e &= 20Q_0^d s + Q_5^d c \\
 Q_6^e &= 59Q_0^d s + Q_6^d c & Q_7^e &= 5 - 8c^2 & Q_8^e &= 9 - 128Q_7^d s^2 \\
 Q_9^e &= 3Q_0^d c + 32Q_8^d s & Q_{10}^e &= 4Q_0^d s - Q_9^d c & Q_{11}^e &= 4Q_0^d s - Q_{10}^d c \\
 Q_{12}^e &= 4Q_0^d s - 3Q_{11}^d c & Q_{13}^e &= 8Q_0^d s - Q_{12}^d c & Q_{14}^e &= 1 - 2c^2 \\
 Q_{15}^e &= 1 - 2Q_{13}^d c^2 & Q_{16}^e &= 12 - Q_{14}^d s c - 6Q_{15}^d c^2 & Q_{17}^e &= 12 - 6Q_{15}^d c^2 + Q_{16}^d s c
 \end{aligned}$$

With their help we derive the correction factors:

$$\begin{aligned}
 \Delta_1^{(4)} M_W &= -\frac{1}{24} B_0^{\text{fin}}(-M_Z^2; 0, 0) \frac{1}{s^2} \frac{c^2}{s^2 - c^2} + \frac{1}{24} B_{0p}^{\text{fin}}(0; 0, M_1) \frac{1}{s^2 - c^2} x_1^4 \\
 &+ \frac{1}{24} a_0^{\text{fin}}(M_1) \frac{1}{s^2 c^2} x_1^4 + \frac{1}{24} [x_1^2 - v_1^2 c^2 x_1^2 - 8s^4 c^2] a_0^{\text{fin}}(M_1) \frac{1}{s^2} \frac{1}{s^2 - c^2} \\
 &+ \frac{1}{24} [x_1^4 + c^2 x_1^2 - 2s c^3] B_0^{\text{fin}}(-M_W^2; 0, M_1) \frac{1}{s^2 c^2} \\
 &+ \frac{1}{144} [6c x_1^2 + Q_6^e + (1 - 6x_1^2) v_1^2 c + 3(v_u^2 + v_d^2) c] \frac{1}{s^2} \frac{c}{s^2 - c^2} \\
 &- \frac{1}{48} [(1 - 4x_1^2) + (1 + 2x_1^2) v_1^2] B_0^{\text{fin}}(-M_Z^2; M_1, M_1) \frac{1}{s^2} \frac{c^2}{s^2 - c^2}
 \end{aligned} \tag{J.2}$$

$$\begin{aligned}
 \Delta_q^{(4)} M_W &= -\frac{1}{8} [x_d^2 - x_u^2] a_0^{\text{fin}}(M_u) \frac{1}{s^2 c^2} x_u^2 + \frac{1}{8} (x_u^2 - x_d^2)^2 B_{0p}^{\text{fin}}(0; M_u, M_d) \frac{1}{s^2 - c^2} \\
 &+ \frac{1}{72} \left[ 9x_u^2 - 9v_u^2 c^2 x_u^2 + 18s^2 x_d^2 - 32s^4 c^2 + 18 \frac{x_d^4}{x_u^2 - x_d^2} s^2 \right] a_0^{\text{fin}}(M_u) \frac{1}{s^2} \frac{1}{s^2 - c^2} \\
 &- \frac{1}{8} [c^2 - (x_d^2 - x_u^2)] a_0^{\text{fin}}(M_d) \frac{1}{s^2 c^2} x_d^2 - \frac{1}{8} [v_u^2 x_u^2 + v_d^2 x_d^2 - (x_d^2 + x_u^2)] \frac{1}{s^2} \frac{c^2}{s^2 - c^2} \\
 &- \frac{1}{72} \left[ 9v_d^2 c^2 x_d^2 + 8s^4 c^2 + 18 \frac{x_d^4}{x_u^2 - x_d^2} s^2 \right] a_0^{\text{fin}}(M_d) \frac{1}{s^2} \frac{1}{s^2 - c^2} \\
 &- \frac{1}{8} [2s c^3 - (x_d^2 - x_u^2)^2 - (x_d^2 + x_u^2) c^2] B_0^{\text{fin}}(M_W^2; M_u, M_d) \frac{1}{s^2 c^2} \\
 &- \frac{1}{16} [(1 - 4x_d^2) + (1 + 2x_d^2) v_d^2] B_0^{\text{fin}}(M_Z^2; M_d, M_d) \frac{1}{s^2} \frac{c^2}{s^2 - c^2} \\
 &- \frac{1}{16} [(1 - 4x_u^2) + (1 + 2x_u^2) v_u^2] B_0^{\text{fin}}(-M_Z^2; M_u, M_u) \frac{1}{s^2} \frac{c^2}{s^2 - c^2}
 \end{aligned} \tag{J.3}$$

$$\begin{aligned}
 \Delta_B^{(4)} M_W &= B_0^{\text{fin}}(-M_W^2; 0, M_W) c^2 - \frac{1}{6} B_{0p}^{\text{fin}}(0; 0, M_W) \frac{s^2 c^4}{s^2 - c^2} \\
 &+ \frac{1}{48} a_0^{\text{fin}}(M_W) \frac{1}{s^2} x_H^2 - \frac{1}{48} Q_1^e B_0^{\text{fin}}(-M_W^2; M_W, M_Z) \frac{1}{s^2 c^2} \\
 &- \frac{1}{48} Q_4^e B_{0p}^{\text{fin}}(0; M_W, M_Z) \frac{1}{s^2 - c^2} s^4 - \frac{1}{48} Q_5^e B_0^{\text{fin}}(-M_Z^2; M_W, M_W) \frac{1}{s^2} \frac{c}{s^2 - c^2} \\
 &- \frac{1}{48} [12 - 4x_H^2 + x_H^4] B_0^{\text{fin}}(-M_Z^2; M_H, M_Z) \frac{1}{s^2} \frac{c^2}{s^2 - c^2} \\
 &- \frac{1}{48} [x_H^4 - 4c^2 x_H^2 + 12s c^3] B_0^{\text{fin}}(-M_W^2; M_W, M_H) \frac{1}{s^2 c^2} \\
 &- \frac{1}{48} [x_H^4 - 2c^2 x_H^2 + c^4] B_{0p}^{\text{fin}}(0; M_W, M_H) \frac{1}{s^2 - c^2} \\
 &+ \frac{1}{48} [3c^2 x_H^2 - 8c^4 - s^2 x_H^4 - 8 \frac{c^6}{x_H^2 - c^2}] a_0^{\text{fin}}(M_H) \frac{1}{c^2} \frac{1}{s^2 - c^2} \\
 &+ \frac{1}{48} [c^4 x_H^2 - Q_3^e] a_0^{\text{fin}}(M_Z) \frac{1}{s^2 c^2} \frac{1}{s^2 - c^2}
 \end{aligned}$$

$$\begin{aligned}
& + \frac{1}{48} \left[ 8 \frac{s^2 c^4}{x_H^2 - c^2} + Q_2^e \right] a_0^{\text{fin}}(M_W) \frac{1}{s^2} \frac{1}{s^2 - c^2} \\
& + \frac{1}{36} \left[ Q_6^e - (100 - 9\Delta_g) s^2 c \right] \frac{1}{s^2} \frac{c}{s^2 - c^2}
\end{aligned} \tag{J.4}$$

$$\begin{aligned}
\Delta_1^{(6)} M_W = & - \frac{1}{48} \left[ 8a_{\phi 1}^{(1)} - a_{\phi D} \right] B_0^{\text{fin}} \left( -M_Z^2; 0, 0 \right) \frac{1}{s^2} \frac{c^2}{s^2 - c^2} \\
& - \frac{1}{144} \left[ 9x_1^2 a_{\phi D} - 8Q_7^e a_{\phi D} + 48(a_{\phi 1}^{(3)} + a_{\phi 1}^{(1)} + a_{\phi 1}) s^2 + 72(a_{\phi 1}^{(1)} - a_{\phi 1}) x_1^2 \right] \frac{1}{s^2} \frac{c^2}{s^2 - c^2} L_R \\
& + \frac{1}{48} \left[ cx_1^2 a_{\phi D} - 8v_1 cx_1^2 a_{\phi 1v} + 8v_1^2 cx_1^2 a_{\phi 1}^{(3)} + 8s^2 c^3 a_{\phi D} + 64s^4 c a_{\phi 1}^{(3)} + Q_{11}^e v_1 x_1^2 a_{\phi D} \right. \\
& \left. + 8(a_{\phi 1}^{(1)} - a_{\phi 1}) cx_1^2 \right] a_0^{\text{fin}}(M_1) \frac{1}{s^2} \frac{c}{s^2 - c^2} \\
& + \frac{1}{288} \left[ 6c^2 x_1^2 a_{\phi D} - Q_8^e c^2 a_{\phi D} - 32Q_9^e s a_{\phi 1}^{(3)} - Q_{10}^e v_d c a_{\phi D} - Q_{13}^e v_u c a_{\phi D} \right. \\
& \left. + 8(1 - 6x_1^2) v_1 c^2 a_{\phi 1v} - 8(1 - 6x_1^2) v_1^2 c^2 a_{\phi 1}^{(3)} \right. \\
& \left. - (1 - 6x_1^2) Q_{11}^e v_1 c a_{\phi D} + 8(122a_{\phi 1}^{(3)} + a_{\phi 1}^{(1)} + a_{\phi 1}) s^2 + 48(a_{\phi 1}^{(1)} - a_{\phi 1}) c^2 x_1^2 \right. \\
& \left. - 24(v_u^2 + v_d^2) c^2 a_{\phi 1}^{(3)} \right] \frac{1}{s^2} \frac{1}{s^2 - c^2} \\
& + \frac{1}{6} (a_{\phi 1}^{(3)} - a_{\phi 1}^{(1)} - a_{\phi 1}) \frac{1}{s^2} + \frac{1}{288} \left[ Q_{10}^e v_d + Q_{13}^e v_u \right] \frac{1}{s^2} \frac{c}{s^2 - c^2} N_{\text{gen}} a_{\phi D} \\
& + \frac{1}{96} \left[ Q_{11}^e v_1 a_{\phi D} + 2Q_{12}^e v_1 x_1^2 a_{\phi D} + (1 - 4x_1^2) c a_{\phi D} - 8(1 + 2x_1^2) v_1 c a_{\phi 1v} \right. \\
& \left. + 8(1 + 2x_1^2) v_1^2 c a_{\phi 1}^{(3)} + 8(a_{\phi 1}^{(1)} - a_{\phi 1})(1 - 4x_1^2) c \right] B_0^{\text{fin}} \left( -M_Z^2; M_1, M_1 \right) \frac{1}{s^2} \frac{c}{s^2 - c^2}
\end{aligned} \tag{J.5}$$

$$\begin{aligned}
\Delta_q^{(6)} M_W = & \frac{1}{3} \left[ c a_{\phi q}^{(3)} - 3s x_d^2 \right] \frac{1}{s} \\
& + \frac{1}{144} \left[ 9c x_d^2 a_{\phi D} - 72v_d c x_d^2 a_{\phi Dv} + 72v_d^2 c x_d^2 a_{\phi q}^{(3)} + 8s^2 c^3 a_{\phi D} + 64s^4 c a_{\phi q}^{(3)} + 3Q_{10}^e v_d x_d^2 a_{\phi D} \right. \\
& \left. + 72(a_{\phi q}^{(1)} - a_{\phi d}) c x_d^2 \right] a_0^{\text{fin}}(M_d) \frac{1}{s^2} \frac{c}{s^2 - c^2} \\
& + \frac{1}{144} \left[ 9c x_u^2 a_{\phi D} - 72v_u c x_u^2 a_{\phi Uv} + 72v_u^2 c x_u^2 a_{\phi q}^{(3)} + 32s^2 c^3 a_{\phi D} + 256s^4 c a_{\phi q}^{(3)} + 3Q_{13}^e v_u x_u^2 a_{\phi D} \right. \\
& \left. - 72(a_{\phi q}^{(1)} - a_{\phi u}) c x_u^2 \right] a_0^{\text{fin}}(M_u) \frac{1}{s^2} \frac{c}{s^2 - c^2} \\
& + \frac{1}{48} \left[ 24v_u^2 c x_u^2 a_{\phi q}^{(3)} + 24v_d^2 c x_d^2 a_{\phi q}^{(3)} + Q_{10}^e v_d x_d^2 a_{\phi D} + Q_{13}^e v_u x_u^2 a_{\phi D} \right. \\
& \left. + 4(1 - 6x_d^2) v_d c a_{\phi Dv} + 4(1 - 6x_u^2) v_u c a_{\phi Uv} \right. \\
& \left. + 4(a_{\phi d\Lambda} + a_{\phi u\Lambda}) c + 24(a_{\phi q}^{(1)} - a_{\phi d}) c x_d^2 - 24(a_{\phi q}^{(1)} - a_{\phi u}) c x_u^2 + 3(x_d^2 + x_u^2) c a_{\phi D} \right] \frac{1}{s^2} \frac{c}{s^2 - c^2} \\
& - \frac{1}{288} \left[ Q_{10}^e v_d + Q_{13}^e v_u \right] \frac{1}{s^2} \frac{c}{s^2 - c^2} N_{\text{gen}} a_{\phi D} \\
& + \frac{1}{96} \left[ 3(1 - 4x_d^2) c a_{\phi D} - 24(1 + 2x_d^2) v_d c a_{\phi Dv} + 24(1 + 2x_d^2) v_d^2 c a_{\phi q}^{(3)} + (1 + 2x_d^2) Q_{10}^e v_d a_{\phi D} \right. \\
& \left. + 24(a_{\phi q}^{(1)} - a_{\phi d})(1 - 4x_d^2) c \right] B_0^{\text{fin}} \left( -M_Z^2; M_d, M_d \right) \frac{1}{s^2} \frac{c}{s^2 - c^2} \\
& + \frac{1}{96} \left[ 3(1 - 4x_u^2) c a_{\phi D} - 24(1 + 2x_u^2) v_u c a_{\phi Uv} + 24(1 + 2x_u^2) v_u^2 c a_{\phi q}^{(3)} + (1 + 2x_u^2) Q_{13}^e v_u a_{\phi D} \right. \\
& \left. - 24(a_{\phi q}^{(1)} - a_{\phi u})(1 - 4x_u^2) c \right] B_0^{\text{fin}} \left( -M_Z^2; M_u, M_u \right) \frac{1}{s^2} \frac{c}{s^2 - c^2} \\
& - \frac{1}{48} \left[ 16(3a_{\phi q}^{(3)} - a_{\phi q}^{(1)} + a_{\phi d} - 2a_{\phi u}) s^2 + 72(a_{\phi q}^{(1)} - a_{\phi d}) x_d^2 - 72(a_{\phi q}^{(1)} - a_{\phi u}) x_u^2 \right.
\end{aligned}$$

$$+9(x_d^2 + x_u^2)a_{\phi_D} \Big] \frac{1}{s^2} \frac{c^2}{s^2 - c^2} L_R \quad (\text{J.6})$$

$$\begin{aligned} \Delta_B^{(6)} M_W = & -\frac{1}{48} Q_1^e B_0^{\text{fin}}(-M_W^2; M_W, M_Z) \frac{1}{s^2 c^2} a_{\phi_D} \\ & -\frac{1}{48} Q_4^e B_{0p}^{\text{fin}}(0; M_W, M_Z) \frac{s^4}{s^2 - c^2} a_{\phi_D} - \frac{1}{48} Q_5^e B_0^{\text{fin}}(-M_Z^2; M_W, M_W) \frac{1}{s^2} \frac{c}{s^2 - c^2} a_{\phi_D} \\ & -\frac{1}{12} Q_{14}^e B_{0p}^{\text{fin}}(0; 0, M_W) \frac{c^4}{s^2 - c^2} a_{\phi_D} - \frac{1}{48} \left[ a_{\phi_D} \right. \\ & \left. + 2a_{\phi_\square} \right] \left[ 12 - 4x_H^2 + x_H^4 \right] B_0^{\text{fin}}(-M_Z^2; M_H, M_Z) \frac{1}{s^2} \frac{c^2}{s^2 - c^2} \\ & + \frac{1}{96} \left[ 33a_{\phi_D} + 8a_{\phi_\square} \right] a_0^{\text{fin}}(M_Z) \frac{1}{s^2} - \frac{1}{96} \left[ 4x_H^2 a_{\phi_\square} + 15c^2 a_{\phi_D} \right] a_0^{\text{fin}}(M_H) \frac{1}{s^2 c^2} x_H^2 \\ & - \frac{1}{24} \left[ x_H^4 - 4c^2 x_H^2 + 12s c^3 \right] B_0^{\text{fin}}(-M_W^2; M_W, M_H) \frac{1}{s^2 c^2} a_{\phi_\square} \\ & - \frac{1}{24} \left[ x_H^4 - 2c^2 x_H^2 + c^4 \right] B_{0p}^{\text{fin}}(0; M_W, M_H) \frac{1}{s^2 - c^2} a_{\phi_\square} \\ & + \frac{1}{48} \left[ 2c x_H^2 a_{\phi_\square} - (39a_{\phi_D} - 4a_{\phi_\square}) s^3 + (51a_{\phi_D} - 4a_{\phi_\square}) s \right] a_0^{\text{fin}}(M_W) \frac{1}{s^2 c} \\ & + \frac{1}{72} \left[ 3s c^3 x_H^2 a_{\phi_D} - 3Q_{17}^e a_{\phi_D} + (19a_{\phi_D} + 4a_{\phi_\square}) s^3 c - (182a_{\phi_D} + 6a_{\phi_\square} - 9\Delta_g a_{\phi_D}) s^3 c^3 \right] \frac{1}{s^3 c} \frac{1}{s^2 - c^2} \\ & - \frac{1}{96} \left[ 32s^2 c^2 a_{\phi_\square} + 32 \frac{s^2 c^4}{x_H^2 - c^2} a_{\phi_\square} + 2(a_{\phi_D} + 2a_{\phi_\square}) c^2 x_H^4 \right. \\ & \left. - 3(5a_{\phi_D} + 4a_{\phi_\square}) s^2 x_H^2 \right] a_0^{\text{fin}}(M_H) \frac{1}{s^2} \frac{1}{s^2 - c^2} \\ & + \frac{1}{48} \left[ 16 \frac{s^3 c^5}{x_H^2 - c^2} a_{\phi_\square} - Q_{16}^e a_{\phi_D} - 2(15a_{\phi_D} + a_{\phi_\square}) s^3 c^3 \right] a_0^{\text{fin}}(M_W) \frac{1}{s^3 c} \frac{1}{s^2 - c^2} \\ & - \frac{1}{96} \left[ 2Q_{15}^e a_{\phi_D} - 2(a_{\phi_D} + 2a_{\phi_\square}) c^4 x_H^2 + (139a_{\phi_D} + 8a_{\phi_\square}) s^2 c^2 \right] a_0^{\text{fin}}(M_Z) \frac{1}{s^2 c^2} \frac{1}{s^2 - c^2} \\ & - \frac{1}{96} \left[ (25 + 9x_H^2) a_{\phi_D} - 4(5a_{\phi_D} - 11a_{\phi_\square}) s^2 \right] \frac{1}{s^2} \frac{c^2}{s^2 - c^2} L_R \\ & + \frac{1}{36} \left[ (a_{\phi_D} - 2a_{\phi_\square}) c - (33a_{\phi_D} + 2a_{\phi_\square}) s^3 + (51a_{\phi_D} + 2a_{\phi_\square}) s \right] \frac{1}{s^2 c} \end{aligned} \quad (\text{J.7})$$

## K T parameter

In this appendix we present explicit results for the T parameter of eq. (6.8). For simplicity we only include PTG operators in loops. We have introduced  $s = \hat{s}_\theta$ ,  $c = \hat{c}_\theta$ ,  $c_2 = \hat{c}_{2\theta}$  and

$$\alpha T = \frac{\alpha}{\pi} \frac{t}{s^2 c^2 c_2} \quad (\text{K.1})$$

- dim = 4 component

$$\begin{aligned} t^{(4)} = & \frac{5}{4} c^2 c_2 - \frac{1}{4} \sum_{\text{gen}} a_0^{\text{fin}}(M_u) \frac{x_u^2 x_d^2}{x_u^2 - x_d^2} c_2 \\ & + \frac{1}{4} \sum_{\text{gen}} a_0^{\text{fin}}(M_d) \frac{x_u^2 x_d^2}{x_u^2 - x_d^2} c_2 - \frac{1}{24} B_{0p}^{\text{fin}}(0; 0, M_1) c_2 \sum_{\text{gen}} x_1^4 \\ & + \frac{1}{6} B_{0p}^{\text{fin}}(0; 0, M_W) s^2 c^4 c_2 - \frac{1}{48} (1 - x_H^2)^2 B_{0p}^{\text{fin}}(0; M_H, M_Z) c_2 \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{48} (5 + 4c_2) \mathbf{B}_{0p}^{\text{fin}}(0; M_W, M_Z) s^4 c_2 - \frac{1}{8} \sum_{\text{gen}} (x_d^2 - x_u^2)^2 \mathbf{B}_{0p}^{\text{fin}}(0; M_u, M_d) c_2 \\
 & + \frac{1}{24} \sum_{\text{gen}} (3x_d^2 + 3x_u^2 + x_1^2) c_2 + \frac{1}{48} (x_H^2 - c^2) (x_H^2 - c^2) \mathbf{B}_{0p}^{\text{fin}}(0; M_W, M_H) c_2 \\
 & - \frac{1}{6} \left[ -c^2 - \frac{c^4}{x_H^2 - c^2} + \frac{x_H^2}{x_H^2 - 1} \right] a_0^{\text{fin}}(M_H) c_2 \\
 & + \frac{1}{24} \left[ 4 \frac{s^2}{x_H^2 - 1} - (9 + 17c_2 + 4c_2^2) \right] a_0^{\text{fin}}(M_Z) \frac{c_2}{s^2} \\
 & - \frac{1}{12} \left[ 2 \frac{s^2 c^2}{x_H^2 - c^2} - 5(2 + c_2) \right] a_0^{\text{fin}}(M_W) \frac{c_2}{s^2} c^2
 \end{aligned} \tag{K.2}$$

- dim = 6 component

$$\begin{aligned}
 t^{(6)} = & -\frac{1}{6} \mathbf{B}_{0p}^{\text{fin}}(0; 0, M_1) c_2 \sum_{\text{gen}} x_1^4 a_{\phi 1}^{(3)} - \frac{1}{12} \mathbf{B}_{0p}^{\text{fin}}(0; 0, M_W) c^6 c_2 a_{\phi D} \\
 & + \frac{1}{96} (5 + 4c_2) \mathbf{B}_{0p}^{\text{fin}}(0; M_W, M_Z) s^4 c_2 a_{\phi D} - \frac{1}{96} (a_{\phi D} + 4a_{\phi \square}) (1 - x_H^2)^2 \mathbf{B}_{0p}^{\text{fin}}(0; M_H, M_Z) c_2 \\
 & - \frac{1}{2} \sum_{\text{gen}} (x_d^2 - x_u^2)^2 \mathbf{B}_{0p}^{\text{fin}}(0; M_u, M_d) c_2 a_{\phi q}^{(3)} - \frac{1}{96} (x_H^2 - c^2) (x_H^2 - c^2) (a_{\phi D} - 4a_{\phi \square}) \mathbf{B}_{0p}^{\text{fin}}(0; M_W, M_H) c_2 \\
 & + \frac{1}{16} \sum_{\text{gen}} \left[ -3a_{\phi D} + 16 \frac{x_d^2}{x_u^2 - x_d^2} a_{\phi q}^{(3)} + 8(2a_{\phi q}^{(3)} - 3a_{\phi q}^{(1)} + 3a_{\phi d}) \right] a_0^{\text{fin}}(M_d) c_2 x_d^2 \\
 & - \frac{1}{16} \sum_{\text{gen}} \left[ a_{\phi D} + 8(a_{\phi 1}^{(1)} - a_{\phi 1}) \right] a_0^{\text{fin}}(M_1) c_2 x_1^2 \\
 & - \frac{1}{16} \sum_{\text{gen}} \left[ 3a_{\phi D} + 16 \frac{x_d^2}{x_u^2 - x_d^2} a_{\phi q}^{(3)} - 24(a_{\phi q}^{(1)} - a_{\phi u}) \right] a_0^{\text{fin}}(M_u) c_2 x_u^2 \\
 & - \frac{1}{96} \left[ 9x_H^2 a_{\phi D} + 8(a_{\phi D} - 4a_{\phi \square}) c^2 + 8(a_{\phi D} - 4a_{\phi \square}) \frac{c^4}{x_H^2 - c^2} + 8(a_{\phi D} + 4a_{\phi \square}) \frac{x_H^2}{x_H^2 - 1} \right] a_0^{\text{fin}}(M_H) c_2 \\
 & - \frac{1}{24} \left[ c_2 x_H^2 a_{\phi D} - 2c^4 a_{\phi \square} + 2s^4 a_{\phi D} - 2(a_{\phi D} - a_{\phi \square}) s^2 c^2 + 2(a_{\phi D} + a_{\phi \square}) c_2 \right] \\
 & - \frac{1}{32} \left[ 6s^2 x_H^2 a_{\phi D} - 3(7 + x_H^2) a_{\phi D} - 8(5a_{\phi D} - 3a_{\phi \square}) s^4 + 2(31a_{\phi D} - 6a_{\phi \square}) s^2 \right] \mathbf{L}_R \\
 & - \frac{1}{24} \left[ 5(1 - 3c_2 - c_2^2) a_{\phi D} - 2(a_{\phi D} - 4a_{\phi \square}) \frac{s^2 c^2}{x_H^2 - c^2} c_2 \right. \\
 & \left. + (a_{\phi D} - a_{\phi \square}) s^2 c^2 - (21a_{\phi D} - a_{\phi \square}) s^4 \right] a_0^{\text{fin}}(M_W) \frac{c^2}{s^2} \\
 & + \frac{1}{192} \left[ (51 - 152c_2 - 3c_2^2 - 16c_2^3) a_{\phi D} + 16(a_{\phi D} + 4a_{\phi \square}) \frac{s^2}{x_H^2 - 1} c_2 + 2(11a_{\phi D} - 4a_{\phi \square}) s^2 c^2 \right. \\
 & \left. - 2(113a_{\phi D} - 4a_{\phi \square}) s^4 \right] a_0^{\text{fin}}(M_Z) \frac{1}{s^2} \\
 & - \frac{1}{48} \sum_{\text{gen}} \left[ -24(a_{\phi q}^{(3)} - 3a_{\phi q}^{(1)} + 3a_{\phi d}) x_d^2 - 24(a_{\phi q}^{(3)} + 3a_{\phi q}^{(1)} - 3a_{\phi u}) x_u^2 - 8(a_{\phi 1}^{(3)} - 3a_{\phi 1}^{(1)} + 3a_{\phi 1}) x_1^2 \right. \\
 & \left. + 3(3x_d^2 + 3x_u^2 + x_1^2) a_{\phi D} \right] c_2 \\
 & + \frac{1}{16} \sum_{\text{gen}} \left[ 24(a_{\phi q}^{(1)} - a_{\phi d}) x_d^2 - 24(a_{\phi q}^{(1)} - a_{\phi u}) x_u^2 + 8(a_{\phi 1}^{(1)} - a_{\phi 1}) x_1^2 + (3x_d^2 + 3x_u^2 + x_1^2) a_{\phi D} \right] \mathbf{L}_R c_2
 \end{aligned} \tag{K.3}$$



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