Rational closure in $\mathcal{SHIQ}$

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Abstract. We define a notion of rational closure for the logic $\mathcal{SHIQ}$, which does not enjoys the finite model property, building on the notion of rational closure introduced by Lehmann and Magidor in [24]. We provide a semantic characterization of rational closure in $\mathcal{SHIQ}$ in terms of a preferential semantics, based on a finite rank characterization of minimal models. We show that the rational closure of a TBox can be computed in EXPTIME using entailment in $\mathcal{SHIQ}$.

1 Introduction

Recently, a large amount of work has been done in order to extend the basic formalism of Description Logics (for short, DLs) with nonmonotonic reasoning features [27, 1, 10, 11, 13, 17, 21, 4, 2, 6, 26, 23]; the purpose of these extensions is that of allowing reasoning about *prototypical properties* of individuals or classes of individuals. In these extensions one can represent, for instance, knowledge expressing the fact that the hematocrit level is *usually* under 50%, with the exceptions of newborns and of males residing at high altitudes, that have usually much higher levels (even over 65%). Furthermore, one can infer that an individual enjoys all the *typical* properties of the classes it belongs to. As an example, in the absence of information that Carlos and the son of Fernando are either newborns or adult males living at a high altitude, one would assume that the hematocrit levels of Carlos and Fernando's son are under 50%. This kind of inferences apply to individual explicitly named in the knowledge base as well as to individuals implicitly introduced by relations among individuals (the son of Fernando).

In spite of the number of works in this direction, finding a solution to the problem of extending DLs for reasoning about prototypical properties seems far from being solved. The most well known semantics for nonmonotonic reasoning have been used to the purpose, from default logic [1], to circumscription [2], to Lifschitz's nonmonotonic logic MKNF [10, 26], to preferential reasoning [13, 4, 17], to rational closure [6, 9].

In this work, we focus on rational closure and, specifically, on the rational closure for $\mathcal{SHIQ}$. The interest of rational closure in DLs is that it provides a significant and reasonable nonmonotonic inference mechanism, still remaining computationally inexpensive. As shown for $\mathcal{ALC}$ in [6], its complexity can be expected not to exceed the one of the underlying monotonic DL. This is a striking difference with most of the other approaches to nonmonotonic reasoning in DLs mentioned above, with some exception such as [26, 23]. More specifically, we define a rational closure for the logic $\mathcal{SHIQ}$, building on the notion of rational closure in [24] for propositional logic. This is a difference with respect to the rational closure construction introduced in [6] for $\mathcal{ALC}$, which is more similar to the one by Freund [12] for propositional logic (for propositional logic, the two definitions of rational closure are shown to be equivalent [12]). We provide a semantic characterization of rational closure in $\mathcal{SHIQ}$ in terms of a preferential semantics, by generalizing to $\mathcal{SHIQ}$ the results for rational closure in $\mathcal{ALC}$ presented in [18]. This generalization is not trivial, since $\mathcal{SHIQ}$ lacks a crucial property of $\mathcal{ALC}$, the finite model property [20]. Our construction exploits an extension of $\mathcal{SHIQ}$ with a typicality operator $\mathbf{T}$, that selects the most typical instances of a concept C, $\mathbf{T}(C)$. We define a *minimal model semantics* and a notion of minimal entailment for the resulting logic, $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$, and we show that the inclusions belonging to the rational closure of a TBox are those minimally entailed by the TBox, when restricting to *canonical* models. This result exploits a characterization of minimal models, showing that we can restrict to models with finite ranks. We also show that the rational closure construction of a TBox can be done exploiting entailment in $\mathcal{SHIQ}$, without requiring to reason in $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$, and that the problem of deciding whether an inclusion belongs to the rational closure of a TBox is in EXPTIME.

Concerning ABox reasoning, because of the interaction between individuals (due to roles) it is not possible to separately assign a unique minimal rank to each individual and alternative minimal ranks must be considered. We end up with a kind of *skeptical* inference with respect to the ABox, whose complexity in EXPTIME as well.

For an extended version of this paper with the proofs of the results see [19].

2 A nonmonotonic extension of $\mathcal{SHIQ}$

Following the approach in [14, 17], we introduce an extension of $\mathcal{SHIQ}$ [20] with a typicality operator $\mathbf{T}$ in order to express typical inclusions, obtaining the logic $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$. The intuitive idea is to allow concepts of the form $\mathbf{T}(C)$, whose intuitive meaning is that $\mathbf{T}(C)$ selects the *typical* instances of a concept C. We can therefore distinguish between the properties that hold for all instances of C ($C \sqsubseteq D$), and those that only hold for the typical such instances ($\mathbf{T}(C) \sqsubseteq D$). Since we are dealing here with rational closure, we attribute to $\mathbf{T}$ properties of rational consequence relation [24]. We consider an alphabet of concept names $\mathcal{C}$, role names $\mathcal{R}$, transitive roles $\mathcal{R}^+ \subseteq \mathcal{R}$, and individual constants $\mathcal{O}$. Given $A \in \mathcal{C}$, $S \in \mathcal{R}$, and $n \in \mathbb{N}$ we define:

$$C_R := A \mid \top \mid \bot \mid \neg C_R \mid C_R \sqcap C_R \mid C_R \sqcup C_R \mid \forall S.C_R \mid \exists S.C_R \mid (\geq nS.C_R) \mid (\leq nS.C_R)$$
$$C_L := C_R \mid \mathbf{T}(C_R) \qquad\qquad S := R \mid R^-$$

As usual, we assume that transitive roles cannot be used in number restrictions [20]. A KB is a pair (TBox, ABox). TBox contains a finite set of concept inclusions $C_L \sqsubseteq C_R$ and role inclusions $R \sqsubseteq S$. ABox contains assertions of the form $C_L(a)$ and $S(a, b)$, where $a, b \in \mathcal{O}$.
The semantics of $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ is formulated in terms of rational models: ordinary models of $\mathcal{SHIQ}$ are equipped with a *preference relation* $<$ on the domain, whose intuitive meaning is to compare the "typicality" of domain elements, that is to say, $x < y$ means that x is more typical than y. Typical instances of a concept C (the instances of $\mathbf{T}(C)$) are the instances x of C that are minimal with respect to the preference relation $<$ (so that there is no other instance of C preferred to x)[4].

[4] As for the logic $\mathcal{ALC}^{\mathsf{R}}\mathbf{T}$ in [15], an alternative semantic characterization of $\mathbf{T}$ can be given by means of a set of postulates that are essentially a reformulation of the properties of rational consequence relation [24].

Definition 1 (Semantics of $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$). *A $\mathcal{SHIQ}^{R}\mathbf{T}$ model $\mathcal{M}$ is any structure $\langle \Delta, <, I\rangle$ where: Δ is the domain; $<$ is an irreflexive, transitive, well-founded, and modular (for all $x, y, z \in \Delta$, if $x < y$ then $x < z$ or $z < y$) relation over Δ; I is the extension function that maps each concept C to $C^I \subseteq \Delta$, and each role R to $R^I \subseteq \Delta^I \times \Delta^I$. For concepts of $\mathcal{SHIQ}$, C^I is defined as usual. For the $\mathbf{T}$ operator, we have $(\mathbf{T}(C))^I = Min_<(C^I)$, where $Min_<(S) = \{u : u \in S$ and $\nexists z \in S$ s.t. $z < u\}$.*

As for rational models in [24] (see Lemma 14), $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ models can be equivalently defined by postulating the existence of a function $k_{\mathcal{M}} : \Delta \longmapsto Ord$ assigning an ordinal to each domain element, and then letting $x < y$ if and only if $k_{\mathcal{M}}(x) < k_{\mathcal{M}}(y)$. We call $k_{\mathcal{M}}(x)$ *the rank of element* x in $\mathcal{M}$. When finite, $k_{\mathcal{M}}(x)$ can be understood as the length of a chain $x_0 < \cdots < x$ from x to a minimal x_0 (an x_0 s.t. for no x', $x' < x_0$).

Definition 2 (Model satisfying a knowledge base). *Given a $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ model $\mathcal{M}$= $\langle \Delta, <, I\rangle$, we say that: - a model $\mathcal{M}$ satisfies an inclusion $C \sqsubseteq D$ if $C^I \subseteq D^I$; similarly for role inclusions; - $\mathcal{M}$ satisfies an assertion $C(a)$ if $a^I \in C^I$; and $\mathcal{M}$ satisfies an assertion $R(a, b)$ if $(a^I, b^I) \in R^I$. Given a KB=(TBox,ABox), we say that: $\mathcal{M}$ satisfies TBox if $\mathcal{M}$ satisfies all inclusions in TBox; $\mathcal{M}$ satisfies ABox if $\mathcal{M}$ satisfies all assertions in ABox; $\mathcal{M}$ is a model of KB if it satisfies both its TBox and its ABox.*

The logic $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$, as well as the underlying $\mathcal{SHIQ}$, does not enjoy the finite model property [20].

Given a KB, let F be an inclusion or an assertion. We say that F is entailed by KB, written KB $\models_{\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}} F$, if for all models $\mathcal{M}$ =$\langle \Delta, <, I\rangle$ of KB, $\mathcal{M}$ satisfies F.
Let us now introduce the notions of rank of a $\mathcal{SHIQ}$ concept.

Definition 3 (Rank of a concept $k_{\mathcal{M}}(C_R)$). *Given a model $\mathcal{M}$ =$\langle \Delta, <, I\rangle$, we define the* rank *$k_{\mathcal{M}}(C_R)$ of a concept C_R in the model $\mathcal{M}$ as $k_{\mathcal{M}}(C_R) = min\{k_{\mathcal{M}}(x) \mid x \in {C_R}^I\}$. If ${C_R}^I = \emptyset$, then C_R has no rank and we write $k_{\mathcal{M}}(C_R) = \infty$.*

Proposition 1. *For any $\mathcal{M}$ =$\langle \Delta, <, I\rangle$, we have that $\mathcal{M}$ satisfies $\mathbf{T}(C) \sqsubseteq D$ if and only if $k_{\mathcal{M}}(C \sqcap D) < k_{\mathcal{M}}(C \sqcap \neg D)$.*

It is immediate to verify that the typicality operator $\mathbf{T}$ itself is nonmonotonic: $\mathbf{T}(C) \sqsubseteq D$ does not imply $\mathbf{T}(C \sqcap E) \sqsubseteq D$. This nonmonotonicity of $\mathbf{T}$ allows to express the properties that hold for the typical instances of a class (not only the properties that hold for all the members of the class). However, the logic $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ is monotonic: what is inferred from KB can still be inferred from any KB' with KB $\subseteq$ KB'. This is a clear limitation in DLs. As a consequence of the monotonicity of $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$, one cannot deal with irrelevance. For instance, KB= $\{$ $\mathit{VIP} \sqsubseteq \mathit{Person}$, $\mathbf{T}(\mathit{Person}) \sqsubseteq\, \leq 1\ \mathit{HasMarried.Person}$, $\mathbf{T}(\mathit{VIP}) \sqsubseteq\, \geq 2\ \mathit{HasMarried.Person}\}$ does not entail KB $\models_{\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}} \mathbf{T}(\mathit{VIP} \sqcap \mathit{Tall}) \sqsubseteq\, \geq 2\ \mathit{HasMarried.Person}$, even if the property of being tall is irrelevant with respect to the number of marriages. Observe that we do not want to draw this conclusion in a monotonic way from $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$, since otherwise we would not be able to retract it when knowing, for instance, that typical tall VIPs have just one marriage (see also Example 1). Rather, we would like to obtain this conclusion in a nonmonotonic way. In order to obtain this nonmonotonic behavior, we strengthen the semantics of $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ by defining a minimal models mechanism which is similar, in spirit, to circumscription. Given a KB, the idea is to: 1. define a *preference relation* among $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ models, giving preference to the model in which domain elements have a lower rank; 2. restrict entailment to *minimal* $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ models (w.r.t. the above preference relation) of KB.

Definition 4 (Minimal models). *Given $\mathcal{M} = \langle \Delta, <, I \rangle$ and $\mathcal{M}' = \langle \Delta', <', I' \rangle$ we say that $\mathcal{M}$ is preferred to $\mathcal{M}'$ ($\mathcal{M} <_{FIMS} \mathcal{M}'$) if (i) $\Delta = \Delta'$, (ii) $C^I = C^{I'}$ for all concepts C, and (iii) for all $x \in \Delta$, $k_{\mathcal{M}}(x) \leq k_{\mathcal{M}'}(x)$ whereas there exists $y \in \Delta$ such that $k_{\mathcal{M}}(y) < k_{\mathcal{M}'}(y)$. Given a KB, we say that $\mathcal{M}$ is a minimal model of KB with respect to $<_{FIMS}$ if it is a model satisfying KB and there is no $\mathcal{M}'$ model satisfying KB such that $\mathcal{M}' <_{FIMS} \mathcal{M}$.*

Proposition 2 (Existence of minimal models). *Let KB be a finite knowledge base, if KB is satisfiable then it has a minimal model.*

The minimal model semantics introduced is similar to the one introduced in [17] for $\mathcal{ALC}$. However, it is worth noticing that the notion of minimality here is based on the minimization of the ranks of the worlds, rather then on the minimization of formulas of a specific kind.

The following theorem says that reasoning in $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ has the same complexity as reasoning in $\mathcal{SHIQ}$, i.e. it is in ExpTime. Its proof is given by providing an encoding of satisfiability in $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ into satisfiability $\mathcal{SHIQ}$, which is known to be an ExpTime-complete problem. The proof is omitted due to space limitations

Theorem 1. *Satisfiability in $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ is an* ExpTime*-complete problem.*

3 Rational Closure for $\mathcal{SHIQ}$

In this section, we extend to $\mathcal{SHIQ}$ the notion of rational closure proposed by Lehmann and Magidor [24] for the propositional case. Given the typicality operator, the typicality inclusions $\mathbf{T}(C) \sqsubseteq D$ (all the typical C's are D's) play the role of conditional assertions $C \mathrel{|\!\sim} D$ in [24]. Here we define the rational closure of the TBox. In Section 6 we will discuss an extension of rational closure that also takes into account the ABox.

Definition 5 (Exceptionality of concepts and inclusions). *Let T_B be a TBox and C a concept. C is said to be* exceptional *for T_B if and only if $T_B \models_{\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}} \mathbf{T}(\top) \sqsubseteq \neg C$. A $\mathbf{T}$-inclusion $\mathbf{T}(C) \sqsubseteq D$ is exceptional for T_B if C is exceptional for T_B. The set of $\mathbf{T}$-inclusions of T_B which are exceptional in T_B will be denoted as $\mathcal{E}(T_B)$.*

Given a DL KB=(TBox,ABox), it is possible to define a sequence of non increasing subsets of TBox $E_0 \supseteq E_1, E_1 \supseteq E_2, \ldots$ by letting $E_0 =$ TBox and, for $i > 0$, $E_i = \mathcal{E}(E_{i-1}) \cup \{C \sqsubseteq D \in$ TBox s.t. $\mathbf{T}$ does not occurr in $C\}$. Observe that, being KB finite, there is an $n \geq 0$ such that, for all $m > n, E_m = E_n$ or $E_m = \emptyset$. Observe also that the definition of the E_i's is the same as the definition of the C_i's in Lehmann and Magidor's rational closure [22], except for that here, at each step, we also add all the "strict" inclusions $C \sqsubseteq D$ (where $\mathbf{T}$ does not occur in C).

Definition 6 (Rank of a concept). *A concept C has* rank i *(denoted by $rank(C) = i$) for KB=(TBox,ABox), iff i is the least natural number for which C is not exceptional for E_i. If C is exceptional for all E_i then $rank(C) = \infty$, and we say that C has no rank.*

The notion of rank of a formula allows to define the rational closure of the TBox of a KB. Let $\models_{\mathcal{SHIQ}}$ be the entailment in $\mathcal{SHIQ}$. In the following definition, by KB $\models_{\mathcal{SHIQ}} F$ we mean $K_F \models_{\mathcal{SHIQ}} F$, where K_F does not include the defeasible inclusions in KB.

Definition 7 (Rational closure of TBox). *Let KB=(TBox,ABox) be a DL knowledge base. We define, $\overline{TBox}$, the* rational closure of TBox*, as $\overline{TBox} = \{\mathbf{T}(C) \sqsubseteq D \mid$ either $rank(C) < rank(C \sqcap \neg D)$ or $rank(C) = \infty\} \cup \{C \sqsubseteq D \mid$ KB $\models_{\mathcal{SHIQ}} C \sqsubseteq D\}$, where C and D are arbitrary $\mathcal{SHIQ}$ concepts.*

Observe that, apart form the addition of strict inclusions, the above definition of rational closure is the same as the one by Lehmann and Magidor in [24]. The rational closure of TBox is a nonmonotonic strengthening of $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$. For instance, it allows to deal with irrelevance, as the following example shows.

Example 1. Let TBox = $\{\mathbf{T}(Actor) \sqsubseteq Charming\}$. It can be verified that $\mathbf{T}(Actor \sqcap Comic) \sqsubseteq Charming \in \overline{TBox}$. This is a nonmonotonic inference that does no longer follow if we discover that indeed comic actors are not charming (and in this respect are untypical actors): indeed given TBox'= TBox $\cup \{\mathbf{T}(Actor \sqcap Comic) \sqsubseteq \neg Charming\}$, we have that $\mathbf{T}(Actor \sqcap Comic) \sqsubseteq Charming \not\in \overline{TBox'}$. Furthermore, as for the propositional case, rational closure is closed under rational monotonicity [22]: from $\mathbf{T}(Actor) \sqsubseteq Charming \in \overline{TBox}$ and $\mathbf{T}(Actor) \sqsubseteq Bold \not\in \overline{TBox}$ it follows that $\mathbf{T}(Actor \sqcap \neg Bold) \sqsubseteq Charming \in \overline{TBox}$.

Although the rational closure $\overline{TBox}$ is an infinite set, its definition is based on the construction of a finite sequence $E_0, E_1, \ldots, E_n$ of subsets of TBox, and the problem of verifying that an inclusion $\mathbf{T}(C) \sqsubseteq D \in \overline{TBox}$ is in EXPTIME. Let us first prove the following proposition:

Proposition 3. *Let KB=(TBox,$\emptyset$) be a knowledge base with empty ABox. $KB \models_{\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}} C_L \sqsubseteq C_R$ iff $KB' \models_{\mathcal{SHIQ}} C'_L \sqsubseteq C'_R$, where KB', C'_L and C'_R are polynomial encodings in $\mathcal{SHIQ}$ of KB, C_L and C_R, respectively.*

Proof. (Sketch) First of all, let us remember that rational entailment is equivalent to preferential entailment for a knowledge base only containing positive non-monotonic implications $A \mathrel{|\!\sim} B$ (see [24]).The same holds in preferential description logics with typicality. Let $\mathcal{SHIQ}^{P}\mathbf{T}$ be the logic that we obtain when we remove the requirement of modularity in the definition of $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$. In this logic the typicality operator has a preferential semantics [22], based on the preferential models of P rather then on the ranked models of R [22]. It is possible to prove that entailment in $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ and entailment in $\mathcal{SHIQ}^{P}\mathbf{T}$ are equivalent if we restrict to KBs with empty ABox, as TBox contains inclusions (positive non-monotonic implications). Hence, to prove the thesis it suffices to show that for all inclusions $C_L \sqsubseteq C_R$ in $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$: $KB \models_{\mathcal{SHIQ}^{P}\mathbf{T}} C_L \sqsubseteq C_R$ iff $KB' \models_{\mathcal{SHIQ}} C'_L \sqsubseteq C'_R$, for some polynomial encoding KB', C'_L, C'_R in $\mathcal{SHIQ}$.

The idea of the encoding exploits the definition of the typicality operator $\mathbf{T}$ introduced in [14] (for $\mathcal{ALC}$), in terms of a Gödel -Löb modality $\Box$ as follows: $\mathbf{T}(C)$ is defined as $C \sqcap \Box\neg C$ where the accessibility relation of the modality $\Box$ is the preference relation $<$ in preferential models.

We define the encoding KB'=(TBox', ABox') of KB in $\mathcal{SHIQ}$ as follows. First, ABox'=$\emptyset$. For each $A \sqsubseteq B \in$ TBox, not containing $\mathbf{T}$, we introduce $A \sqsubseteq B$ in TBox'. For each $\mathbf{T}(A)$ in occurring in the TBox, we introduce a new atomic concept $\Box_{\neg A}$ and, for each inclusion $\mathbf{T}(A) \sqsubseteq B \in$ TBox, we add to TBox' the inclusion: $A \sqcap \Box_{\neg A} \sqsubseteq B$. Furthermore, to capture the properties of the $\Box$ modality, a new role R is introduced to represent the relation $<$ in preferential models, and the following inclusions are introduced in TBox': $\Box_{\neg A} \sqsubseteq \forall R.(\neg A \sqcap \Box_{\neg A})$ and $\neg\Box_{\neg A} \sqsubseteq \exists R.(A \sqcap \Box_{\neg A})$.

For the inclusion $C_L \sqsubseteq C_R$, we let $C'_R = C_R$. For a strict inclusion ($C_L \neq T(A)$), we let $C'_L = C_L$, while for a defeasible inclusion ($C_L = T(A)$), we let $C'_L = A \sqcap \Box_{\neg A}$.

It is clear that the size of KB' is polynomial in the size of the KB. Given the above encoding, it can be proved that: $KB \models_{\mathcal{SHIQ}^P\mathbf{T}} C_L \sqsubseteq C_R$ iff $KB' \models_{\mathcal{SHIQ}} C'_L \sqsubseteq C'_R$. $\Box$

Theorem 2 (Complexity of rational closure over TBox). *Given a TBox, the problem of deciding whether* $\mathbf{T}(C) \sqsubseteq D \in \overline{TBox}$ *is in* EXPTIME.

Proof. Checking if $\mathbf{T}(C) \sqsubseteq D \in \overline{TBox}$ can be done by computing the finite sequence $E_0, E_1, \ldots, E_n$ of non increasing subsets of TBox inclusions in the construction of the rational closure. Note that the number n of the E_i is $O(|KB|)$, where $|KB|$ is the size of the knowledge base KB. Computing each $E_i = \mathcal{E}(E_{i-1})$, requires to check, for all concepts A occurring on the left hand side of an inclusion in the TBox, whether $E_{i-1} \models_{\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}} \mathbf{T}(\top) \sqsubseteq \neg A$. Regarding E_{i-1} as a knowledge base with empty ABox, by Proposition 3 it is enough to check that $E'_{i-1} \models_{\mathcal{SHIQ}} \top \sqcup \Box_{\neg\top} \sqsubseteq \neg A$, which requires an exponential time in the size of E'_{i-1} (and hence in the size of KB). If not already checked, the exceptionality of C and of $C \sqcap \neg D$ have to be checked for each E_i, to determine the ranks of C and of $C \sqcap \neg D$ (which can be computed in $\mathcal{SHIQ}$ as well). Hence, verifying if $\mathbf{T}(C) \sqsubseteq D \in \overline{TBox}$ is in EXPTIME. $\Box$

The above proof also shows that the rational closure of a TBox can be computed simply using the entailment in $\mathcal{SHIQ}$.

4 Infinite Minimal Models with finite ranks

In the following we provide a characterization of minimal models of a KB in terms of their rank: intuitively minimal models are exactly those where each domain element has rank 0 if it satisfies all defeasible inclusions, and otherwise has the smallest rank greater than the rank of any concept C occurring in a defeasible inclusion $\mathbf{T}(C) \sqsubseteq D$ of the KB falsified by the element. Exploiting this intuitive characterization of minimal models, we are able to show that, for a finite KB, minimal models have always a *finite* ranking function, no matter whether they have a finite domain or not. This result allows us to provide a semantic characterization of rational closure of the previous section to logics, like $\mathcal{SHIQ}$, that do not have the finite model property.

Given a model $\mathcal{M} = \langle \Delta, <, I \rangle$, let us define the set $S^{\mathcal{M}}_x$ of defeasible inclusions falsified by a domain element $x \in \Delta$, as $S^{\mathcal{M}}_x = \{\mathbf{T}(C) \sqsubseteq D \in K_D \mid x \in (C \sqcap \neg D)^I\}$.

Proposition 4. *Let $\mathcal{M} = \langle \Delta, <, I \rangle$ be a model of KB and $x \in \Delta$, then: (a) if $k_{\mathcal{M}}(x) = 0$ then $S_x^{\mathcal{M}} = \emptyset$; (b) if $S_x^{\mathcal{M}} \neq \emptyset$ then $k_{\mathcal{M}}(x) > k_{\mathcal{M}}(C)$ for every C such that, for some D, $\mathbf{T}(C) \sqsubseteq D \in S_x^{\mathcal{M}}$.*

Let us define $K_F = \{C \sqsubseteq D \in TBox : \mathbf{T}$ does not occur in $C\} \cup ABox$ and $K_D = \{\mathbf{T}(C) \sqsubseteq D \in TBox\}$, so that KB $= K_F \cup K_D$.

Proposition 5. *Let KB $= K_F \cup K_D$ and $\mathcal{M} = \langle \Delta, <, I \rangle$ be a model of K_F; suppose that for any $x \in \Delta$ it holds that: - if $k_{\mathcal{M}}(x) = 0$ then $S_x^{\mathcal{M}} = \emptyset$; - if $S_x^{\mathcal{M}} \neq \emptyset$ then $k_{\mathcal{M}}(x) > k_{\mathcal{M}}(C)$ for every C s.t., for some D, $\mathbf{T}(C) \sqsubseteq D \in S_x^{\mathcal{M}}$. Then $\mathcal{M} \models$ KB.*

From Propositions 4 and 5, we obtain the following characterization of minimal models.

Theorem 3. *Let KB $= K_F \cup K_D$, and let $\mathcal{M} = \langle \Delta, <, I \rangle$ be a model of K_F. The following are equivalent:*

- *$\mathcal{M}$ is a minimal model of KB*
- *For every $x \in \Delta$ it holds: (a) $S_x^{\mathcal{M}} = \emptyset$ iff $k_{\mathcal{M}}(x) = 0$ (b) if $S_x^{\mathcal{M}} \neq \emptyset$ then $k_{\mathcal{M}}(x) = 1 + max\{k_{\mathcal{M}}(C) \mid \mathbf{T}(C) \sqsubseteq D \in S_x^{\mathcal{M}}\}$.*

The following proposition shows that in any minimal model the *rank* of each domain element is finite.

Proposition 6. *Let KB $= K_F \cup K_D$ and $\mathcal{M} = \langle \Delta, <, I \rangle$ a minimal model of KB, for every $x \in \Delta$, $k_{\mathcal{M}}(x)$ is a finite ordinal ($k_{\mathcal{M}}(x) < \omega$).*

The previous proposition is essential for establishing a correspondence between the minimal model semantics of a KB and its rational closure. From now on, we can assume that the ranking function assigns to each domain element in Δ a natural number, i.e. that $k_{\mathcal{M}} : \Delta \longrightarrow \mathbb{N}$.

5 A Minimal Model Semantics for Rational Closure in $\mathcal{SHIQ}$

In previous sections we have extended to $\mathcal{SHIQ}$ the syntactic notion of rational closure introduced in [24] for propositional logic. To provide a semantic characterization of this notion, we define a special class of minimal models, exploiting the fact that, by Proposition 6, in all minimal $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ models the *rank* of each domain element is always finite. First of all, we can observe that the minimal model semantics in Definition 4 as it is cannot capture the rational closure of a TBox.

Consider the following KB = (TBox, $\emptyset$), where TBox contains: $VIP \sqsubseteq Person$, $\mathbf{T}(Person) \sqsubseteq\ \leq 1\ HasMarried.Person$, $\mathbf{T}(VIP) \sqsubseteq\ \geq 2\ HasMarried.\ Person$. We observe that $\mathbf{T}(VIP \sqcap Tall) \sqsubseteq\ \geq 2\ HasMarried.Person$ does not hold in all minimal $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ models of KB w.r.t. Definition 4. Indeed there can be a model $\mathcal{M} = \langle \Delta, <, I \rangle$ in which $\Delta = \{x, y, z\}$, $VIP^I = \{x, y\}$, $Person^I = \{x, y, z\}$, $(\leq 1\ HasMarried.\ Person)^I = \{x, z\}$, $(\geq 2\ HasMarried.Person)^I = \{y\}$, $Tall^I = \{x\}$, and $z < y < x$. $\mathcal{M}$ is a model of KB, and it is minimal. Also, x is a typical tallVIP in $\mathcal{M}$ (since there is no other tall VIP preferred to him) and has no more than one spouse, therefore $\mathbf{T}(VIP \sqcap Tall) \sqsubseteq\ \geq 2\ HasMarried.Person$ does not hold in $\mathcal{M}$. On the contrary, it can be verified that $\mathbf{T}(VIP \sqcap Tall) \sqsubseteq\ \geq 2\ HasMarried.Person \in \overline{TBox}$.

Things change if we consider the minimal models semantics applied to models that contain a domain element for *each combination of concepts consistent with KB*. We call these models *canonical models*. Therefore, in order to semantically characterize the rational closure of a $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ KB, we restrict our attention to *minimal canonical models*. First, we define $\mathcal{S}$ as the set of all the concepts (and subconcepts) occurring in KB or in the query F together with their complements.

In order to define canonical models, we consider all the sets of concepts $\{C_1, C_2, \ldots, C_n\} \subseteq \mathcal{S}$ that are *consistent with KB*, i.e., s.t. KB $\not\models_{\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}} C_1 \sqcap C_2 \sqcap \cdots \sqcap C_n \sqsubseteq \bot$.

Definition 8 (Canonical model with respect to $\mathcal{S}$). *Given KB=(TBox,ABox) and a query F, a model $\mathcal{M}$ $=\langle \Delta, <, I\rangle$ satisfying KB is* canonical with respect to $\mathcal{S}$ *if it contains at least a domain element $x \in \Delta$ s.t. $x \in (C_1 \sqcap C_2 \sqcap \cdots \sqcap C_n)^I$, for each set of concepts $\{C_1, C_2, \ldots, C_n\} \subseteq \mathcal{S}$ that is consistent with KB.*

Next we define the notion of minimal canonical model.

Definition 9 (Minimal canonical models (w.r.t. $\mathcal{S}$)). *$\mathcal{M}$ is a minimal canonical model of KB if it satisfies KB, it is minimal (with respect to Definition 4) and it is canonical (as defined in Definition 8).*

Proposition 7 (Existence of minimal canonical models). *Let KB be a finite knowledge base, if KB is satisfiable then it has a minimal canonical model.*

To prove the correspondence between minimal canonical models and the rational closure of a TBox, we need to introduce some propositions. The next one concerns all $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ models. Given a $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ model $\mathcal{M}$ $=\langle \Delta, <, I\rangle$, we define a sequence $\mathcal{M}_0$, $\mathcal{M}_1, \mathcal{M}_2, \ldots$ of models as follows: We let $\mathcal{M}_0 = \mathcal{M}$ and, for all i, we let $\mathcal{M}_i = \langle \Delta, <_i, I\rangle$ be the $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ model obtained from $\mathcal{M}$ by assigning a rank 0 to all the domain elements x with $k_{\mathcal{M}}(x) < i$, i.e., $k_{\mathcal{M}_i}(x) = k_{\mathcal{M}}(x) - i$ if $k_{\mathcal{M}}(x) > i$, and $k_{\mathcal{M}_i}(x) = 0$ otherwise. We can prove the following:

Proposition 8. *Let KB= $\langle TBox, ABox\rangle$ and let $\mathcal{M}$ $=\langle \Delta, <, I\rangle$ be any $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ model of TBox. For any concept C, if rank(C) $\geq i$, then 1) $k_{\mathcal{M}}(C) \geq i$, and 2) if $\mathbf{T}(C) \sqsubseteq D$ is entailed by E_i, then $\mathcal{M}_i$ satisfies $\mathbf{T}(C) \sqsubseteq D$.*

Let us now focus our attention on minimal canonical models by proving the correspondence between rank of a formula (as in Definition 6) and rank of a formula in a model (as in Definition 3). The following proposition is proved by induction on the rank i:

Proposition 9. *Given KB and $\mathcal{S}$, for all $C \in \mathcal{S}$, if $rank(C) = i$, then: 1. there is a $\{C_1 \ldots C_n\} \subseteq \mathcal{S}$ maximal and consistent with KB such that $C \in \{C_1 \ldots C_n\}$ and $rank(C_1 \sqcap \cdots \sqcap C_n) = i$; 2. for any $\mathcal{M}$ minimal canonical model of KB, $k_{\mathcal{M}}(C) = i$.*

The following theorem follows from the propositions above:

Theorem 4. *Let KB=(TBox,ABox) be a knowledge base and $C \sqsubseteq D$ a query. We have that $C \sqsubseteq D \in \overline{TBox}$ if and only if $C \sqsubseteq D$ holds in all minimal canonical models of KB with respect to $\mathcal{S}$.*

6 Rational Closure over the ABox

The definition of rational closure in Section 3 takes only into account the TBox. We address the issue of ABox reasoning first by the semantical side: as for any domain element, we would like to attribute to each individual constant named in the ABox the lowest possible rank. Therefore we further refine Definition 9 of minimal canonical models with respect to TBox by taking into account the interpretation of individual constants of the ABox.

Definition 10 (Minimal canonical model w.r.t. ABox). *Given KB=(TBox,ABox), let $\mathcal{M} = \langle \Delta, <, I \rangle$ and $\mathcal{M}' = \langle \Delta', <', I' \rangle$ be two canonical models of KB which are minimal w.r.t. Definition 9. We say that $\mathcal{M}$ is preferred to $\mathcal{M}'$ w.r.t. ABox ($\mathcal{M} <_{ABox} \mathcal{M}'$) if, for all individual constants a occurring in ABox, $k_{\mathcal{M}}(a^I) \leq k_{\mathcal{M}'}(a^{I'})$ and there is at least one individual constant b occurring in ABox such that $k_{\mathcal{M}}(b^I) < k_{\mathcal{M}'}(b^{I'})$.*

As a consequence of Proposition 7 we can prove that:

Theorem 5. *For any KB= $(TBox, ABox)$ there exists a minimal canonical model of KB with respect to ABox.*

In order to see the strength of the above semantics, consider our example about marriages and VIPs.

Example 2. Suppose we have a KB=(TBox,ABox) where: TBox=$\{\mathbf{T}(Person) \sqsubseteq \leq 1\ HasMarried.Person,\ \mathbf{T}(VIP) \sqsubseteq \geq 2\ HasMarried.Person,\ VIP \sqsubseteq Person\}$, and ABox = $\{VIP(demi), Person(marco)\}$. Knowing that Marco is a person and Demi is a VIP, we would like to be able to assume, in the absence of other information, that Marco is a typical person, whereas Demi is a typical VIP, and therefore Marco has at most one spouse, whereas Demi has at least two. Consider any minimal canonical model $\mathcal{M}$ of KB. Being canonical, $\mathcal{M}$ will contain, among other elements, the following:

$x \in (Person)^I$, $x \in (\leq 1\ HasMarried.Person)^I$, $x \in (\neg VIP)^I$, $k_{\mathcal{M}}(x) = 0$;
$y \in (Person)^I$, $y \in (\geq 2\ HasMarried.Person)^I$, $y \in (\neg VIP)^I$, $k_{\mathcal{M}}(y) = 1$;
$z \in (VIP)^I$, $z \in (Person)^I$, $z \in (\geq 2\ HasMarried.Person)^I$, $k_{\mathcal{M}}(z) = 1$;
$w \in (VIP)^I$, $w \in (Person)^I$, $w \in (\leq 1\ HasMarried.Person)^I$, $k_{\mathcal{M}}(w) = 2$.

so that x is a typical person and z is a typical VIP. According to Definition 10, there is a unique minimal canonical model w.r.t. ABox in which $(marco)^I = x$ and $(demi)^I = z$.

We next provide an algorithmic construction for the rational closure of ABox. The idea is that of considering all the possible minimal consistent assignments of ranks to the individuals explicitly named in the ABox. We adopt a skeptical view by considering only those conclusions which hold for all assignments. In order to calculate the rational closure of ABox, written $\overline{ABox}$, for all individual constants of the ABox we find out which is the lowest possible rank they can have in minimal canonical models with respect to Definition 9: the idea is that an individual constant a_i can have a given rank $k_j(a_i)$ just in case it is compatible with all the inclusions $\mathbf{T}(A) \sqsubseteq D$ of the TBox whose antecedent A's rank is $\geq k_j(a_i)$ (the inclusions whose antecedent A's rank is $< k_j(a_i)$ do not matter since, in the canonical model, there will be an instance of A with rank $< k_j(a_i)$ and therefore a_i is not a typical instance of A). The algorithm below computes all minimal rank assignments k_js to all individual constants: μ_i^j contains all the concepts that a_i

would need to satisfy in case it had the rank attributed by k_j ($k_j(a_i)$). The algorithm verifies whether μ^j is compatible with ($\overline{TBox}$, ABox) and whether it is minimal. Notice that, in this phase, all constants are considered simultaneously (indeed, the possible ranks of different individual constants depend on each other).

Definition 11 **($\overline{ABox}$: rational closure of ABox).** *Let $a_1, \ldots, a_m$ be the individuals explicitly named in the ABox. Let $k_1, k_2, \ldots, k_h$ be all the possible rank assignments (ranging from 1 to n) to the individuals occurring in ABox.*

– Given a rank assignment k_j we define:

- *for each a_i: $\mu_i^j = \{(\neg C \sqcup D)(a_i)$ s.t. $C, D \in \mathcal{S}$, $\mathbf{T}(C) \sqsubseteq D$ in $\overline{TBox}$, and $k_j(a_i) \leq rank(C)\} \cup \{(\neg C \sqcup D)(a_i)$ s.t. $C \sqsubseteq D$ in TBox $\}$;*
- *let $\mu^j = \mu_1^j \cup \cdots \cup \mu_m^j$ for all $\mu_1^j \ldots \mu_m^j$ just calculated for all $a_1, \ldots, a_m$ in ABox*

– k_j is minimal and consistent *with ($\overline{TBox}$, ABox), i.e.: (i) TBox $\cup$ ABox $\cup \mu^j$ is consistent in $\mathcal{SHIQ}^{R}\mathbf{T}$; (ii) there is no k_i consistent with ($\overline{TBox}$, ABox) s.t. for all a_i, $k_i(a_i) \leq k_j(a_i)$ and for some b, $k_i(b) < k_j(b)$.*

– The rational closure of ABox ($\overline{ABox}$) is the set of all assertions derivable in $\mathcal{SHIQ}^{R}\mathbf{T}$ from TBox $\cup$ ABox $\cup \mu^j$ for all minimal consistent rank assignments k_j, i.e:

$$\overline{ABox} = \bigcap_{k_j \textit{ minimal consistent}} \{C(a) : \textit{TBox} \cup \textit{ABox} \cup \mu^j \models_{\mathcal{SHIQ}^{R}\mathbf{T}} C(a)\}$$

The example below is the syntactic counterpart of the semantic Example 2 above.

Example 3. Consider the KB in Example 2. Computing the ranking of concepts we get that $rank(Person) = 0, rank(\mathit{VIP}) = 1, rank(Person \sqcap \geq 2\ HasMarried.Person) = 1$, $rank(\mathit{VIP} \sqcap \leq 1\ HasMarried.Person) = 2$. The set μ^1 contains, among the others, $(\neg \mathit{VIP} \sqcup \geq 2\ HasMarried.Person)(demi)$, $(\neg Person \sqcup \leq 1\ HasMarried.Person)(marco)$. It is tedious but easy to check that KB $\cup \mu^1$ is consistent and that k_1 is the only minimal consistent assignment, thus both $(\geq 2\ HasMarried.Person)(demi)$ and $(\leq 1\ HasMarried.Person)\ (marco)$ belong to $\overline{ABox}$.

Theorem 6 **(Soundness and completeness of $\overline{ABox}$).** *Given KB=(TBox, ABox), for each individual constant a in ABox, $C(a) \in \overline{ABox}$ if and only if $C(a)$ holds in all minimal canonical models with respect to ABox of KB.*

Theorem 7 **(Complexity of rational closure over the ABox).** *Given a knowledge base KB=(TBox,ABox) in $\mathcal{SHIQ}^{R}\mathbf{T}$, an individual constant a and a concept C, the problem of deciding whether $C(a) \in \overline{ABox}$ is* EXPTIME*-complete.*

The proof is similar to the one for rational closure over ABox in $\mathcal{ALC}$ (Theorem 5 [18]).

7 Related Works

There are a number of works which are closely related to our proposal.

In [14, 17] nonmonotonic extensions of DLs based on the $\mathbf{T}$ operator have been proposed. In these extensions, focused on the basic DL $\mathcal{ALC}$, the semantics of $\mathbf{T}$ is based on preferential logic P[22]. Moreover and more importantly, the notion of minimal model adopted here is completely independent from the language and is determined only by the relational structure of models.

[6] develop a notion of rational closure for DLs based on the construction of rational closure proposed by Freund [12] at a propositional level. [8] introduces an approach based on the combination of rational closure and *Defeasible Inheritance Networks* (INs).

In [7], a semantic characterization of a variant of the notion of rational closure in [6] is presented, based on a generalization to $\mathcal{ALC}$ of our semantics in [16].

An approach related to ours can be found in [3]. The basic idea of their semantics is similar to ours, but it is restricted to the propositional case. Furthermore, their construction relies on a specific representation of models and it provides a recipe to build a model of the rational closure, rather than a characterization of its properties. Our semantics, defined in terms of standard Kripke models, can be more easily generalized to richer languages, as we have done here for $\mathcal{SHIQ}$.

In [5] the semantics of the logic of defeasible subsumptions is strengthened by a preferential semantics. Furthermore, the authors describe an EXPTIME algorithm in order to compute the rational closure of a given TBox in $\mathcal{ALC}$. In [25] a plug-in for the Protégé ontology editor implementing the mentioned algorithm for computing the rational closure for a TBox for OWL ontologies is described.

Recent works discuss the combination of open and closed world reasoning in DLs. In particular, formalisms have been defined for combining DLs with logic programming rules (see, for instance, [11] and [26]). A grounded circumscription approach for DLs with local closed world capabilities has been defined in [23].

8 Conclusions

In this work we have proposed an extension of the rational closure defined by Lehmann and Magidor to the Description Logic $\mathcal{SHIQ}$, taking into account both TBox and ABox reasoning. One of the contributions is that of extending the semantic characterization of rational closure proposed in [16] for propositional logic, to $\mathcal{SHIQ}$, which does not enjoy the finite model property. We have shown that in all minimal models of a finite KB in $\mathcal{SHIQ}$ the rank of domain elements is always finite, although the domain might be infinite. We have proved an EXPTIME upper bound for both TBox and ABox reasoning with the rational closure shown that the rational closure of a TBox can be computed using entailment in $\mathcal{SHIQ}$.

The rational closure construction in itself can be applied to any description logic. We would like to extend its semantic characterization to stronger logics, such as $\mathcal{SHOIQ}$, for which the notion of canonical model as defined in this paper is too strong due to the interaction of nominals with number restrictions.

It is well known that rational closure has some weaknesses that accompany its well-known qualities. Among the weaknesses is the fact that one cannot separately reason property by property, so that, if a subclass of C is exceptional for a given aspect, it is exceptional "tout court" and does not inherit any of the typical properties of C. Among the strengths there is its computational lightness, which is crucial in Description Logics. Both the qualities and the weaknesses seems to be inherited by its extension to Description Logics. To address the mentioned weakness of rational closure, we may think of attacking the problem from a semantic point of view by considering a finer semantics where models are equipped with several preference relations; in such a semantics it might be possible to relativize the notion of typicality, whence to reason about typical properties independently from each other.

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