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Abstract. We introduce a notion of rational closure for the logic $\mathcal{SHIQ}$ based on the well-known rational closure by Lehmann and Magidor [21]. We provide a semantic characterization of rational closure in $\mathcal{SHIQ}$ in terms of a preferential semantics, based on a finite rank characterization of minimal models.

1 Introduction

The growing interest of defeasible inference in ontology languages has led, in the last years, to the definition of many non-monotonic extensions of Description Logics (DLs) [23, 11, 19, 2, 20]. The best known semantics for nonmonotonic reasoning have been used to the purpose, from default logic [1], to circumscription [2], to Lifschitz's logic MKNF [10, 22], to preferential reasoning [4, 15], and to rational closure [5].

In this work, we focus on rational closure and, specifically, on the rational closure for $\mathcal{SHIQ}$. Rational closure provides a significant and reasonable nonmonotonic inference mechanism for DLs, still remaining computationally inexpensive. As shown for $\mathcal{ALC}$ in [5], its complexity can be expected not to exceed the one of the underlying monotonic DL. This is a striking difference with most of the other approaches to nonmonotonic reasoning in DLs mentioned above, with the exception of some of them, such as [22, 20]. In particular, we define a rational closure for the logic $\mathcal{SHIQ}$ building on the notion of rational closure in [21] for propositional logic. This is a difference with respect to the rational closure construction introduced in [6] for $\mathcal{ALC}$, which is more similar to the one by Freund [12]. We provide a semantic characterization of rational closure in $\mathcal{SHIQ}$ in terms of a preferential semantics, generalizing to $\mathcal{SHIQ}$ the results for rational closure for $\mathcal{ALC}$ in [16]. This generalization is not trivial, since $\mathcal{SHIQ}$ lacks a crucial property of $\mathcal{ALC}$, the finite model property. Our construction exploits an extension of $\mathcal{SHIQ}$ with a typicality operator $\mathbf{T}$, that selects the most typical instances of a concept C, thus allowing defeasible inclusions of the form $\mathbf{T}(C) \sqsubseteq D$ (the typical Cs are Ds) together with the standard (strict) inclusions $C \sqsubseteq D$ (all the Cs are Ds).

We define a *minimal model semantics* and a notion of minimal entailment for the resulting logic, $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$, and we show that the inclusions belonging to the rational closure of a TBox are those minimally entailed by the TBox, when restricting to *canonical* models. This result exploits a characterization of minimal models, showing that we can

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restrict to models with finite ranks. We can show that the rational closure construction of a TBox can be done exploiting entailment in $\mathcal{SHIQ}$, without requiring to reason in $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$, and that the problem of deciding if an inclusion belongs to the rational closure of a TBox is EXPTIME-complete. This abstract is based on the full paper [17].

2 A nonmonotonic extension of $\mathcal{SHIQ}$

Following the approach in [13, 15], we define an extension, $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$, of the logic $\mathcal{SHIQ}$ [18] introducing a typicality operator $\mathbf{T}$ to distinguish defeasible inclusions of the form $\mathbf{T}(C) \sqsubseteq D$, defining the (defeasible) properties of typical instances of C, from strict properties of all instances of C ($C \sqsubseteq D$).

We consider an alphabet of concept names $\mathcal{C}$, role names $\mathcal{R}$, transitive roles $\mathcal{R}^+ \subseteq \mathcal{R}$, and individual constants $\mathcal{O}$. Given $A \in \mathcal{C}$, $R \in \mathcal{R}$, and $n \in \mathbb{N}$ we define:
$C_R := A \mid \top \mid \bot \mid \neg C_R \mid C_R \sqcap C_R \mid C_R \sqcup C_R \mid \forall S.C_R \mid \exists S.C_R \mid (\geq nS.C_R) \mid (\leq nS.C_R)$
$C_L := C_R \mid \mathbf{T}(C_R)$ $\qquad S := R \mid R^-$
As usual, we assume that transitive roles cannot be used in number restrictions [18]. A KB is a pair (TBox, ABox). TBox contains a finite set of concept inclusions $C_L \sqsubseteq C_R$ and role inclusions $R \sqsubseteq S$. ABox contains assertions of the form $C_L(a)$ and $S(a, b)$, where $a, b \in \mathcal{O}$.

The semantics of $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ is formulated in terms of rational models: ordinary models of $\mathcal{SHIQ}$ are equipped with a *preference relation* $<$ on the domain, whose intuitive meaning is to compare the "typicality" of domain elements, that is to say $x < y$ means that x is more typical than y. Typical members of a concept C, that is members of $\mathbf{T}(C)$, are the members x of C that are minimal with respect to this preference relation (s.t. there is no other member of C more typical than x).

Definition 1 (Semantics of $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$). *A model $\mathcal{M}$ of $\mathcal{SHIQ}^{R}\mathbf{T}$ is any structure $\langle \Delta, <, I \rangle$ where: Δ is the domain; $<$ is an irreflexive, transitive, well-founded, and modular (for all x, y, z in Δ, if $x < y$ then either $x < z$ or $z < y$) relation over Δ; I is the extension function that maps each concept C to $C^I \subseteq \Delta$, and each role R to $R^I \subseteq \Delta^I \times \Delta^I$. For concepts of $\mathcal{SHIQ}$, C^I is defined as usual. For the $\mathbf{T}$ operator, we have $(\mathbf{T}(C))^I = Min_<(C^I)$, where $Min_<(S) = \{u : u \in S$ and $\nexists z \in S$ s.t. $z < u\}$.*

$\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ models can be equivalently defined by postulating the existence of a function $k_{\mathcal{M}} : \Delta \longmapsto Ord$, and then letting $x < y$ if and only if $k_{\mathcal{M}}(x) < k_{\mathcal{M}}(y)$. We call $k_{\mathcal{M}}(x)$ *the rank of element* x in $\mathcal{M}$. The rank $k_{\mathcal{M}}(x)$ can be understood as the maximal length of a chain $x_0 < \cdots < x$ from x to a minimal x_0 (s.t. for no x', $x' < x_0$). Observe that because of modularity all chains have the same length.

Definition 2 (Model satisfying a knowledge base). *Given a $\mathcal{SHIQ}^{R}\mathbf{T}$ model $\mathcal{M}$= $\langle \Delta, <, I \rangle$, we say that: - a model $\mathcal{M}$ satisfies an inclusion $C \sqsubseteq D$ if $C^I \subseteq D^I$; similarly for role inclusions; - $\mathcal{M}$ satisfies an assertion $C(a)$ if $a^I \in C^I$; and $\mathcal{M}$ satisfies an assertion $R(a, b)$ if $(a^I, b^I) \in R^I$. Given a KB=(TBox,ABox), we say that: $\mathcal{M}$ satisfies TBox if $\mathcal{M}$ satisfies all inclusions in TBox; $\mathcal{M}$ satisfies ABox if $\mathcal{M}$ satisfies all assertions in ABox; $\mathcal{M}$ satisfies KB if it satisfies both its TBox and its ABox.*

Given a KB, we say that an inclusion $C_L \sqsubseteq C_R$ is derivable from KB, written KB $\models_{\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}} C_L \sqsubseteq C_R$, if ${C_L}^I \subseteq {C_R}^I$ holds in all models $\mathcal{M} = \langle \Delta, <, I \rangle$ satisfying KB; similarly for role inclusions. We also say that an assertion $C_L(a)$, with $a \in \mathcal{O}$, is derivable from KB, written KB $\models_{\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}} C_L(a)$, if $a^I \in {C_L}^I$ holds in all models $\mathcal{M} = \langle \Delta, <, I \rangle$ satisfying KB.

Given a model $\mathcal{M} = \langle \Delta, <, I \rangle$, we define the *rank* $k_{\mathcal{M}}(C_R)$ of a concept C_R in the model $\mathcal{M}$ as $k_{\mathcal{M}}(C_R) = min\{k_{\mathcal{M}}(x) \mid x \in {C_R}^I\}$. If ${C_R}^I = \emptyset$, then C_R has no rank and we write $k_{\mathcal{M}}(C_R) = \infty$. It is immediate to verify that:

Proposition 1. *For any $\mathcal{M} = \langle \Delta, <, I \rangle$, we have that $\mathcal{M}$ satisfies $\mathbf{T}(C) \sqsubseteq D$ if and only if $k_{\mathcal{M}}(C \sqcap D) < k_{\mathcal{M}}(C \sqcap \neg D)$.*

The typicality operator $\mathbf{T}$ itself is nonmonotonic, i.e., $\mathbf{T}(C) \sqsubseteq D$ does not imply $\mathbf{T}(C \sqcap E) \sqsubseteq D$. This nonmonotonicity of $\mathbf{T}$ allows us to express the properties that hold for the typical instances of a class (not only the properties that hold for all the members of the class). However, the logic $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ is monotonic: what is inferred from KB can still be inferred from any KB' with KB $\subseteq$ KB'. This is a clear limitation in DLs. As a consequence of the monotonicity of $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$, one cannot deal with irrelevance. For instance, one cannot derive from KB= $\{ \mathit{VIP} \sqsubseteq \mathit{Person}, \mathbf{T}(\mathit{Person}) \sqsubseteq\ \leq 1\ \mathit{HasMarried.Person}, \mathbf{T}(\mathit{VIP}) \sqsubseteq\ \geq 2\ \mathit{HasMarried.Person}\}$ that KB $\models_{\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}}$ $\mathbf{T}(\mathit{VIP} \sqcap \mathit{Tall}) \sqsubseteq\ \geq\ 2\ \mathit{HasMarried.Person}$, even if the property of being tall is irrelevant with respect to the number of marriages.

In order to overcome this weakness, we strengthen the semantics of $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ by defining a minimal models mechanism which is similar, in spirit, to circumscription. Given a KB, the idea is to: 1. define a *preference relation* among $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ models, giving preference to the model in which domain elements have a lower rank; 2. restrict entailment to *minimal* $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ models (w.r.t. the above preference relation) of KB.

Definition 3 (Minimal models). *Given $\mathcal{M} = \langle \Delta, <, I \rangle$ and $\mathcal{M}' = \langle \Delta', <', I' \rangle$, $\mathcal{M}$ is preferred to $\mathcal{M}'$ ($\mathcal{M} <_{FIMS} \mathcal{M}'$) if (i) $\Delta = \Delta'$, (ii) $C^I = C^{I'}$ for all concepts C, and (iii) for all $x \in \Delta$, $k_{\mathcal{M}}(x) \leq k_{\mathcal{M}'}(x)$ whereas there is $y \in \Delta$ s.t. $k_{\mathcal{M}}(y) < k_{\mathcal{M}'}(y)$. Given a KB, we say that $\mathcal{M}$ is a minimal model of KB w.r.t. $<_{FIMS}$ if it is a model satisfying KB and there is no $\mathcal{M}'$ model satisfying KB s.t. $\mathcal{M}' <_{FIMS} \mathcal{M}$.*

Differently from [15], the notion of minimality here is based on the minimization of the ranks of the worlds, rather then on the minimization of formulas of a specific kind. It can be proved that a consistent KB has at least one minimal model and that satisfiability in $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ is in EXPTIME such as satisfiability in $\mathcal{SHIQ}$.

The logic $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$, as well as the underlying logic $\mathcal{SHIQ}$, does not enjoy the finite model property. However, we can prove that in any minimal model the *rank* of each domain element is finite, which is essential for establishing a correspondence between the minimal model semantics of a KB and its rational closure. From now on, we can assume that the ranking function assigns to each domain element in Δ a natural number.

3 Rational Closure for $\mathcal{SHIQ}$

In this section, we extend to Description Logics the notion of rational closure proposed by Lehmann and Magidor [21] for the propositional case. Given the typicality operator, the typicality assertion $\mathbf{T}(C) \sqsubseteq D$ plays the role of the conditional assertion $C \mathrel{|\!\sim} D$ in Lehmann and Magidor's rational logic R.

Definition 4 (Exceptionality of concepts and inclusions). *Let T_B be a TBox and C a concept. C is said to be* exceptional *for T_B if and only if $T_B \models_{\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}} \mathbf{T}(\top) \sqsubseteq \neg C$. A $\mathbf{T}$-inclusion $\mathbf{T}(C) \sqsubseteq D$ is exceptional for T_B if C is exceptional for T_B. The set of $\mathbf{T}$-inclusions of T_B which are exceptional in T_B will be denoted as $\mathcal{E}(T_B)$.*

Given a DL KB=(TBox,ABox), it is possible to define a sequence of non increasing subsets of TBox $E_0 \supseteq E_1 \supseteq E_2 \supseteq \ldots$ by letting $E_0 =$ TBox and, for $i > 0$, $E_i = \mathcal{E}(E_{i-1}) \cup \{C \sqsubseteq D \in$ TBox s.t. $\mathbf{T}$ does not occurr in $C\}$. Observe that, being KB finite, there is an $n \geq 0$ such that, for all $m > n, E_m = E_n$ or $E_m = \emptyset$. The definition of the E_i's is similar the definition of the C_i's in Lehmann and Magidor's rational closure [21] except for the addition of strict inclusions.

Definition 5 (Rank of a concept). *A concept C has* rank *i ($\mathit{rank}(C) = i$) for KB=(TBox, ABox), iff i is the least natural number for which C is not exceptional for E_i. If C is exceptional for all E_i then $\mathit{rank}(C) = \infty$, and we say that C has no rank.*

The notion of rank of a formula allows us to define the rational closure of the TBox of a KB. We write KB $\models_{\mathcal{SHIQ}} F$ to mean that F holds in all models of $\mathcal{SHIQ}$.

Definition 6 (Rational closure of TBox). *Let KB=(TBox,ABox). We define, $\overline{TBox}$, the* rational closure of TBox, *as $\overline{TBox} = \{\mathbf{T}(C) \sqsubseteq D \mid$ either $\mathit{rank}(C) < \mathit{rank}(C \sqcap \neg D)$ or $\mathit{rank}(C) = \infty\} \cup \{C \sqsubseteq D \mid$ KB $\models_{\mathcal{SHIQ}} C \sqsubseteq D\}$.*

The rational closure of TBox is a nonmonotonic strengthening of $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ which allows us to deal with irrelevance, as the following example shows. Let TBox = $\{\mathbf{T}(\mathit{Actor}) \sqsubseteq \mathit{Charming}\}$. It can be verified that $\mathbf{T}(\mathit{Actor} \sqcap \mathit{Comic}) \sqsubseteq \mathit{Charming} \in \overline{TBox}$. This nonmonotonic inference does no longer follow if we discover that indeed comic actors are not charming (and in this respect are untypical actors): indeed given TBox$'$= TBox $\cup$ $\{\mathbf{T}(\mathit{Actor} \sqcap \mathit{Comic}) \sqsubseteq \neg \mathit{Charming}\}$, we have that $\mathbf{T}(\mathit{Actor} \sqcap \mathit{Comic}) \sqsubseteq \mathit{Charming} \not\in \overline{TBox'}$. Also, as for the propositional case, rational closure is closed under rational monotonicity: from $\mathbf{T}(\mathit{Actor}) \sqsubseteq \mathit{Charming} \in \overline{TBox}$ and $\mathbf{T}(\mathit{Actor}) \sqsubseteq \mathit{Bold} \not\in \overline{TBox}$ it follows that $\mathbf{T}(\mathit{Actor} \sqcap \neg \mathit{Bold}) \sqsubseteq \mathit{Charming} \in \overline{TBox}$.

Theorem 1 (Complexity of rational closure over TBox). *Given a TBox, the problem of deciding whether $\mathbf{T}(C) \sqsubseteq D \in \overline{TBox}$ is in* EXPTIME.

The proof of this result in [17] shows that the rational closure of a TBox can be computed using entailment in $\mathcal{SHIQ}$, through a linear encoding of $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ entailment. EXPTIME-completeness follows from the EXPTIME-hardness result for $\mathcal{SHIQ}$ [18].

4 A Minimal Model Semantics for Rational Closure in $\mathcal{SHIQ}$

To provide a semantic characterization of this notion, we define a special class of minimal models, exploiting the fact that in all minimal $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ models the *rank* of each domain element is always finite. First of all, we observe that the minimal model semantics in Definition 3 as it is cannot capture the rational closure of a TBox.

Consider the TBox containing: $VIP \sqsubseteq Person$, $\mathbf{T}(Person) \sqsubseteq \leq 1\ HasMarried.Person$, $\mathbf{T}(VIP) \sqsubseteq \geq 2\ HasMarried.Person$. We observe that $\mathbf{T}(VIP \sqcap Tall) \sqsubseteq \geq 2\ HasMarried.Person$ does not hold in all minimal $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ models of KB w.r.t. Definition 3. Indeed there can be a model $\mathcal{M} = \langle \Delta, <, I \rangle$ in which $\Delta = \{x, y, z\}$, $VIP^I = \{x, y\}$, $Person^I = \{x, y, z\}$, $(\leq 1\ HasMarried.Person)^I = \{x, z\}$, $(\geq 2\ HasMarried.Person)^I = \{y\}$, $Tall^I = \{x\}$, and $z < y < x$. $\mathcal{M}$ is a model of KB, and it is minimal. Also, x is a typical tallVIP in $\mathcal{M}$ and has no more than one spouse, therefore $\mathbf{T}(VIP \sqcap Tall) \sqsubseteq \geq 2\ HasMarried.Person$ does not hold in $\mathcal{M}$. On the contrary, it can be verified that $\mathbf{T}(VIP \sqcap Tall) \sqsubseteq \geq 2\ HasMarried.Person \in \overline{TBox}$.

Things change if we consider the minimal models semantics applied to models that contain a domain element for *each combination of concepts consistent with KB*. We call these models *canonical models*. Let $\mathcal{S}$ be the set of all the concepts (and subconcepts) occurring in KB or in the query F together with their complements.

Definition 7 (Canonical model). *Given KB=(TBox,ABox) and a query F, a model $\mathcal{M} = \langle \Delta, <, I \rangle$ satisfying KB is* canonical with respect to $\mathcal{S}$ *if it contains at least a domain element $x \in \Delta$ s.t. $x \in (C_1 \sqcap C_2 \sqcap \cdots \sqcap C_n)^I$, for each set of concepts $\{C_1, C_2, \ldots, C_n\} \subseteq \mathcal{S}$ consistent with KB, i.e. KB $\not\models_{\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}} C_1 \sqcap C_2 \sqcap \cdots \sqcap C_n \sqsubseteq \bot$.*

In order to semantically characterize the rational closure of a $\mathcal{SHIQ}^{\mathsf{R}}\mathbf{T}$ KB, we restrict our attention to *minimal canonical models*. Existence of minimal canonical models can be proved for any (finite) satisfiable KB. Let us first introduce the following proposition, which defines a correspondence between the rank of a formula in the rational closure and the rank of a formula in a model (the proof is by induction on the rank i):

Proposition 2. *Given KB and $\mathcal{S}$, for all $C \in \mathcal{S}$, if $rank(C) = i$, then: 1. there is a $\{C_1 \ldots C_n\} \subseteq \mathcal{S}$ maximal and consistent with KB such that $C \in \{C_1 \ldots C_n\}$ and $rank(C_1 \sqcap \cdots \sqcap C_n) = i$; 2. for any $\mathcal{M}$ minimal canonical model of KB, $k_{\mathcal{M}}(C) = i$.*

The following theorem follows from the propositions above:

Theorem 2. *Let KB=(TBox,ABox) be a knowledge base and $C \sqsubseteq D$ a query. We have that $C \sqsubseteq D \in \overline{TBox}$ if and only if $C \sqsubseteq D$ holds in all minimal canonical models of KB with respect to $\mathcal{S}$.*

5 Conclusions and Related Work

In this work we have proposed an extension of the rational closure defined by Lehmann and Magidor to the Description Logic $\mathcal{SHIQ}$, taking into account both TBox reasoning (ABox reasoning is addressed in [17]). There is a number of closely related proposals.

In [13, 15] nonmonotonic extensions of $\mathcal{ALC}$ with the typicality operator $\mathbf{T}$ have been proposed, whose semantics of $\mathbf{T}$ is based on the preferential logic P. The notion of minimal model adopted here is completely independent from the language and is determined only by the relational structure of models.

The first notion of rational closure for DLs was defined by Casini and Straccia in [5], based on the construction proposed by Freund [12] for propositional logic. In [6] a semantic characterization of a variant of the rational closure in [5] has been presented, generalizing to $\mathcal{ALC}$ the notion of minimally ranked models for propositional logic in [14]. Experimental results in [7] show that, from a performance perspective, it is practical to use rational closure as defined in [6]. The major difference of our construction with those is [5, 6] is in the notion of exceptionality: our definition exploits preferential entailment, while [5, 6] directly use entailment in $\mathcal{ALC}$ over a materialization of the KB. In [17] we have shown that our notion of rational closure for the TBox can nevertheless be computed in $\mathcal{SHIQ}$ by exploiting a linear encoding in $\mathcal{SHIQ}$.

The rational closure construction in itself can be applied to any description logic. As future work, we aim to extend it and its semantic characterization to stronger logics, such as $\mathcal{SHOIQ}$, for which the correspondence between the rational closure and the minimal canonical model semantics of the previous sections cannot be established straightforwardly, due to the interaction of nominals with number restrictions. Also, we aim to consider a finer semantics where models are equipped with several preference relations; in such a semantics it might be possible to relativize the notion of typicality, whence to reason about typical properties independently from each other. The aim is to overcome some limitations of rational closure, as done in [8] by combining rational closure and *Defeasible Inheritance Networks* or in [9] with the lexicographic closure.

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