On the topology of the inflaton field in minimal supergravity models

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ABSTRACT: We consider global issues in minimal supergravity models where a single field inflaton potential emerges. In a particular case we reproduce the Starobinsky model and its description dual to a certain formulation of $R + R^2$ supergravity. For definiteness we confine our analysis to spaces at constant curvature, either vanishing or negative. Five distinct models arise, two flat models with respectively a quadratic and a quartic potential and three based on the $\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)}$ space where its distinct isometries, elliptic, hyperbolic and parabolic are gauged. Fayet-Iliopoulos terms are introduced in a geometric way and they turn out to be a crucial ingredient in order to describe the de Sitter inflationary phase of the Starobinsky model.

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Contents

1 Introduction

The accurate results on the CMB power spectrum collected firstly by the WMAP mission and more recently by the PLANCK satellite [1–3] have boosted a new wave of research activities on the theoretical modelling of the inflationary paradigma and seem to favour the scenario based on a single scalar field ϕ (the inflaton) with a suitable potential $V(\phi)$. In the notations adopted in the present paper, the Friedman equations that govern the time evolution of the scale factor $a(t)$ and of the inflaton $\phi(t)$ are written as follows:

$$H^2 = \frac{1}{3}\,\dot{\phi}^2 + \frac{2}{3}\,V(\phi)\,; \quad \dot{H} = -\dot{\phi}^2\,; \quad \ddot{\phi} + 3\,H\,\dot{\phi} + V' = 0 \qquad (1.1)$$

where $H(t) \equiv \frac{\dot{a}(t)}{a(t)}$ is the Hubble function. Equations (2.1) and (2.2) of the recent review [4] of inflationary models coincide with eq.s (1.1) if one chooses the convention $2\,M_{Pl}^2 = 1$. This

observation, together with the statement that the kinetic term of the inflaton is canonical in our lagrangian:

$$\mathcal{L} = \cdots + \frac{1}{2}\,\partial_\mu\phi\partial^\mu\phi + \ldots \tag{1.2}$$

fixes completely all normalizations and allows the comparison of the results we shall present here with any other result in the vast literature on inflation.

After the publication of PLANCK data, the issue whether one inflaton cosmological models with realistic potentials could be embedded into $\mathcal{N} = 1$ supergravity in a minimal way was addressed and resolved in a series of recent papers [5–7]. Any inflation model based on a positive definitive potential can be embedded into $\mathcal{N} = 1$ supergravity, coupled to a single Wess-Zumino multiplet and one massless vector multiplet, which may combine together in a massive vector multiplet with lagrangian specified by a single real function J(C) as shown in [8]. The vector multiplet is utilized to gauge an isometry of the one-dimensional Hodge-Kähler manifold Σ associated with the WZ multiplet. The catch of the method is the supergravity formulation of the Higgs mechanism. The gauging introduces a D-term and the definite potential V is the square of the momentum-map of the Killing vector k^3 that generates the gauged isometry of Σ. One of the two scalar fields of the WZ multiplet, conventionally named B, is eaten up by the vector field $\mathcal{A}_\mu$ which becomes massive and the other one C, after a field transformation $C \to \phi(C)$ that reduces it to have the canonical form (1.2), becomes the inflaton. We name $C(\phi)$ the Van Proeyen coordinate on the Kähler manifold Σ_K since it corresponds to the scalar field in terms of which the $\mathcal{N} = 1$ supergravity lagrangian that turns out to be the minimal one for inflationary models was firstly written by Van Proeyen in [8]. In the sequel we shall emphasize the intrinsic geometrical meaning of the VP coordinate C. The minimal models for inflation of [5] were suggested by the supergravity completion [9] of the $R + R^2$ Starobinski model which, as we will further discuss hereby, corresponds manifold Σ of constant curvature $\nu^2 = \frac{4}{3}$ (see eq. (4.21)) with gauged shift symmetry and and a non vanishing Fayet Iliopoulos term.[1]

In the same period, following work on the phenomenon of climbing scalars [10–12], another series of papers [13, 14] addressed the issue of integrability of the two field[2] dynamical system encoded in the Friedman equations (1.1) and provided a list of 28 integrable potentials $V(\phi)$ in the sense that they provide integrabity of eq.s (1.1). The question if any of these integrable $V(\phi)$ can be embedded into gauged extended supergravity was discussed in [14] and remains partially open, yet as a consequence of the results of [5], the embedding of all positive definite ones among them into $\mathcal{N} = 1$ minimal supergravity is guaranteed and deserves careful considerations for the possibility that this discloses, within a supergravity context of basing Mukhanov-Sasaki equations on exact analytic solutions of the Klein Gordon Einstein system.

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[1]The reader should notice that the field ϕ used in this paper differs from the field $\widetilde{\phi}$ used in many other papers by a factor $\frac{1}{\sqrt{2}}$ as it is evident from the normalization of Friedman equations in eq. (1.1). Correspondingly the exponential $\exp\left[\widetilde{\nu}\widetilde{\phi}\right]$ in other paper normalizations is $\exp\left[\sqrt{2}\widetilde{\nu}\phi\right]$ in our present normalizations. In this way the Starobinsky potential corresponds to $\nu^2 = 4/3$ in our normalizations if it corresponds to $\nu^2 = 2/3$ in other paper normalizations.

[2]The two fields are the scale factor $a(t)$ and the inflaton $\phi(t)$.

Independently from integrability, the geometrical basis of the construction of minimal supergravity models of inflation introduced in [5] was analyzed in another pair of recent papers [15, 16]. It was pointed out that the root from any given positive definite potential $V(\phi)$ to the corresponding minimal supergravity model is a map, named by two of us the D-map, in whose image there is a two-dimensional Kähler surface Σ admitting at least a one-dimensional group of isometries $\mathcal{G}$. Various aspects of this map were explored, but a fundamental question remained so far unanswered about the global topology both of the surface Σ and of its isometry group. This is by no means a marginal issue. Indeed, as we are going to show here, the physical properties and the symmetries of the minimal supergravity lagrangian are significantly different in the two cases of a compact isometry group $\mathcal{G} = \mathrm{U}(1)$ and of a non compact one $\mathcal{G} = \mathrm{SO}(1,1)$ or $\mathcal{G} = \mathbb{R}$. Furthermore the asymptotic behavior of the real function $J(C)$ that defines both the potential and the kinetic terms of the scalars, is distinct in the case of compact and non compact symmetries and actually provides a clue to identify the appropriate global topology. In this paper we exemplify these concepts by classifying all minimal supergravity models and corresponding inflaton potentials that are associated with a Kähler surface Σ of constant curvature R_Σ. We obtain a total of five models, each still depending on one or two parameters, that are associated with $R_\Sigma = 0$ (flat models) and with $R_\Sigma = -4\,\nu^2$. In the latter case the corresponding manifold is always $\Sigma = \mathrm{SL}(2,\mathbb{R})/\mathrm{O}(2)$, but by gauging elliptic, hyperbolic or parabolic subgroups we obtain different families of potentials. The Starobinsky type of potentials [17] are obtained from the parabolic subgroups. Various discussions of Starobinsky potentials and other inflaton potentials within supergravity have been advanced in other papers [18–27].

2 Global structure of the inflaton Kähler surface

As we advocated in the introduction, in the minimal $\mathcal{N} = 1$ supergravity realizations of one-scalar cosmologies the central item of the construction is an axial(-shift) symmetric Kähler surface whose metric can be written as follows:

$$ds_\Sigma^2 = p(U)\,dU^2 + q(U)\,dB^2 \tag{2.1}$$

$p(U), q(U)$ being two positive definite functions of their argument. The manifold Σ is an axial(-shift) symmetric surface, since the metric (2.1) admits the Killing vector $\vec{k}_{[B]} = \partial_B$. This isometry is fundamental since it is by means of its gauging that one produces a D-type positive definite scalar potential that can encode the inflaton dynamics. At the level of the supergravity model that is built by using the Kähler space Σ as the target manifold where the two scalar fields of the inflatonic Wess Zumino multiplet take values, a fundamental question is whether $\vec{k}_{[B]}$ generates a *compact rotation symmetry* or a *non compact shift symmetry*. Indeed the supergravity lagrangian in general and its fermionic sector in particular, display quite different features in the two cases, leading to a different pattern of physical charges and symmetries. Actually, as we are going to illustrate below, when $\Sigma = \Sigma_{\mathrm{max}}$ is a constant curvature surface namely the coset manifold $\frac{\mathrm{SU}(1,1)}{\mathrm{U}(1)} \sim \frac{\mathrm{SL}(2,\mathbb{R})}{\mathrm{O}(2)}$, there is also a third possibility. In such a situation the killing vector $\vec{k}_{[B]}$ can

be the generator of a *dilatation*, namely it can correspond to a non-compact but semi-simple element $\mathbf{d} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ of the Lie algebra $\mathrm{SL}(2,\mathbb{R})$ rather then to a nilpotent one $\mathbf{t} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$. In the ambient algebra $\mathfrak{g} = \mathrm{SL}(2,\mathbb{R})$ this distinction makes sense since by means of internal transformations (conjugations $\mathbf{d}' = \exp[-\mathbf{u}]\,\mathbf{d}\,\exp[\mathbf{u}]$ with $\mathbf{u} \in \mathfrak{g}$) we cannot map $\mathbf{d}$ into $\mathbf{t}$. The main question therefore concerns the global topology of such a group. Is it compact $\mathcal{G} \sim \mathrm{U}(1)$ or is it non-compact $\mathcal{G} \sim \mathbb{R}$? As already advocated, in the two cases the structure of the $\mathcal{N} = 1$ supergravity lagrangian is different and its local and global symmetries are different.

As it was explained in [16], the standard presentation of the geometry of Σ in terms of a complex coordinate and of a Kähler potential is obtained by means of a few standard steps. First one singles out the unique complex structure with vanishing Nienhuis tensor with respect to which the metric is hermitian:

$$\mathfrak{J}_\alpha^\beta \, \mathfrak{J}_\beta^\gamma = -\delta_\alpha^\gamma\,; \quad \partial_{[\alpha}\,\mathfrak{J}_{\beta]}^\gamma - \mathfrak{J}_\alpha^\mu\,\mathfrak{J}_\beta^\nu\,\partial_{[\mu}\,\mathfrak{J}_{\nu]}^\gamma = 0\,; \quad g_{\alpha\beta} = \mathfrak{J}_\alpha^\gamma\,\mathfrak{J}_\beta^\delta\,g_{\gamma\delta}\,. \tag{2.2}$$

In terms of the metric coefficients, such a complex structure is given by the following tensor $\mathfrak{J}$ and leads to the following closed Kähler 2-form K:

$$\mathfrak{J} = \begin{pmatrix} 0 & \sqrt{\dfrac{p(U)}{q(U)}} \\ -\sqrt{\dfrac{q(U)}{p(U)}} & 0 \end{pmatrix} \Rightarrow \mathrm{K} = g_{\alpha\mu}\,\mathfrak{J}_\beta^\mu\,dx^\alpha \wedge dx^\beta = -\sqrt{p(U)\,q(U)}\,dU \wedge dB\,. \tag{2.3}$$

Next one aims at reproducing the Kählerian metric (2.1) in terms of a complex coordinate $\mathfrak{z} = \mathfrak{z}(U, B)$ and a Kähler potential $\mathcal{K}(\mathfrak{z}\,,\,\bar{\mathfrak{z}}) = \mathcal{K}^\star(\mathfrak{z}\,,\,\bar{\mathfrak{z}})$ such that:

$$\mathrm{K} = \partial\,\overline{\partial}\,\mathcal{K} = \mathrm{i}\partial_\mathfrak{z}\,\partial_{\bar{\mathfrak{z}}}\,\mathcal{K}\,d\mathfrak{z} \wedge d\bar{\mathfrak{z}}\,; \qquad ds_\Sigma^2 = \partial_\mathfrak{z}\,\partial_{\bar{\mathfrak{z}}}\,\mathcal{K}\,d\mathfrak{z} \otimes d\bar{\mathfrak{z}}\,. \tag{2.4}$$

As explained in [16] the complex coordinate $\mathfrak{z}$ is necessarily a solution of the complex structure equation:

$$\mathfrak{J}_\alpha^\beta\,\partial_\beta\,\mathfrak{z} = \mathrm{i}\partial_\alpha\,\mathfrak{z} \Rightarrow \sqrt{\frac{p(U)}{q(U)}}\,\partial_B\,\mathfrak{z}(U, B) = \mathrm{i}\,\partial_U\,\mathfrak{z}(U, B)\,. \tag{2.5}$$

The general solution of such an equation is easily found. Define the linear combination:[3]

$$w \equiv \mathrm{i}\,C(U) - B\,; \quad C(U) = \int \sqrt{\frac{p(U)}{q(U)}}\,dU\,, \tag{2.6}$$

and consider any holomorphic function $f(w)$. As one can immediately verify, the position:

$$\mathfrak{z}(U, B) = f(w) \tag{2.7}$$

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[3]As it follows from the present discussion the Van Proeyen coordinate $C(U)$ has an intrinsic geometric characterization as that one which solves the differential equation of the complex structure.

solves eq. (2.5). What is the appropriate choice of the holomorphic function $f(w)$? Locally (in an open neighborhood) this is an empty question, since the holomorphic function $f(t)$ simply corresponds to a change of coordinates and gives rise to the same Kähler metric in a different basis. Suppose we have selected a particular function $f(w)$ and setting $\mathfrak{z} = f(w)$ we have found a Kähler function $\mathcal{K}(\mathfrak{z}, \bar{\mathfrak{z}})$ such that:

$$ds^2 \equiv \partial_{\mathfrak{z}} \, \partial_{\bar{\mathfrak{z}}} \, \mathcal{K} \, d\mathfrak{z} \, d\bar{\mathfrak{z}} = p(U) \, dU^2 + q(U) \, dB^2 \tag{2.8}$$

then, by performing the holomorphic transformation $\hat{\mathfrak{z}} = g(\mathfrak{z}) = g\left(f(w)\right) = \hat{f}(w)$ we obtain a locally equivalent presentation of the same metric. Writing $\hat{\mathcal{K}}(\hat{\mathfrak{z}}, \hat{\bar{\mathfrak{z}}}) = \mathcal{K}\left(g^{-1}(\hat{\mathfrak{z}}), \bar{g}^{-1}(\hat{\bar{\mathfrak{z}}})\right)$ we obviously get:

$$ds^2 \equiv \partial_{\hat{\mathfrak{z}}} \, \partial_{\hat{\bar{\mathfrak{z}}}} \, \hat{\mathcal{K}} \, d\hat{\mathfrak{z}} \, d\hat{\bar{\mathfrak{z}}} = p(U) \, dU^2 + q(U) \, dB^2 \, . \tag{2.9}$$

Globally, however, there are significant restrictions that concern the range of the variables B and $C(U)$, namely the global topology of the manifold Σ. By definition B is the coordinate that, within Σ, parameterizes points along the $\mathcal{G}$-orbits. If $\mathcal{G}$ is compact, then B is a coordinate on the circle and it must be defined up to identifications $B \simeq B + 2 \, n \, \pi$, where n is an integer. On the other hand if B is non compact its range extends on the full real line $\mathbb{R}$. Furthermore, in order to obtain a presentation of the Kähler geometry of Σ that allows to single out a *canonical inflaton* field ϕ with a potential $V(\phi)$ we aim at a Kähler potential $\mathcal{K}(\mathfrak{z}, \bar{\mathfrak{z}})$ that in terms of the variables $C(U)$ and B should actually depend only on C being constant on the $\mathcal{G}$-orbits. Starting from the metric (2.1) we can always choose a canonical variable ϕ defined by the position:

$$\phi = \phi(U) = \int 2 \, \sqrt{p(U)} \, dU \, ; \qquad \frac{1}{2} \, d\phi = \sqrt{p(U)} \, dU \tag{2.10}$$

and assuming that $\phi(U)$ can be inverted $U = \mathrm{U}(\phi)$ we can rewrite (2.1) in the following canonical form:

$$ds^2_{\mathrm{can}} = \frac{1}{4} \left[d\phi^2 + \left(\mathcal{P}'(\phi)\right)^2 \, dB^2 \right] ; \quad \mathcal{P}'(\phi) = 2 \, \sqrt{q\left(\mathrm{U}(\phi)\right)}; \quad \underbrace{\sqrt{p(\mathrm{U}(\phi))} \, \frac{dU}{d\phi} = \frac{1}{2}}_{\text{by construction}} . \tag{2.11}$$

The reason to call the square root of $q\left(\mathrm{U}(\phi)\right)$ with the name $\mathcal{P}'(\phi)$ is the interpretation of such a function as the derivative with respect to the canonical variable ϕ of the momentum map of the Killing vector $\vec{k}_{[B]}$. As it was pointed out in [16] such interpretation is crucial for the construction of the corresponding supergravity model but it is intrinsic to the geometry of the surface Σ.

According to an analysis first introduced in section 4 of [5], by using the canonical variable ϕ, the VP coordinate C defined in equation (2.6) becomes:

$$C(\phi) = C\left(\mathrm{U}(\phi)\right) = \int \frac{d\phi}{\mathcal{P}'(\phi)} \tag{2.12}$$

and the metric $ds^2_\Sigma = ds^2_{\mathrm{can}}$ of the Kähler surface Σ can be rewritten as:

$$ds^2_\Sigma = \frac{1}{4} \frac{\mathrm{d}^2 J}{\mathrm{d}C^2} \left(\mathrm{d}C^2 + \mathrm{d}B^2 \right) \tag{2.13}$$

where the function $J(C)$ is defined as follows:[4]

$$\mathcal{J}(\phi) \equiv \int \frac{\mathcal{P}(\phi)}{\mathcal{P}'(\phi)} \, d\phi \quad ; \quad J(C) \equiv \mathcal{J}\left(\phi(C)\right). \tag{2.14}$$

It appears from the above formula that the crucial step in working out the analytic form of the function $J(C)$ is the ability of inverting the relation between the VP coordinate C, defined by the integral (2.12), and the canonical one ϕ, a task which, in the general case, is quite hard in both directions. The indefinite integral (2.12) can be expressed in terms of special functions only in certain cases and even less frequently one has at his own disposal inverse functions. Yet this is only a technical difficulty. Conceptually, eq.s (2.14) and (2.12) define the function $J(C)$ up to an additive integration constant. The fundamental unanswered question is how to reinterpret eq. (2.13) in terms of a complex coordinate $\mathfrak{z}$ and of a Kähler potential $\mathcal{K}(\mathfrak{z}, \bar{\mathfrak{z}})$. Having already established in eq. (2.6) the general solution of the complex structure equations there are three possibilities that correspond, in the case of constant curvature manifolds Σ_{max} to the three conjugacy classes of $\text{SL}(2, \mathbb{R})$ elements (elliptic, hyperbolic and parabolic). In the three cases $J(C)$ is identified with the Kähler potential $\mathcal{K}(\mathfrak{z}, \bar{\mathfrak{z}})$, but it remains to be decided whether the VP coordinate C is to be identified with the imaginary part of the complex coordinate $C = \text{Im}\,\mathfrak{z}$, with the logarithm of its modulus $C = \frac{1}{2} \log |\mathfrak{z}|^2$, or with a third combination of $\mathfrak{z}$ and $\bar{\mathfrak{z}}$, namely whether we choose the first the second or the third of the options listed below:

$$\mathfrak{z} = \begin{cases} \zeta \equiv \exp\left[-\mathrm{i}\,w\right] & = \underbrace{\exp\left[C(\phi)\right]}_{\rho(\phi)} \exp\left[\mathrm{i}B\right] \\ t \equiv w & = \mathrm{i}\,C(\phi) - B \\ \hat{\zeta} \equiv \mathrm{i} \tanh\left(-\frac{1}{2}\,w\right) = \mathrm{i} \tanh\left(-\frac{1}{2}\left(\mathrm{i}\,C(\phi) - B\right)\right) \end{cases} \quad\Bigg| \quad C(\phi) \equiv \int \frac{1}{\mathcal{P}'(\phi)} \, d\phi. \tag{2.15}$$

If we choose the first solution $\mathfrak{z} = \zeta$, that in [16] was named of *Disk-type*, we obtain that the basic isometry generated by the Killing vector $\vec{k}_{[B]}$ is a compact rotation symmetry and this implies a series of consequences on the supergravity lagrangian and its symmetries that we discuss below. Choosing the second solution $\mathfrak{z} = t$, that was named of *Plane-type* in [16], is appropriate instead to the case of a non compact shift symmetry and leads to different symmetries of the supergravity lagrangian. The third possibility mentioned above which occurs in the case of constant curvature surfaces Σ_{max} and leads to the interpretation of the B-shift as an $\text{SO}(1,1)$-hyperbolic transformation.

In the three cases the analytic form of the holomorphic Killing vector $\vec{k}_{[B]}$ is quite different:

$$\vec{k}_{[B]} = \begin{cases} \mathrm{i}\zeta\,\partial_\zeta & \equiv k^{\mathfrak{z}}\partial_{\mathfrak{z}} \Rightarrow k^{\mathfrak{z}} = \mathrm{i}\,\mathfrak{z} & ; \text{ Disk-type, compact rotation} \\ \partial_t & \equiv k^{\mathfrak{z}}\partial_{\mathfrak{z}} \Rightarrow k^{\mathfrak{z}} = 1 & ; \text{ Plane-type, non-compact shift} \\ \mathrm{i}\left(1 + \hat{\zeta}^2\right)\partial_{\hat{\zeta}} \equiv k^{\mathfrak{z}}\partial_{\mathfrak{z}} \Rightarrow k^{\mathfrak{z}} = \mathrm{i}\left(1 + \mathfrak{z}^2\right) & ; \text{ Disk-type, non-compact dilatation.} \end{cases} \tag{2.16}$$

This has important consequences on the structure of the momentum map leading to the D-type scalar potential and on the transformation properties of the fermions.

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[4]See [16] for more details.

Before proceeding further let us stress once again that the choice of one or the other solution of the complex structure equation, that give to the foliations of Σ into $\mathcal{G}$-orbits a different topology, depends on the global structure of the manifold Σ, whose metric we wrote in eq. (2.1). If we know a priori such a structure from an intrinsic definition of Σ which arizes from other informations, than we know which complex structure is appropriate. Otherwise, choosing the complex structure amounts to the same as introducing one half of the missing information on the global structure of Σ, namely the range of the coordinate B. The other half is the range of the coordinate U. Actually as we shall emphasize by means of the constant curvature examples that we are going to consider a criterion able to discriminate the relevant topologies is encoded in the asymptotic behavior of the function $\partial_C^2 J(C)$ for large and small values of its argument, namely in the center of the bulk and on the boundary.

2.1 The Hodge bundle and Fayet-Iliopoulos term

Let us recall that relevant to supergravity is not only the Kähler structure of the surface Σ rather the full-fledged geometry of the Hodge bundle constructed over it. Our surface Σ is supposed to be Hodge-Kähler and this implies that there exists a line bundle $\mathcal{L} \to \Sigma$ whose Chern class coincides with the Kähler class, namely with the cohomology class of the Kähler two-form $\mathrm{K} = \mathrm{i}\, g_{\mathfrak{z}\bar{\mathfrak{z}}}\, d\mathfrak{z} \wedge d\bar{\mathfrak{z}}$. Explicitly we must have $c_1(L) = [\mathrm{K}]$, where the bracket denotes the cohomology class of the closed p-form embraced by it. The holomorphic sections of this line bundle are the possible superpotentials that encode the self interactions of the Wess-Zumino multiplet and its coupling to supergravity. The exponential of the Kähler potential is a fiber metric on the Hodge bundle: for any holomorphic section $W(\mathfrak{z})$ of such a bundle we define an invariant norm by means of the following position $||W||^2 = \exp[\mathcal{K}]\, W(\mathfrak{z})\, \overline{W}(\bar{\mathfrak{z}})$. A fundamental object entering the construction of matter coupled supergravity is the logarithm of the superpotential norm $G(\mathfrak{z}, \bar{\mathfrak{z}}) = \log ||W||^2 = \mathcal{K} + \log W + \log \overline{W}$. In the present paper, however, we do not consider this type of self-interactions and we put the superpotential to zero $W(z) = 0$ so that we will just work with the Kähler potential $\mathcal{K}(\mathfrak{z}, \bar{\mathfrak{z}})$. Another fundamental ingredient in the matter coupling construction and in its gauging is provided by the prepotentials of the holomorphic Killing vectors. Following the discussion and the conventions of [35], if $k^{\mathfrak{z}}(\mathfrak{z})$, together with its complex conjugate $k^{\bar{\mathfrak{z}}}(\bar{\mathfrak{z}})$, is a holomorphic Killing vector, in the sense that the transformation:

$$\mathfrak{z} \to z + \epsilon\, k^{\mathfrak{z}}(\mathfrak{z}) \qquad (2.17)$$

is an infinitesimal isometry of the Kähler metric for all choices of the small parameters ϵ, then the prepotential of this Killing vector, which realizes the corresponding isometry as a Lie-Poisson flux on the Kähler manifold, is the real function $\mathcal{P}(\mathfrak{z}, \bar{\mathfrak{z}}) = \mathcal{P}(\mathfrak{z}, \bar{\mathfrak{z}})^{\star}$ defined by the following relations:

$$k^{\mathfrak{z}}(\mathfrak{z}) = \mathrm{i}\, g^{\mathfrak{z}\bar{\mathfrak{z}}}\, \partial_{\bar{\mathfrak{z}}}\, \mathcal{P}\,; \qquad k^{\bar{\mathfrak{z}}}(\bar{\mathfrak{z}}) = -\mathrm{i}\, g^{\mathfrak{z}\bar{\mathfrak{z}}}\, \partial_{\mathfrak{z}}\, \mathcal{P}\,. \qquad (2.18)$$

In terms of the Kähler potential $\mathcal{K}$ function, supposedly invariant under the considered isometries, the Killing vector prepotential, satisfying the defining condition (2.18), is con-

structed through the following formula:

$$\mathcal{P} = \mathcal{P}_0 = \mathrm{i}\,\frac{1}{2}\,\left(k^{\mathfrak{z}}\,\partial_{\mathfrak{z}}\,\mathcal{K} - k^{\bar{\mathfrak{z}}}\,\partial_{\bar{\mathfrak{z}}}\,\mathcal{K}\right). \tag{2.19}$$

It should be noted that the solution (2.19) of eq. (2.18) is defined up to an integration constant. Indeed setting:

$$\mathcal{P} = \mathcal{P}_0 + \frac{\mathrm{q_f}}{g} \tag{2.20}$$

where $\mathrm{q_f}$ is an arbitrary constant and g is the gauge coupling constant equations (2.18) are still satisfied. It was first noted in [28] that the above ambiguity is the mechanism behind the introduction of Fayet Iliopoulos terms into supersymmetric lagrangians [29, 30]. The interpretation of Fayet Iliopoulos terms as constant shifts of the momentum maps was later extended to tri-holomorphic momentum maps and to the $\mathcal{N} = 2$ theories in [32]. It should also be noted that the constant term $\frac{\mathrm{q_f}}{g}$ in the momentum can always be reabsorbed into $\mathcal{P}_0$, defined by eq. (2.19), introducing a new Kähler potential which differs from the first by a holomorphic Kähler transformation uneffective on the metric:

$$\widetilde{\mathcal{K}}\left(\mathfrak{z}, \bar{\mathfrak{z}}\right) = \mathcal{K}\left(\mathfrak{z}, \bar{\mathfrak{z}}\right) + f(\mathfrak{z}) + \bar{f}(\bar{\mathfrak{z}})$$
$$\mathrm{i}\,k^{\mathfrak{z}}\,\partial_{\mathfrak{z}}\,f(\mathfrak{z}) = \frac{\mathrm{q_f}}{g}\,. \tag{2.21}$$

Indeed note the second line of eq. (2.21) is a first order holomorphic differential equation that is always immediately solved by quadratures. Hence the appropriate function $f(\mathfrak{z})$ which produces the Fayet Iliopoulos term depends on the chosen Killing vector but the result on the momentum map is always the same: a constant shift.

Upon gauging the isometry $B \to B + c$, the supergravity Lagrangian acquires a D-type potential proportional to the square of the momentum map $\mathcal{P}(\mathfrak{z}, \bar{\mathfrak{z}})$:

$$V_{YM} \propto g^2\,\left(\mathcal{P}(\mathfrak{z}, \bar{\mathfrak{z}})\right)^2. \tag{2.22}$$

We shall come back to the discussion of such potentials. Before doing that we desire to illustrate some general features of the symmetries of the gauged supergravity lagrangian (particularly the fermionic sector) that heavily depend on the nature of the fundamental B-isometry.

2.2 Sections of the Hodge bundle and the fermions

The basic geometric mechanism that allows to gauge the global symmetries of $\mathcal{N} = 1$ supergravity coupled to Wess Zumino multiplets is the so named gauging of the composite connections. Let us recall such a notion. The isometries of the Kähler metric that take the infinitesimal form (2.17) extend to global symmetries of the full theory, including also the fermions, since all the items appearing in the lagrangian transform covariantly. From the geometrical point of view all fields are sections of the tangent bundle to the Kähler manifold and at the same time they are also sections of appropriate powers of the Hodge bundle. The subtle point is that under a holomorphic isometry: $\mathfrak{z} \to \hat{\mathfrak{z}} = f(\mathfrak{z})$ the Kähler potential does not necessarily remain invariant rather it transform as follows:

$$\mathcal{K}\left(\hat{\mathfrak{z}}, \bar{\hat{\mathfrak{z}}}\right) = \mathcal{K}\left(\mathfrak{z}, \bar{\mathfrak{z}}\right) + F(\mathfrak{z}) + \bar{F}(\bar{\mathfrak{z}}) \tag{2.23}$$

where $F(\mathfrak{z})$ is some holomorphic function associated with the considered transformation. By definition a section $\mathfrak{S}_p(\mathfrak{z})$ of weight p of the Hodge-bundle transforms as follows

$$\mathfrak{S}_p(\hat{\mathfrak{z}}) = \mathfrak{S}_p(\mathfrak{z}) \, \exp[-p \, F(\mathfrak{z})]. \qquad (2.24)$$

The fermion fields, namely the gravitino ψ, the chiralinos $\chi^{\mathfrak{z}}$, $\chi^{\bar{\mathfrak{z}}}$ and the gauginos λ^Λ transform as sections of the Hodge bundle, with half integer weights that we presently spell off.

According to [34], we introduce the following notation for the chiral projections of the gravitino one-form ψ and of the gaugino 0-forms λ^Λ that are Majorana:

$$\psi = \psi_\bullet + \psi^\bullet \, ; \qquad \begin{cases} \gamma_5 \, \psi_\bullet = \psi_\bullet \\ \gamma_5 \, \psi^\bullet = -\,\psi^\bullet \end{cases}$$

$$\lambda = \lambda_\bullet + \lambda^\bullet \, ; \qquad \begin{cases} \gamma_5 \, \lambda_\bullet = \lambda_\bullet \\ \gamma_5 \, \lambda^\bullet = -\,\lambda^\bullet \end{cases} \qquad (2.25)$$

while for the complex chiralino we simply have:

$$\gamma_5 \, \chi^{\mathfrak{z}} = \chi^{\mathfrak{z}} \, ; \quad \gamma_5 \, \chi^{\bar{\mathfrak{z}}} = -\chi^{\bar{\mathfrak{z}}} \, . \qquad (2.26)$$

In what follows we just summarize and specialize to the minimal case of supergravity coupled to one vector multiplet $\left(1, \frac{1}{2}\right)$ and one WZ-multiplet $\left(\frac{1}{2}, 0^+, 0^-\right)$ what was described for the general case in [15]. Having clarified the notation, the appropriate Hodge transformations for the fermions are:

$$\psi_\bullet \rightarrow \exp\left[\mathrm{i}\,\frac{1}{2}\,F(z)\right]\psi_\bullet \, ; \qquad \psi^\bullet \rightarrow \exp\left[-\mathrm{i}\,\frac{1}{2}\,F(z)\right]\psi^\bullet$$

$$\lambda_\bullet \rightarrow \exp\left[\mathrm{i}\,\frac{1}{2}\,F(z)\right]\lambda_\bullet \, ; \qquad \lambda^\bullet \rightarrow \exp\left[-\mathrm{i}\,\frac{1}{2}\,F(z)\right]\lambda^\bullet \qquad (2.27)$$

$$\chi^{\mathfrak{z}} \rightarrow \exp\left[-\mathrm{i}\,\frac{1}{2}\,F(z)\right]\chi^{\mathfrak{z}} \, ; \qquad \chi^{\bar{\mathfrak{z}}} \rightarrow \exp\left[\mathrm{i}\,\frac{1}{2}\,F(z)\right]\chi^{\bar{\mathfrak{z}}} \, .$$

These transformations are compensated by the transformation of the Hodge bundle connection which is the following composite one-form:

$$Q \equiv \mathrm{i}\,\frac{1}{2}\,\left(\partial_{\mathfrak{z}}\mathcal{K}\,d\mathfrak{z} - \partial_{\bar{\mathfrak{z}}}\mathcal{K}\,d\bar{\mathfrak{z}}\right) \qquad (2.28)$$

and enters the covariant derivatives of the fermions. For instance the gravitino and gaugino covariant derivatives are defined as follows:

$$\nabla\psi_\bullet = \mathcal{D}\psi_\bullet + \mathrm{i}\,\frac{1}{2}\,Q \wedge \psi_\bullet \, ; \qquad \mathcal{D}\psi_\bullet = d\psi_\bullet - \frac{1}{4}\,\omega^{ab} \wedge \gamma_{ab}\,\psi_\bullet$$

$$\nabla\lambda_\bullet = \mathcal{D}\lambda_\bullet + \mathrm{i}\,\frac{1}{2}\,Q\,\lambda_\bullet \, ; \qquad \mathcal{D}\lambda_\bullet = d\lambda_\bullet - \frac{1}{4}\,\omega^{ab}\,\gamma_{ab}\,\lambda_\bullet \, . \qquad (2.29)$$

The gravitino one-form and the gaugino zero-forms have no indices along the tangent bundle of the Kähler manifold and therefore do not transform in the canonical bundle. On the other hand the chiralino carries a tangent space index and with respect to the canonical

bundle it transforms as a holomorphic vector. Correspondingly it enters the lagrangian covered by a covariant derivative of the form:

$$\nabla \chi^{\mathfrak{z}} \equiv \mathcal{D}\chi^{\mathfrak{z}} + \Gamma^{\mathfrak{z}}{}_{\mathfrak{z}}\chi^{\mathfrak{z}} - \mathrm{i}\frac{1}{2}\,Q\,\chi^{\mathfrak{z}}\,. \tag{2.30}$$

In this way the isometries of the Kähler manifold are promoted to global symmetry of supergravity coupled, in the case under present consideration to just one vector multiplet.

2.3 Gauging of the composite connections

The basic geometric mechanism that allows to gauge the above described global symmetries is the so named *gauging of the composite connections*. Let us recall such a notion, according to the discussion of [35] and [15]. In [35] the construction was applied to $\mathcal{N} = 2$ supergravity so that the composite connections to be gauged were those emerging in Special Kähler Geometry. Here we focus on $\mathcal{N} = 1$ supergravity and we just have Hodge-Kähler manifolds, yet the procedure is completely identical and it was already introduced in [34], but only for symmetries that are linearly realized on the scalars. In [15] it was smoothly generalized to any type of holomorphic isometry, by means of the prepotential of the Killing vectors. In what follows we specialize the formulae of [15] to the minimal case here under discussion, where there is only one Wess-Zumino multiplet and only one isometry is gauged. The connections to be gauged are two: the Hodge-Kähler connection (2.28) and the Levi-Civita connection:

$$\Gamma^{\mathfrak{z}}{}_{\mathfrak{z}} = \left\{ \begin{matrix} \mathfrak{z} \\ \mathfrak{z}\,\mathfrak{z} \end{matrix} \right\}\, d\mathfrak{z} = g^{\mathfrak{z}\bar{\mathfrak{z}}}\,\partial_{\mathfrak{z}}\,g_{\bar{\mathfrak{z}}\mathfrak{z}}\, d\mathfrak{z}\,; \qquad \Gamma^{\bar{\mathfrak{z}}}{}_{\bar{\mathfrak{z}}} = \text{Complex Conjugate of } \Gamma^{\mathfrak{z}}{}_{\mathfrak{z}}\,. \tag{2.31}$$

We set:

$$Q \to \widehat{Q} \equiv \mathrm{i}\,\frac{1}{2}\,(\partial_{\mathfrak{z}}\mathcal{K}\,\nabla_{\mathfrak{z}} - \partial_{\bar{\mathfrak{z}}}\mathcal{K}\,\nabla_{\bar{\mathfrak{z}}})\,; \qquad \Gamma^{\mathfrak{z}}{}_{\mathfrak{z}} \to \widehat{\Gamma}^{\mathfrak{z}}{}_{\mathfrak{z}} = \left\{ \begin{matrix} \mathfrak{z} \\ \mathfrak{z}\,\mathfrak{z} \end{matrix} \right\}\,\nabla_{\mathfrak{z}} \tag{2.32}$$

where

$$\nabla_{\mathfrak{z}} = d\mathfrak{z} + g\,\mathcal{A}\,k^{\mathfrak{z}}(\mathfrak{z}) \tag{2.33}$$

is the covariant derivative of the complex scalar field, g being the gauge coupling constant, $\mathcal{A} = \mathcal{A}_{\mu}\,dx^{\mu}$ the gauge field one-form and $k^{\mathfrak{z}}(\mathfrak{z})$ the Killing vector. It follows from the various identities presented above that:

$$\widehat{Q} = Q + g\,\mathcal{A}\,\mathcal{P}\,; \qquad \widehat{\Gamma}^{\mathfrak{z}}{}_{\mathfrak{z}} = \Gamma^{\mathfrak{z}}{}_{\mathfrak{z}} + g\,\mathcal{A}\,\partial_{\mathfrak{z}}\,k^{\mathfrak{z}}(\mathfrak{z})\,. \tag{2.34}$$

3 General features and symmetries of the minimal supergravity inflationary model

Without entering into the details of any specific model there is a number of features that immediately follow from the formulae presented above, which can be discussed in general terms and significantly distinguish the two cases of gauging either a rotation or a shift symmetry as basic mechanism for the generation of an inflaton potential. These general properties are in our opinion more important and fundamental than the specific form of the inflaton potential obtained from the gauging.

3.1 Compact U(1) case

If the fundamental isometry of Σ is a compact U(1), the Killing vector is given by the first line of eq. (2.16) and we have:

$$\partial_{\mathfrak{z}}\, k^{\mathfrak{z}} = \mathrm{i} \tag{3.1}$$

so that from equation (2.34) we obtain:

$$\widehat{\Gamma}^{\mathfrak{z}}_{\ \mathfrak{z}} = \Gamma^{\mathfrak{z}}_{\ \mathfrak{z}} + \mathrm{i}\, g\, \mathcal{A}\,. \tag{3.2}$$

Furthermore, given a U(1)-invariant Kähler potential $\widetilde{\mathcal{K}}(\mathfrak{z},\bar{\mathfrak{z}})$, namely such that:

$$(\mathfrak{z}\,\partial_{\mathfrak{z}} - \bar{\mathfrak{z}}\,\partial_{\bar{\mathfrak{z}}})\,\widetilde{\mathcal{K}}(\mathfrak{z},\bar{\mathfrak{z}}) = 0 \tag{3.3}$$

and the form of the Killing vector mentioned in the first line of eq. (2.16), the solution of eq. (2.21) is the following one:

$$f(\mathfrak{z}) = \log \mathfrak{z}^{q_{\mathrm{f}}/g} \tag{3.4}$$

where q_{f}/g corresponds to the Fayet Iliopoulus charge introduced in eq. (2.20). Setting $\mathcal{K} = \widetilde{\mathcal{K}} + f(\mathfrak{z}) + \bar{f}(\bar{\mathfrak{z}})$ we obtain

$$\hat{\mathcal{P}}(\mathfrak{z},\bar{\mathfrak{z}}) = \widetilde{\mathcal{P}}(\mathfrak{z},\bar{\mathfrak{z}}) + \frac{q_{\mathrm{f}}}{g}\,. \tag{3.5}$$

Note that if $\widetilde{\mathcal{K}}$ is U(1) invariant the same is true of $\mathcal{K}$. As already stressed the constant shift q_{f}/g of the momentum map has no effect on the Kähler metric and, consequently, on the kinetic terms of the scalar fields. Actually it survives at vanishing scalar fields and it exists even if we completely suppress the Wess-Zumino multiplet.

As a consequence of the above formulae the covariant derivatives of the fermions entering the minimal supergravity lagrangian are the following ones:

$$\nabla\,\psi_\bullet = \widetilde{\nabla}\psi_\bullet - \mathrm{i}\,\frac{1}{2}\,q_{\mathrm{f}}\,\mathcal{A}\wedge\psi_\bullet\,; \qquad \widetilde{\nabla}\psi_\bullet = \mathcal{D}\psi_\bullet + \mathrm{i}\,\frac{1}{2}\,Q\wedge\psi_\bullet + \mathrm{i}\,\frac{1}{2}\,g\,\widetilde{\mathcal{P}}\,\mathcal{A}\wedge\psi_\bullet$$

$$\nabla\,\lambda_\bullet = \widetilde{\nabla}\lambda_\bullet - \mathrm{i}\,\frac{1}{2}\,q_{\mathrm{f}}\,\mathcal{A}\,\lambda_\bullet\,; \qquad \widetilde{\nabla}\lambda_\bullet = \mathcal{D}\lambda_\bullet + \mathrm{i}\,\frac{1}{2}\,Q\,\lambda_\bullet + \mathrm{i}\,\frac{1}{2}\,g\,\mathcal{A}\,\widetilde{\mathcal{P}}\,\lambda_\bullet$$

$$\nabla\,\chi^{\mathfrak{z}} = \widetilde{\nabla}\chi^{\mathfrak{z}} + \mathrm{i}\,\frac{1}{2}\,(q_{\mathrm{f}} + 2\,g)\,\mathcal{A}\,\chi^{\mathfrak{z}}\,; \qquad \widetilde{\nabla}\chi^{\mathfrak{z}} = \mathcal{D}\chi^{\mathfrak{z}} + \Gamma^{\mathfrak{z}}_{\ \mathfrak{z}}\chi^{\mathfrak{z}} - \mathrm{i}\frac{1}{2}\,Q\,\chi^{\mathfrak{z}} - \mathrm{i}\,\frac{1}{2}\,g\widetilde{\mathcal{P}}\,\mathcal{A}\,\chi^{\mathfrak{z}}\,. \tag{3.6}$$

At the same time the covariant derivative of the complex scalar field is:

$$\nabla\mathfrak{z} = d\mathfrak{z} + \mathrm{i}\,g\,\mathcal{A}\,\mathfrak{z}\,. \tag{3.7}$$

Equations (3.6) and (3.7) are sufficient to draw the main conclusions concerning the symmetries of the supergravity lagrangian.

a) There is one chiral global $\mathrm{U}_{\mathrm{R}}(1)$ symmetry (the R-symmetry) with respect to which the gravitino and the gaugino have charge[5] $q_R(\psi) = q_R(\lambda) = \frac{1}{2}$, the chiralino has charge $q_R(\chi) = -\frac{1}{2}$ and the scalar has charge $q_R(\mathfrak{z}) = 0$. Note the R-symmetry charges are the same as the Kähler weights of the corresponding fields under change of trivializations in the Hodge bundle.

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[5]The spelled out charges are those of the holomorphic-chiral fields (left-handed the fermions, holomorphic the scalar). The charges of the antiholomorphic-antichiral fields (right-handed the fermions, antiholomorphic the scalar) are just the opposite ones.

b) There is another chiral global $U_B(1)$ symmetry with respect to which the gravitino and the gaugino have charge $q_B(\psi) = q_B(\lambda) = 0$, the chiralino has charge $q_B(\chi) = 1$ and the scalar has charge $q_B(\mathfrak{z}) = 1$.

c) When we gauge the model in the absence of a Fayet Iliopoulos term the actual gauge algebra is just:

$$\mathfrak{g}_{\text{gauge}} = g\,\mathfrak{u}_B(1)\,. \tag{3.8}$$

d) When we gauge the model in the presence of a Fayet Iliopoulos term the actual gauge group is just:

$$\mathfrak{g}_{\text{gauge}} = -q_{\text{f}}\,\mathfrak{u}_R(1) \,\oplus\, g\,\mathfrak{u}_B(1)\,. \tag{3.9}$$

e) If we put $g = 0$ and we just remove the Wess-Zumino multiplet by setting $\mathfrak{z} = \chi^{\mathfrak{z}} = 0$ we can nonetheless preserve a non vanishing Fayet Iliopoulos charge $q_{\text{f}} \neq 0$. This means that the gauge field $\mathcal{A}$ is utilized to gauge R-symmetry and this produces a positive cosmological constant $\Lambda = q_{\text{f}}^2$ which yields a de-Sitter vacuum where supersymmetry is broken. This is the model constructed by Freedman in [31].

3.2 Non compact shift-symmetry

If the fundamental isometry of Σ is a non-compact translation symmetry $\mathbb{R}$, the Killing vector is given by the second line of eq. (2.16) and we have:

$$\partial_{\mathfrak{z}}\, k^{\mathfrak{z}} = 0 \tag{3.10}$$

so that from equation (2.34) we obtain:

$$\widehat{\Gamma}^{\mathfrak{z}}{}_{\mathfrak{z}} = \Gamma^{\mathfrak{z}}{}_{\mathfrak{z}}\,. \tag{3.11}$$

Furthermore, given a shift-invariant Kähler potential $\widetilde{\mathcal{K}}(\mathfrak{z}, \mathfrak{z})$, namely such that:

$$(\partial_{\mathfrak{z}} + \partial_{\bar{\mathfrak{z}}})\, \widetilde{\mathcal{K}}(\mathfrak{z}, \mathfrak{z}) = 0 \tag{3.12}$$

the solution of eq. (2.21) for the Kähler gauge transformation producing a Fayet Iliopoulos charge is the following one:

$$f(\mathfrak{z}) = \mathrm{i}\,\frac{q_{\text{f}}}{g}\mathfrak{z} \tag{3.13}$$

leading to

$$\mathcal{K}(\mathfrak{z}, \mathfrak{z}) = \widetilde{\mathcal{K}}(\mathfrak{z}, \mathfrak{z}) - 2\,\frac{q_{\text{f}}}{g}\,\mathrm{Im}\,\mathfrak{z} \tag{3.14}$$

and to a momentum map with the same structure as in eq. (3.5). As a consequence of the such formulae, the covariant derivatives of the fermions entering the minimal supergravity lagrangian are now the following ones to be compared with eq.s (3.6):

$$\nabla\psi_{\bullet} = \widetilde{\nabla}\psi_{\bullet} - \mathrm{i}\,\frac{1}{2}\,q_{\text{f}}\,\mathcal{A}\,\wedge\,\psi_{\bullet}\,; \qquad \widetilde{\nabla}\psi_{\bullet} = \mathcal{D}\psi_{\bullet} + \mathrm{i}\,\frac{1}{2}\,Q\,\wedge\,\psi_{\bullet} + \mathrm{i}\,\frac{1}{2}\,g\,\widetilde{\mathcal{P}}\,\mathcal{A}\,\wedge\,\psi_{\bullet}$$

$$\nabla\lambda_{\bullet} = \widetilde{\nabla}\lambda_{\bullet} - \mathrm{i}\,\frac{1}{2}\,q_{\text{f}}\,\mathcal{A}\,\lambda_{\bullet}\,; \qquad \widetilde{\nabla}\lambda_{\bullet} = \mathcal{D}\lambda_{\bullet} + \mathrm{i}\,\frac{1}{2}\,Q\,\lambda_{\bullet} + \mathrm{i}\,\frac{1}{2}\,g\,\mathcal{A}\,\widetilde{\mathcal{P}}\,\lambda_{\bullet}$$

$$\nabla\chi^{\mathfrak{z}} = \widetilde{\nabla}\chi^{\mathfrak{z}} + \mathrm{i}\,\frac{1}{2}\,q_{\text{f}}\,\mathcal{A}\,\chi^{\mathfrak{z}}\,; \qquad \widetilde{\nabla}\chi^{\mathfrak{z}} = \mathcal{D}\chi^{\mathfrak{z}} + \Gamma^{\mathfrak{z}}{}_{\mathfrak{z}}\chi^{\mathfrak{z}} - \mathrm{i}\,\frac{1}{2}\,Q\,\chi^{\mathfrak{z}} - \mathrm{i}\,\frac{1}{2}\,g\,\widetilde{\mathcal{P}}\,\mathcal{A}\,\chi^{\mathfrak{z}} \tag{3.15}$$

and the covariant derivative of the complex scalar field is:

$$\nabla\mathfrak{z} = d\mathfrak{z} + g\,\mathcal{A} \rightarrow \begin{cases} \nabla C = dC \\ \nabla B = dB + g\,\mathcal{A}\,. \end{cases} \tag{3.16}$$

It follows that:

a) Just as before there is one chiral global $U_R(1)$ symmetry (the R-symmetry) with respect to which the gravitino and the gaugino have charge $q_R(\psi) = q_R(\lambda) = \frac{1}{2}$, the chiralino has charge $q_R(\chi) = -\frac{1}{2}$ and the scalar has charge $q_R(\mathfrak{z}) = 0$. Note the R-symmetry charges are the same as the Kähler weights of the corresponding fields under change of trivializations in the Hodge bundle.

b) The second chiral global symmetry $U_B(1)$ which is present in the compact case, here is absent.

d) In the whole lagrangian the B-field appears only under derivatives.

c) When we gauge the model in the absence of a Fayet Iliopoulos term the actual gauge algebra is just:

$$\mathfrak{g}_{\text{gauge}} = g\,\mathbb{R} \tag{3.17}$$

all the fermions are neutral under such an algebra and the gauge field $\mathcal{A}$ appears only in the combination $\mathcal{A} + \frac{1}{g}dB$ that can be renamed $\hat{\mathcal{A}}$ and describes a massive vector field. The massive vector field $\hat{\mathcal{A}}$, the inflaton scalar C and the two fermions λ, χ make up the field content of a massive vector multiplet with 4-bosonic degrees of freedom $\oplus$ 4-fermionic degrees of freedom.

d) When we gauge the model in the presence of a Fayet Iliopoulos term the actual gauge group is just:

$$\mathfrak{g}_{\text{gauge}} = -q_{\text{f}}\,\mathfrak{u}_R(1)\,\oplus\,g\,\mathbb{R}\,. \tag{3.18}$$

d) As in the previous compact case, if we put $g = 0$ and we just remove the Wess-Zumino multiplet by setting $\mathfrak{z} = \chi^{\mathfrak{z}} = 0$ we can nonetheless preserve a non vanishing Fayet Iliopoulos charge $q_{\text{f}} \neq 0$. Also in this case the gauge field $\mathcal{A}$ is utilized to gauge R-symmetry and this produces a positive cosmological constant $\Lambda = q_{\text{f}}^2$ which yields a de-Sitter vacuum where supersymmetry is broken. Actually, once the WZ-multiplet is removed, the distinction between the compact and the shift-symmetry case is removed and we just have the already mentioned gauging of R-symmetry. Once again this is the model constructed by Freedman in [31].

4 Constant curvature models

Having established the above general facts, in the present section we consider explicit examples classified according to the curvature of the Kähler surface Σ.

First we consider flat models where, written, in a standard complex coordinate $\mathfrak{z}$, the Kähler metric is $ds^2 \propto d\bar{\mathfrak{z}}\,d\mathfrak{z}$. Next we consider constant (negative) curvature models, where, written in a disk-type complex coordinate $\mathfrak{z} = \zeta$, the Kähler metric is $ds^2 \propto$

$\left(1 - |\zeta|^2\right)^{-2} d\bar{\zeta}\, d\zeta$. We show that the *a priori* knowledge of the form of the metric in a standard complex coordinate is precisely what allows to determine the appropriate solution of the complex structure equations and, as a by product, to determine the global structure of the isometry group generated by the Killing vector $\vec{k}_B$. In want of this knowledge one has to resort to the criterion of the asymptotic behavior of the function $\partial_C^2 J(C)$ in order to discriminate between the possible topologies. We analyze from this point of view the constant curvature models in order to verify the established criteria.

4.1 The curvature and the Kähler potential

The curvature of an axial (shift) symmetric Kähler manifold can be written in two different ways in terms of the canonical coordinate ϕ or the VP coordinate C. In terms of the VP coordinate C we have the following formula:

$$R = R(C) = \frac{J'''(C)^2 - J''(C)J''''(C)}{2\,J''(C)^3} = -\frac{1}{2}\,\partial_C^2 \log\left[\partial_C^2 J(C)\right]\,\frac{1}{\partial_C^2 J(C)} \qquad (4.1)$$

which can be derived from the standard structural equations of the manifold:

$$0 = \mathrm{d}E^1 + \omega \wedge E^2$$
$$0 = \mathrm{d}E^2 - \omega \wedge E^1$$
$$\mathfrak{R} \equiv \mathrm{d}\omega \equiv R\,E^1 \wedge E^2 \qquad (4.2)$$

by inserting into them the appropriate form of the zweibein:

$$E^1 = \frac{1}{2}\,\sqrt{J''(C)}\,dC\,;\qquad E^2 = \frac{1}{2}\,\sqrt{J''(C)}\,dB \Rightarrow ds^2 = \frac{1}{4}\,J''(C)\left(dC^2 + dB^2\right). \qquad (4.3)$$

Alternatively we can write the curvature in terms of the momentum map $\mathcal{P}(\phi)$ or of the D-type potential $V(\phi) \propto \mathcal{P}^2(\phi)$ if we use the canonical coordinate ϕ and the corresponding appropriate zweibein:

$$E^1 = \frac{1}{2}\,d\phi\,;\qquad E^2 = \frac{1}{2}\,\mathcal{P}'(\phi)\,dB \Rightarrow ds^2 = \frac{1}{4}\left(d\phi^2 + \left(\mathcal{P}'(\phi)\right)^2\,dB^2\right). \qquad (4.4)$$

Upon insertion of eq.s (4.4) into (4.2) we get:

$$R(\phi) = -4\,\frac{\mathcal{P}'''(\phi)}{\mathcal{P}'(\phi)} = -4\left(\frac{V'''}{V'} - \frac{3}{2}\,\frac{V''}{V} - \frac{3}{4}\left(\frac{V'}{V}\right)^2\right). \qquad (4.5)$$

Finally let us compare the above definition (4.1) of the curvature with that utilized in curved index formalism. For instance, if we consider the Disk-type complex structure and we set

$$R = -\frac{1}{2}\,g^{t\bar{t}}\,\partial_t\,\partial_{\bar{t}} \log g_{t\bar{t}} \qquad (4.6)$$

we just reproduce the result (4.1), since $g_{t\bar{t}} = \frac{1}{4}\,J''(C)$ and $\partial_t\,\partial_{\bar{t}} \simeq \frac{1}{4}\,\partial_C^2$. Eq. (4.5) was first derived in [15].[6]

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[6]The curvature R defined as the component of $d\omega$ in the basis $E^1 \wedge E^2$ differs by a factor $\frac{1}{2}$ with respect to the curvature defined in standard curved index tensor calculus. This difference sums up in explicit calculations with the difference in normalization of the scalar field ϕ (see Friedman equations (1.1)).

4.2 Flat models

The above formulae for the curvature easily allow an analysis of the simplest possible supergravity models, namely those based on a flat Kähler manifold where $R = 0$. It is quite instructive to implement the vanishing curvature condition in both formulations (4.1) and (4.5).

4.2.1 Canonical coordinate representation

If we start from eq. (4.5), we see that the most general solution of the vanishing curvature condition is:

$$\mathcal{P}(\phi) = a_0 + a_1\,\phi + \frac{1}{2}\,a_2\,\phi^2 \tag{4.7}$$

where $a_{0,1,2}$ are real constants. By means of the shift $\phi \to \phi - \frac{a_1}{a_2}$, which does not alter the canonical kinetic term of ϕ, we can always suppress the linear term $a_1 = 0$ and we are left with:

$$\mathcal{P}(\phi) = \left(a_0 + \frac{1}{2}\,a_2\,\phi^2\right) \Rightarrow V(\phi) \propto \left(a_0 + \frac{1}{2}\,a_2\,\phi^2\right)^2 = \underbrace{M^4\left[\left(\frac{\phi}{\phi_0}\right)^2 \pm 1\right]}_{M^4 = a_0^2 \; ; \; \phi_0^2 = \left|2\,\frac{a_0}{a_2}\right|} \tag{4.8}$$

where the choice of the sign $\pm$ depends on whether $\frac{a_0}{a_2} > 0$ or $\frac{a_0}{a_2} < 0$. In the second case the obtained potential is the Higgs type of quartic potential that in the classification of inflationary potentials presented in [4] has the name DWI (see table 1 of the quoted reference).

The general case $a_2 \neq 0$. Eq. (4.8) shows that when both a_2 does not vanish the Higgs type of quartic potential can be incorporated into supergravity based on a flat Kähler manifold. Applying eq. (2.12) to the present case we obtain the following relation between the VP coordinate and the canonical coordinate

$$C(\phi) \equiv \int \frac{d\phi}{\mathcal{P}'(\phi)} = \frac{\log(\phi)}{a_2} \Rightarrow \phi = e^{Ca_2} \tag{4.9}$$

while the Kähler potential is given by:

$$J(\phi) = \int \frac{\mathcal{P}(\phi)}{\mathcal{P}'(\phi)} = \frac{\phi^2}{4} + \frac{\log(\phi)a_0}{a_2} + \text{const}$$

$$\Downarrow$$

$$J(C) = Ca_0 + \frac{1}{4}e^{2Ca_2} + \text{const.} \tag{4.10}$$

Correspondingly the metric takes the following form:

$$ds^2 = \frac{a_2^2}{4}\left(\mathrm{d}B^2 + \mathrm{d}C^2\right)e^{2\,a_2\,C} \tag{4.11}$$

and it is turned into the standard form of the flat Kähler metric:

$$ds_{flat}^2 \equiv \frac{1}{4}\,d\mathfrak{z}\,d\bar{\mathfrak{z}} \Leftrightarrow \mathcal{K}\left(\mathfrak{z},\bar{\mathfrak{z}}\right) = \frac{1}{4}\left(\mathfrak{z}\,\bar{\mathfrak{z}} - \frac{2\,a_0}{a_2}\log\left[\mathfrak{z}\,\bar{\mathfrak{z}}\right]\right) \tag{4.12}$$
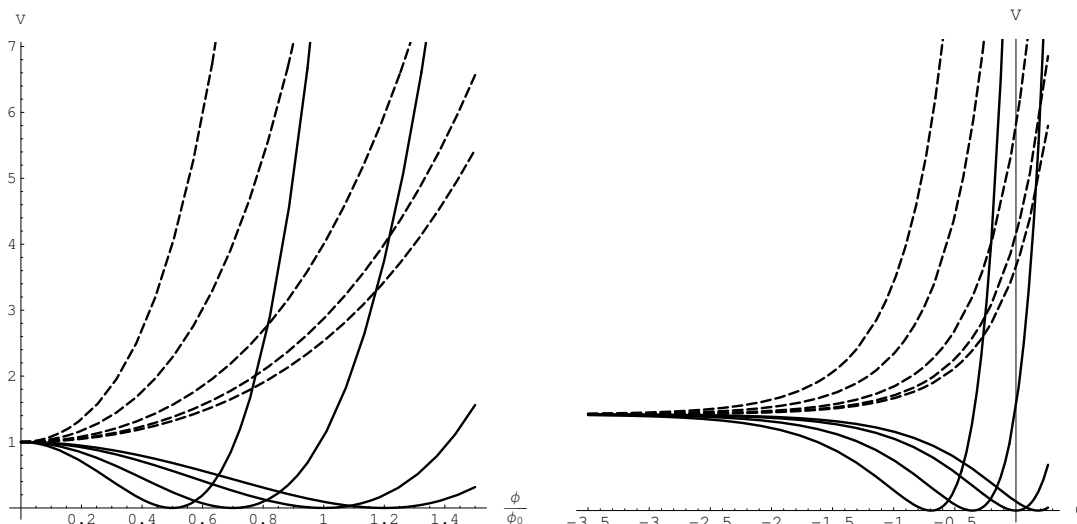
Figure 1. In this figure we show the plots of the inflaton potential obtained from minimal supergravity with a flat Kähler metric in the case of the gauging of a U(1) symmetry. The plots are presented in terms of the canonical variable ϕ (picture on the left) and in terms of the VP coordinate C (picture on the right). The various curves correspond to different values of the parameter $\phi_0 = \sqrt{2 \left| \frac{a_0}{a_2} \right|}$. The solid curves correspond to the cases where $\frac{a_0}{a_2} < 0$, while the dashed curves correspond to the cases $\frac{a_0}{a_2} > 0$. In the first case the plots in terms of ϕ are one half of the familiar mexican hat shaped Higgs potentials. The restriction $\phi \geq 0$ are due to the relation $\phi \propto \exp[C]$.

by the identification:

$$\mathfrak{z} = \exp\left[a_2 \left(C + \mathrm{i}B\right)\right]. \tag{4.13}$$

It follows that when $a_2 \neq 0$, the proper interpretation of the symmetry $B \to B + c$, which is gauged in order to produce the potential, is that of a *compact rotation*. Furthermore the parameter $\frac{a_0}{2\,a_2}$ plays the role of a Fayet-Iliopoulos charge according to the discussion of section 3. The plots of these type of potentials are displayed in figure 1.

The case $a_2 = 0$, $a_1 \neq 0$. As it is evident from the above formulae the limit $a_2 \to 0$ is singular and the case $a_2 = 0$ has to be treated separately. In the case that the momentum map is linear in the canonical coordinate, eqs. (4.9) and (4.10) are replaced by:

$$C(\phi) \equiv \int \frac{d\phi}{\mathcal{P}'(\phi)} = \frac{\phi}{a_1} - \frac{\beta}{a_1^2} \Rightarrow \phi = \frac{C\,a_1^2 + \beta}{a_1} \tag{4.14}$$

and:

$$J(\phi) = \int \frac{\mathcal{P}(\phi)}{\mathcal{P}'(\phi)} = \frac{1}{2}\,\phi^2 + \mathrm{const}$$

$$\Downarrow$$

$$J(C) = \frac{1}{2}\,a_1^2\,C^2 + \beta\,C + \mathrm{const}' \tag{4.15}$$
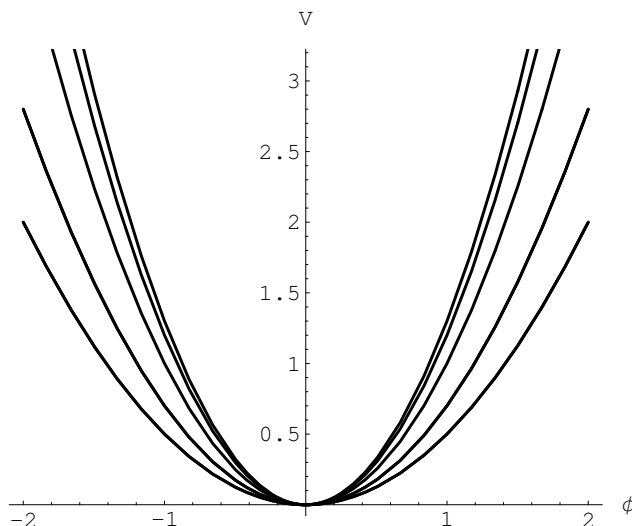
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Figure 2. In this figure we show the plots of the inflaton potential obtained from minimal supergravity with a flat Kähler metric in the case of the gauging of a shift symmetry. In this case the canonical variable ϕ and the VP coordinate C coincide. The various curves correspond to different values of the coefficient a_1. The coefficient a_0 can always be set to zero by means of a constant shift of ϕ which does not alter the metric. In all cases the potentials are quadratic and in the language of inflationary models correspond to *chaotic inflation*.

where $-\frac{\beta}{a_1^2}$ is an arbitrary integration constant. In this case the metric is:

$$ds^2 = a_1^2 \left(\mathrm{d}B^2 + \mathrm{d}C^2 \right) \tag{4.16}$$

which is turned into the standard form of the flat metric:

$$ds_{flat}^2 \equiv a_1^2 \, d\mathfrak{z} \, d\bar{\mathfrak{z}} \Leftrightarrow \mathcal{K}\left(\mathfrak{z}, \bar{\mathfrak{z}}\right) = -\frac{1}{8} \, a_1^2 \left(\mathfrak{z} - \bar{\mathfrak{z}}\right)^2 - \mathrm{i} \, \frac{1}{2} \, \beta \left(\mathfrak{z} - \bar{\mathfrak{z}}\right) \tag{4.17}$$

by the identification:

$$\mathfrak{z} = \mathrm{i} \, C + B \,. \tag{4.18}$$

It follows that when $a_2 = 0$, the proper interpretation of the symmetry $B \to B + c$ which is gauged in order to produce the potential is that of a *non compact translation*, namely of a shift symmetry. The integration constant β plays now the role of Fayet Iliopoulos gauging constant. From the point of view of the scalar potential this case corresponds to a pure mass term:

$$\mathcal{P}(\phi) = a_0 + a_1 \, \phi \Rightarrow V(\phi) = \left(a_0 + a_1 \, \phi\right)^2 \,. \tag{4.19}$$

The plot of these type of potentials are displayed in figure 2.

4.2.2 Retrieving the same result in the VP coordinate representation

If we start from equation (4.1), by imposing the zero curvature condition we obtain that $\log\left(\partial_C^2 J\right)$ should be linear in C, namely

$$\log\left(\partial_C^2 J\right) = 2 \, a_2 \, C + \log\left(a_2^2\right) \Rightarrow \partial_C^2 J = a_2^2 \, \exp\left[2 \, a_2 \, C\right] \Rightarrow J = \frac{1}{4} \, \exp\left[2 \, a_2 \, C\right] + a_0 \, C + \text{const} \tag{4.20}$$

where a_0 is the name given to the integration constant in the second order differential equation displayed above. This makes immediate contact with the result obtained from the momentum map approach. Note that the constant term in the solution of the differential equation for $\log\left(\partial_C^2 J\right)$ simply amounts to the rescaling of the Kähler potential and, hence, of the Kähler metric by an overall constant. The choice of that constant equal to $\log\left(a_2^2\right)$ simply fixes the standard normalization of the scalar field kinetic term.

4.2.3 Asymptotic expansions of the function $\partial_C^2 J(C)$

Let us now discuss the behavior of the function $\partial_C^2 J(C)$ which determines the Kähler metric in real variables in the two instances of flat models discussed above.

The flat U(1) model. For $C \to 0$ the metric coefficient $\partial_C^2 J(C) = a_2^2 \exp\left[2\, a_2\, C\right]$ goes to a constant, while for $a_2\, C \to -\infty$, which we identify with the origin of the field manifold $(\mathfrak{z} \to 0)$, the metric coefficient goes $\partial_C^2 J(C)$ to zero as $|\mathfrak{z}|^2 = \exp\left[2\, a_2\, C\right]$. This asymptotic behavior is essential for the interpretation of the shift $B \to B + c$ as a compact rotation, as we pointed out before.

The flat $\mathbb{R}$ model. In this case the metric coefficient $\partial_C^2 J(C) = a_1^2$ is constant everywhere and for $C \to -\infty$ it does not go to zero. Such behavior selects the interpretation of the shift $B \to B + c$ as a non-compact translation symmetry.

4.3 Constant negative curvature models

In eq. (3.16) of [16] the general solution of the constant curvature equation:

$$R(\phi) = -4\,\nu^2 \tag{4.21}$$

was presented in terms of the momentum map $\mathcal{P}(\phi)$ and of the canonical variable ϕ. We have:[7]

$$\mathcal{P}(\phi) = a\,\exp(\nu\,\phi) + b\,\exp(-\,\nu\,\phi) + c\,; \quad a, b, c \in \mathbb{R}\,. \tag{4.22}$$

In order to convert this solution in terms of the Jordan function $J(C)$ of the VP coordinate C, it is convenient to remark that, up to constant shift redefinitions and sign flips of the canonical variable $\phi \to \pm\phi + \kappa$, which leave its kinetic term invariant, there are only three relevant cases:

A) $a \neq 0$, $b \neq 0$ and $a/b > 0$. In this case, up to an overall constant, we can just set:

$$\mathcal{P}(\phi) = \frac{1}{\nu}\left(\cosh(\nu\,\phi) + \mu\right) \Rightarrow V(\phi) \propto \left(\cosh(\nu\,\phi) + \mu\right)^2. \tag{4.23}$$

B) $a \neq 0$, $b \neq 0$ and $a/b < 0$. In this case we can just set:

$$\mathcal{P}(\phi) = \frac{1}{\nu}\left(\sinh(\nu\,\phi) + \mu\right) \Rightarrow V(\phi) \propto \left(\sinh(\nu\,\phi) + \mu\right)^2. \tag{4.24}$$

C) $a \neq 0$, $b = 0$. In this case we can just set:

$$\mathcal{P}(\phi) = \frac{1}{\nu}\left(\exp(\nu\,\phi) + \mu\right) \Rightarrow V(\phi) \propto \left(\exp(\nu\,\phi) + \mu\right)^2. \tag{4.25}$$

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[7]Note that for the sake of our following arguments the solution of [16] is rewritten here in terms of exponentials rather than in terms of hyperbolic functions cosh and sinh.

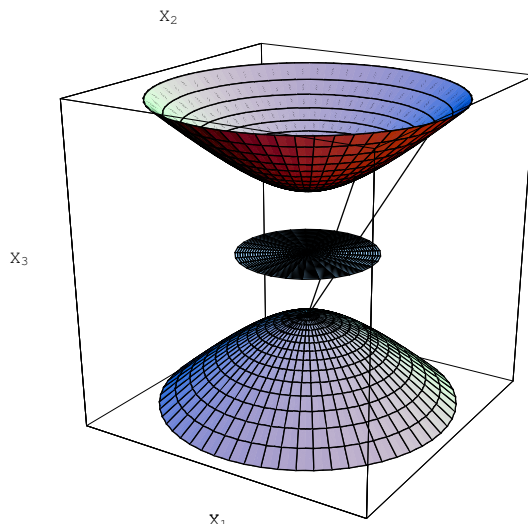**Figure 3**. In this figure we show the hyperboloid ruled by lines of constant ϕ that are circles and of constant B that are hyperbolae. In this figure we also show the stereographic projection of points of the hyperboloid onto points of the unit disk.

4.3.1 Elaboration of case A)

Let us consider the case of the momentum map of eq. (4.23). The corresponding two-dimensional metric is:

$$ds_\phi^2 = \frac{1}{4}\left(d\phi^2 + \sinh^2\left(\nu\,\phi\right)\,dB^2\right) \tag{4.26}$$

which can be shown to be the pull-back of the $(2,1)$-Lorentz metric onto a hyperboloid surface. Indeed setting:

$$
\begin{aligned}
X_1 &= \sinh(\nu\phi)\cos(B\nu) \\
X_2 &= \sinh(\nu\phi)\sin(B\nu) \\
X_3 &= \pm\cosh(\nu\phi)
\end{aligned}
\tag{4.27}
$$

we obtain a parametric covering of the algebraic locus:

$$X_1^2 + X_2^2 - X_3^2 = -1 \tag{4.28}$$

and we can verify that:

$$\frac{1}{4\nu^2}\left(dX_1^2 + dX_2^2 - dX_3^2\right) = \frac{1}{4}\left(d\phi^2 + \sinh^2\left(\nu\,\phi\right)\,dB^2\right) = ds_\phi^2. \tag{4.29}$$

A picture of the hyperboloid ruled by lines of constant ϕ and constant B according to the parametrization (4.27) is depicted in figure 3. Applying to the present case the general rule given in eq. (2.12) that defines the VP coordinate C we get:

$$C(\phi) = \int \frac{d\phi}{\mathcal{P}'(\phi)} = \frac{\log\left(\tanh\left(\frac{\nu\phi}{2}\right)\right)}{\nu^2} \Leftrightarrow \phi = \frac{2\mathrm{Arctanh}\left(e^{C\nu^2}\right)}{\nu} \tag{4.30}$$

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from which we deduce that the allowed range of the flat variable C, in which the canonical variable ϕ is real and goes from 0 to ∞, is the following one:

$$C \in [-\infty, 0]. \tag{4.31}$$

Next applying to the present case the general formula given in eq. (2.14) that yields the Kähler function $J(\phi)$ we obtain:

$$J(\phi) = \int \frac{\mathcal{P}(\phi)}{\mathcal{P}'(\phi)}\, d\phi = \frac{\log(\sinh(\nu\phi))}{\nu^2} + \frac{\mu \log\left(\tanh\left(\frac{\nu\phi}{2}\right)\right)}{\nu^2}. \tag{4.32}$$

Substituting eq. (4.30) into (4.32), after some manipulations we obtain:

$$J(C) = (\mu+1)\, C - \frac{\log\left(1 - e^{2C\nu^2}\right)}{\nu^2} + \frac{\log(2)}{\nu^2} \tag{4.33}$$

which corresponds to the following metric:

$$ds_C^2 = \frac{1}{4} \frac{\partial^2 J(C)}{\partial C^2}\left(dC^2 + dB^2\right) = \frac{1}{4}\left(dB^2 + dC^2\right)\nu^2 \operatorname{csch}^2\left(C\nu^2\right). \tag{4.34}$$

Upon use of the coordinate transformation (4.30) the line element ds_C^2 flows into ds_ϕ^2 and viceversa.

It remains to be seen how such a metric is canonically written in terms of a complex coordinate $\mathfrak{z} = \zeta$. In this case the appropriate relation between ζ in the unit circle and the real variables C, B is the following:

$$\zeta = e^{\nu^2(\mathrm{i}B + C)}. \tag{4.35}$$

With this position we find:

$$ds_C^2 = \frac{1}{\nu^2} \frac{d\zeta\, d\bar{\zeta}}{\left(1 - \zeta\,\bar{\zeta}\right)^2} = \partial_\zeta\, \partial_{\bar\zeta} \underbrace{\left[-\frac{1}{\nu^2}\log\left(1 - \zeta\,\bar\zeta\right) + \frac{\mu+1}{2\,\nu^2}\log|\zeta\,\bar\zeta|^2 \right]}_{J(C) = \mathcal{K}(\zeta,\bar\zeta) = \text{Kähler potential}}\, d\zeta\, d\bar\zeta. \tag{4.36}$$

On the other hand from the position (4.35) it is evident that the shift in B is a compact rotation of the complex coordinate ζ.

For $C \to -\infty$, namely for large and negative values of the argument we have the following expansion of the Kähler potential

$$J(C) = \frac{\log(2)}{\nu^2} + (\mu+1)\, C + \frac{e^{2C\nu^2}}{\nu^2} + \frac{e^{4C\nu^2}}{2\nu^2} + \frac{e^{6C\nu^2}}{3\nu^2} + \mathcal{O}\left(e^{8C\nu^2}\right)$$
$$\Downarrow$$
$$\partial_C^2 J(C) = 4\,\nu^2\, e^{2C\nu^2} + 8\,\nu^2\, e^{4C\nu^2} + 12\,\nu^2\, e^{6C\nu^2} + \mathcal{O}(e^{8C\nu^2}) \overset{C \to -\infty}{\Longrightarrow} 0 \tag{4.37}$$

while for $C \to 0$, which corresponds to the boundary of moduli space, we have a logarithmic singularity:

$$J(C) = \frac{\log(2)}{\nu^2} - \frac{\log\left(-2\nu^2\right)}{\nu^2} - \frac{\log(C)}{\nu^2} + \mu\, C - \frac{1}{6}C^2\nu^2 + \mathcal{O}\left(C^3\right). \tag{4.38}$$

The interpretation of the parameter μ is evident from the above formulae. It introduces a Fayet-Iliopoulos term.

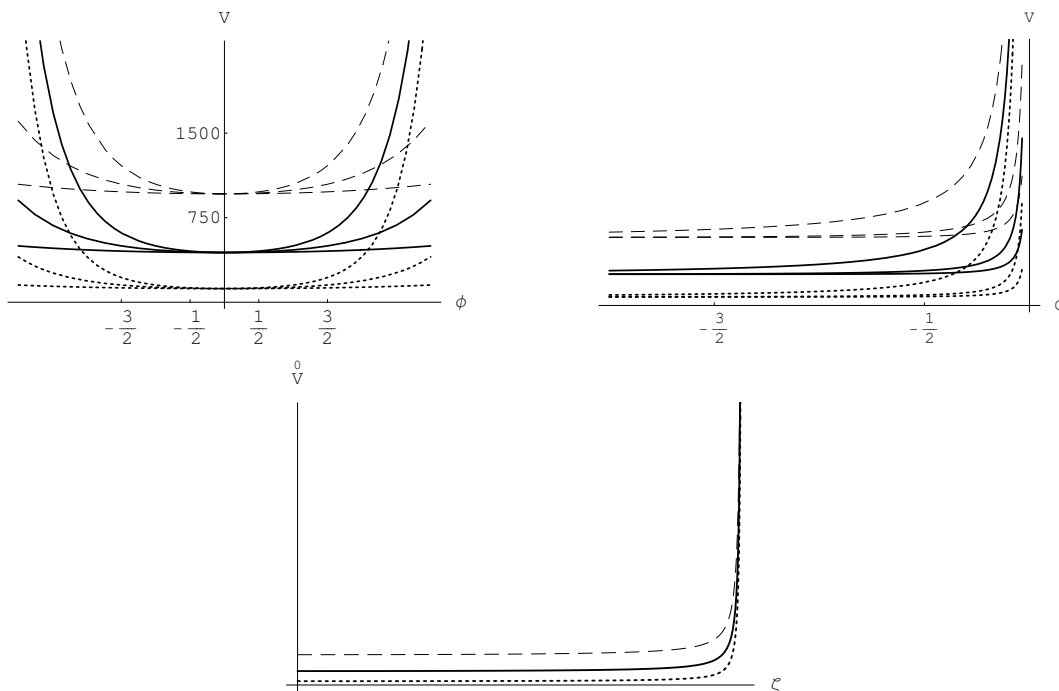**Figure 4**. Plots of the inflaton potential corresponding to the gauging of a compact U(1) isometry inside SL(2, ℝ). The potential is displayed as function of either the canonical coordinate ϕ (first picture) or the VP coordinate C (second picture) or the modulus of the complex coordinate ζ according to the formulae displayed in eq. (4.39). In each picture the curves of the same type (solid, dashed or long dashed) correspond to the same value of the Fayet Iliopoulos parameter μ, but to different values of the curvature parameter ν.

It is useful to write the D-type scalar potential in three different forms, as function of the canonical field ϕ, as function of the VP coordinated C and as function of the complex coordinate ζ:

$$V \propto \left(\cosh\left(\nu\,\phi \right) + \mu \right)^2 = \left(\mu + \frac{2e^{2C\nu^2}}{1 - e^{2C\nu^2}} + 1 \right)^2 = \frac{1}{\nu^4} \left(\frac{\mu + 1 - \mu\,\zeta\,\bar\zeta}{1 - \zeta\,\bar\zeta} \right)^2. \qquad (4.39)$$

The plots of these potential forms are displayed in figure 4.

4.3.2 Elaboration of case B)

Let us now consider the case of the momentum map of eq. (4.24). The corresponding two-dimensional metric is:

$$ds_\phi^2 = \frac{1}{4} \left(d\phi^2 + \cosh^2\left(\nu\,\phi \right) dB^2 \right) \qquad (4.40)$$

which can be shown to be another form of the pull-back of the Lorentz metric onto a hyperboloid surface. Indeed setting:

$$\begin{aligned}
X_1 &= \cosh(\nu\phi)\sinh(B\nu) \\
X_2 &= \sinh(\nu\phi) \\
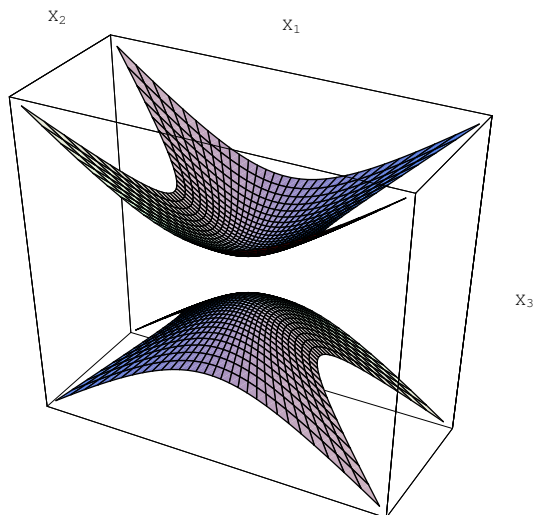X_3 &= \pm\cosh(B\nu)\cosh(\nu\phi)
\end{aligned} \qquad (4.41)$$

Figure 5. The hyperboloid surface displayed in the parametrization (4.41). The lines drawn on the hyperboloid surface are those of constant B and constant ϕ respectively. Both of them are hyperbolae, in this case.

we obtain a parametric covering of the algebraic locus:

$$X_1^2 + X_2^2 - X_3^2 = -1 \tag{4.42}$$

and we can verify that:

$$\frac{1}{4\nu^2} \left(dX_1^2 + dX_2^2 - dX_3^2 \right) = \frac{1}{4} \left(d\phi^2 + \cosh^2 \left(\nu\,\phi \right)\,dB^2 \right) = ds_\phi^2. \tag{4.43}$$

A three-dimensional picture of the hyperboloid ruled by lines of constant ϕ and constant B is displayed in figure 5.

Applying to the present case the general rule given in eq. (2.12) that defines the VP coordinate C we get:

$$C(\phi) = \int \frac{d\phi}{\mathcal{P}'(\phi)} = \frac{2\text{Arctan}\left(\tanh\left(\frac{\nu\phi}{2}\right)\right)}{\nu^2} \Leftrightarrow \phi = \frac{2\text{Arctanh}\left(\tan\left(\frac{C\nu^2}{2}\right)\right)}{\nu} \tag{4.44}$$

from which we deduce that the allowed range of the flat variable C, in which the canonical variable ϕ is real and goes from $-\infty$ to ∞, is the following one:

$$C \in \left[-\frac{\pi}{2\,\nu^2}\,,\,\frac{\pi}{2\,\nu^2} \right]. \tag{4.45}$$

Next applying to the present case the general formula given in eq. (2.14) that yields the Kähler function $J(\phi)$ we obtain:

$$J(\phi) = \int \frac{P(\phi)}{\mathcal{P}'(\phi)}\,d\phi = \frac{2\mu\text{Arctan}\left(\tanh\left(\frac{\nu\phi}{2}\right)\right)}{\nu^2} + \frac{\log(\cosh(\nu\phi))}{\nu^2}. \tag{4.46}$$

Substituting eq. (4.44) into (4.46), after some manipulations we obtain:

$$J(C) = \mu\, C - \frac{1}{\nu^2}\, \log\left(\cos\left(C\nu^2\right)\right) \tag{4.47}$$

which corresponds to the following metric:

$$ds_C^2 = \frac{1}{4}\, \frac{\partial^2 J(C)}{\partial C^2}\left(dC^2 + dB^2\right) = \frac{1}{4}\left(\mathrm{d}B^2 + \mathrm{d}C^2\right)\nu^2 \sec^2\left(C\nu^2\right). \tag{4.48}$$

Upon use of the coordinate transformation (4.44) the line element ds_C^2 flows into ds_ϕ^2 and viceversa.

It remains to be seen how such a metric is canonically written in terms of a complex coordinate ζ. In this case the appropriate relation between ζ in the unit circle and the real variables C, B is different. Setting:

$$\zeta = \mathrm{i}\tanh\left(\frac{1}{2}(B - \mathrm{i}\,C)\nu^2\right) \tag{4.49}$$

which implies:

$$C = -\frac{\mathrm{i}}{\nu^2}\left(\mathrm{ArcTanh}(-\mathrm{i}\,\zeta) - \mathrm{ArcTanh}\left(\mathrm{i}\,\bar\zeta\right)\right)$$

$$B = \frac{1}{\nu^2}\left(\mathrm{ArcTanh}(-\mathrm{i}\,\zeta) + \mathrm{ArcTanh}\left(\mathrm{i}\,\bar\zeta\right)\right) \tag{4.50}$$

we find:

$$J(C) = \mathcal{K}\left(\zeta, \bar\zeta\right) = -\mathrm{i}\,\frac{\mu}{\nu^2}\left(\mathrm{ArcTanh}(-\mathrm{i}\,\zeta) - \mathrm{ArcTanh}\left(\mathrm{i}\bar\zeta\right)\right) - \frac{1}{\nu^2}\, \log\left(1 - \zeta\,\bar\zeta\right) \tag{4.51}$$

$$ds_C^2 = \frac{1}{\nu^2}\, \frac{d\zeta\, d\bar\zeta}{\left(1 - \zeta\,\bar\zeta\right)^2} = \partial_\zeta\, \partial_{\bar\zeta}\, \mathcal{K}\left(\zeta, \bar\zeta\right)\, d\zeta\, d\bar\zeta. \tag{4.52}$$

The identification (4.49) allows us to understand the nature of the isometry $B \to B + c$ which is non compact. To this effect let us consider the image of the $\mathrm{SL}(2,\mathbb{R})$ dilatations inside the $\mathrm{SU}(1,1)$ group:

$$\Lambda_\rho = \begin{pmatrix} 1 & 1 \\ \mathrm{i} & -\mathrm{i} \end{pmatrix} \begin{pmatrix} e^\rho & 0 \\ 0 & e^{-\rho} \end{pmatrix} \begin{pmatrix} \frac{1}{2} & -\frac{\mathrm{i}}{2} \\ \frac{1}{2} & \frac{\mathrm{i}}{2} \end{pmatrix} = \begin{pmatrix} \cosh\rho & -\mathrm{i}\sinh\rho \\ \mathrm{i}\sinh\rho & \cosh\rho \end{pmatrix}. \tag{4.53}$$

The action of this group on the complex coordinate ζ inside the unit circle is given by the following linear fractional transformation:

$$\Lambda_\rho \cdot \zeta = \frac{\zeta\cosh(\rho) - \mathrm{i}\,\sinh(\rho)}{\cosh(\rho) + \mathrm{i}\,\zeta\sinh(\rho)}. \tag{4.54}$$

For an infinitesimal parameter $\rho \ll 1$ we have:

$$\zeta \to \zeta + \delta_\rho\zeta; \quad \delta_\rho\zeta = -\mathrm{i}\left(1 + \zeta^2\right)\rho. \tag{4.55}$$

Consider next the effect of a shift of B on the complex coordinate ζ as given in eq. (4.49). We find:

$$\partial_B\,\zeta = \frac{1}{2}\,\nu^2\,\mathrm{i}\,\mathrm{sech}^2\left(\frac{1}{2}(B - \mathrm{i}\,C)\nu^2\right) = \frac{1}{2}\,\nu^2\mathrm{i}\left(1 + \zeta^2\right). \tag{4.56}$$

This shows that indeed the B-shifts realize the action of the non compact subgroup (4.54) on the complex coordinate ζ.
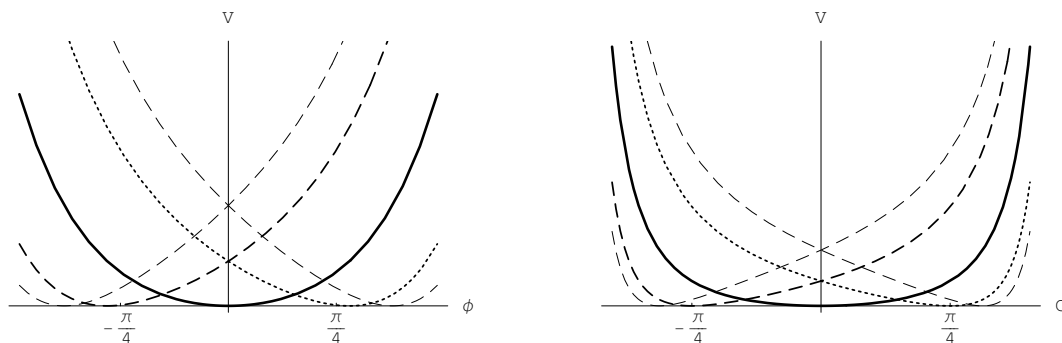
Figure 6. Plots of the inflaton potential corresponding to the gauging of a non-compact $\mathrm{SO}(1,1)$ isometry inside $\mathrm{SL}(2,\mathbb{R})$. The potential is displayed as function of either the canonical coordinate ϕ (first picture) or the VP coordinate C (second picture). In each picture the curves of different type (solid, dashed or long dashed) correspond to different values of the Fayet Iliopoulos parameter μ, and to the same value ν of the curvature parameter.

Knowing the Killing vector we can now write the scalar potential in three different ways as function of the canonical coordinate ϕ, of the VP coordinate C and of the complex coordinate ζ. We find:

$$V \propto (\sinh(\nu\phi)+\mu)^2 = \left(\mu + \tan\left(C\nu^2\right)\right)^2 = \left(\frac{\bar{\zeta}\left(\zeta+\bar{\zeta}\right)\zeta + \zeta + \bar{\zeta} + 2\mu\left(\zeta\bar{\zeta}-1\right)}{4\zeta\bar{\zeta}-4}\right)^2 . \quad (4.57)$$

The behavior of this family of potentials is displayed in figure 6. Note that when written in terms of the complex variable ζ the potential does not appear to depend only on one variable. Yet this is so since the potential depends only from the C-variable defined by eq. (4.50).

Asymptotic behavior. It is now important to consider the behavior of the Kähler function $J(C)$ as given in eq. (4.47) when the VP coordinate approaches the boundary of its own range. The point $C = 0$ is perfectly regular for $J(C)$ and indeed, in consideration of eq. (4.45), it is well inside the range of definition. The boundary is approached when $C \to \pm\frac{\pi}{2\nu^2}$. For this reason we set $C = \frac{\pi}{2\nu^2} - \xi$ and we consider the behavior of the Kähler function for $\xi \simeq 0$. We obtain:

$$J(\xi) = -\xi\mu + \frac{\pi\mu}{2\nu^2} - \frac{\log\left(\nu^2\right)}{\nu^2} - \frac{\log(\xi)}{\nu^2} + \mathcal{O}\left(\xi^2\right) . \quad (4.58)$$

In this way we put into evidence the logarithmic singularity which characterizes the behavior of the Kähler function in at the boundary. Once again it also appears that the parameter μ plays the role of Fayet Iliopoulos term. Considering now the behavior of the function $\partial_C^2 J(C)$ for $C \to 0$, which is the center of the bulk for the field manifold, we find:

$$\partial_C^2 J(C) \overset{C \to 0}{\simeq} \nu^2 + \nu^6 C^2 + \frac{2\nu^{10}C^4}{3} + \mathcal{O}\left(C^5\right) . \quad (4.59)$$

We see that the metric coefficient goes to a constant and this is the obstacle to interpret the symmetry $B \to B + c$ as a compact rotation. Indeed as we have seen it is rather a hyperbolic transformation.

Curv.	Gauge Group	$V(\phi)$	$V(C)$	$V(\mathfrak{z})$	Comp. Struct.
$-\nu^2$	U(1)	$(\cosh(\nu\,\phi)+\mu)^2$	$\left(\mu+\frac{2e^{2C\nu^2}}{1-e^{2C\nu^2}}+1\right)^2$	$\frac{1}{\nu^4}\left(\frac{\mu+1-\mu\zeta\bar\zeta}{1-\zeta\bar\zeta}\right)^2$	$\zeta=e^{C-\mathrm{i}B}$
$-\nu^2$	SO(1,1)	$(\sinh(\nu\,\phi)+\mu)^2$	$(\mu+\tan(C\nu^2))^2$	$\left(\frac{\bar\zeta(\zeta+\bar\zeta)\zeta+\zeta+\bar\zeta+2\mu(\zeta\bar\zeta-1)}{4\zeta\bar\zeta-4}\right)^2$	$\zeta=\mathrm{i}\tanh\left(\frac{1}{2}(B-\mathrm{i}C)\,\nu^2\right)$
$-\nu^2$	parabolic	$(\exp(\nu\,\phi)+\mu)^2$	$\left(\mu+\frac{1}{\nu^2 C}\right)^2$	$\left(\frac{1}{2}\mu+\frac{\mathrm{i}}{\nu^2}\,(t-\bar t)^{-1}\right)^2$	$t=-\mathrm{i}\,C+B$
0	U(1)	$M^4\left[\left(\frac{\phi}{\phi_0}\right)^2\pm1\right]^2$	$M^4\left[\frac{e^{2a_2C}}{\phi_0^2}\pm1\right]^2$	$\frac{1}{4}\left(\mathfrak{z}\,\bar{\mathfrak{z}}-\frac{2a_0}{a_2}\right)^2$	$\mathfrak{z}=\exp[a_2\,(C+\mathrm{i}B)]$
0	parabolic	$(a_0+a_1\,\phi)^2$	$(a_1\,C+\beta)^2$	$\frac{1}{2}\,(a_1\mathrm{Im}\mathfrak{z}+\beta)^2$	$\mathfrak{z}=\mathrm{i}C+B$

Table 1. Summary of the potentials of D-type obtained from constant curvature Kähler manifolds by gauging either a compact or a non compact isometries.

4.3.3   Elaboration of case C)

In the case the momentum map is given by eq. (4.25) by immediate integration of eq. (2.12) we obtain the VP coordinate $C(\phi)$ and its inverse function:

$$C(\phi)=-\frac{e^{-\nu\phi}}{\nu^2}\ \Leftrightarrow\ \phi(C)=-\frac{\log\left(-C\nu^2\right)}{\nu}\,. \tag{4.60}$$

The integration of eq. (2.14) for the Kähler potential is equally immediate and we find:

$$J(\phi)=\frac{\phi-\frac{e^{-\nu\phi}\mu}{\nu}}{\nu}\ \Leftrightarrow\ J(C)=\mu\,C-\frac{1}{\nu^2}\,\log\left(-C\right)+\text{const}\,. \tag{4.61}$$

From the form of equation (4.61) we conclude that the appropriate solution of the complex structure equation in this case is:

$$\mathfrak{z}=t=\mathrm{i}C-B \tag{4.62}$$

so that the Kähler metric becomes proportional to the Poincaré metric in the upper complex plane (note that C is negative definite for the whole range of the canonical variable ϕ):

$$ds^2=\frac{1}{4}\frac{\mathrm{d}^2J}{\mathrm{d}C^2}\,\left(dC^2+dB^2\right)=\frac{1}{4\,\nu^2}\frac{d\bar t\,dt}{(\mathrm{Im}t)^2}. \tag{4.63}$$

As a consequence of equation (4.62), we see that the B-translation happens to be, in this case, a non-compact shift symmetry.

As in the previous cases we can write the potential in three forms:

$$V=(\exp[\nu\,\phi]+\mu)^2=\left(\mu+\frac{1}{\nu^2\,C}\right)^2=\left(\frac{1}{2}\,\mu+\frac{\mathrm{i}}{\nu^2}\,(t-\bar t)^{-1}\right)^2. \tag{4.64}$$

The corresponding plots are displayed in figure 7. The results for the five type of potentials that we have obtained from constant curvature symmetric spaces are summarized in table 1.
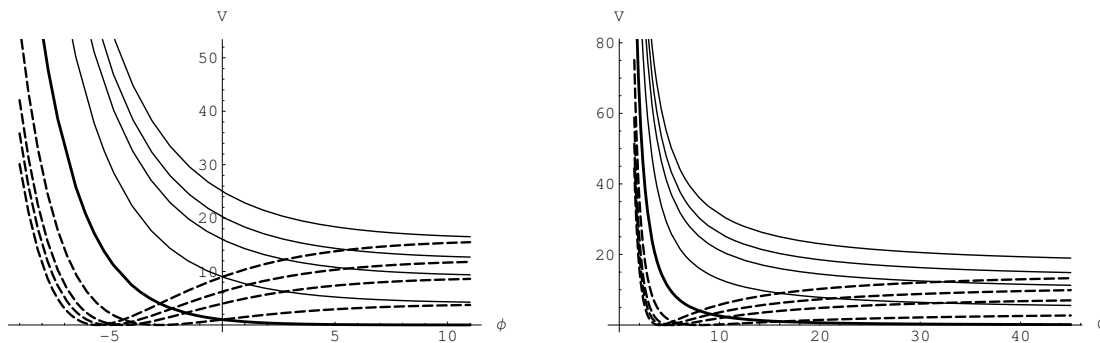
Figure 7. Plots of the inflaton potential corresponding to the gauging of a shift-isometry (parabolic) inside $\mathrm{SL}(2,\mathbb{R})$ (Starobinsky-like models). The potential is displayed as a function of either the canonical coordinate ϕ (first picture) or of the VP coordinate C (second picture). In each picture the solid lines correspond to positive values of the Fayet Iliopoulos parameter μ, while the dashed lines correspond to negative values of μ, which develop a minimum at $V = 0$. The value of ν is the same in all cases and it is negative $\nu = \frac{1}{4}$.

5 Conclusions

Summarizing, the main result of the present paper concerns three related points:

A) The physical properties of the minimal supergravity models that encode one-field cosmologies with a positive definite potential depend in a crucial way on the global topology of the group $\mathcal{G}$ that is gauged in order to produce them. When it is compact we have a certain pattern of symmetries and charge assignments, when it is non-compact we have a different pattern.

B) The global topology of the group $\mathcal{G}$ reflects into a different asymptotic behavior of the function $\partial_C^2 J(C)$ in the region that we can call the origin of the manifold. In the compact case the complex field $\mathfrak{z}$ is charged with respect to U(1) and, for consistency, this symmetry should exist at all orders in an expansion of the scalar field σ-model for small fields. Hence for $\mathfrak{z} \to 0$ the kinetic term of the scalars should go to the standard canonical one:

$$\mathcal{L}_{\mathrm{kin}}^{(\mathrm{can})} = \frac{1}{4} \partial_\mu \mathfrak{z} \, \partial^\mu \bar{\mathfrak{z}} \, . \tag{5.1}$$

Assuming, as it is necessary for the U(1) interpretation of the B-shift symmetry, that $\mathfrak{z} = \zeta = \exp\left[\alpha(\mathrm{i}C - B)\right]$, where α is some real coefficient, eq. (5.1) can be satisfied if and only if we have:

$$\lim_{C \to -\infty} \exp\left[-2\,\alpha\,C\right] \partial_C^2 J(C) = \mathrm{const} \tag{5.2}$$

which therefore is an intrinsic clue to establish the global topology of the inflaton Kähler surface Σ.

C) The Fayet Iliopoulos terms always identified as linear terms in VP coordinate C in the function $J(C)$ are rather different in the complex variable $\mathfrak{z}$, depending on which is the appropriate topology.

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Curv.	Gauge Group	$V(\phi)$	Values of ν	Values of μ	Mother series
$-\nu^2$	U(1)	$(\cosh(\nu\,\phi)+\mu)^2$	$\nu=\frac{\sqrt{3}}{2}$	$\mu=0$	I_1 or I_7 with $\gamma=\frac{1}{2}$
$-\nu^2$	U(1)	$(\cosh(\nu\,\phi)+\mu)^2$	$\nu=\frac{2}{\sqrt{3}}$	$\mu=1$	I_7 with $\gamma=\frac{1}{3}$
$-\nu^2$	U(1)	$(\cosh(\nu\,\phi)+\mu)^2$	$\nu=\frac{2}{\sqrt{3}}$	$\mu=-1$	I_7 with $\gamma=\frac{1}{3}$
$-\nu^2$	SO(1,1)	$(\sinh(\nu\,\phi)+\mu)^2$	$\nu=\frac{\sqrt{3}}{2}$	$\mu=0$	I_1 or I_7 with $\gamma=\frac{1}{2}$
$-\nu^2$	parabolic	$(\exp(\nu\,\phi)+\mu)^2$	$\nu=$ any	$\mu=0$	all pure exp are integ.

Table 2. In this table we mention which particular values of the curvature and of the Fayet Iliopoulos constant yield cosmological potentials that are both associated to constant curvature and integrable according to the classification of [13].

These properties are general and apply to all inflaton models embedded into a minimal $\mathcal{N}=1$ supergravity description. In the particular case of constant cruvature Kähler surfaces we were able to derive five models, two associated with a flat Kähler manifold and three with the unique negative curvature two-dimensional symmetric space $SL(2,\mathbb{R})/O(2)$. Of these five models three correspond to known inflationary potentials: the Higgs potential and the chaotic inflation quadratic potential, coming from a zero curvature Kähler manifold and the Starobinsky-like potentials, coming from the gauging of parabolic subgroups of $SL(2,\mathbb{R})$. These latter potentials were already embedded in supergravity in [5] and [27]. The remaining two potentials, respectively associated with the gauging of elliptic and hyperbolic subgroups so far have not yet been utilized as candidate inflationary potentials and possible they are incompatible with PLANCK data. In any case it is important to emphasize that parabolic Starobinsky-like potentials are associated with higher curvature supergravity models ([9, 39–41]) and it is an obvious question to inquiry what is the origin, in this context, of the elliptic and hyperboic Starobinsky-like potentials we have found. Furthermore let us stress that the Fayet Iliopoulos term (and its sign) drastically changes the behavior of the scalar potential. In the Starobinsky case it is responsible for the de Sitter inflationary phase. It is furthermore interesting to note that for some particular values of the curvature and of the Fayet Iliopoulos parameter the models classified in this paper become integrable. In table 2 we list such cases. They are in the intersection of the list of table 1 with the list of integrable series of potentials classified in [13] and further analyzed in [16]. By means of the arguments contained in this paper we have emphasized the physical relevance of the global topology of the Σ Kähler surface associated with minimal supergravity models of inflations. Global topology amount at the end of the day to giving the precise range of the coordinates C and B labeling the points of Σ. In the five constant curvature cases we presented these ranges are as follows. In the elliptic and parabolic case C is in the range $[-\infty,-0]$ in the elliptic and while it is in the range $[-\infty,+\infty]$ for the flat case and it is periodic in the hyperbolic case. The cooordinate B instead is periodic in the elliptic case, it is unrestricted in the hyperbolic and parabolic cases. The flat case with B periodic is a strip. It is instead the full plane in the parabolic coordinate.

Finally let us stress that the considerations put forward here can be extended to a large class of inflationary models based on non symmetric spaces, namely associated with Kähler surfaces Σ whose curvature is non-constant. Among them a subclass of models are the integrable ones for which a preliminary analysis was given in [16]. In a forthcoming publication [42] we plan to extend and improve such analysis for many models, also of realistic type both non integrable and occasionally integrable.

Acknowledgments

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