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Low energy behaviour of standard model extensions

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Abstract: The integration of heavy scalar fields is discussed in a class of BSM models, containing more that one representation for scalars and with mixing. The interplay between integrating out heavy scalars and the Standard Model decoupling limit is examined. In general, the latter cannot be obtained in terms of only one large scale and can only be achieved by imposing further assumptions on the couplings. Systematic low-energy expansions are derived in the more general, non-decoupling scenario, including mixed tree-loop and mixed heavy-light generated operators. The number of local operators is larger than the one usually reported in the literature.

Keywords: Effective field theories, Beyond Standard Model, Nonperturbative Effects, Renormalization Regularization and Renormalons

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1 Introduction

There are two ways to use effective field theories (EFT), the bottom-up approach and the top-down approach. To apply the first, we must distinguish between two scenarios: a) there is no relevant theory at the energy scale under consideration, in which case one has to construct a Lagrangian from the symmetries that are relevant at that scale, b) there is already some EFT, e.g. Standard Model (SM) EFT or SMEFT, which represents the physics in a region characterized by a cut-off parameter Λ. At higher energies, new phenomena might show up and our EFT does not account for them.

In the top-down approach there is some theory, assumed to be ultraviolet (UV) complete or valid on a given high energy scale (e.g. some BSM model), and the aim is to implement a systematic procedure for getting the low-energy theory. A typical example would be the Euler-Heisenberg Lagrangian. Systematic low-energy expansions are able to obtain low-energy footprints of the high energy regime of the theory.

In the top-down approach the heavy fields are integrated out of the underlying high-energy theory and the resulting effective action is then expanded in a series of local operator terms. The bottom-up approach is constructed by completely removing the heavy fields, as opposed to integrating them out; this removal is compensated by including any new non-renormalizable interaction that may be required. If the UV theory is known, appropriate matching calculations will follow.

In this work we will discuss the integration of heavy fields in a wide class of BSM models, containing more that one representation for scalars, with the presence of mixing. For early work on the subject, see refs. [1–3]. One problem in dealing with BSM models is the absence of a well-defined hierarchy of scales, see ref. [4, 5] for a discussion. A second problem, as observed in ref. [6], is that there are cases where the so-called covariant derivative expansion [7–10] (CDE) does not reproduce all the local operators in the low-energy sector.

In most BSM models, loop effects are certainly suppressed and the leading observable consequences are those generated at tree level. However, considering projections for the precision to be reached in LHC Run-II analysis, LO results for interpretations of the data are challenged by consistency concerns and are not sufficient if the cut off scale is in the few TeV range. Moving to the consistent inclusion of loop effects adds complexity but robustly accommodates the precision projected to be achieved in Run-II analyses.

The aim of this paper is not to guess which is the UV completion of the SM chosen by nature, but rather to present in a systematic way how the calculation of a (top-down) EFT for any realistic model should be done.

The paper is organized as follows: in section 2 we present the general formalism. The low energy behavior for the singlet extension of the SM is discussed in section 3 and the THDM models on section 4.

2 General formalism

The most general Lagrangian that we have in mind contains, after mixing, n heavy scalar fields (charged or neutral) and can be written as

$$\mathcal{L}_{\rm BSM} = \mathcal{L}_{\rm SM} + \Delta\mathcal{L}^{(4)} + \mathcal{L}_{\rm H}^{(4)}\,, \quad \mathcal{L}_{\rm H}^{(4)} = \sum_{i_1=0}^{h} \cdots \sum_{i_k=0}^{{\rm I}_{k-1}} \cdots \sum_{i_{n-1}=0}^{{\rm I}_{n-2}} {\rm F}^{h}_{i_1 \ldots i_n}\, {\rm H}_1^{i_1} \ldots {\rm H}_n^{{\rm I}_{n-1}} + {\rm h.c.}\,, \tag{2.1}$$

where ${\rm I}_k = h - i_1 - \ldots - i_{k-1}$. The term $\mathcal{L}_{\rm SM}$ is the SM Lagrangian and $\Delta\mathcal{L}^{(4)}$ contains light fields only and it is proportional to non-SM couplings (i.e. corrections to SM-couplings, due to the new interactions). Furthermore, ${\rm F}^h$ is a function of the light fields with canonical dimension $4-h$.

Specific examples for the terms in the Lagrangian of eq. (2.1) are: $\Phi^\dagger\,\Phi\,{\rm S}$, where Φ is the standard Higgs doublet and S is a singlet; $\Phi^\dagger\,\tau_{\rm a}\,\Phi\,{\rm T}^{\rm a}$, where $\rm T^a$ is a scalar (real or complex) triplet [11, 12]; $\Phi^\dagger\,\tau_{\rm a}\,\Phi^{\rm c}\,\Xi^\dagger\,{\rm t}^{\rm a}\,{\rm X}$ where Ξ is a zero hypercharge real triplet, X a $Y=1$ complex triplet and $\Phi^{\rm c}$ is the charge conjugate of Φ, the so-called Georgi-Machacek model, see ref. [13]. For a classification of CP even scalars according to their properties

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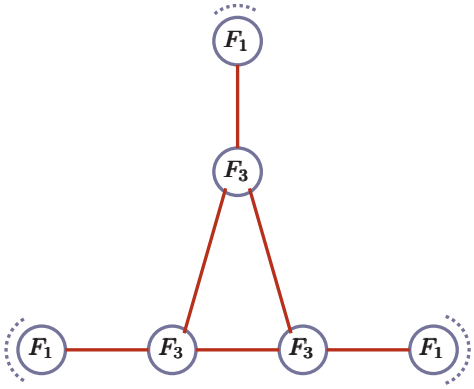

Figure 1. Example of mixed tree-loop-generated operator. Solid (red) lines denote heavy fields, Blobs denote vertices with additional light lines.

under custodial symmetry see refs. [14, 15]. For a discussion on fingerprints of non-minimal Higgs sectors, see ref. [16].

There are two sources of deviations with respect to the SM: new couplings and modified couplings due to VEV mixings, heavy fields. In general, it is not simple to identify only one scale for new physics (NP); it is relatively simple in the unbroken phase using weak eigenstates but it becomes more complicated when EWSB is taken into account and one works with the mass eigenstates. In the second case, one should also take into account that there are relations among the parameters of the BSM model, typically coupling constants can be expressed in terms of VEVs and masses; once the heavy scale has been introduced also these relations should be consistently expanded. Briefly, the SM decoupling limit cannot be obtained by making only assumptions about one parameter. This fact adds additional operators to the SM that are not those caused by integrating out the heavy fields.

There are three reasons why published CDE results do not give the full result in explicit form (e.g. see the $\mathcal{O}(\Phi_c^3)$ terms in eq. (2.7) of ref. [9]).

1. The functions F in eq. (2.1) may contain positive powers of the heavy scale, so that terms of dimension greater than 2 in the heavy fields have been retained in our functional integral (the linear terms as well).

2. The second reason is that there are mixed tree-loop-generated operators, see figure 1, where we show a diagram that, after integration of the internal heavy lines and contraction of the external heavy lines gives a contribution $\mathcal{O}(\Lambda^{-2})$ (here $\mathrm{F}_{1,3} \propto \Lambda$).

3. The third reason is that there are mixed loops, containing both light and heavy particles.

In the following we will discuss the full derivation of the low-energy limit for the case of one (neutral) heavy field, the generalization being straightforward. One important point is that the request of having the heavy state H and small mixing with the light scalar implies that the heavy mass is not equal to the heavy scale (the one controlling the expansion).

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The former, $M_{\rm H}$, is expressed in terms of Λ by

$$M_{\rm H}^2 = \Lambda^2 \sum_{n=0}^{\infty} \xi_n \left(\frac{M^2}{\Lambda^2}\right)^n , \tag{2.2}$$

with coefficients ξ_i that depend on the model. Further, we define M to be the W bare mass. We will show explicit examples in the following sections, e.g. in eq. (3.13). Therefore, we write the Lagrangian as follows:

$$\mathcal{L} = \mathcal{L}_0 + {\rm H}\,\mathcal{O}_2\,{\rm H} + \sum_{n=1}^{4} {\rm F}_n\,{\rm H}^n , \tag{2.3}$$

where $\mathcal{O}_2$ is the Klein-Gordon operator for the heavy field H. In this section we introduce the Lagrangian and the full discussion of tadpoles is postponed until section 3.3.1. For a complete analysis of tadpole terms we refere to ref. [24], in particular section (2.2) and eq. (6). Therefore, without loss of generality, we assume the following behavior

$$\begin{aligned} &{\rm F}_1(x) = \Lambda\,{\rm F}_{10}(x) + \Lambda^{-1}\,{\rm F}_{11}(x) + \Lambda^{-3}\,{\rm F}_{12}(x)\,, \qquad {\rm F}_2(x) = {\rm F}_{20}(x) + \Lambda^{-2}\,{\rm F}_{21}(x) + \Lambda^{-4}\,{\rm F}_{22}(x)\,, \\ &{\rm F}_3(x) = \Lambda\,{\rm F}_{30}(x) + \Lambda^{-1}\,{\rm F}_{31}(x) + \Lambda^{-3}\,{\rm F}_{32}(x)\,, \qquad {\rm F}_4(x) = {\rm F}_{40}(x) + \Lambda^{-2}\,{\rm F}_{41}(x) + \Lambda^{-4}\,{\rm F}_{42}(x)\,, \end{aligned} \tag{2.4}$$

where Λ is the scale controlling the onset of new physics, not necessarily equal to the mass of the heavy field. Furthermore, we have truncated the expansion at the right level to derive dim = 6 operators and: $\dim {\rm F}_{1n} = \dim {\rm F}_{2n} = 2\,(n+1)$, $\dim {\rm F}_{3n} = \dim {\rm F}_{4n} = 2\,n$.

Terms in $\Lambda^3\,{\rm H}$ could be incorporated in $\rm F_1$ but, usually, they arise as (tadpole) beta-terms and their inclusion is postponed until eq. (3.20), extending eq. (2.4). Terms in $\Lambda^2\,{\rm H}^2$ have a double origin, mass terms (from eq. (2.2)) that are kept in the KG operator of eq. (2.3) and will be expanded only after loop integration and, once again, (tadpole) beta-terms (see eq. (7) of ref. [24]) to be inserted and discussed in eq. (3.20).

Terms with negative powers of Λ in eq. (2.4) arise as a consequence of eq. (2.2), i.e. $M_{\rm H}$ expanded in the part of the Lagrangian not containing $\mathcal{O}_2$ and of the corresponding expansion of the mixing angle (see eq. (3.12) for an explicit example). Their field content respects the renormalizability of the UV theory.

The integration of the heavy mode, H, gives an effective Lagrangian and results in the addition of tree-generated, loop-generated, tree-loop-generated and mixed heavy-light loop-generated operators. Actually there are two different ways to construct a low-energy theory: one can integrate the heavy particles by diagrammatic methods, or use functional methods; for both cases see ref. [17]. Our derivation is as follows: consider the functional integral

$${\rm W} = \int [{\rm D\,H}]\,\exp\Big\{ i \int d^4x\,\mathcal{L}_{\rm H} \Big\}\,, \quad \mathcal{L}_{\rm H} = -\frac{1}{2}\,\partial_\mu {\rm H}\,\partial_\mu {\rm H} - \frac{1}{2}\,M_{\rm H}^2\,{\rm H}^2 + \sum_{n=1}^{4} {\rm F}_n\,{\rm H}^n\,. \tag{2.5}$$

Using standard algorithms we obtain

$${\rm W} = \exp\Big\{ i \int d^4y \sum_{n=2}^{4} {\rm F}_n(y)\,\left(-\,i\,\delta_{{\rm F}_1}(y)\right)^n \Big\} \int [{\rm D\,H}]\,\exp\Big\{ i \int d^4x\,\mathcal{L}_{\rm H}^{(0)} \Big\}\,, \tag{2.6}$$

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where we have introduced a free Lagrangian with a source term for the heavy field,

$$\mathcal{L}_{\rm H}^{(0)} = -\frac{1}{2}\,\partial_\mu \,{\rm H}\,\partial_\mu \,{\rm H} - \frac{1}{2}\,M_{\rm H}^2\,{\rm H}^2 + {\rm F}_1\,{\rm H}\,, \tag{2.7}$$

and the functional derivative

$$\delta_{{\rm F}_1}(x) = \frac{\delta}{\delta\,{\rm F}_1(x)}\ . \tag{2.8}$$

It is worth noting that eq. (2.6) is needed in order to reproduce mixed tree-loop-generated operators. Using the well-known result

$$\int [{\rm D\,H}]\,\exp\!\left\{i\int d^4x\,\mathcal{L}_{\rm H}^{(0)}\right\} = {\rm W}_0\,\exp\!\left\{-\frac{1}{2}\int d^4u\,d^4v\,{\rm F}_1(u)\,\Delta_{\rm F}\,(u-{\rm v})\,\,{\rm F}_1(v)\right\}, \tag{2.9}$$

where $\rm W_0$ is the $\rm F_1$ -independent normalization constant and $\Delta_{\rm F}(z)$ is the Feynman propagator,

$$\Delta_{\rm F}(z) = \frac{1}{(2\,\pi)^4\,i}\int d^4p\,\frac{\exp\{i\,p\cdot z\}}{p^2+M_{\rm H}^2-i\,0}\ . \tag{2.10}$$

The effective Lagrangian (up to order Λ^{-2}) becomes

$${\rm W} = {\rm W}_0\,\exp\!\left\{i\int d^4x\,\mathcal{L}_{\rm eff}\right\}, \qquad \mathcal{L}_{\rm eff} = \mathcal{L}_{\rm eff}^{\rm T} + \frac{1}{16\,\pi^2}\,\mathcal{L}_{\rm eff}^{\rm L}\ . \tag{2.11}$$

The tree-generated Lagrangian becomes

$$\mathcal{L}_{\rm eff}^{\rm T} = \frac{1}{2}\,\xi_0^{-1}\,{\rm F}_{10}^2 + \frac{1}{\xi_0^3\,\Lambda^2}\left[{\rm F}_{10}^3\,{\rm F}_{30} + \frac{1}{2}\,\xi_0\left(2\,{\rm F}_{10}^2\,{\rm F}_{20} - M^2\,\xi_1\,{\rm F}_{10}^2 - \partial_\mu {\rm F}_{10}\,\partial^\mu {\rm F}_{10}\right) + \xi_0^2\,{\rm F}_{10}\,{\rm F}_{11}\right] \tag{2.12}$$

It is worth noting that there are terms, e.g. those proportional to $\rm F_{30}$, that are left implicit in the published CDE results.

A construction of tree-generated vertices is shown in figure 2 where the result of functional integration is seen from a different perspective, as a contraction of propagators inside diagrams of the full theory. In eq. (2.4) we see why it is not enough to use the Lagrangian truncated at $\mathcal{O}(\rm H)$ to derive tree-generated operators; for instance, both $\rm F_1$ and $\rm F_3$ start at $\mathcal{O}(\Lambda)$ which is enough to compensate the $M_{\rm H}^{-6}$ from H propagators giving a result at $\mathcal{O}(\Lambda^{-2})$, as shown in the third row of figure 2.

If we restrict to dim = 6 operators (i.e to order Λ^{-2}), loop-induced operators generated by the functional integral belong to three different cases:

1. There are triangles with heavy, internal, lines (third row in figure 3); in the limit of large internal (equal) masses, the corresponding loop integral gives

$${\rm C}_0^{\rm H} = -\frac{1}{2}\,M_{\rm H}^{-2} + \mathcal{O}\left(M_{\rm H}^{-4}\right)\ . \tag{2.13}$$

2. There are also bubbles (first row in figure 3); in the limit of large internal (equal) masses, the corresponding loop integral gives

$${\rm B}_0\left(P^2\,;\,M_{\rm H}^2\,,\,M_{\rm H}^2\right) = -{\rm B}_{00}\left(M_{\rm H}\right) - \frac{1}{6}\,\frac{P^2}{M_{\rm H}^2} + \mathcal{O}\left(\frac{P^4}{M_{\rm H}^4}\right), \tag{2.14}$$

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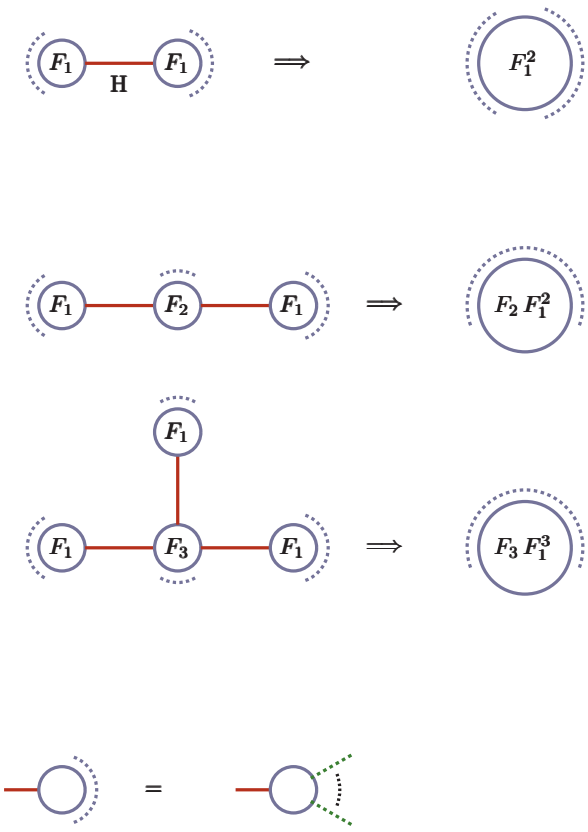

Figure 2. Example of tree-generated operators. Solid (red) lines denote heavy fields, dashed (green) lines denote light fields. Blobs denote vertices with additional light lines.

with $\mathrm{B}_{00}(M_\mathrm{H}) = \overline{\mathrm{A}}_0(M_\mathrm{H}) + 1$ and the (dimensionless) one-point function. $\overline{\mathrm{A}}_0$ is defined in dimensional regularization by

$$\mu_\mathrm{R}^{4-n}\int\frac{d^nq}{q^2+M_\mathrm{H}^2} = i\,\pi^2\overline{\mathrm{A}}_0(M_\mathrm{H})\,M_\mathrm{H}^2 = i\,\pi^2 M_\mathrm{H}^2\left(\frac{2}{n-4}+\gamma+\ln\pi-1+\ln\frac{M_\mathrm{H}^2}{\mu_\mathrm{R}^2}\right), \quad (2.15)$$

where n is the space-time dimension, γ is the Euler-Mascheroni constant and μ_R is the renormalization scale.

3. Finally, there are tadpoles, as shown in the second row of figure 3. The treatment of tadpoles, i.e. their cancellation, is model dependent. Here we present the list of tadpoles and postpone discussing their cancellation until section 3.3.1. Therefore, in writing $\mathcal{L}_\mathrm{eff}^\mathrm{L}$ we split the Lagrangian into two parts: the one containing tadpoles and the one without. Examples are shown in figure 4 for the $\mathrm{h}^2\,\mathrm{Z}^2$ operator: the left diagram is a H tadpole while the right one is a genuine LG operator.

With the Λ power counting of eq. (2.4) and loop power counting of eq. (A.2) it is easily seen that boxes of heavy lines start contributing only at $\mathcal{O}(\Lambda^{-4})$.

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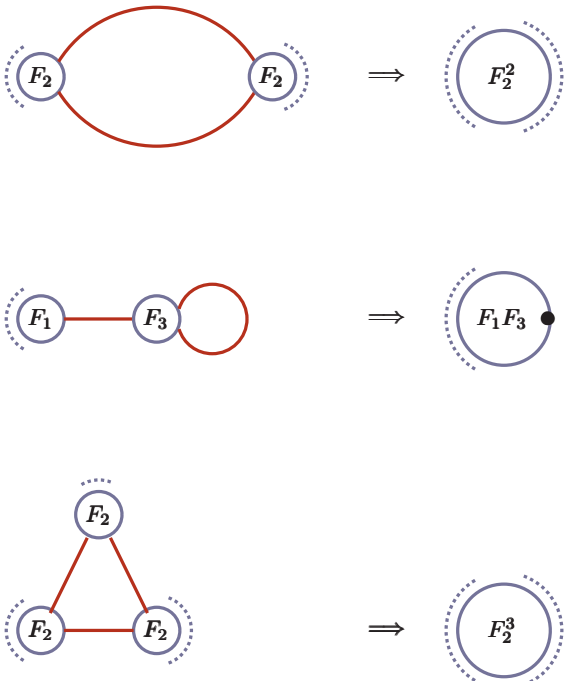

Figure 3. Example of loop-generated operators. Solid (red) lines denote heavy fields, blobs denote vertices with additional light lines. The (black) bullet denotes a tadpole.

The F functions, defined in eq. (2.4), are polynomials in the light fields of the form $\mathrm{F}_{ij} = \mathrm{F}^{\mathrm{h}}_{ij} + \mathrm{F}^{\mathrm{rest}}_{ij}$, where $\mathrm{F}^{\mathrm{h}}_{ij} = \kappa_{ij}\,\mathrm{h}$ and "rest" contains two or more fields. The result, split in non-tadpole (NT) and tadpole (T) contributions, is as follows

$$\mathcal{L}^{\mathrm{L}}_{\mathrm{eff\,NT}} = \xi_0\,\Lambda^2\,\mathcal{L}^{\mathrm{L},2}_{\mathrm{eff\,NT}} + \frac{1}{\xi_0^2}\,\mathcal{L}^{\mathrm{L},0}_{\mathrm{eff\,NT}} + \frac{1}{\xi_0^4\,\Lambda^2}\,\mathcal{L}^{\mathrm{L},-2}_{\mathrm{eff\,NT}} \tag{2.16}$$

$$\begin{aligned}
\mathcal{L}^{\mathrm{L},2}_{\mathrm{eff\,NT}} &= \overline{\mathrm{A}}_0\,\mathrm{F}^{\mathrm{rest}}_{20}\,,\\
\mathcal{L}^{\mathrm{L},0}_{\mathrm{eff\,NT}} &= -\,\mathrm{B}_{00}\left(9\,\mathrm{F}^2_{10}\,\mathrm{F}^2_{30} + 6\,\xi_0\,\mathrm{F}_{10}\,\mathrm{F}_{20}\,\mathrm{F}_{30} + \xi_0^2\,\mathrm{F}^2_{20}\right)\\
&\quad +6\,\overline{\mathrm{A}}_0\,\xi_0\,\mathrm{F}^2_{10}\,\mathrm{F}_{40} + \xi_0^2\,\overline{\mathrm{A}}_0\,\left(\xi_1\,M^2\,\mathrm{F}^{\mathrm{rest}}_{20} + \xi_0\,\mathrm{F}^{\mathrm{rest}}_{21}\right)\,,\\
\mathcal{L}^{\mathrm{L},-2}_{\mathrm{eff\,NT}} &= 18\,(1-3\,\mathrm{B}_{00})\,\mathrm{F}^2_{10}\,\mathrm{F}^2_{30}\,(\mathrm{F}_{10}\,\mathrm{F}_{30} + \xi_0\,\mathrm{F}_{20})\\
&\quad +2\,\xi_0\,\mathrm{B}_{00}\Big[9\,M^2\,\xi_1\,\mathrm{F}^2_{10}\,\mathrm{F}^2_{30} - 3\,\xi_0\,\mathrm{F}_{10}\,\mathrm{F}_{30}\,\left(3\,\mathrm{F}_{10}\,\mathrm{F}_{31} + 3\,\mathrm{F}_{11}\,\mathrm{F}_{30} - M^2\,\xi_1\,\mathrm{F}_{20}\right)\\
&\quad -3\,\xi_0^2\,(\mathrm{F}_{10}\,\mathrm{F}_{21}\,\mathrm{F}_{30} + \mathrm{F}_{10}\,\mathrm{F}_{20}\,\mathrm{F}_{31} + \mathrm{F}_{11}\,\mathrm{F}_{20}\,\mathrm{F}_{30}) - \xi_0^3\,\mathrm{F}_{20}\,\mathrm{F}_{21}\Big]\\
&\quad +6\,\xi_0^2\,(1-2\,\mathrm{B}_{00})\,\mathrm{F}_{10}\,\mathrm{F}^2_{20}\,\mathrm{F}_{30}\\
&\quad -\frac{1}{6}\,\xi_0\left[9\,\mathrm{F}^2_{30}\,\partial_\mu\mathrm{F}_{10}\,\partial^\mu\mathrm{F}_{10} + 6\,\xi_0\,\mathrm{F}_{30}\,\partial_\mu\mathrm{F}_{10}\,\partial^\mu\mathrm{F}_{20} - \xi_0^2\,\left(4\,\mathrm{F}^3_{20} - \partial_\mu\mathrm{F}_{20}\,\partial^\mu\mathrm{F}_{20}\right)\right]\\
&\quad -6\,\overline{\mathrm{A}}_0\,\xi_0^2\,\mathrm{F}_{10}\left[M^2\,\xi_1\,\mathrm{F}_{10}\,\mathrm{F}_{40} - \xi_0\,(\mathrm{F}_{10}\,\mathrm{F}_{41} + 2\,\mathrm{F}_{11}\,\mathrm{F}_{40})\right]\\
&\quad -12\xi_0\mathrm{B}_{00}\,\mathrm{F}_{40}\,\mathrm{F}^2_{10}\,(3\mathrm{F}_{10}\,\mathrm{F}_{30}+\xi_0\mathrm{F}_{20}) + \xi_0^4\overline{\mathrm{A}}_0\left[M^2\,\left(\xi_2 M^2\mathrm{F}^{\mathrm{rest}}_{20} + \xi_1\mathrm{F}^{\mathrm{rest}}_{21}\right) + \xi_0\,\mathrm{F}^{\mathrm{rest}}_{22}\right],
\end{aligned} \tag{2.17}$$

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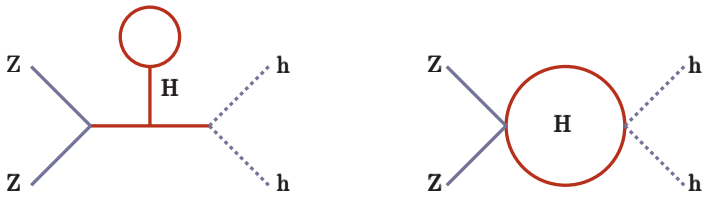

Figure 4. Example of loop-generated operators, Solid (red) lines denote the heavy field H, solid and dashed (blue) lines denote light SM fields. Left figure shows a H tadpole to be canceled in the β-scheme while right figure shows a genuine LG operator.

$$
\begin{aligned}
\mathcal{L}^{\mathrm{L}}_{\mathrm{eff\,T}} &= \overline{\mathrm{A}}_0 \left[\Lambda^2 \, \mathcal{L}^{\mathrm{L},2}_{\mathrm{eff\,T}} + \frac{1}{\xi_0^2} \, \mathcal{L}^{\mathrm{L},0}_{\mathrm{eff\,T}} + \frac{1}{\xi_0^4 \, \Lambda^2} \, \mathcal{L}^{\mathrm{L},-2}_{\mathrm{eff\,T}} \right] \\
\mathcal{L}^{\mathrm{L},2}_{\mathrm{eff\,T}} &= 3\,\mathrm{F}_{10}\,\mathrm{F}_{30} + \xi_0\,\mathrm{F}^{\mathrm{h}}_{20}\,, \\
\mathcal{L}^{\mathrm{L},0}_{\mathrm{eff\,T}} &= 9\,\mathrm{F}^2_{10}\,\mathrm{F}^2_{30} + 6\,\xi_0\,\mathrm{F}_{10}\,\mathrm{F}_{20}\,\mathrm{F}_{30} + \xi_0^2 \left(3\,\mathrm{F}_{10}\,\mathrm{F}_{31} + 3\,\mathrm{F}_{11}\,\mathrm{F}_{30} + M^2\,\xi_1\,\mathrm{F}^{\mathrm{h}}_{20} \right) + \xi_0^3\,\mathrm{F}^{\mathrm{h}}_{21}\,, \\
\mathcal{L}^{\mathrm{L},-2}_{\mathrm{eff\,T}} &= 54\,\mathrm{F}^3_{10}\,\mathrm{F}^3_{30} - 18\,\xi_0\,\mathrm{F}^2_{10}\,\mathrm{F}^2_{30} \left(M^2\,\xi_1 - 3\,\mathrm{F}_{20} \right) \\
&\quad + 6\,\xi_0^2 \Big[\mathrm{F}_{10} \left(3\,\mathrm{F}_{10}\,\mathrm{F}_{30}\,\mathrm{F}_{31} + 3\,\mathrm{F}_{11}\,\mathrm{F}^2_{30} - M^2\,\xi_1\,\mathrm{F}_{20}\,\mathrm{F}_{30} + 2\,\mathrm{F}^2_{20}\,\mathrm{F}_{30} \right) \\
&\quad - \mathrm{F}_{30}\,\partial_\mu \mathrm{F}_{10}\,\partial^\mu \mathrm{F}_{20} \Big] \\
&\quad + 3\,\xi_0^3 \Big[2\,\mathrm{F}_{10}\,(\mathrm{F}_{21}\,\mathrm{F}_{30} + \mathrm{F}_{20}\,\mathrm{F}_{31}) + 2\,\mathrm{F}_{11}\,\mathrm{F}_{20}\,\mathrm{F}_{30} - \partial_\mu \mathrm{F}_{10}\,\partial^\mu \mathrm{F}_{31} \Big] \\
&\quad + \xi_0^4 \left(3\,\mathrm{F}_{10}\,\mathrm{F}_{32} + 3\,\mathrm{F}_{11}\,\mathrm{F}_{31} + 3\,\mathrm{F}_{12}\,\mathrm{F}_{30} + M^4\,\xi_2\,\mathrm{F}^{\mathrm{h}}_{20} + M^2\,\xi_1\,\mathrm{F}^{\mathrm{h}}_{21} \right) + \xi_0^5\,\mathrm{F}^{\mathrm{h}}_{22}\,.
\end{aligned}
\tag{2.18}
$$

We are still missing mixed loop contributions; they are clearly model dependent and will be discussed in details in section 3. The results of eqs. (2.17)–(2.18) form a basis of local operators.

3 Low energy behavior for the singlet extension of the SM

The SM scalar field Φ (with hypercharge 1/2) is defined by

$$
\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \mathrm{h}_2 + \sqrt{2}\,\mathrm{v} + i\,\phi^0 \\ \sqrt{2}\,i\,\phi^- \end{pmatrix}\,, \tag{3.1}
$$

h_2 is the custodial singlet in $(2_{\mathrm{L}} \otimes 2_{\mathrm{R}}) = 1 \oplus 3$. Charge conjugation gives $\Phi^{\mathrm{c}}_{\mathrm{i}} = \varepsilon_{\mathrm{ij}}\,\Phi^*_{\mathrm{j}}$.

3.1 Notations and conventions

The Lagrangian giving the singlet extension [18–22] of the SM (SESM) is

$$
\mathcal{L} = -\left(\mathrm{D}_\mu\,\Phi\right)^\dagger \mathrm{D}_\mu\,\Phi - \partial_\mu\,\chi\,\partial_\mu\,\chi - \mu_2^2\,\Phi^\dagger\,\Phi - \mu_1^2\,\chi^2 - \frac{1}{2}\,\lambda_2 \left(\Phi^\dagger\,\Phi\right)^2 - \frac{1}{2}\,\lambda_1\,\chi^4 - \lambda_{12}\,\chi^2\,\Phi^\dagger\,\Phi\,, \tag{3.2}
$$

where the singlet field and the covariant derivative D_μ are

$$
\chi = \frac{1}{\sqrt{2}}\,(\mathrm{h}_1 + \mathrm{v}_{\mathrm{s}})\,, \quad \mathrm{D}_\mu = \partial_\mu - \frac{i}{2}\,g\,B^a_\mu\,\tau_a - \frac{i}{2}\,g\,g_{_1}\,B^0_\mu\,, \tag{3.3}
$$

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with $g_{_1} = -\mathrm{s}_{_\mathrm{W}}/\mathrm{c}_{_\mathrm{W}}$ and where τ^a are Pauli matrices while $\mathrm{s}_{_\mathrm{W}}\,(\mathrm{c}_{_\mathrm{W}})$ is the sine(cosine) of the weak-mixing angle. Furthermore

$$\mathrm{W}^{\pm}_{\mu} = \frac{1}{\sqrt{2}}\left(B^1_\mu \mp i\, B^2_\mu\right), \qquad \mathrm{Z}_\mu = \mathrm{c}_{_\mathrm{W}}\, B^3_\mu - \mathrm{s}_{_\mathrm{W}}\, B^0_\mu, \qquad \mathrm{A}_\mu = \mathrm{s}_{_\mathrm{W}}\, B^3_\mu + \mathrm{c}_{_\mathrm{W}}\, B^0_\mu, \tag{3.4}$$

$$F^a_{\mu\nu} = \partial_\mu B^a_\nu - \partial_\nu B^a_\mu + g_{_0}\,\epsilon^{abc}\, B^b_\mu\, B^c_\nu, \qquad F^0_{\mu\nu} = \partial_\mu B^0_\nu - \partial_\nu B^0_\mu\,. \tag{3.5}$$

Here $a, b, \cdots = 1, \ldots, 3$. Here, $g_{_0} = g(1 + g^2\Gamma)$ where Gamma is fixed by the request that the $\mathrm{Z} - \gamma$ transition at zero momentum is zero (see section 3 of ref. [24]). Further, we keep M as the W bare mass and add a new mass, $M_{_\mathrm{S}}$, which will play the role of cut-off scale Λ. Also,

$$M^2 = \frac{1}{2}\, g^2\, \mathrm{v}^2, \qquad M^2_{_\mathrm{S}} = \frac{1}{4}\, g^2\, \mathrm{v}^2_{_\mathrm{S}}\,. \tag{3.6}$$

In order to write eq. (3.2) in terms of mass eigenstates we introduce

$$R^2 = \left(\lambda_2\, \mathrm{v}^2 - \frac{1}{2}\,\lambda_2\, \mathrm{v}^2_{_\mathrm{S}}\right)^2 + 2\,\left(\lambda_{12}\, \mathrm{v}\, \mathrm{v}_{_\mathrm{S}}\right)^2\,. \tag{3.7}$$

The mixing angle α is defined in terms of $\sin(\alpha)$ and $\cos(\alpha)$ $(\mathrm{s}_{_\alpha}, \mathrm{c}_{_\alpha})$ by

$$\mathrm{h} = \mathrm{c}_{_\alpha}\, \mathrm{h}_2 - \mathrm{s}_{_\alpha}\, \mathrm{h}_1\,, \qquad \mathrm{H} = \mathrm{s}_{_\alpha}\, \mathrm{h}_2 + \mathrm{c}_{_\alpha}\, \mathrm{h}_1\,, \tag{3.8}$$

$$\sin(2\,\alpha) = \sqrt{2}\,\lambda_{12}\, \mathrm{v}\, \mathrm{v}_{_\mathrm{S}}\, R^{-1}\,, \qquad \cos(2\,\alpha) = \left(-\lambda_2\, \mathrm{v}^2 + \frac{1}{2}\,\lambda_1\, \mathrm{v}^2_{_\mathrm{S}}\right) R^{-1}\,, \tag{3.9}$$

being h and H the fields related to the mass eigenvalues $M_{_\mathrm{h}}$ and $M_{_\mathrm{H}}$. Next, we can eliminate μ_1, μ_2 in eq. (3.2),

$$\mu_2^2 = -2\,\frac{\lambda_2}{g^2}\, M^2 - 2\,\frac{\lambda_{12}}{g^2}\, M^2_{_\mathrm{S}}\,, \qquad \mu_1^2 = -2\,\frac{\lambda_1}{g^2}\, M^2_{_\mathrm{S}} - 2\,\frac{\lambda_{12}}{g^2}\, M^2\,. \tag{3.10}$$

We keep λ_1 and λ_{12} as free parameters and take the limit $M_{_\mathrm{S}} \to \infty$. Following ref. [23] we will assume that the ratio of couplings is of the order of a perturbative coupling, i.e. $\lambda_{12}/\lambda_1^2 < 1/2$. First we eliminate λ_2, in favor of the h mass

$$\lambda_2 = \frac{1}{4}\, g^2\, \frac{M^2_{_\mathrm{h}}}{M^2} + g^2\, \frac{t_3^2}{t_1} + \frac{1}{4}\, g^2\, \frac{t_3^2}{t_1^2}\, \frac{M^2_{_\mathrm{h}}}{M^2_{_\mathrm{S}}} + \mathcal{O}\left(M^{-4}_{_\mathrm{S}}\right), \tag{3.11}$$

where $\lambda_1 = t_1\, g^2$ and $\lambda_{12} = t_3\, g^2$. Similarly, we obtain the expansion for $\sin(\alpha)$ and $\cos(\alpha)$,

$$\mathrm{c}_{_\alpha} = 1 - \frac{1}{2}\,\frac{t_3^2}{t_1^2}\,\frac{M^2}{M^2_{_\mathrm{S}}} + \mathcal{O}\left(M^{-4}_{_\mathrm{S}}\right), \quad \mathrm{s}_{_\alpha} = \frac{t_3}{t_1}\,\frac{M}{M_{_\mathrm{S}}}\left[1 + \left(\frac{t_2}{t_1} - \frac{3}{2}\,\frac{t_3^2}{t_1^2}\right)\,\frac{M^2}{M^2_{_\mathrm{S}}}\right] + \mathcal{O}\left(M^{-5}_{_\mathrm{S}}\right). \tag{3.12}$$

Remark. The behavior of $\mathrm{s}_{_\alpha}$ is not selected a priori but follows from the hierarchy of VEVs. Additional suppression of the heavy mode can be imposed by requiring $\lambda_{12} \propto g^2\, M/M_{_\mathrm{S}}$, i.e. this additional suppression of $\mathrm{s}_{_\alpha}$ is an independent condition. In any case, the SM decoupling limit cannot be obtained by making only assumptions about one parameter. We adopt the more conservative approach, considering the non-decoupling limit and λ_{12} as a free parameter of the effective theory.

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Finally, the relation between the H mass and $M_{\rm s}$ is

$$M_{\rm H}^2 = 4\,t_1\,M_{\rm s}^2\left[1+\frac{t_3^2}{t_1^2}\frac{M^2}{M_{\rm s}^2}+\mathcal{O}\left(M_{\rm s}^{-4}\right)\right], \tag{3.13}$$

comparing with eq. (2.2), the ξ parameters are defined by

$$\xi_0 = 4\,t_1, \qquad \xi_1 = 4\,\frac{t_3^2}{t_1}, \qquad \xi_2 = \frac{t_3^2}{t_1^2}\frac{M_{\rm h}^2}{M^2}\,. \tag{3.14}$$

Assuming that $M_{\rm H} \gg M_{\rm h}$ we construct the corresponding low scale approximation of the model [1, 2]. There are three options that will be discussed in the section 3.2, section 3.3 and section 3.6.

3.2 Integration of the weak eigenstate

Starting from eq. (3.2) we can construct a manifestly SU(2) × U(1) invariant low energy Lagrangian by integrating out the field $\rm h_1$ in the limit $\mu_1 \to \infty$. Note that, from eq. (3.10) the difference between $\mu_1 \to \infty$ and $M_{\rm s} \to \infty$ is sub-leading in $M_{\rm s}$. This is what has been discussed in refs. [4, 23] and we only repeat the observation of refs. [23] that this approach reproduces the effect of scalar mixing on interactions involving one Higgs scalar h, but fails otherwise.

3.3 Integration of the mass eigenstate

In the limit $\Lambda = M_{\rm s} \to \infty$ the structure of the calculation is more complex since the Lagrangian is given by a power expansion even before integrating out the H field, see eq. (2.3). In the following we will describe the steps that are needed to consistently perform the limit.

3.3.1 Tadpoles

Unless the calculation of observables is performed at tree level, tadpoles should be introduced and discussed. Their presence and the heavy-light mixing represent an additional complication. For instance, in the full singlet extension we have H tadpoles and the relations presented in section 3.1 must be modified. Thus, working in the $\beta_{\rm h}$-scheme of ref. [24], we write

$$\mu_2^2 = -2\,\frac{\lambda_2}{g^2}\,M^2 - 2\,\frac{\lambda_{12}}{g^2}\,M_{\rm s}^2 + \beta_2\,, \qquad \mu_1^2 = -2\,\frac{\lambda_1}{g^2}\,M_{\rm s}^2 - 2\,\frac{\lambda_{12}}{g^2}\,M^2 + \beta_1\,. \tag{3.15}$$

Furthermore, we define expansions as follows:

$$\beta_i = \frac{g^2}{16\,\pi^2}\,\beta_i'\,M_{\rm s}^2\,, \qquad \beta_i' = \beta_i^{(0)} + \sum_{n=1}^{\infty}\beta_i^{(n)}\left(\frac{M^2}{M_{\rm s}^2}\right)^n\,. \tag{3.16}$$

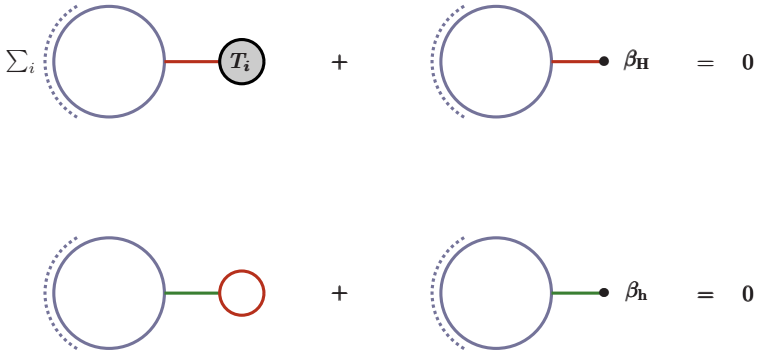

Figure 5. Cancellation of tadpoles. Solid (red) lines denote the heavy H fields, solid (green) lines denote the light h field. Black blobs denote a β-vertex.

The H tadpoles are easily computed in the full theory, giving

$$\begin{aligned}
\mathrm{T_H} = & -i\,\pi^2\,g\,M\,\mathrm{s}_\alpha\Big[2\,M^2+\frac{M^2}{\mathrm{c}_\mathrm{W}^4}+\left(\frac{1}{2}\,\frac{M_\mathrm{H}^2}{M^2}+3\right)\,M^2\,\overline{\mathrm{A}}_0(M)+\left(\frac{1}{4}\,\frac{M_\mathrm{H}^2}{M^2}+\frac{3}{2}\,\frac{1}{\mathrm{c}_\mathrm{W}^2}\right)\,\frac{M^2}{\mathrm{c}_\mathrm{W}^2}\,\overline{\mathrm{A}}_0(M_0)\\
& +\frac{1}{4}\,\mathrm{c}_\alpha\,\left(2\,M_\mathrm{h}^2+M_\mathrm{H}^2\right)\,\left(\frac{\mathrm{c}_\alpha}{M^2}+\frac{\mathrm{s}_\alpha}{M\,M_\mathrm{s}}\right)\,M_\mathrm{h}^2\,\overline{\mathrm{A}}_0(M_\mathrm{h})+\\
& \frac{3}{4}\,\frac{M_\mathrm{H}^2}{\mathrm{s}_\alpha}\,\left(\frac{\mathrm{s}_\alpha^3}{M^2}+\frac{\mathrm{c}_\alpha^3}{M\,M_\mathrm{s}}\right)\,M_\mathrm{H}^2\,\overline{\mathrm{A}}_0(M_\mathrm{H})+\mathrm{T}_\mathrm{H}^\mathrm{f}\Big]\,,
\end{aligned}\tag{3.17}$$

where $\mathrm{T}_\mathrm{H}^\mathrm{f}$ is the part induced by fermion loops and $M_0 = M/\mathrm{c}_\mathrm{W}$. The constants β_i' are used to cancel $\mathrm{T_H}$. Tadpoles cancellation is illustrated in figure 5.

The first step in handling tadpoles requires to fix the coefficients $\beta^{(n)}$ so that $\mathrm{T_H}$ is canceled. Furthermore, when the H field is integrated out we will have to differentiate the h tadpoles, those due to a H (heavy) loop and those due to loops of light particles; therefore, the constants β_2' is split into a part that cancels H tadpole-loops and a part which will be used in performing loop calculations in the low energy theory. We derive

$$\begin{aligned}
\beta_1^{(0)} &= -6\,t_1^2\,\overline{\mathrm{A}}_0(M_\mathrm{H})\,,\\
\beta_1^{(1)} &= -2\,\left(t_3+2\,t_1\right)\,\frac{t_3^2}{t_1}\,\overline{\mathrm{A}}_0(M_\mathrm{H})+\mathrm{T}_1\,,\\
\beta_1^{(2)} &= -\frac{1}{2}\,\left(2\,t_3-t_1\right)\,\frac{t_3^2}{t_1^2}\,x_\mathrm{h}\,\overline{\mathrm{A}}_0(M_\mathrm{H})-\frac{1}{4}\left[t_1\,x_\mathrm{h}+2\,\left(2\,t_3^2+3\,t_1\right)\right]\,\frac{1}{t_1^2}\,\mathrm{T}_1+\mathrm{T}_2\,,\\
\beta_2^{(0)} &= -2\,\left(2\,t_3+t_1\right)\,t_3\,\overline{\mathrm{A}}_0(M_\mathrm{H})\,,\\
\beta_2^{(1)} &= \Delta\,\beta_2^{(1)}-\frac{1}{2}\,\left(12\,t_3^2+5\,t_1\,x_\mathrm{h}\right)\,\frac{t_3^2}{t_1^2}\,\overline{\mathrm{A}}_0(M_\mathrm{H})+\frac{t_3}{t_1}\,\mathrm{T}_1\,,\\
\beta_2^{(2)} &= \Delta\,\beta_2^{(2)}-\frac{1}{2}\left[2\,t_1\,x_\mathrm{h}+\left(7\,t_3-t_1\right)\,t_3\right]\,\frac{t_3^2}{t_1^3}\,x_\mathrm{h}\,\overline{\mathrm{A}}_0(M_\mathrm{H})-\frac{1}{2}\,\left(2\,t_3^2+3\,t_1\right)\,\frac{t_3}{t_1^3}\,\mathrm{T}_1+\frac{t_3}{t_1}\,\mathrm{T}_2\,.
\end{aligned}\tag{3.18}$$

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where the T functions are defined by

$$
\begin{aligned}
\mathrm{T}_1 &= -\frac{1}{2}\,\overline{\mathrm{A}}_0(M_\mathrm{h})\,x_\mathrm{h}\,t_3 - \overline{\mathrm{A}}_0(M)\,t_3 - \frac{1}{2}\,\overline{\mathrm{A}}_0(M_0)\,\frac{t_3}{\mathrm{c}_\mathrm{w}^2}\,,\\
\mathrm{T}_2 &= -\frac{1}{2}\,\frac{1}{M^2}\,\mathrm{T}_\mathrm{H}^\mathrm{f}\,\frac{t_3}{t_1} - \frac{1}{4}\,\overline{\mathrm{A}}_0(M_\mathrm{h})\left[t_1\,x_\mathrm{h} - \left(3\,t_1 - 2\,t_3\,t_1 + 2\,t_3^2\right)\right]x_\mathrm{h}\,\frac{t_3}{t_1^2}\\
&\quad -\frac{3}{4}\,\overline{\mathrm{A}}_0(M_0)\,\frac{\mathrm{s}_\mathrm{w}^2}{\mathrm{c}_\mathrm{w}^4}\,\frac{t_3}{t_1} + \frac{1}{4}\left[t_1\,x_\mathrm{h} + 2\,\left(3\,t_1 + 2\,t_3^2\right)\right]\frac{1}{t_1^2}\,\mathrm{T}_1 - \frac{1}{2}\,\frac{1+2\,\mathrm{c}_\mathrm{w}^4}{\mathrm{c}_\mathrm{w}^4}\,\frac{t_3}{t_1}\,,
\end{aligned}
\tag{3.19}
$$

and where $M_\mathrm{h}^2 = x_\mathrm{h}\,M^2$. Working in the β-scheme we have additional loop-induced contributions and eq. (2.4) is modified into

$$
\begin{aligned}
\mathrm{F}_1(x) &= \Lambda^3\,\overline{\mathrm{F}}_1(x) + \Lambda\left[\mathrm{F}_{10}(x) + \overline{\mathrm{F}}_{10}(x)\right] + \Lambda^{-1}\left[\mathrm{F}_{11}(x) + \overline{\mathrm{F}}_{11}(x)\right] + \Lambda^{-3}\left[\mathrm{F}_{12}(x) + \overline{\mathrm{F}}_{12}(x)\right],\\
\mathrm{F}_2(x) &= \Lambda^2\,\overline{\mathrm{F}}_2(x) + \left[\mathrm{F}_{20}(x) + \overline{\mathrm{F}}_{20}(x)\right] + \Lambda^{-2}\left[\mathrm{F}_{21}(x) + \overline{\mathrm{F}}_{21}(x)\right] + \Lambda^{-4}\left[\mathrm{F}_{22}(x) + \overline{\mathrm{F}}_{22}(x)\right],
\end{aligned}
\tag{3.20}
$$

where the new terms are proportional to β_1 and β_2. Therefore, there is an additional part in the effective Lagrangian,

$$
\mathcal{L}^\mathrm{L}_{\mathrm{eff}\,\beta} = \frac{\Lambda^2}{\xi_0}\,\mathcal{L}^{\mathrm{L},2}_\mathrm{eff} + \frac{1}{\xi_0^3}\,\mathcal{L}^{\mathrm{L},0}_\mathrm{eff} + \frac{1}{\xi_0^5\,\Lambda^2}\,\mathcal{L}^{\mathrm{L},-2}_\mathrm{eff}
\tag{3.21}
$$

$$
\mathcal{L}^{\mathrm{L},2}_\mathrm{eff} = \mathrm{F}_{10}\,\overline{\mathrm{F}}_1\,,
$$

$$
\mathcal{L}^{\mathrm{L},0}_\mathrm{eff} = \left[3\,\mathrm{F}_{10}\,\mathrm{F}_{30}\,\overline{\mathrm{F}}_1 + \xi_0\,\mathrm{F}_{10}\,\left(\mathrm{F}_{10}\,\overline{\mathrm{F}}_2 + 2\,\mathrm{F}_{20}\,\overline{\mathrm{F}}_1 - M^2\,\xi_1\,\overline{\mathrm{F}}_1\right) + \xi_0^2\,\left(\mathrm{F}_{10}\,\overline{\mathrm{F}}_{10} + \mathrm{F}_{11}\,\overline{\mathrm{F}}_1\right)\right],
$$

$$
\begin{aligned}
\mathcal{L}^{\mathrm{L},-2}_\mathrm{eff} &= 18\,\mathrm{F}_{10}^3\,\mathrm{F}_{30}^2\,\overline{\mathrm{F}}_1 + 3\,\xi_0\,\mathrm{F}_{10}^2\,\mathrm{F}_{30}\,\left(2\,\mathrm{F}_{10}\,\overline{\mathrm{F}}_2 + 6\,\mathrm{F}_{20}\,\overline{\mathrm{F}}_1 - 3\,M^2\,\xi_1\,\overline{\mathrm{F}}_1\right)\\
&\quad + \xi_0^2\left[\mathrm{F}_{10}\,\left(3\,\mathrm{F}_{10}\,\mathrm{F}_{30}\,\overline{\mathrm{F}}_{10} + 3\,\mathrm{F}_{10}\,\mathrm{F}_{31}\,\overline{\mathrm{F}}_1 + 4\,\mathrm{F}_{10}\,\mathrm{F}_{20}\,\overline{\mathrm{F}}_2 - 2\,M^2\,\xi_1\,\mathrm{F}_{10}\,\overline{\mathrm{F}}_2 + 6\,\mathrm{F}_{11}\,\mathrm{F}_{30}\,\overline{\mathrm{F}}_1\right.\right.\\
&\quad \left.\left. + 4\,\mathrm{F}_{20}^2\,\overline{\mathrm{F}}_1 - 4\,M^2\,\xi_1\,\mathrm{F}_{20}\,\overline{\mathrm{F}}_1\right) - 2\,\overline{\mathrm{F}}_1\,\partial_\mu\mathrm{F}_{10}\,\partial^\mu\mathrm{F}_{20}\right]\\
&\quad + \xi_0^3\left[\mathrm{F}_{10}\,\left(\mathrm{F}_{10}\,\overline{\mathrm{F}}_{20} + 2\,\mathrm{F}_{11}\,\overline{\mathrm{F}}_2 + 2\,\mathrm{F}_{20}\,\overline{\mathrm{F}}_{10} + 2\,\mathrm{F}_{21}\,\overline{\mathrm{F}}_1 - M^2\,\xi_1\,\overline{\mathrm{F}}_{10}\right)\right.\\
&\quad \left. + 2\,\mathrm{F}_{11}\,\mathrm{F}_{20}\,\overline{\mathrm{F}}_1 - \partial_\mu\mathrm{F}_{10}\,\partial^\mu\overline{\mathrm{F}}_{10}\right]\\
&\quad + \xi_0^4\,\left(\mathrm{F}_{10}\,\overline{\mathrm{F}}_{11} + \mathrm{F}_{11}\,\overline{\mathrm{F}}_{10}\right)\,,
\end{aligned}
\tag{3.22}
$$

with coefficients ξ_i defined in eq. (3.14).

3.3.2 Mixed loops

In a consistent derivation of the low energy limit we must include also mixed (heavy-light) loops. Examples of mixed loops are shown in figure 8. Integration of the heavy fields is performed according to the expansion of three-point functions given in appendix A. Clearly, the result is given by (contact) local operators and by non-local terms that are

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one-loop diagrams in the low energy theory, i.e. loops with internal light lines. We give few examples, restricting the external lines to be physical (no $\phi^0, \phi^\pm$). First we define vertices as follows:

$$
\begin{aligned}
&\mathrm{V}^{2}_{\mathrm{h}} = 2\,g\,M\,t_3\,\left(1-\frac{t_3}{t_1}\right)\,, \qquad \mathrm{V}^{2}_{\mathrm{hh}} = -\frac{1}{2}\,g^2\,t_3\,, \qquad \mathrm{V}^{10}_{\mathrm{hh}} = -2\,g\,t_3\,,\\
&\mathrm{V}^{11}_{\mathrm{hh}} = g\,\frac{t_3}{t_1}\left(M^2\,\frac{t_3^2}{t_1} - 2\,M^2 t_3 - \frac{3}{2}\,M_{\mathrm{h}}^2\right), \quad \mathrm{V}^{11}_{\mathrm{hhh}} = -\frac{3}{4}\,g^2\,\frac{t_3}{t_1}\left(\frac{M_{\mathrm{h}}^2}{M} + 4\,M\,\frac{t_3^2}{t_1} - 4Mt_3\right),\\
&\mathrm{V}^{11}_{\mathrm{hZZ}\,;\,\mu\nu} = -g^2\,\frac{M}{\mathrm{c}_{\mathrm{w}}^2}\,\frac{t_3}{t_1}\,\delta_{\mu\nu}\,, \qquad \mathrm{V}^{11}_{\mathrm{hWW}\,;\,\mu\nu} = -g^2\,M\,\frac{t_3}{t_1}\,\delta_{\mu\nu}\,,\\
&\mathrm{V}^{21}_{\mathrm{h}} = g\,\frac{t_3}{t_1^3}\,M\left[M^2\,t_3^2\,(t_3-t_1) + \frac{1}{4}\,M_{\mathrm{h}}^2\,t_1\,(3\,t_1 - 5\,t_3)\right],\\
&\mathrm{V}^{21}_{\mathrm{hh}} = -\,3\,g^2\,\frac{t_3^2}{t_1^3}\left[M^2\,(t_1-t_3)^2 + \frac{1}{8}\,M_{\mathrm{h}}^2\,t_1\right],\\
&\mathrm{V}^{21}_{\mathrm{ZZ}\,;\,\mu\nu} = -\frac{1}{4}\,g^2\,\frac{M^2}{\mathrm{c}_{\mathrm{w}}^2}\,\frac{t_3^2}{t_1^2}\,\delta_{\mu\nu}\,, \qquad \mathrm{V}^{21}_{\mathrm{WW}\,;\,\mu\nu} = -\frac{1}{4}\,g^2\,M^2\,\frac{t_3^2}{t_1^2}\,\delta_{\mu\nu}\,, \quad \mathrm{V}^{30} = -g\,t_1\,,\\
&\mathrm{V}^{31} = g\,M^2\,\frac{t_3^2}{t_1^2}\left(\frac{1}{2}\,t_1 - t_3\right), \quad \mathrm{V}^{31}_{\mathrm{h}} = \frac{1}{2}\,g^2\,M\,\frac{t_3}{t_1}\,(t_1-t_3)\,,\\
&\mathrm{V}_{\mathrm{hhh}} = -\frac{3}{2}\,g\,\frac{M_{\mathrm{h}}^2}{M}\,, \qquad \mathrm{V}_{\mathrm{hhhh}} = -\frac{3}{2}\,g^2\,\frac{M_{\mathrm{h}}^2}{M^2} - 3\,g^2\,\frac{t_3^2}{t_1}\,.
\end{aligned}
\tag{3.23}
$$

As an example we derive the $\mathrm{h}^2\,\mathrm{Z}^2$ (mixed-loop) vertex

$$
16\,\pi^2\,\mathrm{Q}^{\mathrm{hhZZ}}_{\mu\nu} = \frac{1}{8}\,\mathrm{C}_0^{(2)}(M_{\mathrm{h}})\,\left(\mathrm{V}^{2}_{\mathrm{h}}\,\mathrm{V}^{11}_{\mathrm{hZZ}} + \mathrm{V}^{10}_{\mathrm{hh}}\,\mathrm{V}^{21}_{\mathrm{ZZ}}\right)\,\mathrm{V}^{10}_{\mathrm{hh}}\,\frac{1}{t_1\Lambda^2}\,\delta_{\mu\nu}\,, \tag{3.24}
$$

where the scalar three-point function is given in eq. (A.5).

3.3.3 Field normalization and parameter shift

The Lagrangian for the low energy theory requires canonical normalization of the fields which is a standard procedure when including higher order terms, see refs. [25–27],

$$
\Phi \to \mathrm{Z}_\Phi\,\Phi\,, \qquad \mathrm{Z}_\Phi = 1 + \frac{\mathrm{g}^2}{16\,\pi^2}\,\frac{\mathrm{M}^2}{\Lambda^2}\,\Delta\mathrm{Z}_\Phi\,. \tag{3.25}
$$

In SESM only the h field requires a non-trivial normalization, given by

$$
\Delta\mathrm{Z}_{\mathrm{h}} = -\frac{1}{6}\,\frac{t_3^2}{t_1^3}\,(t_1-t_3)^2\,. \tag{3.26}
$$

Additionally, we can introduce shifts in the Lagrangian parameters,

$$
\begin{aligned}
M_{\mathrm{h}} &= \left[1 + \frac{1}{2}\,\frac{g^2}{16\,\pi^2}\left(\Delta^{(0)}_{M_{\mathrm{h}}}\,\frac{\Lambda^2}{M^2} + \Delta^{(1)}_{M_{\mathrm{h}}} + \Delta^{(2)}_{M_{\mathrm{h}}}\,\frac{M^2}{\Lambda^2}\right)\right]\overline{M}_{\mathrm{h}}\,,\\
M &= \left[1 + \frac{1}{2}\,\frac{g^2}{16\,\pi^2}\left(\Delta^{(1)}_{M} + \Delta^{(2)}_{M}\,\frac{M^2}{\Lambda^2}\right)\right]\overline{M}\,,
\end{aligned}
\tag{3.27}
$$

so that also the bare mass terms (for physical fields) are SM-like. In the following we will drop the bar. These shifts are given by

$$\Delta^{(0)}_{M_{\rm h}} = -2\,\frac{t_1 t_3}{x_{\rm h}}\,\overline{\rm A}_0(M_{\rm H})\,,$$

$$\Delta^{(1)}_{M_{\rm h}} = -8\,\frac{(t_3-t_1)^2}{x_{\rm h}}\,t_m^2 - \frac{1}{2}\left[3\,t_1\,x_{\rm h} + 4\,\left(7\,t_3^2 - 13\,t_1\,t_3 + 7\,t_1^2\right)\right]\frac{t_m^2}{x_{\rm h}}\,\overline{\rm A}_0(M_{\rm H})\,,$$

$$\Delta^{(2)}_{M_{\rm h}} = \frac{1}{3}\left[24\,(t_3-t_1)\,t_3^2 - (29\,t_3 - 17\,t_1)\,t_1\,x_{\rm h}\right]\,(t_3-t_1)\,\frac{t_m^2}{t_1^2 x_{\rm h}}$$
$$-\frac{1}{4}\left[3\,t_1^2\,x_{\rm h}^2 - 56\,(t_3-t_1)^2\,t_3^2 + 2\,\left(26\,t_3^2 - 43\,t_1\,t_3 + 18\,t_1^2\right)\,t_1\,x_{\rm h}\right]\frac{t_m^2}{t_1^2 x_{\rm h}}\,\overline{\rm A}_0(M_{\rm H})\,,$$

$$\Delta^{(1)}_{M} = -\frac{t_3^2}{t_1}\,\overline{\rm A}_0(M_{\rm H})\,,\qquad \Delta^{(2)}_{M} = -\frac{1}{2}\,\frac{t_3^2}{t_1^2}\,x_{\rm h}\,\overline{\rm A}_0(M_{\rm H})\,, \tag{3.28}$$

where we have introduced $t_3 = t_m\,t_1$. It is worth noting that the shifted masses introduced in eq. (3.27) remain bare parameters and are not the physical masses. Furthermore, the shift in $M_{\rm h}$ gives the typical "fine-tuning" that is often present when we "derive" the mass of a low mode (in terms of the scale Λ) from an UV completion.

3.4 The complete Lagrangian

Before introducing the complete Lagrangian we define the concept of (naive) power counting: any local operator in the Lagrangian is schematically of the form

$$\mathcal{O} = \frac{M^l}{\Lambda^n}\,\overline{\psi}^a\,\psi^b\,\partial^c\,\left(\Phi^\dagger\right)^d\,\Phi^e\,{\rm A}^f\,,\quad \frac{3}{2}\,(a+b) + c + d + e + f + l - n = 4\,, \tag{3.29}$$

where Lorentz, flavor and group indices have been suppressed, ψ stands for a generic fermion fields, Φ for a generic scalar and A for a generic gauge field. All light masses are scaled in units of the (bare) W mass M. We define dimensions according to

$$\mathrm{codim}\,\mathcal{O} = \frac{3}{2}\,(a+b) + c + d + e + f\,,\qquad \dim\mathcal{O} = \mathrm{codim} + l\,. \tag{3.30}$$

For a general formulation of power counting see ref. [28]. The SESM Lagrangian can be decomposed as follows,

$$\mathcal{L}_{\rm SESM} = \mathcal{L}_{\rm H=0} + \mathcal{L}_{\rm H}\,,\quad \mathcal{L}_{\rm H=0} = \mathcal{L}_{\rm SM}({\rm h}) + \sum_{n=0,2}\Lambda^{2\,n-2}\,\mathcal{L}_{6-2\,n}\,,\quad \mathcal{L}_{\rm H} \to \mathcal{L}^{\rm H}_{\rm eff} = \mathcal{L}^{\rm T}_{\rm eff} + \mathcal{L}^{\rm L}_{\rm eff} + \mathcal{L}^{\beta}_{\rm eff}\,, \tag{3.31}$$

where $\mathcal{L}_{\rm SM}({\rm h})$ is the SM Lagrangian written in terms of the light Higgs field h. It is worth noting that h, H do not transform under irreducible representations of SU(2) × U(1). In appendix B we present the full list of operators appearing in $\mathcal{L}_{\rm SESM}$, classified according to their dimension ($\dim = 2, 4, 6$) and their codimension ($\mathrm{codim} = 1, \ldots, 6$). As expected only the SM-like operators acquire coefficients that are Λ-enhanced ($\dim = 2$). The local operators that are usually quoted in this context are $\Phi_{\rm h}^6$ and $\partial_\mu\,\Phi_{\rm h}^2\,\partial_\mu\,\Phi_{\rm h}^2$ having $\dim = \mathrm{codim} = 6$ ($\Phi_{\rm h}^2 = {\rm h}^2 + \phi^{0\,2} + 2\,\phi^+\,\phi^-$); however, they should not be confused with $\mathcal{O}_\phi$ and $\mathcal{O}_{\phi\,\Box}$ of the Warsaw basis (see table 2 of ref. [29]), the latter being built with a SU(2) × U(1) scalar doublet while $\Phi_{\rm h}^2$ of eq. (B.1) is not invariant, due to h.

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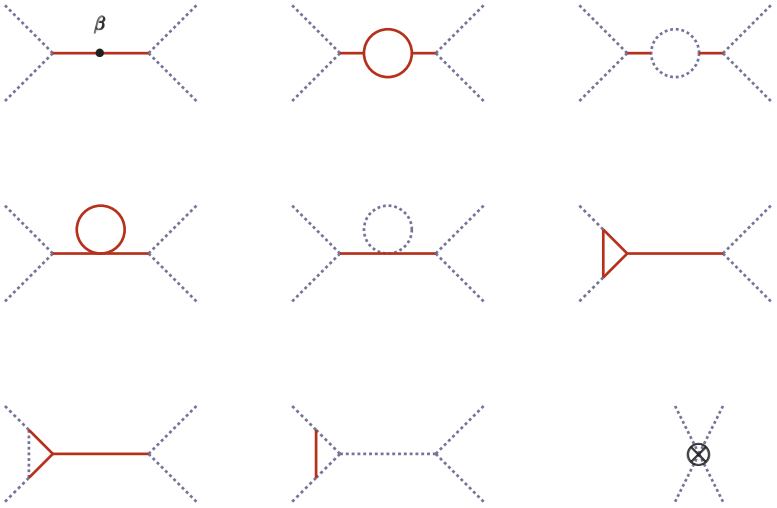

Figure 6. Example of a $2 \to 2$ process in the full SESM involving (dashed blue) light lines. For sake of simplicity we limit the example to scalar lines; moreover, boxes have not been included and vertex corrections have been shown only for the left part of the diagram. Solid (red) lines represent the heavy H field and the last diagram represents counterterms, both UV and finite (in the "on-sell" scheme).

3.4.1 How to use the low energy Lagrangian

The Lagrangian shown in appendix B is ready to use but should be used consistently. No additional problem will arise if we restrict $\mathcal{L}_{\rm SESM}$ to TG operators. When LG operators are included the following strategy must be adopted. Let us distinguish between the full theory (HSESM) and the low energy limit (LSESM). In figure 6 we show a simple example of a process with four external, light, lines; for the sake of simplicity we restrict to scalar lines, do not include boxes and avoid the further complication due to Dyson resummation. There are loops with (solid red) heavy lines and loops with (dashed blue) light lines; furthermore, β cancels H tadpoles, therefore it includes also light loops. The last diagram in figure 6 includes counterterms, both UV and finite, UV counterterms are designed to cancel UV poles and by finite counterterms we mean those that are needed to express bare parameters in terms of experimental quantities (having selected an input parameter set). Therefore, our scheme is "on-shell" (we avoid here complications induced by using the "complex-pole" scheme); the whole procedure is well defined and gauge parameter independent.[1]

When working in the LSESM framework (at the LG level) the Lagrangian $\mathcal{L}_{\rm SESM}$ will generate the diagrams in the first row of figure 7, where dots represent contraction of H propagators. With $\overline{\rm A}_0, {\rm B}_0$ and ${\rm C}_0$ we keep trace of the origin of the loop contraction, i.e. a one-point, two-point and three-point loop in HSESM. To perform a loop calculation in LSESM we must include light loops, as those shown in the second row of figure 7, taking care of avoiding diagrams that would be two loops in HSESM. After having included all contributions we take care of renormalization in LSESM by introducing UV counterterms

[1] For a discussion on the subtleties induced by the tadpoles see section 2.4 of ref. [24].

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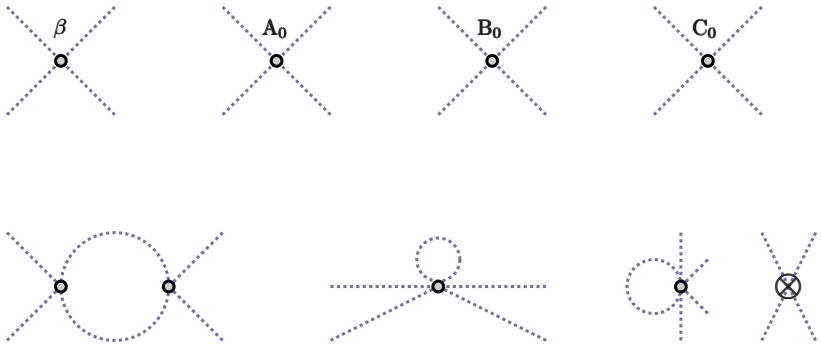

Figure 7. The same process as in figure 6 seen from the low energy side. In the first row we show diagrams that have been generated by contraction of heavy lines and $\overline{\mathrm{A}}_0, \mathrm{B}_0$ and C_0 keep trace of the origin of the loop contraction, i.e. a one-point, two-point and three-point loop in HSESM. In the second row we show the diagrams with light loops that have to be added, including LSESM counterterms in the "on-shell" scheme, in order to have a finite, gauge parameter independent, result.

and finite counterterms in the "on-shell" scheme with a low-energy IPS. Also this procedure is well defined and gauge parameter independent. Any attempt of performing a (simpler) $\overline{\mathrm{MS}}$ renormalization should be handled with great care.

To summarize: when the UV completion is known we have a hierarchy among loops in the low-energy theory. There is a marked contrast between this top-down approach and the bottom-up effective field theory where one cannot unambiguously identify the powers of hypothetical UV couplings present in the Wilson coefficients. In the EFT approach, by performing the calculations without unnecessary assumptions, it is still possible to study the effect of particular hierarchies and specific UV completions (when they are precisely defined) a posteriori. Consider the $h\mathrm{ZZ}$ vertex, we have three contributions (a $\delta_{\mu\nu}$ is left understood);

$$\mathrm{V}^{(0)}_{\mathrm{hZZ}} = -\frac{1}{2}\,\frac{g\,M}{\mathrm{c}_{\mathrm{W}}^2}\,, \qquad \mathrm{V}^{\mathrm{TG}}_{\mathrm{hZZ}} = \frac{1}{4}\,\frac{g\,M^3}{\Lambda^2\,\mathrm{c}_{\mathrm{W}}^2}\,t_m^2\,,$$

$$\mathrm{V}^{\mathrm{LG}}_{\mathrm{hZZ}} = \frac{1}{64}\,\frac{g^3\,M}{\pi^2}\,\frac{t_m^2\,t_1}{\mathrm{c}_{\mathrm{W}}^2}\,\overline{\mathrm{A}}_0(M_{\mathrm{H}}) + \frac{1}{384}\,\frac{g^3}{\pi^2}\,\frac{M^2}{\Lambda^2\,\mathrm{c}_{\mathrm{W}}^2}\Big\{3\left[x_{\mathrm{h}} - \left(18 - 22\,t_m + 7\,t_m^2\right)t_1\right]t_m^2\,\overline{\mathrm{A}}_0(M_{\mathrm{H}})$$
$$+3\,\frac{t_m}{t_1}\,\mathrm{T}_1 + 3\,\frac{t_m^2}{t_1}\,\left(\beta_2^{(0)} - \beta_1^{(0)}\right) - 2\,\left(17 - 22\,t_m + 5\,t_m^2\right)\,t_m^2\,t_1\Big\}\,. \tag{3.32}$$

Here $\mathrm{V}^{(0)}_{\mathrm{hZZ}}$ is SM $\mathcal{O}(g)$; $\mathrm{V}^{\mathrm{TG}}_{\mathrm{hZZ}}$ is power suppressed, $\mathcal{O}(g)$ tree-generated; $\mathrm{V}^{\mathrm{LG}}_{\mathrm{hZZ}}$ is $\mathcal{O}(g^3/\pi^2)$ loop-generated. Clearly, $\mathrm{V}^{\mathrm{TG}}_{\mathrm{hZZ}}$ can be used in any LO/NLO calculation, i.e. it can be consistently inserted in one loop diagrams containing light particles. To the contrary, $\mathrm{V}^{\mathrm{LG}}_{\mathrm{hZZ}}$ can only be used, at tree level, in one loop calculations (i.e. it should not be inserted into loops of light particles). Furthermore it is easily seen that mixed-loop contributions to this coupling are $\mathcal{O}(\Lambda^{-4})$.

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3.5 Gauge invariance

The Lagrangian under consideration is invariant with respect to the following (infinitesimal) transformations,

$$
\begin{aligned}
\mathrm{h} &= \mathrm{h} + \frac{1}{2}\, g\, \mathrm{c}_{_\alpha} \left(\Gamma_{\mathrm{Z}}\, \frac{1}{\mathrm{c}_{_\mathrm{W}}}\, \phi^0 + \phi^+ \Gamma^- + \phi^- \Gamma^+ \right) , \\
\mathrm{H} &= \mathrm{H} + \frac{1}{2}\, g\, \mathrm{s}_{_\alpha} \left(\Gamma_{\mathrm{Z}}\, \frac{1}{\mathrm{c}_{_\mathrm{W}}}\, \phi^0 + \phi^+ \Gamma^- + \phi^- \Gamma^+ \right) , \\
\phi^0 &= \phi^0 - \frac{1}{2}\, g\, \Gamma_{\mathrm{Z}}\, \frac{1}{\mathrm{c}_{_\mathrm{W}}} \left(\mathrm{c}_{_\alpha}\, \mathrm{h} + \mathrm{s}_{_\alpha}\, \mathrm{H} + 2\, \frac{M}{g} \right) + \frac{i}{2}\, g\, \left(\Gamma^- \phi^+ - \Gamma^+ \phi^- \right) , \\
\phi^- &= \phi^- - \frac{1}{2}\, g\, \Gamma^- \left(\mathrm{c}_{_\alpha}\, \mathrm{h} + \mathrm{s}_{_\alpha}\, \mathrm{H} + 2\, \frac{M}{g} + i\, \phi^0 \right) + \frac{i}{2}\, g\, \left[\left(\mathrm{c}_{_\mathrm{W}}^2 - \mathrm{s}_{_\mathrm{W}}^2 \right) \frac{1}{\mathrm{c}_{_\mathrm{W}}}\, \Gamma_{\mathrm{Z}} + 2\, \mathrm{s}_{_\mathrm{W}}\, \Gamma_{\mathrm{A}} \right] \phi^- , \\
\mathrm{A}_\mu &= \mathrm{A}_\mu + i\, g\, \mathrm{s}_{_\mathrm{W}} \left(\Gamma^-\, \mathrm{W}^+{}_\mu - \Gamma^+\, \mathrm{W}^-{}_\mu \right) - \partial_\mu\, \Gamma_{\mathrm{A}} , \\
\mathrm{Z}_\mu &= \mathrm{Z}_\mu + i\, g\, \mathrm{c}_{_\mathrm{W}} \left(\Gamma^-\, \mathrm{W}^+{}_\mu - \Gamma^+\, \mathrm{W}^-{}_\mu \right) - \partial_\mu\, \Gamma_{\mathrm{Z}} , \\
\mathrm{W}^-{}_\mu &= \mathrm{W}^-{}_\mu - i\, g\, \Gamma^- \left(\mathrm{c}_{_\mathrm{W}}\, \mathrm{Z}_\mu + \mathrm{s}_{_\mathrm{W}}\, \mathrm{A}_\mu \right) + i\, g\, \left(\mathrm{c}_{_\mathrm{W}}\, \Gamma_{\mathrm{Z}} + \mathrm{s}_{_\mathrm{W}}\, \Gamma_{\mathrm{A}} \right) \mathrm{W}^-{}_\mu - \partial_\mu\, \Gamma^- ,
\end{aligned}
\tag{3.33}
$$

when we expand $\mathrm{s}_{_\alpha}, \mathrm{c}_{_\alpha}$ to any given order. The gauge invariance of the low energy theory must be understood as follows: the transformations of eq. (3.33) may be seen as generating new vertices in the theory and gauge invariance requires that, for any Green's function, the sum of all diagrams containing one Γ-vertex cancel. When sources are added to the Lagrangian the field transformation generates special vertices that are used to prove equivalence of gauges and simply-contracted Ward-Slavnov-Taylor identities [30]. Therefore, for any "transformed" Green's function we integrate the H field and, order-by-order in Λ, terms containing one Γ-vertex continue to cancel (and WST identities to be valid). For instance, if we set $\mathrm{c}_{_\alpha} = 1$ and $\mathrm{s}_{_\alpha} = 0$ in eq. (3.33), it is easily seen that $\mathcal{L}_{\mathrm{H}=0}$, given in eq. (3.31), is not invariant but the addition of $\mathcal{L}^{\mathrm{T}}_{\mathrm{eff}}$ truncated at $\mathcal{O}(1)$ restores gauge invariance.

3.6 Integration in the non-linear representation

An interesting alternative, see refs. [31–33] is represented by the following Lagrangian

$$
\begin{aligned}
\mathcal{L} = &-\frac{1}{2}\, \partial_\mu \mathrm{h}_2\, \partial_\mu \mathrm{h}_2 - \frac{1}{2}\, F^2(\mathrm{h}_2)\, g_{ab}(\phi)\, \mathrm{D}_\mu \phi^a\, \mathrm{D}_\mu \phi^b - \partial_\mu\, \chi\, \partial_\mu\, \chi - \frac{1}{2}\, \mu_2^2 \left(\mathrm{h}_2 + \sqrt{2}\, \mathrm{v} \right)^2 \\
&-\mu_1^2\, \chi^2 - \frac{1}{8}\, \lambda_2 \left(\mathrm{h}_2 + \sqrt{2}\, \mathrm{v} \right)^4 - \frac{1}{2}\, \lambda_1\, \chi^4 - \frac{1}{2}\, \lambda_{12}\, \chi^2 \left(\mathrm{h}_2 + \sqrt{2}\, \mathrm{v} \right)^2 ,
\end{aligned}
\tag{3.34}
$$

where we have introduced a complex scalar doublet, (see ref. ref. [31] for the complete set of definitions)

$$
\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_{\mathrm{H}}^4 + i\, \phi_{\mathrm{H}}^3 \\ \phi_{\mathrm{H}}^2 + i\, \phi_{\mathrm{H}}^1 \end{pmatrix}
\tag{3.35}
$$

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with "polar" coordinates defined by

$$\phi^i_{\rm H} = (\mathrm{h}_2 + \mathrm{v})\; u^i(\phi)\,, \quad u(\phi)\cdot u(\phi) = 1\,, \quad u^i(0) = \delta^{i4}\,. \tag{3.36}$$

where $i,j,\cdots = 1,\ldots,4$ and $a,b,\cdots = 1,\ldots,3$. A choice for the metric, present in eq. (3.34), is

$$g_{ab}(\phi) = \delta_{ab} + \frac{\phi_a\,\phi_b}{\mathrm{v}^2 - \phi_a\,\phi^a}\,, \tag{3.37}$$

and the covariant derivative in eq. (3.34) is defined in eqs. (15–16) of ref. [31]. When discussing the SM one uses

$$F_{\rm SM}\,(\mathrm{h}_2) = 1 + \frac{\mathrm{h}_2}{\mathrm{v}} \tag{3.38}$$

in eq. (3.34). After mixing with the singlet we obtain

$$\begin{aligned} F^2 &= F^2_{\rm SM}\,(\mathrm{h}) + \frac{2}{\mathrm{v}}\,F_{\rm SM}\,(\mathrm{h})\,\left[(\mathrm{c}_\alpha - 1)\;\mathrm{h} + \mathrm{s}_\alpha\;\mathrm{H}\right] + \ldots \\ &= \left\{1 + \frac{\mathrm{h}}{\mathrm{v}}\left[1 - \frac{1}{2}\,\frac{t_3^2}{t_1^2}\,\frac{M^2}{M_{\rm s}^2} + \mathcal{O}\left(M_{\rm s}^{-4}\right)\right] + \frac{\mathrm{H}}{\mathrm{v}}\,\frac{t_3}{t_1}\,\frac{M}{M_{\rm s}} + \mathcal{O}\left(\mathrm{H}^2\right)\right\}^2 \\ &= \left[1 + f_{\rm h}\,\frac{\mathrm{h}}{\mathrm{v}} + f_{\rm H}\,\frac{\mathrm{H}}{\mathrm{v}} + \mathcal{O}\left(\mathrm{H}^2\right)\right]^2\,. \end{aligned} \tag{3.39}$$

After integrating H we have two effects, a change in eq. (3.34) due to a redefinition of F and the addition of higher dimensional operators, e.g.

$$f_{\rm H}^2\,\left(1 + f_{\rm h}\,\frac{\mathrm{h}}{\mathrm{v}}\right)^2\,\mathcal{O}_\phi^2\,, \qquad \mathcal{O}_\phi = g_{ab}(\phi)\,\mathrm{D}_\mu\phi^a\,\mathrm{D}_\mu\phi^b\,. \tag{3.40}$$

Note that $f_{\rm h} \neq 1$ has an effect on curvatures, see eq. (22) and eq. (27) of ref. [31]. From this point of view the geometric formulation of the Higgs EFT seems the most promising road to account for general mixings in the scalar sector.

4 Low energy behavior for THDM models

We consider THDM with softly-broken Z_2 symmetry [34–36]. The bosonic part of the Lagrangian is given by

$$\begin{aligned} \mathcal{L} = &-\sum_{i=1,2}\left(\mathrm{D}_\mu\,\Phi_i\right)^\dagger\,\mathrm{D}_\mu\,\Phi_i + \sum_{i=1,2}\mu_i^2\,\Phi_i^\dagger\,\Phi_i + \mu_3^2\,\left(\Phi_1^\dagger\,\Phi_2 + \Phi_2^\dagger\,\Phi_1\right) \\ &+\frac{1}{2}\sum_{i=1,2}\lambda_i\left(\Phi_i^\dagger\,\Phi_i\right)^2 + \lambda_3\,\Phi_1^\dagger\,\Phi_1\,\Phi_2^\dagger\,\Phi_2 + \lambda_4\,\Phi_1^\dagger\,\Phi_2\,\Phi_2^\dagger\,\Phi_1 + \frac{1}{2}\,\lambda_5\left[\left(\Phi_1^\dagger\,\Phi_2\right)^2 + \left(\Phi_2^\dagger\,\Phi_1\right)^2\right]. \end{aligned} \tag{4.1}$$

With doublets given by

$$\Phi_i = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathrm{h}_i + \sqrt{2}\,\mathrm{v}_i + i\,\phi^0 \\ \sqrt{2}\,i\,\phi^- \end{pmatrix} \tag{4.2}$$

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The mixing angle β is such that

$$\begin{aligned} \mathrm{h}_1 &= -\mathrm{v}_1 + \cos\beta\,\left(\mathrm{h}_1' + \mathrm{v}\right) - \sin\beta\,\mathrm{h}_2'\,, \quad \phi_1^0 = \cos\beta\,\phi^0 - \sin\beta\,\mathrm{A}^0\,, \quad \phi_1^\pm = \cos\beta\,\phi^\pm - \sin\beta\,\mathrm{H}^\pm \\ \mathrm{h}_2 &= -\mathrm{v}_2 + \sin\beta\,\left(\mathrm{h}_1' + \mathrm{v}\right) - \cos\beta\,\mathrm{h}_2'\,, \quad \phi_2^0 = \sin\beta\,\phi^0 + \sin\beta\,\mathrm{A}^0\,, \quad \phi_2^\pm = \sin\beta\,\phi^\pm + \cos\beta\,\mathrm{H}^\pm. \end{aligned} \tag{4.3}$$

with $\mathrm{v}^2 = \mathrm{v}_1^2 + \mathrm{v}_2^2$. Finally, diagonalization in the neutral sector gives

$$\mathrm{h}_1' = \cos(\alpha-\beta)\,\mathrm{H} - \sin(\alpha-\beta)\,\mathrm{h}\,, \qquad \mathrm{h}_2' = \sin(\alpha-\beta)\,\mathrm{H} + \cos(\alpha-\beta)\,\mathrm{h}\ . \tag{4.4}$$

The first problem in deriving the low energy behavior is represented by the individuation of the cutoff scale. In the unbroken phase one can use the Plehn scale

$$\Lambda^2 = \mu_1^2\,\sin^2\beta + \mu_2^2\,\cos^2\beta + \mu_3^2\,\sin\beta\,, \tag{4.5}$$

whereas in the mass eigenstates, ref. [4] suggests $M_{\mathrm{A}^0}^2$, based on the fact that custodial symmetry requires almost degenerate heavy states. Our procedure is as follows. First we eliminate $\mu_{1,2}^2$ by means of the following transformation:

$$\begin{aligned} \cos^2\beta\,\mu_1^2 &= \beta_1 - \mathrm{v}^2\left[\sin^2\beta\,\cos^2\beta\,\overline{\lambda} + \frac{1}{2}\left(\lambda_2\,\sin^4\beta + \lambda_1\cos^4\beta\right)\right] - 2\sin\beta\cos\beta\,\mu_3^2 + \sin^2\beta\,\mu_2^2\,, \\ \cot\beta\,\mu_2^2 &= \beta_2 - \tan\beta\,\beta_1 + \frac{1}{2}\,\mathrm{v}^2\left[\sin\beta\,\cos\beta\,\overline{\lambda} + (\tan\beta - \sin\beta\cos\beta)\right] + \mu_3^2\,, \end{aligned} \tag{4.6}$$

where $\beta_{1,2}$ are the constants needed to cancel tadpoles and $\overline{\lambda} = \lambda_3 + \lambda_4 + \lambda_5$. Next we introduce new parameters defined by ($\overline{M}$ should not be confused with the redefinition of the bare W mass in eq. (3.27))

$$\begin{aligned} \mu_3^2 &= \sin\beta\,\cos\beta\,\overline{M}^2\,, \quad \mathrm{v}^2\,\lambda_5 = M_{\mathrm{A}\,0}^2 - \overline{M}^2\,, \\ \mathrm{v}^2\,\lambda_4 &= 2\,\beta_1 + 2\,\frac{\cos^2\beta - \sin^2\beta}{\sin\beta\cos\beta}\,\beta_2 + 2\,M_{\mathrm{H}^\pm}^2 - M_{\mathrm{A}\,0}^2 - \overline{M}^2\ . \\ \mathrm{v}^2\,\lambda_2 &= \mathrm{v}^2\,\left(2\,\overline{\lambda} - \lambda_1\right) + \overline{M}^2 - M_{22}^2\sin^2\beta\cos^2\beta\,, \\ \mathrm{v}^2\,\overline{\lambda} &= \frac{1}{2}\,\left(\mathrm{v}^2\,\lambda_1\,\frac{\sin^4\beta - \cos^4\beta}{\sin^2\beta} + \frac{M_{22}^2 - \overline{M}^2}{\cos^2\beta} - \frac{M_{11}^2}{\sin^2\beta}\right)\,, \\ \mathrm{v}^2\,\lambda_1 &= 2\,\tan\beta\,M_{12}^2 + \tan^2\beta\,\left(\overline{M}^2 - M_{22}^2\right) - M_{11}^2\,, \end{aligned} \tag{4.7, 4.8}$$

Requiring

$$\sin(\alpha-\beta)\,\cos(\alpha-\beta)\,\left(M_{11}^2 - M_{22}^2\right) + \left[\sin^2(\alpha-\beta) - \cos^2(\alpha-\beta)\right] = 0\,, \tag{4.9}$$

gives the following result for the neutral masses:

$$M_{\mathrm{h}}^2 = M_{11}^2 - \cot(\alpha-\beta)\,M_{12}^2\,, \qquad M_{\mathrm{H}}^2 = M_{12}^2 + \sin(\alpha-\beta)\,\cos(\alpha-\beta)\,M_{\mathrm{h}}^2 \tag{4.10}$$

Our scenario for the THDM is defined by $\Lambda = \overline{M} \gg \mathrm{v}$ and

$$\beta = \frac{1}{2}\,\left(\pi - \delta_\beta\right)\,, \qquad \alpha = \frac{1}{2}\,\delta_\alpha\ . \tag{4.11}$$

Expanding in v/Λ we obtain:

$$\lambda_2 = -\frac{M_h^2}{v^2} - \frac{1}{2}\left(M_h^2 + v^2\,\overline{\lambda}\right)\frac{v^2}{\Lambda^4}\,, \quad \delta_\beta = \frac{v^2}{\Lambda^2} + \mathcal{O}\left(\frac{v^4}{\Lambda^4}\right), \quad \delta_\alpha = -\frac{v^2}{\Lambda^2} + \mathcal{O}\left(\frac{v^4}{\Lambda^4}\right). \tag{4.12}$$

All masses and angles are re-expressed in term of Λ and couplings.

$$\begin{aligned}
M_{H\,\pm}^2 &= \Lambda^2 + \frac{1}{2}\,v^2\,(\lambda_4+\lambda_5)\,, \qquad M_{A\,0}^2 = \Lambda^2 + v^2\,\lambda_5\,,\\
M_H^2 &= \Lambda^2 - \frac{1}{4}\left[v^2\,\left(\lambda_1 - 2\,\overline{\lambda}\right) - M_h^2\right]\frac{v^4}{\Lambda^4}\\
\sin\beta &= 1 - \frac{1}{8}\,\frac{v^4}{\Lambda^4} + \mathcal{O}\left(\frac{v^6}{\Lambda^6}\right), \qquad \cos\beta = \frac{1}{2}\,\frac{v^2}{\Lambda^2} + \mathcal{O}\left(\frac{v^4}{\Lambda^4}\right),\\
\sin(\alpha-\beta) &= -1 + \mathcal{O}\left(\frac{v^6}{\Lambda^6}\right), \qquad \cos(\alpha-\beta) = -\frac{1}{2}\left(M_h^2 + v^2\,\overline{\lambda}\right)\frac{v^2}{\Lambda^4} + \mathcal{O}\left(\frac{v^6}{\Lambda^6}\right).
\end{aligned} \tag{4.13}$$

Using eq. (4.13) we can expand the Lagrangian in powers of Λ^{-1} and apply the formalism of section 2 to obtain the low energy limit of the model. Also for THDM models the SM decoupling limit cannot be obtained by making only assumptions about one parameter. For a general discussion on alignment and decoupling, see refs. [37, 38].

There are four THDM models that differ in the fermion sector: they are type I, II, X and Y, see ref. [35] for details. The THDM Lagrangian becomes

$$\mathcal{L}_{\rm THDM} = \mathcal{L}_{\rm THDM}\Big|_{\rm heavy=0} + \mathcal{L}_{\rm THDM}^{\rm heavy}\,, \quad \mathcal{L}_{\rm THDM}\Big|_{\rm heavy=0} = \mathcal{L}_0 + \Lambda^{-2}\,\mathcal{L}_2\,, \tag{4.14}$$

with $\mathcal{L}_2 = 0$ for THDM type I. The heavy part of the Lagrangian, $\mathcal{L}_{\rm THDM}^{\rm heavy}$ is given by a sum of terms; we define the set $\{\Phi\} = \{H, A^0, H^\pm\}$ and obtain

$$\mathcal{L}_{\rm THDM}^{\rm heavy} = \sum_n \mathcal{L}_n^{\rm heavy}\,, \tag{4.15}$$

with

$$\mathcal{L}_1^{\rm heavy} = \sum_{\phi\in\{\Phi\}} F_{1\,\phi}\,\phi\,, \qquad F_{1\,H^-} = \left(F_{1\,H^+}\right)^\dagger, \tag{4.16}$$

$$\mathcal{L}_2^{\rm heavy} = \sum_{\phi_i,\phi_j\in\{\Phi\}} \phi_i\,F_{2\,\phi_i\phi_j}\,\phi_j\,, \tag{4.17}$$

where $F_{2\,\phi_i\phi_j}$ contains derivatives and where terms with one or two heavy fields are of $\mathcal{O}(1)$ and $\mathcal{O}(\Lambda^{-2})$. With three fields we have

$$\mathcal{L}_3^{\rm heavy} = \sum_{\phi_i,\phi_j,\phi_k\in\{\Phi\}} F_{3\,\phi_i\phi_j\phi_k}\,\phi_i\phi_j\phi_k\,, \tag{4.18}$$

where F_3 is $\mathcal{O}(\Lambda^{-2})$. Finally we have

$$\mathcal{L}_4^{\rm heavy} = \sum_{\phi_i,\phi_j,\phi_k,\phi_l\in\{\Phi\}} F_{4\,\phi_i\phi_j\phi_k\phi_l}\,\phi_i\phi_j\phi_k\phi_l\,, \tag{4.19}$$

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with contributions of $\mathcal{O}(1)$ and $\mathcal{O}(\Lambda^{-2})$. For THDM type II, X and Y there are terms of $\mathcal{O}(\Lambda^2)$ in F_1.

Due to the SM-like scenario, $\sin(\alpha-\beta) = -1 + \mathcal{O}(\mathrm{v}^6/\Lambda^6)$, h is almost the SM Higgs boson (alignment). If we consider the vertex $\mathrm{h}\gamma\gamma$ the only deviation (at $\mathcal{O}(\Lambda^{-2})$) is given by the $\mathrm{H}^\pm$ loops which, after expansion, contribute to the "contact" term

$$\delta \mathrm{V}^{\mu\nu}_{\mathrm{h}\gamma\gamma} = \frac{1}{3}\, i\, g^3\, (2\,\lambda_5 - \lambda_3)\, \mathrm{s_W}\, \frac{M}{\Lambda^2}\, \left(2\, p_2^\mu\, p_1^\nu + M_\mathrm{h}^2\, \delta^{\mu\nu}\right)\,, \tag{4.20}$$

and there is no contribution from insertion of local operators into SM loops.

There are constraints from electroweak precision data, most noticeably from the ρ parameter. The contribution from scalar loops in THDM is

$$\begin{aligned}
\Delta\rho_\mathrm{THDM} = \frac{G_\mathrm{F}}{8\sqrt{2}\,\pi^2} \Big\{ & \mathrm{F}\left(M_{\mathrm{A}\,0}\,,\, M_{\mathrm{H}\,\pm}\right) - \cos^2(\alpha-\beta)\left[\mathrm{F}\left(M_\mathrm{h},\, M_{\mathrm{A}\,0}\right) - \mathrm{F}\left(M_\mathrm{h},\, M_{\mathrm{H}\,\pm}\right)\right] \\
& - \sin^2(\alpha-\beta)\left[\mathrm{F}\left(M_\mathrm{H}\,,\, M_{\mathrm{A}\,0}\right) - \mathrm{F}\left(M_\mathrm{H}\,,\, M_{\mathrm{H}\,\pm}\right)\right]\Big\}\,, \\
\mathrm{F}\,(m_1\,,m_2) = \frac{1}{2}\,\left(m_1^2 + m_2^2\right) & - \frac{m_1^2\, m_2^2}{m_1^2 - m_2^2}\, \ln\frac{m_1^2}{m_2^2}\,.
\end{aligned} \tag{4.21}$$

In the scenario described by eq. (4.13) we obtain

$$\Delta\rho_\mathrm{THDM} = \frac{G_\mathrm{F}}{96\sqrt{2}\,\pi^2}\, \frac{\mathrm{v}^4}{\overline{M}^2}\, \left(\lambda_4^2 - \lambda_5^2\right)\,, \tag{4.22}$$

i.e. a mass suppressed correction. As before, we define $M_\mathrm{h}^2 = x_\mathrm{h}\, M^2$ and $\lambda_i = t_i\, g^2$. Deriving TG operators is relatively easy; using eq. (4.16) and neglecting quadratic terms, eq. (4.17), we define

$$\mathrm{F}_{1\,\phi} = \mathrm{F}^0_{1\,\phi} + \overline{M}^{\,-2}\, \mathrm{F}^2_{1\,\phi}\,, \tag{4.23}$$

and derive the following result

$$\mathcal{L}^\mathrm{TG}_\mathrm{THDM} = \frac{1}{2}\, \overline{M}^{\,-2}\, \mathcal{L}^\mathrm{TG}_{\mathrm{THDM}\,2} + \overline{M}^{\,-4}\, \mathcal{L}^\mathrm{TG}_{\mathrm{THDM}\,4}\,, \tag{4.24}$$

$$\begin{aligned}
\mathcal{L}^\mathrm{TG}_{\mathrm{THDM}\,2} &= \left(\mathrm{F}^0_{1\,\mathrm{H}}\right)^2 + \left(\mathrm{F}^0_{1\,\mathrm{A}\,0}\right)^2 + 2\,\mathrm{F}^0_{1\,\mathrm{H}^+}\,\mathrm{F}^0_{1\,\mathrm{H}^-}\,, \\
\mathcal{L}^\mathrm{TG}_{\mathrm{THDM}\,4} &= \mathrm{F}^0_{1\,\mathrm{H}}\,\mathrm{F}^2_{1\,\mathrm{H}} + \mathrm{F}^0_{1\,\mathrm{A}\,0}\,\mathrm{F}^2_{1\,\mathrm{A}\,0} + \mathrm{F}^0_{1\,\mathrm{H}^+}\,\mathrm{F}^2_{1\,\mathrm{H}^-} + \mathrm{F}^0_{1\,\mathrm{H}^-}\,\mathrm{F}^2_{1\,\mathrm{H}^+} \\
&\quad - \frac{1}{2}\,\partial_\mu \mathrm{F}^0_{1\,\mathrm{H}}\,\partial_\mu \mathrm{F}^0_{1\,\mathrm{H}} - \frac{1}{2}\,\partial_\mu \mathrm{F}^0_{1\,\mathrm{A}\,0}\,\partial_\mu \mathrm{F}^0_{1\,\mathrm{A}\,0} - \partial_\mu \mathrm{F}^0_{1\,\mathrm{H}^+}\,\partial_\mu \mathrm{F}^0_{1\,\mathrm{H}^-} \\
&\quad - \frac{1}{2}\, g^2\, \mathrm{v}^2 \left[(t_4 + t_5)\,\mathrm{F}^0_{1\,\mathrm{H}^+} + \mathrm{F}^0_{1\,\mathrm{H}^-} + t_5\,\left(\mathrm{F}^0_{1\,\mathrm{A}\,0}\right)^2\right]\,.
\end{aligned} \tag{4.25}$$

Note that $\mathrm{F}^0_{1\,\phi} = 0$ if we do not include β terms; we derive ($\Phi_0^2 = {\phi^0}^2 + 2\,\phi^+\,\phi^-$)

$$\begin{aligned}
\mathrm{F}_{1\,\mathrm{H}} = & -\frac{g\,M}{\Lambda^2}\left[\frac{1}{2}\,M^2\,\left(x_\mathrm{h} + 4\,\bar{t}\right)\,\left(3\,\mathrm{h}^2 + \Phi_0^2\right) + M_\mathrm{l}\,\bar{\mathrm{l}}\,\mathrm{l} + M_\mathrm{u}\,\bar{\mathrm{u}}\,\mathrm{u} + M_\mathrm{d}\,\bar{\mathrm{d}}\,\mathrm{d}\right] \\
& - \frac{1}{4}\,\frac{g^2\,M^2}{\Lambda^2}\,\left(x_\mathrm{h} + 4\,\bar{t}\right)\,\mathrm{h}\,\left(\mathrm{h}^2 + \Phi_0^2\right)\,,
\end{aligned}$$

$$
\begin{aligned}
\mathrm{F}_{1\,\mathrm{A}\,0} &= \frac{g\,M^3}{\Lambda^2}\,\left(x_\mathrm{h}+4\,\bar{t}\right)\,\mathrm{h}\,\phi^0+\frac{1}{4}\,\frac{g^2\,M^2}{\Lambda^2}\,\left(x_\mathrm{h}+4\,\bar{t}\right)\,\phi^0\,\Phi_\mathrm{h}^2\\
&\quad +i\,\frac{g\,M}{\Lambda^2}\left[M_\mathrm{l}\,\bar{\mathrm{l}}\,\gamma^5\,\mathrm{l}-M_\mathrm{u}\,\bar{\mathrm{u}}\,\gamma^5\,\mathrm{u}+M_\mathrm{d}\,\bar{\mathrm{d}}\,\gamma^5\,\mathrm{d}\right],\\
\mathrm{F}_{1\,\mathrm{H}^-} &= \frac{g\,M^3}{\Lambda^2}\,\left(x_\mathrm{h}+4\,\bar{t}\right)\,\mathrm{h}\,\phi^+ +\frac{1}{4}\,\frac{g^2\,M^2}{\Lambda^2}\,\left(x_\mathrm{h}+4\,\bar{t}\right)\,\phi^+\,\Phi_\mathrm{h}^2\\
&\quad -i\,\frac{g\,M}{\sqrt{2}\,\Lambda^2}\left[M_\mathrm{l}\,\bar{\mathrm{l}}\,\gamma_+\,\nu_\mathrm{l}+M_\mathrm{d}\,\bar{\mathrm{d}}\,\gamma_+\,\mathrm{u}-M_\mathrm{u}\,\bar{\mathrm{d}}\,\gamma_-\,\mathrm{u}\right],\\
\mathrm{F}_{1\,\mathrm{H}^+} &= \frac{g\,M^3}{\Lambda^2}\,\left(x_\mathrm{h}+4\,\bar{t}\right)\,\mathrm{h}\,\phi^- +\frac{1}{4}\,\frac{g^2\,M^2}{\Lambda^2}\,\left(x_\mathrm{h}+4\,\bar{t}\right)\,\phi^-\,\Phi_\mathrm{h}^2\\
&\quad +i\,\frac{g\,M}{\sqrt{2}\,\Lambda^2}\left[M_\mathrm{l}\,\bar{\nu}_\mathrm{l}\,\gamma_-\,\mathrm{l}-M_\mathrm{u}\,\bar{\mathrm{u}}\,\gamma_+\,\mathrm{d}+M_\mathrm{d}\,\bar{\mathrm{u}}\,\gamma_-\,\mathrm{d}\right],
\end{aligned}
\tag{4.26}
$$

where $\Lambda^2 = g^2\,\overline{M}^2$ and $\bar{t} = t_3+t_4+t_5$. Finally, in the limit described by eq. (4.13) LG operator are more abundant than TG ones, one-point functions are $\mathcal{O}(\overline{M}^2)$, two-point functions are $\mathcal{O}(1)$ and three-point functions are $\mathcal{O}(\overline{M}^{-2})$. They all involve internal (loop) heavy lines while at tree level any heavy line is quadratically suppressed. To give an example we split the F_2 functions as follows:

$$
\mathrm{F}_{2\,ij} = \mathrm{F}^0_{2\,ij}+\Lambda^{-2}\,\mathrm{F}^2_{2\,ij}+\mathcal{O}(\Lambda^{-4}),\quad \mathrm{F}^0_{2\,ij} = \mathrm{F}^{00}_{2\,ij}+\left(\mathrm{F}^{0\mu}_{2\,ij}\,\overrightarrow{\partial}_\mu-\overleftarrow{\partial}_\mu\,\mathrm{F}^{0\mu}_{2\,ij}\right),
\tag{4.27}
$$

$$
\begin{aligned}
\mathrm{F}_{2\,\mathrm{HH}} &= g\,M\,\bar{t}\,\mathrm{h}+\frac{1}{8}\,g^2\left[2\,\bar{t}\,\Phi_\mathrm{h}^2-4\,t_5\,\phi^{0\,2}-4\,(t_4+t_5)\,\phi^+\,\phi^- -\mathrm{Z}^2\,\frac{1}{\mathrm{c}_\mathrm{W}^2}-2\,\mathrm{W}^{+\mu}\,\mathrm{W}^-{}_\mu\right]\\
&\quad +\frac{1}{8}\,g^3\,\frac{M^3}{\Lambda^2}\,\mathrm{T}_c\,\mathrm{h}+\frac{1}{32}\,g^4\,\frac{M^2}{\Lambda^2}\,\mathrm{T}_c\,\Phi_\mathrm{h}^2,\\
\mathrm{F}_{2\,\mathrm{A}\,0\,\mathrm{A}\,0} &= -g\,M\,(2\,t_5-\bar{t})\,\mathrm{h}+\frac{1}{8}\,g^2\left[2\,\bar{t}\,\Phi_\mathrm{h}^2-4\,t_5\,\mathrm{h}^2-4\,(t_4+t_5)\,\phi^+\,\phi^- -\mathrm{Z}^2\,\frac{1}{\mathrm{c}_\mathrm{W}^2}-2\,\mathrm{W}^{+\mu}\,\mathrm{W}^-{}_\mu\right]\\
&\quad +\frac{1}{8}\,g^3\,\frac{M^3}{\Lambda^2}\,\mathrm{T}_c\,\mathrm{h}+\frac{1}{32}\,g^4\,\frac{M^2}{\Lambda^2}\,\mathrm{T}_c\,\Phi_\mathrm{h}^2,\\
\mathrm{F}_{2\,\mathrm{HA}\,0} &= -2\,g\,M\,t_5\,\phi^0-g^2\,t_5\,\phi^0\,\mathrm{h}+\frac{1}{2}\,g\,(\mathrm{Z}^\mu\,\overrightarrow{\partial}_\mu-\overleftarrow{\partial}_\mu\,\mathrm{Z}^\mu)\,\frac{1}{\mathrm{c}_\mathrm{W}},\\
\mathrm{F}_{2\,\mathrm{HH}^-} &= -g\,M\,(t_4+t_5)\,\phi^+-\frac{1}{2}\,g^2\,(t_4+t_5)\,\phi^+\,\mathrm{h}\\
&\quad +\frac{1}{2}\,i\,g^2\left[(t_5-t_4)\,\phi^0\,\phi^+ +\mathrm{Z}^\mu\,\mathrm{W}^+{}_\mu\,\frac{\mathrm{s}_\mathrm{W}^2}{\mathrm{c}_\mathrm{W}}-\mathrm{A}^\mu\,\mathrm{W}^+{}_\mu\,\mathrm{s}_\mathrm{W}\right]\\
&\quad +\frac{1}{2}\,g\,(\mathrm{W}^{+\mu}\,\overrightarrow{\partial}_\mu-\overleftarrow{\partial}_\mu\,\mathrm{W}^{+\mu}),\\
\mathrm{F}_{2\,\mathrm{A}\,0\,\mathrm{H}^-} &= \frac{1}{2}\,g^2\left[(t_4+t_5)\,\phi^0\,\phi^+-\mathrm{Z}^\mu\,\mathrm{W}^+{}_\mu\,\frac{\mathrm{s}_\mathrm{W}^2}{\mathrm{c}_\mathrm{W}}+\mathrm{A}^\mu\,\mathrm{W}^+{}_\mu\,\mathrm{s}_\mathrm{W}\right]+i\,g\,M\,(t_5-t_4)\,\phi^+\\
&\quad +\frac{1}{2}\,i\,g^2\,(t_5-t_4)\,\phi^+\,\mathrm{h}+\frac{1}{2}\,i\,g\,(\mathrm{W}^{+\mu}\,\overrightarrow{\partial}_\mu-\overleftarrow{\partial}_\mu\,\mathrm{W}^{+\mu}),\\
\mathrm{F}_{2\,\mathrm{H}^-\mathrm{H}^-} &= \frac{1}{2}\,g^2\,t_5\,\phi^+\,\phi^+,
\end{aligned}
$$

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$$
\begin{aligned}
\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-} =& 2\,g\,M\left[\bar{t}-2(t_4+t_5)\right]\mathrm{h}\\
&+\frac{1}{4}\,g^2\left[2\,\bar{t}\,\Phi_{\mathrm{h}}^2-2\,(t_4+t_5)\,\mathrm{h}^2-2\,(t_4+t_5)\,\phi^{0\,2}-4\,t_5\,\phi^+\,\phi^- -\frac{1-4\,\mathrm{s}_{\mathrm{W}}^2\,\mathrm{c}_{\mathrm{W}}^2}{\mathrm{c}_{\mathrm{W}}^2}\,\mathrm{Z}^2\right.\\
&\left.+4\,(1-2\,\mathrm{c}_{\mathrm{W}}^2)\,\frac{\mathrm{s}_{\mathrm{W}}}{\mathrm{c}_{\mathrm{W}}}\,\mathrm{Z}^\mu\,\mathrm{A}_\mu-4\,\mathrm{s}_{\mathrm{W}}^2\,\mathrm{A}^2-\mathrm{W}^{+\mu}\,\mathrm{W}^-{}_\mu\right]\\
&+\frac{1}{4}\,g^3\,\frac{M^3}{\Lambda^2}\,\mathrm{T}_c\,\mathrm{h}+\frac{1}{16}\,g^4\,\frac{M^2}{\Lambda^2}\,\mathrm{T}_c\,\Phi_{\mathrm{h}}^2+\frac{1}{2}\,i\,g\,\frac{1-2\,\mathrm{c}_{\mathrm{W}}^2}{\mathrm{c}_{\mathrm{W}}}\,(\mathrm{Z}^\mu\overrightarrow{\partial}_\mu-\overleftarrow{\partial}_\mu\,\mathrm{Z}^\mu)\\
&-i\,g\,\mathrm{s}_{\mathrm{W}}\,(\mathrm{A}^\mu\overrightarrow{\partial}_\mu-\overleftarrow{\partial}_\mu\,\mathrm{A}^\mu)\ . \qquad (4.28)
\end{aligned}
$$

where we have introduced

$$
\mathrm{T}_c=128\,(t_1-\bar{t})^2\,\frac{t_1}{x_{\mathrm{h}}+4\,t_1}-3\,x_{\mathrm{h}}^2-32\,(t_1-\bar{t})^2-4\,(5\,\bar{t}-2\,t_1)\,x_{\mathrm{h}}\ . \qquad (4.29)
$$

To give an example we show the loop operators generated by integrating heavy bubbles:

$$
\mathcal{L}_b=\frac{1}{16\,\pi^2}\left(\mathcal{L}_b^0+\frac{1}{\Lambda^2}\,\mathcal{L}_b^2\right)\ , \qquad (4.30)
$$

$$
\begin{aligned}
\mathcal{L}_b^0=&-\left(\mathrm{F}_{2\,\mathrm{HH}}^0\right)^2\,\overline{\mathrm{A}}_0(M_{\mathrm{H}})-\left(\mathrm{F}_{2\,\mathrm{A}\,0\,\mathrm{A}\,0}^0\right)^2\,\overline{\mathrm{A}}_0(M_{\mathrm{A}\,0})\\
&-\frac{1}{2}\left[(2\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^+}^0\,\mathrm{F}_{2\,\mathrm{H}^-\mathrm{H}^-}^0+\left(\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^{00}\right)^2-\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^{00}\,\partial_\mu\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^{0\,\mu}\right]\overline{\mathrm{A}}_0(M_{\mathrm{H}\,\pm})\\
&-\frac{1}{2}\,\left(\mathrm{F}_{2\,\mathrm{HA}\,0}^{00}\right)^2\,\mathrm{B}_{00}(\overline{M})-\frac{1}{2}\left[2\,\left(\mathrm{F}_{2\,\mathrm{HH}}^0\right)^2+2\,\left(\mathrm{F}_{2\,\mathrm{A}\,0\,\mathrm{A}\,0}^0\right)^2\right.\\
&\left.+2\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^+}^0\,\mathrm{F}_{2\,\mathrm{H}^-\mathrm{H}^-}^0+\left(\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^{00}\right)^2-\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^{00}\,\partial_\mu\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^{0\,\mu}\right]\,, \qquad (4.31)\\
\mathcal{L}_b^2=&-\frac{1}{4}\,\mathrm{v}^2\,g^2\,\left(\mathrm{F}_{2\,\mathrm{HA}\,0}^{00}\right)^2\,\lambda_5\\
&-\frac{1}{12}\,g^2\left[24\,\mathrm{F}_{2\,\mathrm{HH}}^0\,\mathrm{F}_{2\,\mathrm{HH}}^2+24\,\mathrm{F}_{2\,\mathrm{A}\,0\,\mathrm{A}\,0}^0\,\mathrm{F}_{2\,\mathrm{A}\,0\,\mathrm{A}\,0}^2+12\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^+}^0\,\mathrm{F}_{2\,\mathrm{H}^-\mathrm{H}^-}^2\right.\\
&+12\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^+}^2\,\mathrm{F}_{2\,\mathrm{H}^-\mathrm{H}^-}^0\\
&+12\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^2\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^{00}-6\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^2\,\partial_\mu\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^{0\,\mu}-\partial_\mu\partial_\nu\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^{0\,\nu}\,\partial^\mu\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^{00}\\
&+2\,\partial_\mu\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^+}^0\,\partial^\mu\,\mathrm{F}_{2\,\mathrm{H}^-\mathrm{H}^-}^0+2\,\partial_\mu\,\mathrm{F}_{2\,\mathrm{HH}}^0\,\partial^\mu\,\mathrm{F}_{2\,\mathrm{HH}}^0\\
&\left.+3\,\partial_\mu\,\mathrm{F}_{2\,\mathrm{HA}\,0}^{00}\,\partial^\mu\,\mathrm{F}_{2\,\mathrm{HA}\,0}^{00}+2\,\partial_\mu\,\mathrm{F}_{2\,\mathrm{A}\,0\,\mathrm{A}\,0}^0\,\partial^\mu\,\mathrm{F}_{2\,\mathrm{A}\,0\,\mathrm{A}\,0}^0+\partial_\mu\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^{00}\,\partial^\mu\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^{00}\right]\\
&-2\,g^2\,\mathrm{F}_{2\,\mathrm{HH}}^0\,\mathrm{F}_{2\,\mathrm{HH}}^2\,\overline{\mathrm{A}}_0(M_{\mathrm{H}})-2\,g^2\,\mathrm{F}_{2\,\mathrm{A}\,0\,\mathrm{A}\,0}^0\,\mathrm{F}_{2\,\mathrm{A}\,0\,\mathrm{A}\,0}^2\,\overline{\mathrm{A}}_0(M_{\mathrm{A}\,0})\\
&-\frac{1}{2}\,g^2\left[2\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^+}^0\,\mathrm{F}_{2\,\mathrm{H}^-\mathrm{H}^-}^2+2\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^+}^2\,\mathrm{F}_{2\,\mathrm{H}^-\mathrm{H}^-}^0\right.\\
&\left.+2\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^2\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^{00}-\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^2\,\partial_\mu\,\mathrm{F}_{2\,\mathrm{H}^+\mathrm{H}^-}^{0\,\mu}\right]\overline{\mathrm{A}}_0(M_{\mathrm{H}\,\pm})-g^2\,\mathrm{F}_{2\,\mathrm{HA}\,0}^2\,\mathrm{F}_{2\,\mathrm{HA}\,0}^{00}\,\mathrm{B}_{00}(\overline{M}).
\end{aligned}
$$

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5 Conclusions

In this work we have been mainly interested in the effect of heavy scalars, with masses that are larger than the Higgs VEV and the energies probed by current experimental data. In particular we focused on models where there are mixing effects in the mass matrices. Therefore, we have adopted a top-down approach where there is a model and the aim has been to implement a systematic procedure for getting the low-energy theory, including all loop generated local operators.

We have extended the covariant derivative expansion [9, 10], taking into account SM extensions where heavy fields are coupled to (light) SM fields with linear (or higher) couplings that are proportional to the scale of new physics. Specific examples have been provided for the singlet extension of the SM and for THDM (I, II,X and Y) models [35]. Working in the broken phase, including all contributions and normalizing the kinetic terms considerably increases the number of SM deviations as compared to what is usually reported in the literature.

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A Expansion of loop integrals

Power counting of loop integrals can be summarized as follows: define

$$\int d^n q \frac{q_{\mu_1} \cdots q_{\mu_{2k}}}{(q^2+M^2)^l} = \delta_{\mu_1 \ldots \mu_{2k}} \, \mathrm{I}_{l\,,k}\,, \tag{A.1}$$

where the δ is the fully symmetric combination. In the large M limit one has

$$\mathrm{I}_{1\,,2\,k} \sim M^{2+2\,k} \ln M^2\,, \quad \mathrm{I}_{l\,,0} \sim (M^2)^{2-l} \quad , \quad \mathrm{I}_{2\,,0} \sim \ln M^2\,, \quad \mathrm{I}_{l\,,2\,k} \sim \mathrm{I}_{l-1\,,2\,k-2} \qquad l>1\,. \tag{A.2}$$

We define the following functions:

$$\begin{aligned} i\,\pi^2\,\mathrm{C}_0^{(1)}(m) &= {\mu_\mathrm{R}}^{4-n} \int d^n q \, \frac{1}{(q^2+m^2)\left((q+p_1)^2+M_\mathrm{H}^2\right)\left((q+p_1+p_2)^2+m^2\right)}\,, \\ i\,\pi^2\,\mathrm{C}_0^{(2)}(m) &= {\mu_\mathrm{R}}^{4-n} \int d^n q \, \frac{1}{(q^2+m^2)\left((q+p_1)^2+M_\mathrm{H}^2\right)\left((q+p_1+p_2)^2+M_\mathrm{H}^2\right)}\,, \end{aligned} \tag{A.3}$$

with $P = p_1 + p_2$, μ_R being the renormalization scale. Their M_H expansion is given in terms of two-point functions

$$i\,\pi^2\,\mathrm{B}_0\left(\alpha\,,\,\beta\,;\,P^2\,,m\,,\,m\right) = {\mu_\mathrm{R}}^{4-n} \int d^n q \, \frac{1}{(q^2+m^2)^\alpha \left((q+P)^2+m^2\right)^\beta}\,, \tag{A.4}$$

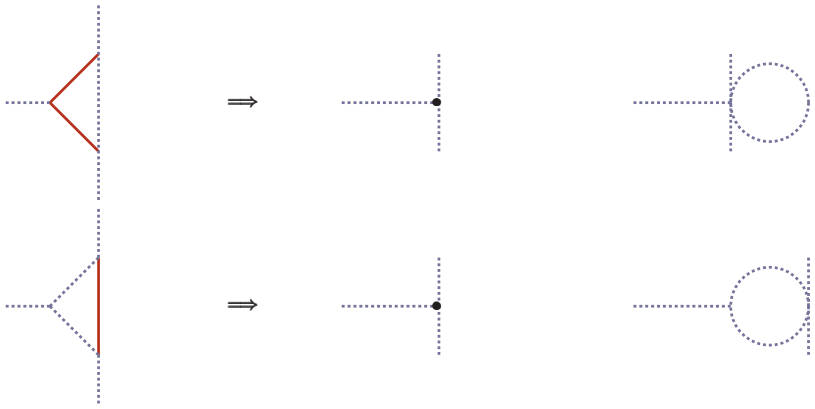

Figure 8. Examples of mixed (heavy-light) loops. Solid (red) lines denote heavy fields, dashed (blue) lines denote light fields. Integrating out the heavy field gives a (contact) local operators plus a non-local term which is interpreted as a one-loop diagram in the low energy theory.

and of one-point functions, defined in eq. (2.15). We obtain

$$
\begin{aligned}
\mathrm{C}_0^{(2)}(m) &= \frac{1}{M_{\mathrm{H}}^2} + \frac{1}{M_{\mathrm{H}}^4}\left\{m^2\left[1-\overline{\mathrm{A}}_0(M_{\mathrm{H}})+\overline{\mathrm{A}}_0(m)\right] - \frac{1}{3}\left(p_1^2+p_2^2+\frac{1}{2}\,p_1\cdot p_2\right)\right\},\\
\mathrm{C}_0^{(2)}(m) &= \frac{1}{M_{\mathrm{H}}^2}\left[\mathrm{B}_0\left(1\,,\,1\,;\,P^2\,,\,m\,,\,m\right)+\overline{\mathrm{A}}_0(m)+\ln\frac{M_{\mathrm{H}}^2}{m^2}\right]\\
&\quad+\frac{1}{M_{\mathrm{H}}^4}\left\{\frac{3}{4}\left(p_1^2+p_2^2-\frac{1}{3}\,P^2\right)-\frac{1}{2}\left(p_1^2+p_2^2-P^2-4\,m^2\right)\ln\frac{M_{\mathrm{H}}^2}{m^2}\right.\\
&\quad+\left[m^2+\frac{1}{2}\left(P^2-p_1^2-p_2^2\right)\right]\overline{\mathrm{A}}_0(m)\\
&\quad\left.+\left[m^2+\frac{1}{2}\left(P^2-p_1^2-p_2^2\right)\right]\mathrm{B}_0\left(1\,,\,1\,;\,P^2\,,\,m\,,\,m\right)\right\},
\end{aligned}
\tag{A.5}
$$

where the B_0 function is given by

$$
\mathrm{B}_0\left(1\,,\,1\,;\,P^2\,,\,m\,,\,m\right) = \frac{2}{4-n}-\gamma-\ln\pi+2-\ln\frac{m^2}{\mu_{\mathrm{R}}^2}-\beta\,\ln\frac{\beta+1}{\beta-1}\,,
\tag{A.6}
$$

with $\beta^2 = 1+4\,m^2/(P^2-i\,0)$, where $P^2=-s$.

In deriving the expansion in eq. (A.5) we also need the following results:

$$
\begin{aligned}
\mathrm{B}_0^{\mathrm{fin}}\left(1\,,\,1\,;\,P^2\,,\,M_{\mathrm{H}}\,,\,m\right) &= 1-\ln\frac{M_{\mathrm{H}}^2}{s}+\frac{s}{M_{\mathrm{H}}^2}\left(\frac{1}{2}-\frac{m^2}{s}\,\ln\frac{M_{\mathrm{H}}^2}{m^2}\right)\\
&\quad+\frac{s^2}{M_{\mathrm{H}}^4}\left[\frac{1}{6}+\frac{3}{2}\,\frac{m^2}{s}-\frac{m^2}{s}\left(1+\frac{m^2}{s}\right)\ln\frac{M_{\mathrm{H}}^2}{m^2}\right],\\
\mathrm{B}_0^{\mathrm{fin}}\left(2\,,\,1\,;\,P^2\,,\,M_{\mathrm{H}}\,,\,m\right) &= \frac{s}{M_{\mathrm{H}}^2}+\frac{s^2}{M_{\mathrm{H}}^4}\left[\frac{1}{2}+\frac{m^2}{s}\left(1-\ln\frac{M_{\mathrm{H}}^2}{m^2}\right)\right].
\end{aligned}
\tag{A.7}
$$

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When all masses are heavy we derive:

$$\mathrm{B}_0\left(1\,,\,1\,;\,P^2\,,\,M_{\mathrm{H}}\,,\,M_{\mathrm{H}}\right) = -\overline{\mathrm{A}}_0(M_{\mathrm{H}}) - 1 - \frac{1}{6}\,\frac{P^2}{M_{\mathrm{H}}^2} + \mathcal{O}\left(\frac{P^4}{M_{\mathrm{H}}^4}\right)\,, \tag{A.8}$$

for the singlet extension and

$$\mathrm{B}_0\Big(1\,,\,1\,;\,P^2\,,\,M_1\,,\,M_1\Big) = -\mathrm{B}_{00}\,(M_1\,,\,M_2) + \frac{\mathrm{v}^2}{\overline{M}^2}\mathrm{B}_{02}\,(M_1\,,\,M_2) + \frac{P^2}{\overline{M}^2}\mathrm{B}_{0p}\,(M_1\,,\,M_2) + \,\cdots\,, \tag{A.9}$$

for THDM models. Using the masses of eq. (4.13) we obtain

$$\begin{aligned}
\mathrm{B}_{00}\,(M_1\,,\,M_2) &= -\mathrm{B}_{00}\,\left(\overline{M}\right)\,, & \mathrm{B}_{0p}\,(M_1\,,\,M_2) &= -\frac{1}{2}\,,\\
\mathrm{B}_{02}\left(M_{\mathrm{A}\,0}\,,\,M_{\mathrm{H}\,\pm}\right) &= -\frac{1}{4}\,\left(\lambda_4 + 3\,\lambda_5\right)\,, & \mathrm{B}_{02}\left(M_{\mathrm{H}}\,,\,M_{\mathrm{H}\,\pm}\right) &= -\frac{1}{4}\,\left(\lambda_4 + \lambda_5\right)\,,\\
\mathrm{B}_{02}\left(M_{\mathrm{H}}\,,\,M_{\mathrm{A}\,0}\right) &= -\frac{1}{2}\,\lambda_5\,. & &
\end{aligned} \tag{A.10}$$

B Complete SESM Lagrangian

In this appendix we give the list of local operators, up to $\mathcal{O}(\Lambda^{-2})$, present in the singlet extension of the SM after integration of the heavy mode H. Field content is: gauge bosons $\mathrm{A}\,\mathrm{Z}\,\mathrm{W}^{\pm}$, light Higgs h, Higgs-Kibble ghosts $\phi^0, \phi^{\pm}$, fermions u, d and Faddeev-Popov ghosts $\mathrm{X}^+, \mathrm{X}^-, \mathrm{Y}_{\mathrm{A}}, \mathrm{Y}_{\mathrm{Z}}$. Few auxiliary quantities are needed:

$$\Phi_0^2 = \phi_0^\dagger\,\phi_{_0} = {\phi^0}^2 + 2\,\phi^+\,\phi^-\,, \qquad \Phi_{\mathrm{h}}^2 = \phi_{\mathrm{h}}^\dagger\,\phi_{_{\mathrm{h}}} = \mathrm{h}^2 + {\phi^0}^2 + 2\,\phi^+\,\phi^-\,. \tag{B.1}$$

We also need U(1) covariant derivatives:

$$\mathrm{D}_\mu\,\phi^\pm = \partial_\mu\,\phi^\pm \pm i\,g\,\mathrm{s}_{\mathrm{W}}\,\mathrm{A}_\mu\,\phi^\pm\,, \qquad \mathrm{D}_\mu\,\mathrm{f} = \partial_\mu\,\mathrm{f} - i\,g\,\mathrm{Q}_{\mathrm{f}}\,\mathrm{s}_{\mathrm{W}}\,\mathrm{A}_\mu\,\mathrm{f}\,. \tag{B.2}$$

Finally, T functions are defined in eq. (3.19), while β coefficients in eq. (3.18); the one-point function $\overline{\mathrm{A}}_0$ is defined in eq. (2.15), in the following, $\overline{\mathrm{A}}_0(M_{\mathrm{H}}) \equiv \overline{\mathrm{A}}_0$. Dimension and codimension of local operators are defined in eq. (3.30). In the following list we give $\mathcal{L}_{\mathrm{dim}\,,\,\mathrm{codim}}$ for one flavor, i.e.

$$\mathcal{L} = \sum_{n=0}^{2} \Lambda^{2\,n-2} \sum_l M^l\,\mathcal{L}_{6-2\,n\,,\,6-2\,n-l}\,. \tag{B.3}$$

- <u>dim = 2</u>

  - $\mathcal{L}_{22} = -\dfrac{1}{32}\,\dfrac{g^2}{\pi^2}\,\Phi_{\mathrm{h}}^2\,\beta_2^{(0} - \dfrac{1}{16}\,\dfrac{g^2}{\pi^2}\,t_m\,t_1^2\,\Phi_0^2\,\overline{\mathrm{A}}_0\,, \quad \mathcal{L}_{23} = \dfrac{1}{32}\,\dfrac{g^3}{\pi^2}\,\dfrac{t_m\,t_1^2}{M}\,\Phi_{\mathrm{h}}^2\,\mathrm{h}\,\overline{\mathrm{A}}_0\,,$
    $\mathcal{L}_{24} = \dfrac{1}{256}\,\dfrac{g^4}{\pi^2}\,\dfrac{t_m\,t_1^2}{M^2}\,\Phi_{\mathrm{h}}^4\,\overline{\mathrm{A}}_0\,,$ (B.4)

- $\underline{\dim = 4}$

► $$\mathcal{L}_{41} = -\frac{1}{8}\,\frac{g\,M^3}{\pi^2}\,\mathrm{h}\,\Delta\beta_2^{(1)}\,, \tag{B.5}$$

► $$\begin{aligned}\mathcal{L}_{42} = &-\frac{1}{32}\,\frac{g^2\,M^2}{\pi^2}\,\Phi_{\mathrm{h}}^2\,\beta_2^{(1)} + \frac{1}{32}\,\frac{g^2\,M^2}{\pi^2}\,t_m^2\,\left(\beta_2^{(0)} - \beta_1^{(0)}\right)\,\mathrm{h}^2\\ &+\frac{1}{16}\,\frac{g^2\,M^2}{\pi^2}\,t_m^2\,t_1\,\left(\overline{\mathrm{X}}^-\,\mathrm{X}^- + \overline{\mathrm{X}}^+\,\mathrm{X}^+\right)\,\overline{\mathrm{A}}_0\\ &-\frac{1}{64}\,\frac{g^2\,M^2}{\pi^2}\,t_m^2\,t_1\,\left(x_{\mathrm{h}} + 4\,t_m^2\,t_1\right)\,\phi^{0\,2}\,\overline{\mathrm{A}}_0\\ &-\frac{1}{32}\,\frac{g^2\,M^2}{\pi^2}\,t_m^2\,t_1\,\left[x_{\mathrm{h}} - 2\,\left(1 - 2\,t_m^2\,t_1\right)\right]\,\phi^+\,\phi^-\,\overline{\mathrm{A}}_0 + \frac{1}{32}\,\frac{g^2\,M^2}{\pi^2}\,t_m\,\Phi_{\mathrm{h}}^2\,\mathrm{T}_1\\ &+\frac{1}{32}\,\frac{g^2\,M^2}{\mathrm{c}_{\mathrm{W}}^2\,\pi^2}\,t_m^2\,t_1\,\left(\phi^{0\,2} + 2\,\overline{\mathrm{Y}}_{\mathrm{Z}}\,\mathrm{Y}_{\mathrm{Z}}\right)\,\overline{\mathrm{A}}_0 - \frac{1}{2}\,\frac{M^2}{\mathrm{c}_{\mathrm{W}}^2}\,\left(\phi^{0\,2} + 2\,\overline{\mathrm{Y}}_{\mathrm{Z}}\,\mathrm{Y}_{\mathrm{Z}}\right)\\ &-\frac{1}{2}\,M^2\,\left[\left(2\,\mathrm{W}_\mu^+\,\mathrm{W}_\mu^- + \frac{1}{\mathrm{c}_{\mathrm{W}}^2}\,\mathrm{Z}_\mu\,\mathrm{Z}_\mu\right) + 2\,\left(\overline{\mathrm{X}}^-\,\mathrm{X}^- + \overline{\mathrm{X}}^+\,\mathrm{X}^+\right) + 2\,\phi^+\,\phi^-\right]\\ &-\frac{1}{2}\,M^2\,x_{\mathrm{h}}\,\mathrm{h}^2\,,\end{aligned} \tag{B.6}$$

► $$\begin{aligned}\mathcal{L}_{43} = &-\left(\partial_\mu\,\overline{\mathrm{X}}^-\,\partial_\mu\,\mathrm{X}^- + \partial_\mu\,\overline{\mathrm{X}}^+\,\partial_\mu\,\mathrm{X}^+ + \partial_\mu\,\overline{\mathrm{Y}}_{\mathrm{Z}}\,\partial_\mu\,\mathrm{Y}_{\mathrm{Z}} + \partial_\mu\,\overline{\mathrm{Y}}_{\mathrm{A}}\,\partial_\mu\,\mathrm{Y}_{\mathrm{A}}\right)\\ &+\frac{1}{64}\,\frac{g^3\,M}{\pi^2}\,t_m^2\,\left(\beta_2^{(0)} - \beta_1^{(0)}\right)\,\Phi_{\mathrm{h}}^2\,\mathrm{h}\\ &+\frac{1}{64}\,\frac{g^3\,M}{\pi^2}\,t_m^2\,t_1\,\left[\left(2\,\mathrm{W}_\mu^+\,\mathrm{W}_\mu^- + \frac{1}{\mathrm{c}_{\mathrm{W}}^2}\,\mathrm{Z}_\mu\,\mathrm{Z}_\mu\right) + \left(\overline{\mathrm{X}}^-\,\mathrm{X}^- + \overline{\mathrm{X}}^+\,\mathrm{X}^+\right)\right]\,\mathrm{h}\,\overline{\mathrm{A}}_0\\ &+\frac{1}{64}\,\frac{g^3\,M}{\pi^2}\,t_m^2\,t_1\,\left[x_{\mathrm{h}}\,\overline{\mathrm{A}}_0 - 8\,\left(1 - t_m\right)\,t_m\,t_1 - \left(13 - 14\,t_m\right)\,t_m\,t_1\,\overline{\mathrm{A}}_0\right]\,\Phi_0^2\,\mathrm{h}\\ &+\frac{1}{64}\,\frac{g^3\,M}{\pi^2}\,t_m^2\,t_1\,\left[x_{\mathrm{h}}\,\overline{\mathrm{A}}_0 - 8\,\left(1 - t_m\right)\,t_m\,t_1 - \left(15 - 14\,t_m\right)\,t_m\,t_1\,\overline{\mathrm{A}}_0\right]\,\mathrm{h}^3\\ &+\frac{1}{64}\,\frac{g^3\,M}{\mathrm{c}_{\mathrm{W}}^2\,\pi^2}\,\overline{\mathrm{Y}}_{\mathrm{Z}}\,\mathrm{Y}_{\mathrm{Z}}\,t_m^2\,t_1\,\mathrm{h}\,\overline{\mathrm{A}}_0 + \frac{1}{2}\,\frac{i\,g\,M}{\mathrm{c}_{\mathrm{W}}}\,\left(1 - 2\,\mathrm{c}_{\mathrm{W}}^2\right)\,\left(\overline{\mathrm{X}}^+\,\phi^+ - \overline{\mathrm{X}}^-\,\phi^-\right)\,\mathrm{Y}_{\mathrm{Z}}\\ &+\frac{1}{2}\,\frac{i\,g\,M}{\mathrm{c}_{\mathrm{W}}}\,\overline{\mathrm{Y}}_{\mathrm{Z}}\,\left(\mathrm{X}^-\,\phi^+ - \mathrm{X}^+\,\phi^-\right)\\ &-\frac{1}{64}\,\frac{i\,g^3\,M}{\pi^2}\,t_m^2\,t_1\,\left[2\,\left(\phi^+\,\mathrm{W}_\mu^- - \phi^-\,\mathrm{W}_\mu^+\right)\,\left(\frac{\mathrm{s}_{\mathrm{W}}^2}{\mathrm{c}_{\mathrm{W}}}\,\mathrm{Z}_\mu - \mathrm{A}_\mu\,\mathrm{s}_{\mathrm{W}}\right)\right.\\ &\left.+\left(\overline{\mathrm{X}}^+\,\mathrm{X}^+ - \overline{\mathrm{X}}^-\,\mathrm{X}^-\right)\,\phi^0\right]\,\overline{\mathrm{A}}_0\\ &-\frac{1}{64}\,\frac{i\,g^3\,M}{\pi^2}\,\left(1 - 2\,\mathrm{c}_{\mathrm{W}}^2\right)\,\frac{1}{\mathrm{c}_{\mathrm{W}}\,t_m^2\,t_1, t_m^2\,t_1}\,\left(\overline{\mathrm{X}}^+\,\phi^+ - \overline{\mathrm{X}}^-\,\phi^-\right)\,\mathrm{Y}_{\mathrm{Z}}\,\overline{\mathrm{A}}_0\end{aligned}$$

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$$
\begin{aligned}
&-\frac{1}{64}\,\frac{i\,g^3\,M}{\pi^2}\,\frac{1}{\mathrm{c_W}\,t_m^2\,t_1\,t_m^2\,t_1}\,\overline{\mathrm{Y}}_{\mathrm{Z}}\,\left(\mathrm{X}^-\,\phi^+-\mathrm{X}^+\,\phi^-\right)\,\overline{\mathrm{A}}_0\\
&+\frac{1}{32}\,\frac{i\,g^3\,M}{\pi^2}\,t_m^2\,t_1\,\mathrm{s_W}\,\left(\overline{\mathrm{X}}^+\,\phi^+-\overline{\mathrm{X}}^-\,\phi^-\right)\,\mathrm{Y_A}\,\overline{\mathrm{A}}_0\\
&-\frac{1}{2}\,\frac{g\,M}{\mathrm{c_W^2}}\,\overline{\mathrm{Y}}_{\mathrm{Z}}\,\mathrm{Y_Z}\,\mathrm{h}-\frac{1}{4}\,g\,M\,x_{\mathrm{h}}\,\Phi_{\mathrm{h}}^2\,\mathrm{h}\\
&+\frac{1}{2}\,i\,g\,M\,\left[2\,\left(\phi^+\,\mathrm{W}_\mu^--\phi^-\,\mathrm{W}_\mu^+\right)\,\left(\frac{\mathrm{s_W^2}}{\mathrm{c_W}}\,\mathrm{Z}_\mu-\mathrm{A}_\mu\,\mathrm{s_W}\right)\right.\\
&\left.+\left(\overline{\mathrm{X}}^+\,\mathrm{X}^+-\overline{\mathrm{X}}^-\,\mathrm{X}^-\right)\,\phi^0\right]\\
&-\frac{1}{2}\,g\,M\,\left[\left(2\,\mathrm{W}_\mu^+\,\mathrm{W}_\mu^-+\frac{1}{\mathrm{c_W^2}}\,\mathrm{Z}_\mu\,\mathrm{Z}_\mu\right)+\left(\overline{\mathrm{X}}^-\,\mathrm{X}^-+\overline{\mathrm{X}}^+\,\mathrm{X}^+\right)\right]\,\mathrm{h}\\
&-M\,x_{\mathrm{d}}\,\overline{\mathrm{d}}\,\mathrm{d}-M\,x_{\mathrm{u}}\,\overline{\mathrm{u}}\,\mathrm{u}-i\,g\,M\,\mathrm{s_W}\,\left(\overline{\mathrm{X}}^+\,\phi^+-\overline{\mathrm{X}}^-\,\phi^-\right)\,\mathrm{Y_A}\,,
\end{aligned}
\tag{B.7}
$$

► $\mathcal{L}_{44}=$

$$
\begin{aligned}
&-\frac{1}{2}\,\left[\partial_\mu\,\mathrm{A}_\nu\,\partial_\mu\,\mathrm{A}_\nu+\partial_\mu\,\mathrm{Z}_\nu\,\partial_\mu\,\mathrm{Z}_\nu+2\,\partial_\mu\,\mathrm{W}_\nu^+\,\partial_\mu\,\mathrm{W}_\nu^-+\partial_\mu\,\mathrm{h}^2+\partial_\mu\,{\phi^0}^2\right.\\
&\left.+2\,\overline{\mathrm{u}}\,\gamma^\mu\,\mathrm{D}_\mu\,\mathrm{u}+2\,\overline{\mathrm{d}}\,\gamma^\mu\,\mathrm{D}_\mu\,\mathrm{d}\right]-\mathrm{D}_\mu\,\phi^+\,\mathrm{D}_\mu\,\phi^-\\
&-\frac{1}{2}\,\frac{g^2\,\mathrm{s_W^2}}{\mathrm{c_W}}\,\left(\phi^+\,\mathrm{W}_\mu^-+\phi^-\,\mathrm{W}_\mu^+\right)\,\phi^0\,\mathrm{Z}_\mu\\
&-\frac{1}{64}\,\frac{g^3}{\pi^2}\,t_m^2\,t_1\,\left(x_{\mathrm{d}}\,\overline{\mathrm{d}}\,\mathrm{d}+x_{\mathrm{u}}\,\overline{\mathrm{u}}\,\mathrm{u}\right)\,\mathrm{h}\,\overline{\mathrm{A}}_0-\frac{1}{512}\,\frac{g^4}{\pi^2}\,t_m^2\,\Phi_{\mathrm{h}}^4\,\beta_1^{(0)}\\
&+\frac{1}{1024}\,\frac{g^4}{\pi^2}\,t_m^2\,t_1\,\left[x_{\mathrm{h}}\,\overline{\mathrm{A}}_0-16\,\left(2-t_m\right)\,t_m\,t_1-28\,\left(2-t_m\right)\,t_m\,t_1\,\overline{\mathrm{A}}_0\right]\,\Phi_0^4\\
&+\frac{1}{512}\,\frac{g^4}{\pi^2}\,t_m^2\,t_1\,\left[x_{\mathrm{h}}\,\overline{\mathrm{A}}_0-16\,\left(2-t_m\right)\,t_m\,t_1-4\,\left(17-7\,t_m\right)\,t_m\,t_1\,\overline{\mathrm{A}}_0\right]\,\Phi_0^2\,\mathrm{h}^2\\
&+\frac{1}{1024}\,\frac{g^4}{\pi^2}\,t_m^2\,t_1\,\left[x_{\mathrm{h}}\,\overline{\mathrm{A}}_0-16\,\left(2-t_m\right)\,t_m\,t_1-4\,\left(20-7\,t_m\right)\,t_m\,t_1\,\overline{\mathrm{A}}_0\right]\,\mathrm{h}^4\\
&+\frac{1}{4}\,\frac{i\,g}{\mathrm{c_W}}\,\left(\overline{\mathrm{u}}\,\gamma^\mu\,\mathrm{u}+\overline{\mathrm{u}}\,\gamma^\mu\,\gamma^5\,\mathrm{u}\right)\,\mathrm{Z}_\mu\\
&+\frac{1}{4}\,\frac{i\,g}{\mathrm{c_W}}\,\left(1-2\,\mathrm{c_W^2}\right)\,\left(\overline{\mathrm{d}}\,\gamma^\mu\,\mathrm{d}+\overline{\mathrm{d}}\,\gamma^\mu\,\gamma^5\,\mathrm{d}\right)\,\mathrm{Z}_\mu\\
&-\frac{1}{2}\,\frac{i\,g}{\mathrm{c_W}}\,\left(1-2\,\mathrm{c_W^2}\right)\,\phi^+\,\mathrm{Z}_\mu\,\mathrm{D}_\mu\,\phi^-+\frac{1}{2}\,\frac{i\,g}{\mathrm{c_W}}\,\left(1-2\,\mathrm{c_W^2}\right)\,\phi^-\,\mathrm{Z}_\mu\,\mathrm{D}_\mu\,\phi^+\\
&-\frac{i\,g\,\mathrm{s_W^2}}{\mathrm{c_W}}\,\left(\mathrm{Q_u}\,\overline{\mathrm{u}}\,\gamma^\mu\,\mathrm{u}+\mathrm{Q_d}\,\overline{\mathrm{d}}\,\gamma^\mu\,\mathrm{d}\right)\,\mathrm{Z}_\mu-\frac{1}{2}\,\frac{i\,g\,\mathrm{s_W^2}}{\mathrm{c_W}}\,\left(\overline{\mathrm{d}}\,\gamma^\mu\,\mathrm{d}+\overline{\mathrm{d}}\,\gamma^\mu\,\gamma^5\,\mathrm{d}\right)\,\mathrm{Z}_\mu\\
&+\frac{1}{2}\,\frac{i\,g^2\,\mathrm{s_W^2}}{\mathrm{c_W}}\,\left(\phi^+\,\mathrm{W}_\mu^--\phi^-\,\mathrm{W}_\mu^+\right)\,\mathrm{h}\,\mathrm{Z}_\mu
\end{aligned}
$$

$$
\begin{aligned}
&-\frac{1}{64}\,\frac{i\,g^3}{\pi^2}\,t_m^2\,t_1\,\left(x_{\rm d}\,\overline{\rm d}\,\gamma^5\,{\rm d}+x_{\rm u}\,\overline{\rm u}\,\gamma^5\,{\rm u}\right)\,\phi^0\,\overline{\rm A}_0\\
&-\frac{1}{64}\,\frac{i\,g^3}{\sqrt{2}\,\pi^2}\,t_m^2\,t_1\,x_{\rm d}\,\left(\overline{\rm u}\,\gamma_-\,\phi^+\,{\rm d}-\overline{\rm d}\,\gamma_+\,\phi^-\,{\rm u}\right)\,\overline{\rm A}_0\\
&+\frac{1}{64}\,\frac{i\,g^3}{\sqrt{2}\,\pi^2}\,t_m^2\,t_1\,x_{\rm u}\,\left(\overline{\rm u}\,\gamma_+\,\phi^+\,{\rm d}-\overline{\rm d}\,\gamma_-\,\phi^-\,{\rm u}\right)\,\overline{\rm A}_0\\
&+\frac{1}{2}\,\frac{i\,g}{\sqrt{2}}\,\left(\overline{\rm u}\,\gamma^\mu\,\gamma_+\,{\rm d}\,{\rm W}_\mu^+ + \overline{\rm d}\,\gamma^\mu\,\gamma_+\,{\rm u}\,{\rm W}_\mu^-\right)\\
&+\frac{1}{2}\,\frac{i}{\sqrt{2}}\,g\,x_{\rm u}\,\left(\overline{\rm u}\,\gamma_+\,\phi^+\,{\rm d}-\overline{\rm d}\,\gamma_-\,\phi^-\,{\rm u}\right)\\
&-\frac{1}{2}\,\frac{i}{\sqrt{2}}\,g\,x_{\rm d}\,\left(\overline{\rm u}\,\gamma_-\,\phi^+\,{\rm d}-\overline{\rm d}\,\gamma_+\,\phi^-\,{\rm u}\right)\\
&-\frac{1}{2}\,\frac{g}{{\rm c_w}}\,\left(\phi^0\,\partial_\mu\,{\rm h}-{\rm h}\,\partial_\mu\,\phi^0\right)\,{\rm Z}_\mu\\
&-\frac{1}{32}\,g^2\,x_{\rm h}\,\Phi_{\rm h}^4+g^2\,{\rm s_w^2}\,\left(\phi^+\,\phi^-\right)\,{\rm Z}_\mu\,{\rm Z}_\mu\\
&-\frac{1}{8}\,g^2\,\left(2\,{\rm W}_\mu^+\,{\rm W}_\mu^-+\frac{1}{{\rm c_w^2}}\,{\rm Z}_\mu\,{\rm Z}_\mu\right)\,\Phi_{\rm h}^2\\
&+\frac{1}{2}\,i\,g\,\left(x_{\rm u}\,\overline{\rm u}\,\gamma^5\,{\rm u}+x_{\rm d}\,\overline{\rm d}\,\gamma^5\,{\rm d}\right)\,\phi^0\\
&-\frac{1}{2}\,i\,g\,{\rm W}_\mu^+\,\phi^0\,{\rm D}_\mu\,\phi^- + \frac{1}{2}\,i\,g\,{\rm W}_\mu^-\,\phi^0\,{\rm D}_\mu\,\phi^+\\
&-\frac{1}{2}\,i\,g\,\Big[\left(\phi^+\,{\rm W}_\mu^- - \phi^-\,{\rm W}_\mu^+\right)\,\partial_\mu\,\phi^0 - 2\,\left({\rm A}_\mu\,{\rm s_w}+{\rm Z}_\mu\,{\rm c_w}\right)\,\partial_\nu\,{\rm W}_\mu^+\,{\rm W}_\nu^-\\
&+2\,\left({\rm A}_\mu\,{\rm s_w}+{\rm Z}_\mu\,{\rm c_w}\right)\,\partial_\nu\,{\rm W}_\mu^-\,{\rm W}_\nu^+\\
&+2\,\left(\overline{\rm X}^+\,{\rm X}^+-\overline{\rm X}^-\,{\rm X}^-\right)\,\left(\partial_\mu\,{\rm A}_\mu\,{\rm s_w}+\partial_\mu\,{\rm Z}_\mu\,{\rm c_w}\right)\\
&+2\,\left(\overline{\rm X}^+\,\partial_\mu\,{\rm X}^+-\overline{\rm X}^-\,\partial_\mu\,{\rm X}^-\right)\,\left({\rm A}_\mu\,{\rm s_w}+{\rm Z}_\mu\,{\rm c_w}\right)\Big]\\
&+i\,g\,{\rm c_w}\,\Big[\left({\rm W}_\mu^+\,\partial_\nu\,{\rm W}_\mu^- - {\rm W}_\mu^-\,\partial_\nu\,{\rm W}_\mu^+\right)\,{\rm Z}_\nu - \left({\rm W}_\mu^+\,{\rm W}_\nu^- - {\rm W}_\mu^-\,{\rm W}_\nu^+\right)\,\partial_\nu\,{\rm Z}_\mu\\
&+\left(\overline{\rm X}^+\,{\rm Y_Z}-\overline{\rm Y}_{\rm Z}\,{\rm X}^-\right)\,\partial_\mu\,{\rm W}_\mu^+ + \left(\overline{\rm Y}_{\rm Z}\,{\rm X}^+-\overline{\rm X}^-\,{\rm Y_Z}\right)\,\partial_\mu\,{\rm W}_\mu^-\\
&+\left(\overline{\rm X}^+\,\partial_\mu\,{\rm Y_Z}-\overline{\rm Y}_{\rm Z}\,\partial_\mu\,{\rm X}^-\right)\,{\rm W}_\mu^+\\
&-\left(\overline{\rm Y}_{\rm Z}\,\partial_\mu\,{\rm X}^+ + \overline{\rm X}^-\,\partial_\mu\,{\rm Y_Z}\right)\,{\rm W}_\mu^-\Big]\\
&+i\,g\,{\rm s_w}\,\Big[\left({\rm W}_\mu^+\,\partial_\nu\,{\rm W}_\mu^- - {\rm W}_\mu^-\,\partial_\nu\,{\rm W}_\mu^+\right)\,{\rm A}_\nu - \left({\rm W}_\mu^+\,{\rm W}_\nu^- - {\rm W}_\mu^-\,{\rm W}_\nu^+\right)\,\partial_\nu\,{\rm A}_\mu\\
&+\left(\overline{\rm X}^+\,{\rm Y_A}-\overline{\rm Y}_{\rm A}\,{\rm X}^-\right)\,\partial_\mu\,{\rm W}_\mu^+ + \left(\overline{\rm Y}_{\rm A}\,{\rm X}^+-\overline{\rm X}^-\,{\rm Y_A}\right)\,\partial_\mu\,{\rm W}_\mu^-\\
&+\left(\overline{\rm X}^+\,\partial_\mu\,{\rm Y_A}-\overline{\rm Y}_{\rm A}\,\partial_\mu\,{\rm X}^-\right)\,{\rm W}_\mu^+
\end{aligned}
$$

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$$
\begin{aligned}
&-\overline{\mathrm{X}}^{-}\,\partial_\mu\,\mathrm{Y}_{\mathrm{A}}\,\mathrm{W}^{-}_{\mu}+\overline{\mathrm{Y}}_{\mathrm{A}}\,\partial_\mu\,\mathrm{X}^{+}\,\mathrm{W}^{-}_{\mu}\Big]\\
&-\frac{1}{2}\,g\,\Big[\Big(\phi^{+}\,\mathrm{W}^{-}_{\mu}+\phi^{-}\,\mathrm{W}^{+}_{\mu}\Big)\,\partial_\mu\,\mathrm{h}-\Big(\mathrm{W}^{+}_{\mu}\,\mathrm{D}_\mu\,\phi^{-}+\mathrm{W}^{-}_{\mu}\,\mathrm{D}_\mu\,\phi^{+}\Big)\,\mathrm{h}\Big]\\
&-\frac{1}{2}\,g\,\left(x_{\mathrm{u}}\,\overline{\mathrm{u}}\,\mathrm{u}+x_{\mathrm{d}}\,\overline{\mathrm{d}}\,\mathrm{d}\right)\,\mathrm{h}
\end{aligned}
\tag{B.8}
$$

► $$\mathcal{L}_{45}=-\frac{1}{256}\,\frac{g^5}{\pi^2}\,\frac{t_m^3\,t_1^2}{M}\,\mathrm{h}\,\Phi_{\mathrm{h}}^4\,\overline{\mathrm{A}}_0\,, \tag{B.9}$$

• <u>dim = 6</u>

► $$\mathcal{L}_{61}=-\frac{1}{8}\,\frac{g\,M^5}{\pi^2}\,\mathrm{h}\,\Delta\beta_2^{(2)}+\frac{1}{16}\,\frac{g\,M^5}{\pi^2}\,t_m^2\,\mathrm{h}\,\Delta\beta_2^{(1)}\,, \tag{B.10}$$

► $$
\begin{aligned}
\mathcal{L}_{62}=&-\frac{1}{32}\,\frac{g^2\,M^4}{\pi^2}\,\Phi_{\mathrm{h}}^2\,\beta_2^{(1)}+\frac{1}{64}\,\frac{g^2\,M^4}{\pi^2}\,\frac{t_m^2}{t_1}\,\left(\beta_2^{(0)}-\beta_1^{(0)}\right)\,\left(x_{\mathrm{h}}-2\,t_m^2\,t_1\right)\,\mathrm{h}^2\\
&+\frac{1}{64}\,\frac{g^2\,M^4}{\pi^2}\,\frac{t_m}{t_1}\,\left(2\,\mathrm{W}^{+}_{\mu}\,\mathrm{W}^{-}_{\mu}+\frac{1}{\mathrm{c}_{\mathrm{W}}^2}\,\mathrm{Z}_\mu\,\mathrm{Z}_\mu\right)\,\mathrm{T}_1\\
&-\frac{1}{64}\,\frac{g^2\,M^4}{\pi^2}\,\frac{t_m}{t_1}\,\left(3+2\,t_m^2\,t_1\right)\,\Phi_0^2\,\mathrm{T}_1\\
&+\frac{1}{64}\,\frac{g^2\,M^4}{\pi^2}\,\frac{t_m}{t_1}\,\Big[x_{\mathrm{h}}-\left(3-2\,t_m\,t_1+4\,t_m^2\,t_1\right)\Big]\,\mathrm{h}^2\,\mathrm{T}_1\\
&+\frac{1}{32}\,\frac{g^2\,M^4}{\pi^2}\,t_m^2\,\Phi_0^2\,\Delta\beta_2^{(1)}\\
&+\frac{1}{32}\,\frac{g^2\,M^4}{\pi^2}\,t_m^2\,\left(\beta_2^{(1)}-\beta_1^{(1)}+\Delta\beta_2^{(1)}\right)\,\mathrm{h}^2\\
&+\frac{1}{32}\,\frac{g^2\,M^4}{\pi^2}\,t_m^2\,x_{\mathrm{h}}\,\left(\overline{\mathrm{X}}^{-}\,\mathrm{X}^{-}+\overline{\mathrm{X}}^{+}\,\mathrm{X}^{+}\right)\,\overline{\mathrm{A}}_0\\
&-\frac{1}{128}\,\frac{g^2\,M^4}{\pi^2}\,t_m^2\,x_{\mathrm{h}}\,\Big[x_{\mathrm{h}}-2\,\left(1-3\,t_m\right)\,t_m\,t_1\Big]\,\phi^{0\,2}\,\overline{\mathrm{A}}_0\\
&-\frac{1}{64}\,\frac{g^2\,M^4}{\pi^2}\,t_m^2\,x_{\mathrm{h}}\,\Big[x_{\mathrm{h}}-2\,\left(1+t_m\,t_1-3\,t_m^2\,t_1\right)\Big]\,\phi^{+}\,\phi^{-}\,\overline{\mathrm{A}}_0\\
&+\frac{1}{32}\,\frac{g^2\,M^4}{\pi^2}\,t_m\,\Phi_{\mathrm{h}}^2\,\mathrm{T}_2+\frac{1}{64}\,\frac{g^2\,M^4}{\mathrm{c}_{\mathrm{W}}^2\,\pi^2}\,t_m^2\,x_{\mathrm{h}}\,\left(\phi^{0\,2}+2\,\overline{\mathrm{Y}}_{\mathrm{Z}}\,\mathrm{Y}_{\mathrm{Z}}\right)\,\overline{\mathrm{A}}_0\,,
\end{aligned}
\tag{B.11}
$$

► $$
\begin{aligned}
\mathcal{L}_{63}=&-\frac{1}{64}\,\frac{g^2\,M^3}{\pi^2}\,\frac{t_m}{t_1}\,\left(\mathrm{W}^{+}_{\mu}\,\mathrm{D}_\mu\,\phi^{-}+\mathrm{W}^{-}_{\mu}\,\mathrm{D}_\mu\,\phi^{+}\right)\,\mathrm{T}_1\\
&+\frac{1}{64}\,\frac{g^2\,M^3}{\pi^2}\,\frac{t_m}{t_1}\,\left(x_{\mathrm{u}}\,\overline{\mathrm{u}}\,\mathrm{u}+x_{\mathrm{d}}\,\overline{\mathrm{d}}\,\mathrm{d}\right)\,\mathrm{T}_1-\frac{1}{64}\,\frac{g^2\,M^3}{\mathrm{c}_{\mathrm{W}}\,\pi^2}\,\frac{t_m}{t_1}\,\mathrm{T}_1\,\mathrm{Z}_\mu\,\partial_\mu\,\phi^0\\
&+\frac{1}{128}\,\frac{g^3\,M^3}{\pi^2}\,\frac{t_m^2}{t_1}\,\left(\beta_2^{(0)}-\beta_1^{(0)}\right)\,\left(2\,\mathrm{W}^{+}_{\mu}\,\mathrm{W}^{-}_{\mu}+\frac{1}{\mathrm{c}_{\mathrm{W}}^2}\,\mathrm{Z}_\mu\,\mathrm{Z}_\mu\right)\,\mathrm{h}
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{128}\,\frac{g^3\,M^3}{\pi^2}\,\frac{t_m^2}{t_1}\,\left(\beta_2^{(0)}-\beta_1^{(0)}\right)\,\left(x_{\rm h}-3\,t_m^2\,t_1\right)\,\Phi_0^2\,{\rm h}\\
&+\frac{1}{128}\,\frac{g^3\,M^3}{\pi^2}\,\frac{t_m^2}{t_1}\,\left(\beta_2^{(0)}-\beta_1^{(0)}\right)\,\left[2\,x_{\rm h}+(2-5\,t_m)\,t_m\,t_1\right]{\rm h}^3\\
&+\frac{1}{128}\,\frac{g^3\,M^3}{\pi^2}\,\frac{t_m}{t_1}\,\left(2\,{\rm W}_\mu^+\,{\rm W}_\mu^-+\frac{1}{{\rm c}_{\rm W}^2}\,{\rm Z}_\mu\,{\rm Z}_\mu\right)\,{\rm h}\,{\rm T}_1\\
&+\frac{1}{256}\,\frac{g^3\,M^3}{\pi^2}\,\frac{t_m}{t_1}\,\left(x_{\rm h}+4\,(1-t_m)\,t_m\,t_1\right)\,\Phi_{\rm h}^2\,{\rm h}\,{\rm T}_1\\
&-\frac{1}{64}\,\frac{g^3\,M^3}{\pi^2}\,t_m^2\,\left(\beta_1^{(1)}-\beta_2^{(1)}\right)\,\Phi_{\rm h}^2\,{\rm h}\\
&+\frac{1}{384}\,\frac{g^3\,M^3}{\pi^2}\,t_m^2\,\left[3\,x_{\rm h}\,\overline{\rm A}_0+2\,(1-t_m)^2\,t_1\right]\,\left(\overline{\rm X}^-\,{\rm X}^-+\overline{\rm X}^+\,{\rm X}^+\right)\,{\rm h}\\
&+\frac{1}{384}\,\frac{g^3\,M^3}{\pi^2}\,t_m^2\,\Big[3\,x_{\rm h}\,\overline{\rm A}_0-2\,(1-t_m)\,(17-5\,t_m)\,t_1\\
&-3\,\left(18-22\,t_m+7\,t_m^2\right)\,t_1\,\overline{\rm A}_0\Big]\,\left(2\,{\rm W}_\mu^+\,{\rm W}_\mu^-+\frac{1}{{\rm c}_{\rm W}^2}\,{\rm Z}_\mu\,{\rm Z}_\mu\right)\,{\rm h}\\
&+\frac{1}{384}\,\frac{g^3\,M^3}{\pi^2}\,t_m^2\,\Big[3\,x_{\rm h}^2\,\overline{\rm A}_0+24\,(1-t_m)\,t_m^3\,t_1^2-\left(1+40\,t_m-53\,t_m^2\right)\,t_1\,x_{\rm h}\\
&+6\,(6-7\,t_m)\,t_m^3\,t_1^2\,\overline{\rm A}_0\\
&-3\,(19-25\,t_m)\,t_m\,t_1\,x_{\rm h}\,\overline{\rm A}_0\Big]\,\Phi_0^2\,{\rm h}\\
&+\frac{1}{384}\,\frac{g^3\,M^3}{\pi^2}\,t_m^2\,\Big[3\,x_{\rm h}^2\,\overline{\rm A}_0+8\,(1-t_m)\,\left(4-20\,t_m+19\,t_m^2\right)\,t_m\,t_1^2\\
&-6\,\left(6+10\,t_m-42\,t_m^2+29\,t_m^3\right)\,t_m\,t_1^2\,\overline{\rm A}_0\\
&-3\,\left(18-3\,t_m-17\,t_m^2\right)\,t_1\,x_{\rm h}\,\overline{\rm A}_0-\left(35-4\,t_m-43\,t_m^2\right)\,t_1\,x_{\rm h}\Big]\,{\rm h}^3\\
&+\frac{1}{384}\,\frac{g^3\,M^3}{{\rm c}_{\rm W}^2\,\pi^2}\,\overline{\rm Y}_{\rm Z}\,{\rm Y}_{\rm Z}\,t_m^2\,\left[3\,x_{\rm h}\,\overline{\rm A}_0+2\,(1-t_m)^2\,t_1\right]\,{\rm h}\\
&-\frac{1}{64}\,\frac{i\,g^3\,M^3}{\pi^2}\,\frac{t_m}{t_1}\,\frac{{\rm s}_{\rm W}^2}{{\rm c}_{\rm W}}\,\left(\phi^+\,{\rm W}_\mu^--\phi^-\,{\rm W}_\mu^+\right)\,{\rm T}_1\,{\rm Z}_\mu\\
&-\frac{1}{128}\,\frac{i\,g^3\,M^3}{\pi^2}\,t_m^2\,x_{\rm h}\,\Big[2\,\left(\phi^+\,{\rm W}_\mu^--\phi^-\,{\rm W}_\mu^+\right)\,\left(\frac{{\rm s}_{\rm W}^2}{{\rm c}_{\rm W}}\,{\rm Z}_\mu-{\rm A}_\mu\,{\rm s}_{\rm W}\right)\\
&+\left(\overline{\rm X}^+\,{\rm X}^+-\overline{\rm X}^-\,{\rm X}^-\right)\,\phi^0\Big]\,\overline{\rm A}_0\\
&-\frac{1}{128}\,\frac{i\,g^3\,M^3}{\pi^2}\,\left(1-2\,{\rm c}_{\rm W}^2\right)\,\frac{1}{{\rm c}_{\rm W}\,t_m^2\,x_{\rm h},t_m^2\,x_{\rm h}}\,\left(\overline{\rm X}^+\,\phi^+-\overline{\rm X}^-\,\phi^-\right)\,{\rm Y}_{\rm Z}\,\overline{\rm A}_0\\
&-\frac{1}{128}\,\frac{i\,g^3\,M^3}{\pi^2}\,\frac{1}{{\rm c}_{\rm W}\,t_m^2\,x_{\rm h},t_m^2\,x_{\rm h}}\,\overline{\rm Y}_{\rm Z}\,\left({\rm X}^-\,\phi^+-{\rm X}^+\,\phi^-\right)\,\overline{\rm A}_0
\end{aligned}
$$

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$$+\frac{1}{64}\,\frac{i\,g^3\,M^3}{\pi^2}\,\mathrm{s_W}\,t_m^2\,x_\mathrm{h}\,\left(\overline{\mathrm{X}}^+\,\phi^+-\overline{\mathrm{X}}^-\,\phi^-\right)\,\mathrm{Y_A}\,\overline{\mathrm{A}}_0$$
$$+\frac{1}{4}\,g\,M^3\,t_m^2\,\left(2\,\mathrm{W}_\mu^+\,\mathrm{W}_\mu^-+\frac{1}{\mathrm{c_W^2}}\,\mathrm{Z}_\mu\,\mathrm{Z}_\mu\right)\,\mathrm{h}+\frac{1}{8}\,g\,M^3\,t_m^2\,x_\mathrm{h}\,\left(\Phi_\mathrm{h}^2+2\,\mathrm{h}^2\right)\,\mathrm{h}\,,\tag{B.12}$$

► $\mathcal{L}_{64}=$
$$-\frac{1}{128}\,\frac{g^3\,M^2}{\pi^2}\,\frac{t_m^2}{t_1}\,\left(\beta_2^{(0)}-\beta_1^{(0)}\right)\,\left(\mathrm{W}_\mu^+\,\mathrm{D}_\mu\,\phi^-+\mathrm{W}_\mu^-\,\mathrm{D}_\mu\,\phi^+\right)\,\mathrm{h}$$
$$+\frac{1}{128}\,\frac{g^3\,M^2}{\pi^2}\,\frac{t_m^2}{t_1}\,\left(\beta_2^{(0)}-\beta_1^{(0)}\right)\,\left(x_\mathrm{u}\,\overline{\mathrm{u}}\,\mathrm{u}+x_\mathrm{d}\,\overline{\mathrm{d}}\,\mathrm{d}\right)\,\mathrm{h}$$
$$-\frac{1}{384}\,\frac{g^3\,M^2}{\pi^2}\,t_m^2\,x_\mathrm{d}$$
$$\times\left[3\,x_\mathrm{h}\,\overline{\mathrm{A}}_0+2\,(1-t_m)\,(17+t_m)\,t_1+3\,\left(18-18\,t_m+t_m^2\right)\,t_1\,\overline{\mathrm{A}}_0\right]\,\overline{\mathrm{d}}\,\mathrm{d}\,\mathrm{h}$$
$$-\frac{1}{384}\,\frac{g^3\,M^2}{\pi^2}\,t_m^2\,x_\mathrm{u}$$
$$\times\left[3\,x_\mathrm{h}\,\overline{\mathrm{A}}_0+2\,(1-t_m)\,(17+t_m)\,t_1+3\,\left(18-18\,t_m+t_m^2\right)\,t_1\,\overline{\mathrm{A}}_0\right]\,\overline{\mathrm{u}}\,\mathrm{u}\,\mathrm{h}$$
$$-\frac{1}{192}\,\frac{g^3\,M^2}{\pi^2}\,t_m^2\,t_1\,\left[3\,t_m^2\,\overline{\mathrm{A}}_0-(1-t_m)^2\right]\,\left(\phi^+\,\mathrm{W}_\mu^-+\phi^-\,\mathrm{W}_\mu^+\right)\,\partial_\mu\,\mathrm{h}$$
$$+\frac{1}{192}\,\frac{g^3\,M^2}{\pi^2}\,t_m^2\,t_1\,\left[(1-t_m)\,(17+t_m)+3\,\left(9-9\,t_m+t_m^2\right)\,\overline{\mathrm{A}}_0\right]$$
$$\times\,\left(\mathrm{W}_\mu^+\,\mathrm{D}_\mu\,\phi^-+\mathrm{W}_\mu^-\,\mathrm{D}_\mu\,\phi^+\right)\,\mathrm{h}$$
$$-\frac{1}{128}\,\frac{g^3\,M^2}{\mathrm{c_W}\,\pi^2}\,\frac{t_m^2}{t_1}\,\left(\beta_2^{(0)}-\beta_1^{(0)}\right)\,\mathrm{h}\,\mathrm{Z}_\mu\,\partial_\mu\,\phi^0$$
$$-\frac{1}{192}\,\frac{g^3\,M^2}{\mathrm{c_W}\,\pi^2}\,t_m^2\,t_1\,\left[3\,t_m^2\,\overline{\mathrm{A}}_0-(1-t_m)^2\right]\,\phi^0\,\mathrm{Z}_\mu\,\partial_\mu\,\mathrm{h}$$
$$+\frac{1}{192}\,\frac{g^3\,M^2}{\mathrm{c_W}\,\pi^2}\,t_m^2\,t_1\,\left[(1-t_m)\,(17+t_m)+3\,\left(9-9\,t_m+t_m^2\right)\,\overline{\mathrm{A}}_0\right]\,\mathrm{h}\,\mathrm{Z}_\mu\,\partial_\mu\,\phi^0$$
$$-\frac{1}{512}\,\frac{g^4\,M^2}{\pi^2}\,\frac{t_m^2}{t_1}\,\left(2\,\mathrm{W}_\mu^+\,\mathrm{W}_\mu^-+\frac{1}{\mathrm{c_W^2}}\,\mathrm{Z}_\mu\,\mathrm{Z}_\mu\right)\,\Phi_0^2\,\beta_1^{(0)}$$
$$+\frac{1}{512}\,\frac{g^4\,M^2}{\pi^2}\,\frac{t_m^2}{t_1}\,\left(2\,\beta_2^{(0)}-3\,\beta_1^{(0)}\right)\,\left(2\,\mathrm{W}_\mu^+\,\mathrm{W}_\mu^-+\frac{1}{\mathrm{c_W^2}}\,\mathrm{Z}_\mu\,\mathrm{Z}_\mu\right)\,\mathrm{h}^2$$
$$-\frac{1}{1024}\,\frac{g^4\,M^2}{\pi^2}\,\frac{t_m^2}{t_1}\,x_\mathrm{h}\,\Phi_0^4\,\beta_1^{(0)}+\frac{1}{1024}\,\frac{g^4\,M^2}{\pi^2}\,\frac{t_m^2}{t_1}\,x_\mathrm{h}\,\left(2\,\beta_2^{(0)}-5\,\beta_1^{(0)}\right)\,\mathrm{h}^4$$
$$+\frac{1}{512}\,\frac{g^4\,M^2}{\pi^2}\,\frac{t_m^2}{t_1}\,x_\mathrm{h}\,\left(\beta_2^{(0}-3\,\beta_1^{(0)}\right)\,\Phi_0^2\,\mathrm{h}^2$$
$$-\frac{1}{512}\,\frac{g^4\,M^2}{\pi^2}\,t_m^2\,\Phi_\mathrm{h}^4\,\beta_1^{(1)}+\frac{1}{512}\,\frac{g^4\,M^2}{\pi^2}\,t_m^2\,\Phi_\mathrm{h}^4\,\mathrm{T}_1$$

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$$
\begin{aligned}
&+\frac{1}{1024}\,\frac{g^4\,M^2}{\pi^2}\,t_m^2\,\Big[x_{\rm h}^2\,\overline{\rm A}_0-32\,(1-t_m)\,\Big(1+3\,t_m-5\,t_m^2\Big)\,t_m\,t_1^2\\
&-8\,\Big(6+3\,t_m-5\,t_m^2\Big)\,t_1\,x_{\rm h}-8\,\Big(21-14\,t_m-38\,t_m^2+35\,t_m^3\Big)\,t_m\,t_1^2\,\overline{\rm A}_0\\
&-2\,\Big(36+17\,t_m-14\,t_m^2\Big)\,t_1\,x_{\rm h}\,\overline{\rm A}_0\Big]\,\Phi_0^2\,{\rm h}^2\\
&+\frac{1}{6144}\,\frac{g^4\,M^2}{\pi^2}\,t_m^2\,\Big[3\,x_{\rm h}^2\,\overline{\rm A}_0+192\,(1-t_m)\,t_m^3\,t_1^2-4\,\Big(1+40\,t_m-29\,t_m^2\Big)\,t_1\,x_{\rm h}\\
&+24\,(13-14\,t_m)\,t_m^3\,t_1^2\,\overline{\rm A}_0-6\,(39-23\,t_m)\,t_m\,t_1\,x_{\rm h}\,\overline{\rm A}_0\Big]\,\Phi_0^4\\
&+\frac{1}{6144}\,\frac{g^4\,M^2}{\pi^2}\,t_m^2\,\Big[3\,x_{\rm h}^2\,\overline{\rm A}_0-192\,(1-t_m)\,\Big(1+3\,t_m-4\,t_m^2\Big)\,t_m\,t_1^2\\
&-168\,\Big(6-4\,t_m-9\,t_m^2+8\,t_m^3\Big)\,t_m\,t_1^2\,\overline{\rm A}_0\\
&-4\,\Big(71-4\,t_m-31\,t_m^2\Big)\,t_1\,x_{\rm h}-6\,\Big(72-5\,t_m-5\,t_m^2\Big)\,t_1\,x_{\rm h}\,\overline{\rm A}_0\Big]\,{\rm h}^4\\
&-\frac{1}{256}\,\frac{g^4\,M^2}{\pi^2}\,t_m^2\,t_1\,\Big[2\,(3-t_m)+\Big(9-2\,t_m+2\,t_m^2\Big)\,\overline{\rm A}_0\Big]\\
&\times\left(2\,{\rm W}_\mu^+\,{\rm W}_\mu^-+\frac{1}{{\rm c}_{\rm W}^2}\,{\rm Z}_\mu\,{\rm Z}_\mu\right)\,\Phi_0^2\\
&-\frac{1}{768}\,\frac{g^4\,M^2}{\pi^2}\,t_m^2\,t_1\,\Big[2\,\Big(26-19\,t_m-t_m^2\Big)+3\,\Big(27-20\,t_m+4\,t_m^2\Big)\,\overline{\rm A}_0\Big]\\
&\times\left(2\,{\rm W}_\mu^+\,{\rm W}_\mu^-+\frac{1}{{\rm c}_{\rm W}^2}\,{\rm Z}_\mu\,{\rm Z}_\mu\right)\,{\rm h}^2\\
&+\frac{3}{32}\,\frac{g^4\,M^2}{\pi^2}\,t_m^3\,t_1^2\,\Big[(1-t_m)^2+(1-t_m)^2\,\overline{\rm A}_0\Big]\,\Phi_{\rm h}^2\,{\rm h}^2\\
&+\frac{1}{256}\,\frac{g^4\,M^2}{\pi^2}\,t_m^3\,\Big(2\,\beta_2^{(0}-3\,\beta_1^{(0)}\Big)\,\Phi_{\rm h}^2\,{\rm h}^2-\frac{1}{512}\,\frac{g^4\,M^2}{\pi^2}\,t_m^4\,\Big(5\,\beta_2^{(0}-8\,\beta_1^{(0)}\Big)\,{\rm h}^4\\
&-\frac{1}{256}\,\frac{g^4\,M^2}{\pi^2}\,t_m^4\,\Big(3\,\beta_2^{(0}-5\,\beta_1^{(0)}\Big)\,\Phi_0^2\,{\rm h}^2-\frac{1}{512}\,\frac{g^4\,M^2}{\pi^2}\,t_m^4\,\Big(\beta_2^{(0}-2\,\beta_1^{(0)}\Big)\,\Phi_0^4\\
&-\frac{1}{4}\,\frac{i\,g^2\,{\rm s}_{\rm W}^2}{{\rm c}_{\rm W}}\,M^2\,t_m^2\,\Big(\phi^+\,{\rm W}_\mu^--\phi^-\,{\rm W}_\mu^+\Big)\,{\rm h}\,{\rm Z}_\mu\\
&-\frac{1}{128}\,\frac{i\,g^3\,M^2}{\pi^2}\,t_m^2\,x_{\rm h}\,\Big(x_{\rm d}\,\overline{\rm d}\,\gamma^5\,{\rm d}+x_{\rm u}\,\overline{\rm u}\,\gamma^5\,{\rm u}\Big)\,\phi^0\,\overline{\rm A}_0\\
&-\frac{1}{128}\,\frac{i\,g^3\,M^2}{\sqrt{2}\,\pi^2}\,t_m^2\,x_{\rm h}\,x_{\rm d}\,\Big(\overline{\rm u}\,\gamma_-\,\phi^+\,{\rm d}-\overline{\rm d}\,\gamma_+\,\phi^-\,{\rm u}\Big)\,\overline{\rm A}_0\\
&+\frac{1}{128}\,\frac{i\,g^3\,M^2}{\sqrt{2}\,\pi^2}\,t_m^2\,x_{\rm h}\,x_{\rm u}\,\Big(\overline{\rm u}\,\gamma_+\,\phi^+\,{\rm d}-\overline{\rm d}\,\gamma_-\,\phi^-\,{\rm u}\Big)\,\overline{\rm A}_0\\
&-\frac{1}{128}\,\frac{i\,g^4\,M^2}{\pi^2}\,\frac{t_m^2}{t_1}\,\frac{{\rm s}_{\rm W}^2}{{\rm c}_{\rm W}}\,\Big(\beta_2^{(0}-\beta_1^{(0)}\Big)\,\Big(\phi^+\,{\rm W}_\mu^--\phi^-\,{\rm W}_\mu^+\Big)\,{\rm h}\,{\rm Z}_\mu
\end{aligned}
$$

$$
\begin{aligned}
&+\frac{1}{192}\,\frac{i\,g^4\,M^2}{\pi^2}\,\frac{\mathrm{s}_\mathrm{w}^2}{\mathrm{c}_\mathrm{w}}\,t_m^2\,t_1\left[(1-t_m)\,(17+t_m)+3\,\left(9-9\,t_m+t_m^2\right)\,\overline{\mathrm{A}}_0\right]\\
&\times\left(\phi^+\,\mathrm{W}_\mu^- - \phi^-\,\mathrm{W}_\mu^+\right)\,\mathrm{h}\,\mathrm{Z}_\mu\\
&+\frac{1}{4}\,\frac{g}{\mathrm{c}_\mathrm{w}}\,M^2\,t_m^2\,\left(\phi^0\,\partial_\mu\,\mathrm{h}-\mathrm{h}\,\partial_\mu\,\phi^0\right)\,\mathrm{Z}_\mu\\
&+\frac{1}{8}\,g^2\,M^2\,t_m^2\,\left(\Phi_\mathrm{h}^2+\mathrm{h}^2\right)\,\left(2\,\mathrm{W}_\mu^+\,\mathrm{W}_\mu^-+\frac{1}{\mathrm{c}_\mathrm{w}^2}\,\mathrm{Z}_\mu\,\mathrm{Z}_\mu\right)\\
&+\frac{1}{32}\,g^2\,M^2\,t_m^2\,x_\mathrm{h}\,\left(\Phi_\mathrm{h}^2+6\,\mathrm{h}^2\right)\,\Phi_\mathrm{h}^2\\
&+\frac{1}{4}\,g\,M^2\,t_m^2\,\left[\left(\phi^+\,\mathrm{W}_\mu^-+\phi^-\,\mathrm{W}_\mu^+\right)\,\partial_\mu\,\mathrm{h}-\left(\mathrm{W}_\mu^+\,\mathrm{D}_\mu\,\phi^-+\mathrm{W}_\mu^-\,\mathrm{D}_\mu\,\phi^+\right)\,\mathrm{h}\right]\\
&+\frac{1}{4}\,g\,M^2\,t_m^2\,\left(x_\mathrm{d}\,\overline{\mathrm{d}}\,\mathrm{d}+x_\mathrm{u}\,\overline{\mathrm{u}}\,\mathrm{u}\right)\,\mathrm{h}
\end{aligned}
\tag{B.13}
$$

► $$
\begin{aligned}
\mathcal{L}_{65}=&\frac{1}{8}\,\frac{g^2}{\mathrm{c}_\mathrm{w}}\,M\,t_m^2\,\left(2\,\phi^0\,\partial_\mu\,\Phi_\mathrm{h}^2-\Phi_\mathrm{h}^2\,\partial_\mu\,\phi^0\right)\,\mathrm{Z}_\mu\\
&-\frac{1}{128}\,\frac{g^3\,M}{\pi^2}\,\frac{t_m^2}{t_1}\,\left(\beta_2^{(0}-\beta_1^{(0)}\right)\,\partial_\mu\,\mathrm{h}\,\partial_\mu\,\Phi_\mathrm{h}^2\\
&-\frac{1}{192}\,\frac{g^3\,M}{\pi^2}\,t_m^2\,t_1\,\left(2-9\,\overline{\mathrm{A}}_0\right)\,(1-t_m)\,\partial_\mu\,\mathrm{h}\,\partial_\mu\,\Phi_\mathrm{h}^2\\
&+\frac{1}{512}\,\frac{g^4\,M}{\pi^2}\,\frac{t_m^2}{t_1}\,\left[\left(\mathrm{W}_\mu^+\,\mathrm{D}_\mu\,\phi^-+\mathrm{W}_\mu^-\,\mathrm{D}_\mu\,\phi^+\right)\,\Phi_\mathrm{h}^2\right.\\
&\left.-2\,\left(\phi^+\,\mathrm{W}_\mu^-+\phi^-\,\mathrm{W}_\mu^+\right)\,\partial_\mu\,\Phi_\mathrm{h}^2\right]\,\beta_1^{(0)}\\
&-\frac{1}{512}\,\frac{g^4\,M}{\pi^2}\,\frac{t_m^2}{t_1}\,\beta_1^{(0)}\,\left(x_\mathrm{u}\,\overline{\mathrm{u}}\,\mathrm{u}+x_\mathrm{d}\,\overline{\mathrm{d}}\,\mathrm{d}\right)\,\Phi_\mathrm{h}^2\\
&+\frac{1}{256}\,\frac{g^4\,M}{\pi^2}\,t_m^2\,t_1\left[6+\left(9+t_m^2\right)\,\overline{\mathrm{A}}_0\right]\,\left[\left(\mathrm{W}_\mu^+\,\mathrm{D}_\mu\,\phi^-+\mathrm{W}_\mu^-\,\mathrm{D}_\mu\,\phi^+\right)\,\Phi_\mathrm{h}^2\right.\\
&\left.-2\,\left(\phi^+\,\mathrm{W}_\mu^-+\phi^-\,\mathrm{W}_\mu^+\right)\,\partial_\mu\,\Phi_\mathrm{h}^2\right]\\
&-\frac{3}{256}\,\frac{g^4\,M}{\pi^2}\,t_m^2\,t_1\,\left(2+3\,\overline{\mathrm{A}}_0\right)\,\left(x_\mathrm{d}\,\overline{\mathrm{d}}\,\mathrm{d}+x_\mathrm{u}\,\overline{\mathrm{u}}\,\mathrm{u}\right)\,\Phi_\mathrm{h}^2\\
&-\frac{1}{512}\,\frac{g^4\,M}{\mathrm{c}_\mathrm{w}\,\pi^2}\,\frac{t_m^2}{t_1}\,\left(2\,\phi^0\,\partial_\mu\,\Phi_\mathrm{h}^2-\Phi_\mathrm{h}^2\,\partial_\mu\,\phi^0\right)\,\mathrm{Z}_\mu\,\beta_1^{(0)}\\
&-\frac{1}{256}\,\frac{g^4\,M}{\mathrm{c}_\mathrm{w}\,\pi^2}\,t_m^2\,t_1\left[6+\left(9+t_m^2\right)\,\overline{\mathrm{A}}_0\right]\,\left(2\,\phi^0\,\partial_\mu\,\Phi_\mathrm{h}^2-\Phi_\mathrm{h}^2\,\partial_\mu\,\phi^0\right)\,\mathrm{Z}_\mu\\
&-\frac{1}{1024}\,\frac{g^5\,M}{\pi^2}\,\frac{t_m^2}{t_1}\,\left(2\,\mathrm{W}_\mu^+\,\mathrm{W}_\mu^-+\frac{1}{\mathrm{c}_\mathrm{w}^2}\,\mathrm{Z}_\mu\,\mathrm{Z}_\mu\right)\,\Phi_\mathrm{h}^2\,\mathrm{h}\,\beta_1^{(0)}\\
&-\frac{1}{2048}\,\frac{g^5\,M}{\pi^2}\,\frac{t_m^2}{t_1}\,\left(x_\mathrm{h}-4\,t_m^2\,t_1\right)\,\mathrm{h}\,\Phi_\mathrm{h}^4\,\beta_1^{(0)}
\end{aligned}
$$

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$$
\begin{aligned}
&-\frac{1}{512}\,\frac{g^5\,M}{\pi^2}\,t_m^2\,t_1\left[6+\left(9+t_m^2\right)\,\overline{\mathrm{A}}_0\right]\left(2\,\mathrm{W}_\mu^+\,\mathrm{W}_\mu^- + \frac{1}{\mathrm{c}_\mathrm{W}^2}\,\mathrm{Z}_\mu\,\mathrm{Z}_\mu\right)\Phi_\mathrm{h}^2\,\mathrm{h}\\
&-\frac{1}{1024}\,\frac{g^5\,M}{\pi^2}\,t_m^2\,t_1\Big[6\,x_\mathrm{h}+4\,(1-t_m)\,\left(5+4\,t_m-4\,t_m^2\right)\,t_m\,t_1\\
&+\left(9+2\,t_m^2\right)\,x_\mathrm{h}\,\overline{\mathrm{A}}_0+2\,\left(15-t_m-26\,t_m^2+14\,t_m^3\right)\,t_m\,t_1\,\overline{\mathrm{A}}_0\Big]\,\mathrm{h}\,\Phi_\mathrm{h}^4\\
&+\frac{1}{1024}\,\frac{g^5\,M}{\pi^2}\,t_m^3\,\left(\beta_2^{(0)}-3\,\beta_1^{(0)}\right)\,\mathrm{h}\,\Phi_\mathrm{h}^4\\
&+\frac{3}{128}\,\frac{g^5\,M}{\pi^2}\,t_m^3\,t_1^2\,\left(1+\overline{\mathrm{A}}_0\right)\,(1-t_m)\,\mathrm{h}\,\Phi_\mathrm{h}^4\\
&-\frac{1}{8}\,\frac{i\,g^3\,\mathrm{s}_\mathrm{W}^2}{\mathrm{c}_\mathrm{W}}\,M\,t_m^2\,\left(\phi^+\,\mathrm{W}_\mu^- - \phi^-\,\mathrm{W}_\mu^+\right)\,\Phi_\mathrm{h}^2\,\mathrm{Z}_\mu\\
&+\frac{1}{512}\,\frac{i\,g^5\,M}{\pi^2}\,\frac{t_m^2}{t_1}\,\frac{\mathrm{s}_\mathrm{W}^2}{\mathrm{c}_\mathrm{W}}\,\left(\phi^+\,\mathrm{W}_\mu^- - \phi^-\,\mathrm{W}_\mu^+\right)\,\Phi_\mathrm{h}^2\,\mathrm{Z}_\mu\,\beta_1^{(0)}\\
&+\frac{1}{256}\,\frac{i\,g^5\,M}{\pi^2}\,\frac{\mathrm{s}_\mathrm{W}^2}{\mathrm{c}_\mathrm{W}}\,t_m^2\,t_1\left[6+\left(9+t_m^2\right)\,\overline{\mathrm{A}}_0\right]\,\left(\phi^+\,\mathrm{W}_\mu^- - \phi^-\,\mathrm{W}_\mu^+\right)\,\Phi_\mathrm{h}^2\,\mathrm{Z}_\mu\\
&-\frac{1}{8}\,g^2\,M\,t_m^2\left[\left(\mathrm{W}_\mu^+\,\mathrm{D}_\mu\,\phi^- + \mathrm{W}_\mu^-\,\mathrm{D}_\mu\,\phi^+\right)\,\Phi_\mathrm{h}^2-2\,\left(\phi^+\,\mathrm{W}_\mu^- + \phi^-\,\mathrm{W}_\mu^+\right)\,\partial_\mu\,\Phi_\mathrm{h}^2\right]\\
&+\frac{1}{8}\,g^2\,M\,t_m^2\,\left(x_\mathrm{d}\,\overline{\mathrm{d}}\,\mathrm{d}+x_\mathrm{u}\,\overline{\mathrm{u}}\,\mathrm{u}\right)\,\Phi_\mathrm{h}^2\\
&+\frac{1}{16}\,g^3\,M\,t_m^2\left(2\,\mathrm{W}_\mu^+\,\mathrm{W}_\mu^- + \frac{1}{\mathrm{c}_\mathrm{W}^2}\,\mathrm{Z}_\mu\,\mathrm{Z}_\mu\right)\Phi_\mathrm{h}^2\,\mathrm{h}+\frac{1}{32}\,g^3\,M\,t_m^2\,x_\mathrm{h}\,\mathrm{h}\,\Phi_\mathrm{h}^4\,,\\
&+\frac{3}{256}\,\frac{g^5\,M}{\pi^2}\,\left(t_m^3\,t_1^2\right)\,(1-t_m)\,\mathrm{B}_{00}\,\mathrm{h}\,\Phi_\mathrm{h}^4
\end{aligned}
\tag{B.14}
$$

$$
\begin{aligned}
\blacktriangleright\ \mathcal{L}_{66} = &-\frac{1}{384}\,\frac{g^4}{\pi^2}\,t_m^2\,t_1\,\partial_\mu\,\Phi_\mathrm{h}^2\,\partial_\mu\,\Phi_\mathrm{h}^2-\frac{1}{4096}\,\frac{g^6}{\pi^2}\,t_m^3\,\Phi_\mathrm{h}^6\,\beta_1^{(0)}\\
&-\frac{1}{3072}\,\frac{g^6}{\pi^2}\,t_m^3\,t_1^2\,\left(5+9\,\overline{\mathrm{A}}_0\right)\,\Phi_\mathrm{h}^6-\frac{1}{8}\,g^2\,t_m^2\,\partial_\mu\,\Phi_\mathrm{h}^2\,\partial_\mu\,\Phi_\mathrm{h}^2\,.\\
&+\frac{3}{1024}\,\frac{g^6}{\pi^2}\,t_m^3\,t_1^2\,\mathrm{B}_{00}\,\Phi_\mathrm{h}^6
\end{aligned}
\tag{B.15}
$$

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