Search for the isospin violating decay $Y(4260) \rightarrow J/\psi \eta \pi^0$
Institute of High Energy Physics, Beijing 100049, People’s Republic of China
Beihang University, Beijing 100191, People’s Republic of China
Beijing Institute of Petrochemical Technology, Beijing 102617, People’s Republic of China
Bochum Ruhr-University, D-44780 Bochum, Germany
Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA
Central China Normal University, Wuhan 430079, People’s Republic of China
COMSATS Institute of Information Technology, Lahore, Defence Road, Off Rawal Road, 54000 Lahore, Pakistan
G.I. Budker Institute of Nuclear Physics SB RAS (BINP), Novosibirsk 630090, Russia
GSI Helmholtzcentre for Heavy Ion Research GmbH, D-64291 Darmstadt, Germany
Guangxi Normal University, Guilin 541004, People’s Republic of China
GuangXi University, Nanning 530004, People’s Republic of China
Hangzhou Normal University, Hangzhou 310036, People’s Republic of China
Helmholtz Institute Mainz, Johann-Joachim-Becher-Weg 45, 55099 Mainz, Germany
Henan Normal University, Xinxiang 453007, People’s Republic of China
Henan University of Science and Technology, Luoyang 471003, People’s Republic of China
Huangshan College, Huangshan 245000, People’s Republic of China
Hunan University, Changsha 410082, People’s Republic of China
Indiana University, Bloomington, Indiana 47405, USA
INFN Laboratori Nazionali di Frascati, I-00044, Frascati, Italy
INFN and University of Perugia, I-06100, Perugia, Italy
INFN Sezione di Ferrara, I-44122, Ferrara, Italy
University of Ferrara, I-44122, Ferrara, Italy
Johannes Gutenberg University of Mainz, Johann-Joachim-Becher-Weg 45, 55099 Mainz, Germany
Joint Institute for Nuclear Research, 141980 Dubna, Moscow region, Russia
Justus Liebig University Giessen, II. Physikalisches Institut, Heinrich-Buff-Ring 16, D-35392 Giessen, Germany
KVI-CART, University of Groningen, NL-9747 AA Groningen, Netherlands
Lanzhou University, Lanzhou 730000, People’s Republic of China
Liaoning University, Shenyang 110036, People’s Republic of China
Nanjing Normal University, Nanjing 210023, People’s Republic of China
Nankai University, Tianjin 300071, People’s Republic of China
Peking University, Beijing 100871, People’s Republic of China
Seoul National University, Seoul, 151-747 Korea
Shandong University, Jinan 250100, People’s Republic of China
Shanghai Jiao Tong University, Shanghai 200240, People’s Republic of China
Shanxi University, Taiyuan 030006, People’s Republic of China
Sichuan University, Chengdu 610064, People’s Republic of China
Soochow University, Suzhou 215006, People’s Republic of China
Sun Yat-Sen University, Guangzhou 510275, People’s Republic of China
Tsinghua University, Beijing 100084, People’s Republic of China
Istanbul Aydin University, 34295 Sefakoy, Istanbul, Turkey
Dogus University, 34722 Istanbul, Turkey
Uludag University, 16059 Bursa, Turkey
University of Chinese Academy of Sciences, Beijing 100049, People’s Republic of China
University of Hawaii, Honolulu, Hawaii 96822, USA
University of Minnesota, Minneapolis, Minnesota 55455, USA
University of Rochester, Rochester, New York 14627, USA
University of Science and Technology of China, Hefei 230026, People’s Republic of China
University of South China, Hengyang 421001, People’s Republic of China
University of the Punjab, Lahore-54590, Pakistan
University of Turin, I-10125, Turin, Italy
University of Eastern Piedmont, I-15121, Alessandria, Italy
INFN, I-10125 Turin, Italy
Uppsala University, Box 516, SE-75120 Uppsala, Sweden
Using data samples collected at center-of-mass energies of $\sqrt{s} = 4.009, 4.226, 4.257, 4.358, 4.416, and 4.599$ GeV with the BESIII detector operating at the BEPCII storage ring, we search for the isospin violating decay $Y(4260) \rightarrow J/\psi\pi^0\eta_0$. No signal is observed, and upper limits on the cross section $\sigma(e^+e^- \rightarrow J/\psi\eta\pi^0)$ at the 90% confidence level are determined to be 3.6, 1.7, 2.4, 1.4, 0.9, and 1.9 pb, respectively.

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I. INTRODUCTION

The $Y(4260)$ charmoniumlike state was first observed in its decay to $\pi^+\pi^- J/\psi$ [1] and has a small coupling to open charm decay modes [2]. $Y(4260)$ is a vector ($J^{PC} = 1^{--}$) state that is only barely observable as an s-channel resonance in $e^+e^-$ collisions and that appears at an energy where no conventional charmonium state is expected. Since its discovery, many theoretical studies have been carried out considering the $Y(4260)$ as a tetraquark state [3], $D_s D$ or $D_s D^*$ hadronic molecule [4], hybrid charmonium [5], baryonium state [6], etc.

Recently, in the study of $Y(4260) \rightarrow \pi^+\pi^- J/\psi$, a charmoniumlike structure, the $Z_c(3900)^\pm$, was observed in the $\pi^\pm J/\psi$ invariant mass spectrum by the BESIII [7] and Belle experiments [8] and confirmed shortly thereafter with CLEO-c data [9]. In the molecule model [10], the $Y(4260)$ is proposed to have a large $D_s \bar{D}$ component, while $Z_c(3900)^\pm$ has a $D \bar{D}^*$ component.

BESIII recently reported the observation of $e^+e^- \rightarrow \gamma X(3872) \rightarrow \gamma \pi^+\pi^- J/\psi$ [11]. The cross section measurements strongly support the existence of the radiative transition $Y(4260) \rightarrow \gamma X(3872)$. One significant feature of the $X(3872)$ that differs from conventional charmonium is that the decay branching fraction of $X(3872)$ to $\pi^+\pi^- J/\psi$ is comparable to $\pi^+\pi^- J/\psi$ [12,13], so the isospin violating process occurs on a large scale.

Isospin violating decays can be used to probe the nature of heavy quarkonium. The hadro-charmonium model [14] and tetraquark models [15,16] predict that the reaction $T(5S) \rightarrow \eta\pi^0$+bottomonium should be observable. The tetraquark model [17] also predicts that $Z^0_c$ can be produced in $Y(4260) \rightarrow J/\psi\eta\pi^0$ with $Z^0_c$ decaying into $J/\psi\pi^0$ and possibly $J/\psi\eta$ in the presence of sizable isospin violation. The molecular model [18] predicts a peak in the cross section of $Y(4260) \rightarrow J/\psi\eta\pi^0$ at the $D_s D^*$ threshold and a narrow peak in the $J/\psi\eta$ invariant mass spectrum at the $DD^*$ threshold.

In this paper, we present results on a search for the isospin violating decay $Y(4260) \rightarrow J/\psi\eta\pi^0$, with $J/\psi \rightarrow e^+e^-/\mu^+\mu^-$, $\pi^0 \rightarrow \gamma\gamma$, and $\eta \rightarrow \gamma\gamma$ (the other decay modes of $\eta$ are not used due to much lower detection efficiency and branching fraction), based on $e^+e^-$ annihilation data collected with the BESIII detector operating at the BEPCII storage ring [19] at center-of-mass energies of $\sqrt{s} = 4.009, 4.226, 4.257, 4.358, 4.416,$ and 4.599 GeV.

II. BESIII DETECTOR AND MONTE CARLO SIMULATION

The BESIII detector, described in detail in Ref. [19], has a geometrical acceptance of 93% of $4\pi$. A small-cell helium-based main drift chamber (MDC) provides a charged particle momentum resolution of 0.5% at 1 GeV/$c$ in a 1 T magnetic field and supplies energy-loss ($dE/dx$) measurements with a resolution of 6% for minimum-ionizing pions. The electromagnetic calorimeter (EMC) measures photon energies with a resolution of 2.5% (5%) at 1.0 GeV in the barrel (end caps). Particle identification is provided by a time-of-flight system with a time resolution of 80 ps (110 ps) for the barrel (end caps). The muon system (MUC), located in the iron flux return yoke of the magnet, provides 2 cm position resolution and detects muon tracks with momentum greater than 0.5 GeV/$c$.

The GEANT4-based [20] Monte Carlo (MC) simulation software BOOST [21] includes the geometric description of the BESIII detector and a simulation of the detector response. It is used to optimize event selection criteria, estimate backgrounds, and evaluate the detection efficiency. For each energy point, we generate large signal MC samples of $e^+e^- \rightarrow J/\psi\pi^0$, $J/\psi \rightarrow e^+e^-/\mu^+\mu^-$,
III. EVENT SELECTION

Events with two charged tracks with a net charge of zero are selected. For each good charged track, the polar angle in the MDC must satisfy $| \cos \theta | < 0.93$, and the point of closest approach to the $e^+e^-$ interaction point must be within $\pm 10$ cm in the beam direction and within $\pm 1$ cm in the plane perpendicular to the beam direction. The momenta of leptons from the $J/\psi$ decays in the laboratory frame are required to be larger than 1.0 GeV/c. $E/p$ is used to separate electrons from muons, where $E$ is the energy deposited in the EMC and $p$ is the momentum measured by the MDC. For electron candidates, $E/p$ should be larger than 0.7, while for muons, it should be less than 0.3. To suppress background from events with pion tracks in the final state, at least one of the two muons is required to have at least five layers with valid hits in the MUC.

Shower candidates identified as photon candidates must satisfy fiducial and shower quality as well as timing requirements. The minimum EMC energy is 25 MeV for barrel showers ($| \cos \theta | < 0.80$) and 50 MeV for end cap showers ($0.86 < | \cos \theta | < 0.92$). To eliminate showers produced by charged particles, a photon must be separated by at least 5 deg from any charged track. The time information from the EMC is also used to suppress electronic noise and energy deposits unrelated to the event. At least four good photon candidates in each event are required.

To improve the momentum resolution and reduce the background, the event is subjected to a four-constraint (4C) kinematic fit under the hypothesis $e^+e^- \rightarrow \gamma \gamma \gamma \gamma l^+l^-$, and the $\chi^2$ is required to be less than 40. For events with more than four photons, the four photons with the smallest $\chi^2$ from the 4C fit are assigned as the photons from $\eta$ and $\pi^0$.

After selecting the $\gamma \gamma \gamma l^+l^-$ candidate, scatter plots $M(\gamma \gamma)$ with all six combinations of photon pairs for events in the $J/\psi$ signal region (both $J/\psi$ and $\pi^0\pi^0$ signal regions for both photon pairs satisfy $| M(\gamma \gamma) - m_{\psi \phi} | < 10$ MeV/$c^2$) for data at $\sqrt{s} = 4.226$ and 4.257 GeV are shown in the left two panels of Fig. 1. Distributions of $M(l^+l^-)$ for events in the $\pi^0\pi^0$ signal region (both photon pairs satisfy $| M(\gamma \gamma) - m_{\psi \phi} | < 10$ MeV/$c^2$) for data at $\sqrt{s} = 4.226$ and 4.257 GeV are shown in the right two panels of Fig. 1. Clear $J/\psi$ peaks are observed, corresponding to $\pi^0\pi^0 J/\psi$ events. To remove this $\pi^0\pi^0$ background, events with any combination of photon pairs in the $\pi^0\pi^0$ region of the scatter plot are rejected.

After rejecting the $\pi^0\pi^0 J/\psi$ background, we choose the combination of photon pairs closest to the $\eta\pi^0$ signal region by minimizing $\sqrt{\left( \frac{M(\gamma \gamma l^+) - m_\eta}{\sigma_\eta} \right)^2 + \left( \frac{M(\gamma \gamma l^-) - m_\pi^0}{\sigma_\pi^0} \right)^2}$, where $\sigma_\eta$ and $\sigma_\pi^0$ are the $\eta$ and $\pi^0$ resolutions obtained from the signal MC, respectively. The scatter plots of $M(\gamma \gamma)$ with the combination closest to the $\eta\pi^0$ signal region for events in the $J/\psi$ signal region for data at $\sqrt{s} = 4.226$ and 4.257 GeV are shown in the top two panels of Fig. 2. No cluster of $\eta\pi^0$ events is observed in the $J/\psi$ signal region, with a vertical band for $\pi^0 \rightarrow \gamma \gamma$ clearly visible, but no prominent band for $\eta \rightarrow \gamma \gamma$ is observed. The projections of the scatter plots on $M(\gamma \gamma l^+)$ with $M(\gamma \gamma l^-)$ in the $\pi^0$ signal region ($| M(\gamma \gamma l^+) - m_\pi^0 | < 10$ MeV/$c^2$) and projections on $M(\gamma \gamma l^-)$ with $M(\gamma \gamma l^+)$ in the $\eta$ signal region ($| M(\gamma \gamma l^-) - m_\eta | < 30$ MeV/$c^2$) are shown for data at $\sqrt{s} = 4.226$ GeV. After imposing all event selection requirements, there are two background

[FIG. 1 (color online). Scatter plot of $M(\gamma \gamma)$ with all six combinations for events in the $J/\psi$ signal region (left) and distribution of $M(l^+l^-)$ for events in the $\pi^0\pi^0$ signal region (right) for data at $\sqrt{s} = 4.226$ GeV (top) and 4.257 GeV (bottom).]
FIG. 2. Scatter plot of $M(\gamma\gamma)$ for the combination closest to the $\eta\pi^0$ signal region for events in the $J/\psi$ signal region (top), projection of the scatter plot on $M(\gamma\gamma)$ with $M(\gamma\gamma)$ in $\pi^0$ signal region (middle), and projection of the scatter plot on $M(\gamma\gamma)$ with $M(\gamma\gamma)$ in $\eta$ signal region (bottom) for data at $\sqrt{s} = 4.226$ GeV (left) and 4.257 GeV (right).

FIG. 3 (color online). Distributions of $M(TT)$ for events in $\eta\pi^0$ signal region and sideband regions for data at $\sqrt{s} = 4.226$ GeV (left) and 4.257 GeV (right). The error bars are the $M(TT)$ distributions for events in the $\eta\pi^0$ signal region, and the shaded histograms are those in the $\eta\pi^0$ sideband regions.

The background can be evaluated with $\eta\pi^0$ sideband events. Distributions of $M(TT)$ for events in the $\eta\pi^0$ signal region for data at $\sqrt{s} = 4.226$ and 4.257 GeV are shown in Fig. 3. Distributions of $M(TT)$ for events corresponding to the normalized two-dimensional $\eta\pi^0$ sidebands are shown as shaded events from $e^+e^- \rightarrow \pi^0\eta^0 J/\psi$ and nine background events arising from $e^+e^- \rightarrow \gamma_{\text{ISR}}\gamma', \gamma_{\text{ISR}}\gamma''$, and $\gamma_{\text{ISR}}\psi(4040)$. No other background survives. The background can be evaluated with $\eta\pi^0$ sideband events. Distributions of $M(TT)$ for events in the $\eta\pi^0$ signal region for data at $\sqrt{s} = 4.226$ and 4.257 GeV are shown in Fig. 3. Distributions of $M(TT)$ for events corresponding to the normalized two-dimensional $\eta\pi^0$ sidebands are shown as shaded histograms. The $\eta$ sideband regions are defined as $0.3978 < M(\gamma\gamma) < 0.4578$ GeV$^2$ and $0.6378 < M(\gamma\gamma) < 0.6978$ GeV$^2$. The $\pi^0$ sideband regions are defined as $0.0849 < M(\gamma\gamma) < 0.1049$ GeV$^2$ and $0.1649 < M(\gamma\gamma) < 0.1849$ GeV$^2$. The counted number of observed events in the $J/\psi\eta\pi^0$ signal region $N_{\text{obs}}$ and number of background events estimated from $\eta\pi^0$ sidebands $N_{\text{bkg}}$ are listed in Table I.

IV. CROSS SECTION UPPER LIMITS

Since no $J/\psi\eta\pi^0$ signal above the background is observed, upper limits on the Born cross section of $e^+e^- \rightarrow J/\psi\eta\pi^0$ at the 90% C.L. are determined using the formula

$$\sigma^{\text{Born}} < \frac{N_{\text{up}}^{\text{obs}}}{\mathcal{L}(1 + \delta')(1 + \delta)^{\delta^{\text{ee}}}(e^{ee} \mathcal{B}^e + e^{\mu\mu} \mathcal{B}^{\mu\mu}) \mathcal{B}^{0\text{ee}} \mathcal{B}^{0\text{ee}}},$$

where $N_{\text{up}}^{\text{obs}}$ is the upper limit on the number of signal events; $\mathcal{L}$ is the integrated luminosity; $(1 + \delta')$ is the radiative correction factor, which is taken from a QED calculation assuming the $e^+e^- \rightarrow J/\psi\eta\pi^0$ cross section is described by a $Y(4260)$ Breit–Wigner line shape with parameters taken from the PDG [23]; $(1 + \delta)$ is the vacuum polarization factor including leptonic and hadronic parts and taken from a QED calculation with an accuracy of 0.5% [28]; $e^{ee}$ and $e^{\mu\mu}$ are the efficiencies for $e^+e^-$ and $\mu^+\mu^-$ modes, respectively; $\mathcal{B}^{e}$ and $\mathcal{B}^{\mu\mu}$ are the branching fractions of $J/\psi \rightarrow e^+e^-$ and $J/\psi \rightarrow \mu^+\mu^-$ [23], respectively; and $\mathcal{B}^{0\text{ee}}$ is the branching fraction of $\eta \rightarrow \gamma\gamma$ and $\pi^0 \rightarrow \gamma\gamma$ [23], respectively.

The efficiency corrected upper limit on the number of signal events $N^{\text{up}} = N_{\text{up}}^{\text{obs}}/\epsilon$ is estimated with $N_{\text{obs}}$ and $N_{\text{bkg}}$ using the profile likelihood method, which is implemented by TROLE in the ROOT framework [29]. The calculation for obtaining $N^{\text{up}}$ includes the background fluctuation and the systematic uncertainty of the cross section measurement. The background fluctuation is assumed to follow a Poisson distribution. The systematic uncertainty of the cross section is taken as a Gaussian uncertainty.

The systematic uncertainty of the cross section measurement in Eq. (1) includes the luminosity measurement, detection efficiency, and intermediate decay branching fractions. The systematic uncertainties of the luminosity, track reconstruction, and photon detection are 1.0% [11], 1.0% per track [30], and 1.0% per photon [31], respectively. The systematic uncertainties from the branching fraction of $\pi^0$ and $\eta$ decays are taken from the PDG [23]. These sources of systematic uncertainty, which are summarized in the top part of Table II, are common for $e^+e^-$ and $\mu^+\mu^-$ modes. The following sources of systematic uncertainty, which are uncorrelated for the $e^+e^-$ and $\mu^+\mu^-$ modes, are summarized in the bottom part of Table II. The systematic
TABLE I. Results on $e^+e^- \rightarrow J/\psi \pi \pi^0$. Listed in the table are the integrated luminosity $L$, radiative correction factor $(1 + \delta')$ taken from QED calculation assuming the $Y(4260)$ cross section follows a Breit–Wigner line shape, vacuum polarization factor $(1 + \delta'')$, average efficiency $(e^+e^- B_{e\gamma} + e^+e^- B_{\mu\mu})$, number of observed events $N_{obs}$, number of estimated background events $N_{Bkg}$, the efficiency corrected upper limits on the number of signal events $N_{up}$, and upper limits on the Born cross section $\sigma_{UL}^{Born}$ (at the 90% C.L.) at each energy point.

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (GeV)</th>
<th>$L$ (pb$^{-1}$)</th>
<th>$(1 + \delta')$</th>
<th>$(1 + \delta'')$</th>
<th>$(e^+e^- B_{e\gamma} + e^+e^- B_{\mu\mu})$ (%)</th>
<th>$N_{obs}$</th>
<th>$N_{Bkg}$</th>
<th>$N_{up}$</th>
<th>$\sigma_{UL}^{Born}$ (pb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.009</td>
<td>482.0</td>
<td>0.838</td>
<td>1.044</td>
<td>$2.1 \pm 0.1$ (sys)</td>
<td>5</td>
<td>1</td>
<td>598.1</td>
<td>3.6</td>
</tr>
<tr>
<td>4.226</td>
<td>1047.3</td>
<td>0.844</td>
<td>1.056</td>
<td>$2.2 \pm 0.1$ (sys)</td>
<td>12</td>
<td>11</td>
<td>592.9</td>
<td>1.7</td>
</tr>
<tr>
<td>4.257</td>
<td>825.6</td>
<td>0.847</td>
<td>1.054</td>
<td>$2.2 \pm 0.1$ (sys)</td>
<td>12</td>
<td>8</td>
<td>654.1</td>
<td>2.4</td>
</tr>
<tr>
<td>4.358</td>
<td>539.8</td>
<td>0.942</td>
<td>1.051</td>
<td>$2.2 \pm 0.1$ (sys)</td>
<td>5</td>
<td>4</td>
<td>283.2</td>
<td>1.4</td>
</tr>
<tr>
<td>4.416</td>
<td>1028.9</td>
<td>0.951</td>
<td>1.053</td>
<td>$2.3 \pm 0.1$ (sys)</td>
<td>5</td>
<td>6</td>
<td>342.7</td>
<td>0.9</td>
</tr>
<tr>
<td>4.599</td>
<td>566.9</td>
<td>0.965</td>
<td>1.055</td>
<td>$2.4 \pm 0.1$ (sys)</td>
<td>6</td>
<td>3</td>
<td>418.4</td>
<td>1.9</td>
</tr>
</tbody>
</table>

TABLE II. Systematic uncertainties in the $J/\psi \eta \pi^0$ cross section measurement at each energy point (in %). The items in parentheses in the bottom part of the table are the uncorrelated systematic uncertainties for the $e^+e^-$ (first) and $\mu^+\mu^-$ (second) modes.

<table>
<thead>
<tr>
<th>Sources/$\sqrt{s}$ (GeV)</th>
<th>4.009</th>
<th>4.226</th>
<th>4.257</th>
<th>4.358</th>
<th>4.416</th>
<th>4.599</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luminosity</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>MDC tracking</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
<tr>
<td>Photon reconstruction</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
<td>4.0</td>
</tr>
<tr>
<td>$B(\pi^0 \rightarrow \gamma\gamma)$, $B(\eta \rightarrow \gamma\gamma)$</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$B(J/\psi \rightarrow l^+l^-)$</td>
<td>(0.5, 0.5)</td>
<td>(0.5, 0.5)</td>
<td>(0.5, 0.5)</td>
<td>(0.5, 0.5)</td>
<td>(0.5, 0.5)</td>
<td>(0.5, 0.5)</td>
</tr>
<tr>
<td>MUC hits</td>
<td>(0, 3.6)</td>
<td>(0, 3.6)</td>
<td>(0, 3.6)</td>
<td>(0, 3.6)</td>
<td>(0, 3.6)</td>
<td>(0, 3.6)</td>
</tr>
<tr>
<td>$J/\psi$ mass resolution</td>
<td>(0.2, 1.3)</td>
<td>(0.8, 1.2)</td>
<td>(0.5, 1.3)</td>
<td>(0.2, 0.7)</td>
<td>(0.7, 1.6)</td>
<td>(0.1, 0.6)</td>
</tr>
<tr>
<td>Decay model</td>
<td>(1.5, 1.9)</td>
<td>(0.9, 1.1)</td>
<td>(0.4, 0.6)</td>
<td>(0.2, 0.7)</td>
<td>(0.7, 0.2)</td>
<td>(0.2, 0.2)</td>
</tr>
<tr>
<td>Kinematic fitting</td>
<td>(1.2, 0.9)</td>
<td>(1.1, 1.2)</td>
<td>(0.9, 0.9)</td>
<td>(0.7, 1.2)</td>
<td>(1.1, 1.0)</td>
<td>(1.0, 1.4)</td>
</tr>
<tr>
<td>Total</td>
<td>5.3</td>
<td>5.3</td>
<td>5.2</td>
<td>5.2</td>
<td>5.3</td>
<td>5.2</td>
</tr>
</tbody>
</table>

The uncertainty from the branching fraction of $J/\psi$ decay is taken from the PDG [23]. The systematic uncertainty from the requirement on the number of MUC hits is 3.6% and estimated by comparing the efficiency of the MUC requirement between data and MC in the control sample $e^+e^- \rightarrow \pi^0\pi^0J/\psi$ at $\sqrt{s} = 4.257$ GeV. The systematic uncertainty from the requirement of the $J/\psi$ signal region is estimated by smearing the invariant mass of $l^+l^-$ of the signal MC with a Gaussian function to compensate for the resolution difference between the data and MC when calculating the efficiency. The parameters for smearing are determined by fitting the $J/\psi$ distribution of data with the MC shape convoluted with a Gaussian function for the control sample $e^+e^- \rightarrow \pi^0\pi^0J/\psi$. The difference in the detection efficiency between signal MC samples with and without the smearing is taken as the systematic uncertainty. The systematic uncertainty from the MC model is estimated by generating a MC sample with the angular distribution of leptons determined from the $\pi^+\pi^-J/\psi$ data. The systematic uncertainty due to kinematic fitting is estimated by correcting the helix parameters of charged tracks according the method described in Ref. [32], where the correction factors are obtained from the control sample $\psi' \rightarrow Y\chi\phi$ and the difference in the detection efficiency between with and without making the correction to the MC is taken as the systematic uncertainty. The uncorrelated systematic uncertainties for the electron and muon channels are combined by taking the weighted average with weights $e^{ee} B_{e\gamma}$ and $e^{ee} B_{\mu\mu}$, respectively. The total systematic uncertainty is obtained by summing all the sources of the systematic uncertainty in quadrature.

The systematic uncertainty on the size of the background is estimated by evaluating $N_{up}$ with different signal and sideband regions for $\eta$ and $\pi^0$. The most conservative $N_{up}$ is taken as the final result, as listed in Table I. The upper limits on the Born cross section of $e^+e^- \rightarrow J/\psi \eta \pi^0(\sigma_{UL}^{Born})$ assuming it follows a $Y(4260)$ Breit–Wigner line shape are listed in Table I.

For comparison, the radiative correction factor and detection efficiency have been recalculated assuming the $e^+e^- \rightarrow J/\psi \eta \pi^0$ cross section follows alternative line shapes. If the cross section follows the line shape of the $Y(4040)$, the upper limit on the Born cross section is 4.1 pb at $\sqrt{s} = 4.009$ GeV. For a $Y(4360)$ line shape, it is 1.6 pb at $\sqrt{s} = 4.358$ GeV. For a $Y(4415)$ line shape, it is
1.5 pb at $\sqrt{s} = 4.358$ GeV and 1.0 pb at $\sqrt{s} = 4.416$ GeV. For a $Y(4660)$ line shape, it is 2.0 pb at $\sqrt{s} = 4.599$ GeV.

It is also possible to set upper limits on $e^+e^- \rightarrow Z_0^0\pi^0 \rightarrow J/\psi\eta\pi^0$. The number of observed events and number of estimated background events in the $Z_0^0$ signal region ($3.850 < M(J/\psi\eta) < 3.940$ GeV/$c^2$) are 7 and 4 $\pm$ 2, respectively, at $\sqrt{s} = 4.226$ GeV, and 8 and 3 $\pm$ 2, respectively, at $\sqrt{s} = 4.257$ GeV. The upper limit on $\sigma(e^+e^- \rightarrow Z_0^0\pi^0 \rightarrow J/\psi\eta\pi^0)$ is determined to be 1.3 pb at $\sqrt{s} = 4.226$ GeV and 2.0 pb at $\sqrt{s} = 4.257$ GeV, where only the statistical uncertainty is given. Compared to the measured cross section of $e^+e^- \rightarrow Z_0^0\pi^0 \rightarrow J/\psi\eta\pi^0$ [33], the upper limit on the ratio of the branching fraction $B(Z_0^0 \rightarrow J/\psi\pi^0)$ at the 90% confidence level is 0.15 at $\sqrt{s} = 4.226$ GeV and 0.65 at $\sqrt{s} = 4.257$ GeV.

V. SUMMARY

In summary, using data collected with the BESIII detector, a search for the isospin violating decay $Y(4260) \rightarrow J/\psi\eta\pi^0$ is performed. No statistically significant signal is observed. The Born cross sections of $e^+e^- \rightarrow J/\psi\eta\pi^0$ at the 90% confidence level limits at $\sqrt{s} = 4.009, 4.226, 4.257, 4.358, 4.416,$ and 4.599 GeV are determined to be 3.6, 1.7, 2.4, 1.4, 0.9, and 1.9 pb, respectively. The upper limits are well above the prediction for the molecule model [18].