Extracting the kaon Collins function from e^+e^- hadron pair production data

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(Received 10 December 2015; published 16 February 2016)

The latest data released by the *BABAR* Collaboration on azimuthal correlations measured for pion-kaon and kaon-kaon pairs produced in e^+e^- annihilations allow, for the first time, a direct extraction of the kaon Collins functions. These functions are then used to compute the kaon Collins asymmetries in semi-inclusive deep inelastic scattering processes, which result in good agreement with the measurements performed by the HERMES and COMPASS collaborations.

DOI: 10.1103/PhysRevD.93.034025

I. INTRODUCTION

In the quest for the understanding of the inner 3D structure of nucleons, the transverse momentum dependent partonic distribution and fragmentation functions (respectively, TMD-PDFs and TMD-FFs) play a fundamental role. In particular, it is inside the TMD-FFs that we encode the nonperturbative, soft part of the hadronization process.

Over the years, combined analyses of semi-inclusive deep inelastic scattering (SIDIS) and $e^+e^- \to \pi^+\pi^- X$ experimental data allowed the extraction of the transversity distribution and the $q^\uparrow \to \pi X$ (pion) Collins functions [1–4]. However, until very recently, no direct experimental information was available on the *kaon* Collins functions, although their effects were clearly evident in SIDIS processes [5–8], both in the $\cos 2\phi_h$ azimuthal modulation of the unpolarized cross section and in the $\sin(\phi_h + \phi_S)$ azimuthal asymmetry, the so-called Collins asymmetry.

The Collins function, in fact, contributes to the $\cos 2\phi_h$ asymmetries in convolution with a Boer-Mulders function, while in the $\sin(\phi_h + \phi_S)$ single spin asymmetries it appears convoluted with the transversity distribution. The kaon $\cos 2\phi_h$ azimuthal asymmetries present some peculiar features: at HERMES [6] K^+ and K^- asymmetries are both sizeable and negative, while the analogous π^+ asymmetries are compatible with zero or slightly negative, and the π^- ones are positive. Looking at the $\sin(\phi_h + \phi_S)$ dependence, instead, we observe that K^+ asymmetries look slightly positive, while K^- data are compatible with zero (within large errors) [5,8].

Clearly, to understand these data better, we have to study the kaon Collins functions. Recent *BABAR* data on pion-pion, pion-kaon and kaon-kaon production from e^+e^- annihilation processes [9] give the opportunity to extract the kaon Collins function, for the first time; moreover, all these results have been presented in the same bins of z_1 and z_2, so that they can be analyzed simultaneously in a consistent way.

In this paper we perform an analysis of the e^+e^- *BABAR* measurements involving kaons, with the aim of extracting the kaon Collins functions. This paper extends a recent study of the Collins functions in e^+e^- and SIDIS data [4] limited to pion production. Our strategy is the following:

(1) When necessary for our analysis (for instance for the description of $e^+e^- \to \pi K X$ data) we employ the favored and disfavored pion Collins functions obtained in Ref. [4]; no free parameters are introduced in this analysis concerning pions.

(2) We parametrize the kaon favored and disfavored Collins functions using a factorized form, similar to that used for pions [4], with an even simpler structure: due to the limitation of the kaon data presently available, we have found out, after several tests, that it suffices for their analysis to consider a model which implies only two free parameters, instead of four. We also do not introduce different parameters between heavy and light flavors in the kaon Collins functions (this point will be further discussed at the end of Sec. III). The free parameters will be determined by best fitting the new $e^+e^- \to \pi K X$ and $e^+e^- \to K^+K^- X$ *BABAR* data sets [9].

(3) The kaon favored and disfavored Collins functions extracted from e^+e^- annihilation data will be used to compute the values of the Collins single spin asymmetries observed in SIDIS processes. As we will discuss in Sec. III, the comparison of our predictions with the measurements performed by the HERMES and COMPASS collaborations confirms, within the precision limits of experimental data, the total consistency of the Collins functions extracted from e^+e^- data with those obtained from SIDIS processes, corroborating their universality [10].

In Sec. II we briefly recall the formalism used in our analysis, while in Sec. III we present the results of our best fits of *BABAR* kaon data and compare them with SIDIS measurements of the kaon Collins asymmetry. Some short final comments and conclusions will be given in Sec. IV.

II. FORMALISM

In this section we briefly summarize the formalism relevant to perform the extraction of the kaon Collins functions using the new data from the *BABAR* Collaboration, which now contain also asymmetries for e^+e^- annihilations into pion-kaon and kaon-kaon pairs. Two methods have been adopted in the experimental analysis, the so-called "thrust-axis method" and the "hadronic-plane method." Here, we concentrate on the latter and refer the reader to our previous simultaneous analyses of SIDIS and $e^+e^- \to \pi\pi X$ data [4] for further details.

A. Parametrization of the kaon Collins function

For the unpolarized parton distribution and fragmentation functions we adopt a simple factorized form, in which longitudinal and transverse degrees of freedom are separated. The dependence on the intrinsic transverse momentum is assumed to have a Gaussian shape,

$$f_{q/p}(x, k_\perp) = f_{q/p}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2\rangle}}{\pi\langle k_\perp^2\rangle} \qquad (1)$$

$$D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}, \qquad (2)$$

with $\langle k_\perp^2\rangle = 0.57\ \mathrm{GeV}^2$ and $\langle p_\perp^2\rangle = 0.12\ \mathrm{GeV}^2$ as found in Ref. [11] by analyzing the HERMES unpolarized SIDIS multiplicities. For the collinear parton distribution and fragmentation functions, $f_{q/p}(x)$ and $D_{h/q}(z)$, we use the GRV98LO PDF set [12] and the de Florian-Sassot-Stratmann (DSS) fragmentation function set from Ref. [13].

For the Collins FF, $\Delta^N D_{h/q^\uparrow}(z, p_\perp)$, we adopt the following parametrization [4],

$$\Delta^N D_{h/q^\uparrow}(z, p_\perp) = \tilde{\Delta}^N D_{h/q^\uparrow}(z) h(p_\perp) \frac{e^{-p_\perp^2/\langle p_\perp^2\rangle}}{\pi\langle p_\perp^2\rangle}, \qquad (3)$$

where

$$\tilde{\Delta}^N D_{h/q^\uparrow}(z) = 2\mathcal{N}_q^C(z) D_{h/q}(z) \qquad (4)$$

represents the z-dependent part of the Collins function at the initial scale Q_0^2, which is then evolved to the appropriate value of $Q^2 = 112\ \mathrm{GeV}^2$. In this analysis, we use a simple model which implies no Q^2 dependence in the $p_\perp$ distribution. As the Collins function in our parametrization is proportional to the unpolarized fragmentation function, see Eqs. (3) and (4), we assume that the only scale dependence is contained in $D(z, Q^2)$, which is evolved with an unpolarized DGLAP kernel, while $\mathcal{N}_q^C$ does not evolve in Q^2. This amounts to assuming that the ratio $\Delta^N D(z, p_\perp, Q^2)/D(z, Q^2)$ is constant in Q^2.

The function $h(p_\perp)$, defined as

$$h(p_\perp) = \sqrt{2e} \frac{p_\perp}{M_C} e^{-p_\perp^2/M_C^2}, \qquad (5)$$

allows for a possible modification of the $p_\perp$ Gaussian width of the Collins function with respect to the unpolarized FF, while fulfilling the appropriate positivity bound: this modification is controlled by the parameter M_C^2.

For the pion $\mathcal{N}_q^C(z)$, we fix the favored and disfavored contributions as obtained from the reference fit of Ref. [4],

$$\mathcal{N}_{\mathrm{fav}}^C(z) = N_{\mathrm{fav}}^\pi z^\gamma (1-z)^\delta \frac{(\gamma+\delta)^{\gamma+\delta}}{\gamma^\gamma \delta^\delta} \qquad (6)$$

$$\mathcal{N}_{\mathrm{dis}}^C(z) = N_{\mathrm{dis}}^\pi, \qquad (7)$$

with $N_{\mathrm{fav}}^\pi = 0.90$, $N_{\mathrm{dis}}^\pi = -0.37$, $\gamma = 2.02$ and $\delta = 0.00$, as reported in Table I.

For the kaon we parametrize the favored and disfavored Collins contributions by setting $\mathcal{N}_q^C(z)$ to a constant,

$$\mathcal{N}_{\mathrm{fav}}^C(z) = N_{\mathrm{fav}}^K \qquad (8)$$

$$\mathcal{N}_{\mathrm{dis}}^C(z) = N_{\mathrm{dis}}^K, \qquad (9)$$

which brings us to a total of two free parameters for the Collins functions. In fact, the experimental data presently available for kaon production do not require a four-parameter fit, as in the pion case. We have indeed explicitly checked that a four-parameter fit does not result in a lower value of the total χ^2.

B. $e^+e^- \to h_1h_2X$ in the hadronic-plane method

In the hadronic-plane method one adopts a reference frame in which one of the produced hadrons (h_2 in our case)

TABLE I. Fixed parameters for the u and d valence quark transversity distribution functions and the favored and disfavored pion-Collins fragmentation functions, as obtained by fitting simultaneously SIDIS data on the Collins asymmetry and Belle and *BABAR* data on A_0^{UL} and A_0^{UC}, for pion-pion pair production, in Ref. [4].

$N_{u_v}^T = 0.61_{-0.23}^{+0.39}$	$N_{d_v}^T = -1.00_{-0.00}^{+1.86}$
$\alpha = 0.70_{-0.63}^{+1.31}$	$\beta = 1.80_{-1.80}^{+7.60}$
$N_{\mathrm{fav}}^\pi = 0.90_{-0.34}^{+0.09}$	$N_{\mathrm{dis}}^\pi = -0.37_{-0.05}^{+0.05}$
$\gamma = 2.02_{-0.33}^{+0.83}$	$\delta = 0.00_{-0.00}^{+0.42}$
$M_C^2 = 0.28_{-0.09}^{+0.20}\ \mathrm{GeV}^2$	

identifies the $\hat{z}$ direction and the $\widehat{xz}$ plane is determined by the lepton and the h_2 directions; the other relevant plane is determined by $\hat{z}$ and the direction of the other observed hadron, h_1, at an angle ϕ_1 with respect to the $\widehat{xz}$ plane; θ_2 is the angle between h_2 and the e^+e^- direction.

In this case, the elementary process $e^+e^- \to q\bar{q}$ does not occur in the $\widehat{xz}$ plane, and thus the helicity scattering amplitudes involve an azimuthal phase, φ_2. The differential cross section reads

$$\begin{aligned}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{dz_1dz_2d^2\boldsymbol{p}_{\perp 1}d^2\boldsymbol{p}_{\perp 2}d\cos\theta_2} &= \frac{3\pi\alpha^2}{2s}\sum_q e_q^2\Big\{(1+\cos^2\theta_2)D_{h_1/q}(z_1,p_{\perp 1})D_{h_2/\bar{q}}(z_2,p_{\perp 2}) \\ &+\frac{1}{4}\sin^2\theta_2\Delta^N D_{h_1/q^\uparrow}(z_1,p_{\perp 1})\Delta^N D_{h_2/\bar{q}^\uparrow}(z_2,p_{\perp 2}) \\ &\times\cos(2\varphi_2+\phi_q^{h_1})\Big\},\end{aligned} \tag{10}$$

where $\phi_q^{h_1}$ is the azimuthal angle of the detected hadron h_1 around the direction of the parent fragmenting quark, q. In other words, $\phi_q^{h_1}$ is the azimuthal angle of $\boldsymbol{p}_{\perp 1}$ in the helicity frame of q. It can be expressed in terms of $\boldsymbol{p}_{\perp 2}$ and $\boldsymbol{P}_{1T}$, the transverse momentum of the h_1 hadron in the hadronic-plane reference frame. At lowest order in $p_\perp/(z\sqrt{s})$ we have

$$\cos\phi_q^{h_1} = \frac{P_{1T}}{p_{\perp 1}}\cos(\phi_1-\varphi_2) - \frac{z_1}{z_2}\frac{p_{\perp 2}}{p_{\perp 1}} \tag{11}$$

$$\sin\phi_q^{h_1} = \frac{P_{1T}}{p_{\perp 1}}\sin(\phi_1-\varphi_2). \tag{12}$$

Using the parametrization of the Collins function given in Eqs. (3)–(5), the integration over $\boldsymbol{p}_{\perp 2}$ in Eq. (10) can be performed explicitly. Moreover, since $\boldsymbol{p}_{\perp 1} = \boldsymbol{P}_1 - z_1\boldsymbol{q}_1$, we can replace $d^2\boldsymbol{p}_{\perp 1}$ with $d^2\boldsymbol{P}_{1T}$. Integrating also over P_{1T}, but not over ϕ_1, we then obtain

$$\frac{d\sigma^{e^+e^-\to h_1h_2X}}{dz_1dz_2d\cos\theta_2d\phi_1} = \frac{3\alpha^2}{4s}\{D^{h_1h_2}+N^{h_1h_2}\cos(2\phi_1)\}, \tag{13}$$

where

$$D^{h_1h_2} = (1+\cos^2\theta_2)\sum_q e_q^2 D_{h_1/q}(z_1)D_{h_2/\bar{q}}(z_2) \tag{14}$$

$$\begin{aligned}N^{h_1h_2} &= \frac{1}{4}\frac{z_1z_2}{z_1^2+z_2^2}\sin^2\theta_2\frac{2e\langle p_\perp^2\rangle M_C^4}{(\langle p_\perp^2\rangle+M_C^2)^3} \\ &\times\sum_q e_q^2\tilde{\Delta}^N D_{h_1/q^\uparrow}(z_1)\tilde{\Delta}^N D_{h_2/\bar{q}^\uparrow}(z_2).\end{aligned} \tag{15}$$

By normalizing this result to the azimuthal averaged cross section

$$\langle d\sigma\rangle = \frac{1}{2\pi}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{dz_1dz_2d\cos\theta_2} = \frac{3\alpha^2}{4s}D^{h_1h_2}, \tag{16}$$

one gets

$$R_0^{h_1h_2} \equiv \frac{1}{\langle d\sigma\rangle}\frac{d\sigma^{e^+e^-\to h_1h_2X}}{dz_1dz_2d\cos\theta_2d\phi_1} = 1+P_0^{h_1h_2}\cos(2\phi_1), \tag{17}$$

having defined

$$P_0^{h_1h_2} = \frac{N^{h_1h_2}}{D^{h_1h_2}}. \tag{18}$$

In our previous analysis [4], we considered the like sign (L), unlike sign (U) and charged (C) combinations for pion-pion pairs, which are constructed by using the appropriate combinations of charged pions, that is, by replacing $P_0^{h_1h_2}$ in Eq. (17) by

$$P_{0L}^{\pi\pi} \equiv \frac{N_L^{\pi\pi}}{D_L^{\pi\pi}} = \frac{N^{\pi^+\pi^+}+N^{\pi^-\pi^-}}{D^{\pi^+\pi^+}+D^{\pi^-\pi^-}} \tag{19a}$$

$$P_{0U}^{\pi\pi} \equiv \frac{N_U^{\pi\pi}}{D_U^{\pi\pi}} = \frac{N^{\pi^+\pi^-}+N^{\pi^-\pi^+}}{D^{\pi^+\pi^-}+D^{\pi^-\pi^+}} \tag{19b}$$

$$P_{0C}^{\pi\pi} \equiv \frac{N_C^{\pi\pi}}{D_C^{\pi\pi}} = \frac{N_L^{\pi\pi}+N_U^{\pi\pi}}{D_L^{\pi\pi}+D_U^{\pi\pi}}. \tag{19c}$$

Analogously, for kaon-kaon pairs,

$$P_{0L}^{KK} \equiv \frac{N_L^{KK}}{D_L^{KK}} = \frac{N^{K^+K^+}+N^{K^-K^-}}{D^{K^+K^+}+D^{K^-K^-}} \tag{20a}$$

$$P_{0U}^{KK} \equiv \frac{N_U^{KK}}{D_U^{KK}} = \frac{N^{K^+K^-}+N^{K^-K^+}}{D^{K^+K^-}+D^{K^-K^+}} \tag{20b}$$

$$P_{0C}^{KK} \equiv \frac{N_C^{KK}}{D_C^{KK}} = \frac{N_L^{KK}+N_U^{KK}}{D_L^{KK}+D_U^{KK}}, \tag{20c}$$

and for pion-kaon production,

$$P_{0L}^{\pi K} \equiv \frac{N_L^{\pi K}}{D_L^{\pi K}} = \frac{N^{\pi^+K^+}+N^{\pi^-K^-}+N^{K^+\pi^+}+N^{K^-\pi^-}}{D^{\pi^+K^+}+D^{\pi^-K^-}+D^{K^+\pi^+}+D^{K^-\pi^-}} \tag{21a}$$

$$P_{0U}^{\pi K} \equiv \frac{N_U^{\pi K}}{D_U^{\pi K}} = \frac{N^{\pi^+K^-}+N^{\pi^-K^+}+N^{K^+\pi^-}+N^{K^-\pi^+}}{D^{\pi^+K^-}+D^{\pi^-K^+}+D^{K^+\pi^-}+D^{K^-\pi^+}} \tag{21b}$$

$$P_{0C}^{\pi K} \equiv \frac{N_C^{\pi K}}{D_C^{\pi K}} = \frac{N_L^{\pi K}+N_U^{\pi K}}{D_L^{\pi K}+D_U^{\pi K}}. \tag{21c}$$

We can now build ratios of unlike/like and unlike/charged asymmetries,

$$\frac{(R_0^{h_1h_2})^U}{(R_0^{h_1h_2})^{L(C)}} = \frac{1 + P_{0U}^{h_1h_2}\cos(2\phi_1)}{1 + P_{0L(C)}^{h_1h_2}\cos(2\phi_1)} \simeq 1 + (P_{0U}^{h_1h_2} - P_{0L(C)}^{h_1h_2})\cos(2\phi_1), \quad (22)$$

where $P_{0U}^{h_1h_2}$, $P_{0L}^{h_1h_2}$ and $P_{0C}^{h_1h_2}$ can be taken from Eqs. (19)–(21). Finally, one can write the asymmetries that are measured experimentally, which correspond to the coefficient of the cosine in Eq. (22):

$$(A_0^{h_1h_2})^{UL} = P_{0U}^{h_1h_2} - P_{0L}^{h_1h_2} \quad (23)$$

$$(A_0^{h_1h_2})^{UC} = P_{0U}^{h_1h_2} - P_{0C}^{h_1h_2}. \quad (24)$$

III. BEST FITTING AND RESULTS

As mentioned above, we have adopted the following procedure:

(1) We employ the pion favored and disfavored Collins functions as obtained in our recent extraction [4] based on *BABAR* [14] and Belle [15,16] $e^+e^- \to \pi\pi X$ data. As far as pions are concerned no free parameters are introduced in this analysis. The fixed values of the pion Collins function parameters are presented in Table I, together with the parameters obtained for the transversity distribution, which are given for later use.

(2) The kaon favored and disfavored Collins functions are parametrized using a factorized form similar to that used for pions, but with a simpler structure: due to the limitations of the kaon data presently available, we introduce only two free parameters in our fit, instead of four, in such a way that the z-dependent part of the Collins functions will simply be proportional to their unpolarized counterparts:

$$\tilde{\Delta}^N D_{K/q^\uparrow}(z) = 2N_i^K D_{K/q}(z), \quad i = \text{fav, dis}. \quad (25)$$

N_{fav}^K and N_{dis}^K are free parameters to be fixed by best fitting the experimental data. In this fit, which we denote as our "reference fit," we make no distinction, for the values of N_i^K, between heavy and light flavors; notice, however, that the favored kaon Collins functions for the s quark will, in fact, be different from that of the u flavor, and this difference is induced by the unpolarized, collinear FFs used in our parametrization, which imply consistently different contributions for heavy and light flavors. The Gaussian width of the kaon Collins function, controlled by the parameter M_C^2, Eq. (5), is assumed to be the same as that of the pion Collins function. Present data are not sensitive enough to the shape of the $p_\perp$ dependence of the Collins functions to make further distinctions. Moreover, for the same reason, no Q^2 dependence of the $p_\perp$ distribution is included in our model. Further considerations on the choice of two parameters will be made at the end of this section.

This reference best fit gives the following results for the two free parameters considered,

$$N_{\text{fav}}^K = 0.41_{-0.10}^{+0.10}, \qquad N_{\text{dis}}^K = 0.08_{-0.26}^{+0.18}, \quad (26)$$

suggesting a solution with a *positive* favored Collins function and a disfavored contribution compatible with zero, within large errors. However, as we will discuss in Sec. III A, a definite conclusion can only be drawn about the positive sign of the favored light-flavor contribution. Note that the pion Collins fragmentation functions extracted in Ref. [4] have opposite signs for favored and disfavored functions, and disfavored functions are definitely nonzero.

The contributions to the total χ^2 of each fitted set of data are given in Table II. It is a good fit, and, as one can see from Figs. 1 and 2, the data are described well. The A_0^{UL} asymmetries for KK production are quite scattered and do not show a definite trend; it is for these data that we obtain the largest χ^2 contribution. The bands shown in Figs. 1 and 2 are obtained by sampling 1500 sets of parameters corresponding to a χ^2 value in the range between χ^2_{min} and $\chi^2_{\text{min}} + \Delta\chi^2$, as explained in Ref. [4]. The value of $\Delta\chi^2$ corresponds to 95.45% confidence level for two parameters; in this case we have $\Delta\chi^2 = 6.18$.

(3) We deliberately choose not to include SIDIS kaon data in the fit at this stage. Including them would, in principle, require a global analysis of both pion and kaon data sets which is beyond the scope of this paper. Moreover, we would like to test the universality of the Collins fragmentation functions in e^+e^- and SIDIS, as proposed in Ref. [10], and check whether the kaon favored and disfavored Collins functions extracted from e^+e^- annihilation data can describe the Collins asymmetries observed in SIDIS processes. We compute the Collins SIDIS asymmetry $A_{UT}^{\sin(\phi_h+\phi_S)}$, using the kaon Collins functions given by our reference fit, Eqs. (8), (9) and (26),

TABLE II. χ^2 values obtained in our reference fit. See the text for details.

Data set	χ^2	Points	χ^2/points
$K\pi$ production A_0^{UL}	14.6	16	0.91
$K\pi$ production A_0^{UC}	7.4	16	0.46
KK production A_0^{UL}	23.6	16	1.48
KK production A_0^{UC}	9.4	16	0.59
Total	55.0	64	$\chi^2_{\text{d.o.f.}} = 0.89$

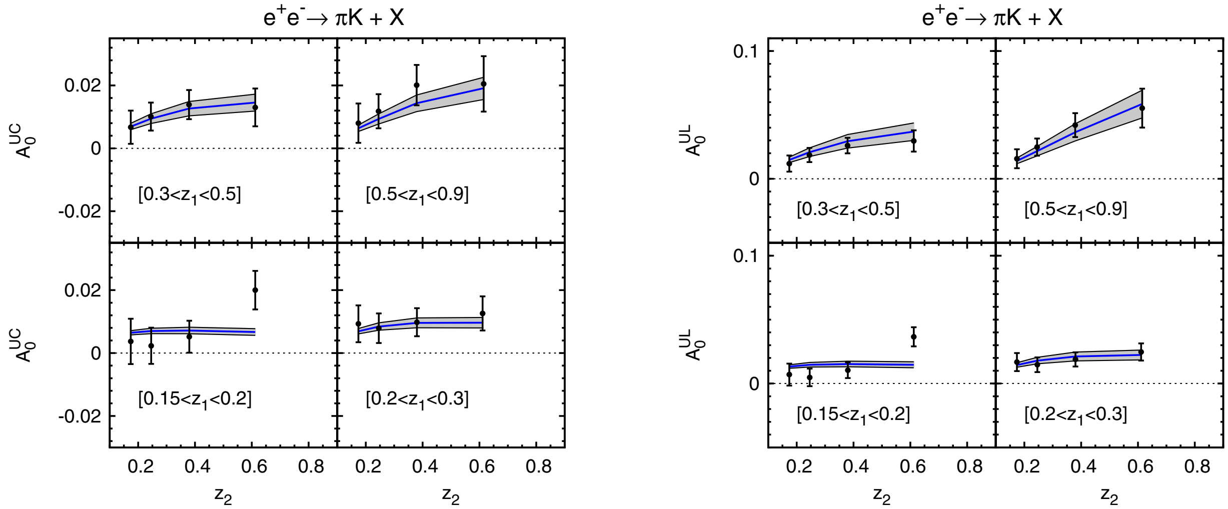

FIG. 1. The experimental data on the azimuthal correlations A_0^{UC} and A_0^{UL} as functions of z_1 and z_2 in unpolarized $e^+e^- \to \pi K X$ processes, as measured by the *BABAR* Collaboration, are compared to the curves obtained from our reference fit, given by the parameters shown in Eq. (26). The shaded area corresponds to the statistical uncertainty on these parameters.

and the transversity distributions obtained in Ref. [4] and given in Table I. The comparison of our predictions with the measurements performed by the HERMES and COMPASS collaborations is shown in Figs. 3 and 4, respectively. The good agreement confirms, within the precision limits of experimental data, the consistency of the Collins functions extracted from e^+e^- data with those active in SIDIS processes.

A. Fits with additional parameters

Looking at the results of our reference fit, Eq. (26), the disfavored Collins function appears to be quite undetermined and compatible with zero, while the favored one is definitely nonzero and positive. However, we have assumed that the heavy (s quark) and light (u quark) favored contributions are controlled by the same parameter. We wonder whether, by disentangling these two contributions, one can confirm the results obtained above.

An inspection of the analytical formulas, Eqs. (20) and (21) and (14) and (15), shows that the sign of the light-flavor favored contribution is determined by the πK data, where it appears convoluted with the pion Collins function, which is fixed. Most of the information, in particular, comes from the A_0^{UL} asymmetries, which are dominated by doubly favored terms of the type $\tilde{\Delta}^N D_{\pi^+/u^\uparrow} \tilde{\Delta}^N D_{K^-/\bar{u}^\uparrow}$.

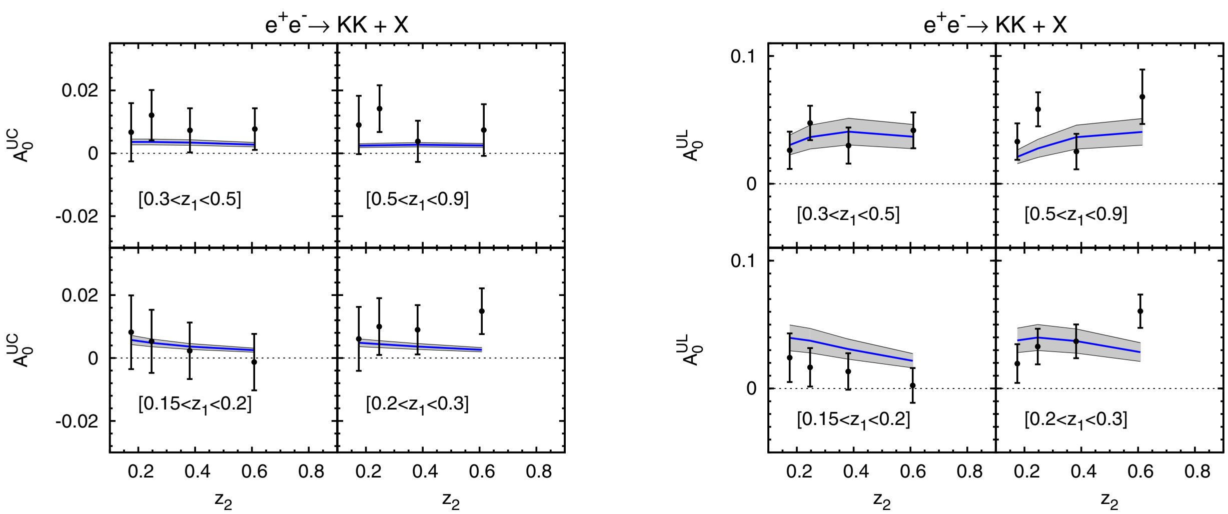

FIG. 2. The experimental data on the azimuthal correlations A_0^{UC} and A_0^{UL} as functions of z_1 and z_2 in unpolarized $e^+e^- \to K^+K^-X$ processes, as measured by the *BABAR* Collaboration, are compared to the curves obtained from our reference fit, given by the parameters shown in Eq. (26). The shaded area corresponds to the statistical uncertainty on these parameters.

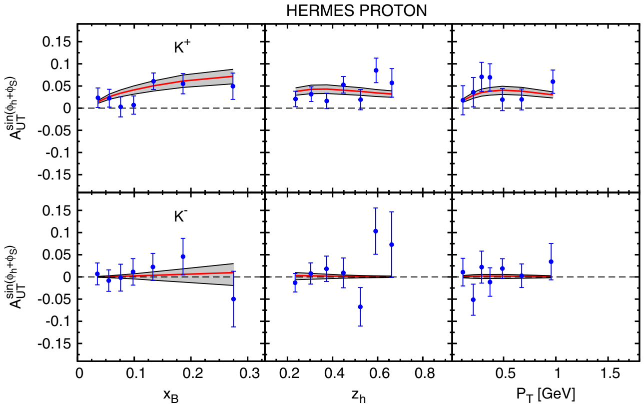

FIG. 3. The experimental data on the SIDIS azimuthal moment $A_{UT}^{\sin(\phi_h+\phi_S)}$ as measured by the HERMES Collaboration [5] are compared with our computation of the same quantity. The solid (red) lines correspond to our reference fit, with the parameters given in Eq. (26). The shaded area corresponds to the statistical uncertainty on these parameters. For the transversity distributions we used the fixed parameters reported in Table I.

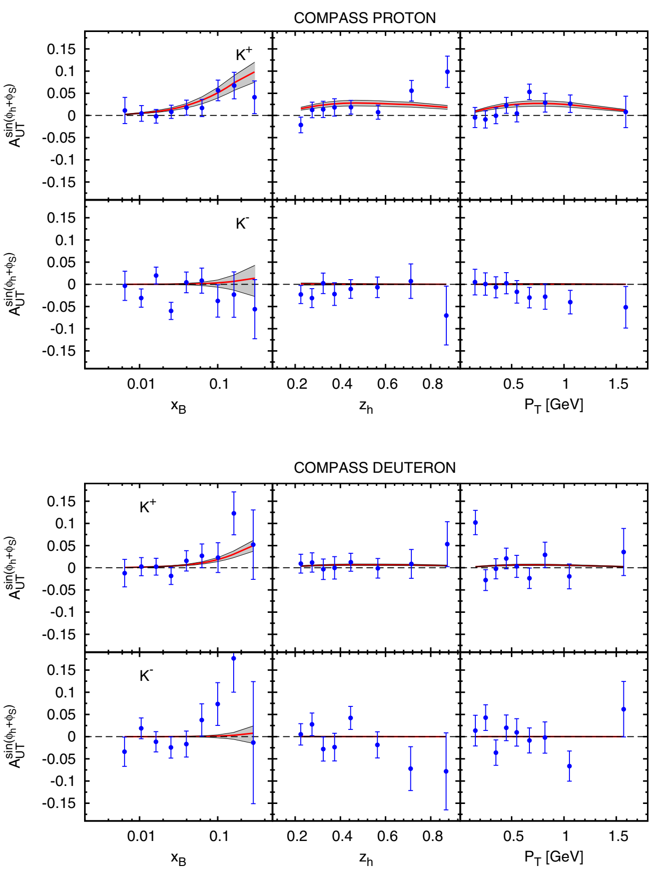

FIG. 4. The experimental data on the SIDIS Collins single spin asymmetry (SSA) $A_{UT}^{\sin(\phi_h+\phi_S)}$ as measured by the COMPASS Collaboration on proton (upper panel) [8] and deuteron (lower panel) targets [7] are compared with our computation of the same quantity. The solid (red) lines correspond to our reference fit, with the parameters given in Eq. (26). The shaded area corresponds to the statistical uncertainty on these parameters. For the transversity distributions we used the fixed parameters reported in Table I.

The heavy-flavor contribution, instead, is not determined by the data (not even in sign); this is due to the fact that, in KK production processes, it appears in doubly favored terms where it is convoluted with itself and is therefore insensitive to the sign choice, while in πK production processes it appears only in *subleading* combinations, such as $\tilde{\Delta}^N D_{\pi^-/s^\uparrow} \tilde{\Delta}^N D_{K^+/\bar{s}^\uparrow}$.

To study this in more detail, we have performed a series of fits allowing for up to three free parameters, i.e. one normalization constant for the favored light flavor, $N_{\rm fav}^{\rm light}$; one for the favored heavy flavor, $N_{\rm fav}^{\rm heavy}$; and one for the disfavored, $N_{\rm dis}$, contributions. The results, with the $\chi^2_{\rm d.o.f.}$ for each of the fits, are presented in Table III, while some correlations between the parameters are studied in Fig. 5. Let us comment on such results:

(i) The first clear conclusion is that it is not possible to fit the data with one and only one of the parameters $N_{\rm fav}^{\rm light}$, $N_{\rm fav}^{\rm heavy}$, $N_{\rm dis}$, as shown in the upper panel of Table III.

(ii) Regarding the two parameter fits (central panel of Table III), we see that the data can be successfully described only by including the light favored contribution together with either the heavy favored or the disfavored Collins function. Notice that the sign of the heavy contribution can be either positive or negative, leading to equally good fits (first two lines of the central panel in Table III). The sign of $N_{\rm fav}^{\rm light}$ turns out to be always positive, with its best value in the approximate range between 0.3 and 0.6 (see the left panel of Fig. 5). Instead, fitting the data without any light quark favored contribution appears not to

TABLE III. χ^2/d.o.f. for different scenarios for the kaon Collins functions: one-parameter (upper panel), two-parameter (central panel) and three-parameter (lower panel) fits. The symbol filled circle (•) means that the corresponding parameter is actually used in the fit, while the symbol open circle (∘) means that the contribution to the Collins asymmetry corresponding to that parameter is not included in the fit. For $N_{\rm fav}^{\rm heavy}$, we explicitly indicate the two different constraints we use: $N_{\rm fav}^{\rm heavy} > 0$ and $N_{\rm fav}^{\rm heavy} < 0$.

$N_{\rm fav}^{\rm light}$	$N_{\rm fav}^{\rm heavy} > 0$	$N_{\rm fav}^{\rm heavy} < 0$	$N_{\rm dis}$	$\chi^2_{\rm d.o.f.}$
•	∘	∘	∘	1.83
∘	•	∘	∘	3.32
∘	∘	•	∘	5.68
∘	∘	∘	•	3.94
•	•	∘	∘	0.89
•	∘	•	∘	0.88
•	∘	∘	•	0.98
∘	•	∘	•	2.00
∘	∘	•	•	4.00
•	•	∘	•	0.90
•	∘	•	•	0.89

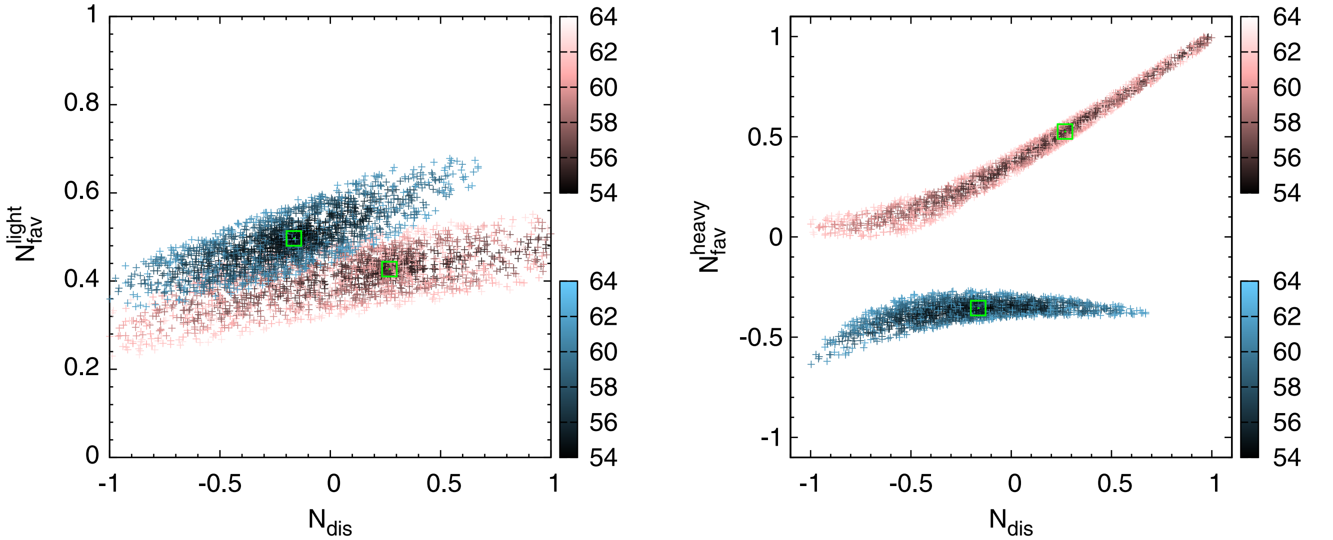

FIG. 5. Correlation between the parameters: $N_{\rm fav}^{\rm light}$ and $N_{\rm dis}$ (left panel) and $N_{\rm fav}^{\rm heavy}$ and $N_{\rm dis}$ (right panel). Red points represent solutions with positive $N_{\rm fav}^{\rm heavy}$, while blue points represent solutions with negative $N_{\rm fav}^{\rm heavy}$. All points in the figure correspond to a total χ^2 included between $\chi^2_{\rm min}$ and $\chi^2_{\rm min}+\Delta\chi^2$; the spread of the points indicates the statistical error which affects the two parameters. Lighter (darker) shades of color represent higher (lower) values of χ^2. The points in which $\chi^2 \equiv \chi^2_{\rm min}$ are shown as green squares.

be possible (last two lines of the central panel in Table III).

(iii) Fits with three parameters (bottom panel of Table III) result in good values of $\chi^2_{\rm d.o.f.}$. These fits allow us to study the correlation among the free parameters. We, in fact, observe a very strong correlation between the heavy-flavor (favored) and the disfavored contributions to the kaon Collins functions: values of $N_{\rm fav}^{\rm heavy}$ with opposite sign can easily be compensated by different values of $N_{\rm dis}$, resulting in fits of equal quality, as shown in the last part of Table III. We actually find two distinct solutions resulting from the present data, one with positive and one with negative heavy-flavor Collins FFs.

Figure 5 (right panel) illustrates this correlation. Two distinct distributions are clearly evident: red (blue) points represent solutions with positive (negative) $N_{\rm fav}^{\rm heavy}$. All points in the figure correspond to a total χ^2 included between $\chi^2_{\rm min}$ and $\chi^2_{\rm min}+\Delta\chi^2$; for a three parameter fit $\Delta\chi^2 = 8.02$. The spread of the points indicates the statistical error which affects the two parameters. Lighter (darker) shades of color represent higher (lower) values of χ^2. The points in which $\chi^2 \equiv \chi^2_{\rm min}$ are shown as green squares.

Notice that model calculations predict the same sign of light- and heavy-flavor Collins FF; see for instance Ref. [17].

In Fig. 6 we show the lowest $\boldsymbol{p}_\perp$-moment of the light-flavor favored kaon Collins function, as extracted in our reference fit [with the parameters of Eq. (26)]. Note that, in the case of a factorized Gaussian shape, Eqs. (3), (4) and (5), the lowest $\boldsymbol{p}_\perp$-moment of the Collins function,

$$\Delta^N D_{h/q^\uparrow}(z,Q^2) = \int d^2\boldsymbol{p}_\perp \Delta^N D_{h/q^\uparrow}(z,p_\perp,Q^2), \quad (27)$$

is related to the z-dependent part of the Collins function, $\tilde{\Delta}^N D_{h/q^\uparrow}(z,Q^2)$, by

$$\Delta^N D_{h/q^\uparrow}(z,Q^2) = \frac{\sqrt{\pi}}{2}\frac{\langle p_\perp^2\rangle_C^{3/2}}{\langle p_\perp^2\rangle}\frac{\sqrt{2e}}{M_C}\tilde{\Delta}^N D_{h/q^\uparrow}(z,Q^2), \quad (28)$$

where $\langle p_\perp^2\rangle_C$ is defined in Eq. (14) of Ref. [4].

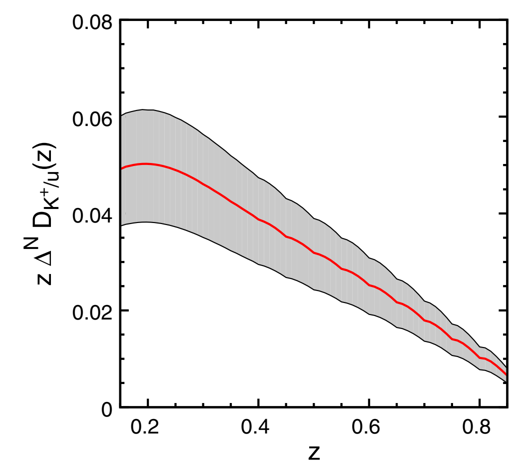

FIG. 6. Plot of z times the lowest $\boldsymbol{p}_\perp$-moment, Eqs. (27) and (28), of the $u^\uparrow \to K^+X$ Collins function, as extracted in our reference fit [with the parameters of Eq. (26)]. The analogous plots for heavy-flavor favored and (all flavor) disfavored Collins functions are not shown; in fact, it is not possible to reliably distinguish between these two contributions to the available *BABAR* data. Furthermore, not even the sign of the heavy-flavor favored Collins function can be determined.

The heavy-flavor favored and (all flavor) disfavored results are not shown; in fact, the study performed above shows that it is not possible to reliably distinguish between these two contributions to the available data. Furthermore, not even the sign of the heavy-flavor favored Collins function can be determined.

IV. COMMENTS AND CONCLUSIONS

We have extracted, for the first time, the kaon Collins functions, $q^\uparrow \to KX$, by best fitting recent *BABAR* data [9]. This paper extends a recent study of the Collins functions in e^+e^- and SIDIS processes [4] limited to pion production.

It turns out that a simple phenomenological parametrization of the Collins function, Eqs. (3) and (4), is quite adequate to describe the data. When comparing with the pion Collins functions [4], due to the limited amount and relatively big errors of data, an even smaller number of parameters suffices to describe the experimental results. Indeed, we find that kaon Collins functions of two kinds, favored and disfavored, both simply proportional to the unpolarized TMD fragmentation functions, describe well the *BABAR* data.

As a result of the attempted fits, we can conclude that a definite outcome of this study is the determination of a positive $u^\uparrow \to K^+X = \bar{u}^\uparrow \to K^-X$ Collins function, assuming a positive favored pion Collins function [4]. No definite independent conclusion, based on the available data, can be drawn on the signs of $s^\uparrow \to K^-X = \bar{s}^\uparrow \to K^+X$ Collins functions nor on the disfavored ones.

The extracted kaon Collins functions, together with the transversity distributions obtained in Ref. [4], give a very good description, within the rather large experimental uncertainties, of SIDIS data on kaon Collins asymmetries measured by COMPASS [7,8] and HERMES [5] collaborations. This points toward a consistent and universal role of the Collins effect in different physical processes, which should be further explored in the future.

ACKNOWLEDGMENTS

M. A., M. B., J. O. G. H. and S. M. acknowledge the support of “Progetto di Ricerca Ateneo/CSP” (codice TO-Call3-2012-0103).

[1] M. Anselmino, M. Boglione, U. D’Alesio, A. Kotzinian, F. Murgia, A. Prokudin, and C. Türk, Phys. Rev. D **75**, 054032 (2007).
[2] M. Anselmino, M. Boglione, U. D'Alesio, A. Kotzinian, F. Murgia, A. Prokudin, and S. Melis, Nucl. Phys. B, Proc. Suppl. **191**, 98 (2009).
[3] M. Anselmino, M. Boglione, U. D’Alesio, S. Melis, F. Murgia, and A. Prokudin, Phys. Rev. D **87**, 094019 (2013).
[4] M. Anselmino, M. Boglione, U. D’Alesio, J. O. Gonzalez Hernandez, S. Melis, F. Murgia, and A. Prokudin, Phys. Rev. D **92**, 114023 (2015).
[5] A. Airapetian *et al.* (HERMES Collaboration), Phys. Lett. B **693**, 11 (2010).
[6] A. Airapetian *et al.* (HERMES Collaboration), Phys. Rev. D **87**, 012010 (2013).
[7] M. Alekseev *et al.* (COMPASS Collaboration), Phys. Lett. B **673**, 127 (2009).
[8] C. Adolph *et al.* (COMPASS Collaboration), Phys. Lett. B **744**, 250 (2015).
[9] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. D **92**, 111101 (2015).
[10] J. C. Collins and A. Metz, Phys. Rev. Lett. **93**, 252001 (2004).
[11] M. Anselmino, M. Boglione, J. Gonzalez Hernandez, S. Melis, and A. Prokudin, J. High Energy Phys. 04 (2014) 005.
[12] M. Gluck, E. Reya, and A. Vogt, Eur. Phys. J. C **5**, 461 (1998).
[13] D. de Florian, R. Sassot, and M. Stratmann, Phys. Rev. D **76**, 074033 (2007).
[14] J. Lees *et al.* (*BABAR* Collaboration), Phys. Rev. D **90**, 052003 (2014).
[15] R. Seidl *et al.* (Belle Collaboration), Phys. Rev. D **78**, 032011 (2008).
[16] R. Seidl *et al.* (Belle Collaboration), Phys. Rev. D **86**, 039905 (2012).
[17] A. Bacchetta, L. P. Gamberg, G. R. Goldstein, and A. Mukherjee, Phys. Lett. B **659**, 234 (2008).