Time-average velocity estimation through surface-wave analysis: Part 2 — P-wave velocity

Laura Valentina Socco¹ and Cesare Comina²

ABSTRACT

Surface waves (SWs) in seismic records can be used to extract local dispersion curves (DCs) along a seismic line. These curves can be used to estimate near-surface S-wave velocity models. If the velocity models are used to compute S-wave static corrections, the required information consists of S-wave time-average velocities that define the one-way time for a given datum plan depth. However, given the wider use of P-wave reflection seismic with respect to S-wave surveys, the estimate of P-wave time-average velocity would be more useful. We therefore focus on the possibility of also extracting time-average P-wave velocity models from SW dispersion data. We start from a known 1D S-wave velocity model along the line, with its relevant DC, and we estimate a wavelength/depth relationship for SWs. We found that this relationship is sensitive to Poisson’s ratio, and we develop a simple method for estimating an “apparent” Poisson’s ratio profile, defined as the Poisson’s ratio value that relates the time-average S-wave velocity to the time-average P-wave velocity. Hence, we transform the time-average S-wave velocity models estimated from the DCs into the time-average P-wave velocity models along the seismic line. We tested the method on synthetic and field data and found that it is possible to retrieve time-average P-wave velocity models with uncertainties mostly less than 10% in laterally varying sites and one-way traveltime for P-waves with less than 5 ms uncertainty with respect to P-wave tomography data. To our knowledge, this is the first method for reliable estimation of P-wave velocity from SW data without any a priori information or additional data.

INTRODUCTION

The present paper is the companion paper of Socco et al. (2017), which will be referred to as paper 1. In paper 1, we outline the use of surface waves (SWs) in seismic exploration records to directly estimate the S-wave time-average velocity that can be used to directly compute \( V_S \) static corrections. Several authors (Al Dulaijan and Stewart, 2010; Boiero et al., 2013; Gendrin et al., 2015) state that the use of SW for \( V_S \) static estimation is a standard method currently applied in industrial processing workflows. However, because most hydrocarbon seismic exploration data are P-wave data, the most useful information to be estimated would be \( V_p \) statics. SWs are usually considered not suitable for this purpose because it has been shown that the sensitivity of SW dispersion curves (DCs) to \( V_p \) is insufficient for a reliable \( V_p \) estimation through DC inversion. Nazarian (1984) showed that the effect of the Poisson’s ratio variation, from 0.15 to 0.49, on the DC phase velocity is less than 10%. Xia et al. (1999) showed that the variability of the DC due to a variation in P-wave velocity in the model is negligible with respect to \( V_S \) and thicknesses. For this reason, in the standard procedure of SW inversion, the value of \( V_p \) (or Poisson’s ratio) for the layered system is usually assumed a priori and only S-wave velocity and thickness of the layers are estimated.

Nevertheless, several attempts have been made to estimate \( V_p \) near-surface velocity models from SW DCs. In particular, the role of higher modes to improve sensitivity toward \( V_p \) has been investigated. Ernst (2008) claimed that higher modes have higher sensitivity to \( V_p \), contrary to Xia et al. (2003), who showed that individual higher modes have lower variability than fundamental mode for \( V_p \) variations in the model. Barreto (2013) performed an extensive analysis on synthetic data to investigate the possibility of exploiting the distance between modal curves to estimate \( V_p \), concluding that for a layered system, Poisson’s ratio cannot be reliably estimated even by including higher modes. Bergamo and
that of data sets. Documented otherwise, has not received enough attention and remains poorly theoretical dispersion relationships, associated with the inversion process. Karray and Lefebvre (2008) stated cannot be used to directly estimate (Badji et al., 2016). Materials can make the P-wave first-break autopicking unreliable significantly improve the reliability of P-wave tomograms, particularly (2014) showed that the constraints coming from SW can signifi- cantly affect the goodness of the investigation depth expressed as a ratio between the investigated thickness and the SW wavelength. Pelekis and Athanasopoulos (2011) proposed a simplified SW inversion technique based on the empirical estimation of an “investigation depth coefficient” that is used to directly convert the DC into an approximated VS model. They also showed that this coefficient depends on Poisson’s ratio. Pan et al. (2013) also proposed an empirical linear relationship between a penetration depth coefficient and Poisson’s ratio. Although these works again did not aim to estimate Poisson’s ratio or VP, they pointed out that the sensitivity of the investigation depth to Poisson’s ratio is evident and has a simple pattern.

In paper I, we exploited the concept of penetration depth of SW for a simplified time-average VS model estimation. Our approach was based on the estimation of a piecewise polynomial relationship between the wavelength and the depth (W/D). In this paper, we in- vestigate the sensitivity of the W/D relationship to Poisson’s ratio and show that this sensitivity can be used to estimate Poisson’s ratio to transform the time-average VS model into a time-average VP model. The time-average velocity, consistently with paper I, is defined as:

\[ V_z = \frac{\sum h_i V_i}{\sum h_i} \]  

where \( n \) is the number of layers down to the depth \( z \), and \( h_i \) and \( V_i \) are the thickness and the velocity of the \( i \)th layer, respectively. This velocity provides directly the one-way time at a selected depth. The proposed approach is already introduced by Socco et al. (2016) and applied to a synthetic data set. Here, we describe the method in detail and justify its validity. We then test it on two field data sets.

Because we will estimate Poisson’s ratio to transform the time average S-wave velocity \( V_{Sz} \) into the time average P-wave velocity \( V_{Pz} \), it is useful to examine the expected values of Poisson’s ratio in the near-surface materials and how accurate its estimate should be to provide a reliable \( V_{Pz} \). Moreover, it is also important to address the expected spatial variability of Poisson’s ratio in near-surface layers.

First, it should be stressed that when we relate Poisson’s ratio to seismic velocities, we are implicitly assuming to deal with the so-called dynamic Poisson’s ratio, i.e., Poisson’s ratio at small strain level and in undrained conditions. The dynamic Poisson’s ratio \( \nu \) (in the following Poisson’s ratio) is related to seismic velocities as shown in equation 2:

\[ \nu = \frac{1}{2} \left( \frac{V_P}{V_S} \right)^2 - 1. \]  

Figure 1. Poisson’s ratio versus \( V_P/V_S \) (equation 2).
where $V_p$ and $V_S$ are the P- and S-wave velocities, respectively. Hence, Poisson’s ratio is a nonlinear scaling factor between $V_p$ and $V_S$. By plotting equation 2, we can see (Figure 1) that the sensitivity of $V_p/V_S$ to Poisson’s ratio varies for different ranges of Poisson’s ratio value. In particular, at Poisson’s ratio values less than 0.4, the $V_p/V_S$ varies from 1.4 to 2.5 and small variations in Poisson’s ratio will induce very small variations in $V_p/V_S$. Hence, for materials in this range of Poisson’s ratio, if $V_S$ is available and Poisson’s is estimated and used to retrieve $V_p$, an error of 0.01 in Poisson’s ratio estimation will cause an error from 0.5% to 3.9% on $V_p$, whereas an error of 0.1 on Poisson’s ratio will cause an error from 6% to 24% on $V_p$. Conversely, in the range above 0.4, the error grows up to almost 50% for a Poisson’s ratio of 0.48 and reaches almost 100% for a Poisson’s ratio equal to 0.499.

Hence, in different ranges of Poisson’s ratio, different accuracies are required. In the literature, there are relatively few data collections of Poisson’s ratio values. One of the largest to our knowledge is a series of seismic borehole measurements performed by the USGS over decades and reported by Boore (2007). The available data from 274 downhole tests and 53 suspension logging tests show that in saturated soils, Poisson’s ratio is strongly governed by saturating fluids and assumes poorly dispersed values, which are very close to 0.5. In the case of dry soils or weathered rocks, the values appear to be more variable with the majority of field data in the range of 0.25–0.4. Along the vertical direction, abrupt variations in Poisson’s ratio can be encountered at the water table and at the bedrock interfaces. Even though at the water table the variation is mainly due to a variation in $V_p$ with almost unchanged $V_S$, at the bedrock, both $V_p$ and $V_S$ change. Along the lateral direction, abrupt variations in Poisson’s ratio are generally related to sharp variations in the solid matrix properties. Thus, the variations will concern both $V_p$ and $V_S$. This is an important point for the method we propose because we estimate one Poisson’s ratio vertical profile in correspondence with one reference DC and we use it to convert the $V_S$ estimated along the line into $V_{pS}$. Hence, we make the assumption that lateral variations in Poisson’s ratio are expected to be negligible along the line. We also assume that if a significant lateral variation in Poisson’s ratio is present along the line, this will correspond to a significant variation in $V_S$ that will show up in the experimental DCs. In this case, the data set can be split into subdata sets and a reference Poisson’s ratio profile can be estimated for each of them.

This paper is organized as follows: First, we outline the method using a single synthetic 1D model example with the constant Poisson’s ratio with depth, and then, we test the method on more challenging synthetic profiles with a variable Poisson’s ratio with depth and also in the presence of a low-velocity layer. We finally apply the method to the same field data shown in paper 1, and we compare the estimated $V_{pS}$ models with those obtained through travelt ime tomography and local downhole tests.

**METHOD**

In paper 1, we have introduced the W/D relationship that can be used to directly estimate the time average S-wave velocity ($V_{Sb}$) given the SW DC. As for paper 1, in this work, we refer to fundamental mode only; hence, for “dispersion curve,” we intend “dispersion curve of the fundamental mode of propagation of Rayleigh waves.” Here, we show that the W/D relationship is sensitive to Poisson’s ratio and can therefore also be used to estimate the time average P-wave velocity $V_{pS}$. We use the same synthetic $V_S$ model used in paper 1 to outline the method. We start by assuming a constant Poisson’s ratio of 0.33 for all of the layers. In Figure 2a, we report the $V_p$ and $V_S$ models, the resulting $V_{pS}$ and $V_{Sb}$, and the relevant fundamental mode DC plotted as a function of wavelength. From the time-average $V_S$ model and the DC, we compute the W/D relationship by searching, for each $V_{Sb}$ value, the wavelength at which the phase velocity of the DC is equal to $V_{Sb}$. We take the $V_{pS}$ model and the W/D relationship as our reference (we call them “experimental” data), and we show that the W/D relationship can be used to estimate $V_{pS}$.

We then keep the $V_S$ model as it is and change the Poisson’s ratio for the whole model, ranging from 0.1 to 0.45, and we compute the theoretical DCs corresponding to different values of Poisson’s ratio. We use these DCs and the $V_{Sb}$ to compute the W/D relationships for the different values of Poisson’s ratio. In Figure 2b, we report all the obtained W/D relationships together with the one of the reference $V_p$ model in Figure 2a. It can be noted that Poisson’s ratio acts on the slope of W/D relationship. In particular, the slope decreases when Poisson’s ratio increases.

By comparing the experimental W/D relationship with the theoretical ones, we can identify the Poisson’s ratio value that matches the experimental data. The comparison is performed, similarly to Socco et al. (2016), by associating to each depth the value of Poisson’s ratio that corresponds to the linear interpolation between the values of $\nu^+$ and $\nu^-$, where $\nu^+$ and $\nu^-$ are the values at the upper and lower nearest constant Poisson’s ratio W/D relationship, respec-

![Figure 2. (a) The synthetic $V_p$ and $V_S$ model (black and gray solid), the corresponding $V_{pS}$ and $V_{Sb}$ (black and gray dashed line), the estimated $V_{pS}$ (gray solid), and Rayleigh wave DC as a function of wavelength (gray dots). The vertical axis corresponds to $\nu$ for the velocity models and to wavelength for the DC; the horizontal axis corresponds to $V_S$ and $V_p$ for the velocity models and to the Rayleigh wave phase velocity for the DC. (b) The W/D relationship for the reference model of plot (a) (black) compared with the ones obtained with different Poisson’s ratio values (gray). Some reference Poisson’s ratio values are indicated on the right of the plot.](http://library.seg.org/)
Then, we use the estimated Poisson’s ratio \( \nu_z \) to calculate the \( V_{Pz} \) starting from the \( V_{Sz} \) using equation 3:

\[
V_{Pz} = V_{Sz} \sqrt{\frac{2(\nu_z - 1)}{2\nu_z - 1}}
\]

We compare the estimated and true \( V_{Pz} \) models in Figure 2a. In this case, the retrieved Poisson’s ratio has a constant value with depth that corresponds to the true value, and hence, the estimated and true \( V_{Pz} \) are exactly coincident.

In Figure 2, we have shown that the W/D relationship is sufficiently sensitive to Poisson’s ratio to identify the true Poisson’s ratio and estimate \( V_{Pz} \). This sensitivity is the result of the link between Poisson’s ratio and the investigation depth. This can be shown by considering the Rayleigh wave vertical displacement pattern with depth. In Figure 3a, we show the normalized vertical displacements of the fundamental mode of Rayleigh wave propagation computed for the reference \( V_S \) model and different values of Poisson’s ratio, for a frequency equal to 6 Hz. The vertical displacements are computed using the code implemented by Maraschini (2008) and based on the work of Herrmann (2002) who uses the method introduced by Thomson (1950) and Haskell (1953) and modified by Dunkin (1965). It can be seen that the propagation spans different depths, and for a higher Poisson’s ratio, the investigation depth is higher.

The link between W/D relationship slope and Poisson’s ratio is shown by plotting the normalized amplitude of vertical displacements at different depths for each propagating wavelength. This is shown in Figure 3b–3f for the \( V_S \) reference model in Figure 2a and different constant values of Poisson’s ratio. As already shown in paper 1, the W/D relationships roughly fall within the zone where the amplitude of the displacements becomes negligible (approximately 10%). Here, we observe that this boundary clearly changes according to Poisson’s ratios and that the slope decreases when Poisson’s ratio increases.

Thus far, we have used a simple example with a constant Poisson’s ratio value for the whole model to show the sensitivity of the W/D relationship toward Poisson’s ratio. In the following, we present three examples of 1D models with variable Poisson’s ratio with depth and growing complexity. The first two models are in the range of Poisson’s ratio values typical of dry sediments or soft rocks: Poisson’s ratio is equal to 0.33 in the first four layers and
is equal to 0.2 for the deeper layers. In the first model \( V_S \) always grows with depth, while in the second model there is a velocity inversion at the interface corresponding to Poisson’s ratio change. The third synthetic model is more challenging and has Poisson’s ratio varying in each layer ranging from 0.2 to 0.45. In Figure 4, we report the results of our analyses for the three models. In

Figure 4a–4c, we show the \( V_p \) models with the corresponding \( V_{pz} \) and the relevant DC plotted as a function of wavelength.

In Figure 4d–4f, we report the experimental W/D relationship of each reference model, compared with the W/D relationships computed with constant Poisson’s ratios. We estimate Poisson’s ratio with depth as explained above. The estimated \( \nu_z \) for the different

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**Figure 4.** Synthetic examples with variable Poisson’s ratio with depth: (a–c) The synthetic \( V_p \) models (black solid), the corresponding true \( V_{pz} \) (black dashed line), the estimated \( V_{pz} \) (gray solid), and Rayleigh wave DC as a function of wavelength (gray dots), the vertical axis corresponds to \( z \) for the velocity models and to wavelength for the DC; the horizontal axis corresponds to \( V_p \) for the velocity models and to the Rayleigh wave phase velocity for the DC. (d–f) The W/D relationships for the reference models (black) compared with the ones obtained with different Poisson’s ratio values (gray); some reference Poisson’s ratio values are indicated on the right of the plots. (g–i) The estimated (gray) and true (black) apparent Poisson’s ratios; for the example in plot (i), Poisson’s ratio values of each layer are also reported.
models are reported in Figure 4g, 4h, and 4i and compared with the Poisson’s ratio profile obtained from true $V_{Sz}$ and $V_{Pz}$ values at each depth using equation 4:

$$\nu'_{z} = \frac{1}{2} \left( \frac{V_{Pz}}{V_{Sz}} \right)^2 - 1.$$

We call this $\nu'_{z}$ the “apparent” Poisson’s ratio. Equation 4 is the same as equation 2, where $V_{P}$ and $V_{S}$ are substituted with the time average $V_{Pz}$ and $V_{Sz}$. The estimated apparent Poisson’s ratios are finally used to transform the $V_{Sz}$ profile into $V_{Pz}$ using equation 3 and compared with the true $V_{Pz}$ (Figure 4a-4c).

It is worth noting that for the first two examples, the true apparent Poisson’s ratio corresponds with the layered Poisson’s ratio in the upper part of the profile in which Poisson’s ratio is constant, and then, it tends to the new value without reaching it in the considered investigation depth. Therefore, an abrupt change in the Poisson’s ratio of the layered model corresponds to a smooth variation in the apparent Poisson’s ratio. In these two synthetic examples, the estimated Poisson’s ratio correctly retrieves the value in the uppermost portion of the model and then also tends to the lower value in the deeper portion. Nevertheless, the transition between the two apparent Poisson’s ratio values is slightly oversmoothed. This effect is stronger in the second model that presents the velocity inversion. For the third example, again, the sharp variation in the layered Poisson’s ratio corresponds to a smooth transition of apparent Poisson’s ratio (Figure 4i); it can also be observed that in this situation, the estimated apparent Poisson’s ratio shows a reduced capability of evidencing this transition.

In spite that the estimated Poisson’s ratio is smoother than the true apparent Poisson’s ratio, the resulting errors in the $V_{Pz}$ are small. In Table 1, we report the values of the estimation error on the velocity for the three synthetic cases. The greatest error is found at 20 m of depth for the profile of the third case, for which the average error is less than 4%.

We have shown that using the W/D relationship in the case of a constant Poisson’s ratio with depth, it is possible to recover it and to provide a very accurate estimate of $V_{P}$. In the case of a variable Poisson’s ratio with depth, we have introduced the concept of “apparent” Poisson’s ratio and we have shown that the Poisson’s ratio variability can be recovered from the W/D relationship. Nevertheless, with the proposed approach, the estimate shows uncertainty in the transition zone that produces low errors in the $V_{Pz}$. This method is, to our knowledge, the first presented method that is able to estimate the $V_{Pz}$ of a 1D profile from SW data with this level of accuracy.

In paper 1, we showed that the W/D relationship estimated for one known $V_{S}$ model and the corresponding DC can be applied to a set of DCs extracted along a seismic line to directly estimate the $V_{Sz}$ for the whole data set. We also showed that this method is able to reconstruct the $V_{S}$ lateral variability at the site. Here, we have shown that the same W/D relationship is sensitive to Poisson’s ratio variability with depth and can be used to obtain an apparent Poisson’s ratio profile that allows the $V_{Sz}$ model to be converted into a $V_{Pz}$ model. In Socco et al. (2016), we have extended this approach to estimate $V_{Pz}$ models along the whole seismic line starting from the estimated $V_{Sz}$ models and the estimated apparent Poisson’s ratio for the reference profile. Using a synthetic data set, we have shown that the $V_{Pz}$ of all the profiles could be recovered. Nevertheless, in those synthetic models, the Poisson’s ratio was chosen to be laterally constant within each layer. We think that this assumption can be transferred to the real world with some care. As we stated in the “Introduction” section, it is reasonable to expect a stronger variability of Poisson’s ratio along the vertical direction than along the horizontal direction. In particular, we expect that if a significant lateral variation in Poisson’s ratio occurs, this is likely to correspond to a variation in both $V_{P}$ and $V_{S}$ and will therefore show up in the DC trend along the line. In this case, we can split the data set into subdata sets in which we assume negligible lateral variations in the Poisson’s ratio and we estimate the Poisson’s ratio for each of them. It is worth noting that when Poisson’s ratio is expected to vary significantly, the method could also be applied to each individual DC curve along the line. In the following, we apply the method to the two field data sets used in paper 1. The first is processed assuming negligible lateral variations in the Poisson’s ratio, and through the second field case we address the lateral variability of Poisson’s ratio.

### FIELD CASES

The two data sets are the same used in paper 1. For a brief description of their geological setting...
and of the SW data, please refer to paper 1 and the references therein. We here focus on the P-wave data and on the estimation of the apparent Poisson’s ratio for the reference profiles. Then, we use it to transform the $V_S$ models along the line into $V_P$ models. We compare the $V_P$ estimated only by the use of SWs to benchmark results obtained on the two data sets using P-wave traveltomeography and P-wave downhole tests. As stated in paper 1, where we compared our results with those obtained through DC inversion, we are conscious that P-wave tomography results cannot be considered as the ground truth. Nevertheless, P-wave tomography is an accepted method for near-surface P-wave velocity estimation, and being a 2D approach based on P-wave data, we think that showing that our method is able to replicate tomography results is a significant validation. Moreover, for the two field cases, the dense receiver and source layout provided a very large amount of traveltime data and the maximum offset (approximately 500 m) guaranteed a very high coverage (thousands of rays/cell) within the depth of interest for the present work (see details in Socco et al., 2008). We hence assume that the tomograms can be used as a reliable benchmark.

Field 1: La Salle

The site is located in the Alps, on the alluvial fan of La Salle (Northwest Italy). The deposit was the object of a geophysics campaign in 2007, and details about the site and DC data can be found in paper 1 and in Socco et al. (2008). The P-wave traveltimes were picked from two orthogonal high-resolution P-wave seismic reflection lines and used to estimate near-surface $V_P$ models using a commercial traveltime tomography code (see Socco et al. [2008] for details). Here, we focus on the DCs and P-wave tomogram along line 1. As already specified in paper 1, the DC data set has a quite good quality with a dense set of 60 smooth and continuous DCs along the line. The trend of the DCs along the line did not evidence significant variations. We used the same reference model and W/D relationship used in paper 1, which is not part of the data set, but had been obtained through Monte Carlo inversion of one of the broadband DCs from the merging of SW active and passive data (located at site A in Socco et al., 2008). For the inversion of the reference DC, we used a Monte Carlo inversion algorithm (Socco and Boiero, 2008) in which model parameters are randomly sampled from uniform distributions. Usually, only $V_S$ and layer thickness are free to vary whereas the density and Poisson’s ratio are fixed a priori. Here, instead, we defined a wide model space also for Poisson’s ratio with boundaries from 0.2 to 0.49 for each layer. This was done to avoid that Poisson’s ratio of the reference $V_S$ model could be biased by the a priori assumption of a fixed Poisson’s ratio. In the Monte Carlo results (not shown), as expected, the sensitivity to $V_S$ strongly prevails and the accepted models have a very narrow $V_S$ range. On the contrary, Poisson’s ratio cannot be identified from the accepted models because the obtained values span the whole model space range.

In Figure 5a, we compare the W/D relationship obtained for the reference model with the W/D relationship obtained by simulating DCs for the reference $V_S$ model at constant Poisson’s ratio values. The W/D relationship was used in paper 1 to directly transform all the DCs along the line into $V_S$ models. Here, we have used the same W/D relationship to estimate the apparent Poisson’s ratio for the reference model and we have applied it to transform all the $V_S$ models along the line into $V_P$ models. We have, therefore, assumed

![Figure 6. Field case 1: (a) $V_P$ obtained from P-wave traveltime tomography and DHT results, (b) $V_P$ obtained using the estimated apparent Poisson’s ratio and the $V_S$ estimated in paper 1 and DHT results, the location of the reference model (site A) is indicated by the arrow. (c) Normalized difference between the two $V_P$ models and DH results. The black crosses represent the DC data points of all the DCs plotted as a function of wavelength. Please notice that here the normalized difference range is higher than in paper 1. d) One-way time along the line for datum plan depth reported in panels (a and b) for tomographic and approximated $V_P$.](image-url)
that the lateral variation in Poisson’s ratio was negligible along the line. In Figure 5b, we show the estimated apparent Poisson’s ratio for the reference model.

We show the results in terms of $V_{Pz}$ along the line in Figure 6 compared with the $V_{Pz}$ models obtained by transforming the $V_P$ model from P-wave traveltome tomography into a time-average $V_{Pz}$. We show the normalized difference between the two results, and consistently with paper 1, we superimpose to the difference plot the data points of the experimental DCs as a function of wavelength. We also report the results of the local downhole tests (DHT) in terms of $V_{Pz}$ using the same color scale used for the velocity sections and the normalized difference of our results with respect to the DHT. In Figure 6d, we compare the one-way time (static shift) at a floating datum at 55 m depth along the line computed using the tomography results and our results.

The difference along the 2D section is within 10% for most of the estimated values. There are some higher difference values at very shallow depths in which the DC data points have no coverage. The slight lateral variability is properly reconstructed as already pointed out in paper 1 for $V_{Sz}$. Our results are also in better agreement with the DHT with respect to tomography data. The static shifts computed at a depth of 55 m (below the top of the first high-velocity layer) show maximum difference of the order of 5 ms and provide very similar trends. This field case shows that for good-quality data and smoothly laterally varying sites, the proposed direct estimation of P-wave time-average velocity can be conveniently applied.

Field 2: Torre Pellice

The site is located in an alpine valley in the town of Torre Pellice (northwest of Italy). The deposit was the object of a geophysics campaign in 2007, and details about the site, data acquisition and processing can be found in paper 1, in Socco et al. (2009), and in Boiero and Socco (2014). In this case, the benchmark is a tomographic P-wave velocity model that was obtained through a joint inversion with DCs by Boiero and Socco (2014). On this site, there is a significant lateral variability because the seismic line crosses two different geologic environments: the zone close to the river (approximately from 0 to 250 m) in the south and the zone over an alluvial fan in the north (from 320 m to the end of the line). Consistently with paper 1, we used two reference models: one from the north zone and one from the south zone, and we estimated two W/D relationships to be applied in the two zones. As specified in paper 1, for this data set, the data quality is critical: The DCs are discontinuous and noisy, with gaps in significant frequency bands. The reference DCs have been chosen as a compromise among frequency band, smoothness, continuity, and representativeness.

We performed the same analysis shown for field case 1. The W/D relationships for the reference models were estimated in paper 1. The reference DC curves were chosen among the data set, and the reference models were the corresponding $V_S$ models obtained through the joint inversion (Boiero and Socco, 2014). The W/D relationships of these models were used in paper 1 to directly transform the DCs of the two zones along the line into $V_{Sz}$ models.
In Figure 7a and 7b, we compare the W/D relationships obtained for the reference models with the W/D relationships obtained by simulating DCs for the reference $V_S$ models at constant Poisson’s ratio values. We used the W/D relationships to estimate the apparent Poisson’s ratio for the reference models, and we applied them to transform all the $V_S$ models along the line into $V_p$ models. We show the estimated apparent Poisson’s ratios for the reference models in Figure 7c and 7d. The patterns of the apparent Poisson’s ratio with depth in the two zones are different: In the south, the apparent Poisson’ ratio reaches very high values at shallow depths, whereas in the north, the apparent Poisson’s ratio assumes lower values. For the north zone, an abrupt variation in the estimated apparent Poisson’s ratio at shallow depths can be observed. This can be related to the frequency gap contained in the reference DC (see Figure 10 in Socco et al., 2017).

We show the results in terms of $V_p$ along the line in Figure 8 compared with the $V_p$ models obtained from P-wave tomography. We show the normalized difference between the two results in Figure 8c, and, consistent with paper 1, we superimpose the data points of the experimental DCs as a function of wavelength. Also in this case, we report the DHT results and compare them with the tomography and our results.

The poor data coverage of the DC points has a significant impact on the quality of the final result that presents some zones of high difference with respect to benchmark. The very high difference of the profile at 320 m can be related to an anomaly in the joint inversion results. The differences with respect to the DHT results are small (mostly less than 10%) in the north zone, whereas they show a similar trend of joint inversion data in the south zone. In Figure 8d, we show the one-way time along the line for the datum plan indicated in Figure 8a and 8b (located at a depth of approximately 45 m in the north zone and 30 m in the south zone) computed from joint-inversion results and our direct estimation results. Also in this case, the trends are in good agreement and the difference is mostly within 5 ms. However, a few zones with higher differences can be observed. In Figure 8d, we also report the DHT traveltimes at datum plan depth; in the north zone, the DHT traveltime is in between our estimation and joint-inversion data.

**DISCUSSION**

We have proposed a new method to estimate the time average $V_p$ ($V_{p2}$) from SW dispersion data along a seismic line. The method relies on the knowledge of one $V_S$ model that is used, together with its corresponding DC, to estimate the experimental W/D relationship. The W/D relationship is then used to estimate the apparent Poisson’s ratio with depth by comparison with theoretical W/Ds for the same reference $V_S$ model at different Poisson’s ratio values. The same W/D relationship was used in paper 1 to estimate the $V_S$ along the line and, by using the estimated apparent Poisson’s ratio, the $V_S$ is transformed here into a $V_{p2}$ model. The $V_{p2}$ can then be used to directly compute the one-way time at the desired datum plan depth within the investigation limit.

Using 1D synthetic models, we showed that, if Poisson’s ratio is constant with depth, the Poisson’s ratio value retrieved with this method is the correct one. In the case of a Poisson’s ratio that varies with depth, we have introduced the apparent Poisson’s ratio concept that relates, at each depth, the $V_p$ models and $V_p$ estimated using the W/D relationship and the true apparent Poisson’s ratio show a smooth variability. Furthermore, the $V_{p2}$ value estimated has maximum errors less than 15% with respect to the true one, even in challenging situations. As the depth increases,
the reciprocal distance among the W/D relationships at a constant Poisson’s ratio increases as well. This means that the sensitivity to Poisson’s ratio increases with depth (see for instance Figures 2b, 4d, 4e, and 4f), and hence the estimation improves at greater depths. In particular, the synthetic examples show that the estimation of $V_{p_z}$ becomes very accurate below the bedrock: For the first two synthetic models, the error of the estimated P-wave one-way time at 70 m is 0.15 and 0.44 ms, respectively, whereas for the more challenging third model, it is 5.5 ms (still near the time sampling for seismic exploration data). The synthetic 1D cases have evidenced that our direct estimation tends to oversmooth the apparent Poisson’s ratio profile with respect to the true profile and that this effect is more significant for large contrasts of Poisson’s ratio among layers (such as in the third synthetic case in which Poisson’s ratio varies between 0.2 and 0.45). Nevertheless, the corresponding errors in the obtained $V_{p_z}$ are low (see Figure 4 and Table 1). The synthetic models used have been defined according to typical Poisson’s ratio values of near-surface materials reported in the literature. Nevertheless, the strong contrasts of our synthetic models are not very common according to our experience. In particular, in dry loose granular materials (such as in sand dunes), the value of Poisson’s ratio can easily be almost constant with depth and can assume typically values in the range 0.2–0.3 (Bachrach et al., 2000; Zimmer et al., 2007). Hence, even the simple example reported in Figure 2, that concerns a velocity profile with constant Poisson’s ratio with depth, can be significant. As we stated in the “Introduction” section, the required accuracy in the estimate of Poisson’s ratio increases for a higher value of Poisson’s ratio, hence in saturated media. For this case, it is interesting to analyze the pattern of Poisson’s ratio for the reference model in the south zone of field case 2, in which the W/D relationship below the water table tends rapidly to a value of almost 0.5. In this situation, even a slight overestimation of Poisson’s ratio can lead to large errors in the $V_{p_z}$. Hence, caution is required in the application of the proposed method in these situations.

Considering the application to individual velocity profiles, it is important to specify that the proposed approach is a very simple method to exploit the sensitivity of the W/D relationship to Poisson’s ratio. Other, more sophisticated methods than the proposed interpolation (for instance inversion) could be implemented to improve the accuracy of the apparent Poisson’s ratio estimate for the cases in which there are strong Poisson’s ratio contrasts with depth. Nevertheless, to our knowledge, this is the very first method described in the literature to directly estimate the P-wave velocity from SW DC without any additional information. So, the novelty of this work lies more in the new opportunities for P-wave velocity estimation than in the specific method we used.

Following the approach of paper 1 and synthetic tests performed by Socco et al. (2016), we used the apparent Poisson’s ratio estimated for one reference model to estimate the $V_{p_z}$ along a line where several DCs are available. This means that we assumed that the apparent Poisson’s ratio profile has negligible variations along the line. This hypothesis has been motivated in the “Introduction” section and is quite consistent in shallow sedimentary environments where vertical variability of Poisson’s ratio is mainly due to water table and/or bedrock top whereas lateral variability is mainly related to changes in the solid matrix properties. As already pointed out in paper 1, the extension to 2D using a single reference model for estimating the W/D relationship and the apparent Poisson’s ratio should be checked by analyzing the variability of the DCs within the data set. In our experience, if significant lateral variations exist along the line, they will be evident in the DCs. As an example, we refer to Konstantaki et al. (2013), in which a set of DCs is estimated from seismic data crossing a shallow fault. The DC trend clearly evidenced two different regions at the two sides of the faults, similarly to what happens for field case 2 in this paper.

Because for the two field cases we had independent $V_S$ models, from laterally constrained inversions of the DCs, and $V_P$ models, from the P-wave traveltime tomographies, we have used these results to compute the 2D apparent Poisson’s ratio along the survey lines. We compare these 2D apparent Poisson’s ratios from independent results with the one estimated with our method for the reference models in Figure 9. We also report in Figure 9 the apparent Poisson’s ratio profiles from the DHTs. The two field data sets are characterized by two different conditions in terms of lateral variations. In field case 1, Poisson’s ratio does not present significant lateral variations and the estimate of one Poisson’s ratio profile can be enough to provide a sufficiently accurate estimate of $V_{p_z}$. In the second field data set, the geologic setting originates two different zones with different kinds of sediments and water table depths and hence different characteristic Poisson’s ratio profiles. These two different zones are clearly evident in the DC data set that presents two well-identified trends (see Figure 10, paper 1). This suggests to use two reference models and to estimate two W/D relationships. In both field cases, our estimated apparent Poisson’s ratio compares well with the independent data (Figure 9); in the first meters of the zone north of field case 2, our Poisson’s ratio profile shows a greater contrast with respect to the 2D benchmark, but this trend is in better agreement with DHTs than the 2D benchmark. Another significant difference with respect to DHT results is in the first 10 m in the south zone of field case 2, in
which the DHT shows very high values of Poisson’s ratio up to the ground surface whereas both our estimates and the 2D benchmark show a lower value. Nevertheless, this difference is fully due to water table level variations. In fact, DHT was acquired in the spring (during the melting period) whereas the seismic line was acquired in the winter (during the dry period). The expected variability of water table level in the south zone is of several meters, contrary to what observed in the north zone where there is very limited seasonal change.

In the two presented field cases, the reference $V_S$ models were estimated with no bias on Poisson’s ratio. In field case 1, the inversion of the DC used to estimate the reference $V_S$ was performed through a Monte Carlo inversion algorithm in which the Poisson’s ratio was set free to vary in a wide range (0.2–0.49). In field case 2, the $V_S$ reference model was estimated through the joint inversion of DC and P-wave traveltimes with no constraints on the values of the Poisson’s ratio of the initial model. This is very important because if the reference model is obtained through the inversion of a DC using a priori fixed Poisson’s ratio value, this will affect the estimate performed using the W/D relationship.

It is important to stress that the proposed approach consists in a data transform: The knowledge of one $V_S$ model along the line is used to estimate the W/D relationship that represents the function for data transform from DC into $V_{Sz}$. The W/D relationship is then used to estimate the apparent Poisson’s ratio that represents the function for the data transform from $V_{Sz}$ into $V_p$. Hence, the method is completely data driven and this also affects the final uncertainties. The uncertainties of the estimated $V_{Sz}$ model have been extensively discussed in paper 1. Here, we focus on the uncertainties on $V_p$, which are a function of the uncertainty on $V_{Sz}$ and those on the apparent Poisson’s ratio. The two field cases have clearly shown a dependency of the result quality to the data quality and coverage. However, in spite of all the approximations involved in the method, the final $V_p$ sections appear to be a good estimation when compared with the benchmark obtained through P-wave traveltomeography. In particular, field case 1 shows a difference within 10% with respect to the benchmark and case 2 shows similar differences in most of the zones with good data coverage. Our results are also in good agreement with the ones of DHTs, and the velocity models and traveltimes directly picked from borehole data tend to be in better agreement with our results than with tomograms. It has also to be noted that the used benchmarks do not represent the ground truth, but these are the result of an inversion process and, hence, are affected by an unknown uncertainty that is not considered here. Consistently with paper 1, however, we also used the P-wave traveltome inversion results as a reference for the computation of one-way time. We have shown (Figures 6d and 8d) that the static shift computed from the velocity models obtained through inversion and those obtained from the direct estimation have similar spatial trends and have differences mostly less than 5 ms. Moreover, for field case 2, the comparison of one-way times obtained with our approach with the available DHT traveltimes (Figure 8d) showed that our estimate is in better agreement with the DHT and that both are slightly higher than those obtained from tomography, particularly in the north zone. In Figure 10, we show the difference between the P-wave one-way time for the two sections in Figures 6 and 8. For field case 1, for a floating datum plan at a depth of 55 m, most of the one-way time values (80%) present a difference with a benchmark of less than 2 ms. Hence, they can be considered of the same order of typical time sampling for seismic exploration data. For field case 2, in which the data quality is critical, for a fixed datum plan (at 45 m depth in the north zone and 30 m in the south zone), the difference is greater and the $V_p$ directly estimated with W/D relationship shows a globally lower value with respect to P-wave traveltime inversion. This reflects in an overestimation of the one-way time. Nevertheless, this overestimation is partly a consequence of the overestimation of $V_{Sz}$ shown in paper 1, and our results are supported by DHT data.

It is important to stress that the two field data sets were acquired for high-resolution shallow seismic investigations. Light sources and dense spatial sampling were used in both cases. This produced high-frequency data that resulted in short wavelengths compared with data normally acquired in hydrocarbon exploration. This limited the investigation depth to few tenths of meters, and in neither of the two cases the seismic models have reached the bedrock top. In the synthetic examples, we have shown that the estimation improves with depth, and this makes the results even more promising in the case of greater investigation depth than the one reached here.

This new method is very fast, even though approximated, way to directly estimate $V_p$ statics from SW dispersion data. As for the direct estimate of $V_S$ statics proposed in paper 1, inversion is not needed and only a local 1D $V_S$ reference model is required for the processing after the DCs are extracted from the data.

As stated in paper 1, because the analysis is performed on local 1D models, there is no difference in applying this method to 2D or 3D data set. The lateral resolution and investigation depth depend on the way DCs have been estimated and on the frequency band of the propagating wavefield. For optimal data processing and requirements on acquisition parameters, one can refer to Socco et al. (2010).
CONCLUSIONS

We have extended the work presented in paper 1 to the direct estimation of P-wave time-average velocity. We have shown that the relationship between SW DC wavelength and the time-average $V_S$ model depth, introduced in paper 1, is sensitive to Poisson’s ratio. We have shown that this sensitivity increases with depth, and it can be used to estimate Poisson’s ratio for models with constant and variable Poisson’s ratio with depth.

For a constant Poisson’s ratio with depth, the estimate provides the exact value. For a variable Poisson’s ratio, we have introduced the concept of the apparent Poisson’s ratio that relates $V_{S1}$ and $V_{P2}$, and we have applied it to the transformation of the $V_{S1}$ estimated in paper 1 into $V_{P2}$. We obtained synthetic and field data results with $V_{P2}$ uncertainty within 10% for good quality data. When lateral variations are depicted through the analysis of DC trends, we recommend to split the data set into subdata sets and to extract more than one W/D relationship and Poisson’s ratio estimation.

The method proposed in this paper, integrated with that proposed in paper 1 for the estimation of $V_{S1}$, represents a double data transformation that allows S- and P-wave statics at a datum plan to be estimated from SW DC with no need for inversion, within the investigation depth of SWs. Being a data transformation, the results of this method are fully data dependent. The investigation depth depends on the retrieved wavelength of SW data and Poisson’s ratio.

This represents the main limitation of the proposed method.

A possible further development of this work is the inversion of the W/D relationship to estimate Poisson’s ratio of the layered system with greater accuracy. Nevertheless, we think that if the task is the estimation of P-wave statics, the uncertainty obtained with the simple interpolation proposed here is low enough and that if the data quality is poor, inversion of W/D relationship will suffer from strong nonuniqueness.

Besides the computation of statics, this approach to estimate an approximated $V_P$ model from SW DCs also offers interesting possible developments for other processing needs, such as defining initial models for full-waveform inversions or complementing deep velocity models for any processing applications requiring accurate velocity starting models.

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