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(Article begins on next page)
Regulation versus Taxation*

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Abstract

We study which policy tool and at what level a majority chooses in order to reduce activities with negative externalities. We consider three instruments: a rule, that sets an upper limit to the activity which produces the negative externality, a quota that forces a proportional reduction of the activity, and a proportional tax on it. For all instruments the majority chooses levels which are too restrictive when the activity is performed mainly by a small fraction of the population, and when costs for reducing activities or paying taxes are sufficiently convex. Also a majority may prefer an instrument different than what a social planner would choose; for instance a rule when the social planner would choose a tax.

Keywords: Externalities, Voting, Rules, Taxation.

JEL Codes: H23, P16, D62, D72, L51, K32.

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1 Introduction

Three ways of reducing the level of an activity generating negative externalities are routinely used: a rule that sets an upper bound to this activity, a proportional tax on it, a compulsory proportional reduction of the activity for everybody.\footnote{A fourth way of curbing negative externalities are tradeable permits. We do not study them in the present paper, but we briefly discuss them in the Conclusion.} This paper investigates which policy and at what level would be chosen by majority voting. The latter does not deliver the optimal policy choice for two reasons. First, for given policy instrument, majority voting does not yield the optimal level of it. Second, and perhaps more interestingly, when choosing amongst alternative instruments, majority voting in general does not lead to the choice of the optimal one. For instance, the majority may choose a rule instead of a proportional tax because a rule concentrates on the minority the burden of the reduction of the activity which generates negative externalities. A social planner would instead choose a tax and, if he were constrained to choose a rule, he would choose one which was more permissive than the one chosen by the majority. We thus have a “double distortion” caused by voting. This case arises when those who generate a negative externality are a minority. The opposite double distortion occurs when the activity with negative externality is enjoyed by many. In this case a social planner may choose a restrictive rule, while a majority may choose a lenient tax.

These insights are consistent with the evidence that in many cases we observe regulation while the optimal policy would be taxation, or vice versa. For example, in agriculture the limits in the use of pesticides are quite frequent whereas taxes on them are less common. In the case of air pollution, there is a sharp contrast between the use of taxes and the use of emission standards. The latter are preferred when polluters are concentrated in specific industries or plants, such as emissions of pollutants by power generation industries or by steel and cement makers.\footnote{On November 22, 2010 the Wall Street Journal reported that since Mr. Obama took office, the US Environment Protection Agency (EPA) had proposed or finalized 29 major regulations and 172 major policy rules. Requiring energy sources to install the best available control technology to limit greenhouse gas emissions, would impose the electric industry costly capital expenditures to meet the increasingly strict burden.} Anti smoking regulations became very strict as the number of smokers declined. We see low levels of taxation when the polluters are the majority; for instance low
taxes on gasoline in the US or on heating fuel.\textsuperscript{3} Policymakers may choose quotas when tax collection is costly or simply impossible, or because they are perceived as a fair method of sharing the sacrifices of curbing externalities (e.g. international agreements, like the Kyoto protocol, or in many cases in the European Union).

We label our negative externality “pollution” for brevity. However our discussion of instrument choice applies to many other policy issues, which may include construction rules, speed limits, rules of behavior in communities like condominiums, prohibition (or very strict regulation) of certain activities, from gambling to selling of organs, to prostitution to free acquisition of guns and many others. Masciandaro and Passarelli (2013) apply the model of the present paper to discuss issues of financial regulation. Thus we believe that our model is sufficiently general to be applied to a variety of different cases. In some of those, the externality has the straightforward interpretation of monetary costs inflicted on others. In other cases, it may take the form of a negative “utility cost” inflicted on others, who engage in certain activities which they find objectionable, like gambling or prostitution. Baron (2003) claims that “moralistic” goals regarding how others should behave are prominent in how people vote. Roth (2007) in his discussion of organ exchanges argues that repugnance of certain transactions related to trades in organs, implies relevant social costs.

This is why we feel that it is appropriate to use a majority rule voting model. Much of the literature on “pollution” strictly defined adopts lobbying models, as discussed in the next section. While lobbying pressures are clearly important, especially for legislation which affects one particular sector, clearly decisions regarding the list of activities mentioned above, from smoking to gun control etc., involve voting in legislatures or even in private associations, e.g. owners’ associations. Our contribution is on the voting aspect of the issue at hand, future research could merge the two approaches, lobbying and majority voting.\textsuperscript{4} Take, for instance, smoking regulations. Clearly the decision regarding smoking age, taxation

\textsuperscript{3}According Parry and Small (2005) the optimal gasoline tax in the US is $1.01/gal, more than twice the current rate. Lin and Prince (2009) find that for California this tax should be $1.37/gal (over three times the current level). The International Center for Technology Assessment computed that indirect costs to society total around $12/gal. ($3.17 per liter; cf. www.icta.org).

Parry and Small (2005) also find that the gasoline tax is above the socially optimal level in the UK.

\textsuperscript{4}For a model which incorporates voting and lobbying, although not about externalities and instrument choice, see Alesina and Tabellini (2008).
over cigarettes etc., is influenced by the lobby of the tobacco industry. But the fraction of
individuals smoking will also influence the legislative choice regarding regulation and taxation
of smoking. The same applies to gun control: the gun lobby is strong but different states in
the US have different regulations as a function of the preferences of the voters.5

We should make clear from the outset that we consider only proportional taxes on the
polluting activities. By allowing any type of curvature on the tax schedule, including cor-
ners, one could reproduce patterns which approximate a rule, and are quite far from the
allocation generated by a proportional tax. In a “positive” politico economic model we need
to worry about the existence of a Condorcet winner. While we can prove its existence with
a proportional tax, in general one cannot do that with any curvature of the tax schedule.
Thus all of our positive results would be interpreted as comparing rules and quotas versus
a proportional tax on the polluting activities. Realistically speaking these are the kind of
policies routinely discussed in this area. We briefly return to this issue in the conclusions.

The present paper is organized as follows. In Section 2 we review the relevant literature.
In Section 3 we set up the basic model of the activity which produces negative externalities.
Then we study the majority vote equilibrium when the policy instrument is a rule (Section
4) a quota (Section 5) and a tax (Section 6). In Section 7 we study the choice of the policy
instrument by majority voting. Section 8 concludes and illustrates several extensions of the
model. All the proofs are in Appendix.

2 Review of the literature

The dilemma between regulation and taxation is old in the literature, but it has been tradi-
tionally posed in a normative context. The idea that the two instruments perform differently
when uncertainty regards either costs or benefits dates back to Weitzman (1974).

The literature which introduces political economy considerations in this area is confined
to environmental issues.6 Buchanan and Tullock (1975) compare environmental taxes with
a proportional reduction of polluting activities, which they define “regulation”. There is no

5See Knight (2013) for a discussion of the efficacy of such regulations.
6For a survey in favor or against environmental taxes and quantitative regulations, with some reference
to political economy issues, see Hepburn (2006).
voting stage or any specification of the political process in their work. They offer several arguments in favor of taxes, but they claim that people are more likely to prefer proportional reductions. Congleton (1992) focuses on how political institutions affect the enactment of environmental regulations. Schneider and Volkert (1999) claim that differentiated interests between voters, politicians, interest groups and bureaucrats may lead to suboptimal instrument choice or to inefficient implementation.

We share an interest in the connection between redistributive policies and regulation with Coate and Morris (1995), who claim that inefficient environmental policies are frequently adopted as redistribution schemes even when more efficient redistribution instruments are available. Fredriksson and Sterner (2005) argue that “clean” firms may, somehow surprisingly, according to their analysis, lobby in favor of higher taxes in order to benefit from larger refunds. Our majority voting model yields a similar result when the polluters are a minority in the society. In fact this result is not surprising in a voting model.

Cremer, De Donder and Galvari (2004) study the efficiency of majority voting on an environmental tax when the proceeds of the latter are used to reduce income and capital taxes. If labor and capital taxes are rebated in the same proportion, the majority chooses an environmental tax which is too low.7 MacKenzie and Onhorf (2012) argue that the distributional conflict is harsher with revenue-rising instruments (e.g. ecotaxes or tradeable-permit auctions) than with non-revenue-raising instruments. In Kawara (2012) voters cannot observe the type of politicians and the environmental damage. In a pooling equilibrium, pro-industry politicians implement too low taxes in order to please polluters; in a separating equilibrium pro-environmental politicians choose too high the tax in order to signal their stand. Aidt (2010) argues that when income taxation is highly distortionary and the political environment is highly competitive, the polluter group lobbies in favor of refunding all ecotax revenues to citizens-voters.

A related strand of the environmental policy literature studies the instrument choice.8 We share with this literature the idea that, whenever regulation and taxes are available policy options, majorities may prefer regulation to taxes even when the latter would be

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7 In a related work they consider the role of militants and opportunists within political parties (Cremer, De Donder and Galvari, 2008).  
8 For a survey, see Aidt (2013).
socially optimal (cf. Keohane et al., 1998). Dijkstra (1998) claims that in the presence of rent seeking taxes or other financial instruments are rarely applied in environmental policy. Damania (1999) shows that emission standards are more frequent when interest groups are at work, whereas emission taxes are more likely when parties represent environmental interests. In fact, we show that majority voting yields a different result: a majority of low polluters has stronger incentive to adopt a strict standard, whereas a majority of large polluters would be better off with a tax. Aidt and Dutta (2004) study the transition from command-and-control instruments, usually adopted when environmental targets are lax, towards either an emission tax or tradeable permits. The latter are supported by the lobby of polluting firms, the former is preferred by citizens, interested in tax rebates. We do not study neither lobbies nor tradeable permits. However, a general insight is that as far as political distortion is concerned, lobby models yield opposite results with respect to our majority voting model. For example, a highly interested minority of polluters can be very efficient in lobbying self-interested politicians, and this can improve welfare. By contrast, under majority voting, the minority is without any defense. As a result, political distortions under majority voting can be larger than under lobbying, especially if the polluters (or the polluted) are quite a small minority.

Our work is also related to a small literature which studies how voting rules affect environmental policies. Fredriksson et al. (2010) look at the efficiency of an environmental tax that is implemented by a federal legislature. Boyer and Laffont (1999) look at the optimal level of flexibility that should be delegated to the majority. Different majorities have different stakes in the rents of a polluting monopolist, and there is asymmetry in information. Fluctuating majorities determine excessive fluctuation in environmental policy. Thus constitutional constraints may be desirable.
3 The model

Our model is in the tradition of the political economy literature on redistributive fiscal policy;\textsuperscript{9} but we focus upon rules and externalities. Consider a society with a continuum of individuals/voters of size one; each individual has an exogenously given location in the interval $[0, 1]$. Define those locations “types”: $t_i$ for individual $i$. $t_i$ represents the behavior that $i$ can assume at no cost. A behavior different from $t_i$ entails for $i$ an “adjustment cost”, which depends on the distance between type $t_i$ and his behavior, denoted by $b_i$. Types and behaviors are constrained in the unit interval: $t_i, b_i \in [0, 1]$. We can think of $t_i$ as the level of the activity that maximizes profits (in case of a firm) or utility (in case of a consumer).

The adjustment cost function, $c$, is the same for all individuals:

$$c(|b_i - t_i|)$$

(1)

with $c(0) = 0$ and $c'(0) = 0$; $c(.) > 0$, $c'(.) > 0$, $c''(.) > 0$, $\forall b_i \neq t_i$.\textsuperscript{10} Let $\varepsilon(b_i)$ be the social damage produced by an individual with behavior $b_i$, with $\varepsilon'(b_i) > 0$ and $\varepsilon''(b_i) > 0$. If we denote with $G(b)$ the cumulative distribution of behaviors, the total (per capita) loss is:

$$-\int_0^1 \varepsilon(b)dG(b)$$

(2)

For any behavioral profile $G(b)$ everyone receives the same externality. The utility of individual $i$, $U_i$, is given by the difference between the total externality received and the private adjustment cost:

$$U_i(G(b)) = -\int_0^1 \varepsilon(b)dG(b) - c(t_i - b_i)$$

(3)

As it will become clear later an individual never has incentive to choose a behavior which is higher than his type. This is why we suppressed the absolute value sign in the cost function

\textsuperscript{9}This literature was pioneered by Roberts (1977), Romer (1977) and Meltzer and Richard (1981). For a survey, see Persson and Tabellini (2000).

\textsuperscript{10}The simplifying assumption that $c'(0) = 0$ avoids corner solutions, but does not alter in any way the nature of our results.
of (3). Thus, hereafter $t_i \geq b_i, \forall i$. Each individual is infinitely small, thus he does not take into account his own effect on the aggregate level of negative externality. Then if $F(t)$ is the cumulative distribution of types, in an unregulated economy without policy intervention,

$$U_i(F(t)) = -\int_0^1 \varepsilon(t) dF(t)$$

There is scope for policy intervention: we consider in turn rules, quotas and taxes.

4 Voting on a rule

A rule fixes an upper bound, $\rho$, to the behavior of all individuals. The timing is as follows: first, individuals vote on the rule in pairwise comparisons, and then they choose their behavior as a function of the rule chosen by majority rule, namely $\hat{\rho}$ (fully enforced). All types higher than $\hat{\rho}$ have to adjust and pay the cost; all types below (or equal to) $\hat{\rho}$ can adopt their preferred behavior at no costs. Any individual knows that, by voting for a rule $\rho$, he affects the behavior of $1 - F(\rho)$ individuals whose types are above $\rho$, and can benefit from the reduction of their negative externalities. However, if $\rho$ is lower than his type, he has to bear a private adjustment cost. The individual indirect utility function can be then written as

$$V_i(\rho) = -\varepsilon(\rho) \cdot (1 - F(\rho)) - \int_0^{\rho} \varepsilon(t) dF(t) - I_i(\rho) \cdot c(t_i - \rho)$$

where $I_i(\rho) = 1$ if $t_i > \rho$ and $I_i(\rho) = 0$ if $t_i \leq \rho$. The first term in the RHS of (4) is the externality produced by all the affected individuals (i.e. those with $t > \rho$); the second term is the externality produced by the non-affected individuals below $\rho$; the third term is $i$’s private compliance cost, which is different from zero only if $t_i > \rho$.

Call $\rho_i^*$ the most preferred rule, or $i$’s bliss point. If $\rho_i^* \in (0, t_i)$, it solves the following FOC:

$$-(1 - F(\rho)) \cdot \varepsilon'(\rho) = -c'(t_i - \rho)$$

Equation (5) shows that the most preferred rule for voter $i$ is the one which equalizes the marginal benefit of affecting the behavior of $1 - F(\rho_i)$ individuals to the marginal cost due
to complying with the rule. Nobody would prefer a rule higher than his type. Moreover, because of $c'(0) = 0$, an individual always prefers a rule that is strictly lower than his type: $\rho_i^* \in [0, t_i)$. If $F(.)$ is sufficiently “smooth” overall, the SOC is satisfied.\footnote{The second order derivative of $V_i(\rho)$ is $-\varepsilon''(\rho_i) \cdot (1 - F(\rho_i)) + \varepsilon'(\rho_i) \cdot f(\rho_i) - c''(t_i - \rho_i)$. By “sufficiently smooth” we mean that the second term (which is positive) is small, so that the second order derivative is negative overall.} Thus any individual $i$ has a uniquely preferred rule and a Condorcet winner exists (Black, 1948). Moreover, lower types prefer lower rules:

**Lemma 1** For any two individuals $i$ and $j$, if $t_i > t_j$, then $\rho_i^* \geq \rho_j^*$.

Call $t_m$ the median type and let $\rho_m^*$ be his bliss point. By Lemma 1, under majority rule the voting outcome, $\hat{\rho}$, is the bliss point of the median type: $\hat{\rho} = \rho_m^*$.

The socially optimal rule $\rho^*$ in general differs from the voting outcome, $\hat{\rho}$. In fact $\rho^*$ maximizes the following social welfare schedule, $W(\rho)$, that is the sum of all individuals’ indirect utilities:

$$W(\rho) = -\varepsilon(\rho) \cdot (1 - F(\rho)) - \int_0^\rho \varepsilon(t)dF(t) - \int_\rho^1 c(t - \rho)dF(t)$$

If the solution is interior, the following FOC pins down $\rho^*$:

$$- (1 - F(\rho)) \cdot \varepsilon'(\rho) = -ac'(\rho)$$

\footnote{To be precise, $ac'(\rho) = \int_\rho^1 c'(t - \rho)dF(t) + c(t - \rho)f(\rho)$ represents the average marginal cost over the entire population.\footnote[12]{To be precise, $ac'(\rho) = \int_\rho^1 c'(t - \rho)dF(t) + c(t - \rho)f(\rho)$ represents the average marginal cost over the entire population.}}

where $ac'(\rho) = \int_\rho^1 c'(t - \rho)dF(t) + c(t - \rho)f(\rho)$ represents the average marginal cost over the entire population.\footnote[12]{To be precise, $ac'(\rho) > 0$ is the average cost of a marginal decrease in $\rho$.}

We label a rule as too restrictive if $\hat{\rho} < \rho^*$; it is too permissive if $\hat{\rho} > \rho^*$.

**Proposition 1**

i) The majority chooses a rule which is too restrictive if and only if at the point $\hat{\rho}$ the median’s marginal cost is lower than the average.  

ii) The majority chooses a rule which is too permissive if and only if at the point $\hat{\rho}$ the median’s marginal cost is higher than the average.
Two factors determine whether a rule is too restrictive or not and of how much. First, if the cost function is very convex, the median voters’ marginal cost may be substantially lower than the average. Second, if the median voter is a low type, lowering the rule is highly beneficial for him since he can affect the behavior of many individuals with a relatively limited personal adjustment cost.

5 Voting on a quota

We now analyze a policy which requires a reduction of the activity by a proportion $\theta \in [0, 1]$ that we call “quota”. Once $\theta$ has been decided by the majority, any individual $i$ has to lower his behavior from $t_i$ to $b_i = (1 - \theta) \cdot t_i$. Thus:

$$V_i(\theta) = -\int_0^1 \varepsilon((1 - \theta) \cdot t)dF(t) - c(\theta t_i)$$

(7)

$V_i(\theta)$ is concave, and each voter’s most preferred quota, $\theta^*_i$, is negatively related to his type. Under majority rule, the chosen policy is the one most preferred by the median: $\hat{\theta} = \theta^*_m$.

In the voting equilibrium,

$$a \varepsilon_{\hat{\theta}} = c_{\hat{\theta} t_m}$$

(8)

where $a \varepsilon_{\hat{\theta}} = \int_0^1 \varepsilon'((1 - \theta) t)dF(t)$ is the (positive per capita) marginal externality produced, after the quota has been enforced; $c_{\hat{\theta} t_m} = t_m \cdot c'(\theta t_m)$ is the median’s marginal cost.\footnote{Examples of quotas include the “20-20-20” plan of emission reduction adopted by the European Union in 2007 or the California’s tailpipe standards which require a 30% reduction in emissions from new cars by 2016.}

The social welfare function is:

$$W(\theta) = -\int_0^1 \varepsilon((1 - \theta) t)dF(t) - \int_0^1 c(\theta t)dF(t)$$

\footnote{We are considering interior solutions. There might be corner bliss points, $\theta^*_i = 1$, which are likely to concern low types, large externalities and low marginal costs.}
$W(\theta)$ is concave and the social optimum, $\theta^*$, satisfies:

$$a\varepsilon_{\theta}(\theta^*) = ac_{\theta}(\theta^*)$$  \hspace{1cm} (9)

Again, social optimum is reached where the marginal benefit from a quota, $a\varepsilon_{\theta}(\theta)$, equals the average marginal cost, $ac_{\theta}(\theta) = \int_0^1 c'(\theta t)tdF(t)$.

As for the case of rules we define a quota as too restrictive if $\theta > \theta^*$ and too permissive if $\theta < \theta^*$.

**Proposition 2** i) The majority chooses a quota which is too restrictive (too permissive) if and only if in equilibrium the median voter’s marginal cost is lower (higher) than the average marginal cost. ii) If the adjustment cost function is linear, a median in the average position chooses the social optimum.

### 6 Voting on a tax

We now examine a proportional tax on polluting activities (tax for brevity).\(^{15}\) Tax revenues can either be redistributed or used to provide public goods or to rebate other taxes. Here we analyze the case of lump sum redistribution. As above, voters first choose the optimal level of the policy (the tax rate in this case), then they choose their behavior and pay their taxes. Let $\tau$ be the tax rate ($\tau \geq 0$) so that the tax burden for individual $i$ is $\tau \cdot b_i$. The government budget is balanced: $\int_0^1 \tau \cdot b(t, \tau)dF(t) = \tau \cdot \bar{b}$, where $\bar{b} = \bar{b}(\tau)$ is the “after-tax” average behavior in the society. Each individual receives a transfer of $\tau \cdot \bar{b}(\tau)$. We assume that utility is quasi-linear in income.\(^{16}\) Thus the net cost that $i$ bears from paying the tax

\(^{15}\)Examples are the so-called ecotaxes, intended to promote ecologically sustainable activities. Environmental taxes target a broad array of bases (e.g. fertilizer, pesticides, plastic bags, landfill waste, batteries, etc.). Taxes on motor fuels and vehicles represent almost 90% of the revenue from environmentally related taxes in Europe. In the US motor fuel taxes remain substantially below the European levels. Revenues from federal environmentally related taxes represent 3.5% of total tax revenues, compared to an average of 7% for the OECD countries (OECD (2011): Database on Instruments Used for Environmental Policy and Natural Resources Management).

\(^{16}\)Under quasi-linearity there is no endowment effect of paying taxes. We do not include any individual income or wealth in the model since no result would change.
when his behavior is \( b_i \) is \( \tau \cdot (b_i - \bar{b}) \). Given a behavior profile \( G(b) \) and a tax \( \tau \), individual utility is:

\[
U_i = -\int_0^1 \varepsilon(b) dF(t) - c(t_i - b_i) - \tau \cdot (b_i - \bar{b})
\]

Individuals choose after-tax behavior in order to minimize costs: \( b_i \in \arg \min \left\{ c(t_i - \bar{b}_i) + \tau \cdot (\bar{b}_i - \bar{b}) \right\} \).

Therefore, individual behavior satisfies the optimality condition, \( c'(t_i - b_i) = \tau \),\(^{17}\) or:

\[
b_i = t_i - r(\tau)
\]  

where \( r(\tau) \equiv c'^{-1}(\tau) \), with \( r' \geq 0 \) and \( r'' \leq 0 \). Thus, i’s indirect preferences for the tax rate are:

\[
V_i(\tau) = -\int_0^1 \varepsilon(t - r(\tau))dF(t) - c(r(\tau)) - \tau \cdot (t_i - \bar{t})
\]  

where \( \bar{t} \) is the average type. The following FOC pins down agent i’s bliss point \( \tau_i^* \):

\[
a\varepsilon_r = c'_r + (t_i - \bar{t})
\]  

where \( a\varepsilon_r = \int_0^1 \varepsilon r' f(t) dt \) is the per capita private marginal benefit from externality reduction and the RHS is the private marginal cost of reducing behavior and paying (net) taxes. The SOC is satisfied thanks to the convexity of \( \varepsilon \) and \( c \), which ensure that \( V_i(\tau) \) is concave. By implicit differentiation of (12), we get \( \partial \tau^* / \partial t_i = (a\varepsilon_{\tau\tau} - c_{\tau\tau})^{-1} < 0 \).\(^{18}\) This means that higher types want lower tax rates. Bliss points monotonicity and the concavity of \( V_i(\tau) \) are sufficient to say that the voting equilibrium, \( \hat{\tau} \), is the median’s most preferred tax rate, \( \hat{\tau} = \tau_m^* \). In equilibrium,

\[
a\varepsilon_r = c_r + (t_m - \bar{t})
\]  

The policy benchmark maximizes the following social preference function, subject to (10)

\(^{17}\)The SOC is satisfied thanks to the convexity of \( c \). Here we assume that optimal after-tax behavior is interior for all \( i \).

\(^{18}\)Consider that \( a\varepsilon_{\tau\tau} = \int_0^1 (-\varepsilon'' r'^2 + \varepsilon' r''') f(t) dt < 0 \).
for all $i$: 19

$$W(\tau) = -\int_0^1 \varepsilon(b(\tau, t))dF(t) - c(r(\tau))$$

Of course also $W(\tau)$ is concave; thus the socially optimal tax, $\tau^*$, is computed from the following equation:

$$a\varepsilon_r = c_r$$  (14)

The social planner would choose a tax such that per-capita marginal benefits are equal to per-capita (or average) marginal costs. Compare (12) with (14). The voting equilibrium coincides with the social optimum when the median type is also the average type. A median that pollutes less than the average chooses too high a tax, and vice versa.

As for the case of a rule and a quota, we label a tax as too permissive if $\tau < \tau^*$ or too restrictive if $\tau > \tau^*$.

**Proposition 3** i) A tax is too restrictive if and only if the median type is lower than the average type. ii) A tax is too permissive if and only if the median type is higher than the average type.

Differently from the other two instruments, the convexity of adjustment costs does not affect the political distortion. Independently of $c''$, the policy outcome is optimal as soon as $t_m = \bar{\tau}$. This result, which perfectly parallels the classical result by Meltzer and Richards (1981) in public finance, is a consequence of quasi-linearity of the utility function. 20

### 7 The choice of a policy instrument

#### 7.1 Preliminaries: comparing instruments

##### 7.1.1 Taxes vs rules

A rule imposes all the costs of the reduction of the externality only on types above the threshold. A tax distributes these costs more evenly. In fact, for any amount of externality

19 Observe that by the balanced budget constraint, the average net cost of paying taxes is zero.

20Supplementary Material, available from the authors, presents a more general model in which taxes are used to provide public goods and preferences are not quasi-linear.
reduction, a low median pays relatively more with a tax than with a rule. As a consequence, a low median has a stronger tendency to prefer a restrictive rule rather than a tax. For the same reason, a high median has a stronger incentive to choose a tax that is socially too low. This incentive asymmetry yields the following:

**Proposition 4** In equilibrium, i) a tax cannot be too restrictive whenever a rule is too permissive; ii) a tax may be too permissive when a rule is too restrictive.

Proposition 4 yields the general idea that whenever a democratic society adopts rules instead of taxes, it does so because it wants to impose very restrictive behavior on a minority of “polluters”. This result holds for any given externality or cost function, and any distribution of types.

### 7.1.2 Rules vs quotas

The comparison of rules versus quotas is similar to that of rules versus taxes. If a majority chooses a quota that is socially too restrictive it cannot choose a rule that is too permissive. As in the previous subsection, rules that are imposed by majority are always more likely to be too restrictive compared to quotas.

**Proposition 5** In equilibrium, i) a quota cannot be too restrictive whenever a rule is too permissive; ii) a quota may be too permissive when a rule is too restrictive.

### 7.1.3 Taxes vs quotas

With a tax, the tax burden is shared proportionally, whereas with a quota the costs are more concentrated on high types. As a consequence, other things being equal, the median never chooses a quota that is too permissive when he would choose a tax that is too restrictive, or socially optimal.

**Proposition 6** In equilibrium i) a tax cannot be too restrictive whenever a quota is too permissive. ii) A tax may be too permissive when a quota is too restrictive.
7.2 The majority’s choice

The decision takes place in two stages. In the first stage a majority chooses the instrument; in the second stage a possibly different majority chooses the level. Voters in the first stage know that, whatever the instrument, the outcome of the second stage will be the level preferred by the median, i.e. $\rho^*_m$, $\theta^*_m$ or $\tau^*_m$. They compare their indirect utilities in those three cases, and choose which instrument to vote for. We show below that a Condorcet winner always exists.

7.2.1 Rule vs Tax

Individual $j$ prefers the rule if his utility is higher: $V_j(\rho^*_m) \geq V_j(\tau^*_m)$. Low types tend to prefer a rule. They have zero (or little) adjustments to make with a rule. Moreover a rule produces more benefits since it concentrates reductions on top polluters. Lemma 2 below shows that when the costs of the externalities are sufficiently convex then all types under a given level, that we call $t_1$, prefer the rule, and all types above $t_1$ prefer the tax. The intuition can be found in Figure 1.1, where $C(\rho^*_m, t)$ is the private cost of the rule as a function of type, and $C(\tau^*_m, t)$ is the cost of the tax,

$$
C(\rho^*_m, t) = \begin{cases} c(t - \rho^*_m) & \text{for } t > \rho^*_m \\ 0 & \text{for } t \leq \rho^*_m \end{cases} \\
C(\tau^*_m, t) = \tau^*_m \cdot t
$$

In the figure, a term $A$ has been added to the latter cost function. This term represents the difference between the benefits of the rule and the benefits of the tax (see Appendix for the definition of $A$). Since benefits are the same for all, $A$ is constant in $t$. If externalities are sufficiently convex, a rule is more effective in curbing externalities, thus $A$ is positive, a sort of opportunity cost that, if one chooses a tax, must be added to the cost of a tax.$^{21}$ Observe that $t_1$ is determined by the intersection between the two curves. Of course, if the median is to the left of $t_1$ the majority chooses the rule, otherwise the majority prefers the tax. Thus

$^{21}$By (4) and (11), and using the definition of $A$ in Appendix (cf. proof of Lemma 2), it is easy to see that $V_j(\rho^*_m) > V_j(\tau^*_m)$ if and only if $C(\rho^*_m, t) < C(\tau^*_m, t) + A.$
we will need to know if $t_1$ is above or below the median.

### 7.2.2 Rule vs Quota

Individual $j$ prefers a rule if utility is higher: $V_j(\rho_m^*) \geq V_j(\theta_m^*)$. Lemma 2 shows that if externalities are sufficiently convex, all types under a level $t_2$ prefer the rule and all types above that level prefer the quota. Thus, also when compared to a quota, a rule is preferred by low types. The reason is the same, a rule forces top polluters to drastic reductions and it is cheap for low polluters.

Figure 1.2, shows that $t_2$ comes out of the intersection between the cost of the rule, $C(\rho_m^*, t)$, and $C(\theta_m^*, t) + B$, which is the cost of the quota, plus a constant $B$ that represents
the additional benefits of the rule.\footnote{Specifically, \( C(\theta_m^*, t) = c(\theta_m^* \cdot t) \) and \( B \) is defined in Appendix. By (4) and (7), \( V_j(\rho_m^*) > V_j(\theta_m^*) \Leftrightarrow C(\rho_m^*, t) < C(\theta_m^*, t) + B. \)}

7.2.3 Quota vs Tax

Low types prefer the quota when externalities are sufficiently convex; in particular, there exists a \( t_3 \) such that \( \forall j \) with \( t_j \leq t_3 \), \( V_j(\theta_m^*) \geq V_j(\tau_m^*) \) and \( \forall j \) with \( t_j > t_3 \), \( V_j(\tau_m^*) > V_j(\theta_m^*). \) The idea is that since a tax obliges everyone to the same behavioral reduction it is less effective than a quota in curbing high types’ externalities. If \( \varepsilon \) is sufficiently convex, then low types prefer the quota, despite they have to give up tax transfers.

Figure 1.3 shows that the preference of low types for the quota can be drawn by comparing the cost of the quota, \( C(\theta_m^*, t) \), with \( C(\tau_m^*, t) + D; \) i.e. the cost of the tax plus a constant \( D \) which represents the additional benefits of a quota.\footnote{\( D \) is defined in Appendix. By (7) and (11), \( V_j(\theta_m^*) > V_j(\tau_m^*) \Leftrightarrow C(\theta_m^*, t) < C(\tau_m^*, t) + D. \)}

7.2.4 Condorcet winners

The instrument choice at the first stage will depend on the position of the median with respect to \( t_1, t_2 \) and \( t_3 \). Of course we do not know much about the orderings in which \( t_1, t_2 \) and \( t_3 \) may occur. All the arguments above regarding pairwise instrument choice can be summarized in the following lemma proven in Appendix.\footnote{Hereafter we assume that Lemma 2 holds. Cases in which this lemma does not apply are discussed in the Supplementary Material available from the authors upon request.}

**Lemma 2** If the externality function is sufficiently convex, then \( t_1, t_2 \) and \( t_3 \) exist and are unique. The only two orderings that do not violate transitivity of preferences for any voter are \( a \) and \( b \) in Figure 7.2.4.

7.2.5 Ordering \( a \)

Clearly the position of the median is the key issue. Suppose that \( t_m \leq t_3 \). and thus \( t_m \leq t_1. \) The majority prefers a rule when posed against the tax. Also since \( t_m \leq t_2 \), the majority
prefers the rule against the quota. The rule wins. Instead a tax wins if \( t_1 < t_m \leq t_2 \), or \( t_m > t_2 \). Thus under ordering \( a \), the rule is voted when the median is a low polluter and the tax is voted when the medial pollutes a lot. The quota is never chosen.

But when does ordering \( a \) occur? The graphical intuition for the answer can be found in Figure 1. Ordering \( a \) requires that \( t_3 \) is small compared to \( t_1 \) and \( t_2 \). Roughly speaking, \( A \) and \( B \) have to be rather large, compared to \( D \). Recall that \( A \) and \( B \) represent the relative advantage of a rule with respect to a tax and a quota, respectively. Since a rule forces top polluters to larger reductions, this relative advantage of the rule is big when externalities are quite convex. In fact the Appendix proves that:

**Proposition 7** When externalities are quite convex ordering \( a \) occurs. In this case:

i) the majority chooses a rule if the median is a low type \( (t_m \leq t_1) \);

ii) the majority chooses a tax if the median is high type \( (t_m > t_1) \).

iii) A quota is never chosen.

7.2.6 Ordering \( b \)

Ordering \( b \) occurs with a low \( B \), a high \( D \), and a moderately high \( A \). A low \( B \) means that the advantage of the rule over the quota is small. Thus \( \varepsilon'' \) has to be low. A high \( D \) means that the advantage of the quota over the tax is large (low utility from transfers compared to the benefits from curbing externalities). When \( \varepsilon'' \) is low, externalities grow rather linearly with behavior. A proportional quota performs well. Summing up:

**Proposition 8** When externalities are not too convex ordering \( b \) occurs. In this case:

i) the majority chooses a rule if the median is a low type \( (t_m \leq t_2) \);
ii) the majority chooses a tax if the median is high type ($t_m > t_3$);
iii) the majority chooses a quota if the median is an intermediate type ($t_2 < t_m \leq t_3$).

7.3 Deviations from optimality

Suppose that the median is “low” and costs are quite convex. According to Propositions 7 and 8 the majority chooses a rule. Due to convexity of the costs, the level of rule set at the second voting stage is too restrictive because high types pay too much. In this case a social planner maximizing average utility would choose a tax, which shares costs more evenly.

**Proposition 9** If the social planner would choose a tax and the majority prefers a rule, the chosen rule is more restrictive than what a social planner would choose if he were restricted to use a rule.

The idea is that when polluting activities are concentrated in a minority of high types and costs are sufficiently convex, the majority chooses a sub-optimal instrument, namely a rule rather than a tax, and the “wrong” level of the instrument. Suppose instead that the majority “enjoys” the polluting activity, namely the median is a high type. By Propositions 7 and 8, the majority prefers a tax which by Proposition 3, is too permissive. However, if cost convexity is low the social planner would choose the rule.

**Proposition 10** If the social planner prefers a rule and the majority prefers a tax, the outcome is a tax which is too permissive, namely the tax is lower than what a social planner would choose if he were restricted to use a tax.

These propositions suggest a relationship between the choice of the instrument and the nature of the political distortion. When the activities that cause externalities are mainly due minorities, we observe overly restrictive rules. If the externalities derive from activities enjoyed by the majority, then the choice will be taxes which are too permissive.

8 Conclusion

We have examined the political economy of how to curb activities which generate negative externalities. Our main result can be summarized in a “double distortion”. Under certain
conditions, when the individuals producing a negative externality (broadly defined) are a relatively small minority, the voters would choose a rule while a social planner would choose a tax. In addition, the rule chosen by the majority is more restrictive than the level that the social planner would choose if constrained to use a rule as his only instrument. Conversely, when the activity with negative externality is enjoyed by many, majority voting would select a tax even when a social planner would choose a rule. In this case, the tax chosen by majority voting would be lower than the level that the social planner would choose if he were restricted to use a tax as his only policy instrument.\(^{25}\) The majority chooses a quota only for a small set of parameter values and the quota is generally, but not always, dominated by a rule or a tax. This opens the question of why quotas are so broadly used in practice.

One could explore several extensions. First, we have not studied tradeable permits in this paper. The political economy aspects would concern the assignment of property rights (i.e. the rights to pollute), which crucially determines who can sell permits and who needs to buy them.\(^{26}\) Second, some activities with negative externalities (but not all) impose cost on future generations who do not vote, at least not directly except for the intergenerational altruism. Third, one could extend the analysis to more sophisticated tax schedules allowing for some curvature in the tax rate. Our hunch is that when the population is concentrated on low types the majority would choose a more “progressive” tax than the social planner. Fourth, thus far we have imposed that rules and quotas are self enforcing. This equilibrium is equivalent to assuming perfect monitoring (or imperfect monitoring with such a high fine if caught that nobody cheats in equilibrium). In reality, rules can be broken. The social choice involves a certain amount of investment in costly monitoring activities and the selection of a fine. The revenue from the fine could be used to finance monitoring and, if anything is left over, to provide public goods. With imperfect monitoring and a fine, individual polluters

\(^{25}\)This possibly explains why motor fuel taxation is too low, as discussed earlier. This is also consistent with the recent debate on obesity policies, which is a major concern in the US compared to Europe. What we observe is that the EU prefers regulation (e.g. more standards on fat contents, clearer food labelling, improving the nutritional content of school and office meals). According to this model, the reason is that obese people are a minority (at least for now) in Europe. The US is more oriented toward soft taxation, possibly because obese people are a larger share of the population. In fact, in the US Congress there have been recent proposals for an obesity tax, which is expected to be low compared to social costs.

\(^{26}\)Aidt and Dutta (2004) use a lobby model to explain the transition from command-and-control instruments to tradeable permits.
would choose how much to pollute and how much risk of being caught is worth taking. This would lead to a less sharp distinction between a rule (or a quota) and a tax. The fifth extension relates to voting rules. In our model any possible form of tyranny does not come from direct expropriation of the minority but rather from the fact that, within the political process, the majority ignores the costs incurred by the minority. This may result in decisions that are too costly from a social viewpoint. If for example the median’s policy were too restrictive, efficiency would be enhanced by giving the minority of high types some amount of blocking power. This is frequently done by adopting super-majorities.\textsuperscript{27} The problem is that a super-majority assigns blocking power not only to high types, but also to low types. If the objective is avoiding that the median is the pivot, a super-majority may not work. A potential alternative is giving the minority more voting weight.\textsuperscript{28} The idea is simple: when the median’s policy is too restrictive we must “shift the pivot” towards a higher type, whose bliss point is at the socially efficient level. The issue here is not equity: assigning more power to the most concerned individuals in order to counter balance the power of the least concerned ones improves efficiency. Implementation problems of such schemes are, however, extremely severe.

}\textsuperscript{27}Literature on super-majorities is vast and belongs to the normative analysis of constitutions. The focus is mainly on distributional issues (see Mueller (2003) for an extensive survey). Aghion and Bolton (2003) suggest that, when preferences are not single-peaked, higher super-majorities lower the risk of Condorcet cycles, but also lower the chance of circumventing ex-post vested interests; the solution of this trade-off yields the optimal majority threshold. Dixit, Grossman and Gul (2000) argue that super-majority rules may reduce compromise; as a consequence, the incidence of majority tyranny may increase. Aghion, Alesina and Trebbi (2004) analyze the constitutional choice about the level of super-majority needed to block policies of elected political leaders. Di Giannatale and Passarelli (2013) argue that, compared to weighted votes, a system based on the probability of being selected for voting generates less political distortion.

}\textsuperscript{28}The literature on weighted voting is possibly less developed, and mostly concerned with problems of equal representation in indirect democracies. Barbera and Jackson (2006) suggest a mixture of weights and super-majority that allows sticking with the status quo, unless at least a threshold of weighted votes is cast for change.
9 Appendix

9.1 Proofs of Lemmas and Propositions

Proof. Lemma 1. Implicit differentiating (5) at the point \( \rho_i^* \in (0, 1] \) yields, for any \( i \),

\[
\frac{\partial \rho_i^*}{\partial t_i} = -\frac{c''(t_i - \rho_i^*)}{-\varepsilon' \cdot (1 - F(\rho_i^*)) + \varepsilon' \cdot f(\rho_i^*) - c''(t_i - \rho_i^*)}
\] (15)

The denominator in the RHS of (15) is the second derivative of \( V_i(\rho) \), which is negative by assumption. Thus the sign of \( \frac{\partial \rho_i^*}{\partial t_i} \) is positive, since \( c''(t_i - \rho_i^*) > 0 \). In case of a corner solution, the above derivative is zero. The relationship between type and bliss point is weakly monotone. QED.

Proof. Proposition 1. Recall that \( \rho^* \) solves (6) and that \( ac'(\rho) \) is decreasing in \( \rho \), and that, by the concavity of \( W(\rho) \), \( (1 - F(\rho)) \cdot \varepsilon'(\rho) < ac'(\rho) \) for any \( \rho < \rho^* \). Consider a too restrictive rule: \( \hat{\rho} = \rho_m^* < \rho^* \). We have that

\[
(1 - F(\rho_m^*)) \cdot \varepsilon'(\rho_m^*) < ac'(\rho_m^*)
\]

and

\[
(1 - F(\rho_m^*)) \cdot \varepsilon'(\rho_m^*) = c'(t_m - \rho_m^*)
\]

Therefore,

\[
c'(t_m - \rho_m^*) < ac'(\rho_m^*)
\]

or,

\[
\frac{c'(t_m - \rho_m^*)}{ac'(\rho_m^*)/(1 - F(\rho_m^*))} < 1 - F(\rho_m^*)
\]

where \( ac'(\rho_m^*)/(1 - F(\rho_m^*)) \) is the average marginal cost computed over the affected population.

Equivalently, the condition for a too permissive rule is the following:

\[
\frac{c'(t_m - \rho_m^*)}{ac'(\rho_m^*)/(1 - F(\rho_m^*))} > 1 - F(\rho_m^*)
\]
QED. ■

**Proof. Proposition 2.** i) By the convexity of \(c(.)\), it follows that \(ac_\theta\) is increasing in \(\theta\). Moreover, by the concavity of \(W(\theta)\), \(a\varepsilon_\theta(\theta) < ac_\theta(\theta)\) for any \(\theta > \theta^*\). Let us consider the case of a too restrictive quota, \(\hat{\theta} > \theta^*\). By (8) and (9) we have:

\[
a\varepsilon_\theta(\hat{\theta}) < ac_\theta(\hat{\theta})
\]

and

\[
a\varepsilon_\theta(\hat{\theta}) = c_\theta(\hat{\theta}_m)
\]

Therefore,

\[
\frac{c_\theta(\hat{\theta}_m)}{ac_\theta(\hat{\theta})} < 1
\]

The vice versa holds for a too permissive quota.

ii) Let \(\bar{t}\) be the average type. When the adjustment cost function is linear, \(c(\theta t) = k \cdot \theta t\) (where \(k\) is a positive parameter), then marginal costs are linear in \(t\): \(c_\theta(t) = k \cdot t\). In this case, if \(t_m = \bar{t}\), then \(ac_\theta(\bar{\theta}) = c_\theta(\bar{\theta}_m) = k \cdot \bar{\theta}\). A median in the average position chooses the social optimum. By Jensen’s inequality, if \(c''(\theta t) > 0\), then \(c_\theta(\hat{\theta}t) < ac_\theta(\hat{\theta})\), then the quota is too restrictive even if \(t_m = \bar{t}\). QED. ■

**Proof. Proposition 3.** The proof is trivial thus we omit it. ■

**Proof. Proposition 4.** Suppose that \(t_m = \bar{t}\) and consider the “worst” case in which cost convexity is very low. Say \(c'' = \zeta\), where \(\zeta\) is a very low positive constant. In this case, \(c'(t - \rho) = \zeta \cdot (t - \rho)\). We know that with \(t_m = \bar{\theta}\) the tax is optimal (Proposition 3). We have to show that the rule is too restrictive. At the equilibrium point \(\hat{\rho}\), the median’s and the average marginal costs are the following:

- \(c'(t_m - \hat{\rho}) = \zeta \cdot (t_m - \hat{\rho})\)
- \(ac'(\hat{\rho}) = \int_\hat{\rho}^1 \zeta \cdot (t - \hat{\rho})dF(t) = \zeta \cdot [\bar{t} - (1 - F(\hat{\rho}))\hat{\rho}]\), where \(\bar{t}\) is the average type in \([\hat{\rho}, 1]\).

Observe that \(\bar{t} > \bar{\theta}\) and \((1 - F(\hat{\rho}))\hat{\rho} < \hat{\rho}\). Therefore,

\[
\frac{c'(t_m - \hat{\rho})}{ac'(\hat{\rho})} = \frac{\zeta \cdot (t_m - \hat{\rho})}{\zeta \cdot [\bar{t} - (1 - F(\hat{\rho}))\hat{\rho}]} < 1
\]

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The rule is too restrictive (Proposition 1). Despite convexity is very low, even a median who is in the average (i.e. \( t_m = i \)) chooses too restrictive a rule. In general, with any degree of cost convexity, one can find a distribution with a median below the average who chooses too restrictive a rule, whereas, by Proposition 3 this is not possible when the instrument is a tax. QED.

**Proof. Proposition 5.** If an interior rule is too permissive, then

\[
\frac{c'(t_m - \hat{\rho})}{ac'\hat{\rho})} > 1
\]

We want to show that in this case also the quota chosen by the majority is too permissive; i.e.

\[
c_{\theta}(\hat{\theta}t_m)/ac_{\theta}(\hat{\theta}) > 1
\]

Consider that \( ac'\hat{\rho}) \) is an average in which the only non-zero elements are the \((1 - F(\hat{\rho})\) marginal costs of the affected people above \( F(\hat{\rho})\); where \( 1 - F(\hat{\rho}) > 0.5 \). Moreover 50\% of the elements in \( ac'\hat{\rho}) \) are larger than \( c'(t_m - \hat{\rho}) \). Further consider that in equilibrium the median has stronger incentive to bear private costs when a rule is adopted, thus \( c'(t_m - \hat{\rho}) > c_{\theta}(\hat{\theta}t_m) \). Thus, with a quota the median’s marginal cost is lower. We now show that also the average is lower, but it decreases by a larger amount.

Split the population in two sets: the 50\% above the median and the 50\% below the median. With a quota, all individuals in the first set lower their behavior, and at most all individuals in the second set increase their behavior. However, the behavior reduction of any individual in the first set is larger than the behavior increase of the individuals in the second set (since the quota affects behavior proportionally). By cost convexity, it follows that the marginal costs of all individuals in the first set decrease *on average* by a larger amount with respect of the marginal cost increase in the second set. Thus the average marginal cost over the entire population decreases. Moreover, the average decrease in the first set is larger than the decrease in the median’s marginal cost, and the average increase in the second set is smaller. Thus, when a quota instead of a rule is adopted, the marginal cost over the entire population, \( ac_{\theta}(\theta) \), decreases by a larger amount than the median’s marginal cost, \( c_{\theta}(\hat{\theta}t_m) \). This implies that if \( c'(t_m - \hat{\rho}) > ac'(\hat{\rho}) \) then \( ac_{\theta}(\theta) \) cannot be larger than the median’s marginal cost,
Proof. Proposition 6. By Propositions 2 and 4, if \( c'' > 0 \) and utility is quasi-linear, then \( t_m > \bar{t} \) is a sufficient condition for a too permissive tax whereas it is not sufficient for a too permissive quota. Thus we might have cases in which the tax is too permissive and the quota is too restrictive, but the vice versa is impossible. QED.

Proof. Lemma 2. The first part of the proof consists of showing that the crossing points of curves \( C(\rho^*_m, t), C(\tau^*_m, t) \) and \( C(\theta^*_m, t) \) are as represented in Figure 1. We do this in two steps. First, we show that for sufficiently convex externality functions \( A, B, \) and \( D \) are positive. Second, we show that, given \( \rho^*_m, \tau^*_m, \) and \( \theta^*_m, C(\rho^*_m, t) \) is steeper than \( C(\tau^*_m, t) \) and \( C(\theta^*_m, t) \) for a sufficiently large \( t \), and \( C(\theta^*_m, t) \) is steeper than \( C(\tau^*_m, t) \) for any \( t \).

Specifically, \( A, B, \) and \( D \) are the following:

\[
A = \left[ -\varepsilon(\rho^*_m) \cdot (1 - F(\rho^*_m)) - \int_0^{\rho^*_m} \varepsilon(t) dF(t) \right] - \\
- \left[ - \int_0^1 \varepsilon(t - r(\tau^*_m)) dF(t) - c(r(\tau^*_m)) + \tau^*_m \cdot \bar{t} \right]
\]

\[
B = \left[ -\varepsilon(\rho^*_m) \cdot (1 - F(\rho^*_m)) - \int_0^{\rho^*_m} \varepsilon(t) dF(t) \right] - \\
- \left[ - \int_0^1 \varepsilon((1 - \theta^*_m) t) dF(t) \right]
\]

and

\[
D = \left[ - \int_0^1 \varepsilon((1 - \theta^*_m) t) dF(t) \right] - \\
- \left[ - \int_0^1 \varepsilon(t - r(\tau^*_m)) dF(t) - c(r(\tau^*_m)) + \tau^*_m \cdot \bar{t} \right]
\]

Observe that when \( \varepsilon'' \) is large enough the first squared brackets in \( A, B, \) and \( D \) are larger than the second ones. The reason is the same: a rule forces high types to larger adjustments with respect to the other two instruments; and a quota does the same, with respect to a tax.
With sufficiently large $\varepsilon''$, these effects overcome the benefits of the transfers in $A$ and $D$. Therefore $A$, $B$ and $D$ are larger than zero.

Let us consider the second step. The cost functions are the following:

$$C(p_m^*, t) = \begin{cases} \frac{c(t - p_m^*)}{t} & \text{for } t > p_m^* \\ 0 & \text{for } t \leq p_m^* \end{cases}, \quad C(p_m^*, t) = \frac{p_m^*}{t}, \quad C(p_m^*, t) = \frac{c(p_m^*)}{t}$$

The derivatives are the following:

$$\frac{\partial C}{\partial t} = \begin{cases} \frac{c'}{t} & \text{for } t > p_m^* \\ 0 & \text{for } t \leq p_m^* \end{cases}, \quad \frac{\partial C}{\partial t} = \frac{p_m^*}{t}, \quad \frac{\partial C}{\partial t} = \frac{c'}{t} \cdot p_m^*$$

Since $p_m^* < 1$, then for a sufficiently large $t > p_m^*$, $\frac{\partial C}{\partial t} > \frac{\partial C}{\partial t}$ and $\frac{\partial C}{\partial t} < \frac{\partial C}{\partial t}$. Moreover, for any sufficiently large $t$, $\frac{\partial C}{\partial t} > \frac{\partial C}{\partial t}$ and $\frac{\partial C}{\partial t} < \frac{\partial C}{\partial t}$. Then, $t_1$, $t_2$ and $t_3$ are positive and unique.

The second part of the proof consists of showing that any ordering other than $a$ and $b$ violate transitivity conditions. The full set of possible orderings is as follows:

- Ordering $a$: $t_3 < t_1 < t_2$
- Ordering $b$: $t_2 < t_1 < t_3$
- Ordering $c$: $t_3 < t_2 < t_1$
- Ordering $d$: $t_1 < t_3 < t_2$
- Ordering $e$: $t_1 < t_2 < t_3$
- Ordering $f$: $t_2 < t_3 < t_1$

i) Ordering $c$. Take an individual $j$ with type $t_j \in (t_2, t_1]$. He prefers the rule to the tax; the quota to the rule; the tax to the quota. His preferences clearly do not meet transitivity. Ordering $c$ is impossible. Applying the same argument, one can easily see that:

ii) Ordering $d$ cannot occur because preferences of types in $(t_1, t_3)$ are not transitive.

iii) Ordering $e$ cannot occur because preferences of types in $(t_1, t_2)$ are not transitive.

iv) Ordering $f$ cannot occur because preferences of types in $(t_3, t_1]$ are not transitive.

QED. ■

**Proof. Proposition 7.** By Lemma 2, it is easy to see that when externalities are
quite convex, $A$ and $B$ are large, compared to $D$. Thus ordering $a$ occurs. In this case, the majority choice is the following:

- $a.1$, if $t_m \leq t_3$, then Rule $\succ$ Tax, Rule $\succ$ Quota, and Quota $\succ$ Tax $\to$ **Rule wins.**
- $a.2$, if $t_3 < t_m \leq t_1$, then Rule $\succ$ Tax, Rule $\succ$ Quota and Tax $\succ$ Quota $\to$ **Rule wins.**
- $a.3$, if $t_1 < t_m \leq t_2$, then Tax $\succ$ Rule, Rule $\succ$ Quota and Tax $\succ$ Quota $\to$ **Tax wins.**
- $a.4$, if $t_m > t_2$, then Tax $\succ$ Rule, Quota $\succ$ Rule and Tax $\succ$ Quota $\to$ **Tax wins.**

Thus under ordering $a$ the rule is voted when the median is a low polluter and the tax is voted when the medial pollutes a lot. The quota never comes about. **QED.**

**Proof.** 8. Externalities are sufficiently convex, so that Lemma 2 holds. However, if convexity is not too high, $B$ is relatively small wrt $A$ and $D$. In this case ordering $b$ occurs. Then, the majority choice is the following:

- $b.1$, if $t_m \leq t_2$, then Rule $\succ$ Tax, Rule $\succ$ Quota, and Quota $\succ$ Tax $\to$ **Rule wins.**
- $b.2$, if $t_2 < t_m \leq t_1$, then Rule $\succ$ Tax, Quota $\succ$ Rule and Quota $\succ$ Tax $\to$ **Quota wins.**
- $b.3$, if $t_1 < t_m \leq t_3$, then Tax $\succ$ Rule, Quota $\succ$ Rule and Quota $\succ$ Tax $\to$ **Quota wins.**
- $b.4$, if $t_m > t_3$, then Tax $\succ$ Rule, Quota $\succ$ Rule and Tax $\succ$ Quota $\to$ **Tax wins.**

The quota then is an equilibrium when the median is in an intermediate position and he is not strongly distorted toward a tax or a rule. **QED.**

**Proof. Proposition 9.** The social planner prefers a tax to a rule when externality convexity is low and cost convexity is high. With low convexity of $\varepsilon$, ordering $b$ occurs and Propositions 1 and 8 apply: a majority with a low median chooses a rule and the rule is very restrictive. Due to high cost convexity, there is a large difference between the average and the median marginal costs. Then the median’s choice is quite different from to the rule level that would be socially optimal. This causes a large welfare loss. **QED.**

**Proof. Proposition 10.** The social planner prefers a rule when externality convexity is high and cost convexity is low. If $\varepsilon''$ is high, Proposition 7 applies (i.e. ordering $a$ occurs). A majority with a high median chooses a tax. By Proposition 3, if $t_m > \tilde{t}$, the level is too low. **QED.**
References


