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Luxury Consumption, Precautionary Savings and Wealth Inequality

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Abstract

Most macroeconomic models are based on the assumption of a single homogeneous consumption good. In the present paper we consider a model with two goods: a basic good and a luxury good. We then apply this assumption to a standard general equilibrium heterogeneous agent model. We find a substantial reduction in precautionary savings compared to a standard model. The effect on wealth inequality turns out to be ambiguous and to depend on the size of the assumed earnings risk.

Keywords: precautionary savings, wealth inequality, luxury consumption, non-homothetic utility

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1 Introduction

Models with a large number of ex-ante identical agents with standard preferences subject to uninsurable, idiosyncratic shocks to earnings are a key tool used to study, among else, two questions that are relevant for macroeconomics, that is, what is the size of precautionary savings and what accounts for the large observed differences in wealth holdings among households. In this paper we use the general framework mentioned above to assess the role played by non-homothetic preferences that make a distinction between basic and luxury consumption, in determining the volume of precautionary savings and in shaping the distribution of wealth, and hence, their contribution to answering those two important macroeconomic questions.

Non homothetic preferences have been recently used to improve the performance of portfolio choice models by Wachter and Yogo (2010) and to help understanding the equity premium puzzle by Ait-Sahalia, Parker and Yogo (2004). Other authors like Ikeda (2006) studied models where luxury consumption induces a preference for wealth and explored the consequences of taxation of luxury consumption for capital accumulation. The distinction between luxury and basic consumption has even been used by Dalgin, Mitra and Trindade (2004) to improve the understanding of international trade patterns. Despite all this work with non-homothetic preferences, their implications for the determination of precautionary savings and for the shape of the wealth distribution have been neglected. Filling up this gap is precisely the purpose of the present paper.

The economies studied in this research are in most respect standard. They feature a continuum of infinitely lived ex-ante identical agents. Agents receive an exogenous stochastic stream of earnings that cannot be insured due to incomplete markets. They have access to a single risk-free asset that they can use to smooth consumption in the face of variable earnings, subject to a borrowing constraint. As a result of different histories of realizations of the earnings shock they will be ex-post heterogeneous. The model is closed by a standard neoclassical production function. It does not feature any aggregate uncertainty, hence we can focus on studying stationary equilibria. The key innovation of the model is the assumption that there are two consumption goods: a basic good and a luxury good. The basic good has an expenditure elasticity less than unity while the luxury good has an expenditure elasticity greater than unity so that its share is rising in total expenditures. In order to obtain this result we use an additive logarithmic intra-temporal utility function defined over the two goods. Following Wachter and Yogo (2010) we calibrate its parameters so as to match the median share of basic consumption in total expenditures and the gradient of the share with respect to quartiles of the expenditure distribution. We then simulate several economies characterized by a different amount of earnings risk under the standard assumption of homothetic preferences over a single good and under the assumption of non-homothetic preferences over two goods. These economies are otherwise equally parameterized. We find that precautionary savings is reduced under all earnings processes. The reduction is always significant and may be very large when earnings risk is large. With respect to the wealth distribution, the sign of the effect of introducing this class of preferences is not univocally determined: wealth concentration decreases for moderate levels of earnings risk but increases when earnings risk is large. In both cases though, quantitatively the change in wealth concentration is not big. Overall then we can say that these preferences are important to understand aggregate savings but do not seem to represent a clear step forward towards the explanation of the long standing issue of wealth concentration, especially at the top of the distribution.

The intuition for the reduction in the size of precautionary savings is that if the model with homothetic utility and the one with non-homothetic utility are equally parameterized, agents in the latter model will be on average less risk-averse because risk aversion declines from a common value as wealth, hence the share of luxuries in total consumption, increases. Wealthy agents, who carry out most of the saving are less risk-averse in the non-homothetic model, hence they reduce their precautionary savings, thus depressing savings in the aggregate. As for the wealth distribution, the two goods with non-homothetic utility introduce two forces operating in opposite directions. On the one hand poor agents who consume a smaller proportion of luxuries are more risk averse than richer agents hence have stronger precautionary motives. This works towards a more equal distribution of wealth. On the other hand the elasticity of inter temporal substitution is higher for wealthier agents who can thus more easily cut consumption in the face of negative earnings shocks. This factor works towards a more concentrated wealth distribution. Which of the two forces prevails is a quantitative issue that turns out not to be univocally determined.

The current paper is related to several strands of literature. First it is related to the literature that has tried to quantify the impact of earnings risk on wealth accumulation in the context of models with a large number of ex-ante identical agents subject to uninsurable idiosyncratic risk. This literature includes early work in partial equilibrium like Hubbard, Skinner and Zeldes (1994) and Carroll and Samwick (1997) and in general equilibrium like Aiyagari (1994) and Huggett (1996). It also includes more recent work that uses a structural estimation approach in life-cycle partial equilibrium economies like Cagetti (2000) and Gourinchas and Parker (2002). Second it is related to the large body of literature that, in the same framework, has tried to understand the wealth distribution. This was started by the already mentioned works by Aiyagari (1994) and Huggett (1996) who first showed how models with uninsurable idiosyncratic shocks to earnings in general equilibrium could generate the observed fact that wealth is more concentrated than earnings. The failure to match the observed wealth concentration spurred a number of papers that, building on the original framework, added several possible explanatory mechanisms. These included entrepreneurship, like in Quadrini (2000) and Cagetti and De Nardi (2006), bequest motives like in De Nardi (2004), heterogeneous preferences like in Krusell and Smith (1998) or the assumption of earnings risk large enough to match the observed earnings variability, like in Castañeda, Díaz-Giménez and Ríos-Rull (2003). Among the body of research mentioned above, the current paper is particularly related to works of Díaz, Pijoan-Mas and Ríos-Rull (2003), Carroll (2000) and Francis (2009). The relationship with Díaz et al. (2003) stems from the fact that both papers analyze the effect of an alternative preference specification on both precautionary savings and the concentration of wealth. The main difference with that paper is that they explore the role of habit formation while here we focus on the introduction of multiple consumption goods with non-homothetic preferences defined over the goods.[1] On the other hand Carroll (2000) and Francis (2009) both propose non-homothetic preferences as a possible explanation of wealth inequality. In their work though, these are introduced in the form of having wealth in the utility function, which gives another saving motive on top of precautionary saving. Also, Carroll (2000) does not explore the quantitative implications of his idea, while Francis (2009) calibrates the model by picking the parameters of the utility function over wealth to match the Gini index of the wealth distribution itself. The present paper, besides focusing only on the precautionary saving motive, also chooses a rigorous calibration procedure where the parameters of the utility function are based on independent empirical evidence on the share of the two goods in total consumption.

The third strand of literature that is related to the current research consists of macroeconomic models that have put to different uses the assumption of multiple, non-durable consumption goods. Among them Ait-Sahalia et al. (2004) using a more restrictive definition of luxury goods — high end luxuries — show that the empirical correlation between the equity return and consumption of these goods may reconcile the observed equity premium with a limited degree of risk aversion. Wachter and Yogo (2010) use non-homothetic preferences to explain why the share of risky assets is increasing in wealth. Finally Aguiar and Bils (2014) and Aguiar and Hurst (2013) use the multiple goods framework to address issues of consumption inequality. Aguiar and Bils (2014) find empirically that once this distinction is made consumption inequality tracks very closely the increase in income inequality observed

[1] Díaz and Luengo-Prado (2010) study the wealth distribution in a two goods model. In their case though the second good is a durable, more precisely housing.

in the recent decades. Aguiar and Hurst (2013) focus on goods with different elasticity of substitution of time and expenditures in their production to explain the evolution of consumption inequality over the life-cycle.

The rest of the paper is organized in the following way. Section 2 presents the model, section 3 is devoted to explaining the calibration strategy, section 4 presents the results. In section 5 a brief conclusion is outlined. Finally, an appendix describes the numerical methods used to solve the model.

2 Model

We consider a standard neoclassical economy with no aggregate uncertainty which allows us to focus on steady-states. We assume that there is a measure one of infinitely lived agents. Agents are ex-ante identical and receive idiosyncratic shocks to their endowment of labor units. This feature is standard and is the source of microeconomic uncertainty that leads to precautionary savings. We assume that there are two types of goods: basic goods and luxury goods or simply luxuries.

A description of the production side of the economy is given in section 2.1, more details about the household's preferences are given in section 2.2, the household's optimal decision problem is described in section 2.3, while the definition of the general equilibrium of the economy is given in section 2.4.

2.1 Technology

Each period households receive a shock to their efficiency units of labor that we denote with e. We assume that e belongs to a finite set $E = \{e_1, ..., e_n\}$ and that it follows a first-order Markov process that we can describe with a transition probability matrix π. Aggregate output is produced via a standard neoclassical, constant return to scale production function $Y = F(K, L)$, where Y is aggregate output, K is the total amount of capital and L is the total amount of labor used in production. The output can be indifferently used for investment and as both a luxury and a basic consumption good. Hence the relative price of those goods will be 1 and the quantity produced and consumed will be entirely demand driven. As it will become clear in the next subsection, in a stationary environment like the one considered in this paper, the relative price of the two goods cannot be separately identified, hence this assumption is innocuous.[2] Capital depreciates at a rate $\delta \in [0, 1]$.

2.2 Preferences

Households are endowed with one unit of time. They do not value leisure hence they supply this unit inelastically to the market. They can consume two types of goods which we call basic and luxury goods. In each period consumption of quantity b of basic goods and of quantity l of luxury goods

[2] In other words, one could introduce a friction in the model that makes the relative price of the two goods different from 1, but this price would be constant and its effect would not be identified separately from the effect of one of the preference parameters. For this reason it is legitimate to assume no friction which implies a relative price of basics and luxuries equal to 1.

gives utility

$$U(b,l) = \left[b^{1-\lambda} + \frac{\eta(1-\lambda)}{1-\phi} l^{1-\phi}\right]^{\frac{1}{1-\lambda}} \tag{1}$$

where $\eta \geq 0$ is the utility weight on l and the two curvature parameters satisfy the restriction $\lambda \geq \phi > 1$. The intra-temporal utility index is embedded in the household's inter-temporal utility as:

$$u(b,l) = \frac{U(b,l)^{1-\gamma}}{1-\gamma} \tag{2}$$

where $\gamma > 1$ controls risk-aversion. This specification follows Wachter and Yogo (2010) and has been used in a number of settings to model the fact that as households grow wealthier, they spend relatively more on some goods than on others.

The allocation of total consumption expenditures between basic and luxury goods is entirely determined by the properties of the intra-temporal utility index. If we let q be the relative price of l in units of b, then the household will solve in each period the problem:

$$\max_{b,l} U(b,l) \tag{3}$$

subject to the constraint:

$$b + ql = c \tag{4}$$

where c denotes total consumption expenditures. This problem has first order conditions:

$$\frac{U_l}{U_b} = \frac{\eta l^{-\phi}}{b^{-\lambda}} = q \tag{5}$$

which once substituted in the constraints and then in the intra-temporal utility index allow us to write the latter as a function of the relative price q and of the total amount of consumption expenditures c as:

$$\widehat{U}(c,q) = \max_{b+ql=c} U(b,l) \tag{6}$$

In turn this leads to the following period utility function:

$$\widehat{u}(c,q) = \frac{\widehat{U}(c,q)^{1-\gamma}}{1-\gamma} \tag{7}$$

Two observations can be made about this representation of utility.[3] First when $\lambda = \phi$ it collapses to the usual homothetic case. Second this utility function displays declining relative risk aversion.[4] When the agent is very poor it consumes mostly basic goods and its risk aversion is close to γ. As the household becomes wealthier the share of luxury consumption increases and risk aversion converges towards the value:

$$\frac{\gamma(1-\phi)+\phi-\lambda}{1-\lambda} < \gamma \tag{8}$$

Also, the relative price of luxuries q and the parameter η both determine the average share of luxury in total consumption, hence in a stationary environment like the one considered here they cannot be separately identified. For this reason we can simply set $q = 1$ and drop it from the maximized intra-temporal utility function that can then be written $\widehat{u}(c)$. [5]

2.3 Household's optimal program

There are no state contingent markets to insure the household specific shock e. The household then has access to a single asset that pays interest at a rate r. We denote the amount of the asset held by the household by a and we assume that $a \in A \equiv [\underline{a}, \infty)$, thus we assume an exogenous borrowing constraint. The absence of state-contingent markets and the presence of

[3]The function $\widehat{U}(c,q)$ does not have an analytical representation and is thus evaluated numerically as it will be explained in the Appendix.

[4]See Wachter and Yogo (2010).

[5]The fact that $q = 1$ is insured by the assumption about technology in section 2.1.

borrowing constraints are the necessary ingredients to depart from the standard representative agent framework which enables us to study distributional issues.

Given the preferences and asset structure specified above we directly write the household's optimization problem in dynamic programming form. Since we only look at steady-states, the individual household's state variables are the shock to its endowment of efficiency units of labor and its assets, that is the pair $\{e, a\}$. The problem that the household solves reads:

$$v(a,e) = \max_{c \geq 0, a' \geq \underline{a}} \widehat{u}(c) + \beta \sum_{e'} \pi_{e,e'} v(a', e') \tag{9}$$

subject to:

$$a' = ew + (1+r)a - c \tag{10}$$

where w is the rental rate for efficiency units of labor, r is the rental rate on capital and the function $\widehat{u}$ has been defined in the previous section.

This problem gives rise to two decision rules: $a' = g^a(a,e)$ for asset holdings and $c = g^c(a,e)$ for consumption expenditures. The exogenous Markov process for the efficiency units of labor together with the individual decision rules determine a Markov process for the state variables. Letting $s = \{a, e\}$ be the set of individual state variables and $S = \{A \times E\}$ be the set of values that those state variables can take, define $\mathcal{B}$ the σ-algebra generated in S by the open sets. A probability measure x over $\mathcal{B}$ can be used to describe the economy by stating how many households are of each type. Define a function $Q(s, B)$ that gives the probability that a type $\{s\}$ has of becoming a type in a set $B \subset \mathcal{B}$. The function Q can then be used to describe how the economy evolves over time by giving a probability measure for tomorrow x' given the probability measure x today. The rule that governs this transition from x to x' is given by:

$$x'(B) = \int_S Q(s,B)dx \tag{11}$$

Ríos-Rull (1999) shows that if the earnings shock process has a unique stationary distribution, so has the economy. Moreover this unique stationary distribution is the limit to which the economy converges, starting from any initial distribution.

2.4 Equilibrium

Given the description of the model in the previous subsections we are now ready to define the steady-state equilibrium for this economy. Formally a steady-state equilibrium will be a set of value and decision functions $\{v, g^a, g^c\}$ and a measure of households x such that: (i) Factor inputs are obtained by aggregating over households, that is, $K = A = \int_S adx$ and $L = \int_S edx$; (ii) factor prices are equal to the factor's marginal productivity, formally: $r = F_1(K,L) - \delta$ and $w = F_2(K,L)$; (iii) given K and L — hence factor prices — the functions $\{v, g^a, g^c\}$ solve the household's optimal program described in Section 2.3; (iv) the goods market clears: $\int_S [g^c(s) + g^a(s)]dx = F(K,L) + (1-\delta)K$, where $c(s) = b(s) + l(s)$ and (v), the measure of households is stationary: $x(B) = \int_S Q(s,B)dx$ for all $B \subset \mathcal{B}$.

3 Calibration

This paper explores the role of luxury consumption in the quantitative determination of precautionary savings and in shaping the wealth distribution in an infinite horizon framework. We thus take as a point of departure in the calibration the model economies studied in Aiyagari (1994), the seminal paper in this area. The model period is taken to be one year and the subjective discount factor β is set at 0.96. The coefficient γ that determines the curvature of the period utility index defined over the maximized intra-temporal utility of consumption expenditures is set at a value of 3, corresponding to the intermediate risk-aversion case in Aiyagari (1994). Thus in the limit a very poor agent that consumed almost entirely basic goods would exhibit a coefficient of relative risk aversion of 3. The capital share of income α is taken to be 0.36 and the depreciation rate of capital δ is set at 0.08. With respect to the labor earnings process we define a benchmark case where we take it to follow an $AR(1)$ process in logarithms and then discretize it using the procedure outlined in Tauchen (1986).[6] The $AR(1)$ process is completely defined by two parameters, the autocorrelation coefficient and the coefficient of variation. We take the former to be 0.6 and the latter to be 0.4, both are values among those used by Aiyagari (1994).

The parameters related to the specific contribution of the paper are those that determine the share of basic and luxury goods in total consumption expenditures. To calibrate those parameters we start from data presented in Wachter and Yogo (2010). The two authors use the Consumer Expenditures Survey for the period 1982-2003 as a source of detailed data on consumption of different non-durable goods and services. They categorize them into several groups and for each they estimate a regression of the expenditure share on several explanatory variables, including total consumption expendi-

[6]Strictly speaking in the description of the model we directly used the discretized form. Here we explain how the finite-state Markov chain is obtained

tures.[7] Following standard demand analysis they then classify luxury goods and services as those whose share raises with total consumption expenditures and basic goods and services as those whose share decreases with total consumption expenditures. Aggregating over the different categories they can thus provide expenditure shares for basics and luxuries at different points of the consumption expenditures distribution. They estimate that the share of basics goods for the median household is 47 percent with a variation from 57 percent in the bottom quartile of the consumption distribution to 35 percent for the top quartile. We take those shares as the targets for our calibration and proceed as follows. We try to match the median share as precisely as possible. This share can be matched basically by an appropriate choice of the parameter η. The parameters λ and ϕ on the other hand are mainly involved in the determination of how the expenditure share of basic goods varies over the consumption distribution. In practice we follow Wachter and Yogo (2010) in setting a low value of 1.1 for ϕ. Then we determine λ so that the difference between the share of basic consumption in total expenditures at the top and bottom quartile of the consumption distribution is the same 22 percent value found in the data. The chosen values in the baseline case are 75 for η and 11.5 for λ.

The parameter values for this benchmark calibration are reported in table 1 which is divided in two panels, the top panel with the parameters that are common to both the homothetic and the non-homothetic model, and the bottom panel with the parameters specific to the non-homothetic

[7] They identify the following categories. For goods: Food at home, food away from home, clothing and shoes, fuel oil and coal, gasoline, other non durable goods. For services: Housing, household operation, transportation, personal care, personal business, recreation.

Table 1: **Parameters**

Parameter	Value
β	0.96
γ	3
α	0.36
δ	0.08
ρ_e	0.6
σ_e	0.4
η	75
λ	11.5
ϕ	1.1

model. As it will turn out, moving to non-homothetic preferences changes the equilibrium interest rate, hence the incentive to save at different points of the wealth distribution. Since we want to study the effect of this class of preferences on the wealth distribution, and in order to separate the direct effect of preferences from the indirect effect of the change in the equilibrium interest rate on wealth dispersion, we also run a second version of the model with non homothetic preferences where the discount rate is adjusted to keep the capital-output ratio — hence the interest rate — at the same level as in the benchmark homothetic model. In this latter case β needs to take a value of 0.9652.

In later sections we also provide results for economies where the earnings process shows a lot more dispersion than the one in the seminal Aiyagari's paper. This is a needed extension since the wealth dispersion generated in Aiyagari (1994) is substantially less than in the data among else precisely because the earnings dispersion is too small under standard $AR(1)$ processes.[8] Given our focus on the marginal effect of introducing non-homothetic preferences on the the wealth distribution we think it is a necessary step to also explore this issue with a model that can get closer to the empirical wealth dispersion. Details of the earnings process and the necessary adjustment in parameters are given in the corresponding result sections.

4 Results

Results are presented in two sections. In section 4.1 we describe the result of the model that uses the labor earnings process taken from Aiyagari (1994). In section 4.2 we explore the results of two model economies that use two alternative earnings processes that generate a higher earnings variability, one consistent with SCF data and the second consistent with PSID data. In each case we report results from three different economies. First we consider an economy with standard homothetic, constant relative risk aversion preferences. Second we explore an economy that is identical to the former except that it has non-homothetic preferences as specified in section 2.2. Notice that the first economy can be obtained as a special case of the second one by setting λ equal to ϕ. Third we simulate an economy that has non-homothetic

[8] Budriá Rodríguez et al. (2002) report that the coefficient of variation of earnings in the U.S. Survey of Consumer Finances (1998) is 2.65 while earnings dispersion in Aiyagari (1994) — and in our benchmark calibration – is only 0.4.

preferences with the same preference parameters as the second one but whose subjective discount factor is adjusted so that the capital output ratio is the same as the one in the first economy. In this way we can isolate the direct effect of non-homothetic preferences on the wealth distribution from the indirect effect induced by the change in the size of precautionary savings.

4.1 Baseline

In this subsection we report results for the economies that use the Aiyagari's earnings process. Given the parametrization described in the calibration section agents in the homothetic preference economy exhibit a relative risk aversion coefficient of 3. In the non-homothetic economy this will be the limit of the coefficient of relative risk-aversion for a very poor agent that consumes virtually only basic goods. This coefficient declines as the share of luxuries increases. Substituting the calibrated parameters in equation (8), we can see that it converges to 1.019.

Table 2 reports average statistics for the baseline homothetic economy and for comparison for the corresponding representative agent economy.[9] All quantities are normalized by output in the deterministic economy for easiness of comparison. Aggregate assets are 2.96 and the interest rate is 4.17 percent in the deterministic economy. In the benchmark homothetic economy assets go up to 3.51 and the interest rate is reduced to 2.92 percent. Capital thus increases by 18.5 percent. This increase can be attributed to precautionary savings. Measures of precautionary savings in the literature vary quite a bit.

[9]The representative agent economy has the same parameters as the baseline economy and the labor endowment fixed at the unconditional mean of the earnings process of the model with uninsurable earnings risk.

Table 2: **Main statistics: Benchmark and deterministic economies**

	Deterministic	Benchmark	% change
Aggregate assets	2.96	3.51	+ 18.5 %
Output	1.00	1.06	+ 6.3 %
Capital/output ratio	2.96	3.30	+ 11.5 %
Interest rate (%)	4.17	2.92	- 42.8 %
C.V. of wealth	–	0.696	–
Gini of wealth	–	0.372	–

For example Cagetti (2000) finds that up to age 50 savings for precautionary reasons represent almost all savings and that at the time of retirement, wealth is twice as high than in a world without precautionary savings. On the other hand Huggett (1996) finds a result of just a few percentage points. Our estimate thus falls in between those. This said, in our context the measure of precautionary savings would have varied substantially, had we chosen a more extreme parametrization of earning risk and risk aversion among those presented by Aiyagari (1994). Thus we see this figure essentially as a benchmark against which to compare the effect of introducing non-homothetic preferences.

In table 3 we thus report results for the benchmark homothetic model with the Aiyagari's earnings process and for two versions of the corresponding non-homothetic model. The labels B, NH and NHA in this table —like in the subsequent tables 6 and 8 — stand respectively for benchmark homothetic model, non-homothetic model and non-homothetic model with the subjective discount factor adjusted so as to keep the wealth-output ratio constant. Hence columns 2 and 3 consider a version of the model whose parameters are exactly the same as the ones of the homothetic model, except of course for those defining preferences over basic and luxury goods. Total asset holdings move from 3.51 to 3.31 and the interest rate increases from 2.92 percent to 3.38 percent. In terms of precautionary savings this means that moving from the homothetic case to the non-homothetic case reduces them from 18.5 percent to 11.8 percent, that is, a reduction of about a third. Qualitatively this result could be anticipated since in the non-homothetic model risk aversion almost coincides with the one of the homothetic model for very poor agents that consume almost entirely basic goods and then declines as the share of luxuries increases, hence the average agent is less risk averse. The interest of the result is thus in the quantitative figure, which shows a substantial change.

The last two rows of table 3 show results concerning the wealth distribution. Both measures of wealth concentration show a more equal distribution in the non-homothetic model. The coefficient of variation declines from 0.696 to 0.648, that is, by about 7 percent and the Gini index of wealth declines from 0.372 to 0.349 a decline of about 6 percent. These figures are non negligible albeit not big. A priori the sign of the variation is not clear. On the one hand in the non-homothetic model risk-aversion declines with the share of luxury consumption, hence with wealth; thus suggesting relatively more savings for precautionary reasons for the poor agents. On the other

Table 3: **Main statistics: Benchmark and non-homothetic economies**

	B	NH	% change	NHA	% change
			$\frac{NH-B}{B}$		$\frac{NHA-B}{B}$
Aggregate assets	3.51	3.31	-5.6 %	3.51	–
Output	1.06	1.05	-1.6 %	1.06	–
Capital/output ratio	3.30	3.16	-4.2 %	3.30	–
Interest rate (%)	2.92	3.38	+15.8%	2.91	–
Precautionary savings	18.5 %	11.8 %	-36.2%	11.8 %	–
C.V. of wealth	0.696	0.648	-6.9 %	0.654	-6.0 %
Gini of wealth	0.372	0.349	-6.2 %	0.351	-5.7 %

hand, as it was pointed out by Browning and Crossley (2000) the elasticity of inter-temporal substitution increases with the share of luxuries. In their words “luxuries are easier to postpone”, hence a wealthy agent facing a negative earnings shock might more easily cut consumption slowing down wealth decumulation. The balance of these two forces at different points of the distribution determines the effect on wealth concentration which in this example is reduced.

The last two columns of the table report results for an economy with non-homothetic preferences where the subjective discount factor is changed so as to restore the same interest rate as the benchmark economy. This would isolate the effect of preferences on the wealth distribution from the indirect effect of the change in the interest rate. To obtain this, given the reduction in aggregate assets in the non-homothetic model one needs to make agents a little more patient and raise β to 0.9652. The coefficient of variation of wealth is in this case 0.654 and the Gini index is 0.351, both representing a reduction in the measure of wealth concentration of about 6 percent compared to the benchmark homothetic model. These figures are very close to those for the constant β economy, hence most of the reduction in wealth concentration is a direct effect of the change in preferences. Finally figure 1 reports the Lorenz curves for the three economies. Consistent with the values of the Gini index and coefficient of variation we see that the continuous curve for the homothetic model lies further from the 45 degree line that signals perfectly equally distributed wealth, while the dashed and dash-dotted lines lie closer indicating less wealth concentration.

The models considered so far exhibit a Gini coefficient of wealth in the

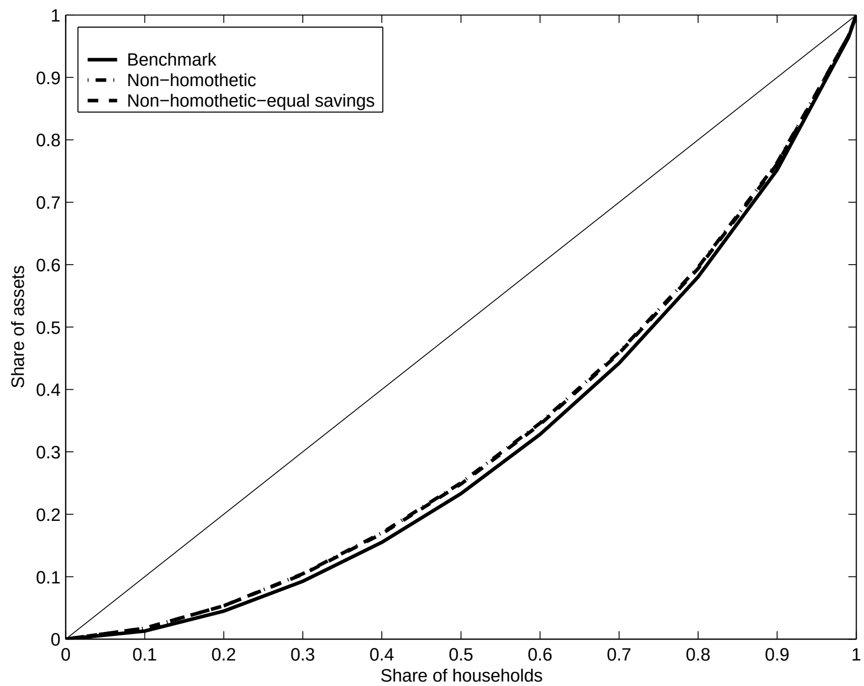

Figure 1: Lorenz curve for assets: Benchmark earnings process economies

range of 0.35 to 0.37, well below the value in the data that according to Budría-Rodríguez et al. (2002) was 0.803 in the 1998 issue of the Survey of Consumer Finance. A key reason for this discrepancy is that earnings concentration is much lower in the process we have used than in the data: the Gini index is 0.22 in the model versus 0.611 in the data. For this reason we revisit the results thus far obtained under two alternative earnings processes that display substantially greater volatility.

4.2 Economies with high earnings variability

In this section we consider two economies that display higher variability of earnings, matching respectively those that can be found in the SCF and in the PSID.

4.2.1 SCF earnings process

In the economy based on the SCF we take the labor earnings process used by Díaz et al. (2003). The realizations of the earnings process and the transition probabilities between states are reported in table 4. The implied Gini index for earnings is 0.588, very close to the value of 0.611 in the 1998 issue of the SCF. More details about the distributional properties of this process in comparison with the data are given in Table 5 where selected points of the Lorenz curve for earnings are given.[10] As it can be seen the process matches closely the share of earnings of the top 10 and especially the top 20 percent of the distribution. It overestimates somewhat the share of the

[10]The empirical distribution is taken from Budría-Rodríguez et al. (2002) and is based on the 1998 issue of the SCF.

bottom 40 percent — by 3.4 percentage points —, and it underestimates a bit the share of the top 1 percent, in this case by 7 percentage points. Overall we can say that it does a quite satisfactory job, taking into account that with 3 states only it is very parsimonious. The choice of this process must be taken as illustrative of what the implications of non-homothetic preferences are for precautionary savings and the wealth distribution when earnings risk is taken to the extreme. The adoption of this process requires two further adjustments in the choice of parameters. First we want the baseline economy with homothetic preferences to generate the same capital output ratio as the benchmark economy with the Aiyagari's earnings process, hence we need to reduce the subjective discount factor β to 0.852 to compensate for the much higher desire to save in the face of the heightened earnings risk. Second, with higher earnings variability also comes higher variability in consumption expenditures. For this reason, in order to match the usual targets of the median share of expenditures on basic goods and its gradient across consumption quartiles an adjustment in the parameters η and λ is also needed. The chosen values, as shown in the last row of table 4, are 1.7 and 1.55 respectively. With these parameters, risk aversion for an agent that consumed virtually only luxuries would converge to 1.36.

Table 6 reports the results. In the first column we can see the main statistics for the benchmark economy with homothetic preferences. The values of assets, output, the capital-output ratio and the interest rate are by construction equal to those of the benchmark economy with the Aiyagari's earnings process.[11] Precautionary savings though, now stands at the very large value

[11]Occasionally one may see tiny differences in the tables for values that are supposed to be equal by construction. These are due to the precision of the root finding algorithm,

Table 4: **Parameters: Benchmark and non-homothetic economies. High earnings variability I**

e_1, e_2, e_3		0.179	0.949	8.351
		0.992	0.008	0.000
$\pi_{e,e'}$		0.009	0.980	0.011
		0.000	0.083	0.917
	β	η	ϕ	λ
	0.852	1.7	1.1	1.55

Table 5: **The earnings distribution: data and high earnings risk model I**

Percentiles	bottom 40	top 20	top 10	top 1
Data	3.8	60.2	42.9	15.3
Model	7.2	62.7	53.2	8.4

of 156.2 percent, a consequence of the very high risk entailed by the earnings process.[12] The measures of inequality, have now increased substantially and the Gini index, at 0.819 is close to the 0.803 in the SCF data. Moving to the second and third column of the table we see that once we introduce non-homothetic preferences, aggregate capital and the capital-output ratio fall dramatically by about 52 and 37 percent. Consequently precautionary saving is reduced to about 80 percent. The interpretation is that with non-homothetic preferences risk aversion declines with the share of luxuries in total expenditures, hence also with wealth. With the high earnings volatility process wealth is highly concentrated in the hands of the very rich. In turn these will hold a large amount of wealth, devote a large fraction of expenditures to luxury goods, hence they will also be little averse to risk. As a result the reduction in aggregate savings and total capital will be large. The most interesting result though, concerns the measures of inequality. As we can see from the bottom two rows, the coefficient of variation and the Gini index of wealth increase by 7.1 and 4 percent respectively. This result stands in contrast with the model that used the low variability earnings process where measures of inequality decreased when we moved from the homothetic to the non-homothetic model.

The last two columns of the table present the results for the model with non-homothetic preferences but with the subjective discount factor adjusted

compounded by rounding in reporting the results.

[12]Recall here that we compute precautionary savings as the percentage difference in total assets between the benchmark economy and an otherwise equally parameterized deterministic economy. The latter now features the low value of 0.852 for β, needed to keep capital constant across all the homothetic benchmark economies that we consider.

Table 6: **Main statistics: Benchmark and non-homothetic economies. High earnings variability I**

	B	NH	% change	NHA	% change
			$\frac{NH-B}{B}$		$\frac{NHA-B}{B}$
Aggregate assets	3.49	1.69	-51.6%	3.48	–
Output	1.06	0.82	-23.0%	1.06	–
Capital/output ratio	3.29	2.07	-37.1%	3.28	–
Interest rate (%)	2.94	9.41	+220.1%	2.96	–
Precautionary savings	156.2%	80.1%	-48.7%	156.2%	–
C.V. of wealth	2.424	2.596	+7.1%	2.494	+2.89%
Gini of wealth	0.819	0.852	+4.0%	0.840	+2.56%

to 0.918 so as to keep aggregate capital equal to the one in the homothetic model. Clearly all aggregate statistics are equal to those of the benchmark homothetic model. The only difference regards the Gini index and the coefficient of variation of wealth. As we can see from the last two rows we still notice an increase of these two measures over the homothetic model, the increase though is smaller: 2.9 and 2.5 percent respectively. Figure 2 shows the Lorenz curves for the three high variability economies. Consistently with the Gini index we see that the curves referring to the non-homothetic economies are further from the 45 degree lines, showing that the latter economies produce more wealth inequality than the homothetic economy when earnings risk is very high. Also visual inspection of the curves suggests that it is the

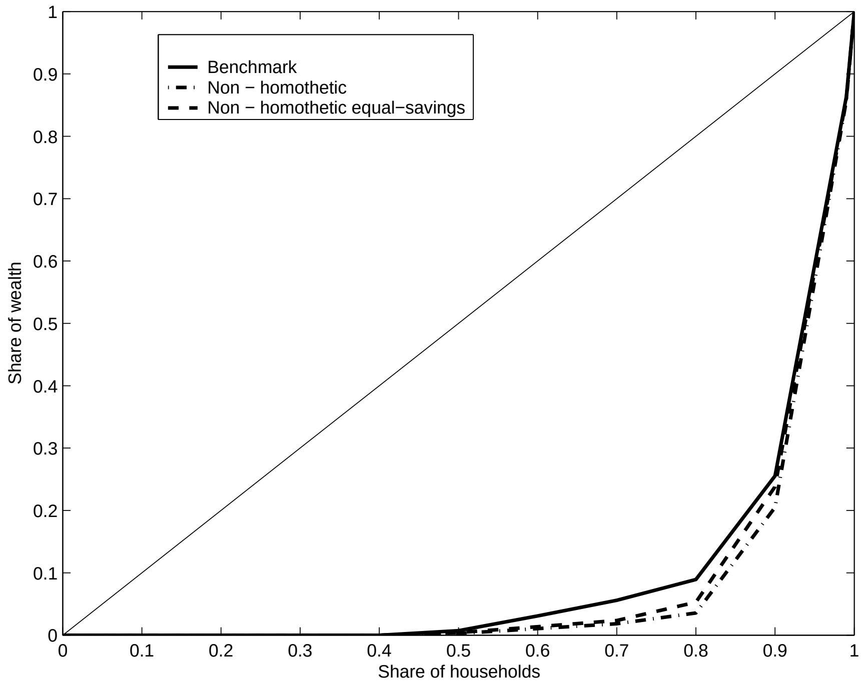

Figure 2: Lorenz curve for assets: High earnings variability process (SCF) economies

90-99 percentiles where the gains in shares of wealth are concentrated.[13]

4.2.2 PSID earnings process

In this second economy we take the earnings process used by Quadrini (2000). Quadrini estimates an $AR(1)$ process on PSID data, assuming that agents are of two types with different average lifetime earnings. He then characterizes the infinite living agents of his model as a succession of generations of expected duration 35 years and sets the probability of transition across earnings types so as to match the intergenerational correlation of earnings of

[13]This statement is backed by the data used to construct the figure, which are not reported for the sake of brevity.

0.5 reported for example in Solon (1992).[14] The transition matrix and the value of the endowment of labor efficiency units in each state are reported in table 7. The resulting Gini coefficient for earnings is 0.39 which matches very closely the one for the PSID in the years 1970-1992 reported in Quadrini (2000). This value falls between the one in the Aiyagari's economy and the one in the SCF economy and must be taken as illustrative of a case where dispersion in wealth, hence expenditures is closer to the data while not being so extreme as in the process used in Díaz et al. (2003).

Like in the previous case we first simulate an economy with homothetic preferences where we adjust β so that we keep the same aggregate assets of the benchmark economy with Aiyagari's earnings process. In this case the value of β needs to be set at 0.928. Then we run two versions of the non-homothetic economy: one that keeps the same parametrization as the benchmark and the second that further adjusts the subjective discount factor so as to keep aggregate capital constant. The change in the distribution of consumption expenditures also requires a change in the parameters η and λ that determine the median share of basic goods in total consumption and its variation between the bottom and top quartile of the consumption distribution, that is, our targets in the calibration. As shown in the last row of table 7 we now set $\eta = 1.7$ and $\lambda = 1.85$. Given equation 8 this implies that the limiting value of risk aversion for an agent consuming almost entirely luxury goods is 1.23.

Results for the economies described in this section are reported in table 8. Looking at the first column of the table we see that precautionary savings in the homothetic economy amount to 76.8 percent. As for the inequality

[14]See Quadrini (2000) for details on how the process is constructed.

statistics the coefficient of variation of wealth is 0.939 and the Gini index is 0.516. Looking at the second column we see that when we move to the non-homothetic model the capital-output ratio falls by 21.3 percent and aggregate wealth falls by 31.2 percent, leaving precautionary savings at only 21.7 percent. Again this is a consequence of the fact that earnings, hence wealth, display a substantial degree of concentration. With non-homothetic preferences risk aversion declines with consumption expenditures leading to a substantial reduction in savings by the wealthy. The measures of wealth inequality increase as well but the increase is in this case modest: 2.2 percent for the coefficient of variation and 0.4 percent for the Gini index. The fourth column of table 8 shows the results for the non-homothetic model where we readjust the subjective discount factor to keep aggregate assets unchanged. Looking at the last two rows we see that the coefficient of variation of wealth is 0.953 and the Gini index is 0.518, representing an increase over the benchmark model of 1.5 and 0.4 percent respectively. Also in this case then the introduction of two goods with different expenditures elasticities leads to an increase in wealth inequality, even though in this case the increase is small. This is confirmed by figure 3 that displays the Lorenz curve for the three economies. As it can be seen the three curves are almost overlapping and indeed do cross each other.

5 Conclusions

There is by now a large literature devoted to the study of precautionary savings and wealth inequality based on models with exogenous stochastic earnings that are uninsurable. This literature typically assumes one single

Table 7: **Parameters: Benchmark and non-homothetic economies. High earnings variability II**

e_1, e_2, e_3, e_4	0.246	0.531	1.023	2.201
$\pi_{e,e'} =$	0.743	0.250	0.005	0.002
	0.250	0.743	0.002	0.005
	0.005	0.002	0.743	0.250
	0.002	0.005	0.250	0.743
	β	η	ϕ	λ
	0.928	1.7	1.1	1.85

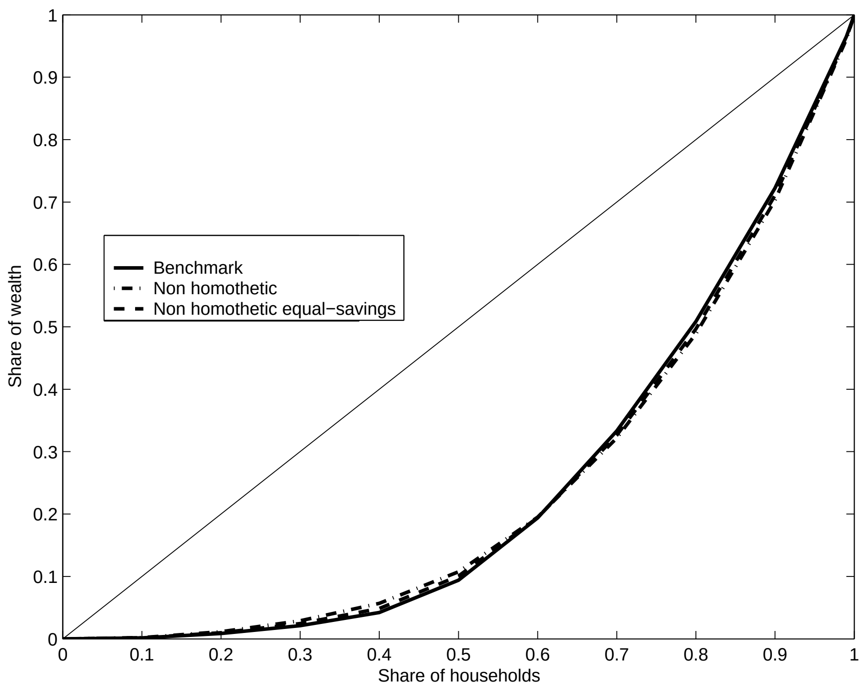

Figure 3: Lorenz curve for assets: High earnings variability process (PSID) economies

Table 8: **Main statistics: Benchmark and non-homothetic economies. High earnings variability II**

	B	NH	% change	NHA	% change
			$\frac{NH-B}{B}$		$\frac{NHA-B}{B}$
Aggregate assets	3.49	2.40	-31.2%	3.50	–
Output	1.06	0.93	-12.6%	1.06	–
Capital/output ratio	3.29	2.59	-21.3%	3.30	
Interest rate (%)	2.94	5.89	+100.3%	2.92	
Precautionary savings	76.8%	21.7%	-71.7%	+76.8%	–
C.V. of wealth	0.939	0.960	+2.24%	0.953	+1.49%
Gini of wealth	0.516	0.518	+0.39%	0.518	+0.30%

homogeneous good. In the current paper we have moved one step beyond by assuming that there are two goods in the economy — a basic and a luxury consumption good — with non-homothetic intra-temporal preferences over the two goods. Based on work by Wachter and Yogo (2010) we have calibrated those preferences so as to match the share of expenditures on basic goods for the median consumer and its variation over quartiles of the consumption distribution.

We find that compared to an otherwise equally parameterized model with homothetic preferences, the two goods model generates a lower amount of precautionary savings. The reduction in precautionary saving is always significant. The effect on the wealth distribution on the other hand is ambiguous. In the non-homothetic model risk aversion declines with the share of luxury in total consumption, hence with total consumption expenditures and wealth. This works towards a less concentrated wealth distribution. At the same time the elasticity of inter-temporal substitution increases with wealth, so that wealthier agents can more easily cut consumption in the face of negative earnings shocks. This second force works towards a more concentrated wealth distribution. As it turns out the overall effect depends on the amount of earnings risk: wealth concentration is reduced for a moderate amount of risk but is increased for large values of earnings risk. Whatever the sign of the change though, quantitatively this change is not big, hence these preferences do not seem to be likely candidate to substantially improve our understanding of the wealth distribution.

There are several natural extensions of the ideas in this paper. One is the application to a life-cycle setting to study savings over the life-cycle following the structural estimation approach of Gourinchas and Parker (2002). Also, there is a considerable literature on consumption inequality, the nature of earnings risk and the extent to which available insurance translate the latter into the former. In light of the work by Aguiar and Bils (2013) it seems promising to apply the current framework to this area of macroeconomics. The latter is the object of an ongoing research effort but it is outside the scope of this paper.

Appendix: Numerical solution method

In this appendix we briefly describe the numerical method adopted to solve the model studied in this paper. The model and the associated solution method are standard in most respects. The algorithm can be summarized by the following steps:

(i) Guess an initial value for the interest rate r.

(ii) Solve the dynamic optimization problem described by equations 9 and 10.

(iii) Use the decision rule $g^a(a, e)$ and realizations of the exogenous process for e to simulate the economy and obtain total asset holdings.

(iv) If total asset holdings match capital demand, then stop. Otherwise go back to step (i) with a new guess for the interest rate.

The dynamic optimization problem is solved by value function iteration, the value function is approximated by cubic splines and the static maximization to be performed at each state space point uses Brent's (1973) algorithm.[15] Successive guesses on the interest rate r are obtained following a simple bisection scheme.

Where the solution method needs to deviate from standard practice is at the evaluation of the flow of utility from current consumption. This is because for each level of consumption utility cannot be simply evaluated as

[15] Given that the time to solve the household's dynamic programming problem with the algorithm described in the text is small and comparable to the simulation time we refrain from using faster methods, like Carroll's (2006) endogenous gridpoints algorithm.

the power utility value of that level of consumption. This flow of utility is instead the result of the solution of the problem:

$$\max_{b,l} U(b,l)$$

subject to the constraint:

$$b + l = c$$

Solving this problem each time the maximization routine in the dynamic programming problem needs to evaluate the utility of consumption level c would make the algorithm exceedingly slow. For this reason we work with an approximation of the period utility function that can be computed once and for all at the beginning of the algorithm. In practice we define a grid for consumption expenditures $\{c_i\}_{i=1}^n$. For each value of expenditures c_i we solve the problem:

$$\max_{b,l} U(b,l)$$

subject to the constraint:

$$b + l = c_i$$

The solution to this problem delivers a value of the maximized intra-temporal utility function $\widehat{u}(c_i)$ and a cubic spline is fitted on the sequence of points $\{\widehat{u}(c_i)\}_{i=1}^n$ thus obtained. This cubic spline is then used to evaluate the period utility flow from consumption expenditures.

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