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# Temporal distortion of the annual modulation signal of weakly interacting massive particles at low recoil energies

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**Abstract**

We show that the main features of the annual modulation of the signal expected in a WIMP direct detection experiment, i.e., its sinusoidal dependence with time, the occurrence of its maxima and minima during the year and (under some circumstances) even the one-year period, are modified for anisotropic galactic halos. The most relevant effect is a peculiar distortion of the time-behaviour at low recoil energies for tangential anisotropies. The observation of these effects could provide a direct way of measuring (or setting limits on) the local degree of anisotropy of the velocity distribution of the dark matter particles.


## 1. Introduction

It is widely believed, as suggested by a host of independent cosmological and astrophysical observations, that most of the matter in the Universe is not visible, revealing its existence only through gravitational effects. In particular, data on the rotational curves of galaxies indicate that the galactic visible parts are surrounded by dark halos which extend up to several times the size of the luminous components.

The best candidates to provide dark matter in galaxies are Weakly Interacting Massive Particles (WIMP). Several WIMP direct-detection experiments are operating [1], with the goal of measuring the nuclear recoil energy (in the keV range) expected to be deposited in solid, liquid or gaseous targets by the scattering of the non-relativistic dark halo WIMPs. Expected rates are small and the exponential decay of the WIMP recoil-spectrum mimics that of the background at low energies. However, a specific signature can be exploited in order to disentangle a WIMP signal from the background: the annual modulation of the rate [2]. This effect, expected to be of the order of a few per cent, is induced by the rotation of the Earth around the Sun, which periodically changes the WIMP flux relative to the detector. The observation of the annual modulation effect is widely known to be a smoking gun for the discovery of galactic dark matter. The annual modulation effect

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has been experimentally investigated by the DAMA Collaboration, which has indeed reported a positive evidence by using a 100 kg sodium iodide detector [3].

In the present Letter we show that the annual modulation signature has additional features as compared to what is usually assumed, features that have the remarkable property of being sensitive to the shape of our galactic halo in velocity space. The usual sinusoidal time-dependence of the rate with maximum (or minimum) around June 2nd is altered in a peculiar way for local anisotropies in the velocity distribution. The type and amount of anisotropy is observable in the specific distortions of the time-dependence of the rate. The study of the time-behaviour of the annual modulation signal is therefore a unique probe of the local degree of anisotropy of the dark matter particles bounded to our Galaxy.

## 2. The annual modulation effect

Due to the rotation of the disk around the galactic center, the Solar system moves through the WIMP halo, assumed to be at rest in the galactic rest frame. In the following, we will assume a right-handed system of orthogonal coordinates: the $x$ axis in the galactic plane, pointing radially outward; the $y$ axis in the galactic plane, pointing in the direction of the disk rotation; the $z$ axis directed upward, perpendicular to the galactic plane. Notice that our system differs from standard "galactic coordinates" by the different choice of the $x$ axis, which for us is directed outward.

The relative velocity between the WIMP halo and the detector is given by the Earth velocity $\vec{v}^E$, as seen in the galactic rest frame. It is the sum of three components: the galactic rotational velocity $\vec{v}^G = (0, v_0, 0)\ \mathrm{km\,s^{-1}}$ (we will assume: $v_0 = 220\ \mathrm{km\,s^{-1}}$), the Sun proper motion $\vec{v}^S = (-9, 12, 7)\ \mathrm{km\,s^{-1}}$ [4] and the Earth orbital motion $\vec{u}^E(t)$ [4]:

$$v_x^E = v_x^G + v_x^S + u^E(\lambda)\cos\beta_x \cos\big[\omega(t - t_x)\big], \tag{1}$$

$$v_y^E = v_y^G + v_y^S + u^E(\lambda)\cos\beta_y \cos\big[\omega(t - t_y)\big], \tag{2}$$

$$v_z^E = v_z^G + v_z^S + u^E(\lambda)\cos\beta_z \cos\big[\omega(t - t_z)\big], \tag{3}$$

where $\lambda$ is the ecliptic longitude, which is function of time. We can express $\lambda$ as [4]:

$$\lambda = L + 1.915^\circ \sin g + 0.020^\circ \sin 2g,$$

where

$$L = 280.460^\circ + 0.9856474^\circ\, t,$$
$$g = 357.528^\circ + 0.9856003^\circ\, t,$$

and $t$ denotes the time expressed in days relative to UT noon on December 31. In Eqs. (1)–(3) $u^E(\lambda) = \langle u^E \rangle [1 - e\sin(\lambda - \lambda_0)]$ is the modulus of the Earth rotational velocity, which slightly changes with time due to the small ellipticity $e$ of the Earth orbit ($\langle u^E \rangle = 29.79\ \mathrm{km\,s^{-1}}$, $e = 0.016722$ and $\lambda_0 = 13^\circ \pm 1^\circ$) [4]. In Eqs. (1)–(3) the $\beta_i$ denote the ecliptic latitudes and the $t_i$ are the phases of the three velocity components: $\beta_x = 174.4697^\circ$, $\beta_y = 59.575^\circ$, $\beta_z = 29.812^\circ$ and $t_x = 76.1$ day, $t_y = 156.3$ day, $t_z = 352.4$ day. The angular velocity has a period of 1 year, and is given by $\omega = 2\pi/(365\ \text{days})$. With the numbers given above, the modulus of the Earth velocity changes in time as:

$$v_E \equiv |\vec{v}_E| \simeq 233.5 + 14.4\cos[\omega(t - t_0)]$$

(in $\mathrm{km\,s^{-1}}$), where $t_0 \simeq 152$ days, i.e., June 2nd. Notice the slight offset between $t_0$ and $t_y$, due to the composition of the velocity components. As a good approximation, $v_E$ is usually taken as: $v_E = (v_0 + v_y^S) + \langle u^E \rangle \cos\beta_y \cos[\omega(t - t_0)]$.

The direct detection differential rate $dR/dE_R$ is proportional to the integral:

$$\mathcal{I}(v_{\min}) = \int\limits_{w \geqslant v_{\min}} d\vec{w}\, \frac{f_{\mathrm{ES}}(\vec{w})}{w}, \tag{4}$$

where $f_{\mathrm{ES}}$ and $w$ are the local WIMP velocity DF and the WIMP velocity, respectively, in the Earth's rest frame; $v_{\min}$ is the minimum value of $w$ for a given recoil energy $E_R$, WIMP mass $m_W$ and nuclear target mass $m_N$ and is given by

$$v_{\min} \equiv \sqrt{E_R/(2m_N)}\,(m_W + m_N)/m_W.$$

By indicating with $f(\vec{v})$ and $\vec{v}$ the local WIMP velocity DF and the WIMP velocity in the galactic reference frame, the following transformations hold:

$$\vec{v} \to \vec{w} = \vec{v} - \vec{v}^E(t), \tag{5}$$

$$f(\vec{v}) \to f_{\mathrm{ES}}(\vec{w}) = f\big(\vec{w} + \vec{v}^E(t)\big), \tag{6}$$

which imply that $\mathcal{I}(v_{\min})$, and so $dR/dE_R$, develops a time dependence induced by $\vec{v}_E(t)$.

Direct detection is sensitive to local (solar neighbourhood) galactic properties: the rate $R$ is in fact proportional to the local dark matter density $\rho_l$ [5] and probes the *local* velocity distribution of dark matter particles although, in general, the velocity DF is a function of the position in the Galaxy [5]. Therefore, in our discussion $f(\vec{v})$ denotes the dark matter velocity DF in the neighbourhood of the Sun.

In our analysis, the properties that we will derive on the time-behaviour of the direct detection rate are independent of the value of the local matter density: the relevant function which describes the time-behaviour, $\mathcal{I}(v_{\min})$, depends only on $f(\vec{v})$ and not on $\rho_l$. Therefore, the features that we will show are pure properties on the shape of the behaviour of the annual modulation effect as a function of time, and they are decoupled from the absolute normalization given by $\rho_l$.

The two quantities relevant to direct detection, $\rho_l$ and $f(\vec{v})$, can be determined in a self-consistent way by means of a complete halo modeling (see, for instance, Ref. [5] and references therein): in this way, a correlation between $\rho(\vec{r})$ and $f(\vec{v})$ can be found. Out of the two, only $f(\vec{v})$ matters for determining the time-behaviour of the annual modulation effect.

## 3. The isotropic isothermal sphere

In the case of the isothermal sphere model the DF is given by a truncated isotropic Maxwellian which depends only on $v \equiv |\vec{v}|$:

$$f(v) = N \exp\left(-\frac{3v^2}{2\sigma^2}\right)\bigg|_{v \leqslant v_{\rm esc}}, \tag{7}$$

where $\sigma$ is the WIMP r.m.s. velocity, given by $\sigma^2 = 3v_0^2/2$. It is clear that, through the change of reference frame of Eq. (6), $\mathcal{I}(v_{\min})$ depends on time only through $v_E$. Since the relative change of $v_E$ during the year is of the order of a few per cent, we can approximate $\mathcal{I}(v_{\min})$ with its first-order expansion in the small parameter $\epsilon \equiv \delta v_E/v_E$, around its mean value $\mathcal{I}_0(v_{\min})$:

$$\mathcal{I}(v_{\min}) - \mathcal{I}_0(v_{\min}) \propto \cos\big[\omega(t - t_0)\big].$$

The well-known result is then obtained that the WIMP rate has a sinusoidal time-dependence with the same phase ($t_0 \simeq$ June 2nd) as $v_E$, for all values of $v_{\min}$. This is shown in Fig. 1, where $\mathcal{I}(v_{\min})$ is plotted as a function of time for various values of $v_{\min}$.

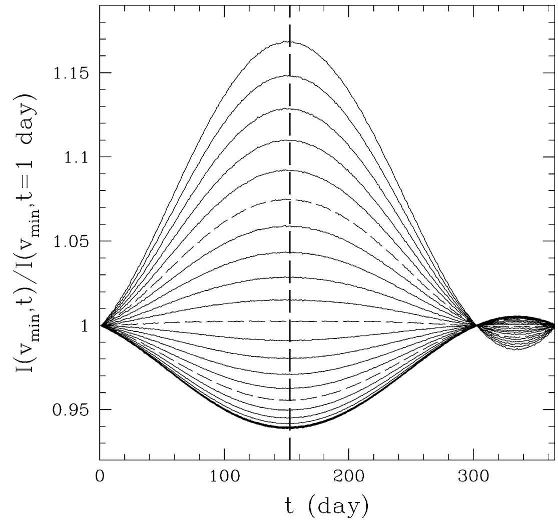


Fig. 1. Time-dependence of $\mathcal{I}(v_{\min})$ for an isotropic isothermal sphere. The different curves refer to values of $v_{\min}$ ranging from 0 (lower) to 400 (upper) km s$^{-1}$, in steps of 20 km s$^{-1}$ (the dashed curves correspond to $v_{\min} = 100$, 200 and 300 km s$^{-1}$). The vertical dashed line denotes $t = 152$ days.

## 4. Anisotropic models

As a simple generalization of the isothermal sphere model to a triaxial system in velocity space, we use a multivariate Gaussian:

$$f(\vec{v}) = N \exp\left(-\frac{v_x^2}{2\sigma_x^2} - \frac{v_y^2}{2\sigma_y^2} - \frac{v_z^2}{2\sigma_z^2}\right)\bigg|_{v \leqslant v_{\rm esc}}, \tag{8}$$

where $N$ is the normalization constant. For $v_{\rm esc} \to \infty$, then

$$N = \left[\left(2\pi\sigma_x^2\right)\left(2\pi\sigma_y^2\right)\left(2\pi\sigma_z^2\right)\right]^{-1/2}.$$

The usual isothermal sphere is the spherical limit of Eq. (8), obtained with: $\sigma_x = \sigma_y = \sigma_z \equiv \sigma/\sqrt{3}$. In order to discuss the effect of anisotropy at fixed WIMP mean kinetic energy, we will fix $\sigma^2 = \sigma_x^2 + \sigma_y^2 + \sigma_z^2$ as in the isothermal case ($\sigma^2 = 3v_0^2/2$) and discuss our results in terms of the two independent parameters: $\lambda_{12} \equiv \sigma_x/\sigma_y$ and $\lambda_{32} \equiv \sigma_z/\sigma_y$.

Eq. (8) is a useful parametrization to study velocity anisotropies at the Solar system position. This distribution function may not hold everywhere in the Galaxy, since the class of models described by Eq. (8) do not satisfy the collisionless Boltzmann equation, but only the lowest order moments. However, since direct detection can only probe local halo properties, Eq. (8)

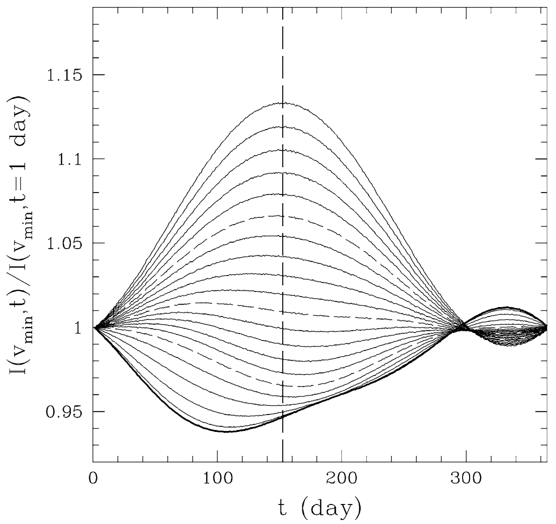


Fig. 2. The same as in Fig. 1, for an anisotropic model with $\lambda_{12} \equiv \sigma_x/\sigma_y = 0.2$ and $\lambda_{32} \equiv \sigma_z/\sigma_y = 0.8$.

represents a useful parametrization to illustrate the effects induced by local anisotropies in velocity space.

At variance with the isothermal sphere, now $\mathcal{I}(v_{\min})$ depends in general on all the three components of $\vec{v}_E$, and not simply on $v_E$. We can write:

$$f_{\mathrm{ES}}(\vec{w}) = N \exp\left[-(\vec{r}+\vec{\eta})^2\right], \tag{9}$$

where we have defined the reduced (dimensionless) variables:

$$r_i = \frac{w_i}{\sqrt{2}\sigma_i},$$
$$\eta_i = \frac{v_i^E}{\sqrt{2}\sigma_i} = \eta_i^0 + \delta\eta_i \cos\left[\omega(t-t_i)\right] \quad (i = x, y, z).$$

In the isotropic case (isothermal sphere) one has $\eta_{x,z}^0 \ll 1$, $\eta_y^0 \sim 1$ and $\delta\eta_i \ll 1$. The presence of the order-one parameter $\eta_y^0$ and of the small oscillation amplitudes $\delta\eta_i$ allows the Taylor-expansion of $\mathcal{I}(v_{\min})$ in terms of the $\epsilon_i \equiv \delta\eta_i/\eta_y^0$ parameters. A straightforward calculation shows that the conclusions of the previous section are recovered.

On the contrary, allowing anisotropies such that $\lambda_{12} < 1$ and/or $\lambda_{32} < 1$, the values of the parameters $\eta_{x,z}$ and $\delta\eta_{x,z}$ are enhanced, and the time-dependence of $v_x^E$ and $v_z^E$ in $\mathcal{I}(v_{\min})$ may become important. An example of this situation is shown in Fig. 2, where the time evolution of $\mathcal{I}(v_{\min})$ is plotted for $\lambda_{12} = 0.2$ and $\lambda_{12} = 0.8$ (i.e.: $(\sigma_x, \sigma_y, \sigma_z) = (42, 208, 166)\ \mathrm{km\,s^{-1}}$). This choice of $\lambda_{12}$, $\lambda_{32}$, which refers to a tangential anisotropy, corresponds to triaxial models discussed, for instance, in Refs. [5,6]. The amount of tangential anisotropy used in Fig. 2 is similar to the one discussed in the triaxial halo models of Ref. [6], where a sizeable velocity anisotropy was obtained for halos with mild anisotropic dark matter density profile: $\sigma_R/\sigma_T \simeq 0.16$ ($\sigma_R$ and $\sigma_T$ denote the radial and tangential velocity dispersions). A distortion of the curves of Fig. 2, as compared to the familiar sinusoidal time-dependence, appears: this effect may be explained by the fact that now $\delta\eta_x \sim 1$, a Taylor-expansion of the type used in the isothermal sphere case breaks down and a full numerical calculation of the integral of Eq. (4) is required. The final result is not sinusoidal. This peculiar behaviour is more pronounced at low values of $v_{\min}$ (i.e., low recoil energies) namely for $v_{\min} \lesssim 80\ \mathrm{km\,s^{-1}}$, for which the distortion is strong and the maxima (in absolute value) of the rate are shifted as compared to the standard case [7]. For larger values of $v_{\min}$ the distortion is less pronounced, and it dies away when $v_{\min} \gtrsim 230\ \mathrm{km\,s^{-1}}$. This may be explained by the fact that, as $v_{\min}$ grows, the integral of Eq. (4) becomes less sensitive to the parameter $\eta_x$ since it gets increasingly dominated by WIMPs with velocities along the $y$ axis, which is the one along which the boost due to the galactic rotation is directed. We notice that for values of $v_{\min}$ around (100–200) $\mathrm{km\,s^{-1}}$ a distortion is present, but the amplitude of the modulation is suppressed and therefore difficult to detect.

As a second example, in Fig. 3 we plot $\mathcal{I}(v_{\min})$ as a function of time for the case $\lambda_{12} = 10$, $\lambda_{32} = 3$ (i.e.: $(\sigma_x, \sigma_y, \sigma_z) = (257, 26, 77)\ \mathrm{km\,s^{-1}}$). This situation is representative of a radial anisotropy. In this case, we have further enhanced the contribution of $\delta\eta_y$ over $\delta\eta_{x,z}$, so that one should expect to draw the same conclusions as in the isothermal sphere case, with the usual sinusoidal time-dependence of the rate and a phase close to $t_y \sim t_0$. This is indeed the case, except for a narrow interval of values of $v_{\min}$ around $v_{\min} \simeq 210\ \mathrm{km\,s^{-1}}$. In this range, $\mathcal{I}(v_{\min})$ develops two maxima because, for that particular choice of $v_{\min}$, there is an exact cancellation in the first term of the expansion in $\epsilon_y = \delta\eta_y/\eta_y^0$, so that the term $\mathcal{O}(\epsilon_y^2)$ proportional to $\cos^2[\omega(t-t_y)]$ sets in. This particular cancellation of the first term in the Taylor expansion of $\mathcal{I}(v_{\min})$ happens also for the isothermal-sphere model,

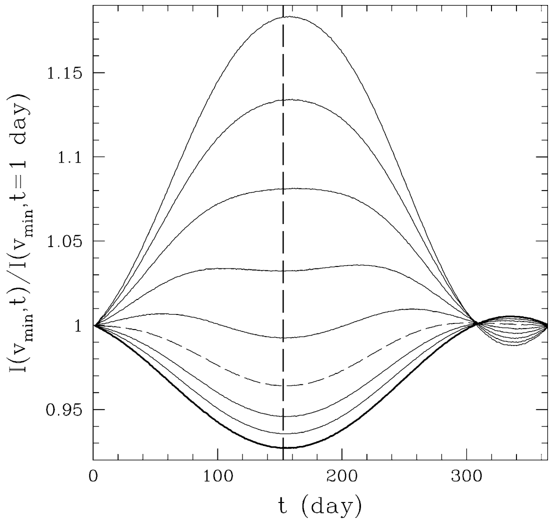


Fig. 3. The same as in Fig. 1, for an anisotropic model with $\lambda_{12} \equiv \sigma_x/\sigma_y = 10$ and $\lambda_{32} \equiv \sigma_z/\sigma_y = 3$. The curves refer to: $v_{\rm min} = 0$ (lower), 180, 190, 200 (dashed), 210, 220, 230, 240, 250 (upper) $\rm km\,s^{-1}$.

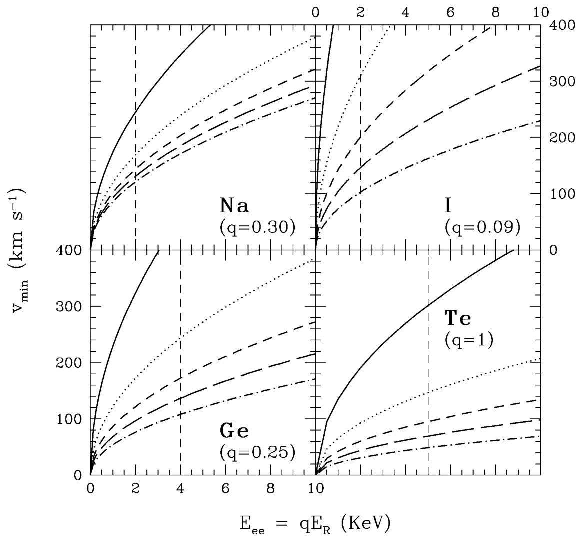


Fig. 4. Values of $v_{\rm min}$ as a function of the quenched nuclear recoil energies $E_{ee} = qE_R$, for: NaI scintillators [3], Ge ionization detectors [1] and Te bolometers [1]. For each panel, the different curves refer to WIMP masses of: 20 GeV (solid), 50 GeV (dotted), 100 GeV (dashed), 200 GeV (long-dashed) and 1 TeV (dot-dashed). The dashed vertical lines denote the current energy thresholds.

but in that case the size of the quadratic term is strongly suppressed because $\epsilon_y$ is much smaller. In a WIMP direct detection experiment this effect would show up in a very peculiar way: a halving of the modulation period of the rate in a narrow range of recoil energies. Notice, however, that in order to have some realistic chance to detect this effect, it should show up in one of the experimental energy bins *just above threshold*, where the highest signal/background ratio is usually attained. This peculiar effect due to radial anisotropies would be in any case very difficult to detect, since it would require large radial anisotropies in order to be visible. The values of $\lambda_{12}$ and $\lambda_{32}$ used in Fig. 3, which show the effect as visible at the percent level, are actually large: $\sigma_R/\sigma_T \simeq 3$. Systems where the radial velocity dispersion is much larger than the tangential velocity dispersion may be instable, for instance, they may suffer "firehose" instability [8] for axis ratios in excess of 2 : 5. These considerations imply that this type of distortion of annual modulation is likely to be suppressed and therefore difficult to observe, unless the experimental sensitivity is at a level better than a percent.

In order to establish a link between our discussion and WIMP direct detection experiments [1], in Fig. 4 we plot $v_{\rm min}$ as a function of the quenched nuclear recoil energy $E_{ee} \equiv qE_R$ ($q$ is the quenching factor) for the target nuclei: Na, I, Ge, Te and for different WIMP masses. The vertical dashed lines show current energy thresholds achieved by each type of detector. Fig. 4 shows that values of $v_{\rm min} \lesssim 80\ \rm km\,s^{-1}$, i.e., sufficiently low to observe a sizeable distortion effect as the one discussed for tangential anisotropy, correspond to WIMP recoil energies below the threshold of present direct detection experiments, and that the effect would be more easily detected at higher WIMP masses. However, a foreseeable lowering of the threshold, down to 0.5–1 KeV, could be enough to observe the distortion. On the other hand, for radial anisotropy, we can conclude that the recoil energy corresponding to a halving of the modulation period can actually coincide with the experimental thresholds within the reach of present-day detectors for 20 GeV $\lesssim m_W \lesssim$ 100 GeV, depending on the particular target nucleus. In this respect, we note that the properties of the annual modulation effect observed by the DAMA/NaI experiment [3] (a one-year-period sinusoidal behaviour in the 2–6 KeV energy bins [3,5]) implies that the DAMA/NaI experiment is potentially able to set constraint on strong radial anisotropies.

## 5. Conclusions

In the present Letter we have shown that the typical features of the annual modulation of the signal of WIMP direct searches, i.e., the sinusoidal dependence of the rate with time, the position of its maxima and minima during the year and even the period, are altered in dark matter halos with anisotropic velocity distributions. Depending on the pattern of anisotropy, different situations may occur: local tangential anisotropies induce a peculiar departure at low energies from the usual sinusoidal time-dependence, along with a shift in the position of the extrema of the signal during the year; local radial anisotropies may produce a halving of the modulation period for a specific and restricted range of recoil energies, which depends on the amount of anisotropy. The former effect turns out to be relevant at low recoil energies, actually close or below the threshold of present-day experiments, while the latter should be already within the reach of current detectors. In particular, the properties of the annual modulation effect observed by the DAMA/NaI experiment [3] may indicate that strong radial local anisotropies are excluded, while tangential anisotropies may be even favoured over the isotropic case, since the best fit to the modulation phase is smaller than the isotropic value of 152 days [3]: this is compatible with our discussion on local tangential anisotropy for a NaI detector with a 2 KeV energy threshold.

We conclude that the annual modulation effect, in addition to being a powerful signature of the existence of dark matter particles in the Galaxy, offers also a unique possibility of measuring local properties of the galactic halo which are otherwise hardly testable directly, namely, the local distribution of dark matter particles in velocity space and specifically their degree of anisotropy. The features discussed in this Letter, which up to now have been neglected, will have to be taken into account in present and future dark matter direct searches when anisotropic models are considered.

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