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HighLights

- Reputation can reduce the risks of the buyer in the e-commerce environment
- Reputation management systems can guarantee the reliability of the transactions
- Game theory can model the interaction between the seller and the buyer
- Agent Based Simulation is adopted to deal with the game theory approach complexity
- A hybrid model is proposed and an extensive analysis proves its feasibility

Modelling the rational behaviour of individuals on an e-commerce system

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Abstract

With the increasing popularity of e-commerce systems, commercial transactions are becoming more and more frequent. Such transactions are not direct but mediated, putting the buyer in a position of weakness with respect to the seller, especially in the case of a failure of a transaction. The literature showed that the reputation can play an important role to reduce the risks of the buyer in the current e-commerce environment. An online reputation management system (RMS) maintains the reputation, made of beliefs and/or opinions, that are generally held about someone or something, and it can guarantee the reliability of the transactions that take place in an e-commerce system. Despite of the fact that the basic element of a RMS – the interaction between the seller and the buyer – is a classical field of application of the Game Theory (GT) methodologies, the use of a GT approach in this context seems quite limited and this is probably due to its solution complexity. A way to deal with such a complexity is by exploiting the capability of the agent based simulation (ABS) approach. In this paper, we propose a hybrid GT and ABS model for the analysis of an e-commerce system in which a centralized reputation system is maintained by a trusted third party. We report an extensive quantitative analysis in order to

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validate the proposed model, and to evaluate the impact of a set of buyers' and sellers' policies on the behavior of the e-commerce system.

Keywords: Game theory, Simulation, Behavioral OR, Decision Making

1. Introduction

With the increasing popularity of e-commerce systems, commercial transactions become more and more frequent. Such transactions are not direct but mediated by the supporting online platforms, that is the payment and delivery of the good (or the use of the service) are not at the same time. In the current practice, the seller delivers the good only after receiving the proof of payment from the buyer. In this context, the buyer is in a position of weakness with respect to the seller, especially in the case of a failure of the transaction.

Reputation is *an aggregate composite of all previous transactions over the life of the entity, a historical notion, and requires consistency of the entity's actions over a prolonged time* [1]. Reputation includes not only the direct experiences of the buyer but also any other form of communication – reviews, scores – that provides information about the seller [2]. Reputation can play an important role to reduce the risks of the buyer in the current e-commerce environment. In [3], the authors showed that positive online review scores can positively influence the firm financial performance while the heterogeneity of different product classes moderates the relationship between review score and performance. Furthermore, in [4], the authors reported that a limited number of fake reviews can determine a consistent reduction of the reputation of a competitor.

In order to limit the impact of malicious behaviors, online reputation management systems (RMS) have been developed over the years. RMS is a system that maintains the beliefs or the opinions that are generally held about someone or something. Such a RMS can provide a solution to guarantee the reliability of the transactions that take place in an e-commerce system [5, 6, 7]. Several RMS are proposed in the literature: those systems are based on different methodologies, such as artificial intelligence, multi-agent systems, cognitive science, game

theory, and the social and organizational sciences [8]. In computer science, particular attention has been dedicated to the analysis of the RMSs operating on a peer-to-peer systems [9, 10, 11, 12, 13].

The interaction between the seller and the buyer, which is the basic element of a RMS, is a classical field of application of the Game Theory (GT) methodologies, which allow modeling the rational behavior of the individuals [14]. On the contrary, the use of a GT approach in this context seems quite limited despite of its potential (see, e.g., [15, 16, 17]).

This is probably due to the resulting solution complexity of the GT approach. Such a complexity does not depend on the complexity of each single transaction: actually, the strategic interaction model of a single transaction between seller and buyer is extremely simple as the buyer has to decide whether to buy or not, while the alternative of the seller are to fully comply with the request or not. On the contrary, the complexity relies on the fact that the reputation is the result of (i) a number of repeated transactions between pairs of sellers and buyers, not necessarily the same, and (ii) the sharing with other sellers and buyers of the outcomes of the transaction. Note that the sharing of the outcomes of the transaction represents the learning effect that is typical of repeated games.

A way to deal with such a complexity is by exploiting the capability of the agent based simulation (ABS) approach, widely applied in economics [18, 19]. An ABS model allows tracking the behavior of each individual acting in the simulated environment [20]. A set of rules describes the agent behavior and its interaction with the environment; as a consequence, the state of each agent is determined [21].

In this paper, we propose a hybrid game theory and agent based simulation model for the analysis of an e-commerce system in which a centralized reputation system is maintained by a trusted third party. The individuals' behavior is modeled with a game with incomplete information, which is then *solved* through an agent based simulation model. In order to validate the proposed hybrid model, we assume equal prices for all the sellers. In this way the behavior of the whole system is more predictable to get information about the quality of

the results. Then, we relax such an assumption considering variable prices and evaluating the introduction of an insurance system.

The paper is organized as follows. The game theoretic approach and its complexity are discussed in Section 2. The proposed hybrid model is presented in Section 3: first, we report a basic model in Section 3.1 in such a way to ease the validation, and to introduce the basic notation; then, we extended such a model including the items with variable prices and an insurance system in Sections 3.2 and 3.3, respectively. An extensive quantitative analysis is reported and discussed in Section 4 evaluating the model behavior on several scenario and under the application of several buyers' and sellers' policies. Section 5 closes the paper.

2. The game theoretic approach

In this section we recall the basic notion and notations of non-cooperative games and present the game theoretic model. In the literature there exist both cooperative (see, e.g., [22]) and non-cooperative (see e.g., [23]) models for market situations. Here we consider a non-cooperative model because, in our setting, the buyer and the seller may have different objectives, making impossible the agreement that is at the basis of a cooperative model. Even if, we suppose that the two individuals have the common aim of increasing the number of transactions, they have difficulties in trusting each other.

2.1. Preliminaries

We start by recalling some basic definitions on non-cooperative games, i.e., when interacting individuals, or *players*, cannot subscribe binding agreements.

First, we consider a game in *extensive form*; more precisely, we refer to the tree representation where each node, but the leaves, represents a possible situation of the game and is associated to the player that has the role of moving in that situation, the outgoing arcs are associated to the possible choices, or *moves*, that are available to that player in that situation and each terminal node,

i.e., a leave, represents an exit of the game; the terminal nodes are associated with no player, but to a tuple of real values, each representing the payoff of the corresponding player when the game ends with that exit. This way to represent a game is sometimes cumbersome, but on the other hand it provides a very detailed description of all the possible developments of the game according to all the possible choices of the players.

In order to reduce the amount of data necessary for describing the game, often it is represented in *strategic form*. In this case the game is formally described by a triple $G = (N, (\Sigma_i)_{i \in N}, (u_i)_{i \in N})$ where $N = \{1, 2, ..., n\}$ is the set of players, $\Sigma_i = \{\sigma_i^1, \sigma_i^2, ..., \sigma_i^{k_i}\}$ is the set of pure strategies of player $i \in N$, where a strategy is an ordered sequence of moves of player i, one for each situation in which s/he has to move, and $u_i : E \to \mathbf{R}$ is the utility function of player $i \in N$, i.e., a function that associates to each possible termination of the game in the set of exits E the payoff of player i. Sometimes, we use the preference relations, $\succ_i$, $i \in N$ of the players instead of the utility functions, where $\alpha \succ_i \beta$ means that player $i \in N$ prefers the exit α to the exit β. In fact, the individuals are able to say which exit they prefer for any pair of exits, but it may be very difficult to define the utility associated to an exit. The two concepts are related in the sense that a utility function has to assign a higher utility to a preferred exit, i.e., $\alpha \succ_i \beta \iff u_i(\alpha) > u_i(\beta)$ for each $\alpha, \beta \in E$ for every $i \in N$. The possible exits may be associated, not biunivocally, with a *strategy profile* $(\sigma_1, \sigma_2, ..., \sigma_n) \in \prod_{i \in N} \Sigma_i$, where $\sigma_i \in \Sigma_i, i \in N$ is a strategy of player i. The correspondence is not biunivocal as different strategy profiles may lead to the same exit of the game.

More generally, we can introduce the set of *mixed strategies* for player $i \in N$, that is a probability distribution over the set of her/his pure strategies Σ_i. We denote a mixed strategy by $p_i = (p_i(\sigma_i^1), p_i(\sigma_i^2), ..., p_i(\sigma_i^{k_i}))$ where $p_i(\sigma_i^j) \geq 0$ represents the probability of choosing the pure strategy $\sigma_i^j \in \Sigma_i$, with the condition $p_i(\sigma_i^1) + p_i(\sigma_i^2) + ... + p_i(\sigma_i^{k_i}) = 1$; the set of mixed strategies of player $i \in N$ is denoted by $\Delta(\Sigma_i)$.

Given a mixed strategy profile $p = (p_1, p_2, ..., p_n)$, where $p_i \in \Delta(\Sigma_i), i \in N$,

the corresponding utility for player $i \in N$ is:

$$u_i(p) = \sum_{(\sigma_1,\sigma_2,...,\sigma_n)\in\prod_{i\in N}\Sigma_i} \left(\prod_{i\in N} p_i(\sigma_i)\right) u_i(\sigma_1,\sigma_2,...,\sigma_n)$$

The most usual solution concept for a non-cooperative game is the *Nash Equilibrium* (NE) [24]. A NE in mixed strategies is a strategy profile $(p_1^*,...,p_n^*)$ such that $u_i(p_1^*,...,p_i^*,...,p_n^*) \geq u_i(p_1^*,...,p_i,...,p_n^*)$, for each $p_i \in \Delta(\Sigma_i)$ and for every $i \in N$, i.e., no player has an incentive to unilaterally deviate from $(p_1^*,...,p_n^*)$. For further details we address the interested reader to the book of [25].

2.2. The Model

The basic scheme of a simple e-commerce situation may be represented using a 2-person game where the players are the buyer (B) and the seller (S). Considering just one transaction of an item from the seller to the buyer, the buyer has to choose among purchasing the item (P) or refusing it (R); successively, if the choice of the buyer is P, the seller has to decide if delivering the item (D) or keeping the item and the money (K). These assumptions lead to a non-cooperative game with player set is $N = \{B, S\}$; the set of strategies of B is $\Sigma_B = \{P, R\}$ and the set of strategies of S is $\Sigma_S = \{D, K\}$. Using an extensive form representation we may consider the tree reported in Figure 1.

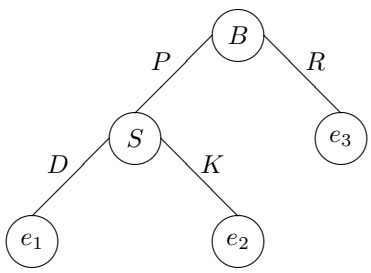


Figure 1: Tree representation of the extensive form.

The possible exits e_1, e_2, e_3, depicted in Figure 1, correspond to the following situations:

- e_1: the buyer decides to purchase the item, pays for it and receives it;
- e_2: the buyer decides to purchase the item and pays for it but the seller keeps the item and the money;
- e_3: the buyer decides not to purchase the item.

Note that the exit e_2 may include also other situations in which the buyer pays for the item but the transaction is not satisfactory for her/him. For instance, we may think of a situation in which the item is delivered by the seller but the buyer does not receive it, or the item is damaged or different from the expectation of the buyer; the formulation we used emphasizes the role of decision-maker of the seller, that could be responsible also when apparently s/he is not, e.g., for choosing a low quality (and cheap) carrier.

To complete the game we need the preferences of the two players on the exits. It seems obvious that exit e_2 is the most profitable for the seller and the less preferable for the buyer. On the other hand, it seems obvious that both players prefer exit e_1 to exit e_3; the last result relies on the assumption that the selling price is higher than the valuation of the item given by the seller and lower than the value that the buyer assigns to the item, so they both prefer to conclude the transaction. We may assign the following ordering to the exits:

$$\text{for the buyer:} \quad e_1 \succ_B e_3 \succ_B e_2\,;$$

$$\text{for the seller:} \quad e_2 \succ_S e_1 \succ_S e_3\,.$$

As the number of moves of the two players is finite, this basic game could be solved by *backward induction*, i.e., in the last situation the seller chooses K that leads to the preferred exit among e_1 and e_2 and consequently, in the first situation the buyer decides for R that leads to the preferred exit among e_2 and e_3. It should be clear that the consequence is that the transaction never take place. The main point is that the seller prefers the exit e_2 if only one

transaction is possible (and also if s/he is not honest). In the real world the seller is interested in carrying out the transaction, and often more than one. This implies that the previous model should be modified in order to account that the honest seller prefers exit e_1, as the buyer, even if the buyer may be not aware of this.

When the risk-aversion of the buyer is sufficiently high, then the best choice is R, so that the exit e_2 cannot realism, unless it is possible to provide enough guarantee that the seller chooses D. Of course, the final decision of the buyer is influenced not only by her/his level of risk-aversion but also by the guarantee that the seller may offer that the item will be delivered. This is the point where reputation plays a role.

The main difficulty to extend the basic game depicted in Figure 1 to a real-world situation arises from two points:

1. the concept of reputation requires that a seller is involved in a high number of transactions; consequently, the decisional tree results very large;
2. even fixing a reasonable number of transactions with a unique seller, it is not possible to consider it as a repeated game, because the buyer is generally different at each transaction; consequently, we cannot exploit the results available in the literature about repeated games.

The second point means also that we have to deal with a very complex game with several players, sellers and buyers: in view of this, the simulation approach seems particularly interesting and advantageous.

We would emphasize two main points: 1) the basic model is extremely simple but becomes very complicated when we introduce the repetition of the game, which is necessary in our context; 2) there exists an exit of the game that both agents prefer with respect to the most cautious exit of not to start the transaction; this is true also for other models, e.g. the investment game (see [26]), but it is hidden by the huge amount of exits.

In situations similar to the previous one in which a player chooses sometimes a strategy and some other times a different one, good results were obtained

looking for a solution in mixed strategies. More precisely, a mixed strategy for the seller is a probability distribution (d, k) with $d, k \in [0, 1], d + k = 1$ where d represents the probability that the seller satisfies the buyer and k the probability that the seller does not satisfy the buyer. This probability distribution accounts for the frequency with which in the past the seller chose D or K; analogously, we may account for the risk-aversion of the buyer using a mixed strategy (p, r) with $p, r \in [0, 1], p + r = 1$, again referring to the frequency with which in the past s/he chose P or R (the more p is close to 1 the less is the risk-aversion, the more p is close to 0 the greater is the risk-aversion).

Anyhow, the probabilities d, k, p, r summarizes the frequencies, but do not provide a detailed history of the agents, differently from the simulative approach.

Even if we decide to represent the probabilities with which the players will choose their strategies in the future, the mixed strategy approach is, in a sense, static, because it considers one transaction at each time, instead of a sequence of transactions, possibly involving different buyers, where it is difficult to account the previous transactions of the seller; on the opposite, the simulation approach allows accounting for dynamic aspects in the behavior of the individuals according to their historical and recent experiences in other transactions.

A possible modification of the model is the definition of a Bayesian game (see [27]), that allows accounting for uncertain elements, associating to each different behavior of the players a *type*, according to a given probability distribution. In our situation, for instance, the buyer may consider two types of sellers: the honest, S', and the thief, S'', whose preferences are $e_1 \succ_{S'} e_3 \succ_{S'} e_2$ and $e_2 \succ_{S''} e_1 \succ_{S''} e_3$, respectively. Note that the honest seller S' has the same preferences of the buyer, so the transaction is more possible, if the buyer trusts that the seller is honest. In order to have a realistic model, the different types of the seller have to represent different degrees of honesty, including not only elements depending on the seller, e.g., the choice of a faithful description of the item, or of a high quality transportation service, but also independent from him/her, e.g., the failure of the item. Analogously, a realistic model should consider different types of buyers, depending of their degree of risk-aversion.

The consequence is that the Bayesian model results to be intractable due to the difficulty of getting suitable data and to the high computational complexity.

3. The Hybrid Model

In this section, we first describe the basic hybrid model (sect. 3.1) whose main assumption is to consider equal-price transactions, that is each seller sells the item or service at the same price. The main reason of this assumption is to ease the model description and its validation. Note that such a type of assumption is not new in the literature: for instance, we can mention the duopoly model by Cournot [28], as a historical case in which the equal-price assumption was introduced in order to simplify the model. Then, we extend the model in order to take into account a more realistic scenario: indeed, we introduce the transactions with variable prices (sect. 3.2) and an insurance system to guarantee the transactions (sect. 3.3).

3.1. Basic Model

Let us suppose to consider a population of n individuals partitioned into a set of n_B buyers and a set of n_S sellers.

We suppose that m_t transactions are undertaken at each time interval $t = 1, 2, 3, \ldots, T$. Among them, m_t^c are those completed, and m_t^h are those completed correctly. Clearly, it results that $m_t^h \leq m_t^c \leq m_t$. We remark that $m_t - m_t^c$ are the transactions in which the buyer refuses to buy in accordance with the exit e_3 depicted in Figure 1.

The overall system reputation β is defined as follows:

$$\beta(0) = \beta_0 \tag{1a}$$

$$\beta(t+1) = (1 - w_\beta)\beta(t) + w_\beta \frac{m_t^h}{m_t^c}, \qquad t = 1, 2, \ldots, T\,. \tag{1b}$$

The value of $\beta(t+1)$ evaluates the trustworthiness of the overall system after the completion of the n_t^c transactions. The values $\beta_0 \in [0, 1]$ and $w_\beta \in [0, 1]$

represent respectively the initial trustworthiness and the *weight* given to the outcomes of the last transactions with respect to those in the past.

The infopoint maintains the centralized RMS. Upon request, the RMS provides the reputation ρ of a seller, which is periodically updated after the end of each transaction as soon as the infopoint receives the feedback. The reputation of the seller s is defined in a similar manner to the trustworthiness, that is the reputation $\rho_s(t+1)$ of the seller s at the beginning of the time interval $t+1$ depends on her/his reputation $\rho_s(t)$ at the beginning of the time interval t and on the positive outcomes of the transactions involving him/her. More formally, we have

$$\rho_s(0) = \rho_0^s \tag{2a}$$

$$\rho_s(t+1) = (1 - w_\rho^s)\rho_s(t) + w_\rho^s T_t^s, \qquad t = 1, 2, \ldots, T\,, \tag{2b}$$

where T_t^s is the ratio between the number of transactions completed correctly and the number of transaction completed at the time t considering only the transactions involving the seller s. As in (1b), $\rho_0^s \in [0,1]$ and $w_\rho^s \in [0,1]$ are the initial reputation value and the weight of the last transaction outcomes, respectively.

Let us define the personal experience $\gamma_b(t)$ of the buyer b as

$$\gamma_b(0) = \gamma_0^b \tag{3a}$$

$$\gamma_b(t+1) = (1 - w_\gamma^b)\gamma_b(t) + w_\gamma^b T_t^b, \qquad t = 1, 2, \ldots, T\,, \tag{3b}$$

where T_t^b is equal to: 1 if the last exit was e_1, 0 if the last exit was e_2, and $\gamma_b(t)$ otherwise (exit e_3). Again, $\gamma_0^b \in [0,1]$ and $w_\gamma^b \in [0,1]$ are the initial personal experience value and the weight of the last transaction outcomes, respectively.

Finally, we can model how a buyer takes her/his decision. Under the equal-price assumption, the decision of a buyer b is only based on the trustworthiness of the system, on her/his own personal experience, and on the reputation of the seller. Our idea is to model the willingness of the buyer b to complete the

transaction during the interval $[t, t+1]$ through the value W_b defined as

$$W_b = p^b_\beta \beta(t) + p^b_\gamma \gamma_b(t) + (1 - p^b_\gamma - p^b_\beta)\rho_s(t)\,, \tag{4}$$

where p^b_β and $p^b_\gamma \in [0,1]$ are the weights given to the system trustworthiness and to the personal experience by the buyer b in such a way that $p^b_\beta + p^b_\gamma \leq 1$.

The buyer b decides to complete the transaction with the seller s with probability $(1-z)$ if $W_b > 1 - R_b$, or with probability z otherwise, where $z \in [0,1]$ is a coefficient of *irrationality* – the probability that a buyer decides otherwise with respect to her/him normal decision – and R_b is the willingness to take risks of the buyer b. The idea is to compare the evaluated risk of the current transaction with the buyer's willingness to take risks, and to decide accordingly.

3.1.1. Agents and environment

The basic ABS model is composed of three types of agents modeling the buyers, the sellers, and the infopoint, respectively. The buyers and the sellers are embedded on a *small world* network modeling the possible connections among them. Each agent is also connected to the infopoint. The network models the possible transactions between each buyer and all the sellers that can offer the required item or service. Since the needs of the buyer can change, at each time interval $t = 1, 2, \ldots, T$, the connections starting from the same buyer can change. The small network topology has been used to model the fact that only a subset of sellers can have the item required since the others are out of stock at the moment of the current transaction.

In the ABS framework, a statechart models the agent behavior and its interaction with the simulated environment describing the transitions among different states. Figure 2 reports the statechart of the agent modeling a buyer. Further, the figure highlights its relationships with the game in the extensive form depicted in Figure 1.

At each time interval $t = 1, 2, \ldots, T$ and for each transactions, the buyer b collects the information ρ_s about the sellers s involved in one of the buyer's transactions. After computing the value W_b, the buyer decides to trust the seller,

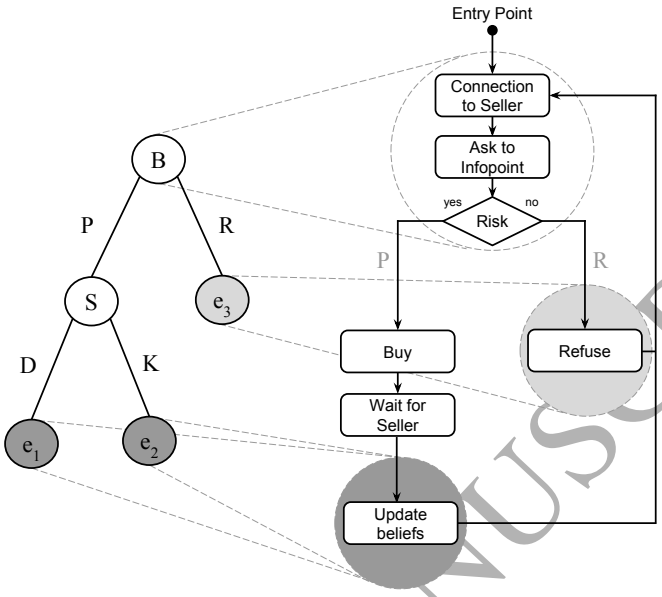


Figure 2: The statechart of a buyer and its relationships with the game in the extensive form.

and to complete the transaction, or to wait the next time interval. Whenever a transition is completed, the buyer provides to the infopoint a feedback that allows him updating the RMS. Further, her/his personal experience γ_b has been updated. The statechart of the infopoint is described in Figure 3.

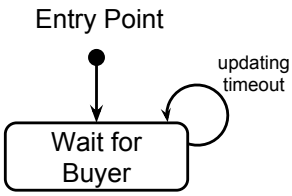


Figure 3: The statechart of the infopoint.

Figure 4 reports the statechart of the agent modeling a seller, highlighting its relationships with the game in the extensive form (Figure 1). Basically, the seller s waits for the transaction approval from the buyer, and then s/he decides to honor the transaction agreement with probability $H_s(t)$, or to cheat with

probability $1 - H_s(t)$. The probability $H(t)$ changes over the time following one of the policies reported in Section 3.1.2 and represent by the transition *refresh*. Further, the transitions between the state "Wait for a buyer" and "Go offline" model the fact that the seller goes offline after completing L transactions, until the next time interval.

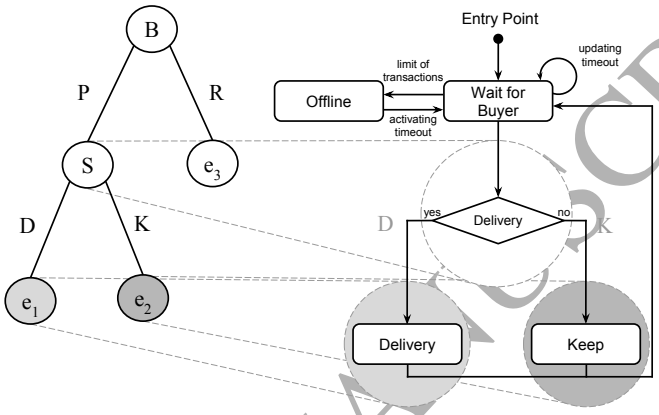


Figure 4: The statechart of a seller and its relationships with the extensive form.

3.1.2. Buyers' and sellers' policies

The basic model can be customized applying different policies both for the buyers and for the sellers.

Regarding the buyers, we implemented two different, but very simple, policies in order to select a seller for starting a transaction: the former is a random selection among those connected with the buyer while the latter select the connected seller s with the best reputation ρ_s.

The seller's policies consist in different ways to update the probability $H_s(t)$, that is the probability that the seller s honors the transaction, taking into account her/his current reputation ρ_s in the system.

Let H_0 be the initial honesty of a seller, that is the probability that the seller completes the first transaction honestly. The first policy is called *steady* since

the honesty is constant over the time, and is defined as

$$H(t+1) = H(t) = H_0\,. \tag{5}$$

Let $\varrho_{\min}$ and $\varrho_{\max}$ be the values for which a given percentile f of sellers are such that their reputation ρ_s is less than and greater than $\varrho_{\min}$ and $\varrho_{\max}$, respectively. The following two policies are based on the following idea: the seller decides to increase $H_s(t)$ when $\rho_s < \varrho_{\min}$, that is the seller is forced to increase her/his honesty to have more buyers; on the contrary, the buyer decides to decrease $H_s(t)$, when $\rho_s > \varrho_{\max}$, to take advantage of her/his reputation to make a higher profit:

- *α-aggressive policy*:

$$H(t+1) = \begin{cases} 1 & \text{if } \rho_s < \varrho_{\min} \\ \max\{0, H(t) - \alpha\} & \text{if } \rho_s > \varrho_{\max} \\ H(t) & \text{otherwise} \end{cases} \tag{6}$$

  with $\alpha \in (0,1)$ with the value closes to 0;

- *a-public consciousness policy*:

$$H(t+1) = \begin{cases} \min\left\{1, H(t) + a\sigma_{\rho(t)}\right\} & \text{if } \rho_s < \varrho_{\min} \\ \max\left\{0, H(t) - \dfrac{\sigma_{\rho(t)}}{a}\right\} & \text{if } \rho_s > \varrho_{\max} \\ H(t) & \text{otherwise} \end{cases} \tag{7}$$

  with $a \in \mathbb{N}$ and $\sigma_{\rho(t)}$ is the standard deviation of the sellers' reputation.

The policy (6) drastically changes the honesty value when the reputation of the seller is under the threshold $\varrho_{\min}$, while the values is slowly decreased when the reputation of the seller is over the threshold $\varrho_{\max}$. The policy (7), taking into account the average and the standard deviation of the sellers' reputation, strongly increase the lower honesties and slightly decrease the higher ones.

3.2. Variable Prices

The main assumption of the basic model reported in Section 3.1 is that the required item or service has the same price for all the sellers. Clearly, this is a quite narrow assumption adopted only for validation purposes. Here we extend the model in order to consider variable prices. This imposes to evaluate the impact of possible buyers' savings during the transaction and to establish new policies both for the buyers and the sellers.

Let C and $P_{\max}$ be respectively the cost and the maximum selling price of the item or service sold during the transaction. Let $\Pi_s(t)$ be the price proposed by the seller s at the time interval $t = 1, 2, \ldots, T$. The price is initialized to Π_0^s, that is $\Pi_s(0) = \Pi_0^s$ such that $C + \epsilon < \Pi_0^s \leq P_{\max}$, where ϵ is the minimum earning requested by the seller.

3.2.1. Saving based policies for buyers

The buyer should consider the possible saving before deciding to buy or not. We propose three different policies including the saving criteria to select the seller, and to evaluate her/his willingness to complete the transaction.

- *minimum-price policy*: the buyer selects the seller offering the item or the service at the best price;
- *lower-risk-good-price policy*: the buyer considers all the sellers whose price is lower than the average price π_t selecting the one with the best reputation, that is
$$\max_{\Pi_s(t) \leq \pi_t} \rho_s(t);$$
- *lower-price-good-reputation policy*: the buyer considers the sellers whose reputation is higher than the overall system reputation $\beta(t)$ selecting the one with the best price, that is
$$\min_{\rho_s(t) \geq \beta(t)} \Pi_s(t).$$

We remark that if a buyer is interested in saving money, then s/he could be more willing to risk. Let $r \in (0,1)$ be a coefficient of *rashness* to model this fact. We redefine the equation (4) as follows

$$W_b = (1-r)\left(p^b_\beta \beta(t) + p^b_\gamma \gamma_b(t) + (1 - p^b_\gamma - p^b_\beta)\rho_s(t)\right) + r\,. \tag{8}$$

Given the new value of W_b, the buyer decides to end the transaction as in the case with equal-price assumption.

3.2.2. Price based policies for sellers

The seller can operate on the prices in order to increase the number of transactions. The following price based policies increases or decreases the price as soon as the seller reputation is good ($\rho_s > \varrho_{\max}$) or not ($\rho_s < \varrho_{\min}$). We introduce two new policies:

- *α-price-based policy*:

$$H(t+1) = H(t) \tag{9a}$$

$$\Pi(t+1) = \begin{cases} \max\{(1-\alpha)\Pi(t), C+\epsilon\} & \text{if } \rho_s < \varrho_{\min} \\ \min\{(1+\alpha)\Pi(t), P_{\max}\} & \text{if } \rho_s > \varrho_{\max} \\ \Pi(t) & \text{otherwise} \end{cases} \tag{9b}$$

  with α close to 0;

- *δ-honesty-and-price based policy*:

$$H(t+1) = \begin{cases} \max\{1 - w^s_\delta \delta, H(t)\} & \text{if } \rho_s < \varrho_{\min} \\ \max\{0, H(t) - w^s_\delta \delta\} & \text{if } \rho_s > \varrho_{\max} \\ H(t) & \text{otherwise} \end{cases} \tag{10}$$

  with $\delta \in (0,1)$ and the price $\Pi(t+1)$ determined as in the equation (9b) setting $\alpha = (1-w^s_\delta)\delta$, where the weight w^s_δ is a value in $[0,1]$ that indicates the leaning of the seller s to operate on the honesty more than on the price, or vice versa.

3.3. Insurance on Transactions

In this section, we extend the model with prices including an insurance system. The basic idea is to provide an option to the buyer that guarantees to get the money back at the price of an insurance premium when the seller decides to cheat.

Let us introduce four rules to determine the value $I(t;s)$ of the insurance premium during the time interval $t = 1, 2, \ldots, T$:

1. *fixed-percentage policy*: the premium is equal to a given percentage q of the selling price, that is

$$I(t;s) = q\,\Pi_s(t) \tag{11}$$

   with $q \in (0,1)$;

2. *global reputation-based policy*: the premium is proportional to the reputation of the system, that is

$$I(t;s) = (1-\beta(t))q_{\max}\,\Pi_s(t) \tag{12}$$

   where $q_{\max} \in (0,1)$ is the maximum insurance percentage allowed;

3. *seller reputation-based policy*: the premium is proportional to the reputation of the single seller s involved in the transaction, that is

$$I(t;s) = (1-\rho_s(t))q_{\max}\,\Pi_s(t)\,; \tag{13}$$

4. *mixed-reputation policy*: the premium is a function of both the system reputation and seller reputation, that is

$$I(t;s) = \left[k_I(1-\beta(t)) + (1-k_I)(1-\rho_s(t))\right] q_{\max}\,\Pi_s(t) \tag{14}$$

   where $k_I \in (0,1)$ is a weighting factor.

The final price paid by the buyer b to end a transaction with the seller s during the time interval $t = 1, 2, \ldots, T$ is therefore

$$P_b^F = \Pi_s(t) + d_b(t;s)\,I(t;s)\,,$$

where $d_b(t; s)$ is a binary variable whose value 1 models the decision of the buyer b to insure the transaction with the seller s performed in the time interval t, 0 otherwise. Such a variable depends on the parameter c_b of inclination of the buyer b equal to the minimum seller reputation threshold to buy without insure the transaction, that is $d_b(t; s) = 1$ with probability $1 - z$ if $\rho_s(t) < c_b$ and with probability z otherwise.

When the buyer b decides to insure the transaction, s/he should consider her/his willing to save money. To this end, at the time of choosing the best seller, the buyer has to take into account the final price P_b^F.

4. Quantitative analysis

In this section, we report an extensive quantitative analysis of our model. In Section 4.1 we report a series of computational experiments devoted to validate the model, while in Section 4.2 we consider different policies of the sellers in the case of fixed price. In Sections 4.2, 4.3 and 4.4, we provide the analysis of the model after the introduction of the variable prices and of the insurance system, respectively.

The model has been developed using AnyLogic 6.9 [29] whose agent based library has been exploited for the implementation of the hybrid model. Each computational test consists in running the model 30 times on a given scenario and, each time, starting from a different initial condition. Roughly, each computational test requires about 10 seconds of running time to be executed on a standard laptop. Each run replicates $T = 300$ time intervals. The population is composed of $n = 1000$ individuals, of which $n_B = 900$ buyers and $n_S = 100$ sellers. Further, each buyer is connected to one third of the sellers, and the reputation of the sellers will be updated each 10 time intervals. We recall that m_t, m_t^c and m_t^h are respectively, at each time interval $t = 1, \ldots, T$, the number of transactions undertaken, those that have been completed, and those that have been completed correctly (without any scam). Finally, to evaluate the outcomes of each computational test, we adopt the indices reported in Table 1.

Table 1: Performance indices to evaluate the outcomes of the computational tests.

m^c_{avg}	average number of transactions completed in a single time interval
γ_{avg}	average value of $\gamma_b(t)$ over time and for all buyers
ν	percentage of transactions completed correctly over all the completed transactions
μ	average gain of a single seller in a single time interval
η	ratio between the total amount of money spent by the sellers and m^h_T
$t_{\beta>0.9}$	time interval t s.t. $\beta(t) > 0.9$ for the first time, `nil` otherwise
$t_{\beta<0.3}$	time interval t s.t. $\beta(t) < 0.3$ for the first time, `nil` otherwise
ν_{ins}	percentage of insured transactions
ν_{rep}	percentage of insured transactions refunded
ζ_{avg}	average insurance balance in a single time interval

4.1. Validation

The main characteristic of an agent based model is to analyze the behavior of the whole system starting from the behavior of its individuals. Therefore, it is not an easy task to validate such a model since the system behavior is unknown. Furthermore, we can not compare the hybrid model results with those provided by another validated model or with those obtained by the analysis of a realistic situation. Our choice is therefore to consider a set of scenarios devised in such a way that the system behavior can be foreseeable, and to verify if the model behaves as one would expect [30, 31].

Table 2: Scenarios: parameters.

Common parameters							
β_0	0.5	w_β	0.1	ρ^s_0	0.9	w^b_γ	0.1
p^b_β	0.25	p^b_γ	0.25	z	0.02	L	100
$\min R_b$	0	$\max R_b$	1	c	0.5	Π^s_0	1

Scenario parameters		
	$\min H_0$	$\max H_0$
awful (1)	0.0	0.3
almost perfect (2)	0.7	1.0
likely (3)	0.4	0.9

We introduce three scenarios (see Table 2), that is an *almost perfect* world, an *awful* world, and a *likely* world. Each scenario is characterized by a set of parameters: in the scenario "almost perfect world" the behavior of the seller is almost always honest; on the contrary, the scenario "awful" world has dishonest sellers; finally, the third scenario should represent an average situation.

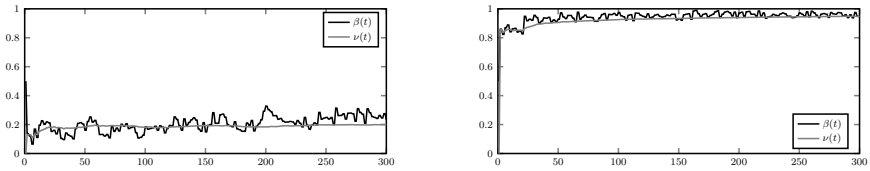
Table 3: Validation: results of the computational tests (buyers).

scenario	**buyers' policy**	m^c_{avg}	γ_{avg}	ν	μ	η	$t_{\beta>0.9}$	$t_{\beta<0.3}$
(1)	best ρ_s	206	0.21	21%	1.84	4.71	`nil`	2
(2)	best ρ_s	515	0.96	96%	2.68	1.04	16	`nil`
(3)	best ρ_s	477	0.84	84%	2.75	1.18	130	`nil`
(3)	random	379	0.66	66%	2.54	1.51	`nil`	`nil`

Table 3 reports the results of our computational tests on the three different scenarios, varying the buyers' policy while the seller behaves following the steady policy depicted in (5). Note that, in the case of equal-price transactions with $\Pi^s_0 = 1$, the value of η is given by the ratio between m^c_t and m^h_t.

Considering the two opposite scenarios, that is the "awful world" and "almost perfect world", the results are those that one would expect: for instance, in the scenario "awful world", the value of ν is very low while it is higher in the scenario "almost perfect world"; similar considerations hold for the other indices reported in Table 3. In Figure 5, we report the plots of the values of β and ν over the time: Figures 5(a) and 5(b) report such values for the scenarios (1) and (2), respectively. Both plots show that the percentage of transactions completed correctly follows the overall system reputation, as expected.

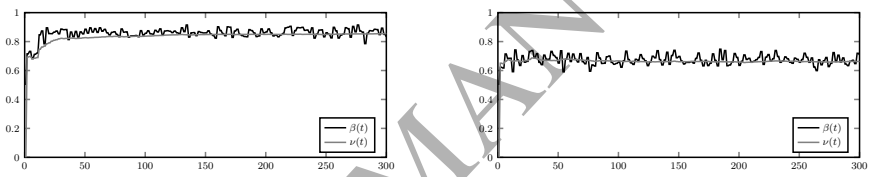
Clearly, the results regarding to the "likely world" scenario stay in between those of the "awful" and "almost perfect" work, as one would expects. In Figure 6, we report the plots of the values of β and ν over the time regarding both the scenario (3) but considering the two different policies for the seller selection by the buyer, which is the one having the best reputation 6(a) and randomly 6(b). Again, both plots show that the percentage of transactions



(a) Scenario (1) (b) Scenario (2)

Figure 5: Plotting β and ν over the time horizon: scenario (1) and (2)

completed correctly follows the overall system reputation, as one would expect. Further, the gap between plots 6(a) and 6(b) measures the quality improvement due to the more rational choice of selecting the seller taking into account her/his reputation.



(a) Scenario (3): best ρ (b) Scenario (3): random

Figure 6: Plotting β and ν over the time horizon: scenario (3), different buyer's policies.

4.2. Fixed prices

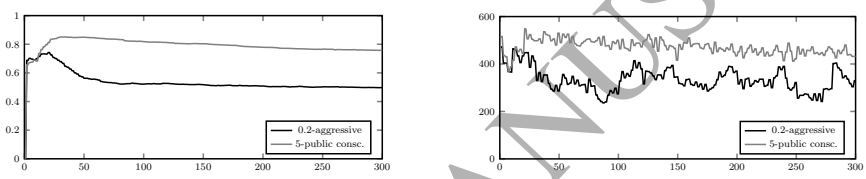
In the previous section, we assumed that the seller adopted always the same policy, which is the steady policy depicted in (5). The aim of this section is to analyze how the system behaves when the seller changes her/his behavior according to the two policies depicted in (6) and (7). In the following, the settings are the same of those reported in Section 4.1. Further, we will consider the "likely world" scenario, and the best ρ policy for the buyers.

Table 4 reports the results of comparing the two policies of the sellers with $f = 0.2$. From the results, it is evident that the policy (7) dominates the policy (6), giving advantages to both sellers (see μ) and buyers (see η) because of the higher number of transactions completed. This is more evident from

Table 4: Validation: results of the computational tests (sellers).

sellers' policy	m^c_{avg}	γ_{avg}	ν	μ	η
0.2-aggressive	367	0.54	56%	2.65	1.80
5-public consciousness	461	0.74	74%	2.91	1.35

the analysis of the Figure 7, which reports both the plots of the percentage of the transactions completed correctly (7(a)) and the number of transactions performed (7(b)) with respect to the two policies considered.



(a) value of $\nu(t)$ (b) number of transactions

Figure 7: Plotting ν and number of transactions over the time horizon.

4.3. Variable prices

Now we consider the situation in which the item or the service is offered on the market at different prices. In the following, we take into account the point of view of the sellers whose main aim is to maximize her/his profits.

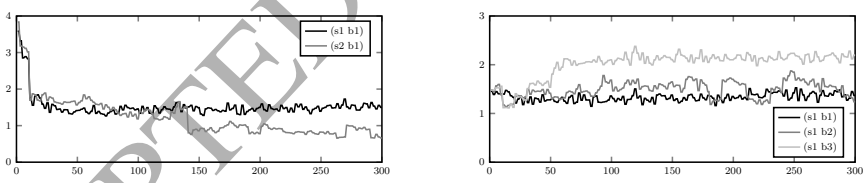
With respect to the common parameters reported in Table 2, the new parameters are $P_{\max} = 2$, $\epsilon = 0.1$ and $r = 0.2$. The scenario considered is again the "likely world". Note that Π^s_0 is now the initial price. We denote the policies of the buyers with (b1) minimum-price, (b2) lower-risk-good-price and (b3) lower-price-good-reputation, while those of the sellers with (s1) 0.2-price-based and (s2) 0.2-honesty-and-price based.

Table 5 reports the results of our computational tests over all the possible combinations of buyers' and sellers' policies. Even if determining a reduced number of transaction completed during each time interval, the pair of policies

Table 5: Variable prices: results of the computational tests.

sellers' policy	**buyers' policy**	m^c_{avg}	γ_{avg}	ν	μ	η
(s1)	(b1)	369	0.49	48%	1.51	1.35
(s1)	(b2)	428	0.65	65%	2.80	1.50
(s1)	(b3)	441	0.70	71%	4.97	2.09
(s2)	(b1)	488	0.91	92%	1.12	0.75
(s2)	(b2)	439	0.74	74%	2.46	1.26
(s2)	(b3)	367	0.64	64%	2.36	1.50

(s1-b1) results the best for the system since it allow maximizing the gain of the sellers, and at the same time, to minimize the average amount of money spent by the buyers. From the point of view of the sellers, the policy (s1) dominates the policy (s2): as a matter of fact, given a policy of the buyers, the value of μ in (s1) is always better that in (s2). This remark is also confirmed by Figure 8, which reports the plots of the value of μ and η over the time horizon, for different policy combinations.



(a) value of μ (b) value of η

Figure 8: Plotting μ and η over the time horizon.

4.4. Insurance

The introduction of the insurance system offers to the buyer the opportunity to protect the transactions, and to get the money back in the case of a failure for the dishonesty of the seller. So, we would evaluate the impact of such insurance system on the behavior of the whole e-commerce system.

Considering the analysis in the previous section, we consider the "likely world" scenario with policy (s1) for the sellers, and (b1) for the buyers. The weight k_I of (14) is set to 0.5. Please note that q_{max} has been introduced as the maximum insurance percentage allowed. Clearly, the maximum percentage will be applied only when the reputation is equal to 0, but it never happens in our settings.

Table 6: Insurance: results of the computational tests.

insurance policy	q_{max}	m^c_{avg}	γ_{avg}	ν	μ	η	ν_{ins}	ν_{rep}	ζ_{avg}
fixed-percentage (11)	0.5	376	0.50	50%	1.53	1.29	44%	51%	5.10
global-reputation (12)	1	381	0.49	50%	1.52	1.30	44%	51%	4.76
seller-reputation (13)	1	382	0.52	52%	1.51	1.23	44%	47%	6.20
mixed-reputation (14)	1	378	0.51	52%	1.51	1.24	44%	48%	6.30

Table 6 reports the results of our computational tests to evaluate the different insurance policies proposed. Such results show clearly the dominance of the seller reputation based policy depicted by the equation (13).

The model allow evaluating of the case in which the insurance system can incur into a bankruptcy. Figure 9 reports the balance of the insurance over the time for three different values of q_{max} ($q_{max} = 0.8, 0.9, 1.0$) when the insurance adopts the seller reputation based policy.

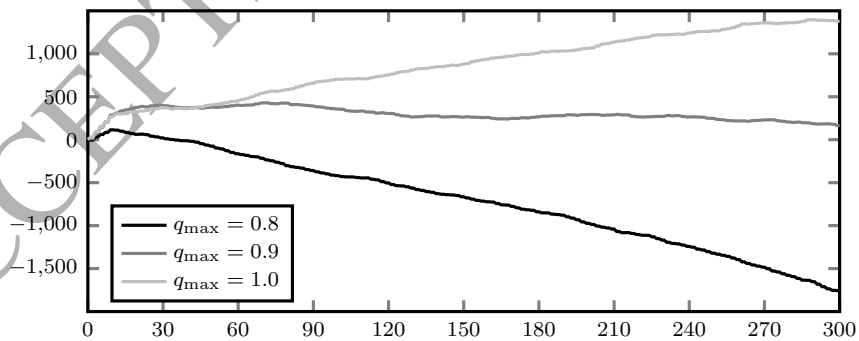


Figure 9: Plotting the insurance balance over the time varying the value of $q_{\max}$.

5. Conclusions

With the increasing popularity of e-commerce systems, commercial transactions become more and more frequent. Such transactions are not direct but mediated putting the buyer is in a position of weakness with respect to the seller, especially in the case of a failure of the transaction. Literature showed that the reputation can play an important role to reduce the risks of the buyer in the current e-commerce environment. An online RMS maintains the reputation, made of beliefs and/or opinions, that are generally held about someone or something, and it can guarantee the reliability of the transactions that take place in an e-commerce system.

In this paper we presented a hybrid model, based on game theory and agent based simulation, to analyze an e-commerce system in which a centralized reputation system is maintained by a trusted and third party. Game theory has been adopted to model the rational behavior of the buyers and the sellers while agent based simulation allows modeling the whole e-commerce system and the underlying network. Such an approach mitigates the complexity of a pure game theory approach. We reported an extensive quantitative analysis in order to validate the proposed model, and to evaluate the impact of a set of buyers' and sellers' policies on the behavior of the e-commerce system.

The results of the quantitative analysis confirm the capability of the model to represent the rational behavior of the buyers and the sellers, and the positive impact of an online RMS on the whole number of transactions under different policies and scenarios. In particular, the model allows us estimating the value of the insurance premium in order to avoid the insurance bankruptcy. More generally, our work proves the feasibility of *solving* a complex game theory model using an appropriate agent based simulation model.

Our hybrid model can be extended to deal with different research questions, both from a modeling and an application point of view.

From a modeling point of view, we can consider a market with many items in which the population of individuals plays, in different moments, both the

role of a buyer and the role of a seller. It could be worthy of investigation the evaluation of particular malicious behaviors such as (i) the tentative of a seller to boost her/his reputation trough a series of low-value transactions and then to cheat, in a short period, some buyers with high-value transactions, and (ii) to evaluate and to compare the benefits of fake positive and negative scores to boost or to drop the reputation of an individual, respectively.

From an application point of view, it is worth noting that the inherent flexibility of our modeling approach can be exploited to evaluate the effectiveness of a RMS with a particular attention to the trust management in cloud computing [32]. Cloud computing is a new computing model that involves outsourcing of computer technologies due to the lack of their availability in certain locations. In this context, the concept of reputation is connected to the reliability, quality and performance of the services being offered.

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