Global Types for Open Systems *

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Global-type formalisms enable to describe the overall behaviour of distributed systems and at the same time to enforce safety properties for communications between system components. Our goal is that of amending a weakness of such formalisms: the difficulty in describing open systems, i.e., systems which can be connected and interact with other open systems. We parametrically extend, with the notion of interface role and interface connection, the syntax of global-type formalisms. Semantically, global types with interface roles denote open systems of communicating finite state machines connected by means of gateways obtained from compatible interfaces. We show that safety properties are preserved when open systems are connected that way.

1 Introduction

The intrinsic difficulties programmers have to face when developing and verifying distributed applications have been variously attacked by the theoretical computer science community with the aim of devising formal systems enabling (1) to describe in a structured way the overall behavior of a system, and (2) to steer the implementation of the system components, guaranteeing their compliance with the overall behaviour together with some relevant properties of communications.

Several formalisms based on the notion of global type have been proposed in the literature to pursue such an aim [2, 3, 5]. The expressiveness of the investigated formalisms kept on increasing during the last decade, recently leading to representations of global behaviours as graphs [8, 13, 16], where the local end-point projections are interpreted by communicating finite state machines (CFSMs), a widely investigated formalism for the description and the analysis of distributed systems [1]. For systems of CFSMs, most of the relevant properties of communications are, in general, undecidable [4] or computationally hard. Instead, systems of CFSMs obtained by projecting the generalised global types of [8] or the global graphs of [13, 16] (more precisely those which satisfy a well-formedness condition) are guaranteed to satisfy desired properties of communications like deadlock-freeness, that any sent message is eventually consumed or that each participant will eventually receive any message s/he is waiting for [8].

The centralised viewpoint offered by the global type approaches makes them naturally suitable for describing closed systems. This prevents a system described/developed by means of global types to be looked at as a module that can be connected to other systems. The description and analysis of open systems has been investigated, instead, in the context of CFSMs in [12, 11] and for synchronous communication in the context of interface automata in [6, 7]. In the present paper we address the problem of generalising the notion of global type in order to encompass the description of open systems and, in particular, open systems of CFSMs; so paving the way towards a fruitful interaction between the investigations on open systems carried out in automata theory and those on global types.

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In our approach, an “open global type” – that we dub “global type with interface roles” (GTIR) – denotes a number of connected open systems of CFSMs where some participants (roles) are identified as interfaces rather than proper participants. We have no necessity to stick to any particular global type formalism as a basis for our GTIRs, as long as the local end-point behaviours of a global type $G$ can be interpreted as CFSMs. So we introduce a parametric syntax which, given a global type formalism $\mathcal{G}$, extends its syntax by essentially enabling to identify some roles as interface roles and to define a composition of open global types, semantically interpreted by systems of CFSMs. We call $\mathcal{G}$-IR ($\mathcal{G}$-with-Interface-Roles) the so obtained formalism.

Syntactically, a GTIR is either a global type $G$ (formulated in $\mathcal{G}$) together with a distinguished subset of the roles of $G$ declared as interface roles, or it is a composite expression where two GTIRs are composed via compatible interfaces. The non-connected interface roles remain open after composition. The semantics of a GTIR is always a set of CFSMs. In the case of a basic GTIR, i.e. a global type $G$ equipped with interface roles, it is just the set of CFSMs obtained by projecting $G$ to its end-point CFSMs. Those CFSMs interpreting interface roles model the expected behaviour of an external environment of the open system. Interface roles are compatible if their CFSMs have no mixed states, are input and output deterministic and if their languages are dual to each other. If a GTIR $G$ is a composite expression, composing GTIRs $G_1$ and $G_2$ via compatible interface roles $H$ and $K$, then the semantics of $G$ is the union of the two CFSM systems denoted by $G_1$ and $G_2$ where the CFSMs $M_H$ and $M_K$, interpreting the interface roles $H$ and $K$, are replaced by appropriate gateway CFSMs $gw(M_H,K)$ and $gw(M_K,H)$. These gateways are constructed by a simple algorithm out of $M_H$ and $M_K$.

A main objective of our work is to study the preservation of safety properties under composition. We consider three kinds of properties: deadlock-freedom, freedom of orphan messages and freedom of unspecified receptions following the definitions in [8] (which in turn follow definitions in [4]). The main result of the present paper is that these safety properties hold for the CFSM system $S$ denoted by a GTIR $G$ whenever they hold singularly for all subsystems $S_i$ obtained by the semantics of the global types $G_i$ that are used for the construction of $G$. In particular, it has been shown in [8] that the safety properties are ensured whenever the $G_i$’s are well-formed generalised global types

**Overview.** In Section 2 the main definitions concerning CFSMs and systems of CFSMs are recalled, together with the definitions of safety properties. Syntax and semantics of global types with interface roles are introduced in Section 3. Section 4 studies the preservation of safety properties for systems connected via gateways and proves our main result. Section 5 concludes by pointing to related work and describing directions for further investigations.

## 2 Systems of Communicating Finite State Machines

In this section we recall (partly following [4,8,13]) the definitions of communicating finite state machine (CFSM) and systems of CFSMs. Throughout the paper we assume given a countably infinite set $\mathbb{P}$ of role (participant) names (ranged over by $p,q,r,s,A,B,H,I,\ldots$) and a countably infinite alphabet $\mathbb{A}$ (ranged over by $a,b,c,\ldots$) of messages.

**Definition 2.1 (CFSM).** Let $\mathbb{P}$ and $\mathbb{A}$ be finite subsets of $\mathbb{P}$ and $\mathbb{A}$ respectively.

i) The set $\mathbb{C}$ of channels over $\mathbb{P}$ is defined by

$$\mathbb{C} = \{pq \mid p,q \in \mathbb{P}, p \neq q\}$$

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1We prefer to use the word role rather than participant since interface role sounds more suitable for the present setting than interface participant.
ii) The set $\text{Act}_{\mathcal{P},\mathcal{A}}$ of actions over $\mathcal{P}$ and $\mathcal{A}$ is defined by
\[ \text{Act}_{\mathcal{P},\mathcal{A}} = \mathcal{C}_P \times \{!,?\} \times \mathcal{A} \]

iii) A communicating finite-state machine over $\mathcal{P}$ and $\mathcal{A}$ is a finite transition system given by a tuple
\[ M = (Q,q_0,\mathcal{A},\delta) \]

where $Q$ is a finite set of states, $q_0 \in Q$ is the initial state, and $\delta \subseteq Q \times \text{Act}_{\mathcal{P},\mathcal{A}} \times Q$ is a set of transitions.

Notice that the above definition of a CFSM is generic w.r.t. the underlying sets $\mathcal{P}$ of roles and $\mathcal{A}$ of messages. This is necessary, since we shall not deal with a single system of CFSMs but with an arbitrary number of open systems that can be composed. We shall write $C$ and $\text{Act}$ instead of $\mathcal{C}_P$ and $\text{Act}_{\mathcal{P},\mathcal{A}}$ when no ambiguity can arise. We assume $l,l',\ldots$ to range over $Act$; $\varphi,\varphi',\ldots$ to range over $Act^*$ (the set of finite words over $Act$); and $w,w',\ldots$ to range over $\mathcal{A}^*$ (the set of finite words over $\mathcal{A}$). $\varepsilon (\notin \mathcal{A} \cup Act)$ denotes the empty word and $|\,v\,|$ the length of a word $v \in Act^* \cup \mathcal{A}^*$. The transitions of a CFSM are labelled by actions; a label $s!a$ represents the asynchronous sending of message $a$ from machine $s$ to $r$ through channel $sr$ and, dually, $sr?a$ represents the reception (consumption) of $a$ by $r$ from channel $sr$.

We write $\mathcal{L}(M) \subseteq Act^*$ for the language over $Act$ accepted by the automaton corresponding to machine $M$, where each state of $M$ is an accepting state. A state $q \in Q$ with no outgoing transition is final; $q$ is a sending (resp. receiving) state if all its outgoing transitions are labelled with sending (resp. receiving) actions; $q$ is a mixed state otherwise.

A CFSM $M = (Q,q_0,\mathcal{A},\delta)$ is:

a) deterministic if for all states $q \in Q$ and all actions $l$: $(q,l,q'),(q,l,q'') \in \delta$ imply $q' = q''$;

b) $?!$-deterministic (resp. $!?$-deterministic) if for all states $q \in Q$ and all actions $(q,rs?a,q'),(q,pq?a,q'') \in \delta$ (resp. $(q,rs!a,q'),(q,pq!a,q'') \in \delta$) imply $q' = q''$;

c) $?!$-deterministic if it is both $?!$-deterministic and $!?$-deterministic.

The notion of $?!$-deterministic machine is more demanding than in usual CFSM settings. It will be needed in order to guarantee safety-properties preservation when systems are connected.

**Definition 2.2** (Communicating systems and configurations). Let $\mathcal{P}$ and $\mathcal{A}$ be as in Def. 2.1.

i) A communicating system (CS) over $\mathcal{P}$ and $\mathcal{A}$ is a tuple $S = (M_p)_{p \in \mathcal{P}}$ where
for each $p \in \mathcal{P}$, $M_p = (Q_p,q_{0p},\mathcal{A},\delta_p)$ is a CFSM over $\mathcal{P}$ and $\mathcal{A}$.

ii) A configuration of a system $S$ is a pair $s = (\vec{q},\vec{w})$ where
- $\vec{q} = (q_p)_{p \in \mathcal{P}}$ with $q_p \in Q_p$;
- $\vec{w} = (w_{pq})_{pq \in \mathcal{C}}$ with $w_{pq} \in \mathcal{A}^*$.

The component $\vec{q}$ is the control state of the system and $q_p \in Q_p$ is the local state of machine $M_p$.

The component $\vec{w}$ represents the state of the channels of the system and $w_{pq} \in \mathcal{A}^*$ is the state of the channel for messages sent from $p$ to $q$. The initial configuration of $S$ is $s_0 = (\vec{q}_0,\vec{w})$ with $\vec{q}_0 = (q_{0p})_{p \in \mathcal{P}}$.

**Definition 2.3** (Reachable configurations). Let $S$ be a communicating system, and let $s = (\vec{q},\vec{w})$ and $s' = (\vec{q}',\vec{w}')$ be two configurations of $S$. Configuration $s'$ is reachable from $s$ by firing a transition with action $l$, written $s \xrightarrow{l} s'$, if there is $a \in \mathcal{A}$ such that one of the following conditions holds:

1. $l = sr!a$ and $(q_p,l,q_p') \in \delta_a$ and
   a) for all $p \neq s$ : $q_p' = q_p$ and
b) \( w'_{sr} = w_{sr} \cdot a \) and for all \( pq \neq sr : w'_{pq} = w_{pq} \);

2. \( l = sr ? a \) and \( (q_x, l, q'_x) \in \delta_x \) and
   a) for all \( p \neq x : q'_p = q_p \) and
   b) \( w_{sr} = a \cdot w'_{sr} \) and for all \( pq \neq sr : w'_{pq} = w_{pq} \).

We write \( s \rightarrow s' \) if there exists \( l \) such that \( s l \rightarrow s' \).

As usual, we denote the reflexive and transitive closure of \( \rightarrow \) by \( \rightarrow^* \).

3. The set of reachable configurations of \( S \) is \( RS(S) = \{ s | s_0 \rightarrow^* s \} \).

According to the last definition, communication happens via buffered channels following the FIFO principle.

**Definition 2.4** (Safety properties [8, 4]).

Let \( S \) be a communicating system, and let \( s = (q, w) \) be a configuration of \( S \).

i) \( s \) is a deadlock configuration if

\[ \tilde{w} = \tilde{\varepsilon} \land \forall p \in P, q_p \text{ is a receiving state} \]

i.e. all buffers are empty, but all machines are waiting for a message.

We say that \( S \) is deadlock-free whenever for any \( s \in RS(S) \), \( s \) is not a deadlock configuration.

ii) \( s \) is an orphan-message configuration if

\[ (\forall p \in P, q_p \text{ is final}) \land \tilde{w} \neq \tilde{\varepsilon} \]

i.e. each machine is in a final state, but there is still at least one non-empty buffer.

iii) \( s \) is an unspecified reception configuration if

\[ \exists r \in P. q_r \text{ is a receiving state} \land \forall s \in P. [(q_r, sr? a, q'_r) \in \delta_r \implies (|w_{sr}| > 0 \land w_{sr} \notin a_\mathcal{A}^*)] \]

i.e. there is a receiving state \( q_r \) which is prevented from receiving any message from any of its buffers. (In other words, in each channel \( sr \) from which role \( x \) could consume there is a message which cannot be received by \( x \) in state \( q_x \).

iv) \( S \) is safe if, for each \( s \in RS(S) \), \( s \) is neither a deadlock, nor an orphan-message, nor an unspecified reception configuration.

The above definitions of safety properties are the same as in [8]. They follow, for the notions of deadlock and unspecified reception, the definitions in [4]. The deadlock definition in [13] is slightly weaker, but coincides with [8] if the local CFSMs have no final states. Still weaker definitions of deadlock are used in [16] and in [1].

## 3 Global Types with Interface Roles

Our aim is the development of a formalism \( \mathcal{GT}-IR \) suitable for the composition of open systems which ensures preservation of safety properties and hence is suitable for modular system construction. The idea is that our approach should be usable for any global type formalism \( \mathcal{GT} \) which satisfies the following assumptions: For each global type \( G \) in \( \mathcal{GT} \),

1. there is associated a finite set of roles \( P(G) \subset P_\mathcal{A} \) and a finite set of actions \( A(G) \subset A_\mathcal{A} \).
2. there is a projection function, denoted by \( P \), such that for any \( p \in P(G), G \bowtie p \) is a CFSM over \( P(G) \) and \( \bowtie(G) \).

Global types are considered as syntactic objects while the projection function yielding a communicating system \( G \bowtie p \) over \( P(G) \) and \( \bowtie(G) \) is considered as the semantics of a global type \( G \). A \( \mathcal{GT} \) formalism satisfying the above requirements could be, for instance, the formalism of generalised global types in \([8]\) or that of global graphs in \([13,16]\). Under certain conditions on the form of a global type \( G \), safety properties are guaranteed for the system obtained by projecting all roles of \( G \) to a CFSM. For instance, Theorem 3.1 in \([8]\) states that a communicating system generated from the projections of a well-formed global type is safe in the sense of Def. 2.4. Therefore, if we can assure that safety is preserved by composition, we get a safe system whenever the underlying global types denote safe (sub)systems.

In order to look at global types as open, we shall identify some of their roles as interface roles. An interface roles represents (part of) the expected communication behaviour of the environment. Interface roles are the basis to compose systems. In this section, we introduce Global Types with Interface Roles (GTIRs) and provide their syntax and semantics. First we define the syntactic notion of a pre-GTIR. Then we present our working example which already points out that we need a semantic compatibility relation between interfaces for safe composition. Interface compatibility and a few additional conditions must be respected to get a proper GTIR. The semantics of a GTIR is then defined in terms of a system of CFSMs obtained, in the base case, by the projections of a global type with distinguished interface roles, and, in the composite case, by composing open CFSM systems by means of suitable “gateway” CFSMs.

### 3.1 Pre-GTIRs

A pre-GTIR is either just a global type where some roles are declared as interface roles or it is a syntactic expression composed from two pre-GTIRs by connecting certain interface roles. The non-connected interface roles remain open.

**Definition 3.1** (pre-GTIR). The set of pre-GTIR expressions \([G]^I\) with set of interface roles \( I \) is defined by simultaneous induction together with their sets of roles \( P([G]^I) \) and components \( C([G]^I) \):

i) \([G]^I \in \text{pre-GTIR and } P([G]^I) = P(G) \text{ and } C([G]^I) = \{G\} \text{ if}

(a) \( G \) is a global type of \( \mathcal{GT} \),

(b) \( I \subseteq P(G) \).


C([[G_1]^H_{\bowtie \leftarrow \mathcal{K}} [G_2]^K]^I) = C([G_1]^H) \cup C([G_2]^K) \text{ if}

(a) \([G_1]^H, [G_2]^K \in \text{pre-GTIR with } H \in H, K \in K,

(b) \( I = (H \cup K) \setminus \{H,K\},

(c) P([G_1]^H) \cap P([G_2]^K) = \emptyset \).

Notice that in a composed pre-GTIR, the notation of the set \( I \) is actually redundant and it is used just to immediately spot the interface roles. By the above definition, a pre-GTIR is an expression formed by either a global type in \( \mathcal{GT} \) or a number of global types in \( \mathcal{GT} \) “composed” via symbols of the form \( H_{\bowtie \leftarrow \mathcal{K}} \), and where sets of interface roles are identified by means of superscripts. These global types are what we have defined as the components of the pre-GTIR.
3.2 Working example

We introduce the compatibility relation we have in mind and the composition operator that we want to use for constructing GTIRs by means of a working example inspired by one in [7].

Let us assume we wish to develop an open system, let us dub it \( S \), which can receive a text message from the outside. Once a text is received, the system tries to transmit it at most \( n \) times (where also the number \( n \) of possible trials is provided from the outside when the system is initialized). A successful transmission is acknowledged by an \( \text{ack} \) message; a \( \text{nack} \) message represents instead an unsuccessful transmission. An \( \text{ok} \) message is sent back in case of a successful transmission; a \( \text{fail} \) message in case of \( n \) unsuccessful trials. Before any transmission trial, a semantically-invariant transformation is applied to the message. The system can hence be used to send messages to social networks which are particularly strict for what concerns propriety of language. If the message is not accepted by the social network, our system automatically transforms it maintaining its sense, and sending it again and again up to \( n \) times, invariantly transforming it each time. A counter is used to keep track of the number of trials and it is reset to \( n \) each time a message is successfully transmitted. It is instead automatically reset to \( n \) each time \( 0 \) is reached, before issuing a failure message and restarting the protocol with some new message.

If we consider the formalism of generalized global types of [8], the overall behaviour of the above system \( S \) can be described by the graph \( G \) shown in Fig. 1 where the roles (participants) are:

- \( \mathcal{M} \): the manager of the system;
- \( T \): the process implementing the semantically-invariant message transformation;
- \( C \): the trials counter;
- \( I \), \( J \) and \( H \): the roles (that we identify as interface roles) representing those parts of the environment which, respectively: initializes the system; sends the text message and receives back the \( \text{ok} \) or \( \text{fail} \) message; receives the messages transmitted by the systems and acknowledges its propriety, if so.

Informally, in the graph \( G \) in Fig. 1 a label \( s \rightarrow r : a \) represents an interaction where \( s \) sends a message \( a \) to \( r \). A vertex with label \( \circ \) represents the source of the graph and \( \bigotimes \) marks vertexes corresponding to branch or merge points, or to entry points of loops. In our formalism, we can look at \( G \) as a global type with interface roles by identifying the roles \( I \), \( J \) and \( H \) as interface roles. We do that by writing, according to the syntax of Def. 3.1(i):

\[
\left[ G \right]_{\langle \{I, J, H\} \rangle}
\]

Given a global type with interface roles, it is reasonable to expect all its roles to be implemented but the interface ones, since they are actually used to describe the behaviour of the “environment” of the system. The projections of global types with interface roles onto their interface roles yield CFSMs which are used instead to check whether two systems can be connected in a safe way. In particular, to check whether interface roles are “compatible”. According to the projection algorithm for generalized global types (see §3.1 and Def. 3.4 of [8]), the projection on role \( J \) of the graph \( G \) of Fig. 1 is the following CFSM \( M_J = G \upharpoonright J \) (see (1)) which describes the behaviour of that part of the environment of system \( S \) which sends a text and waits for a positive or negative answer.

![Diagram](image-url)

(1)
Going on with our example, let us consider now another open system $S'$, having, among others, roles $A$, $B$ and $K$, where $K$ is one of its interface roles. In $S'$, the roles $A$ and $B$ keep on sending, in an alternating manner, a text message to $K$, which replies with a positive or negative acknowledgement (ok or fail, respectively). Role $B$ can send its message only after a successful sending by $A$, and vice versa. A fail message from $K$ forces the resending of the message. Let us assume that the behavior of interface role $K$ is given by the following CFSM $M_K$ (see (2)):
The interface roles $J$ and $K$ are compatible, in that the text message asked for by $K$ can be the one provided by $J$ to system $S$, whereas the ok and fail messages $J$ receives can be the ones that $K$ sends to system $S'$. In a nutshell, if we do not take into account channels in the labels, the language accepted by $J$ is the dual (i.e. ‘!’ and ‘?’ are exchanged) of that accepted by $K$.

Once the compatibility of $J$ and $K$ is ascertained, the behaviours of two gateways processes could be easily constructed from $M_J$ and $M_K$. The idea is to insert an intermediate state with appropriate transitions in the middle of any transition of $M_J$ (and similarly of $M_K$) enabling to pass messages from the interface role of one system to the other. For instance, the transition of $M_J$ from state 1 to state 2 labelled with JM!text is split into two transitions (see Fig. 2, left), where $J$ first receives a text from $K$ and then sends it to $M$.

Such gateways processes can be constructed by means of an algorithm that we dub $gw(\cdot)$. It takes two arguments: the CFSM to be transformed and the name of the interface role of the other system, and returns a “gateway” CFSM which enables systems to interact. For what concerns our example, by applying $gw(\cdot)$ to $M_J$ and $K$ and by applying $gw(\cdot)$ to $M_K$ and $J$, we get the two CFSMs depicted in Figure 2.

By assuming $[G']^I(J,K)$ to be the GTIR denoting the open system $S'$ above, the pre-GTIR
\[ [[G']^I(J,K)]_J \uplus K [G']^I(J,K)]_I \uplus \{J,R\} \]
is actually a proper GTIR, since the interface roles $J$ and $K$ are compatible. Its semantics is the system obtained by connecting $S$ and $S'$ by means of the gateways $gw(M_J,K)$ and $gw(M_K,J)$. Notice that the GTIR now exposes the remaining interface roles $I \uplus \{J,R\}$.

**Remark 3.2.** We could choose to connect systems by implementing a single “two-sided” gateway process, but this would imply to change all the names in the channels of $S$ and $S'$ from and to $J$ and $K$. This is not feasible if, as it is likely, $S$ and $S'$ have been separately implemented. Also there would be no straightforward generation of a single “two-sided” gateway process.

**Remark 3.3.** One could wonder what would change if, instead of using gateways, one simply renamed the target of communications to interface nodes. That is unfeasible (unless a rather strict relation of compatibility were used). It is enough to take into account our working example: if we tried to rename the target of communications between $M_R$ and $M_J$, machine $M_R$ should be completely rewritten. In fact, whereas both in $S$ and in the new system with gateways $M_R$ receives the text from $M_J$, in a new system without gateways $M_R$ would receive the text both from $M_A$ and $M_B$.

### 3.3 Interface compatibility and GTIRs

The following definitions are preliminary to the formal definition of interface compatibility and hence of a proper GTIR. First, we extend the projection function of the global type formalism $\mathcal{G}T$ to pre-GTIRs. The projection of interface roles to CFSMs can then be used below to check interface compatibility.

**Definition 3.4.** Let $[G]^I$ be a pre-GTIR and let $p \in P([G]^I)$. We define
\[ [G]^I \mid_p = G \mid_p \] where $G \in \mathcal{C}([G]^I)$ such that $p \in P(G)$.

At next we want to consider the dual of the language accepted by a CFSM when input and output are reversed and the names of communication channels are forgotten. For that purpose we need the following definition:

**Definition 3.5.**

i) Let $\varphi \in Act^*$, we define $\varphi^C \in (\{!, ?\} \times \mathbb{A})^*$ inductively by:
\[ \varepsilon^C = \varepsilon \quad (pq?a \cdot \varphi)^C = ?a \cdot \varphi^C \quad (pq!a \cdot \varphi)^C = !a \cdot \varphi^C. \]
Moreover, for $A \subseteq Act^*$, $A^C = \{\varphi^C \mid \varphi \in A\}$. 

Barbanera, de’Liguoro & Hennicker

11
ii) We define \( \overline{\cdot} : \{!, ?\} \times \mathcal{A} \rightarrow \{!, ?\} \times \mathcal{A} \) by: \( \overline{!a} = ?a \) and \( \overline{?a} = !a \). This function \( \overline{\cdot} \) is then straightforwardly extended also to words and finite sets of words.

Finally, we define interface compatibility by requiring that the CFSMs of two interface roles are dual to each other. Additionally we require the absence of mixed states as well as input and output determinism for each of the two CFSMs. In fact, if either of these two conditions were omitted one can provide counterexamples showing that our results on preservation of safety properties would generally no longer be valid.

**Definition 3.6** (Interface compatibility).

i) Let \( M \) and \( M' \) be two CFSMs over \( \mathcal{P} \) and \( \mathcal{A}' \) (\( \mathcal{P}' \) and \( \mathcal{A}' \) resp.). \( M \) and \( M' \) are compatible, denoted by \( M \leftrightarrow M' \), whenever

1) \( \mathcal{L}(M)^C = \mathcal{L}(M')^C \).

2) \( M \) and \( M' \) do not contain mixed states.

3) \( M \) and \( M' \) are \( ?! - \)deterministic.

ii) Let \( [G_1]^{[H]} \) and \( [G_2]^{[K]} \) be two pre-GTIRs. Two interface roles \( H \in H \) and \( K \in K \) are interface compatible, denoted by \( H \leftrightarrow K \), if \( [G_1]^{[H]} | H \leftrightarrow [G_2]^{[K]} | K \).

It is easy to check that in our working example we have \( J \leftrightarrow H \), since the CFSMs (1) and (2) are compatible, i.e. one accepts the dual language of the other if channel names are not taken into account; they have no mixed states and are \( ?! - \)deterministic.

We are now ready to introduce our notion of GTIR. The syntactic construction of proper GTIRs follows the construction of pre-GTIRs but imposes two semantic conditions for the underlying CFSMs formulated in i) and ii) of the next definition.

**Definition 3.7** (GTIR).

i) A pre-GTIR \( [G]^{[I]} \) formed by a global type \( G \) and interface roles \( I \subseteq \mathcal{P}(G) \) is a GTIR if no communication between interface roles (i.e. roles in \( I \)) is present, i.e. for each \( p \in \mathcal{P}(G) \) the projection \( G|_p \) has no transition with a label of the form \( 1J!a \) or \( 1J?a \) with \( I, J \in I \).

ii) A pre-GTIR \( [[G_1]^{[H]}],_H \leftrightarrow [[G_2]^{[K]}],_K^{[I]} \) obtained by the composition of two GTIRs \( [G_1]^{[H]} \) and \( [G_2]^{[K]} \) via interface roles \( H \in H, K \in K \) is a GTIR if \( H \) and \( K \) are interface compatible, i.e. \( H \leftrightarrow K \).
3.4 Semantics of GTIRs

To provide semantics for GTIRs we first define the gateway transformation $gw(\cdot)$ previously mentioned. By means of such a function it is possible to construct the gateway processes enabling the CFSM systems described by GTIRs to be connected.

The $gw(\cdot)$ function takes as input a CFSM $M_H$ of some role $H$ and a role name $K$. In our application $H$ and $K$ will be interface roles where $H$ is the interface role to be connected to the interface role $K$. The gateway function transforms $M_H$ by “inserting” a new state “in between” any transition. In such a way a transition from $q$ to $q'$ receiving a message $a$ from a role $s(\neq K)$ is transformed into two transitions: one from $q$ to the new state $\hat{q}$ receiving $a$ from $s$, and one from $\hat{q}$ to $q'$ sending $a$ to $K$. Conversely, a transition from $q$ to $q'$ sending a message $a$ to a role $s(\neq K)$ is transformed into two transitions: one from $q$ to the new state $\hat{q}$ receiving $a$ from $K$, and one from $\hat{q}$ to $q'$ sending $a$ to $s$. We distinguish the new “inserted” states by superscripting them by the transition they are “inserted in between”.

Definition 3.8 (The $gw(\cdot)$ transformation).

Let $M_H = (Q, q_0, A, \delta)$ be the CFSM of a role $H$, and let $K$ be a role name. We define

$$gw(M_H,K) = (Q', q_0, A, \delta')$$

where

- $Q' = Q \cup \hat{Q}_\delta$, with $\hat{Q}_\delta = \bigcup_{q \in Q} \{ q'(q,l,q') \mid (q,l,q') \in \delta \}$, and
- $\delta' = \{ (q,Hs!a,q'!,Hs'!a,q') \mid (q,Hs!a,q') \in \delta \}$
  $\cup \{ (q,sH?a,q'!,sH?a,q') \mid (q,sH?a,q') \in \delta \}$

For the sake of readability, we shall often denote elements $q'(q,l,q'), q'(q,l',q'')$, $q'(q,l'',q''')$, … of $\hat{Q}_\delta$ by $\hat{q}, \hat{q}', \hat{q}'', …$. We shall also refer to $\hat{Q}_\delta$ simply as $\hat{Q}$ when clear from the context.

We can now define the composition of two communicating systems $S_1$ and $S_2$ w.r.t. compatible interface roles $H$ and $K$. We take the union of the CFSMs of $S_1$ and $S_2$ but replace the CFSMs $M_H$ and $M_K$ of the interface roles $H$ and $K$ by their gateway CFSMs $gw(M_H,K)$ and $gw(M_K,H)$.

Definition 3.9 (Composition of communicating systems).

Let $S_1 = (M_1^1)_{p \in P_1}$ and $S_2 = (M_2^2)_{q \in P_2}$ be two communicating systems over $P_1$ and $\hat{A}_1$ ($P_2$ and $\hat{A}_2$ resp.) such that $P_1 \cap P_2 = \emptyset$. Moreover, let $H \in P_1$ and $K \in P_2$ be such that $H \leftrightarrow K$ (i.e. $M_H \leftrightarrow M_K$).

The composition of $S_1$ and $S_2$ w.r.t. $H$ and $K$ is the communicating system

$$S_1 \otimes_s S_2 = (M_p^p)_{p \in P_1 \cup P_2}$$

over $P_1 \cup P_2$ and $\hat{A}_1 \cup \hat{A}_2$ where $M_H = gw(M_H^K)$, $M_K = gw(M_K^K)$, $M_p = M_p^1$ for all $p \in P_1$ and $M_p = M_p^2$ for all $p \in P_2$.

The semantics of a GTIR is inductively defined following its syntactic construction. In the base case, its semantics is the communicating system obtained by the projections of the underlying global graph. The semantics of a composite GTIR is the composition of the CFSM systems denoted by its constituent parts.

Definition 3.10 (GTIR semantics).

The communicating system $\llbracket G \rrbracket^H$ denoted by a GTIR $\llbracket G \rrbracket^H$ is inductively defined as follows:

- $\llbracket G \rrbracket^H = (G, p \in P(G))$ where $G$ is a global type in $G\mathcal{T}$;
- $\llbracket G \rrbracket^H \otimes_s \llbracket G_1 \rrbracket^H = \llbracket [G_1]^{H \otimes_s} [G_2]^{K} \rrbracket^{H \otimes_s} [G_2]^{K}$.

\footnote{The CFSMs over $P_1$ and $\hat{A}_1$ ($P_2$ and $\hat{A}_2$ resp.) are considered here as CFSMs over $P_1 \cup P_2$ and $\hat{A}_1 \cup \hat{A}_2$.}
It is immediate to check that the operation of “connecting” GTIRs is semantically commutative and associative, i.e. the following holds:

\[
\begin{aligned}
\text{(comm)} & \quad \left[\left[\left[ G_1 \right]_{H \leftarrow K} \left[ G_2 \right]_{K} \right]_J \right]_I = \left[\left[\left[ G_2 \right]_{K} \left[ G_1 \right]_{H \leftarrow K} \right]_J \right]_I \\
\text{(ass)} & \quad \left[\left[\left[ G_1 \right]_{H \leftarrow K} \left[ G_2 \right]_{K} \right]_I \right]_J = \left[\left[\left[ G_2 \right]_{K} \left[ G_1 \right]_{H \leftarrow K} \right]_J \right]_I \right]
\end{aligned}
\]

where \( J' = K \cup J \setminus \{I, J\} \)

### 4 Preservation of Safety-Properties

In the present section we show that if we take two safe communicating systems \( S_1 \) and \( S_2 \) such that \( S_1 \) possesses a CFSM \( M_1^I \) and \( S_2 \) a CFSM \( M_2^I \) which is compatible with \( M_1^I \), replace both CFSMs by their gateway transformations and then join the resulting systems, we get a a safe system.

**General assumption:** In the following of this section we generally assume given a system \( S = S_1 \bowtie K S_2 \) composed as described in Def. 3.9 from systems \( S_1 \) and \( S_2 \) with compatible CFSMs \( M_1^I \) and \( M_2^I \).

**Notation:** The channels of \( S \) are \( C = \{pq \mid p, q \in P, p \neq q\} \) and the channels of \( S_i \) are \( C_i = \{pq \mid p, q \in P, p \neq q\} \) for \( i = 1, 2 \). If \( s = (\vec q, \vec w) \) is a configuration of \( S \), where \( \vec q = (q_p)_{p \in P} \) and \( \vec w = (w_{pq})_{pq \in C} \), we write \( s_i \) for \( (\vec q_i, \vec w_i) \) where \( \vec q_i = (q_p)_{p \in P_i} \) and \( \vec w_i = (w_{pq})_{pq \in C_i} \) (\( i = 1, 2 \)). Notice that \( s_i \) is not necessarily a configuration of \( S_i \), because of possible states in \( \hat Q \), which are the additional states of the gateways.

The following technical properties easily descend from the definition of gw(\( \cdot \)). In particular from the fact that the gateway transformation of a machine \( M \) does insert an intermediate state between any pair of states of \( M \) connected by a transition. By definition, the intermediate state possesses exactly one incoming transition and one outgoing transition.

**Fact 4.1.** Let \( s = (\vec q, \vec w) \) be a reachable configuration of \( S = S_1 \bowtie K S_2 \).

1. If \( q_H = \vec q \in \hat Q_H \) then \( q_H \) is not final and there exists a unique transition \( (q_H, \rightarrow, q_f') \) with \( q_f' \) \( \notin \hat Q_H \).

   *Moreover such a transition is of the form \( (q_H, Hs!a, q_f') \) with \( q_f' \neq G_H \).

   *Similarly for \( K \).

2. If \( q_H \notin \hat Q_H \) then either \( q_H \) is final, or any transition \( (q_H, \rightarrow) \in \delta_H \) is an input one, that is of the form \( (q_H, sH?a, \vec q_H') \) with \( \vec q_H' \in \hat Q_H \). Similarly for \( K \).

3. If \( q_H \notin \hat Q_H \) then

   a) If \( (q_H, Hs!a, \vec q_H') \in \delta_H \) then there exists \( (\vec q_H', Hs!a, q_f'' \in \delta_H \) with \( s \neq K \) such that \( (q_H, Hs!a, q_f'') \in \delta_H \). The same holds for \( \delta_K^2 \) by exchanging \( H \) with \( K \) and vice versa.

   b) If \( (q_H, sH?a, \vec q_H') \in \delta_H \) with \( s \neq K \) then there exists \( (\vec q_H', HK!a, q_f'' \in \delta_H \) such that \( (q_H, sH?a, q_f'' \in \delta_H \). The same holds for \( \delta_K^2 \) by exchanging \( H \) with \( K \) and vice versa.

If a reachable configuration of the connected system \( S = S_1 \bowtie K S_2 \) does not involve an intermediate state of the gateway \( M_H = gw(M_1^I, K) \), then by taking into account only the states of machines of \( S_1 \) and disregarding the channels between the gateways, we get a reachable configuration of \( S_1 \). Similarly for \( S_2 \).

**Lemma 4.2.**

Let \( s = (\vec q, \vec w) \) be a reachable configuration of \( S = S_1 \bowtie K S_2 \).
Lemma 4.3. Let \( q_H \in \widehat{Q}_H \) and \( q_K \in \widehat{Q}_K \) be participants. This property relies on compatibility.

Proof. [Sketch] (i) If \( s \in RS(S) \), then there exists \( s_0 \rightarrow s_1 \rightarrow \ldots \rightarrow s_{n-1} \rightarrow s_n = s \). Let \( s_i = (\vec{q}_i, \vec{w}_i) \) \((i = 0, \ldots, n)\). Let \( j \geq 0 \) be the smallest index such that \( q_{j+1} \in \widehat{Q}_H \) and \( q_{j+1} \in \widehat{Q}_H \) (if there is not such a \( j \), then the thesis follows immediately). By definition of \( gw(\cdot) \) we have that \( s_j \rightarrow_{\vec{r}_H} s_{j+1} \) for a certain \( \vec{r} \).

Now let \( t \geq j + 1 \) be the smallest index such that \( q_H = q_{j+1} \) and \( q_{j+1} \in \widehat{Q}_H \). Such an index \( t \) does exist because of the hypothesis \( q_H \in \widehat{Q}_H \) (moreover, notice that no-self loop transitions are possible out of a state in \( Q_H \)). By definition of \( gw(\cdot) \) we have that \( s_i \rightarrow_{\vec{r}_H} s_{i+1} \) for a certain \( \vec{r} \). It is now possible to build a configuration-transitions sequence like the following one

\[
\begin{align*}
0 & \rightarrow 1 \\
\vdots & \vdots \\
1 & \rightarrow 2 \\
\vdots & \vdots \\
H & \rightarrow 1 = s
\end{align*}
\]

where \( \vec{r}_H \).

By iterating this procedure, we can get a sequence

\[
\begin{align*}
0 & \rightarrow 1 \\
\vdots & \vdots \\
1 & \rightarrow 2 \\
\vdots & \vdots \\
H & \rightarrow 1 = s
\end{align*}
\]

such that any transition of the form \( \vec{r}_H?a \) is immediately followed by a transition \( \vec{s}_H?a \).

Now, it is possible to check that

a) for \( z = 0, \ldots, j - 1 \), either \( s_{z+1} \rightarrow_{\vec{s}_H} s_{z+1} \) or \( s_{z+1} = s_{z+1} \), and

b) \( s_{j+1} \rightarrow_{\vec{s}_H} s_{j+1} \).

By doing that for any transition of the form \( \vec{r}_H?a \) which is immediately followed by a transition \( \vec{s}_H?a \), we can get a sequence \( s_0 \rightarrow_{\vec{s}_H} s_n \). So \( s_1 \in RS(S) \).

(ii) This case can be treated similarly to (i). \( \square \)

In a reachable configuration of a connected system, if the states of the gateways are not among those introduced by the transformation \( gw(\cdot) \) and not final, and if the channels between the gateways are empty, then one of the two gateways is ready to receive messages that the other gateway is ready to receive from its system’s participants. This property relies on compatibility.

Lemma 4.3. Let \( s = (\vec{q}, \vec{e}) \in RS(S) \) be a reachable configuration of \( S = S_1?S_2 \) such that

1. \( q_H \notin Q_H \) and \( q_K \notin Q_K \), and
2. \( q_H \) and \( q_K \) are not final.

Then either

a) all the transitions from \( q_H \) in \( \delta_H \) are of the form \( (q_H, \vec{r}_H?\cdot) \) and all the transitions from \( q_K \) in \( \delta_K \) are of the form \( (q_K, \vec{s}_K?\cdot) \) with \( \vec{s} \notin H \), or

b) all the transitions from \( q_K \) in \( \delta_K \) are of the form \( (q_K, \vec{r}_K?\cdot) \) and all the transitions from \( q_H \) in \( \delta_H \) are of the form \( (q_H, \vec{s}_H?\cdot) \) with \( \vec{s} \notin K \).

Proof. [Sketch] Let

\[
\begin{align*}
0 & \rightarrow 1 \\
\vdots & \vdots \\
1 & \rightarrow 2 \\
\vdots & \vdots \\
H & \rightarrow 1 = s
\end{align*}
\]

be a configuration-transitions sequence leading to \( s \in RS(S) \). Let \( \vec{s}_H \) be the sequence of transitions \( l_1 \rightarrow \ldots \rightarrow l_m \) of the above sequence such that, for any \( m \in \{l_1, \ldots, l_m\} \), \( l_m \in \delta_H \). We define similarly the sequence \( \vec{r}_K \). By definition of \( gw(\cdot) \), and by the fact that \( q_H \notin Q_H \), we have that \( \vec{s}_H \) is made of consecutive pairs of the form \( (\vec{r}_H?s_{j+1}) \), with \( \vec{s} \notin K \), or \( (\vec{r}_H?s_n) \), with \( \vec{s} \notin K \). Similarly for \( \vec{r}_K \).
Since \( w_{KH} = w_{KH} = \varepsilon \) (which immediately follows from the hypothesis \( s = (q, \bar{e}) \)), the number of pairs \( (s^H \rightarrow s^K \rightarrow s^R) \) in \( s^R \) is equal to the number of pairs \( (s^R \rightarrow s^K \rightarrow s^H) \) in \( s^H \); and vice versa. This implies that \( |s^R| = |s^K| \).

Now, by extending to sequences \( s^R \) and \( s^K \) the following symbols function on pairs

\[
\text{symb}(s^H \rightarrow s^K \rightarrow s^R) = \lambda a \quad \text{symb}(s^R \rightarrow s^K \rightarrow s^H) = ?a
\]

(and the clauses got by exchanging \( H \) and \( K \)), we get \( \text{symb}(s^R) \in \mathcal{L}(M^1_H) \) and \( \text{symb}(s^K) \in \mathcal{L}(M^2_K) \).

Moreover, \( \text{symb}(s^R) = \text{symb}(s^K) \). Notice that, by \(!\)-determinism of \( M^1_H \) and \( M^2_K \) and absence of mixed states, there are no other sequences of pairs for which the previous properties hold.

Now, by contradiction, and by recalling that \( q_H \) and \( q_K \) are not final, let us assume that either

1) all the transitions from \( q_H \) in \( \delta_H \) are of the form \( (q_H, KH?\_\_, \_) \) and all the transitions from \( q_K \) in \( \delta_K \) are of the form \( (q_K, HK?\_\_, \_) \) or

2) all the transitions from \( q_H \) in \( \delta_H \) are of the form \( (q_H, sH?\_\_, s) \) with \( s \neq H \) and all the transitions from \( q_K \) in \( \delta_K \) are of the form \( (q_K, sK?\_\_, s) \) with \( s \neq H \).

Notice that no other possibilities are given because, by compatibility, \( M^1_H \) and \( M^2_K \) have no mixed state.

If \( \text{I} \) holds, we get a contradiction, since, by Fact \( 4.1(3) \), we would get both \( \text{symb}(s^R) \cdot \lambda a \in \mathcal{L}(M^1_H) \) and \( \text{symb}(s^K) \cdot \lambda b \in \mathcal{L}(M^2_K) \) for some \( a \) and \( b \), which is impossible by compatibility.

In case \( \text{II} \) we get a contradiction by arguing analogously as in the previous case.

**Lemma 4.4.** Let \( s = (q, \bar{e}) \in RS(S) \) be a deadlock configuration for \( S \). Then, either \( s_i \in RS(S_1) \) is a deadlock configuration for \( S_1 \) or \( s' \in RS(S_2) \) is a deadlock configuration for \( S_2 \) (or both).

**Proof.** By definition of deadlock configuration and by Fact \( 4.1(1) \), we have that neither \( q_H \in Q_H \) nor \( q_K \in \hat{Q}_K \). Otherwise there will be an output transition from either \( q_H \) or \( q_K \), contradicting \( s \) to be a deadlock configuration. Hence necessarily \( q_H \not\in Q_H \) and \( q_K \not\in \hat{Q}_K \). So, by Lemma \( 4.2 \) we get \( s_i \in RS(S_1) \) for \( i = 1, 2 \).

We show that under the assumption either \( s_i \in RS(S_1) \) or \( s' \in RS(S_2) \) is a deadlock configuration.

Since \( s = (q, \bar{e}) \in RS(S) \) is a deadlock configuration for \( S \), we have that for all \( r \in R \), \( q_r \) is a receiving state of \( M_r \). In particular, for all \( r \in P_1 \setminus \{H\} \), \( q_r \) is a receiving state of \( M_r = M^1_r \) and for all \( r \in P_2 \setminus \{K\} \), \( q_r \) is a receiving state of \( M_r = M^2_r \). It remains to show that either \( q_H \) is a receiving state of \( M^1_H \) or \( q_K \) is a receiving state of \( M^2_K \), whereby we can assume that \( q_H \) is a receiving state of \( M_H = gw(M^1_H, K) \) and \( q_K \) is a receiving state of \( M_K = gw(M^2_K, H) \).

Without loss of generality we consider \( q_H \). The proof for \( q_K \) is analogous. By Lemma \( 4.3 \) we have two possibilities to take into account

All the transitions from \( q_H \) in \( \delta_H \) are of the form \( (q_H, KH?\_\_, \_) \).

In such a case, still resorting to Lemma \( 4.1(3) \), we have that all the transitions from \( q_H \) in \( \delta_H \) are of the form \( (q_H, sH?\_\_, s) \) with \( s \neq H \) and, by Fact \( 4.1(3) \),

\[
(q_H, sH?a, q_H') \in \delta_H \text{ implies } (q_H, KH!a, q_H'') \in \delta_K \text{ and } (q_H, sH?a, q_H'') \in \delta_K.
\]

Hence \( q_H \) is a receiving state of \( M^2_K \). In summary, \( s_i \) is a deadlock configuration for \( S_1 \).

All the transitions from \( q_H \) in \( \delta_H \) are of the form \( (q_H, sH?\_\_, s) \) with \( s \neq H \).

In such a case, by Fact \( 4.1(3) \), we have that

\[
(q_H, sH?a, q_H') \in \delta_H \text{ implies } (q_H, KH!a, q_H'') \in \delta_H \text{ and } (q_H, sH?a, q_H'') \in \delta_H.
\]

Hence \( q_H \) is a receiving state of \( M^1_H \). In summary, \( s_i \) is a deadlock configuration for \( S_1 \).
Corollary 4.5 (Preservation of deadlock-freeness). Let $S_1$ and $S_2$ be deadlock-free. Then $S = S_1^{\text{Hi-K}} S_2$ is deadlock-free.

Proof. By contradiction, let us assume there is an $s \in RS(S)$ such that $s = (\overline{q}, \overline{w})$ is a deadlock configuration. We get immediately a contradiction by Lemma 4.4 and the fact that $S_1$ and $S_2$ are two deadlock-free systems.

Compatibility of interface roles forces all the messages sent by a gateway to be correctly received by the other one. This implies that if the gateways both reach final states, the channels connecting them are empty.

Lemma 4.6. If $s = (\overline{q}, \overline{w}) \in RS(S)$ is a reachable configuration of $S = S_1^{\text{Hi-K}} S_2$ such that both states $q_{\text{Hi}}$ and $q_{\text{K}}$ are final, then $w_{\text{Hi}} = w_{\text{K}} = \varepsilon$.

Proof. [Sketch] By Fact 4.11, $q_{\text{Hi}} \not\in \overline{Q}_H$ and $q_{\text{K}} \not\in \overline{Q}_K$. We now proceed as in the first part of the proof of Lemma 4.3. Let $s_0 \xrightarrow{l_1} s_1 \xrightarrow{l_2} \cdots \xrightarrow{l_{n-1}} s_{n-1} \xrightarrow{l_n} s_n = s$ be a configuration-transitions sequence leading to $s \in RS(S)$.

Let $s^{\text{Hi}}$ be the sequence of transitions $\xrightarrow{l_1} \cdots \xrightarrow{l_m}$ of the above sequence such that, for any $m \in \{i_1, \ldots, i_8\}$, $l_m \in \delta_H$. We define similarly the sequence $s^{\text{K}}$. By definition of $gw(\cdot)$, and by the fact that $q_{\text{Hi}} \not\in \overline{Q}_H$, we have that $s^{\text{Hi}}$ is made of consecutive pairs of the form $\xrightarrow{\left(\overline{aK}q_{\alpha}\right)} \xrightarrow{\left(\overline{HK}r_{\alpha}\right)}$, with $s \not\equiv K$, or $\xrightarrow{\left(\overline{aH}q_{\alpha}\right)} \xrightarrow{\left(\overline{HR}p_{\alpha}\right)}$, with $s \not\equiv K$. Similarly for $s^{\text{K}}$.

Without loss of generality, let us assume $s^{\text{Hi}}$ to begin with a pair of the form $\xrightarrow{\left(\overline{aH}q_{\alpha}\right)} \xrightarrow{\left(\overline{HR}p_{\alpha}\right)}$. (Otherwise $s^{\text{Hi}}$ would begin with a pair of the form $\xrightarrow{\left(\overline{aK}q_{\alpha}\right)} \xrightarrow{\left(\overline{HK}r_{\alpha}\right)}$ since $L(M^1_H) = L(M^2_K)$. Hence, up to the role $\approx$, the symbols of $s^{\text{Hi}}$ are uniquely determined by $s^{\text{Hi}}$ because of $?!$-determinism and absence of mixed states. Then $\text{symb}(s^{\text{Hi}}) \in L(M^1_H)$ and $\text{symb}(s^{\text{Hi}}) \in L(M^2_K)$. Moreover, $\text{symb}(s^{\text{Hi}}) = \text{symb}(s^{\text{K}})$. By assuming either $w_{\text{Hi}} \not\equiv \varepsilon$ or $w_{\text{Hi}} \not\equiv \varepsilon$ we would get a contradiction. In fact, by the above, $|s^{\text{Hi}}| = |s^{\text{K}}|$. □

Lemma 4.7. Let $s = (\overline{q}, \overline{w}) \in RS(S)$ be an orphan-message configuration for $S$. Then, either $s_{i_4}$ is an orphan-message configuration for $S_1$ or $s_{i_4}$ is an orphan-message configuration for $S_2$.

Proof. By Fact 4.11, no state in $\overline{Q}_H \cup \overline{Q}_K$ can be final. So, by definition of orphan-message configuration, $q_{\text{Hi}} \not\in \overline{Q}_H$ and $q_{\text{K}} \not\in \overline{Q}_K$. Hence, for $i = 1, 2$, $s_i \in RS(S_i)$ by Lemma 4.2. Now, by definition of orphan-message configuration, both $q_{\text{Hi}}$ and $q_{\text{K}}$ are final in $M_H$ and $M_K$ respectively. Hence, by Lemma 4.6, $w_{\text{Hi}} = w_{\text{Hi}} = \varepsilon$. This implies that either $w_{i_4} \not\equiv \varepsilon$ or $w_{i_4} \not\equiv \varepsilon$. Moreover, $q_{\text{Hi}}$ and $q_{\text{K}}$ must also be final in $M^1_H$ and $M^2_K$ respectively. The rest of the thesis follows then by definition of orphan-message configuration. □

Corollary 4.8 (Preservation of no orphan-message). Let $S_1$ and $S_2$ be such that both $RS(S_1)$ and $RS(S_2)$ do not contain any orphan-message configuration. Then there is no orphan-message configuration in $RS(S)$.

Proof. By contradiction, let us assume there is an $s \in RS(S)$ which is an orphan-message configuration. We get immediately a contradiction by Lemma 4.7. □

Proposition 4.9 (Preservation of no unspecified reception). Let $S_1$ and $S_2$ be such that both $RS(S_1)$ and $RS(S_2)$ do not contain any unspecified reception configuration. Then there is no unspecified reception configuration in $RS(S)$. □
Proof. [Sketch] By contradiction, let us assume there is an \( s = (q, w) \in RS(S) \) which is an unspecified reception configuration. Moreover, let \( r \in P \) and \( q_k \) be the receiving state of \( M_k \) prevented from receiving any message from any of its buffers (Definition 2.4(iii)). Without loss of generality, we assume \( r \in P_1 \).

The following cases can occur:

\[ q_k \notin \hat{Q}_k. \]

By Lemma 4.2, we get \( s_1 \in RS(S_1) \). Two sub-cases are possible:

- \( r \neq H \)
  - In such a case we get immediately a contradiction by the hypothesis that \( RS(S_1) \) does not contain any unspecified reception configuration.
  - \( r = H \)
    \[ q_\hat{H} (= q_k) \] is hence a receiving state. So let \( \{ (q_\hat{H}, s_j, H?a_j, \hat{q}_j) \}_{j=1..m} \) be the set of all the outgoing transitions from \( q_\hat{H} \) in \( \delta_{\hat{H}} \).
    By definition of unspecified reception configuration, for any \( j = 1..m, |w_{\hat{H}H}| > 0 \) and \( w_{\hat{H}} \notin a_j \cdot H^* \). By compatibility, and in particular by the absence of mixed states, we have just the following two possibilities:
    \( s_j \neq K \) for any \( j = 1..m \).
    By Fact 4.1(3) and definition of \( gw(\cdot) \) we have that
    \[
    [(q_\hat{H}, s_j, H?a_j, \hat{q}_j) \in \delta_{\hat{H}} \land s_j \neq K] \iff (q_\hat{H}, sH?a_j, q_j) \in \delta_{\hat{H}}^\Gamma
    \]
    This implies \( s_1 \) to be an unspecified reception configuration for \( S_1 \). Contradiction.
    \( s_j = K \) for any \( j = 1..m \).
    Let \( s^H \) and \( s^K \) be defined as in the proofs of Lemmas 4.3 and 4.6. We define now
    - \( s^H \) as the sequence made of the transition pairs in \( s^H \) of the form \( (\_H\mu, a\_\mu) \)
      with \( s \neq K \), and
    - \( s^K \) as the sequence made of the transition pairs in \( s^K \) of the form \( (\_K\mu, a\_\mu) \)
      with \( s \neq H \).
    Let now \( n = |s^H| \) and let \( s^H_n \) be the sequence of the messages of the first \( n \) elements of \( s^H \). By compatibility, in particular \( \mathcal{L}(M^1_\hat{H}) = \mathcal{L}(M^2_\hat{K}) \), it follows that, if \( w_{\hat{H}H} \in b \cdot H^* \), then \( s^H_n = s^H_n \cdot b \) and \( \exists j. a_j = b. \) So contradicting that, for any \( j = 1..m, w_{\hat{H}} \notin a_j \cdot H^* \).

\[ q_\hat{H} = \hat{q} \in \hat{Q}_H. \]

By Fact 4.1(1) \( q_\hat{H} \in \hat{Q}_H \) is a sending state such that \( (q_\hat{H}, H?a, q''_H) \in \delta_{\hat{H}} \) and it is impossible that \( r = H \). So, let \( r \neq H \). In such a case, by definition of \( gw(\cdot) \), we have necessarily a unique transition of the form \( (q_\mu, H?a, q_\mu) \in \delta_{\hat{H}} \). Moreover, \( q_\mu \notin \hat{Q}_H \). So there exists necessarily an element \( s' \in RS(S) \) such that \( s' = (q_\mu, w' \mu) \) and \( q''_H \notin \hat{Q}_H \). It follows that also \( s' \) is an unspecified-reception configuration and \( s_1 \in RS(S_1) \). Then we get a contradiction by arguing like in the first case, sub-case \( r \neq H \).

We are now ready to state our main results.

**Corollary 4.10** (Safety properties preservation).

1. Let \( S = S_1^H \cdot K \cdot S_2 \) be the system composed from systems \( S_1 \) and \( S_2 \) with compatible CFSMs for the roles \( H \) and \( K \). If \( S_1 \) and \( S_2 \) are both safe, then \( S \) is safe.

2. Let \( (G)|^H = [(G)^H_1 \cdot K](G)^H_2 \cdot K \) be the GTIR composed from GTIRs \( (G)^H_1 \) and \( (G)^H_2 \) via compatible interface roles \( H \) and \( K \). If \( [(G)^H_1] \) and \( [(G)^H_2] \) are both safe, then \( [(G)|^H] \) is safe.
As a consequence, by induction on the pairwise composition of GTIRs, we obtain the following desired result.

**Corollary 4.11.** Let $[G]^I$ be a GTIR such that, for any global graph $G \in \mathcal{G}([G]^I)$, the system $(G|p)_{p \in P(G)}$ is safe. Then $\llbracket [G]^I \rrbracket$ is safe.

### 5 Conclusions

We have proposed the GTIR formalism (Global Types with Interface Roles) to support the usage of global types in the context of open systems, whenever the underlying global type formalism, like $\mathcal{G}$, allows the interpretation of global types in terms of systems of communicating finite state machines (CF-\ SMs). Our main result is that safety properties (deadlock-freeness, no orphan messages, no unspecified receptions) are preserved by composing open systems when interface roles are compatible.

In [9, Sect. 6], the authors use the same compatibility notion for CFSMs showing that a system made of two CFSMs, which both are deterministic and do not have mixed states, is free from deadlocks and unspecified receptions. The general aim of enhancing the expressive power of global type formalisms has been variously pursued in the literature. For example, in [15] the authors define a formalism where global types with initial and end points can be combined. The results in [14] are slightly more related to our approach. It is shown how to define choreographies partially specified, where only some processes are provided. In case two choreographies are composable, some completely specified process of one can be used instead of the unspecified ones in the other.

Even if some loose connections can be envisaged with the approach of *interface automata* of [6, 7], our approach to open global types diverges from them in many relevant points: First of all, an interface automaton describes the communication abilities of an automaton with its environment in terms of input and output actions while internal behavior is described by internal actions. GTIRs, however, emulate the expected behavior of the environment by providing distinguished interface roles and their CFSMs, while internal behavior is modelled by the CFSMs of the other roles. Interface automata rely on synchronous communication while we consider asynchronous communication via FIFO buffers. The crucial idea of compatibility for interface automata is that no error state should be reachable in the synchronous product of two automata. An error state is a state, in which one automaton wants to send a message to the other but the other automaton is not ready to accept it. This situation is related to unspecified reception in the asynchronous context. The speciality of interface automata is, however, that an error state must be autonomously reachable, i.e. without influence of the environment. Since interface automata use synchronous message passing, the problem of orphans is empty. Moreover, the theory of interface automata does not consider deadlock-freedom. On the other hand, interface automata consider also refinement and preservation of compatibility by refinement.

In the future, we first want to study whether our conditions for compatibility could be relaxed still guaranteeing preservation of safety. Moreover, it would be worth taking into account, besides safety properties, also liveness properties. In particular, the generalised global types of $\mathcal{G}$, at the cost of being less expressive than global types in [13, 15], guarantee also liveness properties. Properties preserved by connecting CFSM systems via *gateways* are worth to be investigated also for systems unrelated to global type formalisms. For instance, a variety of communication properties are formalised for asynchronous I/O-transition systems in [10]. Preservation by composition is shown there but using bags instead of FIFO buffers for communication. Finally, the current composition operator for GTIRs is binary and thus can only lead to tree-like compositions of global graphs. Therefore it would be challenging to see, how we could get cyclic architectures.
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References


