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Information about CIEAEM 71 and presentation of the Volume
Informations sur la CIEAEM 71 et présentation du Volume

The 71st CIEAEM conference was held from 22nd to 26th July 2019 at the Instituto de Educação da Universidade do Minho, Campus de Gualtar, Braga. Several participants from many countries all over the world, actively participated to the Conference. The Theme “Connections and understanding in mathematics education: Making sense of a complex world” was really appreciated by participants; researchers, teachers, educators, and students worked together in the true CIEAEM spirit, with a collaborative and inspiring attitude.

How can we re-conceptualise learning with understanding in a complex world?, What kind of mathematics training should teachers have in order to be able to promote learning with understanding?, In relation to connections and understanding, what kind of teaching methods are more appropriate?, How do we evaluate and/or research the resources from the perspective of the connections and the understanding they try to promote?, How can ICTs contribute to learning rich in connections, in an increasingly complex world?, How can ICT be used in teacher training to promote understanding in mathematics?, Is it possible to understand peoples’ lives from an ethnomathematics perspective?, How can school mathematics take into account the culture developed by young people in their everyday lives?, How to take advantage of cultural aspects to enrich the teaching and learning of mathematics?
These are only some of important questions on which all the participants had the possibility to fruitfully discuss in critical and constructive ways.
Many of the Conference participants were also authors of significant papers presented during the working groups, workshops, and poster (forum of ideas) activities. This volume contains all final versions of these papers.

We thank all the contributors and all the participants to the conference and we are grateful to Pedro Palhares and all the members of the International Programme Committee and the Local Organizing Committee for the realization of the 71st CIEAEM conference. Particularly we want to thank the Working Group animators, who, as in previous CIEAEM meetings, worked in a well high-quality way.

March, 2020

Benedetto Di Paola,
Pedro Palhares
Translating practices for reflecting on ourselves: Lesson Study

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Abstract. Lesson Study (LS) is a collaborative methodology for teachers’ professional development (TPD) rooted in Japan. In LS, a group of teachers and experts collaborates to the detailed planning of a one-hour lesson. The difference between LS and other methodologies is the collaborative foundation of the experience, there is no evaluation on the performance of a single member of the group. We believe that, in the Italian context, LS can be an appropriate tool to answer in an efficient way the Ministry’s demands for a “mandatory, permanent and strategic” TPD and for the “establishment of adequate networks for professional collaboration”, while maintaining the focus on teachers’ needs. Discussing, observing and reflecting on your own and others’ practices can help in re-thinking your own professionalism while relating to a community of peers. In the workshop we will work on one of the main tools in LS, the Lesson Plan, and discuss how the encounter of such a foreign tool can lead to self-reflection on one’s own practices.

1. Lesson Study

Lesson Study (LS) is a collaborative methodology for teachers’ professional development (TPD) rooted in Japan. Since 1999, researchers in TPD and didactics from all over the world have started studying the methodology (Huang & Shimizu, 2016), and since 2003 the Asia-Pacific Economic Cooperation (APEC) has been following its international diffusion. Catherine Lewis, vice-president of WALS, has had an essential role in the world-wide diffusion of LS (Bartolini Bussi & Ramploud, 2018).

In a LS, a group of at least three teachers (which we will call Lesson Study Group; it can include one or more student teachers or university experts) collaborates to the detailed planning of a one-hour lesson, to be taught in one of the teachers’ classroom observed by the other teachers, and discussed by the group. The difference between LS and other methodologies is the collaborative
foundation of the experience, which leads to the establishment of a sense of diffused responsibility between the members of the group. Moreover, the observation of the lesson indicates LS as a form of action-research. There is no evaluation on the performance of a single member of the group: the focus is on the lesson and the students, not on teachers’ individual ability.

The National strategies for Teachers’ Professional Development document of the Italian Ministry of Public Education, covering the three-year period 2016 – 2019, stresses the importance of addressing issues such as: teachers’ isolation in managing pupils’ learning; connecting work and professional development; difficulties in applying in a real-classroom context the didactic innovations proposed by universities. We believe that LS can support the researchers’ and teachers’ communities in answering the Ministry’s demands for a “mandatory, permanent and strategic” TPD and for the “establishment of adequate networks for professional collaboration”, while maintaining the TPD focus on teachers’ needs. Discussing, observing and reflecting on your own and others’ practices can help in re-thinking your own professionalism while relating to a community of peers. The encounter with others, from this point of view, is one’s self-rediscovery (Mellone, Ramploud, Di Paola, & Martignone, 2018).

2. Italian Lesson Study: ideas from Turin

As shown from the literature review (Fernandez & Yoshida, 2004; Minisola, 2016; Robutti, et al., 2016), LS is generally a three-step cycle aimed at creating a virtuous process in which teachers can grow continuously (Ramploud & Munarini Frenesi, 2015). In its (cultural) transposition to Western cultures – particularly the Italian one – these steps can be defined the “essentials” of LS: establishment of long-term learning goals and lesson planning, implementation and observation of the research lesson, discussion on the lesson. These steps can be repeated, like a life cycle in which each lesson is the foundation for subsequent growth. In the Italian context, the time expected for each step is: at least 2 hours for goals establishing, 2 hours for lesson planning, 1 hour for the lesson, 2 to 4 hours for the discussion. The overall commitment for teachers is predicted in 7 to 10 hours.

Initial findings mandate to clarify that, in LS, a lesson is a specific moment in the classroom routine (i.e. the mathematics lesson in class 3B held from 9 to 10 a.m. on February the 3rd, 2019). The group of lessons dedicated to a specific topic (i.e. continued fractions) is called teaching unit. The existence of a LS-Group is tied to that of the observed lesson. Even so, the same group of people can participate in more study cycle and establish a stable-over-time community of practice. The aim is to build and institutionalize a collaborative methodology, which can sustain teachers in both their job and professional development, focusing on the new multicultural context we all are living in.

The tentative structure of Lesson Study in the Italian context is: Definition of long-term educational objectives: LS is a form of action-research, in which teachers collaborate to improve their professionalism in accordance to the context in which they work. The reasons to engage in LS might stem from different teachers’ needs: i.e., difficulties in confronting with certain mathematical topics, improving strategies to involve students, experimenting new didactical methodologies. A research question is formulated by the group in accordance to these needs, and exploring the possible answers is the objective of the LS. Moreover, Italian secondary school teachers have autonomy on defining the educational plan, referring to Indicazioni Nazionali – the national curriculum by Ministry of Education – containing knowledge and competences related to the specific kind of school. Moreover, each school has its Piano Triennale dell’Offerta Formativa or PTOF (“three-year plan of the educational offer”), in which more specific educational objectives are described. Thus, in the first phase of LS in Italy, the teachers choose a teaching unit and the related long-term learning objectives, in accordance to Indicazioni Nazionali and PTOF: these objectives should be relevant to the whole group (i.e. because they are difficult to attain) to
promote engagement, and related to the research question(s). One (or two) demonstrating teachers should be chosen, to develop a lesson aimed at a specific context, and to investigate the answer to specific (and yet shared by the group) needs.

Lesson Planning: The demonstrating teacher(s) prepares – on his/her own or working with colleagues – a draft of the Lesson Plan, describing: the class context (such as the general level of knowledge and competencies, or the presence of students with special educational needs: the Italian school is inclusive, meaning that in Italy there are no special schools for students with learning difficulties, physical disabilities or behavioural problems); the teaching unit in which the lesson is inscribed; a proposal for the 1-hour lesson in accordance – as much as possible – to the class’ didactic contract. The tentative Lesson Plan is given to the whole LS-Group before the planning meeting, in which the group discusses the details and decides: the phases of the lesson, the time to allocate for each phase, the teacher’s requests to students, how the teacher should react to some students’ expected reactions, what are the educational aim of each phase, which classroom grouping strategies to apply. The plan is carefully fitted on both the classroom’s pupils and the demonstrating teacher’s disposition. The group proposes ideas, techniques, strategies, but ultimately it is the demonstrating teachers’ choice what to implement and what is not doable in his/her classroom. Observational focuses are established in accordance to the initial aims and to the group’s decisions in planning the lesson: i.e. the group might decide to focus on the efficacy of artefacts proposed by the teacher to the class, of the grouping strategies, of the problem structure, etc. The group may decide to use a table to guide the observation using some learning descriptors, and/or to focus on some students considered representative of the classroom situation. In this phase, appointing a secretary to record the discussion, and a moderator to the discussion, is useful in terms of time management.

Lesson implementation and observation: The teacher and the observers enter into the classroom to teach and observe the prepared lesson. A series of preliminary encounters might be necessary to get students accustomed to the presence of other people. The observers are silent and should not influence the class’ practice. The presence of all the members of the LS-Group is not necessary, but a video record of the lesson is recommended.

Discussion: LS methodology is focused on the efficacy of the prepared lesson in accordance to the established objectives, not on the ability of the individual teacher. Before the discussion meeting, the whole LS-Group (including the teacher) has shared and studied the observers’ reports and possible videos. The discussion is opened by the teacher, who shares his/her impressions and observations on what occurred in the classroom. The whole group discusses how to fix what did not work, improve what did, reflect on how to deal with (and consider the possibility of) the unexpected: it is not possible to plan for every instance that may occur. Missing something is a “mistake” of the whole group, even if the teacher in charge of the lesson was the one who had to respond to the unexpected event(s). As different teachers make different kind of expertise available for the group, this is the opportunity for both the demonstrating teacher and the whole group to learn how to manage unexpected situations or improve non-optimal behaviours, absorbing new ideas from others’ experience. This discussion may or may not result in a new “improved” Lesson Plan, which can be taught by the same or another teacher in a different classroom, bringing about a new study cycle. As in the planning phase, choosing a secretary and a moderator is advised.

3. Plan for the workshop
The workshop will be organised in the following steps:
1. A 10 minutes introduction on Lesson Study and the Lesson Plan
2. The participants will be divided in small groups of 3-4 persons each. The groups should be organised as much as possible according to nationality and school level. Each group will have about 10 minutes to decide, possibly according to the participants’ training needs, which activity they will work on among the proposed ones (differentiated by school level).

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3. 60 minutes will be dedicated to the lesson planning, using the empty Lesson Plan form provided. Each group is asked to keep track of the encountered difficulties, whether they are educational, planning-related, organizational, and also relationship-related difficulties. Since the proposed Lesson Plan structure is adequate to the Italian context, the focus will be especially on those related to the different cultural context the participants are used to.

4. A 10 minutes meta-discussion on the activity: what are the most striking differences between this planning methodology and those used in the participants’ usual contexts? Are there any analogies? Does the encounter with the Lesson Plan, a tool coming from a different cultural and institutional context, bring a reflection on the participants’ own practices?

4. The Workshop

The workshop lasted 70’ and the tentative times were adjusted accordingly: 7’ for the introduction, 8’ for deciding the teaching material, 45’ for lesson planning, 10’ for the discussion. 17 people attended the workshop, from a number of nationalities, ages and professional backgrounds. In particular, 7 of them were university students and declared no teaching experience. Audio recording the session was not possible due to GDPR limitations; the presenters noted arguments and comments on a notebook.

During step 1, material on Lesson Study was presented to the attendants. The presentation focused on Lesson Study in relation with the Japanese context, analysed the similarities with the Italian context, and the peculiarities of the latter (i.e.: both contexts focus on the pupils and design educational plans in terms of long-term goals; Japanese teachers mainly work inside schools, Italian ones mainly work at home; Italian school is inclusive, Japanese school is not). A possible adaptation of LS in the Italian context was proposed, and a copy of an empty Lesson Plan (cfr. Appendix) was distributed to each participant. An overview of the proposed structure for the Lesson Plan was discussed, and the participants were proposed the task of the workshop for step 2. The groups resulting by the participants self-organisations were more numerous than expected and heterogeneous, differently from what asked by the presenters. Rearranging them was deemed not necessary. In the case that some of the participants needed support in planning for a lesson, two teaching activities were proposed on different mathematical topics: Heron’s problem and its generalization for high school, exploration on non-planar surfaces for middle school.

At the beginning of step 3, the participants were asked to work and discuss within their groups, and three questions were presented to guide the work: (1) What are the most striking differences between this planning methodology and those used in the participants’ usual contexts? (2) Are there any analogies? (3) Does the encounter with the Lesson Plan, a tool coming from a different cultural and institutional context, bring a reflection on the participants’ own practices? About one third of the participants continued working on the teaching activities; after other 10’ the presenter decided to ask to concentrate on the provided Lesson Plan, as it was the focus of the workshop.

By the beginning of the whole-group discussion (step 4), no one had compiled a Lesson Plan: when asked for a reason, a participant commented “analysing and discussing the Lesson Plan is more interesting, as we didn’t have enough time to study the material”, at which the others agreed. Three participants expressed curiosity about the structure of the lesson proposed in the Lesson Plan, and the presenters explained that it was inspired to Calvani (2014) and Bartolini Bussi et al. (2017).

Participants from France and Spain noted that the philosophy of the Lesson Plan is not far from the work teachers usually do in their context. In Spain, for example, “our teachers do this kind of work for each lesson, and lesson plans are uploaded online for families and others to be consulted”; others from Holland and Switzerland explained that detailed lesson planning is usually mandatory for pre-service teachers during their training but not for in-service ones, and it is usually kept as internal documentation for the school. Two participants from Poland commented that “[detailed
lesson planning] is unusual for both prospective and in-service [teachers], especially with all the details proposed in this Lesson Plan”. Spain followed that “estimating the time for each phase, we don’t do that” and suggested a modification in the table to highlight this characteristic of the proposed Lesson Plan.

All the participants agreed that specifying the “educational intentionality” for each phase was “the real peculiarity of this document”. When asked for clarification, one young participant from Poland explained: “I believe that this is very important when doing pre-service training. It makes you aware that everything you do in classroom is relevant for your students”. A participant from Belgium agreed: “it is something you tend to forget even when you are an experienced teacher, so it would be nice to have a reminder every now and then. It makes you aware”. The participant from France concluded: “detailed lesson planning is very difficult no matter the subject, but I believe that in Mathematics it is especially important: Mathematics gives students critical thinking, develops their cognitive abilities, is the basis for their scientific approach. We all need to be very careful when we teach it, we are shaping the future… and we need all the help we can get”.

5. Discussion

Designing a detailed lesson plan is no easy task, as proven by the fact that no participant produced a complete Lesson Plan, not even those accustomed to detailed documentational work. Furthermore, the task of planning and compiling the plan for a lesson depends on the documentational work that may vary from context to context, for cultural and institutional reasons.

The proposed Lesson Plan is the result of the reflection on the Italian culture, institutional context, usual practices. It embeds Italian institutional peculiarities, such as the different stance on educational objectives: namely, long-term objectives are presented in the National Guidelines, whereas lesson objectives are decided by each teacher individually. A Lesson Plan fitted to a certain context is not immediately effective: its conscious use requires familiarity with the mathematical knowledge, curriculum, teaching traditions, institutional context.

In this sense, we might say that the workshop evolved unexpectedly. The focus shifted from the analysis of Lesson Plan’s specific steps to its general issue as a design tool in a school. This new focus nourished the participants’ confrontation. The self-organized heterogeneity enriched the discussion, albeit some realism got lost in the transition (namely, no Lesson Plan was compiled), and answered to the needs of the participants.

In conclusion, preparing and studying a detailed plan for an effective Mathematics lesson is perceived both like a challenge and a necessity. Discussing and sharing educational experiences, provided they happen within customary practices of designing and programming, might sustain to collaboratively overcome the perceived challenges of teachers' professionalism. To achieve this, it is necessary that researchers in mathematics education deepen their studies of interaction with teachers in order to improve the collaboration with them in concrete teaching activities.

Appendix

In this appendix, the empty Lesson Plan used in the workshop is presented. Please note that this is a condensed version. The printed version provides enough space to write. This version of the Lesson Plan is inspired by Bartolini Bussi et al. (2017) and Fernandez & Yoshida (2004).
References


