Splitting Recursion Schemes into Reversible and Classical Interacting Threads

Splitting recursion schemes into reversible and classical interacting threads

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Abstract. Given a simple recursive function, we show how to extract from it a reversible and an classical iterative part. Those parts can synchronously cooperate under a Producer/Consumer pattern in order to implement the original recursive function. The reversible producer is meant to run on reversible hardware. We also discuss how to extend the extraction to a more general compilation scheme.

1 Introduction

Our goal is to compile a class of recursive functions in a way that parts of the object code produced can leverage the promised green foot-print of truly reversible hardware. This work illustrates preliminary steps towards that goal. We focus on a basic class of recursive functions in order to demonstrate its feasibility.

Contributions. Let `recF[p,b,h]` be a recursive function defined in some programming formalism, where `p` is a *predecessor* function, `h` a *step* function, and `b` a *base* function. We show how to compile `recF[p,b,h]` into `itFCls[b,h]` and `itFRev[p,pInv]` such that:

$$\texttt{recF[p,b,h]} \simeq \texttt{itFCls[b,h]} \parallel \texttt{itFRev[p,pInv]} \quad , \tag{1}$$

where: (i) "$\simeq$" stands for "*equivalent to*"; (ii) `itFCls[b,h]` is a classical `for`-loop that, starting from a value produced by `b`, iteratively applies `h`; (iii) `itFRev[p, pInv]` is a reversible code with two `for`-loops in it one iterating `p`, the other its inverse `pInv`; (iv) "$\parallel$" is interpreted as an *interaction* between `itFCls[b,h]` and `itFRev[p,pInv]`, according to a Producer/Consumer pattern, where `itFRev[p, pInv]` produces the values that `itFCls[b,h]` consumes to implement the initially given recursion `recF[p,b,h]`. In principle, `itFRev[p,pInv]` can drive a real reversible hardware to exploit its low energy consumption features.

In this work we limit the compilation scheme (1) to use: (i) a predecessor `p` such that the value `p(x)-x` is any *constant* $\Delta_{\texttt{p}}$ equal to, or smaller than, `-1`; (ii) recursion functions `recF[p,b,h]` whose *condition* identifying the base case is `x<=0` instead than the more standard `x==0`; this means that more than one base *non positive* value for `recF[p,b,h]` exists in the interval $[\Delta_{\mathtt{p}}+1, 0]$. This slight generalization will require a careful management of the reversible behavior of `itFRev[p,pInv]` and its interaction with `itFCls[b,h]` in order to reconstruct `recF[p,b,h]`.

Contents. Section 2 sets the stage to develop the main ideas about (1), restricting `recF[p,b,h]` to a recursive function that identifies its base case by means of the standard condition `x==0`; this ease the description of how `itFRev[p,pInv]` and `itFCls[b,h]` interact. Section 3 extends (1) to deal with `recF[p,b,h]` having `x<=0`, and not `x==0`, to identify its base case(s); this impacts on how `itFRev[p,pInv]` must work. In both cases, the programming syntax we use can be interpreted into the reversible languages SRL [3,4] and RPP [5,6,4], up to minor syntactic details. Section 4 addresses future work.

```
Fix recF(x)                          {
    if   (c(x)) { b(x);           }
    else        { h(x,recF(p(x))); } }
```

Fig. 1. The recursive function `recF`.

```
 /*** Assumption: the inital value of x is 3 */
 x = p(x)       // ==2
 x = p(x)       // ==1
 x = p(x)       // ==0
 y = b(x)       // ==b(p(p(p(3))))
 y = h(x,y)     // ==h(p(p(p(3))),b(p(p(p(3)))))
 x = pInv(x)    // ==pInv(p(p(p(3))))==p(p(3))
 y = h(x,y)     // ==h(p(p(3)),h(p(p(p(3))),b(p(p(p(3))))))
 x = pInv(x)    // ==pInv(p(p(3)))==p(3)
 y = h(x,y)     // ==h(p(3),h(p(p(3))
                //           ,h(p(p(p(3))),b(p(p(p(3)))))))
 x = pInv(x)    // ==pInv(p(3))==3
 y = h(x,y)     // ==h(3,h(p(3),h(p(p(3))
                //        ,h(p(p(p(3))),b(p(p(p(3))))))))
```

Fig. 2. Iterative unfolding `recF(3)`: the bottom-up part.

2 The driving idea

Let `recF[p,b,h]` in (1) have a structure as in **Fig. 1** where `b(x)` is the *base* function, `h(x,y)` the *step* function, `p(x)` the *predecessor* `x-1`, and `c(x)` the *condition* `x==0` to identify a unique base case.

Fig. 2 details out `h(3,h(p(3),h(p(p(3)), h(p(p(p(3))),b(p(p(p(3))))))))`, unfolding of `recF(3)`. Every comment asserts a property of the values that `x` or `y` stores. Lines 2–4 unfold an iteration that computes `p(p(p(3)))`, which eventually sets the value of `x` to `0`. Line 5 starts the construction of the final value of `recF(3)` by applying the base case of `recF`, i.e. `b(x)`. By definition, let `pInv` denote the inverse of `p`, i.e. `pInv(p(z))==p(pInv(z))==z`, for any `z`. Clearly, in our running example, the function `pInv(x)` is `x+1`. Lines 6–13 alternate `h(x,y)`, whose result `y`, step by step, gets closer to the final value `recF(3)`, and `pInv(x)`, which produces a new value for `x`.

```
1  s = 0, e = 0, g = 0, w = 0
2  w = w + x;
3  for (i = 0; i<=w; i++)      {
4    if      (x> 0) { g++; }
5    else if (x==0) { e++; }
6    else           { s++; }
7    x = p(x);              }
8
9  for (i = 0; i<=w; i++)                 {
10   x = pInv(x);
11   if      (x> 0) { g--; y = h(x,y); }
12   else if (x==0) { e--; y = b(x);   }
13   else           { s--;             } }
14 w = w - x;
```

Fig. 3. Iterative `itF` equivalent to `recF`.

Let us call `itF` the code in **Fig. 3**. It implements `recF` by means of finite iterations only. Continuing with our running example, if we run `itF` here above starting with `x==3`, then `x==0` holds at line 8, just after the first `for`-loop; after the second `for`-loop `y==recF(3)` holds at line 14.

The code of `itF` has two parts. Through lines 2–7 the variable `g` counts how many times `x` remains positive, the variable `e` how many it stays equal to `0`, and the variable `s` how many it becomes negative. In this running example we notice that `x` never becomes negative, for the iteration at lines 3–7 is driven by the value of `x` which, initially, we can assume non negative, and which `p(x)` decreases of a single unity. We shall clarify the role of `s` later. Lines 9–13 undo what lines 2–7 do by executing `pInv(x)`, `g--`, `e--`, `s--`, i.e. the inverses, in reversed order, of `p(x)`, `g++`, `e++`, `s++`. So the correct values of `x` are available at lines 12, and

11, ready to be used as arguments of `b(x)` and `h(x,y)` to update `y` as in **Fig. 3**, according to the results we obtain by the recursive calls to `recF`.

```
1  s = 0, e = 0, g = 0, w = 0
2  w = w + x;
3  for (i=0; i<=w; i++)      {
4    if      (x> 0) { g++; } //number of times x is 'g'reater than 0
5    else if (x==0) { e++; } //number of times x is 'e'qual to 0
6    else           { s++; } //number of times x is 's'maller than 0
7    x = p(x);               }
8
9  for (i=0; i<=w; i++)                                          {
10   x = pInv(x);
11   if      (x> 0) { g--; /* Value of x for h availabe here */ }
12   else if (x==0) { e--; /* Value of x for b availabe here */ }
13   else           { s--;                                     } }
14 w = w - x;
```

Fig. 4. Reversible side of `itF`.

Now, let us focus on the main difference between **Fig. 4** and **Fig. 3**.

Both `x=b(x)` and `y=h(x,y)` at lines 12, and 11 of **Fig. 3** are missing from lines 12, and 11 of **Fig. 4**. Dropping them let **Fig. 4** be the *reversible side* of `itF`; calling `b(x)` and `h(x,y)` in it generates`y`, which is the result we need, so preventing the possibility to reset the value of every variable dealt with in **Fig. 4** to their initial value. This is why we also need a *classical side* of `itF` that generates `y` in collaboration with the *reversible side* in order to implement the initial `recF` correctly.

```
 /*** Assumption. The value of the input x is available here */
 /* Inject the current x at line 2 of itFRev to let it start */
 iterations = /* Probe line 9 of itFRev to get the
                 number of iterations to execute   */
 y = b(/* Probe line 14 of itFRev to get the argument */);
 for (i = 0; i<iterations; i++)                  {
   y = h(/* Probe line 12 itFRev to get
            the first argument of h     */ , y); }
```

Fig. 5. Classical side of `itF`: the consumer `itFCls`.

The previous observations lead to **Fig. 5** which defines the *classical side* `itFCls` of `recF`, and to **Fig. 6** which defines the *reversible side* `itFCRev` of `recF`.

```
1  s = 0, e = 0, g = 0, w = 0;
2  x = /* Inject here the value of x at line 2 of itFCls */
3  w = w + x;
4  for (i = 0; i<=w; i++)      {
5    if      (x> 0) { g++; }
6    else if (x==0) { e++; }
7    else           { s++; }
8    x = p(x);                 }
9  /* itFCls probes here g which has the number of iterations */
10 for (i = 0; i<=w; i++)                                   {
11   x = pInv(x);
12   if      (x> 0) { g--; /* itFCls probes here the
13                             first argument value of h */ }
14   else if (x==0) { e--; /* itFCls probes here the
15                             argument value of b       */ }
16   else           { s--;                                  } }
17 w = w - x;
```

Fig. 6. Reversible side of `itF` updated to be the producer `itFRev` of the values that the consumer `itFCls` needs.

So, here below we can illustrate how `itFCls` and `itFRev` synchronously interact, `itFRev` producing values, `itFCls` consuming them as arguments of `b(x)` and `h(x,y)`.

Line 2 of `itFCls` is the starting point of the synchronous interaction between `itFCls` and `itFRev`; its comment:

```
/* Inject the current x at line 2 of itFRev to let it start */
```

describes what, in a fully implemented version of `itFCls`, we expect in that line of code. The comment says that `itFCls` injects (sends, puts) its input value `x` to line 2 of the *reversible side* `itFRev` (cf. **Fig. 6**). Once `itFRev` obtains that value at line 2, as outlined by:

```
/* Inject here the value of x from line 2 of itFCls */
```

its `for`-loop at lines 4–8 executes.

After line 2, `itFCls` stops at line 3. It waits for `itFRev` to produce the number of times that `itFCls` has to iterate line 7. Accordingly to:

```
/* Probe line 9 of itFRev to get the number of iterations to execute */
```

`itFRev` makes that value available in its variable `g` at line 9:

```
/* itFCls probes here g which has the number of iterations */ .
```

Once gotten the value in `iterations`, `itFCls` proceeds to line 5 and stops, waiting for `itFRev` to produce the argument of `b` which is eventually available for probing at line 14 of `itFRev`.

Once the argument becomes available `b` is applied, and `itFCls` enters its `for` -loop, stopping at line 7 at every iteration. The reason is that `itFCls` waits for line 12 in `itFRev` to produce the value of the first argument of `h(x,y)`. This interleaved dialog between line 7 of `itFCls` and line 12 of `itFRev` lasts `iterations` times.

```
Fix recG(x)                          {
  if (x<=0) { b(x);               }
  else      { h(x,recG(p(x))); } }
```

Fig. 7. The generic structure of `recG`.

3 From recursion to iteration

We now generalize what we have seen in Section 2. Inside (1) we use `recG` of **Fig. 7** instead than `recF` of **Fig. 1**. This requires to generalize **Fig. 6**.

From the introduction we recall that, given a *predecessor* `p(x)`, we define $\Delta_{\texttt{p}}$ = `p(x)-x`, which is a negative value. In this section $\Delta_{\texttt{p}}$ can be any *constant* k `<=-1`, not only k `==-1`; this requires to consider the slightly more general *condition* `x<=0` in `recG`. For example, let `p(x)` be `x-2`. The computation of `recG(3)` is `h(3,h(p(3),h(p(p(3)),b(p(p(3))))))` which looks for the least n of iterated applications of `p(x)` such that `p(...p(3)...)<=0`; in our case we have `2==` n `<3`.

Fig. 8 introduces `itG` which generalizes `itF` in **Fig. 3**.

The scheme `itG` iteratively implements any recursive function whose structure can be brought back to the one of `recG`. We remark that line 1 in **Fig. 8** initializes ancillae `s`, `e`, `g`, and `w`, like **Fig. 3** initializes the namesake variables of `itF`, but line 2 of `itG` has new ancillae `z`, `predDivX`, and `predNotDivX`.

We also assume an initial *non negative* value for `x`. The reason is twofold. Firstly, it keeps our discussion as simple as possible, with no need to use the absolute value of `x` to set the upper limit of every index `i` in the `for`-loops that occur in the code. Second, negative values of `x` would widen our discussion about what a classical recursive function on negative values is and about what its reversible equivalent iteration has to be; we see this as a very interesting subject connected to [1], which is much more oriented than us to optimization issues of recursively defined functions.

We start observing that line 3 of `itG` sets `w` to the initial value of `x`; the reason is that every `for`-loop, but the one at lines 10–12, has to last `x+1` iterations, and `x` changes in the course of the computation; so, `w` stores the initial value of `x` and stays constant from line 4 through line 21. In fact it can change at lines 22–33. We will see why, but `w` is eventually reset to its initial value `0` at line 36.

```
s = 0, e = 0, g = 0, w = 0;
z = 0, predDivX = 0, predNotDivX = 1;
w = w + x; /* x is assumed to be the input */
for (i = 0; i <= w; i++)   {
  if      (x >  0) { g++; }
  else if (x == 0) { e++; }
  else             { s++; }
  x = p(x);                  }

for (i = 0; i < e; i++)                 {
  predDivX = predDivX + predNotDivX;
  predNotDivX = predDivX - predNotDivX; }

for (j = 0; j < predDivX; j++)            {
  for (i = 0; i <= w; i++)               {
    x = pInv(x);
    if      (x >  0) { g--; y = h(x,y); }
    else if (x == 0) { e--; y = b(x);   }
    else             { s--;             }}}

for (j = 0; j < predNotDivX; j++)                           {
  w++;
  for (i = 0; i <= w; i++)                                 {
    x = pInv(x);
    if      (x >  0) { g--;
                       x = p(x);
                       if      (z <  0) {                }
                       else if (z == 0) { y = b(x); z++; }
                       else             { y = h(x,y);    }
                       x = pInv(x);                       }
    else if (x == 0) { e--;             }
    else             { s--;             }}
  w--;                                                      }
for (i = 0; i < predNotDivX; i++)  {
  z--;                             }
w = w - x;
/* y carries the output */
```

Fig. 8. The iterative function `itG`.

With the here above assumptions, given a non negative `x`, and in analogy to `itF`, the `for`-loop at lines 4–8 of `itG` iterates the application of `p(x)` as many times as `w+1`, i.e. the initial value of `x` plus `1`. So, the value of `x` at line 9 is equal to `w+(w+1)*`$\Delta_{\mathtt{p}}$ which cannot be positive. In particular, all the values that `x` assumes in the `for`-loop at lines 4–8 belong to the following interval:

$$I(\mathtt{w}) \triangleq [\mathtt{w+(w+1)*}\Delta_{\mathtt{p}}, \mathtt{w+w*}\Delta_{\mathtt{p}}, \ldots, \mathtt{w+}\Delta_{\mathtt{p}}, \mathtt{w}] \tag{2}$$

from the least to the greatest; the counters `g`, `e`, `s` say how many elements of $I(\mathtt{x})$ are greater, equal or smaller than `0`, respectively. Depending on `0` to belong to $I(\mathtt{x})$ determines the behavior of the reminder part of `itG`, i.e. lines 10–36.

We distinguish two cases in order to illustrate them.

First case. Let `w%`$\Delta_{\mathtt{p}}$ `== 0`, i.e. the integer value $\Delta_{\mathtt{p}}$ divides with no reminder the initial value of `x` that we find in `w`. So, $\mathtt{0} \in I(\mathtt{x})$, which implies the following relations hold at line 9:

$$\mathtt{e\ ==\ 1} \qquad\qquad \mathtt{g\ ==\ -}\frac{\mathtt{w}}{\Delta_{\mathtt{p}}} \qquad\qquad \mathtt{s\ ==\ (w+1)-g-e}\ . \tag{3}$$

```
if      (e <  0) {                                    }
else if (e == 0) { predDivX = predDivX+predNotDivX;
                   predNotDivX = predDivX - predNotDivX; }
else             {                                    }
```

Fig. 9. A possible replacement of lines 10–12 in **Fig. 8**.

Lines 10–12 execute exactly once, swapping `predDivX` and `predNotDivX`. As a remark, we could have well used the `if`-selection in **Fig. 9** (a construct of RPP) in place of the `for`-loop at lines 10–12, but we opt for a more compact code.

Swapping `predDivX` and `predNotDivX` sets `predDivX==1` and `predNotDivX==0`, computationally exploiting that $\Delta_{\mathtt{p}}$ divides `w` with no reminder: the `for`-loop body at lines 15–19 becomes accessible, while lines 22–33, with `for`-loops among them, do not. Lines 15–19 are identical to lines 10–16 of `itF` in **Fig. 4** which we already know to correctly apply `b(x)` and `h(x,y)` in order to simulate the recursive function we start from.

As a second case. Let `w%`$\Delta_{\mathtt{p}}$ `!= 0`, i.e. the integer value $\Delta_{\mathtt{p}}$ divides the initial value of `x` that `w` stores, *but with some reminder*. So, $\mathtt{0} \notin I(\mathtt{x})$, which imply:

$$\mathtt{e\ ==\ 0} \qquad\qquad \mathtt{g\ ==\ -}\left\lfloor\frac{\mathtt{w}}{\Delta_{\mathtt{p}}}\right\rfloor \qquad\qquad \mathtt{s\ ==\ (w+1)-g-e} \tag{4}$$

hold at line 9. Lines 11–12 cannot execute, leaving `predDivX` and `predNotDivX` as they are: lines 22–33 become accessible and the `for`-loop at lines 15–19 does not. Line 22 increments `w` to balance the information loss that the rounding of `g` in (4) introduces; line 33 recovers the value of `w` when the outer `for`-loop starts. The `if`-selection at lines 25–32 identifies when to apply `b(x)`, which must be followed by the required applications of `h(x,y)`. We know that $0 \notin I(\texttt{x})$, so `x==0` can never hold. Clearly, `s--` is executed until `x>0`. But the *first* time `x>0` holds true we must compute `b(p(x))`, because the *base* function `b(x)` *must be used the last time* `x` assumes a negative value, *not the first time* it gets positive; lines 26–30 implement our needs. Whenever `x>0` is true, the value of `x` is one step ahead the required one: we get one step back with line 26 and, if it is the first time we step back, i.e. `z==0` holds, then we must execute line 28. If not, i.e. `z!=0`, we must apply the *step* function at line 29. Line 30, restores the right value of `x`. Finally, the `for`-loop at line 34 sets `z` to its initial value.

At this point, in order to obtain the fully reversible version of **Fig. 8** we must think of replacing the calls to `h(x,y)` and `b(x)` at lines in 28, and 29 by means of actions that probe the value of `x`, in analogy to **Fig. 6**, lines 12 and 14. The full details are in [7] which we look as a playground with Java classes that implement **Fig. 8** and **Fig. 5** as synchronous and parallel threads, acting as a producer and a consumer.

4 Future work

We have shown that we can decompose every classical recursive function, based on a *predecessor* that decreases every of its input by a constant value, into reversible and classical components that cooperate to implement the original recursive functions under a Producer/Consumer pattern (see (1)).

Firstly, we plan to extend (1) to recursive functions `recF` based on predecessors `p` not limited to a constant $\Delta_{\texttt{p}}$ not greater than `-1`. A predecessor `p` should be at least such that:

1. $\Delta_{\texttt{p}}$ is not necessarily a constant. For example, $\Delta_{\texttt{p}}$ `== -3` on even arguments, and `-2` on odd ones can be useful;
2. the predecessor can be an integer division `x/k`, for some given `k>0`, like in a dichotomic search, which has `k==2`.

Secondly, we aim at generalizing (1) to a compiler $[\![\cdot]\!]$:

$$\begin{aligned} [\![\texttt{p}]\!] &= \texttt{some implementation code} \\ [\![\texttt{pInv}]\!] &= \texttt{!}[\![\texttt{p}]\!]\texttt{, i.e. implementation that inverts } [\![\texttt{p}]\!] \\ [\![\texttt{recF[p,b,h]}]\!] &= \texttt{itFCls[}[\![\texttt{b}]\!]\texttt{,}[\![\texttt{h}]\!]\texttt{]} \parallel \texttt{itFRev[}[\![\texttt{p}]\!]\texttt{,}[\![\texttt{pInv}]\!]\texttt{]}\ . \end{aligned} \tag{5}$$

The domain of $[\![\cdot]\!]$ should be a class R of recursive functions built by means of standard composition schemes, starting from a class of predecessors `p1`, `p2`, ... each of which must have the corresponding inverse function `p1Inv`, `p2Inv`, ...

In these lines we want to explore interpretations of $||$ more liberal than the essentially obvious synchronous Producer/Consumer that we implement in [7]. We shall very likely take advantage of parallel discrete events simulators as described in [8,9] in order to get rid of any explicit synchronization between the pairs of reversible-producer/classical-consumer that (5) would recursively generate when applied to an element in R.

We also plan to follow a more abstract line of research. The compilation scheme (5) recalls Girard's decomposition $A \to B \simeq !A \multimap B$ of a classical computation into a linear one that can erase/duplicate computational resources. Decomposing `recF[p,b,h]` in terms of `itFCls[b,h]` and `itFRev[p,pInv]` suggests that the relation between reversible and classical computations can be formalized by a linear isomorphism $A^n \multimapboth B^n$ between tensor products A^n, and B^n of A, and B, in analogy to [2]. Then we can think of recovering classical computations by some functor, say γ, whose purpose is, at least, to forget, or to inject replicas, of parts of A^n, and B^n in a way that $(\gamma A^n \to \gamma A^n) \uplus (\gamma A^n \leftarrow \gamma A^n)$ can be their type. The type says that we move from a reversible computation to a classical one by choosing which is input and which is output, so recovering the freedom to manage computational resources as we are used to when writing classical programs.

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