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We explore the constraints on dark sector models imposed by the recent observation of coincident gravitational waves and gamma rays from a binary neutron star merger, GW170817. Rather than focusing on specific models as has been considered by other authors, we explore this in the context of the equation of state approach of which the specific models are special cases. After confirming the strong constraints found by others for Horndeski, Einstein-Aether, and massive gravity models, we discuss how it is possible to construct models which might evade the constraints from GW170817 but still leading to cosmologically interesting modifications to gravity. Possible examples are "miracle cancellations" such as in $f(R)$ models, non-local models and higher-order derivatives. The latter two rely on the dimensionless ratio of the wave number of the observed gravitational waves to the Hubble expansion rate being very large ($\sim 10^{19}$) which is used to suppress modifications to the speed of gravitational waves.

I. INTRODUCTION

The detection of gravitational waves from a source almost coincident with a gamma ray burst suggests that the two come from the merger of a binary neutron star system [1,2]. The measured time difference between the two is $\Delta t_{\mathrm{obs}} = (1.75 \pm 0.05)$ sec and the distance inferred to the source is $D = 40^{+8}_{-14}$ Mpc [3]. The difference between two waves emitted a time Δt_{emit} apart is given by

$$\Delta t = \Delta t_{\mathrm{obs}} - \Delta t_{\mathrm{emit}}$$
$$= \frac{D}{c_{\mathrm{G}}} - \frac{D}{c_{\gamma}} = \frac{D}{c_{\gamma}} \left[\left(1 + \frac{\Delta c}{c_{\gamma}} \right)^{-1} - 1 \right], \quad (1.1)$$

where c_{γ} and c_{GW} are the propagation speeds of the photons and gravitational waves respectively, and $\Delta c = c_{\mathrm{GW}} - c_{\gamma}$. By making the assumptions that $-10 < \Delta t_{\mathrm{emit}}/\mathrm{sec} < 0$ and $\Delta c / c_{\gamma} \ll 1$, and also conservatively using the lower bound on the distance, $D \approx 26$ Mpc, one obtains a very strong constraint on the difference between the speed of propagation of gravitational waves and photons

$$-3 \times 10^{-15} < \frac{\Delta c}{c_{\gamma}} < 7 \times 10^{-16}. \quad (1.2)$$

One might question this constraint in that the precise numbers depend very strongly on the unknown Δt_{emit}.

However, any value for which one might imagine that it was possible to make a definite association between the gravitational wave signal and the counterpart photons still leads to a very strong constraint on $\Delta c / c_{\gamma}$ due to the large distance over which the signals have propagated. For example, if $|\Delta t_{\mathrm{emit}}| < 1$ day then $|\Delta c / c_{\gamma}| < 10^{-9}$ which is already a very stringent limit. Similar bounds are obtained by the lack of gravitational Cherenkov radiation [4–7].

A number of authors [8–13] have pointed out that this constraint has very severe implications for many, but not all, modified gravity models considered in the literature as possible origins of the cosmic acceleration.[1] The focus of these discussions is mainly on the generalized scalar-tensor (ST) models known as Horndeski and beyond Horndeski theories, although there is also some discussion on vector-tensor (VT), massive gravity and Hořava models. If these works are to be taken at face value they appear to rule out all but the simplest—and observationally least interesting—modified gravity models, implying that observational programs aimed at constraining them using cosmological observations might be wasting their time and significant amounts of taxpayer funding.

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[1]We note that the constraining power of a simultaneous detection of gravitational waves and electromagnetic counterpart was anticipated by [14–18]. In particular, it was shown that large scale structure observations would not be able to unequivocally distinguish Horndeski models from the ΛCDM model, but that gravitational waves could break what the authors call "dark degeneracy" [15,16].

In our contribution to this discussion[2] we do not question the specific calculations presented in these earlier works. However, we do note that dark sector models are designed to modify gravity on scales $\sim H_0^{-1}$ whereas the scales relevant to the observations of the binary neutron star merger GW170817 are $\sim 10^{14}$ sec $\sim 10^{-4} H_0^{-1}$ (the lookback time inferred from the distance) and $\sim 10^{16}$ Mpc$^{-1} \sim 10^{19} H_0$ (the wave number computed from the frequency of gravitational waves detected). This means that in the context of gravitational wave sources, such as GW170817, there is a large dimensionless number $K_{grav} = k_{grav}/H_0$ which in principle might be used to suppress modifications to gravity on small scales, but which can be very different on large scales. In its very simplest terms our argument is that the very wide range of scales between those probed by cosmology and those relevant for the detection of gravitational waves means that there is significant room for the construction of models that avoid these constraints. In what follows we flesh out our arguments within the equation of state approach to cosmological perturbations in dark sector models.[3]

II. CONSTRAINING THE EQUATION OF STATE APPROACH

The equation of state approach [19,20] is a phenomenological idea for describing perturbations in dark sector models whereby whatever is causing the cosmic acceleration is modelled as an isotropic fluid with equation of state $P_{ds} = w_{ds}\rho_{ds}$ where P_{ds} and ρ_{ds} are the pressure and the density of the dark sector fluid at the background level, respectively, and w_{ds} is not necessarily a constant, but is often considered to be so within the context of present observations. Such a description is sufficient for describing observations that are only sensitive to the expansion rate of the Universe. If one wants to also include observations sensitive to perturbations, such as those for the cosmic microwave background (CMB) or cosmic shear, then it is necessary to also provide an equation of state for the perturbations which encodes how the dark sector perturbations respond, allowing the linearised conservation equations for the dark sector fluid to become closed and, hence, be solved using standard codes (see, for example, the discussions presented in [21,22]).

Most work to date has focused on the scalar perturbations since they are most relevant to cosmological observations, but it can be adapted to the tensor (gravitational wave) sector and indeed the simplicity of the idea is even more clear there due to the reduced number of degrees of freedom. Assuming that the $+$ and $\times$ modes of gravitational

wave evolve identically—which need not be the case—the equation for the evolution of the transverse-traceless component of the metric in an Friedmann-Robertson-Walker universe with a dark sector producing cosmic acceleration is given by

$$\ddot{h} + 3H\dot{h} + \frac{k^2}{a^2}h = 16\pi G_N P_{ds}\Pi_{ds}^T, \qquad (2.1)$$

where $H = \dot{a}/a$ is the Hubble parameter, G_N is Newton's constant, and Π_{ds}^T is the tensor component of the anisotropic stress. In general, $\Pi_{ds}^T \equiv 0$ does not need to imply $\Pi_{ds}^S \equiv 0$ and this could be seen as a simple way to avoid all constraints from GW170817. We would, however, see $\Pi_{ds}^T \equiv 0$ and $\Pi_{ds}^S \neq 0$ as being a little unnatural, but not necessarily impossible. In order to solve this equation it is necessary to specify Π_{ds}^T and by similar arguments to those applied to the scalar sector we can write

$$8\pi G_N P_{ds}\Pi_{ds}^T = C_{\ddot{h}}\ddot{h} + C_{\dot{h}}H\dot{h} + C_h H^2 h, \qquad (2.2)$$

where $C_{\ddot{h}}$, $C_{\dot{h}}$ and C_h are all functions of a and k.

When discussing constraints imposed by GW170817, one needs to solve (2.1) inserting the expression in (2.2) which leads to

$$\ddot{h} + \frac{3 - 2C_{\dot{h}}}{1 - 2C_{\ddot{h}}}H\dot{h} + \frac{K^2 - 2C_h}{1 - 2C_{\ddot{h}}}H^2 h = 0, \qquad (2.3)$$

where $K = k/(aH)$. In what follows it is more convenient to rewrite (2.3) in a simpler form

$$\ddot{h} + [3 + \beta_M(a, K)]H\dot{h} + \beta_T(a, K)H^2 h = 0, \qquad (2.4)$$

where in general the dimensionless coefficients β_M and β_T can be a function of both time and scale and are related to C_i where $i = \ddot{h}, \dot{h}, h$, via

$$\beta_M = \frac{2(3C_{\ddot{h}} - C_{\dot{h}})}{1 - 2C_{\ddot{h}}}, \qquad \beta_T = \frac{K^2 - 2C_h}{1 - 2C_{\ddot{h}}}. \qquad (2.5)$$

Specific models for the dark sector predict different forms for the coefficients $\beta_M(a, K)$ and $\beta_T(a, K)$ and those already in the literature are presented in Appendix A and we note that for $\beta_M = 0$ and $\beta_T = K^2$ we recover the standard general relativistic result.

The specific choice typically assumed, is that of the Horndeski class of models[4] which leads to the specific forms $\beta_M = \alpha_M(a)$ and $\beta_T = [1 + \alpha_T(a)]K^2$, and it is this specific choice that leads to the very strong conclusions reported in [8–13], for example. In particular it has been

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[2]From now on we will use natural units where $c_\gamma = \hbar = k_B = 1$.
[3]We use the term dark sector to refer to whatever causes cosmic acceleration encompassing both dark energy and modified gravity models.

[4]We have shown in Appendix A that generalized Einstein-Aether models fall into this category, but that massive gravity and elastic dark energy models do not.

argued that the constraints from GW170817 imply that $|\alpha_T| < 10^{-15}$ and hence that it is reasonable to assume that $\alpha_T \equiv 0$ in these models. What we have argued here is that this specific form could be too restrictive and in particular there is room for the speed of gravitational waves being dependent on K. In Sec. III we will investigate how it might be possible to avoid these conclusions.

Before doing this we will address the solution of (2.4) using the Wentzel-Kramers-Brillouin (WKB) approximation as recently done in detail, including source terms, in [23,24], to which we refer for more details and for a more general discussion. Assuming a solution of the form $h = A(t) \exp[\iota\psi(t)]$, Eq. (2.4) is equivalent to the following two sets of equations:

$$\ddot{A} - A\dot{\psi}^2 + (3 + \beta_M)H\dot{A} + \beta_T H^2 A = 0, \qquad (2.6)$$

$$2\dot{A}\dot{\psi} + A\ddot{\psi} + (3 + \beta_M)HA\dot{\psi} = 0, \qquad (2.7)$$

where the two equations are derived from the real and imaginary parts, respectively. The condition we impose is that the amplitude of the gravitational wave is slowly changing with respect to the frequency of the wave itself ψ, therefore it is reasonable to assume that $\dot{\psi}^2 \gg \ddot{A}/A$ and $\dot{\psi}^2 \gg (3 + \beta_M)H\dot{A}/A$ which is equivalent to the oscillation timescale being much faster than the Hubble rate. This would be true for gravitational waves from GW170817, and similar objects, but is not necessarily relevant on cosmological scales. Under these conditions, the first equation reduces to $\dot{\psi} = \sqrt{\beta_T}H$ whose solution is

$$\psi = \int_a \sqrt{\beta_T} \frac{da'}{a'}, \qquad (2.8)$$

and the second one to $\partial_t \ln(A^2\dot{\psi}) = -(3 + \beta_M)H$ whose solution is

$$A = \frac{\exp\left[-\frac{1}{2}\int_a (3 + \beta_M)\frac{da'}{a'}\right]}{(\sqrt{\beta_T}H)^{1/2}}. \qquad (2.9)$$

The full WKB solution is

$$h(K, t) = \frac{h_0}{(\sqrt{\beta_T}H)^{1/2}} \exp\left[-\frac{1}{2}\int_{a_i}^{a(t)} (3 + \beta_M)\frac{da'}{a'}\right]$$
$$\times \exp\left[\iota \int_{a_i}^{a(t)} \sqrt{\beta_T}\frac{da'}{a'}\right], \qquad (2.10)$$

where h_0 represents the amplitude of the wave at $a = a_i = a(t_i)$.

We now evaluate the dispersion relation for gravitational waves and derive expressions for the phase $v_p = \omega/k$ and the group velocity $v_g = d\omega/dk$. The frequency is $\omega(K) = \dot{\psi} = \sqrt{\beta_T}H$, which leads to

$$v_p(k) = \frac{\sqrt{\beta_T}}{aK}, \qquad v_g(k) = \frac{\beta_T'}{2a^2 K v_p} = v_p \frac{K\beta_T'}{2\beta_T}, \qquad (2.11)$$

where a prime denotes the derivative with respect to K. These expressions are very simple and encompass a wide range of dark sector models. For a more general discussion on the group velocity of gravitational waves, we refer the reader to [25], but from the point of view of the present discussion it is important to note two points. First, the speed of gravitational waves only depends on β_T and β_M is unconstrained.[5] In addition it is clear that, for a general dependence of β_T on K, $v_p \neq v_g$. The observations of coincident electromagnetic and gravitational waves refer to the coincidence of detection of energy and hence refer specifically to the group velocity and not to the phase velocity. This distinction is not relevant in the Horndeski case where $v_p = v_g$, but we need to be slightly more careful here.

All the models discussed in the Appendix can be parametrized by the form $\beta_T = (1 + \alpha_T)K^2 + M_{GW}^2$, where α_T is a function of time and $M_{GW} = m_{GW}/H$ is the time dependent, dimensionless graviton mass. In this case the two velocities read

$$v_p(K) = \frac{1}{a}\sqrt{1 + \alpha_T + \frac{M_{GW}^2}{K^2}}, \qquad v_g(K) = \frac{1 + \alpha_T}{a^2 v_p}. \qquad (2.12)$$

If $M_{GW} \ll K_{grav} \sim 10^{19}$, which one would naturally expect, then we have that $v_p = v_g = \frac{1}{a}\sqrt{1 + \alpha_T}$ and hence we derive the constraint $|\alpha_T| < 10^{-15}$ as previously deduced. However, we see that there is no extra constraint imposed by GW170817 on M_{GW}. This is due to the suppression of this quantity by the large dimensionless number K_{grav}. However, the massive graviton could still have significant cosmological effects. In the next section we will attempt to develop this line of argument to more general dark sector models.

III. FORM OF THE EQUATION OF STATE FOR THE DARK SECTOR

In the previous section we have argued that the evolution of cosmological gravitational waves in the most general dark sector models can be parameterized by $\beta_M \equiv \beta_M(a, K)$ and $\beta_T = \beta_T(a, K)$ and the very specific case of $\beta_M \equiv \alpha_M(a)$ and $\beta_T \equiv [1 + \alpha_T(a)]K^2$ assumed by most authors leads to very strong constraints from GW170817.

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[5]It is possible for the observations of coincident gravitational and electromagnetic waves to be used to infer a distance measure and a redshift and hence for the construction of a Hubble diagram based on these "standard sirens". Indeed this method has already been used to infer a measurement of the Hubble constant [3]. In future it might be possible to use this approach to infer constraints on β_M [16,26].

In this section we will explore how it might be possible to evade these constraints in more general models.

Before this we should make an important point concerning our choice to parameterise these functions in terms of the dimensionless combination $K = k/(aH)$ which is $\gg 1$ in the regime relevant to gravitational waves from GW170817. All dark sector models could be considered to be unnatural in some way since the timescale of the age of Universe, H_0^{-1}, has been introduced to them by hand. This is manifest even in models with a cosmological constant where $\Lambda \propto H_0^2$—this is often known as the timescale problem or "why is $\Omega_\Lambda \sim \Omega_m$ today?" We do not attempt to solve this problem, but our argument is that once one accepts the addition of this new dimensionful quantity into the problem, one is not further increasing the complexity by reusing it. The significant consequence of this is that it is natural for cosmological observations to probe in the regime $K \ll 1$, while the solar system, where there are very stringent constraints on the nature of gravitational interactions [27,28], and GW170817 are in the regime $K \gg 1$. Hence, the constraints imposed by GW170817, while extremely strong in the regime of validity, only impose constraints in a regime very different to that probed by cosmological observations and hence one does not have to work too hard to construct a dark sector model capable of explaining large-scale cosmic acceleration while still being compatible with measurements on smaller scales.

In order to understand how one might avoid the constraints imposed by GW170817, let us consider the case where the dispersion relation is parametrized by some function $\chi(K)$ defined by

$$\omega^2 = K^2 H^2 [1 + \chi(K)], \qquad (3.1)$$

in which case the coefficients of the equation of state can be written as

$$C_h = K^2 \left[C_{\ddot{h}} + \chi \left(C_{\ddot{h}} - \frac{1}{2} \right) \right]. \qquad (3.2)$$

With this form for the dispersion relation, $\beta_T = K^2[1 + \chi(K)]$ and

$$v_p = \frac{1}{a}\sqrt{1 + \chi}, \qquad v_g = \frac{1}{a}\left[\sqrt{1 + \chi} + \frac{K\chi'}{2\sqrt{1 + \chi}} \right]. \qquad (3.3)$$

For the case below ensuring that $v_g \approx 1/a$, which is what the observations require, is equivalent to $v_p \approx 1/a$ and therefore we will concentrate on the simpler case of ensuring $v \approx 1/a$.

The form of $\chi(K)$ in the regime $K \gg 1$ governs the evolution of gravitational waves in the regime relevant to GW170817. If spatial derivatives enter in second order combinations (for example, $(\nabla_i F)^2$, $\nabla_i \nabla_j F$ for some scalar

function F) then it seems reasonable to expand $\chi(K)$ as a power series in K^2. The observed properties of gravitational waves suggest that terms with positive powers of K^2 are excluded and therefore we consider[6]

$$\chi(K) = \sum_{n=0}^{\infty} \frac{\chi_n}{K^{2n}}, \qquad (3.4)$$

where the dimensionless coefficients $\chi_n \equiv \chi_n(a)$ are chosen so that $1 + \chi$ remains > 0 for all K. The first two coefficients have physical interpretations: $\chi_0 = \Delta c/c_\gamma$ is the modification to the speed of propagation of gravitational waves constrained to be $|\chi_0| \ll 10^{-15}$ and $\chi_1 = M_{GW}^2 = m_{GW}^2/H^2$ is the dimensionless mass associated with a graviton mass m_{GW}. Observations of the gravitational waves event GW150914 lead to a relatively weak limit of $m_{GW} \leq 1.2 \times 10^{-22}$ eV which implies that $M_{GW}(a = 1) \lesssim 10^{10}$ [29]. We note that there is a stronger constraint of $m_{GW} \lesssim 10^{-30}$ eV, $M_{GW}(a = 1) \lesssim 10^3$ enforced by consideration of gravity in the solar system [30] and from weak lensing data [31–33].

In order to investigate possible models that might be able to avoid constraints from GW170817 it is interesting to consider some special cases.

(i) The simplest possible case is where $\chi \equiv 0$ which implies that $C_h = K^2 C_{\ddot{h}}$. An example of such a model is the $f(R)$ gravity model, or indeed any Horndeski model with $\alpha_T \equiv 0$. We describe models with this property as having a "miracle cancellation," in that they have $\alpha_T \equiv 0$ without having $\Pi_{ds}^T = 0$ and more importantly from the point of view of having interesting observational signatures due to the evolution of dark sector perturbations. In fact all Horndeski models with $G_4 \equiv G_4(\phi)$ and G_5 constant lead to such miracle cancellations. The conditions required for these miracle cancellations in generic scalar-tensor theories were determined in [34].

(ii) If $C_{\ddot{h}}$ is independent of K and consider the possibility of $\chi = \chi_1/K^2 + \chi_2/K^4$ as the simplest case which gives something beyond the graviton mass, then $C_h = B_2 K^2 + B_0 + B_{-2}/K^2$ for some coefficients B_2, B_0 and B_{-2} which are functions of the scale factor. In order to construct such a model with negative powers of K it may be necessary to introduce nonlocal modifications to gravity so that the equation of state contains terms such as $\nabla^{-2} h$. To see this more explicitly, let us consider for simplicity a model where the graviton mass is zero and the only term in the

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[6]This choice, written as a power series, appears to diverge as $K \to 0$. It is necessary the actual function which this power series represents would have a finite limit and is regularized in some way in order to avoid extreme behavior in the infrared regime of the theory. Such behavior would lead to a violation of causality. Simple function which has this property is $\chi(K) \propto (K_0^2 + K^2)^{-1}$ for some constant K_0.

series expansion is χ_1. The equation of motion for the transverse-traceless degrees of freedom h is

$$\ddot{h} + 3H\dot{h} + K^2H^2h + \frac{\chi_1}{K^2} = 0$$

$$\leftrightarrow \ddot{h} + 3H\dot{h} - \frac{1}{a^2}\nabla^2 h + \int d^3\mathbf{x}' K(\mathbf{x}-\mathbf{x}')h(\mathbf{x}',t) = 0,$$

(3.5)

where $K(\mathbf{x}) = \chi_1|\mathbf{x}|^{-1}$ would give rise to such a behavior and other suitably regularized kernels could be computed to achieve other limiting behaviors for $K \gg 1$ (i.e., higher order inverse powers of K). Constructing a Lagrangian which leads to this kind of evolution for the gravitational waves may be quite challenging, but it is not obviously impossible. We note that a model containing a nonlocal "mass" term $\propto R\Box^{-2}R$ where R is the Ricci scalar has been studied in a number of works with the conclusion that the model gives rise to a local equation for the traceless-transverse degrees of freedom where gravitational waves propagate at the speed of light [for example [35,36]]. Since this model has a nonvanishing $\Pi^{\mathrm{T}}_{\mathrm{ds}}$, it can be seen as another example, together with $f(R)$ models, of miracle cancellation. This happens because the model can be recast into a multiscalar-tensor theory. We feel though, this issue warrants further investigation since it has interesting cosmological consequences while at the same time surviving the constraints of GW170817.

In general it is not known how to build a nonlocal Lagrangian that gives rise to an integral term of the form as in (3.5), but one can follow the approach of [37,38] and enforce it at the level of the equations of motion. As shown in these works, this nonlocal term will propagate up to the equations of motion for the gravitational waves. Since there is no underlying physical argument which leads to a form for the kernel function $K(\mathbf{x} - \mathbf{x}')$, it is necessary to use phenomenological parametrizations, which can, nevertheless, be constrained by data, as for example where a nonlocal Poisson equation is valid; we refer to [38] for details of specific models.

(iii) A more general form for $C_{\ddot{h}} = A_0 + A_2K^2$ and the same form for χ as in the last example in which case $C_h = B_4K^4 + B_2K^2 + B_0 + B_{-2}/K^2$ with B_4, B_2, B_0 and B_{-2} again scale factor dependent coefficients. If one were to make the particular choice $A_0 = 1/2$ then one finds that $B_{-2} \equiv 0$ removing, by a specific cancellation, the need for nonlocal inverse powers of K and $\beta_{\mathrm{T}} \approx B_4K^2/A_2$ at large K so observations require $B_4/A_2 = 1 + \mathcal{O}(10^{-15})$. In this case, it would be necessary for the equation of state to contain terms such as $\nabla^2\ddot{h}$ and $\nabla^4 h$. This is for example the case for higher-order-derivatives

theories (see, for example, [39]). One might be concerned that such models might suffer from Ostrogradsky ghosts or other instabilities since these often appear in theories with higher order derivatives. Of course, one can easily construct models without them, $f(R)$ and more general Horndeski models being examples, and by construction—since we have defined a positive definite dispersion relation—our suggestions would automatically avoid them.

A simple example of such an equation of motion for the transverse-traceless degrees of freedom is

$$\ddot{h} + \frac{\nabla^2\ddot{h}}{a^2H^2} + 3H\dot{h} + 3\frac{\nabla^2\dot{h}}{a^2H} - \frac{\nabla^2 h}{a^2} + \frac{\nabla^4 h}{a^4H^2} + m^2_{\mathrm{GW}}h = 0$$

$$\leftrightarrow \ddot{h} + 3H\dot{h} + \frac{M^2_{\mathrm{GW}} + K^2 + K^4}{1 - K^2}H^2h = 0,$$

(3.6)

where we have specifically chosen the functional form of $C_{\dot{h}}$ to recover the standard friction term of general relativity, which need not be the case. Finally, ∇^4 represents the biharmonic operator.[7]

We note that this list of possibilities is far from exhaustive and indeed the details of the last two depend quite strongly on the choice of χ. Nonetheless we believe that one would come to similar qualitative conclusions in more general cases.

We note that an approach very similar to ours has been suggested by [40–42] to take into account quantum-mechanical effects which predict a small amount of violation to the otherwise accepted Lorentz covariance of physical laws. In this approach, the modified dispersion relation is defined by

$$E^2 = p^2 + m^2_{\mathrm{GW}} + \mathbb{A}p^\alpha, \qquad (3.7)$$

where $\mathbb{A}$ defines the magnitude of the deviations from the standard picture (with units $[\mathrm{energy}]^{2-\alpha}$) and α is a dimensionless constant. The models become particularly appealing for $\alpha < 2$ as they provide a screening length. In addition to the Compton length $\lambda_{\mathrm{GW}} = 1/m_{\mathrm{GW}}$ associated to the graviton mass, there is a characteristic scale $\lambda_{\mathbb{A}} = \mathbb{A}^{1/(\alpha-2)}$ associated with Lorentz violation [42]. Despite being phenomenological, the parametrized form of the dispersion relation in (3.7) can accommodate some particular classes of models, as described in [40,41].

Let us now rewrite the dispersion relation in a form more suitable for the goals of this work. Upon the following identifications, $E = \omega$ and $p = k$, we obtain

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[7]In three dimensions, we have $\nabla^4 = \frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial y^4} + \frac{\partial^4}{\partial z^4} + 2\frac{\partial^4}{\partial x^2\partial y^2} + 2\frac{\partial^4}{\partial x^2\partial y^2} + 2\frac{\partial^4}{\partial y^2\partial z^2}$.

$$\omega^2 = \mathrm{K}^2 H^2 \left(1 + \frac{\mathrm{M}_{\mathrm{GW}}^2}{\mathrm{K}^2} + \frac{\mathbb{A}}{(H\mathrm{K})^{2-\alpha}} \right), \qquad (3.8)$$

and from (3.1), assuming $\alpha = -2$, we can read off $\chi(\mathrm{K}) = \mathrm{M}_{\mathrm{GW}}^2/\mathrm{K}^2 + \mathbb{A}/(H\mathrm{K})^4$. We can easily see that $\chi_1 = \mathrm{M}_{\mathrm{GW}}^2$ and $\chi_2 = (\lambda_\mathbb{A}/H)^4$ is the term arising from Lorentz violation.

IV. CONCLUSIONS

In this paper we have attempted to address the question of whether it is possible to construct dark sector models which can naturally evade the very strong constraints imposed by GW170817 while still giving rise to cosmologically interesting signatures. Within the Horndeski class of scalar tensor models usually considered there is a strong constraint which restricts the space of models. This restriction prima-facie forces one into the regime where $G_4 \equiv G_4(\phi)$ and $G_5 \equiv 0$. Models where G_4 is a constant which fall into this class are much less observationally interesting since they do not have anisotropic stress and indeed they could be thought of as dark energy models, as opposed to a genuine modified gravity model where the cosmic acceleration is a self-acceleration effect [15,16]. An alternative that avoids this constraint is the introduction of the mass for the graviton or an equivalent effect due to elastic dark energy. The generalization we have advocated is to allow the coefficients describing the evolution of cosmological gravitational waves (2.4) to have arbitrary dependence on K parameterized by $\beta_{\mathrm{M}}(a, \mathrm{K})$ and $\beta_{\mathrm{T}}(a, \mathrm{K})$. The specific choice $\beta_{\mathrm{T}} = \mathrm{K}^2(1 + \alpha_{\mathrm{T}}) + \mathrm{M}_{\mathrm{GW}}^2$ is the one which is strongly constrained as described in previous works and we concur with these conclusions. More generally, observations force $\beta_{\mathrm{T}} \approx H^2\mathrm{K}^2$ at $\mathrm{K} = \mathrm{K}_{\mathrm{grav}} \approx 10^{19}$, but say nothing about the larger scales relevant to cosmology and, at least at this level of sophistication, it seems perfectly reasonable to imagine a simple functional form leading to this kind of behavior.

The strong constraints on $\Delta c/c_\gamma$ come from the large distance between the source of the gravitational waves and their detection on earth by LIGO. In order to avoid this constraint we have suggested to use the small dimensionless number $\mathrm{K}_{\mathrm{grav}}^{-1}$ to suppress the effects of a modification of gravity that might lead to cosmologically interesting effects on the scales relevant to gravitational wave sources. We have only talked about the basic idea behind this suppression mechanism. We have not constructed explicit models at the level of a Lagrangian and indeed we acknowledge that it might be difficult to achieve in practice. Other than a miracle cancellation similar to that found in $f(R)$ models, we identified two possible directions for further exploration: nonlocal models and higher-order derivatives, providing concrete examples of equations of motions which lead to such dispersion relations.

One thing that we should point out is that the suppression mechanism used for the gravitational wave sector of the theory could also operate in the scalar density perturbation sector and in principle be used to suppress modifications to gravity on solar system scales characterized by $\mathrm{K}_{\mathrm{solar}} \approx 4 \times 10^{14}$ (corresponding to lengthscales ~ 10 au). In Appendix B we have outlined some of the basics behind this idea. In its simplest possible terms, the coefficients C_{ij} from (B7) are chosen so that modifications to gravity quantified by two of μ_ψ, μ_ϕ, η and Σ are equal to their general relativistic values when $\mathrm{K} \sim \mathrm{K}_{\mathrm{solar}}$, but the cancellations of coefficients required to achieve this would only be true up to inverse powers of K.

Since we have only outlined the basic ideas, there is clearly much detailed work to be done to develop fully fledged theories. Nonetheless we believe we have made a simple argument that one can develop theories which are compatible with general relativity on scales $\sim \mathrm{K}_{\mathrm{grav}}$ and $\mathrm{K}_{\mathrm{solar}}$ while being interestingly different for $\mathrm{K} \sim 1$. Indeed the fact that $\mathrm{K}_{\mathrm{grav}} > \mathrm{K}_{\mathrm{solar}}$ suggests that it is not at all unreasonable to think that a suppression mechanism which works on solar system scales would allow one to avoid the constraints from GW170817.

ACKNOWLEDGMENTS

APPENDIX A: EXAMPLE EQUATIONS OF STATE

In this Appendix we present a survey of the coefficients C_{ij} for some of the modified gravity models which have been already evaluated in literature and show that these results lead to the conclusions that match the results found by others.

(1) *Horndeski theories*. These are the most general scalar-tensor theories compatible with second-order time evolution. They are specified in terms of four free functions $G_i(\phi, X)$ for $i = 2, 5$ where ϕ is the scalar field and $X = -\frac{1}{2}\nabla_\mu\phi\nabla^\mu\phi$ is the canonical kinetic term. The equation of state for the tensor sector in these models is given by

$$8\pi G_{\mathrm{N}} P_{\mathrm{ds}} \Pi_{\mathrm{ds}}^{\mathrm{T}} = -\frac{1}{2} \left\{ \left(\frac{m^2}{m_{\mathrm{pl}}^2} - 1 \right) \ddot{h} + \left[\frac{m^2}{m_{\mathrm{pl}}^2}(3 + \alpha_{\mathrm{M}}) - 3 \right] H\dot{h} \right.$$
$$\left. + \left[\frac{m^2}{m_{\mathrm{pl}}^2}(1 + \alpha_{\mathrm{T}}) - 1 \right] \mathrm{K}^2 H^2 h \right\}, \qquad (A1)$$

and hence we can read off

$$C_{\ddot{h}} = -\frac{1}{2}\left(\frac{m^2}{m_{pl}^2} - 1\right), \qquad C_{\dot{h}} = -\frac{1}{2}\left[\frac{m^2}{m_{pl}^2}(3 + \alpha_M) - 3\right],$$

$$C_h = -\frac{1}{2}\left[\frac{m^2}{m_{pl}^2}(1 + \alpha_T) - 1\right]K^2, \qquad (A2)$$

where m represents the effective Planck mass which can be, in general, a function of time, α_T the excess speed of gravitational waves and $m_{pl} = G_N^{-1/2}$ the bare Planck mass. $\alpha_M = \frac{1}{H}\frac{d\ln m^2}{dt}$ is the logarithmic time variation of the effective Planck mass. These parameters, together with α_B and α_K (these last two important for the scalar sector) completely define Horndeski theories and have been introduced for the first time in [43]. The identification with the β_i functions introduced in (2.4) is now trivial: $\beta_M = \alpha_M(a)$ and $\beta_T = [1 + \alpha_T(a)]K^2$ and the observations of GW170817 imply that $|\alpha_T| < 10^{-15}$.

We can express α_T in terms of the functions G_4 and G_5 as[8]

$$\alpha_T = \frac{X[2G_{4,X} - 2G_{5,\phi} - (\ddot{\phi} - H\dot{\phi})G_{5,X}]}{G_4 - 2XG_{4,X} + XG_{5,\phi} - \dot{\phi}HXG_{5,X}}, \qquad (A3)$$

which reduces to

$$\alpha_T = \frac{2XG_{4,X}}{G_4}\left(1 - \frac{2XG_{4,X}}{G_4}\right)^{-1}, \qquad (A4)$$

when G_5 is a constant, which is equivalent to setting $G_5 \equiv 0$ by integration by parts. From this we can deduce that $\alpha_T \ll 1$ can be achieved when $XG_{4,X}/G_4 \ll 1$ (i.e., the slope of G_4 with respect to X is close to zero). The most natural way to achieve this is when $G_4 \equiv G_4(\phi)$ although there are other possibilities.

There are two interesting and well studied subclasses of the Horndeski model:

(i) Quintessence [47–51], k-essence [52–57] and kinetic gravity braiding (KGB) models [58,59] are subclasses of the Horndeski theory with G_4 constant and $G_5 = 0$ and hence $C_{\ddot{h}} = C_{\dot{h}} = C_h \equiv 0$. All of these minimally coupled scalar field models predict no modifications to the evolution of gravitational waves and, therefore, survive constraints from GW170817. Of course, this should be no surprise since such models have no anisotropic stress at all, but this also implies that they only weakly impact on cosmological observables such as the CMB and cosmic shear [21].

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[8]Expressions for m and all the α_i functions can be found in [43–46] in terms of the G_i functions.

(ii) $f(R)$ models are also a subclass for which $m^2 = m_{pl}^2(1 + \frac{df}{dR})$ and $\alpha_T = 0$ where $f(R)$ is the modification to the Einstein-Hilbert action. In this class of models, $C_h = K^2 C_{\ddot{h}}$ which is the miracle cancellation discussed in Sec. III and hence this class of models survives the constraints imposed by GW170817 by having $\alpha_T \equiv 0$, but $\Pi_{ds}^T \neq 0$.

(2) *Generalized Einstein-Aether theories.* Einstein-Aether theories [60] are vector-tensor theories of gravity which involve the addition of a timelike unit normalized vector field A^μ, such that $A^\mu A_\mu + 1 = 0$, with a Lagrangian described by a generalized function $F(\mathcal{K})$ where

$$\mathcal{K} = \frac{1}{m_K^2}K^{\alpha\beta}{}_{\mu\nu}\nabla_\alpha A^\mu \nabla_\beta A^\nu, \qquad (A5)$$

and the rank-4 tensor is defined as

$$K^{\alpha\beta}{}_{\mu\nu} = c_1 g^{\alpha\beta}g_{\mu\nu} + c_2\delta_\mu^\alpha\delta_\nu^\beta + c_3\delta_\nu^\alpha\delta_\mu^\beta + c_4 A^\alpha A^\beta g_{\mu\nu}. \quad (A6)$$

The c_i are dimensionless constants and m_K has dimensions of mass. The timelike unit norm constraint ensures only one scalar degree of freedom propagates which makes this theory similar to the scalar-tensor theories discussed above. It can be shown that

$$8\pi G_N P_{ds}\Pi_{ds}^T = -\frac{1}{2}c_{13}\left[\frac{dF}{d\mathcal{K}}\ddot{h} + \left(3\frac{dF}{d\mathcal{K}}H + \frac{d^2F}{d\mathcal{K}^2}\dot{\mathcal{K}}\right)\dot{h}\right], \tag{A7}$$

where $c_{13} = c_1 + c_3$ from which we can read off

$$C_{\ddot{h}} = -\frac{1}{2}c_{13}\frac{dF}{d\mathcal{K}}, \qquad C_{\dot{h}} = -\frac{1}{2}c_{13}\left(3\frac{dF}{d\mathcal{K}} + \frac{d^2F}{d\mathcal{K}^2}\frac{\dot{\mathcal{K}}}{H}\right),$$

$$C_h = 0. \tag{A8}$$

In terms of the β_i parameters, we find $\beta_M = \alpha_M = \frac{1}{H}\frac{d\ln m^2}{dt}$ with an effective Planck mass $m^2 = m_{pl}^2(1 + c_{13}\frac{dF}{d\mathcal{K}})$ and $\beta_T = K^2(1 + \alpha_T)$ where $\alpha_T = -c_{13}\frac{dF}{d\mathcal{K}}(1 + c_{13}\frac{dF}{d\mathcal{K}})^{-1}$.

From this we can deduce that $v_p = (1 + c_{13}\frac{dF}{d\mathcal{K}})^{-1/2}$. The tight constraints $\Delta c/c_\gamma$ suggest that the only models in this class which would survive—should the generalized Einstein-Aether model apply on the scales relevant to observations of gravitational waves—are those with $c_{13} \equiv 0$ and hence $\Pi_{ds}^T \equiv 0$. If $1 + w_{de} = 0$, it can be shown that if $c_{13} = 0$ then the scalar sector will be observationally equivalent to a cosmological constant. This is because c_{13} also sets $\Pi_{ds}^S = 0$. It is possible that if $1 + w_{ds} \neq 0$ then this equivalence will be broken and will lead to interesting observational consequences [61]. We also note the striking analogy for α_M between $f(R)$ and $F(\mathcal{K})$ models.

(3) *Massive gravity theories.* Differently from the models above, these theories consider the graviton to be massive

$(m_{GW} \neq 0)$ and in general the mass could be a function of time (and space) but the scalar and vector sectors are unaffected by this choice [62]. From the general equation describing the propagation of gravitational waves [63,64]

$$\ddot{h} + (3 + \alpha_M) H \dot{h} + [(1 + \alpha_T) K^2 H^2 + m_{GW}^2] h = 0, \quad (A9)$$

we can deduce a modification to the coefficients in the Horndeski model

$$\delta C_h = -\frac{1}{2} \frac{m^2}{m_{pl}^2} \frac{m_{GW}^2}{H^2}. \quad (A10)$$

The β_i functions read: $\beta_M = \alpha_M$ and $\beta_T = (1 + \alpha_T) K^2 + M_{GW}^2$ where $M_{GW} = m_{GW}/H$.

(4) *Elastic dark energy models.* These models represent a generalization of the perfect fluid approach to dark energy where the rigidity of the medium is taken into account. In their simplest formulation elastic dark energy models are analogous to massive gravity models albeit the mass term introduced is not linked to the graviton itself. It was shown that [65,66]

$$8\pi G_N P_{ds} \Pi_{de}^T = \left(\frac{\mu}{m_{pl}^2} + 2aH \frac{\nu}{m_{pl}^2}\right)(h_i - h) - a\frac{\nu}{m_{pl}^2} \dot{h}, \quad (A11)$$

where μ, identified as the rigidity modulus, and ν as the viscosity, are parameters with dimensions M^4 and M^3 respectively. The previous expression reduces to what found in [65] for $\nu = 0$. The C_i coefficients are

$$C_{\dot{h}} = 0, \quad C_{\dot{h}} = -\frac{a\nu}{m_{pl}^2 H}, \quad C_h = -\frac{\mu + 2aH\nu}{m_{pl}^2 H^2}. \quad (A12)$$

The additional term h_i takes into account the formation time of the elastic medium.

APPENDIX B: SUPPRESSING MODIFIED GRAVITY EFFECTS IN THE SCALAR SECTOR

In this Appendix we will discuss the principle of applying the same approach to suppressing modified gravity effects as $K \to \infty$ in the gravitational waves sector to the scalar sector. First let us define some parameters commonly used to quantify deviations from Einstein gravity. We will use a metric of the form

$$ds^2 = -(1 + 2\phi)c^2 dt^2 + a(t)^2 (1 - 2\psi)\delta_{ij} dx^i dx^j. \quad (B1)$$

The two of the Einstein equations yield

$$-\frac{2}{3} K^2 \psi = \sum_i \Omega_i \Delta_i = \mu_\psi(a, K)\Omega_m \Delta_m, \quad (B2)$$

$$-\frac{1}{3} K^2 (\psi - \phi) = \sum_i \Omega_i w_i \Pi_i^S, \quad (B3)$$

where the summation is over $i = $ m and ds and the relative contributions to the critical density are $\Omega_i \equiv \Omega_i(a)$. In what follows we will assume that $\Pi_m^S \equiv 0$ which is the case for a perfect pressureless fluid. The Weyl potential $\Psi = \frac{1}{2}(\phi + \psi)$ is the quantity which leads to a number of observational effects notably lensing and is often parametrized as

$$-\frac{2}{3} K^2 \Psi = \Sigma(a, K)\Omega_m \Delta_m. \quad (B4)$$

The functions μ_Ψ and Σ have been introduced to encode the effects of modifications to gravity. In principle other parameters can be used to describe this but they are all related to μ_Ψ and Σ; any two independent parameters are needed to fully describe the theory. One notable alternative often used is the gravitational slip

$$\eta(a, K) = \frac{\psi}{\phi} = \left(1 - 2\frac{\sum_i \Omega_i w_i \Pi_i^S}{\sum_i \Omega_i \Delta_i}\right)^{-1}, \quad (B5)$$

while one can also define μ_ϕ according to

$$-\frac{2}{3} K^2 \phi = \mu_\phi(a, K)\Omega_m \Delta_m, \quad (B6)$$

where $\mu_\psi(a, K) = \eta(a, K)\mu_\phi(a, K)$ and $\Sigma(a, K) = \frac{1}{2}\mu_\phi(a, K)[1 + \eta(a, K)] = \frac{1}{2}[\mu_\phi(a, K) + \mu_\psi(a, K)]$. If the dark sector were to only comprise a cosmological constant then $\mu_\Psi = \mu_\Phi = \eta \equiv 1$ and $\Sigma = 1$.

Using the equation of state approach in the scalar sector it is necessary to specify two functions and it has been argued that the natural ones to specify are the entropy perturbation, $w_{ds}\Gamma$, and the scalar anisotropic stress, $w_{ds}\Pi_{ds}^S$ which are both gauge invariant. Since the perturbations are linear these functions must be linear functions of the other perturbation variables and can be written (using the Einstein and conservation equations to remove metric perturbations and time derivatives) as

$$w_{ds}\Gamma_{ds} = C_{\Gamma_{ds}\Delta_{ds}}\Delta_{ds} + C_{\Gamma_{ds}\Theta_{ds}}\Theta_{ds} + C_{\Gamma_{ds}\Delta_m}\Delta_m + C_{\Gamma_{ds}\Theta_m}\Theta_m$$
$$+ C_{\Gamma_{ds}\Gamma_m}\Gamma_m,$$
$$w_{ds}\Pi_{ds}^S = C_{\Pi_{ds}^S\Delta_{ds}}\Delta_{ds} + C_{\Pi_{ds}^S\Theta_{ds}}\Theta_{ds} + C_{\Pi_{ds}^S\Delta_m}\Delta_m$$
$$+ C_{\Pi_{ds}^S\Theta_m}\Theta_m + C_{\Pi_{ds}^S\Pi_m^S}\Pi_m^S. \quad (B7)$$

Here, we are describing the system where Δ_i and Θ_i are density and velocity perturbations in the dark (ds) and matter (m) sectors using the same notation as in [67]. For completeness we have also included the entropy perturbation Γ_m and the anisotropic stress Π_m^S for the

matter component which are typically negligible in the regime relevant to observations of cosmic acceleration; in the subsequent discussions we will ignore these terms. The coefficients (C_{ij}) have been computed for k-essence [68–70], kinetic gravity braiding [19], $f(R)$ [67], Horndeski theories [45], generalized Einstein-Aether [61], elastic dark energy [65,66], and Lorentz-violating massive gravity models [71]. In full generality they are free functions of the scale factor (and hence cosmic time) and scale via the wave number, usually entering as a k^2 term due to the presumed dependence on second order combinations of spatial derivatives.

In order to establish a relationship between Δ_m and Δ_{ds} we will now assume that the approach to understanding perturbations in the scalar sector which works in $f(R)$ models (see, for example, [72]) works in more general models. We would assume that this is a good approximation to a wide range of models, but not all cases. In particular, we will assume that one can ignore the contributions from Θ_m and Θ_{ds} in (B7) and construct a second order differential equation describing the evolution of Δ_{ds} which is sourced by matter perturbations

$$\ddot{\Delta}_{ds} + (2 - 3w_{de} - 2C_{\Pi^s_{ds}\Delta ds}) H \dot{\Delta}_{ds}$$
$$+ \frac{1}{3}(3w_{ds} + 2C_{\Pi^s_{ds}\Delta ds} + 3C_{\Gamma_{ds}\Delta ds})H^2 K^2 \Delta_{ds}$$
$$= -\frac{1}{3}(2C_{\Pi^s_{ds}\Delta m} + 3C_{\Gamma_{ds}\Delta m})H^2 K^2 \Delta_m, \quad \text{(B8)}$$

so that the relation between Δ_{ds} and Δ_m (the attractor solution) is

$$\Delta_{de} = -\frac{2C_{\Pi^s_{ds}\Delta m} + 3C_{\Gamma_{ds}\Delta m}}{3w_{ds} + 2C_{\Pi^s_{ds}\Delta ds} + 3C_{\Gamma_{ds}\Delta ds}}\Delta_m. \quad \text{(B9)}$$

When this attractor solution applies, we can deduce that

$$\mu_\psi = 1 - \frac{\Omega_{ds}}{\Omega_m}\kappa,$$

$$\Sigma = 1 - \frac{\Omega_{ds}}{\Omega_m}[C_{\Pi^s_{ds}\Delta m} + \kappa(1 - C_{\Pi^s_{ds}\Delta ds})]. \quad \text{(B10)}$$

where we have defined

$$\kappa = \frac{2C_{\Pi^s_{ds}\Delta m} + 3C_{\Gamma_{ds}\Delta m}}{3w_{ds} + 2C_{\Pi^s_{ds}\Delta ds} + 3C_{\Gamma_{ds}\Delta ds}}. \quad \text{(B11)}$$

Using these expressions, we see that one experiences general relativity on small scales if, in the limit $K \to \infty$, we have that $\mu_\psi \to 1$, which implies $\kappa \to 0$, and $\Sigma \to 1$, and this can be achieved if $\kappa = 0$ and $C_{\Pi^s_{ds}} = 0$. One specific way of enforcing this is by setting $C_{\Pi^s_{ds}\Delta m} = C_{\Gamma_{ds}\Delta m} = 0$ in

this limit, although there are other possible ways of achieving this. The suppression mechanism we are suggesting would require the zero in these conditions to be replaced by $\mathcal{O}(K^{-2})$ so that they are effectively zero for $K \sim K_{solar} \approx 4 \times 10^{14}$. We will discuss the details of how this might be achieved in practice in future work.

The attractor solution arises naturally when writing the equation for Δ_{ds} with Δ_m as source term. The expression in (B9) is valid provided that the attractor solution is attained for each Fourier mode before the dark energy component starts to dominate. Under the assumption that any modification of gravity is relevant only at late times, one would expect this will be true in the matter dominated era since we can in general assume that at very early times, i.e., in the radiation dominated era, perturbations in the dark sector are negligible. However, this need not to be the case when the dark energy component is not negligible at early times, such as in early dark energy models. In this case the attractor solution would have to be obtained during radiation dominated era and its validity would need to be checked carefully. If the field does not reach the attractor solution sufficiently fast to make exact initial conditions unimportant, then the full equations of motions need to be solved. For a deeper discussion on the issue we refer to [73] where this issue is discussed in detail for a perfect fluid.

It is interesting to calculate the expressions for $f(R)$ models since they exhibit some of the properties we are looking for, but not all. We will use approximations for the C_{ij} coefficients presented in [72] which appear to give a good description of the full problem on all but the very largest scales when $f_R \ll 1$ (which one would expect to be the case),

$$C_{\Pi^s_{ds}\Delta m} = 0, \qquad C_{\Pi^s_{ds}\Delta ds} = 1,$$
$$C_{\Gamma_{ds}\Delta m} = \frac{1}{3}\frac{\Omega_m}{\Omega_{ds}}, \qquad C_{\Gamma_{ds}\Delta ds} = \left(\frac{1}{3} - w_{ds} + \frac{M^2}{K^2}\right), \quad \text{(B12)}$$

where $M^2 \equiv \dot{R}/(3H\dot{H}B)$ and $B = f_{RR}H\dot{R}/[\dot{H}(1 + f_R)]$ with $f_R = \frac{df}{dR}$ and $f_{RR} = \frac{d^2f}{dR^2}$. Using these expressions we can deduce that

$$\kappa = \frac{1}{3}\frac{K^2}{K^2 + M^2}\frac{\Omega_m}{\Omega_{ds}}, \quad \mu_\psi = \frac{2K^2 + 3M^2}{3(K^2 + M^2)},$$
$$\mu_\phi = \frac{4K^2 + 3M^2}{3(K^2 + M^2)}, \qquad \eta = \frac{2K^2 + 3M^2}{4K^2 + 3M^2}, \quad \Sigma = 1. \quad \text{(B13)}$$

In the small scale limit $K \gg M \gg 1$: $\mu_\phi \to 4/3$ and this represents the well known effective gravitational constant in $f(R)$ models leading to an increase of clustering on small scales; $\eta \to 1/2$ and therefore the two Bardeen potentials

differ from each other by a factor of two (another well known result [74]). Finally we see at work the screening mechanism which reduces $\mu_\psi \to 2/3$ and more importantly $\Sigma = 1$ which is what would be expected for a model recovering general relativity on small scales, despite the fact that the $f(R)$ does not. This model achieves $\Sigma = 1$ by having $C_{\Pi^S_{ds}\Delta_m} = 0$ and $C_{\Pi^S_{ds}\Delta_{ds}} = 1$ and does not have $\kappa = 0$ and hence $\mu_\psi \neq 1$ and indeed it is believed that the $f(R)$ model can be compatible with the solar system scales by the nonlinear chameleon mechanism [75].

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